

Simulation 3

Lecture 24

STA 371G

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- If you win, you win what you bet.
- Suppose there is a 40% chance that you will win at any given hand.

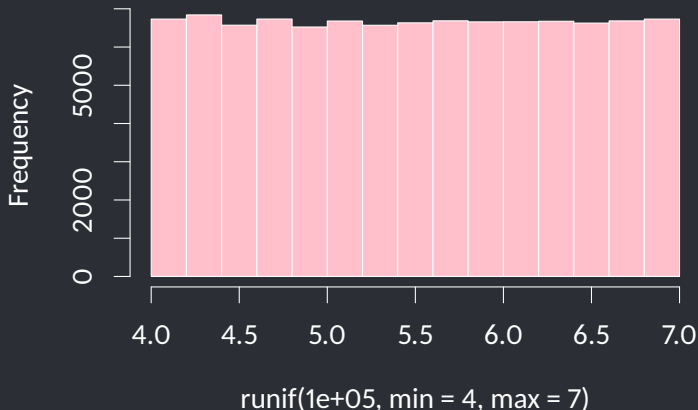
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- What will your bank look like after 20 hands of blackjack?

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- Without any parameters, it samples from the range $[0, 1]$.

```
hist(runif(100000, min=4, max=7), col="pink", main="")
```



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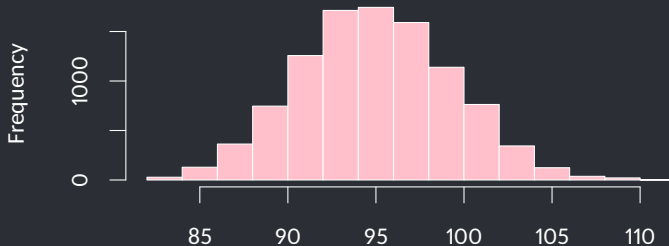
- `runif` is often useful for simulating an event that happens with a certain probability.
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- `runif` is often useful for simulating an event that happens with a certain probability.
- For example, `runif(1)` randomly selects one number uniformly from $[0, 1]$, so it will be less than 0.2 about 20% of the time.
- To simulate an event that has a 20% chance of happening:

```
if (runif(1) < 0.2) {  
  # The event happened.  
} else {  
  # The event did not happen.  
}
```

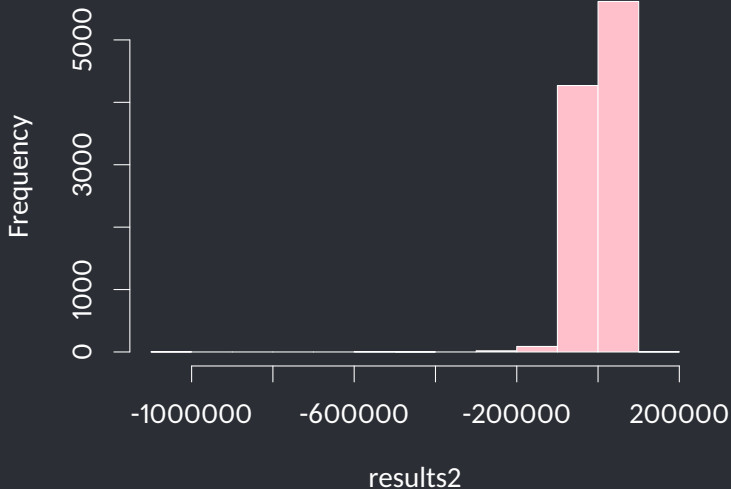
```
results <- replicate(10000, {  
  bank <- 100  
  bet <- 1  
  for (hand in 1:20) {  
    if (runif(1) < 0.4) {  
      bank <- bank + bet  
    } else {  
      bank <- bank - bet  
    }  
  }  
  return(bank)  
})  
hist(results, col="pink", main="")
```



