THE UNIVERSITY OF TEXAS AT AUSTIN
McCOMBS SCHOOL OF BUSINESS

Model building: dummy variables

Lecture 11

STA 371G

Let’s predict fuel economy (miles per gallon) for different car models of the 70s.

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- Cylinders
- Displacement
- Horsepower
- Weight
- Acceleration
- Year (After 1975 or not)

Exploring the data

```
head(cars)
```

```
  MPG Cylinders Displacement  HP Weight Acceleration After1975 Origin
1  18         8          307 130   3504         12.0        No     US
2  15         8          350 165   3693         11.5        No     US
3  18         8          318 150   3436         11.0        No     US
4  16         8          304 150   3433         12.0        No     US
5  17         8          302 140   3449         10.5        No     US
6  15         8          429 198   4341         10.0        No     US
```

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```r
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```
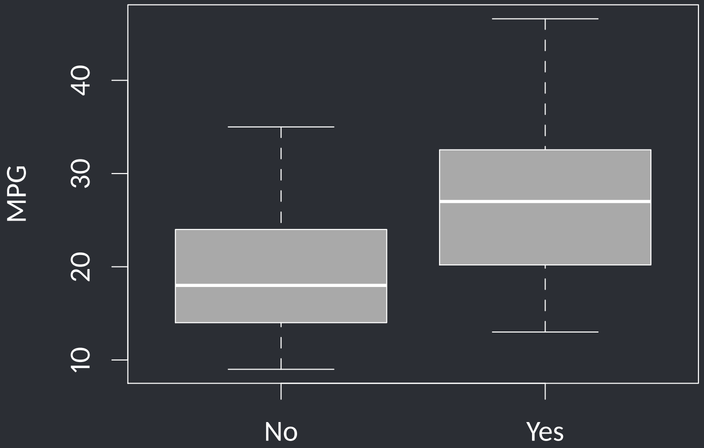

How do we handle the Yes/No data in the `After1975` column?

Are late-model cars different?

```r
boxplot(MPG ~ After1975, data=cars, ylab="MPG",
              xlab="After 1975", col='darkgray')
```



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Now let's a regression model using the predictors `Cylinders`, `Displacement`, `HP`, `Weight`, `Acceleration`, and `LateModel`.

R will actually create this “dummy” (0/1) variable for us automatically, when you put a categorical variable (what R calls a “factor” into a model).

```
summary(lm(MPG ~ Cylinders + Displacement + HP + Weight +
    Acceleration + After1975, data=cars))
```

```
Call:
lm(formula = MPG ~ Cylinders + Displacement + HP + Weight + Acceleration
    After1975, data = cars)

Residuals:
    Min      1Q  Median      3Q     Max
-8.9302 -2.5727 -0.2574  2.0630 15.0381

Coefficients:
               Estimate Std. Error t value Pr(>|t|)
(Intercept)  42.1939988  2.3687735  17.813  < 2e-16 ***
Cylinders    -0.5840362  0.3601220  -1.622    0.106
Displacement  0.0074811  0.0079791   0.938    0.349
HP           -0.0198909  0.0147848  -1.345    0.179
Weight       -0.0059904  0.0007194  -8.327 1.46e-15 ***
Acceleration  0.0354327  0.1103506   0.321    0.748
After1975Yes  4.3590043  0.4016220  10.853  < 2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 3.721 on 385 degrees of freedom
```

Dummy variables

`After1975Yes` is 1 whenever `After1975` is "Yes," and 0 otherwise:

MPG	…	Acceleration	After1975	After1975Yes
…	…	…	…	…
25	…	13.5	No	0
33	…	17.5	No	0
28	…	15.5	Yes	1
25	…	16.9	Yes	1
…	…	…	…	…

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MPG	…	Acceleration	After1975	After1975Yes
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33	…	17.5	No	0
28	…	15.5	Yes	1
25	…	16.9	Yes	1
…	…	…	…	…

Notice that we do not have a `After1975No` variable; it would cause problems because it would be perfectly correlated with `After1975Yes`.

Interpretation of the $\hat{\beta}$ of the dummy variable

Our regression equation is:

$$\widehat{\text{MPG}} = 41.71 - 0.02 \cdot \text{HP} - 0.01 \cdot \text{Weight} + 4.33 \cdot \text{After1975Yes}.$$

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Let's interpret the coefficient 4.33. Consider this:

- Model A and B have the same HP and Weight.
- Model A was manufactured before 1975, whereas B was manufactured after 1975.
- We predict Model B will have a MPG that is 4.33 higher than Model A.

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R has assigned “Yes” to 1 and “No” to 0 in our dummy variable, so the “reference level” is cars manufactured before 1975.

