THE UNIVERSITY OF TEXAS AT AUSTIN
McCOMBS SCHOOL OF BUSINESS

Logistic Regression 1

Lecture 16

STA 371G

okcupid

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DATING DESERVES BETTER

On OkCupid, you're more than just a photo. You have stories to tell, and passions to share, and things to talk about that are more interesting than the weather. Get noticed for who you are, not what you look like. Because you deserve what dating deserves: better.

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The OkCupid data set

- The OkCupid data set contains information about 59946 profiles from users of the OkCupid online dating service.
- We have data on user age, height, sex, income, sexual orientation, education level, body type, ethnicity, and more.
- OkCupid often publishes their own analyses of their data—see `https://theblog.okcupid.com/tagged/data`.
- Let's see if we can predict the sex/gender of the user based on their height.

What’s wrong with this regression?

$$\widehat{\text{sex}} = \hat{\beta}_0 + \hat{\beta}_1 \cdot \text{height}$$

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The Y variable here is categorical (two levels—everyone in this data set is either labeled male or female), so regular linear regression won't work here.

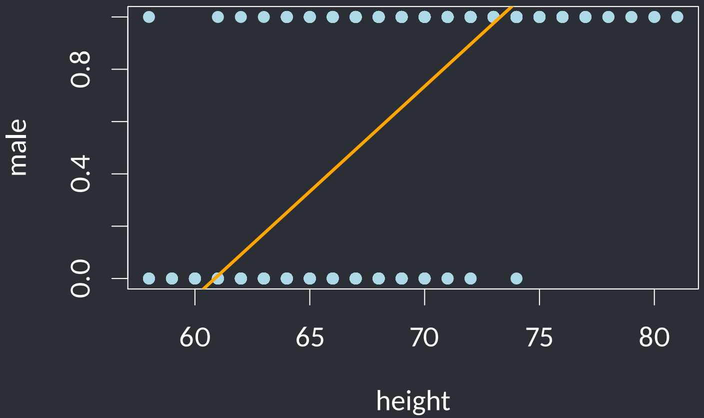
But what if we just do it anyway?

Let's first create a dummy variable to convert sex to a quantitative dummy variable:

```
profiles$male <- ifelse(profiles$sex == "m", 1, 0)
```

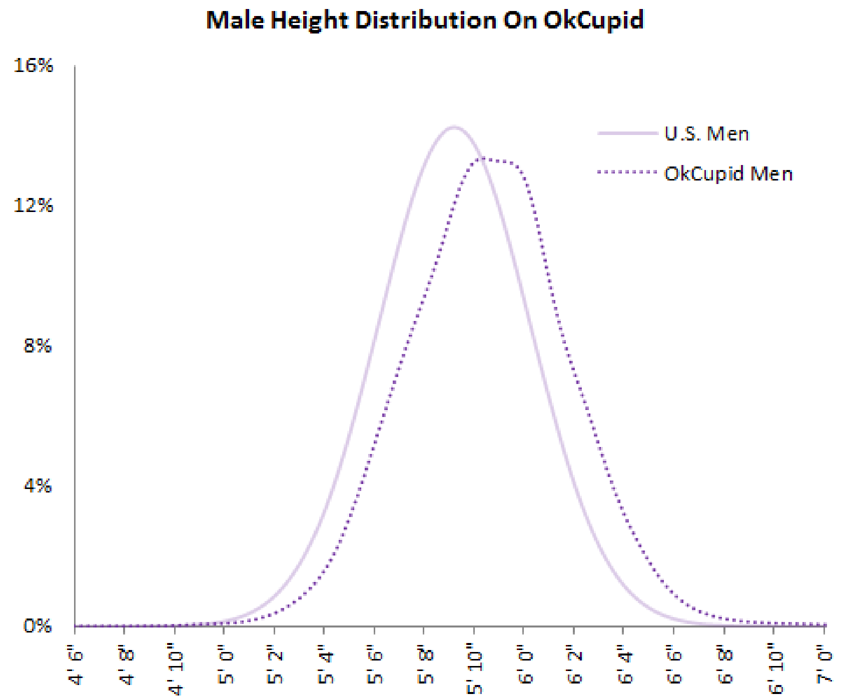
We could do this with 1 representing either male or female (it wouldn't matter).