- Let's model another betting strategy. Suppose that if we lose, we double our bet in an effort to recover the previous loss.
- This way, if we lose, a win on the next hand would cancel out the losses.
- This is called a *Martingale betting strategy*.

```
results2 <- replicate(10000, {  
  bank <- 100  
  bet <- 1  
  for (hand in 1:20) {  
    win <- (runif(1) < 0.4)  
    if (win) {  
      bank <- bank + bet  
    } else {  
      bank <- bank - bet  
      bet <- 2 * bet  
    }  
  }  
  return(bank)  
})
```

```
hist(results2, col="pink", main="")
```



Examining percentiles of outcomes

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- To look at the 5th, 50th (median), and 95th percentile result:

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- In other words: under the Martingale strategy, 5% of the time our bank will end up less than $-\$31001.1$, 50% of the time we will end up with less than $\$271$, etc.

Comparing the strategies

Strategy	Original	Martingale
Expected total after 20 hands	95.96	-4026.97
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You could do better with the Martingale strategy, but you'll probably end up doing *much* worse!

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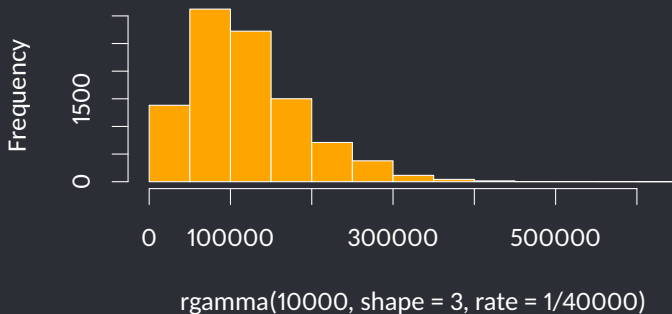
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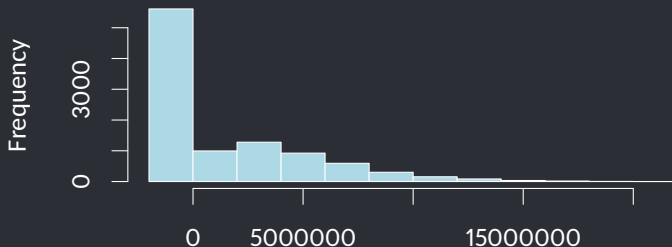
- Let's model the problem of oil drilling: you are planning to drill for oil in a newly-discovered field.
- Setting up the drilling equipment costs \$1M.
- There's a 45% chance that you strike oil.
- If you strike oil, you will generate money—but how much depends on the price of oil and how much demand there is.

- Suppose that the price of oil is normally distributed, with a mean of \$45/barrel and an SD of \$8.
- Suppose that the demand (the number of barrels we can sell) has a *gamma distribution* with shape 3 and rate 1/40000 (the gamma distribution can be used to model many processes, like waiting times and consumer demand, which must be ≥ 0 and has a long right tail):

```
hist(rgamma(10000, shape=3, rate=1/40000),  
     col="orange", main="")
```



```
results <- replicate(10000, {  
  if (runif(1) < 0.45) {  
    price <- rnorm(1, mean=45, sd=8)  
    demand <- rgamma(1, shape=3, rate=1/40000)  
    revenue <- price * demand - 1000000  
  } else {  
    revenue <- -1000000  
  }  
  return(revenue)  
})  
hist(results, col="lightblue", main="")
```



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- It's more realistic to not assume this will necessarily be the case.
- Suppose that production also has a gamma distribution with shape 3 and rate $1/40000$.
- How much do we make if we decide to drill for oil?

```
results <- replicate(10000, {  
  if (runif(1) < 0.45) {  
    price <- rnorm(1, mean=45, sd=8)  
    demand <- rgamma(1, shape=3, rate=1/40000)  
    production <- rgamma(1, shape=3, rate=1/40000)  
    barrels.sold <- min(production, demand)  
    revenue <- price * barrels.sold - 1000000  
  } else {  
    revenue <- -1000000  
  }  
  return(revenue)  
})  
hist(results, col="lightblue", main="")
```

