UNIT-04 10/12/2024

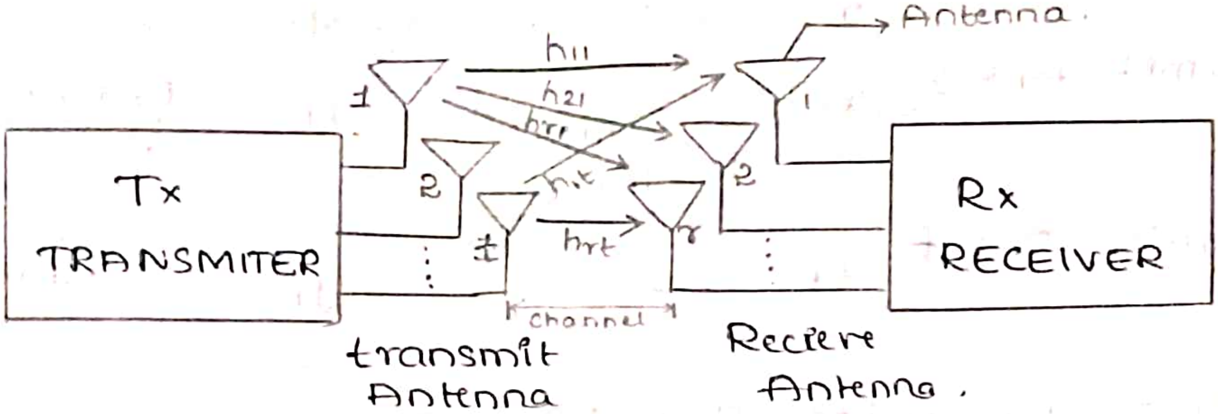
Don't study co-variance of MIMO

Multiple Antenna Systems

- 6.1 Introduction
- 6.2 MIMO model
- 6.3 SVD of MIMO channel
- 6.4 SVD MIMO capacity
- 6.5 Power allocation
- 6.6 Alamouti space-Time Codes.

(Q) With block diagram explain MIMO w.c system.

⟹



- MIMO antennas are used for high speed data rate in wireless communication and increases reliability.
- Spatial multiplexing → SISO, SIMO, MISO, MIMO.
- High data rate converted to low data rate
- Transmitted through different antennas.
- At R_x to increase diversity gain → div. combiner (EGC, MRC)

(Q) Design and develop the basic MIMO transmission mode

⟹ wrt above fig,

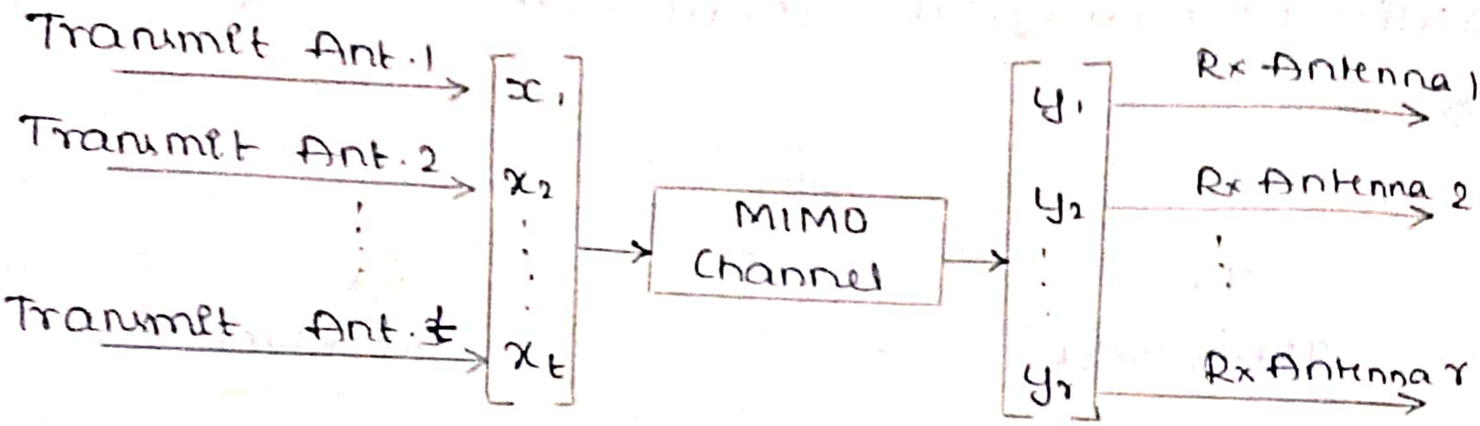
Consider MIMO wireless system with 't' Transmit antennas and 'R' Reciere antennas as shown in the fig.