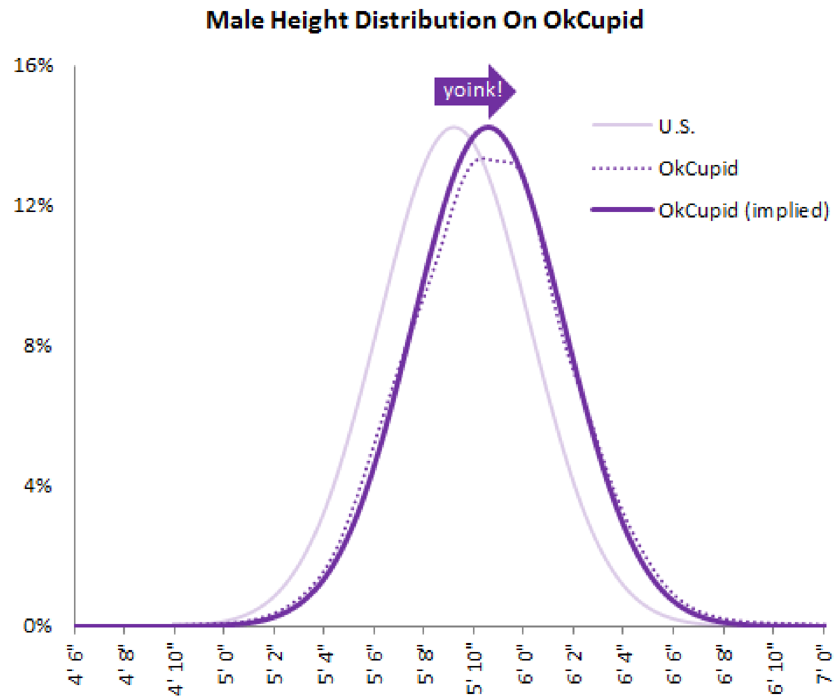
But what if we just do it anyway?



A line is a spectacularly bad fit to this data—it's not even close to linear. And what does it mean to predict that male = 0.7 (or 1.2)?

What challenges might we run into with this data?


Male Height Distribution On OkCupid
16%
12%
8%
4%
0%
U.S. Men
OkCupid Men
4' 6"
4' 8"
4' 10"
5' 0"
5' 2"
5' 4"
5' 6"
5' 8"
5' 10"
6' 0"
6' 2"
6' 4"
6' 6"
6' 8"
6' 10"
7' 0"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yoink!
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So men lie about their height—by an average of about 2 inches! And many men round up to 6 feet.

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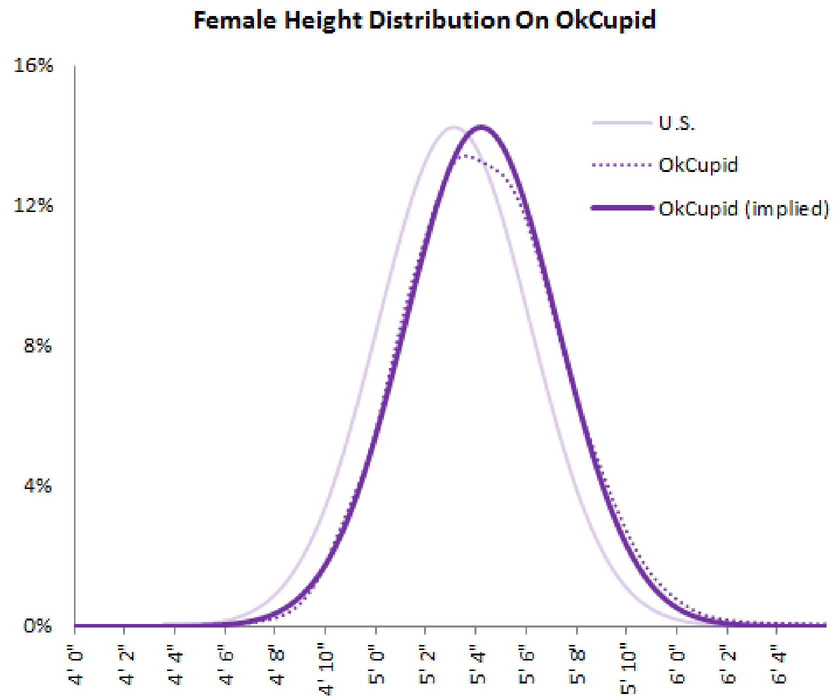
Women do too!



