



THE UNIVERSITY OF TEXAS AT AUSTIN  
McCOMBS SCHOOL OF BUSINESS

# Simple regression

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## Lecture 2

STA 371G

## About the course staff

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  - Office hours: M 11:30 AM-1:30 PM, 2-4 PM, W 12-2 PM, Th 4-6 PM in CBA 4.304
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## National Longitudinal Study of Adolescent to Adult Health

Nationally representative sample of US students in grades 7-12 were surveyed in the 1994-95 school year

(<http://www.cpc.unc.edu/projects/addhealth>)

Students were followed up on with subsequent in-home interviews four times (most recently 2008)

This is an **awesome** data set, with data on:

- family
- relationships
- health
- military service
- religion
- sex and STDs
- economics
- education
- personality
- criminality
- tobacco
- drugs
- alcohol
- pregnancy
- sleep
- daily activities

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when they become adults?

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when they become adults?

We want to know:

- What is our best **prediction** of alcohol consumption if we know at what age had their first drink?
- How good is that prediction?
- What is the **relationship** between alcohol consumption and age of first drink?



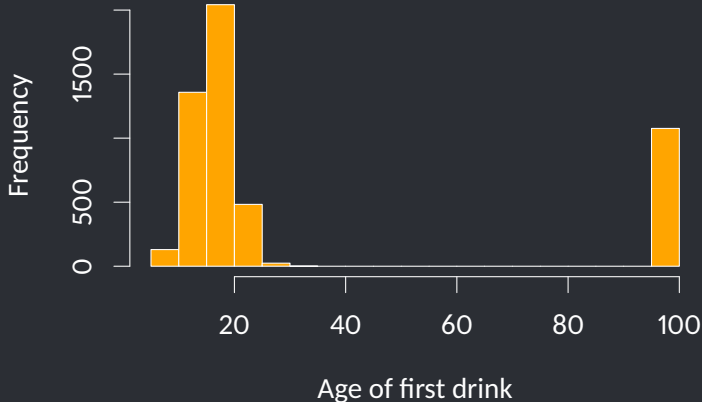
Age of first drink

**Predictor variable**

Number of drinks consumed as adult

**Response variable**

```
> hist(addhealth$h4to34,  
+      main='', xlab='Age of first drink',  
+      col='orange')
```

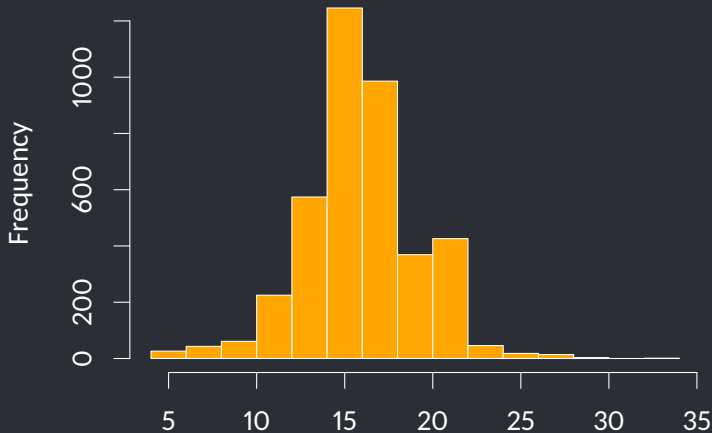


## Let's examine our variables

*If Q.33 = 1, ask Q.34, else skip to Q.63.*

H4TO34		Num	34. How old were you when you first had an alcoholic drink? By drink, we mean a glass of wine, a can or bottle of beer, a wine cooler, a shot glass of liquor, or a mixed drink, not just sips or tastes from someone else's drink. NOTE: Smallest 5 and largest 5 values are displayed.
Frequency	Percent	Value	Label
56	0.4%	5	5 years
30	0.2%	6	6 years
21	0.1%	7	7 years
71	0.5%	8	8 years
52	0.3%	9	9 years
12014	76.5%	10-31	NOTE: Range of values omitted from display
1	0.0%	32	32 years
2	0.0%	33	33 years
21	0.1%	96	refused
3322	21.2%	97	legitimate skip
111	0.7%	98	don't know

```
> age <- addhealth$h4to34  
> age[age >= 96] <- NA  
> hist(age, main='', xlab='', col='orange')
```