Let $x_1, x_2, x_3 \ldots x_t$ denotes 't' symbols transmitted from 't' transmit antennas in MIMO system i.e., x_i denotes symbol transmitted from i^{th} transmit antenna $(1 \le i \le t)$.

$\therefore$ The transmit vector, $x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_t \end{bmatrix} \rightarrow (1)$

Illy Let $y_1, y_2, y_3 \ldots y_r$ denotes 'r' recived symbols accross the 'r' recieve antennas in the MIMO system.

$\therefore$ The recieved vector, $y = \begin{bmatrix} y_1 \\ y_2 \\ y_r \end{bmatrix} \rightarrow (2)$



MIMO system with INPUT - OUTPUT Schematic

Let the complex co-efficient h_{ij} represent fading channel co-efficient from i^{th} recieve antenna & j^{th} tranumit antenna.

$\therefore$ The channel co-efficient in this scenario,

$$H = \begin{bmatrix} h_{11} & h_{12} & h_{13} & \ldots & h_{1t} \\ h_{21} & h_{22} & h_{23} & \ldots & h_{2t} \\ \vdots & \vdots & \vdots & & \\ h_{r1} & h_{r2} & h_{r3} & \ldots & h_{rt} \end{bmatrix}_{(r \times t)} \rightarrow (3)$$

Matrix $H = r \times t$ dimensional.

Let the additive noise at the Rx antenna $i = N_i$

$N = \begin{bmatrix} N_1 \\ N_2 \\ \vdots \\ N_r \end{bmatrix} \rightarrow (4)$ // Denotes additive noise at r^{th} rx antenna.

∴ The net MIMO i/p - o/p system model is,

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_r \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & \cdots & h_{1t} \\ h_{21} & h_{22} & \cdots & h_{2t} \\ \vdots & \vdots & & \\ h_{r1} & h_{r2} & \cdots & h_{rt} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_t \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \\ \vdots \\ n_r \end{bmatrix}$$

Rx vector (Y) | Channel co-efficient (H) | Tx matrix (X) | Additive Noise (n)

∴ The net MIMO matrix expression is,

$$\boxed{Y = HX + n} \rightarrow (4)$$

The first recieved symbol,

$$\underbrace{\boxed{y_1 = h_{11} x_1}}_{\text{Signal}} + \underbrace{h_{12} x_2 + \cdots h_{1t} x_t + n_1}_{\text{Interference}}$$

From this expression it is noted, all signals x_1, $x_2 \ldots x_t$, interfere at y_1; recieved at Rx 1.
It holds true for all Rx antennas.

(i) when $t = 1$, <u>SIMO</u>

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_r \end{bmatrix} = \begin{bmatrix} h_{11} \\ h_{21} \\ \vdots \\ h_{r1} \end{bmatrix} x + \begin{bmatrix} n_1 \\ n_2 \\ \vdots \\ n_r \end{bmatrix}$$

Also called as recieve diversity.

(ii) when $r = 1$, <u>MISO</u>

$$y = \begin{bmatrix} h_{11} & h_{12} & \cdots & h_{1t} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_t \end{bmatrix} + n$$

Also called as transmit diversity.

(iii) when $r = t = 1$, SISO

$$y = hx + n$$

(Q) Using mathematical expression discuss SVD (singular value decomposition).

↳ used to optimize channel co-efficient & S.P.

In MIMO system, SVD is a powerful mathematical tool for analysing and optimizing channel capacity and signal processing. It decomposes MIMO channel matrix into orthogonal components i.e., SVD transforms the MIMO channel into ||el independent channels which are non-process communication H.

Explanation:

Consider '$r \times t$' MIMO channel, with $r \geq t$ i.e., the number of recieved antennas are $\geq t$.

The SVD of the channel matrix 'H' is given as,

$$H = \underbrace{\begin{bmatrix} u_{11} & u_{12} & \cdots & u_{1t} \\ u_{21} & u_{22} & \cdots & u_{2t} \\ \vdots & \vdots & & \vdots \\ u_{r1} & u_{r2} & \cdots & u_{rt} \end{bmatrix}}_{U} \underbrace{\begin{bmatrix} \sigma_1 & 0 & 0 & \cdots & 0 \\ 0 & \sigma_2 & \cdots & & 0 \\ \vdots & & & & \\ 0 & 0 & \cdots & & \sigma_r \end{bmatrix}}_{\Sigma} \underbrace{\begin{bmatrix} V_{11}^* & V_{12}^* & \cdots & V_{1t}^* \\ V_{21}^* & V_{22}^* & \cdots & V_{2t}^* \\ \vdots & \vdots & & \vdots \\ V_{r1}^* & V_{r2}^* & \cdots & V_{rt}^* \end{bmatrix}}_{V^H}$$

V_1^H ... V_r^H

$$\therefore \boxed{H = U \Sigma V^H}$$

H → Hermitian

Transverse of a matrix (T)

Where the matrix U, Σ & V are $r \times t$, $t \times t$; $t \times t$ respectively

1) The column of matrix U & V are unit ~~orthog~~

$$\|u_i\|^2 = \|v_i\|^2 = 1 \;;\; 1 \leq i \leq t$$

(2) The column of matrix. U & V are orthogonal

$$u_i^H u_j = V_i^H V_j = 0 \;;\; i \neq j,\; 1 \leq i,\; 1 \leq j$$

③ The quantity $\sigma_1, \sigma_2 \ldots \sigma_t$ are known as singular value of the matrix.