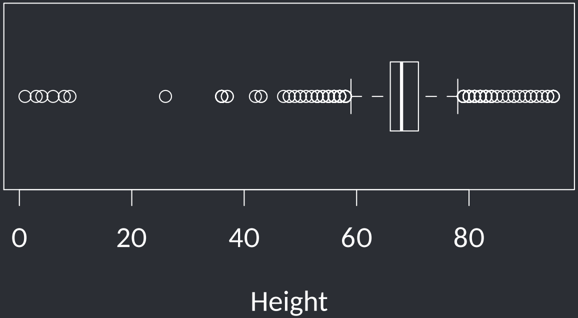
Female Height Distribution On OkCupid
16%
12%
8%
4%
0%
U.S.
OkCupid
OkCupid (implied)
4' 0"
4' 2"
4' 4"
4' 6"
4' 8"
4' 10"
5' 0"
5' 2"
5' 4"
5' 6"
5' 8"
5' 10"
6' 0"
6' 2"
6' 4"

We don’t really have any tools at our disposal to correct for this, but let’s still proceed with the analysis (with some caution) since the exaggeration seems about the same regardless of gender.

Cleaning the data

There are definitely some weird values for height:

```
boxplot(profiles$height, horizontal=T, xlab="Height")
```



Cleaning the data

Let’s consider only heights between 55 and 80 inches (4’7" and 6’8"), inclusive. This is arbitrary, but it excludes only 117 cases out of 59946.

```
my.profiles <- subset(profiles,
                      height >= 55 & height <= 80)
```

The idea behind logistic regression

- Instead of predicting whether someone is male, let's predict the *probability* that they are male
- In logistic regression, one level of Y is always called "success" and the other called "failure." Since $Y = 1$ for males, in our setup we have designated males as "success." (You could also set $Y = 1$ for females and call females "success.")
- Let's fit a curve that is always between 0 and 1.

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- When something has “2/1 odds,” the probability of success is 2/3
- When something has “3/2 odds,” the probability of success is 3/5
- In general, the odds of something happening are $p/(1-p)$

The logistic regression model

Logistic regression models the log odds of success p as a linear function of X:

$$\log\left(\frac{p}{1-p}\right) = \beta_0 + \beta_1 X + \epsilon$$

This fits an S-shaped curve to the data (we'll see what it looks like later).

Let's try it

```r
model <- glm(male ~ height, data=my.profiles,
             family=binomial)
summary(model)
```

How to interpret the curve?

The regression output tells us that our prediction is

$$\log \text{odds} = \log\left(\frac{P(\text{male})}{1 - P(\text{male})}\right) = -44.45 + 0.66 \cdot \text{height}.$$

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Let's solve for $P(\text{male})$:

$$\widehat{P(\text{male})} = \frac{e^{-44.45+0.66 \cdot \text{height}}}{1 + e^{-44.45+0.66 \cdot \text{height}}}$$

Making predictions

We can use `predict` to automate the process of plugging into the equation:

```
predict(model, list(height=69), type="response")

   1
0.77
```

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We predict that someone that is 5’9" has a 77% chance of being male.