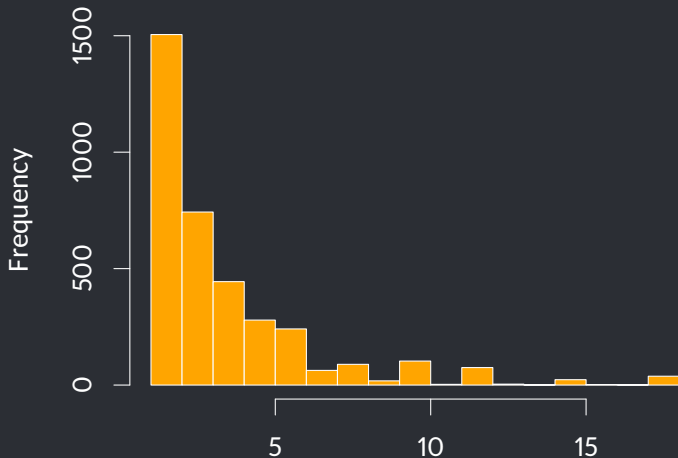


## Let's examine our variables

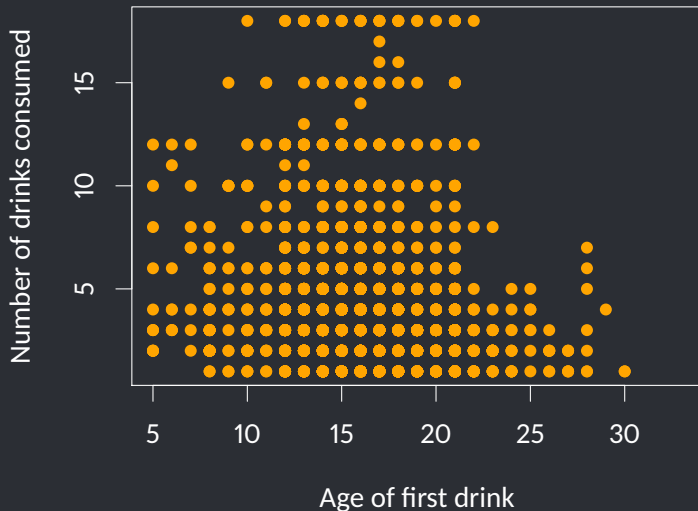
*If Q.35 not equal 0, ask Q.36, else if Q.35 = 0, then skip to Q.43.*

<b>H4TO36</b>		Num	36. Think of all the times you have had a drink during the past 12 months. How many drinks did you <b>usually</b> have each time? A 'drink' is a glass of wine, a can or bottle of beer, a wine cooler, a shot glass of liquor, or a mixed drink. NOTE: Smallest 5 and largest 5 values are displayed.
Frequency	Percent	Value	Label
1651	10.5%	1	1 drink
3051	19.4%	2	2 drinks
2274	14.5%	3	3 drinks
1343	8.6%	4	4 drinks
891	5.7%	5	5 drinks
1815	11.6%	6-16	NOTE: Range of values omitted from display
4	0.0%	17	17 drinks
108	0.7%	18	18 drinks
27	0.2%	96	refused
4427	28.2%	97	legitimate skip
110	0.7%	98	don't know

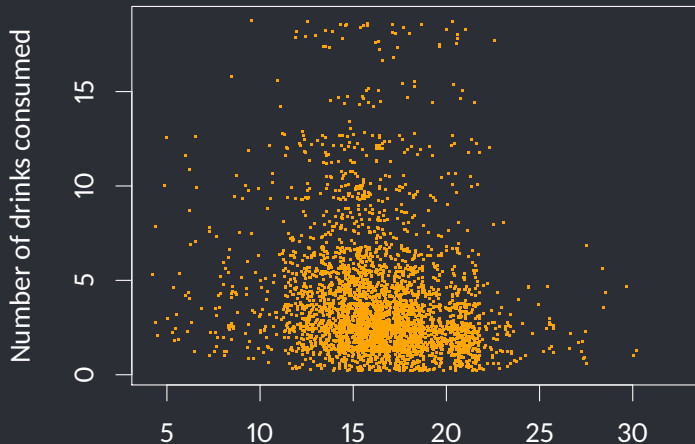
```
> num.drinks <- addhealth$h4to36  
> num.drinks[num.drinks >= 96] <- NA  
> hist(num.drinks, main='', xlab='How many drinks',  
+   col='orange')
```



```
> plot(num.drinks ~ age, pch=16, col='orange',  
+      xlab='Age of first drink',  
+      ylab='Number of drinks consumed')
```