④ If $r = t$, it is called unitary matrix. It contains left singular vector H. It represents orthogonal direction in the recievers space.

⑤ V is unitary matrix containing right singular vector of H. It represents orthogonal direction in the transmitter space and 'Σ' diagonal matrix containing singular value.

Numericals on SVD :

(Q) Consider (2×1) SIMO wireless system channel and verify it.

$\downarrow$ Tx t.

$$H = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

STEPS TO BE FOLLOWED :

Step(1) : Find H^T

Step(2) : Find $H^T \cdot H$

Step(3) : Find Eigen values i.e., λ_1, λ_2.

Step(4) : Find singular value of H ($\sigma_1 = \sqrt{\lambda_1}, \sigma_2 = \sqrt{\lambda_2}$

Step(5) : Find Eigen vector $H - \lambda I = 0$

Step(6) : Find vector V_1 & V_2

$$V_1 = \frac{\lambda_1}{||\lambda_1||}, \quad V_2 = \frac{\lambda_2}{||\lambda_2||}$$

Step(7) : Find $V = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$

Step(8) : Find u_1 & u_2

$$u_1 = \frac{HV_1}{\sigma_1}, \quad u_2 = \frac{HV_2}{\sigma_2}$$

Step(9) : Find $u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$

Step(10) : $\Sigma = \begin{bmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \end{bmatrix}$

Step(11) : Find SVD, $SVD = U \Sigma V^H$

Step (12) : Verification $SVD \Rightarrow H$.

<u>Soln.</u> :- $H = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

<u>Step(1):</u> $H^T = [1 \quad 1]$

<u>Step(2):</u> $H^T.H = [1 \quad 1] \begin{bmatrix} 1 \\ 1 \end{bmatrix} = 1+1 = [2]$

<u>Step(3):</u> Eigen values $|H^TH - \lambda I| = 0$

$$\begin{bmatrix} 2-\lambda & 0 \\ 0 & 0 \end{bmatrix} = 0 \Rightarrow \lambda_1 = 2, \ \lambda_2 = 0$$

<u>Step(4):</u> $\sigma_1 = \sqrt{\lambda_1} = \sqrt{2}$, $\sigma_2 = 0$

<u>Step(5):</u> $v_1 = \dfrac{\lambda_1}{\|\lambda_1\|} = \dfrac{2}{\sqrt{4}} = 1$

<u>Step(6):</u> $V^T = [1]$

<u>Step(7):</u> $u_1 = \dfrac{Hv_1}{\sigma_1} = \dfrac{\begin{bmatrix} 1 \\ 1 \end{bmatrix} . [1]}{\sqrt{2}} = \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix}$

<u>Step(8):</u> $\Sigma = [\sqrt{2}]$

<u>Step(9):</u> $SVD = U \Sigma V^H$

$$= \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix} [\sqrt{2}] \ [1]$$

<u>Verify:</u> $SVD = \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix} [\sqrt{2}][1] = \begin{bmatrix} 1 \\ 1 \end{bmatrix} = H$ //

SVD working properly
$SVD \Rightarrow H$

(Q) Consider 2x2 MIMO W.C system & verify

Soln. $H = \begin{bmatrix} 1 & 2 \\ 1 & -2 \end{bmatrix}$

S1: $H^T = \begin{bmatrix} 1 & 1 \\ 2 & -2 \end{bmatrix}$

S2: $H^T \cdot H = \begin{bmatrix} 2 & 0 \\ 0 & 8 \end{bmatrix}$

S-3: $|H^TH - \lambda I| = 0$ $(2-\lambda)(8-\lambda) = 0$

$\begin{bmatrix} 2-\lambda & 0 \\ 0 & 8-\lambda \end{bmatrix} = 0 \Rightarrow$ $\lambda^2 - 10\lambda + 16 = 0$

$\lambda_1 = 8, \lambda_2 = 2$

$\lambda_1 > \lambda_2$ **

S-4: $\sigma_1 = \sqrt{\lambda_1} = \sqrt{8}$

$\sigma_2 = \sqrt{\lambda_2} = \sqrt{2}$

S-5: $\cancel{V_1 = \frac{\lambda_1}{||\lambda_1||} = \frac{2}{\sqrt{4}} = 1}$

$\cancel{V_2 = \frac{\lambda_2}{||\lambda_2||} = \frac{8}{\sqrt{64}} = 1}$

$\Sigma = \begin{bmatrix} \sqrt{8} & 0 \\ 0 & \sqrt{2} \end{bmatrix}$ //

To find V_1; Eigen vector.