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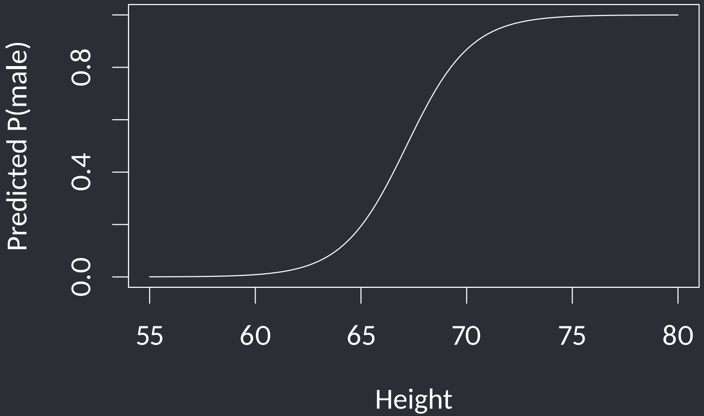

Interpreting the coefficients

Our prediction equation is:

$$\log\left(\frac{P(\text{male})}{1 - P(\text{male})}\right) = -44.45 + 0.66 \cdot \text{height}.$$

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Let's start with some basic, but not particularly useful, interpretations:

- When height $= 0$, we predict that the log odds will be -44.45 , so the probability of male is predicted to be very close to 0%.
- When height increases by 1 inch, we predict that the log odds of being male will increase by 0.66.

Interpreting the coefficients

Let's rewrite the prediction equation as:

$$\text{Predicted odds of male} = e^{-44.45+0.66\cdot\text{height}}$$

Increasing height by 1 inch will *multiply* the odds by $e^{0.66} = 1.94$; i.e., increase the odds by 94%.

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Let's rewrite the prediction equation as:

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Increasing height by 1 inch will *multiply* the odds by $e^{0.66} = 1.94$; i.e., increase the odds by 94%.

Increasing height by 2 inches will *multiply* the odds by $e^{2\cdot 0.66} = 3.76$; i.e., increase the odds by 276%.

Testing the null hypothesis

As in regular linear regression, the overall null hypothesis is that $\beta_1 = 0$; we can test this by using the p-value for that variable on the output.

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Since p is very small, we can reject the null hypothesis that $\beta_1 = 0$; i.e., there is a statistically significant relationship between height and sex.

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- However, there are many "pseudo-R^2" metrics that indicate model fit.
- But: most of these pseudo-R^2 metrics are difficult to interpret, so we'll focus on something simpler to interpret and communicate.

How many cases did we accurately predict?

We could use our model to make a prediction of sex based on the probability. Suppose we say that our prediction is:

$$\text{Prediction} = \begin{cases} \text{male}, & \text{if } \widehat{P(\text{male})} \geq 0.5, \\ \text{female}, & \text{if } \widehat{P(\text{male})} < 0.5. \end{cases}$$

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Now we can compute the fraction of people whose sex we correctly predicted:

```
predicted.male <- (predict(model, type="response") >= 0.5)
actual.male <- (my.profiles$male == 1)
sum(predicted.male == actual.male) / nrow(my.profiles)

[1] 0.83
```

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```

In other words, our model provided a “lift” in accuracy from 60% to 83%.

The confusion matrix

Sometimes it is useful to understand what kinds of errors our model is making:

- True positives: predicting male for someone that is male
- True negatives: predicting female for someone that is female
- False positives: predicting male for someone that is female
- False negatives: predicting female for someone that is male

(If we had designated female as 1 and male as 0, these would have switched!)

The confusion matrix

```
table(predicted.male, actual.male)

               actual.male
predicted.male FALSE  TRUE
         FALSE 19466  5494
         TRUE   4623 30243

prop.table(table(predicted.male, actual.male), 2)

               actual.male
predicted.male FALSE TRUE
         FALSE  0.81 0.15
         TRUE   0.19 0.85
```

What else can we use logistic regression for?

- **Finance:** Predicting which customers are most likely to default on a loan
- **Advertising:** Predicting when a customer will respond positively to an advertising campaign
- **Marketing:** Predicting when a customer will purchase a product or sign up for a service