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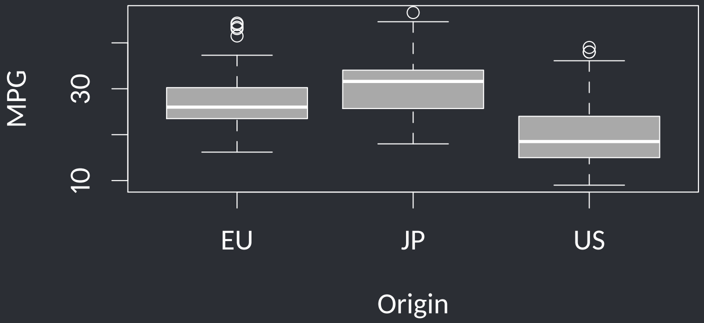
R has assigned “Yes” to 1 and “No” to 0 in our dummy variable, so the “reference level” is cars manufactured before 1975.

If we created a dummy variable `After1975No` that is 1 for cars manufactured *before* 1975, what would the regression look like?

What if there are more than two categories?

The `Origin` variable represents the country of manufacture:

```
boxplot(MPG ~ Origin, data=cars, ylab="MPG",
              xlab="Origin", col='darkgray')
```



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- The category left out is the reference level and all slope coefficients for dummy variables are interpreted as the difference between that category and the reference level.
- If *any* of the dummy variables are significant for a particular categorical variable, we consider the entire categorical variable to be significant!

```r
origin.model <- lm(MPG ~ HP + Weight + After1975 + Origin, data=cars)
summary(origin.model)
```

```


Call:
lm(formula = MPG ~ HP + Weight + After1975 + Origin, data = cars)

Residuals:
   Min     1Q Median     3Q    Max 
-9.705 -2.243 -0.199  1.816 13.789 

Coefficients:
              Estimate Std. Error t value Pr(>|t|)    
(Intercept)  40.181974   0.874271   45.96   <2e-16 ***
HP           -0.027984   0.009865   -2.84   0.0048 ** 
Weight       -0.005160   0.000477  -10.81   <2e-16 ***
After1975Yes  4.334280   0.392849   11.03   <2e-16 ***
OriginJP      1.000586   0.611954    1.64   0.1029    
OriginUS     -1.592573   0.561900   -2.83   0.0048 ** 
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 3.6 on 386 degrees of freedom
Multiple R-squared:  0.787,Adjusted R-squared:  0.784 
F-statistic:  285 on 5 and 386 DF,  p-value: <2e-16
```

A warning about categorical variables with numeric representations

- In the original dataset, the origin was represented as 1 for US, 2 for EU and 3 for JP.

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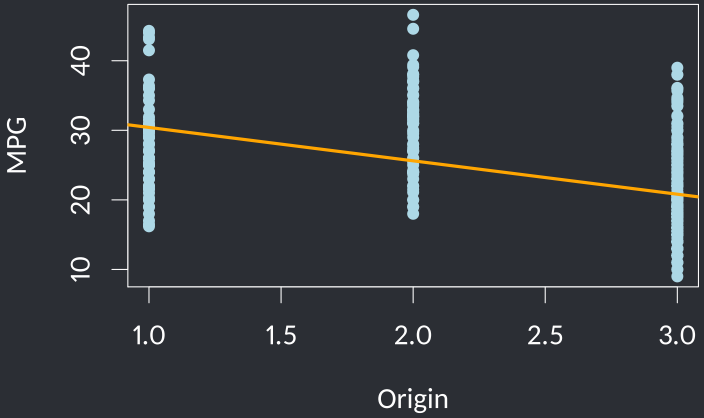
- We would not want to just put these numbers in the regression as numbers, because then regression would treat this as if it were a quantitative variable!
- Even though the representation in the file is numeric, it is still a categorical variable and should be treated as such.

A warning about categorical variables with numeric representations



Statistical significance of a categorical variable

While dealing with categorical variables, we want to look at the significance of the categorical variable as a whole, rather than looking at *p*-values of individual dummy variables.

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We want to test the compound null hypothesis

$$H_0 : \beta_{\mathrm{US}} = \beta_{\mathrm{JP}} = 0.$$

Statistical significance of a categorical variable

To do this, we look at the ANOVA table; the *p*-value on the Origin line (2.4×10^{-5}) is the *p*-value for the compound null hypothesis $H_0 : \beta_{\text{US}} = \beta_{\text{JP}} = 0$.

```
anova(origin.model)

Analysis of Variance Table

Response: MPG
           Df Sum Sq Mean Sq F value  Pr(>F)    
HP          1  14433   14433  1096.0 < 2e-16 ***
Weight      1   2392    2392   181.6 < 2e-16 ***
After1975   1   1623    1623   123.2 < 2e-16 ***
Origin      2    288     144    10.9 2.4e-05 ***
Residuals 386   5083      13                    
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Practical significance of a categorical variable

Since $p < .05$, we can conclude that Origin is a statistically significant predictor of MPG. But is it a *practically* significant predictor?

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To do this, compare R^2 values, or standard error of residuals:

Model	R^2	**Residual standard error**
`Origin` not included in model	0.77	3.72
`Origin` included in model	0.79	3.63

We have to decide if the increased precision is worth the extra complexity in the model.