$\lambda = 8$, $\begin{bmatrix} 2-8 & 0 \\ 0 & 8-8 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$

$\begin{bmatrix} -6 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$

Divide by 6 $\Rightarrow$ $\begin{bmatrix} -1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0 \Rightarrow \boxed{y - x = 0}$

Let $x = 0, y = 1 \Rightarrow -x + y = 0$

$\therefore V_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ //

IIly To find V_2

$\lambda = 2$ $\begin{bmatrix} 2-2 & 0 \\ 0 & 8-2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0 \Rightarrow V_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ //

S-6 - $V = [V_1 \quad V_2] = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

$$V^H = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

S-7 - $S = U \Sigma V^H$.

To find u_1, $u_1 = \dfrac{HV_1}{\sigma_1}$

$$u_1 = \frac{\begin{bmatrix} 1 & 2 \\ 1 & -2 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix}}{\sqrt{8}} = \frac{\begin{bmatrix} 2 \\ -2 \end{bmatrix}}{2\sqrt{2}} = \begin{bmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \end{bmatrix}$$

To find u_2, $u_2 = \dfrac{HV_2}{\sigma_2}$

$$u_2 = \frac{\begin{bmatrix} 1 & 2 \\ 1 & -2 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix}}{\sqrt{2}} = \frac{\begin{bmatrix} 1 \\ 1 \end{bmatrix}}{\sqrt{2}} = \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix}$$

$\therefore$ $U = [u_1 \quad u_2] = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ -1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}$

$$SVD = U \Sigma V^H$$

$$= \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ -1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}\begin{bmatrix} \sqrt{8} & 0 \\ 0 & \sqrt{2} \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} //$$

Verify $\Rightarrow$ $\begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ -1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}\begin{bmatrix} \sqrt{8} & 0 \\ 0 & \sqrt{2} \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

$$= \begin{bmatrix} 1 & 2 \\ 1 & -2 \end{bmatrix}$$

$\therefore$ $SVD = H //$

(Q) Find SVD and verify it for 2x2 MIMO w.c.

$H = \begin{bmatrix} 1 & 0 \\ 0 & \sqrt{5} \end{bmatrix}$

Soln

S-1 : $H^T = \begin{bmatrix} 1 & 0 \\ 0 & \sqrt{5} \end{bmatrix}$

S-2 : $H^T . H = \begin{bmatrix} 1 & 0 \\ 0 & 5 \end{bmatrix}$

S-3 :- $|H^T H - \lambda I| = 0$

$\begin{bmatrix} 1-\lambda & 0 \\ 0 & 5-\lambda \end{bmatrix} = 0$

$(1-\lambda)(5-\lambda) = 0$

$5 - \lambda - 5\lambda + \lambda^2 = 0$

$\lambda^2 - 6\lambda + 5 = 0$

$\lambda_1 = 5$

$\lambda_2 = 1$ //

S-4 :- $\sigma_1 = \sqrt{\lambda_1} = \sqrt{5}$

$\sigma_2 = \sqrt{\lambda_2} = \sqrt{1}$

S-5 :- To find v_1

$\lambda = 5$, $\begin{bmatrix} 1-5 & 0 \\ 0 & 5-5 \end{bmatrix} \Rightarrow \begin{bmatrix} -4 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$

$\begin{bmatrix} -1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$

$-x + 0y = 0 \Rightarrow v_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

To find v_2.

$\lambda = 1$, $\begin{bmatrix} 0 & 0 \\ 0 & 4 \end{bmatrix} \Rightarrow v_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

S-6 $V = [v_1 \; v_2] = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ & $V^H = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

S-7 :- $S = U \Sigma V^H$

To find $u_1 = \frac{HV_1}{\sigma_1} = \frac{\begin{bmatrix} 1 & 0 \\ 0 & \sqrt{5} \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix}}{\sqrt{5}} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$\Rightarrow U = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

To find $u_2 = \frac{HV_2}{\sigma_2} = \frac{\begin{bmatrix} 1 & 0 \\ 0 & \sqrt{5} \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix}}{\sqrt{1}} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$.

S-8 :- SVD = $U \Sigma V^H$.

$$= \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} \sqrt{5} & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}.$$

S-9 :- Verify :-

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} \sqrt{5} & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & \sqrt{5} \end{bmatrix} = H //$$

(Q) Show how the information stream can be transmitted in ||ᵘ with MIMO channel system.

