



THE UNIVERSITY OF TEXAS AT AUSTIN
McCOMBS SCHOOL OF BUSINESS

Inference for simple regression 2

Lecture 4

STA 371G

Announcements and Logistics

- HW 1 is now posted on Canvas/MyStatLab, and is due on Thursday.
- In addition to completing the assignment in MyStatLab, submit an R script file through Canvas that contains the commands you used to solve the problems.

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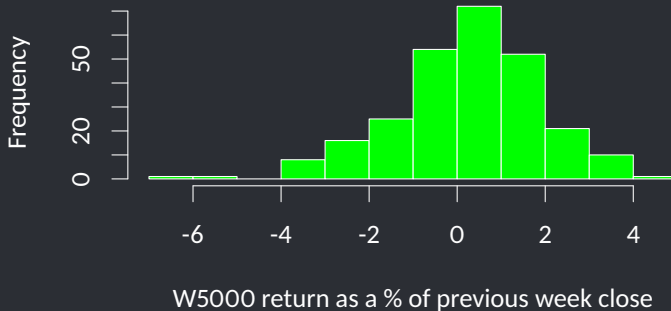
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β is just the slope of the regression line (i.e. $\hat{\beta}_1$) when we regress the asset's weekly returns against the weekly returns of a market index.

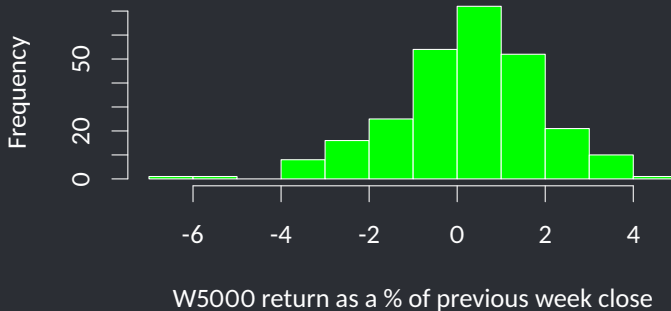
W5000 (Wilshire 5000, a broad market index)

```
> hist(stock.market$W5000, col='green',  
+   main='', xlab='W5000 return as a % of previous week close')
```



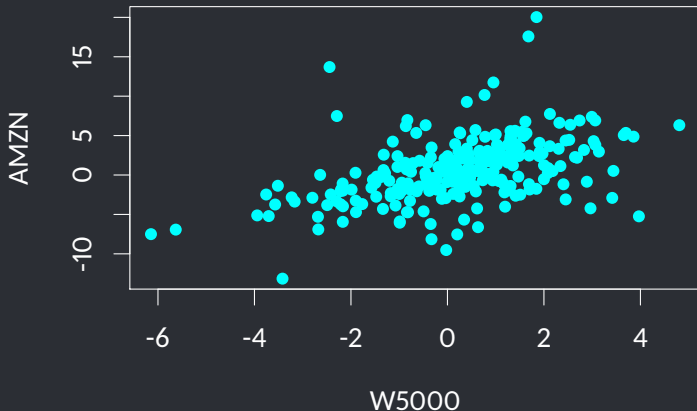
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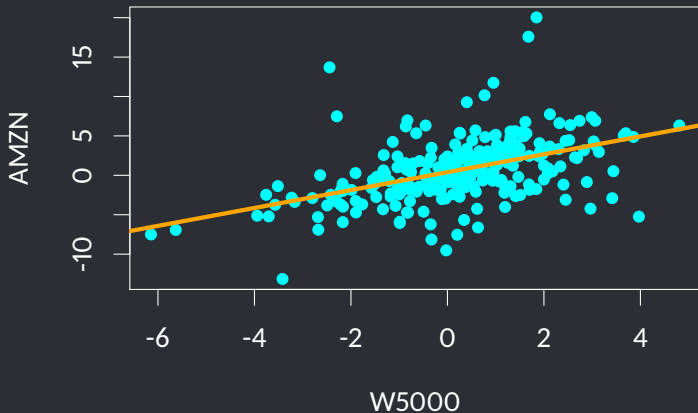
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Amazon (AMZN)

```
> plot(AMZN ~ W5000, data=stock.market,  
+      pch=16, col='cyan')
```





The regression line is

$$\widehat{\text{AMZN}} = 0.4 + 1.13 \cdot \text{W5000},$$

with $R^2 = 0.22$ and $p = 7.8 \times 10^{-16}$.