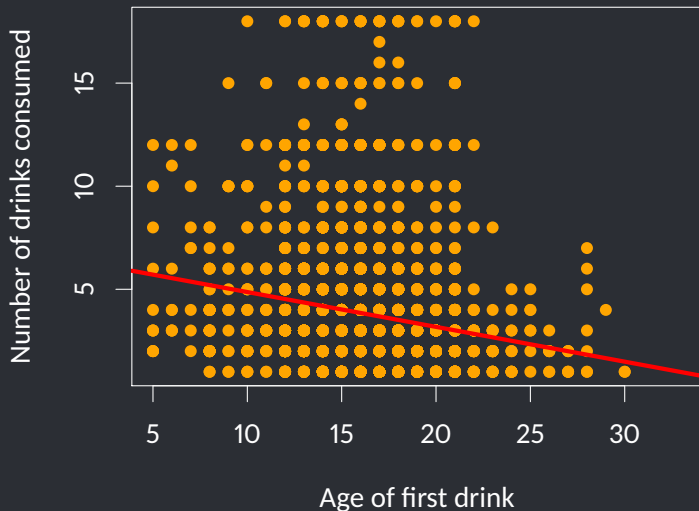


```
> plot(jitter(num.drinks, 4) ~ jitter(age, 4),  
+      pch=46, col='orange',  
+      xlab='Age of first drink',  
+      ylab='Number of drinks consumed')
```





The regression line is the line of “best fit” through this plot:



## What is linear regression doing?

We model each case ( $x_i$  = age for  $i$ th person,  $y_i$  = number of drinks for  $i$ th person) as a linear relationship plus some error:

$$y_i = \beta_0 + \beta_1 x_i + \epsilon_i$$

$\beta_0$  and  $\beta_1$  are the intercept and slope, respectively.

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We find estimates for  $\beta_0$  and  $\beta_1$  in our sample that *minimize* the errors:

$$\hat{Y} = \hat{\beta}_0 + \hat{\beta}_1 X$$

This is the regression (best fit) line.

```
> model <- lm(num.drinks ~ age)
> summary(model)
```

Call:

```
lm(formula = num.drinks ~ age)
```

Residuals:

	Min	1Q	Median	3Q	Max
	-4.2035	-1.8528	-0.8528	0.8095	15.1602

Coefficients:

	Estimate	Std. Error	t value	Pr(> t )	
(Intercept)	6.55417	0.26532	24.70	<2e-16	***
age	-0.16883	0.01588	-10.63	<2e-16	***

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 2.963 on 3600 degrees of freedom  
(2902 observations deleted due to missingness)

Multiple R-squared: 0.03044, Adjusted R-squared: 0.03017

F-statistic: 113 on 1 and 3600 DF, p-value: < 2.2e-16



This translates to a regression line of:

$$\widehat{\text{num drinks}} = 6.55 - 0.17 \cdot \text{age}$$



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Predict number of drinks for age = 21:

$$\widehat{\text{num drinks}} = 6.55 - 0.17 \cdot 21 = 3.01$$

Or we can use R to do the work for us:

```
> predict(model, list(age=21))
```



## How good are our predictions?

$R^2$  quantifies how closely the model fits the data.

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- $R^2 = 0$  when the model has no predictive power at all.
- $R^2 = 1$  when the model yields perfect predictions every time.
- $R^2 = \text{cor}(Y, \hat{Y})^2$ , i.e., the squared correlation between the actual and predicted values of  $Y$ .



```
> model <- lm(num.drinks ~ age)
> summary(model)
```

Call:

lm(formula = num.drinks ~ age)

Residuals:

Min	1Q	Median	3Q	Max
-4.204	-1.853	-0.853	0.810	15.160

Coefficients:

	Estimate	Std. Error	t value	Pr(> t )
(Intercept)	6.5542	0.2653	24.7	<2e-16 ***
age	-0.1688	0.0159	-10.6	<2e-16 ***

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 3 on 3600 degrees of freedom  
(2902 observations deleted due to missingness)

Multiple R-squared: 0.0304, Adjusted R-squared: 0.0302

F-statistic: 113 on 1 and 3600 DF, p-value: <2e-16

In our regression,  $R^2 = 0.03$ , so  $r = \sqrt{0.03} = -0.17$  (negative since the slope is negative).

Is this “significant?”

In our regression,  $R^2 = 0.03$ , so  $r = \sqrt{0.03} = -0.17$  (negative since the slope is negative).

Is this “significant?” We'll discuss this next time!