(Q) Derive the expression for total MIMO capacity.

(Q) With mathematical expⁿ explain optimal MIMO capacity

(Q) Derive the expression for optimal power allocation or water filling technique.

(Q) With a schematic diagram discuss MIMO water filling capacity.

①

Consider $r \times t$ MIMO system, i.e., $y = Hx + n \rightarrow (1)$

Let SVD of channel matrix, $H = U \Sigma V^H \rightarrow (2)$

The MIMO system model is, $y = U \Sigma V^H x + n \rightarrow (3)$

At the receiver multiply U^H, it is called receive processing.

$$U^H y = \underbrace{U^H . U}_{I} \Sigma V^H x + U^H n .$$

$$\underbrace{U^H y}_{\tilde{y}} = \Sigma V^H x + \underbrace{U^H n}_{\tilde{n}}$$

$$\boxed{\tilde{y} = \Sigma V^H x + \tilde{n}} \rightarrow (4)$$

This operation is termed as transmit processing.

$$x = V \tilde{x} \rightarrow (5)$$

Subs (5) in (4),

$$\tilde{y} = \Sigma \cancel{V^H V} \tilde{x} + \tilde{n}$$

$$\boxed{\tilde{y} = \Sigma \tilde{x} + \tilde{n}} \rightarrow (6)$$

After receive & Tx operation, the equivalent system model for MIMO when transmitting 't' information

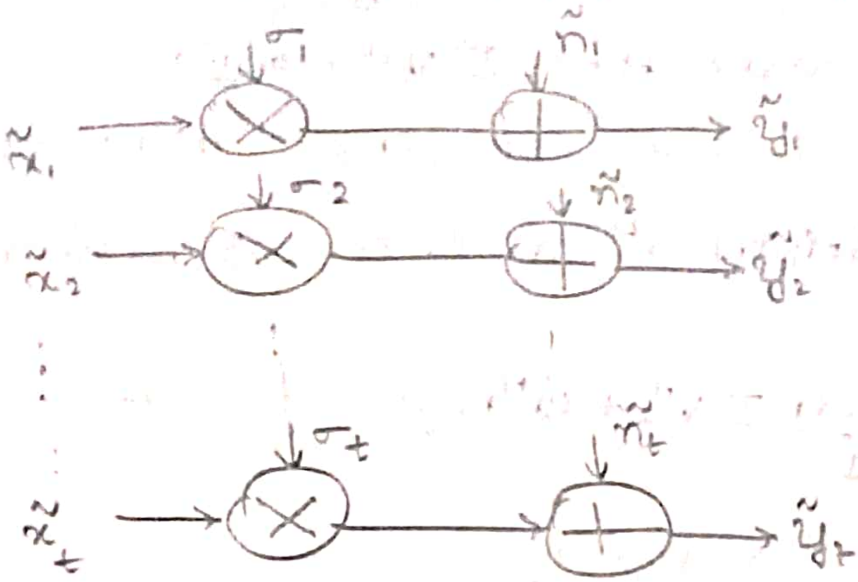
$$\begin{bmatrix} \tilde{y}_1 \\ \tilde{y}_2 \\ \vdots \\ \tilde{y}_t \end{bmatrix} = \begin{bmatrix} \sigma_1 & 0 & \cdots & 0 \\ 0 & \sigma_2 & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & \sigma_t \end{bmatrix} \begin{bmatrix} \tilde{x}_1 \\ \tilde{x}_2 \\ \vdots \\ \tilde{x}_t \end{bmatrix} + \begin{bmatrix} \tilde{n}_1 \\ \tilde{n}_2 \\ \vdots \\ \tilde{n}_t \end{bmatrix} \rightarrow (7)$$

It can be written as,

$$\tilde{y}_1 = \sigma_1 \tilde{x}_1 + \tilde{n}_1$$
$$\tilde{y}_2 = \sigma_2 \tilde{x}_2 + \tilde{n}_2$$
$$\vdots$$
$$\tilde{y}_t = \sigma_t \tilde{x}_t + \tilde{n}_t$$ //

∴ Thus, the system represents parallelization of MIMO channel with 't' information stream being transmitted ||ly. This is termed as <u>spatial multiplexing</u>.

MIMO SVD parallel channel :



(2) Consider i^{th} ||el MIMO channel, $\tilde{y}_i = \sigma_i \tilde{x}_i + \tilde{n}_i$

SNR for each ||el channel,

$n \rightarrow$ constant (same channel)

$$SNR = \frac{\sigma_i^2 P_i}{\sigma_n^2}$$

where σ_i^2 is the singular value corresponding to i^{th} parallel channel (power gain)

σ_n^2 is the noise variance (noise power)

P_i is the power of the i^{th} transmitter.

