THE UNIVERSITY OF TEXAS AT AUSTIN
McCOMBS SCHOOL OF BUSINESS

Model building: interactions 1

Lecture 12

STA 371G

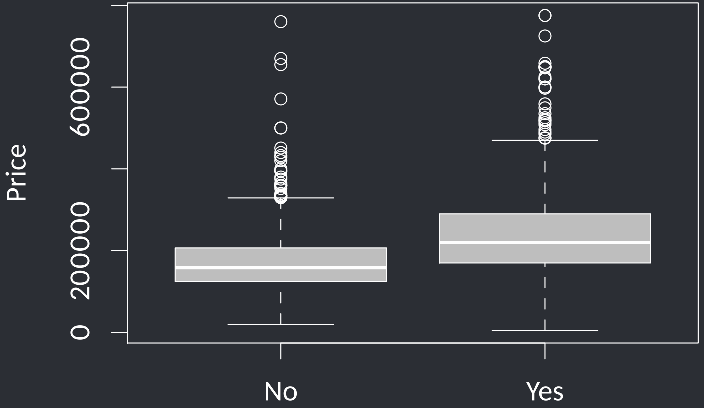
Housing price data

Today we'll consider 2007 housing price data set from Saratoga County, NY.

- **Price**: price of house ($)
- **Living.Area**: amount of living space (sq ft)
- **Fireplace**: whether house has a fireplace (yes/no)

How much is a fireplace worth?

```r
boxplot(Price ~ Fireplace, data=houses,
  col='gray', ylab='Price')
```



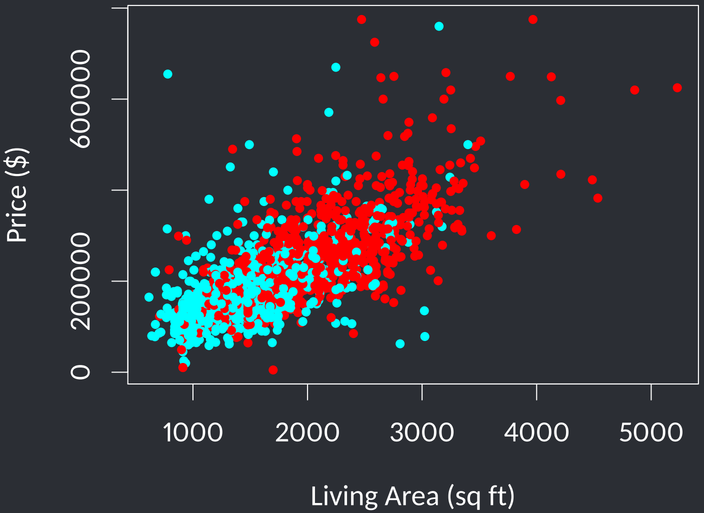
How much is a fireplace worth?

If we regress Price on Fireplace, we get the regression equation

$$\widehat{\text{Price}} = 174653 + 65261 \cdot (\text{Fireplace = Yes})$$

The average difference between houses with and without a fireplace is $65261.

What is the relationship between price and size?



Predicting price from living area

Let’s start by creating a simple regression predicting price from living area (in sq ft).

```r
model1 <- lm(Price ~ Living.Area, data=houses)
summary(model1)
```

```

Call:
lm(formula = Price ~ Living.Area, data = houses)

Residuals:
    Min      1Q  Median      3Q     Max 
-277022  -39371   -7726   28350  553325 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) 13439.39    4992.35    2.69   0.0072 ** 
Living.Area   113.12       2.68   42.17   <2e-16 ***
---
Signif. codes:  0 '***' 0 '**' 0 '*' 0 '.' 0 ' ' 1

Residual standard error: 69100 on 1726 degrees of freedom
Multiple R-squared:  0.507,Adjusted R-squared:  0.507 
F-statistic: 1.78e+03 on 1 and 1726 DF,  p-value: <2e-16
```

Can we do better by adding a dummy variable for fireplace to the model?

```r
model2 <- lm(Price ~ Living.Area + Fireplace, data=houses)
summary(model2)
```

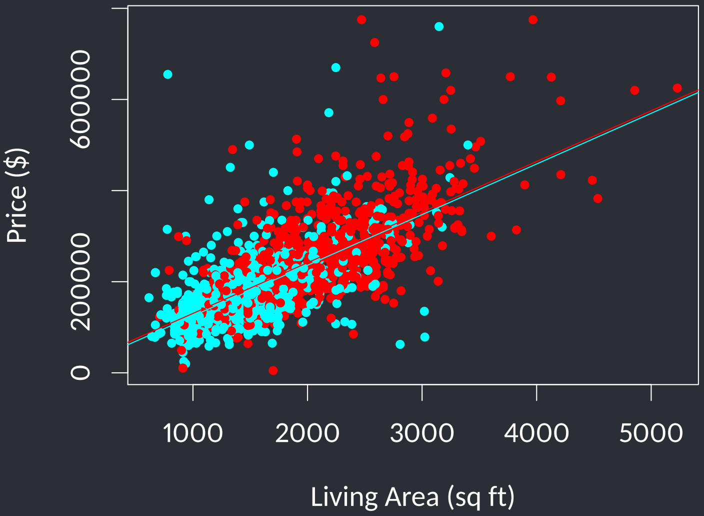
```
Call:
lm(formula = Price ~ Living.Area + Fireplace, data = houses)

Residuals:
    Min      1Q  Median      3Q     Max
-271421  -39935   -7887   28215  554651

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  13599.16    4991.70    2.72   0.0065 **
Living.Area    111.22       2.97   37.48   <2e-16 ***
FireplaceYes  5567.38    3716.95    1.50   0.1344
---
Signif. codes:  0 '***' 0 '**' 0 '*' 0 '.' 0 ' ' 1

Residual standard error: 69100 on 1725 degrees of freedom
Multiple R-squared:  0.508,Adjusted R-squared:  0.508
F-statistic:  891 on 2 and 1725 DF,  p-value: <2e-16
```

By adding the dummy variable, we are essentially fitting two regression lines:



They have the same slope, but different intercepts

Interactions

Our regression equation is

$$\widehat{\text{Price}} = 13599 + 111 \cdot \text{Living.Area} + 5567 \cdot \text{FireplaceYes}.$$

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What if the *slope* of the best-fit line is different for houses with a fireplace than for houses without?