Interpreting the regression statistics

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Simple regression assumptions

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1. The errors are independent.
2. Y is a linear function of X (except for the errors).
3. The errors are normally distributed.
4. The variance of Y is the same for any value of X (“homoscedasticity”).

Assumption 1: Independence

Independence means that knowing the value of a variable for one case doesn't tell you anything about another case.

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- From today: Knowing the return this week doesn't tell us anything about the return next week, if we believe the efficient market hypothesis.
- **But:** Time-series data often violates the independence assumption!
- We can only check this assumption by thinking about the situation conceptually.

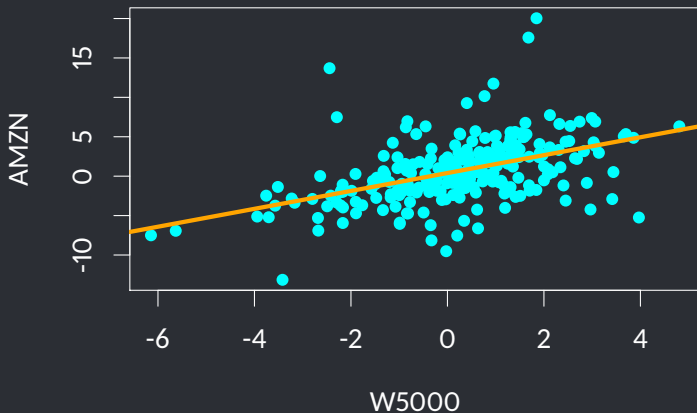
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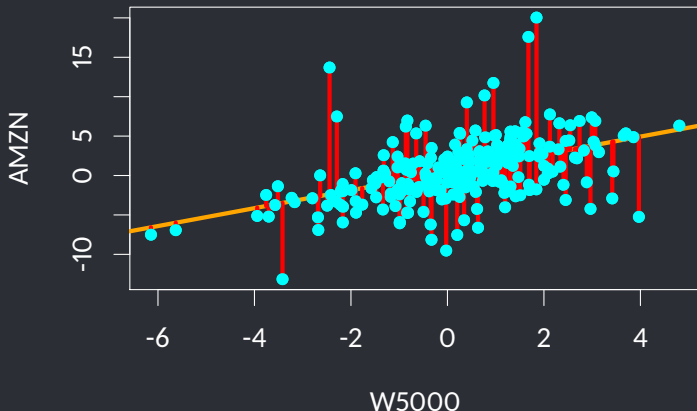
Assumption 2: Linearity

Step 1: Visually examine to ensure a line is a good fit for the data:



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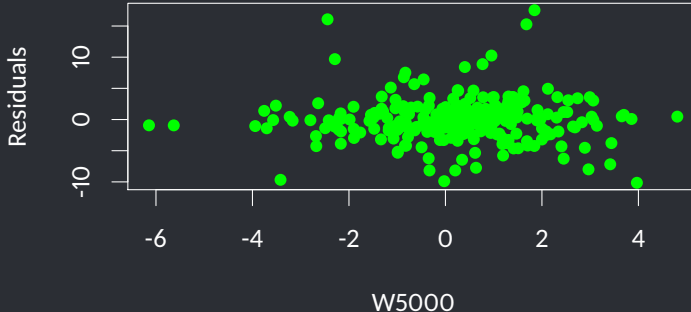
Each point has a **residual** ($Y - \hat{Y}$); this is the over/under-prediction of the model (red lines).