The shannon capacity of the channel,

$$C_i = \log_2(1+SNR)$$

$$C_i = \log_2\left[1+\frac{\sigma_i^2 P_i}{\sigma_n^2}\right] \rightarrow (8)$$

MIMO system consists of 't' ||el data stream,

$$\therefore C_1 = \log_2\left(1+\frac{\sigma_1^2 P_1}{\sigma_n^2}\right), \; C_2 = \log_2\left[1+\frac{\sigma_2^2 P_2}{\sigma_n^2}\right] \ldots$$

$$C_t = \log_2\left[1 + \frac{\sigma_t^2 P_t}{\sigma_n^2}\right]$$

∴ The total MIMO capacity is,

$$C_i = \sum_{i=1}^{t} \log_2\left[1 + \frac{\sigma_i^2 P_i}{\sigma_n^2}\right]$$

$$P_1 + P_2 + \dots P_t \le P$$

where $P \rightarrow$ total power at Tx, can be allocated to the individual stream to maximize the net capacity

③ <u>optimal MIMO capacity</u> :-

Let us consider optimization problem, where maximization of total capacity (C_{total}) of the MIMO system is subjected to power constraint.

$$C = \sum_{i=1}^{t} \log_2\left[1 + \frac{\sigma_i^2 P_i}{\sigma_n^2}\right].$$

The power constraint is,

$$\sum_{i=1}^{t} P_t \le P.$$

This means that, the sum of power allocated to all $||^{el}$ channel is less than the total power available for transmission

Hence, Lagrange's multiplier technique is used in power constraint optimization problem (λ) //

The function,

$$f(P, \lambda) = \sum_{i=1}^{t} \log_2\left[1 + \frac{\sigma_i^2 P_i}{\sigma_n^2}\right] + \lambda\left[P - \sum_{i=1}^{t} P_i\right]$$

D/F above expression wrt P_i & maximize,

$$\frac{\delta f(P,\lambda)}{\delta P_i} = 0 .$$

$$\left[\frac{\sigma_i^2/\sigma_n^2}{1+\frac{\sigma_i^2 P_i}{\sigma_n^2}}\right] + \lambda(0-1) = 0$$

$$\left[\frac{\sigma_i^2/\sigma_n^2}{1+\frac{\sigma_i^2 P_i}{\sigma_n^2}}\right] = \lambda$$

$$\left[\frac{\sigma_i^2/\cancel{\sigma_n^2}}{\frac{\sigma_n^2+\sigma_i^2 P_i}{\cancel{\sigma_n^2}}}\right] = \lambda$$

$$\frac{\sigma_i^2}{\lambda} = \sigma_n^2 + \sigma_i^2 P_i ,$$

$$\frac{\sigma_i^2}{\lambda\sigma_i^2} - \frac{\sigma_n^2}{\sigma_i^2} = P_i$$

$$\boxed{\left[\frac{1}{\lambda} - \frac{\sigma_n^2}{\sigma_i^2}\right] = P_i}$$

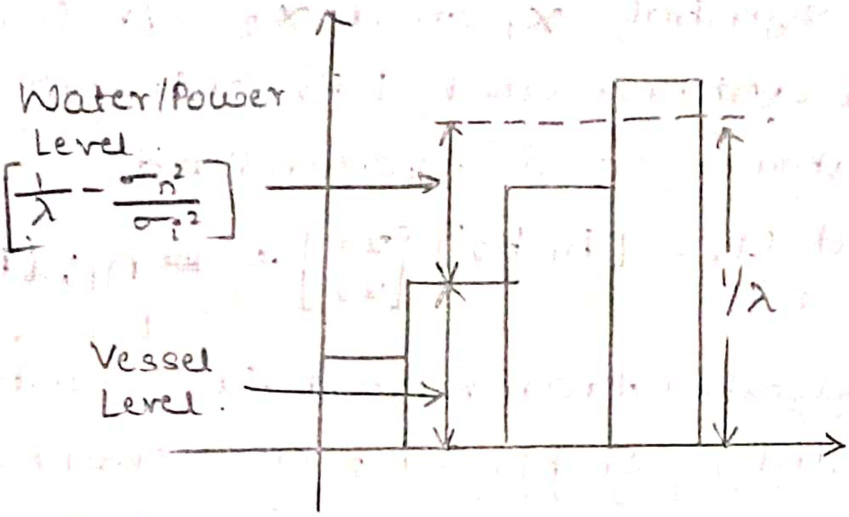
$$P_i = \begin{cases} 1/\lambda - \sigma_n^2/\sigma_i^2 , & \text{for } 1/\lambda - \sigma_n^2/\sigma_i^2 \geq 0 \\ 0 , & \text{otherwise} \end{cases}$$

$$\Rightarrow \sum_{i=1}^{t} \left(1/\lambda - \sigma_n^2/\sigma_i^2\right] = P //$$

This expression is the <u>optimum power allocation</u>.