Equivalently, what if the *effect* of having a bigger house is different for houses with fireplaces than for houses without fireplaces?

Interactions

To model this, we can add an interaction term that consists of the product of the two predictors:

$$\begin{aligned}\text{Price} = \beta_0 + \beta_1 \cdot \text{Living.Area} + \beta_2 \cdot \text{FireplaceYes} \\ + \beta_3 \cdot \text{Living.Area} \cdot \text{FireplaceYes} + \epsilon_i.\end{aligned}$$

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Now, the *slope* of Living.Area depends on the *value* of Fireplace!

Houses with a fireplace have a slope of of $\beta_1 + \beta_3$, houses without have a slope of β_1.

```r
model3 <- lm(Price ~ Living.Area * Fireplace, data=houses)
summary(model3)
```

```
Call:
lm(formula = Price ~ Living.Area * Fireplace, data = houses)

Residuals:
    Min      1Q  Median      3Q     Max
-241710  -39588   -7821   28480  542055

Coefficients:
                          Estimate Std. Error t value   Pr(>|t|)
(Intercept)               40901.29    8234.66    4.97 0.00000075 ***
Living.Area                  92.36       5.41   17.07    < 2e-16 ***
FireplaceYes             -37610.41   11024.85   -3.41    0.00066 ***
Living.Area:FireplaceYes     26.85       6.46    4.16 0.00003376 ***
---
Signif. codes:  0 '***' 0 '**' 0 '*' 0 '.' 0 ' ' 1

Residual standard error: 68800 on 1724 degrees of freedom
Multiple R-squared:  0.513,Adjusted R-squared:  0.512
F-statistic:  605 on 3 and 1724 DF,  p-value: <2e-16
```

This corresponds to the regression equation:

$$\widehat{\text{Price}} = 40901 + 92 \cdot \text{Living.Area} - 37610 \cdot \text{FireplaceYes} + 27 \cdot \text{Living.Area} \cdot \text{FireplaceYes}$$

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In other words, for houses without a fireplace:

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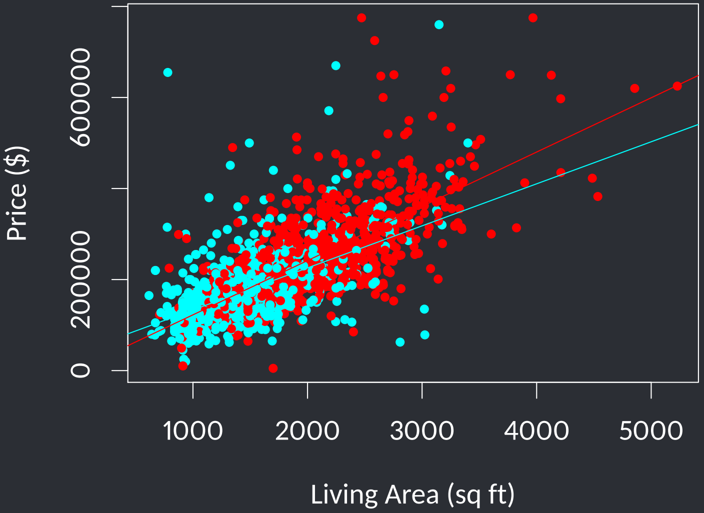
In other words, for houses without a fireplace:

$$\widehat{\text{Price}} = 40901 + 92 \cdot \text{Living.Area}$$

And for houses with a fireplace:

$$\widehat{\text{Price}} = (40901 - 37610) + (92 + 27) \cdot \text{Living.Area}$$


Price ($)
0
200000
600000
1000
2000
3000
4000
5000
Living Area (sq ft)

Main effects and interaction effects

In the output, the coefficients for Living.Space and Fireplace are main effects, and the coefficient for Living.Space · Fireplace is an interaction effect.

```
summary(model3)$coefficients
```

```
                         Estimate Std. Error t value Pr(>|t|)
(Intercept)                 40901     8234.7     5.0  7.5e-07
Living.Area                    92        5.4    17.1  1.8e-60
FireplaceYes               -37610    11024.9    -3.4  6.6e-04
Living.Area:FireplaceYes       27        6.5     4.2  3.4e-05
```

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The main effect for Living.Area (92.36) represents the predicted incremental effect of each additional square foot of living space, when there is no fireplace present.

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The main effect for Living.Area (92.36) represents the predicted incremental effect of each additional square foot of living space, when there is no fireplace present.

When we have an interaction term in the model, we *must* include the main effect as well!

Making predictions

Let's make predictions for the price of a 2500 sq ft house, both with and without a fireplace:

```
predict.lm(model3, list(Living.Area=2500, Fireplace='Yes'),
  interval='prediction')

     fit    lwr    upr
1 301331 166362 436300

predict.lm(model3, list(Living.Area=2500, Fireplace='No'),
  interval='prediction')

     fit    lwr    upr
1 271811 136405 407217
```