Assumption 2: Linearity

A **residual plot** (of residuals vs X) helps us ensure that there is not subtle nonlinearity. We want to see **no trend** in this plot:

```
> model <- lm(AMZN ~ W5000, data=stock.market)
> plot(stock.market$W5000, resid(model),
+       pch=16, col='green', xlab='W5000', ylab='Residuals')
```



Simple regression assumptions

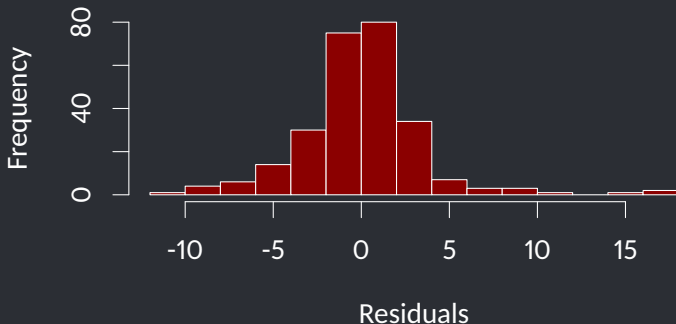
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Assumption 3: Errors are normally distributed

Step 1: Look at a histogram of the residuals and ensure they are approximately normally distributed:

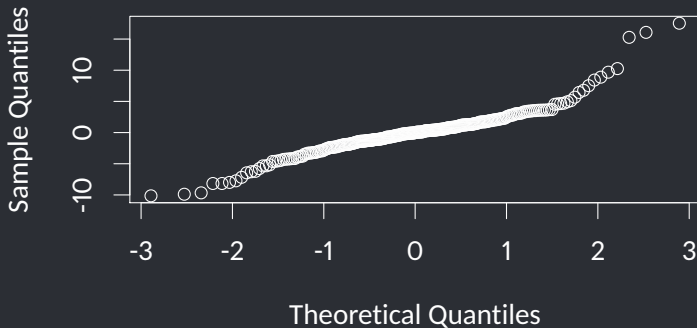
```
> hist(resid(model), col='darkred',  
+       xlab='Residuals', main='')
```



Assumption 3: Errors are normally distributed

Step 2: Look at a Q-Q plot of the residuals and look for an approximately straight line:

```
> qqnorm(resid(model), main='')
```



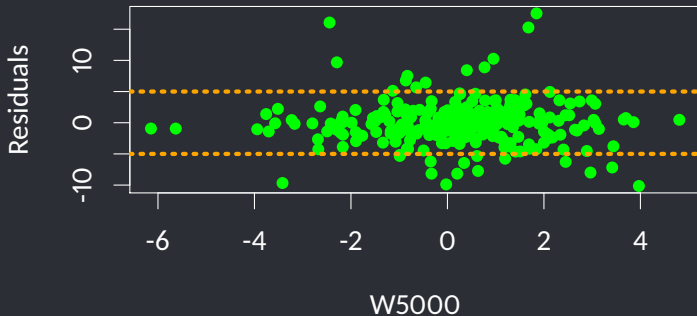
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Assumption 4: The variance of Y is the same for any value of X

Look for the residual plot to have roughly equal vertical spread all the way across:



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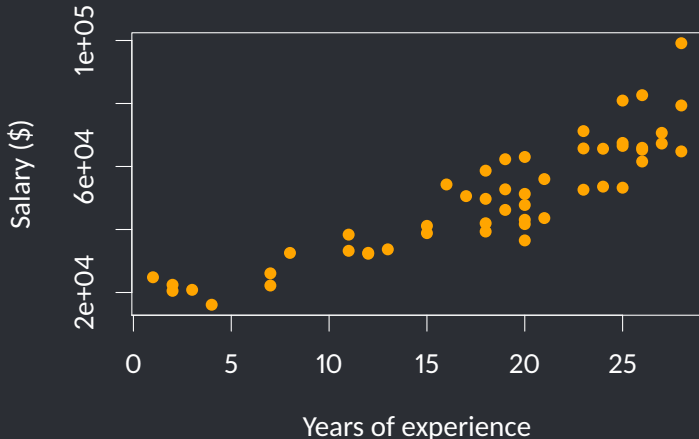
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We always need to check these assumptions before interpreting p -values or confidence intervals!



An example where an assumption fails

This is a data set of social worker salaries based on years of experience. Which assumption might be violated here?



An example where an assumption fails