It is termed as <u>water filling technique</u>.

(5) Schematic diagram for MIMO water filling capacity



Consider a vessel with 't' bars height of the i^{th} bar is (σ_n^2/σ_i^2), as the water is poured into the vessel, the level is $1/\lambda$, the power level at the i^{th} bar is $(1/\lambda - \sigma_n^2/\sigma_i^2)$ as shown in the above figure, The power allocated is propotional to the singular value i.e.,

Large is σ_i, large is the power allocated

weak channel with low σ_i are not allocated any power.

If $(1/\lambda - \sigma_n^2/\sigma_i^2) \geq 0$, this condition is satisfied produces desired power allocation.

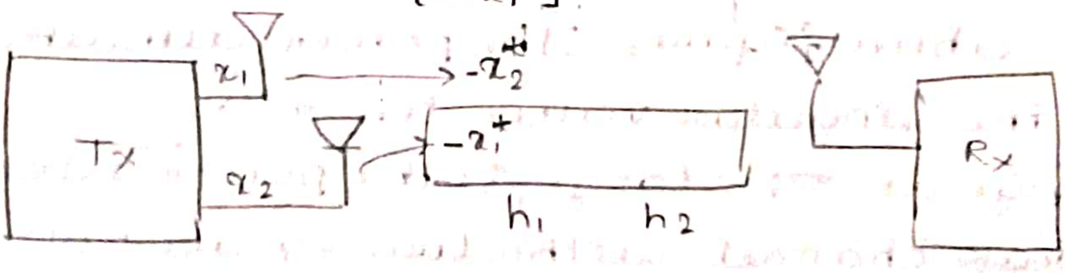
If $(1/\lambda - \sigma_n^2/\sigma_i^2) < 0$, then power is 0.

- **Alamouti and space-time codes** -

Consider the symbol x_1 and x_2, it is a MISO system with 2 Tx antenna and 1 Rx antenna.
At first time instance, x_1 is transmitted
∴ symbol received $y_1 = [h_1 \; h_2] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + n_1$; $(T=1)$ → (1)

x_1 & x_2 are the symbol transmitted from antenna 1 and 2 at time slot 1 & H_1 & h_2 are channel co-efficient b/w antenna 1 & 2 and single R_x antenna
n is the additive gaussian noise at the reciever.

At the second transmit period, antenna 1 transmits $-x_2^*$, and antenna 2 transmits $-x_1^*$
The received symbol after a time period is,
$$y_2 = [h_1 \; h_2] \begin{bmatrix} -x_2^* \\ -x_1^* \end{bmatrix} + n_2 \rightarrow (2)$$



$$y_2 = -h_1 x_2^* + h_2 x_1^* + n_2$$
$$y_2^* = -h_1^* x_2 + h_2^* x_1 + n_2^* = h_2^* x_1 - h_1^* x_2 + n_2^*$$
$$y_2^* = [-h_2^* \; -h_1^*] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + n_2^* \rightarrow (3)$$

The reciever combines the signal using channel information to recover the original signal.

$$\begin{bmatrix} y_1 \\ y_2^* \end{bmatrix} = \begin{bmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2^* \end{bmatrix}$$

$$y = Hx + n$$

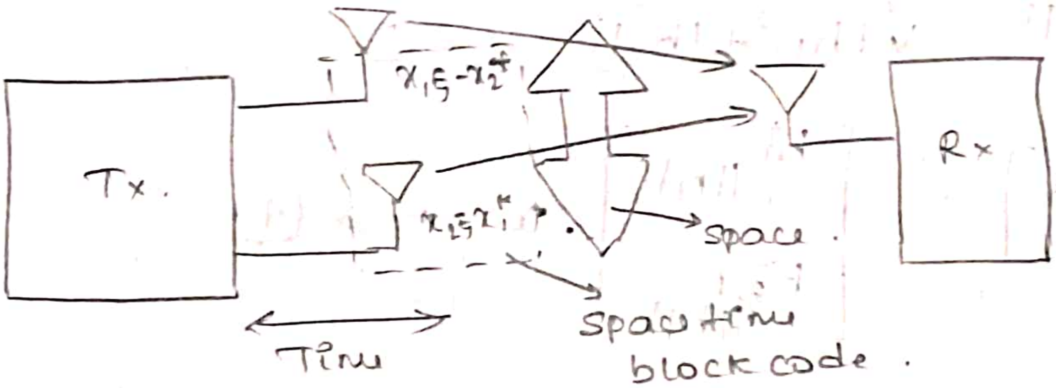
Let $c_1 = \begin{bmatrix} h_1 \\ h_2^* \end{bmatrix}$; $c_2 = \begin{bmatrix} h_2 \\ -h_1^* \end{bmatrix}$

$$(c_1^H)^* c_2 = [h_1 \;\; h_2^*]^* \begin{bmatrix} h_2 \\ -h_1^* \end{bmatrix} \cancel{= h_1 h_2 - h_1^* h_2^*}$$

$$c_1^H c_2 = [h_1^* \;\; h_2] \begin{bmatrix} h_2 \\ -h_1^* \end{bmatrix} = h_1^* h_2 - h_1^* h_2 = 0 //$$

