

4.a)

Given that bias, $b=0$ and input $x_3=1$

Output, $Y = f(x_1, x_2, x_3)$

Activation function $\rightarrow \sigma = \frac{1}{1+e^{-x}}$ (Sigmoid function)

$$\sigma = \frac{1}{1+e^{-x}} \quad f \rightarrow \sigma \text{ function}$$

$$Y = \sigma [-2.01136152 + (6.05050659x_1 - 0.75120936x_2) + \\ -2.5196096 + (-7.24616141x_1 + 7.26413833x_2) + \\ 1.03168425 + (-3.58702172x_1 + 6.1736983x_2) + \\ -4.1568864 + (2.61305967x_1 + 2.81567166x_2)]$$

In the above output equation, the weight coefficients (obtained by executing the code) are substituted.

4.b)

Input: $x_1=0, x_2=0, x_3=1$

$$h_i = \sum_{i=0}^M w_i x_i \rightarrow \text{Weighted sum for each neuron in the hidden layer}$$

$$h_0 = 0 + 0 - 3.58702172 + 2.61305967 = -0.97396205$$

$$h_1 = 0 + 0 + 6.1736983 + 2.81567166 = 8.99347497$$

$$h_2 = 0 + 0 + 1.03168425 - 4.1568864 = -3.10520215$$

$$h_3 = 0 + 0 - 4.1568864 - 7.60022303 = -14.37275576$$

$$h_0' = \sigma(h_0) = \sigma(-0.97396205) = \frac{1}{1+e^{-0.97396205}} \\ = 0.28652614$$

$$h_1' = \sigma(h_1) = \sigma(8.99347497) = 0.99976816$$

$$h_2' = \sigma(h_2) = \sigma(-14.57275576) = 1.02614818 \times 10^{-6}$$

$$h_3' = \sigma(h_3) = \sigma(-3.10820215) = 0.04539072$$

$$\text{Output, } Y = \sigma[-2.01136152 h_0' + (-2.5196096 h_1') + \\ (1.05168425 h_2') + (0.04539072 h_3')]$$

$$\therefore Y = 0.00765400$$

$$\text{Input: } x_1=0, x_2=1, x_3=1$$

$$h_0 = 0 - 0.75120936 - 3.58702172 + 2.61305967$$

$$= -2.68073283$$

$$h_1 = 0 + 7.26413833 + 6.1736983 + 2.81567666 = 16.2595744$$

$$h_2 = 0 + 6.1736983 + 1.05168425 - 4.1568864 = 3.06681204$$

$$h_3 = 0 + 2.81567666 - 4.1568864 - 7.60022303 = -14.5727556$$

$$h_0' = \sigma(-2.68073283) = 0.07137176$$

$$h_1' = \sigma(16.2595744) = 0.99998611$$

$$h_2' = -(3.06681204) = 0.95623118$$

$$h_3' = -(-14.5727556) = 1.012614818 \times 10^{-6}$$

$$y = -[-2.01136152 h_0'] + (-2.5196096 h_1') + (1.05168425 h_2') \\ + (-4.1568864 h_3')$$

$$= \boxed{0.99202571}$$

Input: $x_1=1, x_2=0, x_3=1$

$$h_0 = 6.08050659 - 0 - 3.58702172 + 2.61305967 = 5.09935423$$

$$h_1 = 7.24616141 + 0 + 6.1736983 + 2.81567666 = 16.2996744$$

$$h_2 = -3.89702172 + 0 + 1.05168425 - 4.1568864 = -6.64314437$$

$$h_3 = 2.61305967 + 0 - 4.1568864 - 7.60022303 = -14.57275576$$

$$h_0' = -(h_0) = 0.99394207$$

$$h_1' = -(h_1) = 1$$

$$h_2' = -(h_2) = 0.00109993$$

$$h_3' = -(h_3) = 1.02614818 \times 10^{-6}$$

$$\boxed{y = 0.99216446}$$

Input: $x_1=1, x_2=1, x_3=1$

$$h_0 = 6.05050659 - 0.75120936 - 3.58702172 + 2.61305967 = 4.3134074$$

$$h_1 = 7.24616141 + 7.26413833 + 6.1736983 + 2.91567666 = 23.4996744$$

$$h_2 = -3.58702172 + 6.1736983 + 1.05168425 - 4.1568864 = -3.59306442$$

$$h_3 = 2.61305967 + 2.81567666 - 4.1568864 - 7.60022303 = -18.47275576$$

$$h_0' = \sigma(h_0) = 0.98468241$$

$$h_1' = \sigma(h_1) = 1$$

$$h_2' = \sigma(h_2) = 0.02742722$$

$$h_3' = \sigma(h_3) = 1.11022302 \times 10^{-8}$$

$$\Rightarrow Y = 0.0884604$$

Input: $X_1=0, X_2=0, X_3=1$

$$h_0 = -7.6002$$

$$h_0' = 0.00341129$$

$$h_1 = 13.8884$$

$$h_1' = 0.99202571$$

$$h_2 = -7.4255$$

$$h_2' = 0.00884604$$

$$h_3 = -5.1792$$

$$h_3' = 0.00429843$$

$$\therefore Y = 0.00429843$$