∴ The column c_1 & c_2 are orthogonal, hence alamouti code is known as orthogonal space time block code. (OSTBC) or orthogonal space .
Hence channel information state. is not required

Alamouti OSTBC -



Alamouti code transmits x_1 & x_2 symbols one symbol per time slot, ∴ The net rate (R),
i.e., no. of information symbol divided by no. of time slot

$$R = \frac{\text{no. of info symbol}}{\text{no. of time slot}} = \frac{2}{2} = 1$$

⇒ The coding scheme allows transmission of one information symbol per time. Hence it is called as full rate code.

BEAM FORMING :

To focus the transmitted / received signal in a specific direction to improve SNR. It needs CSI (channel state information)

→ Used in multiuser system (MIMO)

→ Interference is reduced

Consider x is transmitted as

(i) x_1 is generated $= \frac{h_1^*}{\|h\|} . x \rightarrow (1)$

x_2 is generated $= \frac{h_2^*}{\|h\|} . x \rightarrow (2)$

$\|h\| \Rightarrow$ norm of vector 'h'.

$$\|h\| = \sqrt{|h_1|^2 + |h_2|^2} \longrightarrow (3)$$

(ii) $$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} h_1^*/\|h\| \\ h_2^*/\|h\| \end{bmatrix} . x \longrightarrow (4)$$

This is termed as transmit beam form

∴ The transmitting symbol 'x' in the direction of vector $\begin{bmatrix} \frac{h_1^*}{\|h\|} \\ \frac{h_2^*}{\|h\|} \end{bmatrix} \rightarrow (5)$.

The o/p $$y = [h_1 \; h_2] \begin{bmatrix} h_1^*/\|h\| \\ h_2^*/\|h\| \end{bmatrix} x + n \longrightarrow (6)$$

$$y = \begin{bmatrix} \frac{|h_1|^2}{\|h\|} & \frac{|h_2|^2}{\|h\|} \end{bmatrix} x + n$$

$$y = \|h\| x + n \longrightarrow (7)$$

The SNR of the system is given by,

$$\boxed{SNR = \frac{\|h^2\| P}{\sigma_n^2}} \rightarrow (8)$$

It uses multiple antenna at the transmitter & at the reciever. Hence from (4) we can conclude that transmitter requires knowledge of channel co-efficient h_1 h_2, this is termed as CSI (channel state informⁿ) i.e., channel co-efficient h_1 & h_2 should be estimated at the reciever. To impliment beam forming, CSI should be fed back to the transmitter. It is a challenging task. ∴ Alamouti code is used to overcome these constraints.

<u>Beam Forming using weighted vector</u> -

Let w_1 be the weighted vector of beam forming

$$w_1 = \frac{c_1}{\|c_1\|}, \quad c_1 = \begin{bmatrix} h_1 \\ h_2^* \end{bmatrix}; \quad c_2 = \begin{bmatrix} h_2 \\ -h_1^* \end{bmatrix}$$

$$w_1 = \frac{1}{\|h\|}\begin{bmatrix} h_1 \\ h_2^* \end{bmatrix}$$

To derive the processed symbol,

$$w_1^H y = \begin{bmatrix} \frac{h_1^*}{\|h\|} & \frac{h_2}{\|h\|} \end{bmatrix} \begin{bmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{bmatrix} x + w_1^H n$$

$$= \begin{bmatrix} \|h\| & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \tilde{n}_1$$

$$\therefore SNR = \frac{\|h^2\| P_1}{\sigma_n^2}$$

II^{ly} at Rx 2, $w_2 = \frac{c_2}{\|c_2\|} = \frac{1}{\|h\|}\begin{bmatrix} h_2 \\ -h_1^* \end{bmatrix}$

$$\therefore \ SNR = \frac{||h^2|| P_2}{\sigma_n^2}$$

where P_1 & P_2 are the power allocated to x_1 & x_2 respectively.

The total power 'P' is fixed and allocated for individual streaming,

$$P_1 = P_2 = P/2$$

$$\boxed{SNR = \frac{1}{2} \cdot \frac{||h^2|| P}{\sigma_n^2}}$$