



THE UNIVERSITY OF TEXAS AT AUSTIN
McCOMBS SCHOOL OF BUSINESS

Multiple regression 1

Lecture 7

STA 371G

Why do some colleges have higher graduation rates than others?

- What factors do you think impact the graduation rate of a college?

Why do some colleges have higher graduation rates than others?

- What factors do you think impact the graduation rate of a college?
- It seems like there is no *one* factor that dominates—it is probably true that to make a good prediction we need to put a lot of variables together, so simple regression is likely not sufficient.

Why do some colleges have higher graduation rates than others?

- What factors do you think impact the graduation rate of a college?
- It seems like there is no *one* factor that dominates—it is probably true that to make a good prediction we need to put a lot of variables together, so simple regression is likely not sufficient.
- **Multiple regression** allows us to build on simple regression by predicting one Y variable using multiple X variables.

The colleges data set

Today's data set is a sample of 1302 colleges with various factors about the colleges, including SAT scores, student/faculty ratios, tuition rates, acceptance rates, etc.

SAT scores and (in-state) tuition were the two best single predictors, with R^2 values of 0.353 and 0.325, respectively. Can we combine these together and get an R^2 that is better than either predictor would produce on its own?

Using multiple predictors to predict graduation rate

The simple regression models were:

$$Y_i = \beta_0 + \beta_1(\text{SAT}) + \epsilon_i$$

and

$$Y_i = \beta_0 + \beta_1(\text{tuition}) + \epsilon_i.$$

The multiple regression model is

$$Y_i = \beta_0 + \beta_1(\text{tuition}) + \beta_2(\text{SAT}) + \epsilon_i.$$

```
> model <- lm(Graduation.rate ~ Average.combined.SAT + In.state.tuition, data=my.  
> summary(model)
```

Call:

```
lm(formula = Graduation.rate ~ Average.combined.SAT + In.state.tuition,  
    data = my.sample)
```

Residuals:

Min	1Q	Median	3Q	Max
-45.53	-9.18	0.05	8.70	43.66

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	-8.324646	4.370828	-1.9	0.057 .
Average.combined.SAT	0.061122	0.004888	12.5	<2e-16 ***
In.state.tuition	0.001249	0.000111	11.2	<2e-16 ***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 13.7 on 709 degrees of freedom

(19 observations deleted due to missingness)

Multiple R-squared: 0.447, Adjusted R-squared: 0.445

F-statistic: 286 on 2 and 709 DF, p-value: <2e-16

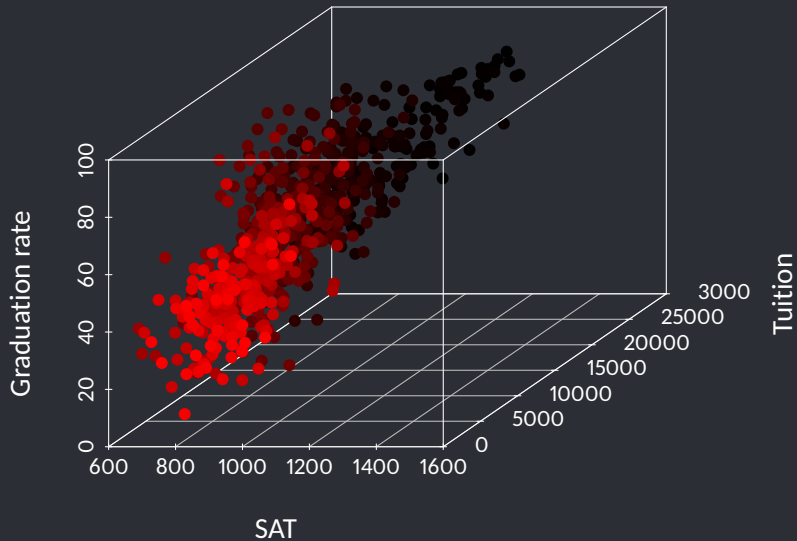
The multiple regression prediction equation is:

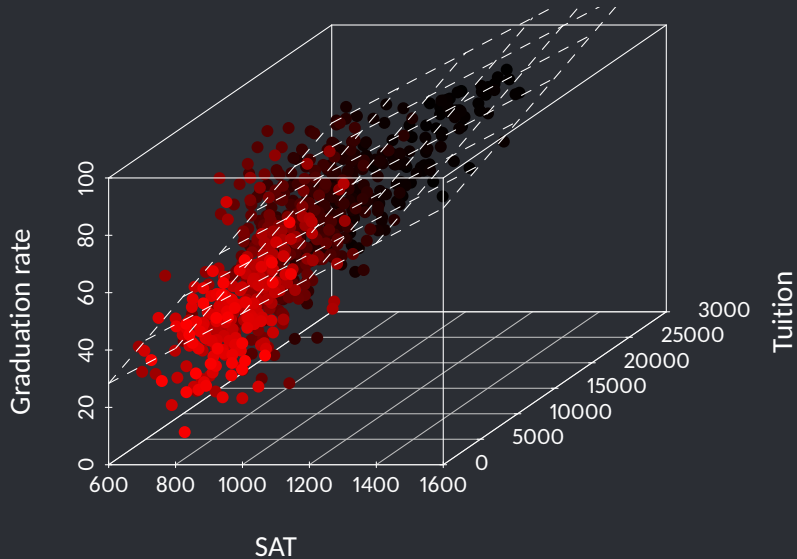
$$\widehat{\text{Graduation rate}} = -8.3246 + 0.0611(\text{SAT}) + 0.0012(\text{tuition})$$

The multiple regression prediction equation is:

$$\widehat{\text{Graduation rate}} = -8.3246 + 0.0611(\text{SAT}) + 0.0012(\text{tuition})$$

We can use this to make predictions like we would for a simple regression!





Interpreting the coefficients: intercept

Let's interpret the intercept coefficient of -8.3246 :

- The predicted graduation rate when the average SAT score is 0 and the in-state tuition is \$0 is -8.3246 .

Interpreting the coefficients: intercept

Let's interpret the intercept coefficient of -8.3246 :

- The predicted graduation rate when the average SAT score is 0 and the in-state tuition is \$0 is -8.3246 .
- This is not a meaningful number on its own in this case, since there will never be a school with those particular predictor values! (The intercept might be interpretable for other models.)

Interpreting the coefficients: SAT

Let's interpret the SAT coefficient of 0.0611:

- Holding tuition constant, each additional SAT score point increases our predicted graduation rate by 0.0611 percentage points.

Interpreting the coefficients: SAT

Let's interpret the SAT coefficient of 0.0611:

- **Holding tuition constant**, each additional SAT score point increases our predicted graduation rate by 0.0611 percentage points.
- **Among colleges that have the same tuition**, an increase in SAT of 1 point would result in a predicted graduation rate that is 0.0611 percentage points higher.

Interpreting the coefficients: SAT

Let's interpret the SAT coefficient of 0.0611:

- Holding tuition constant, each additional SAT score point increases our predicted graduation rate by 0.0611 percentage points.
- Among colleges that have the same tuition, an increase in SAT of 1 point would result in a predicted graduation rate that is 0.0611 percentage points higher.
- If we compared two colleges that have the same tuition but differ in average SAT scores by 1 point, the college with the higher SAT score would be predicted to have a graduation rate that is 0.0611 percentage points higher.

Interpreting the coefficients: tuition

Let's interpret the tuition coefficient of 0.0012:

- **Holding SAT constant**, each additional dollar of in-state tuition increases our predicted graduation rate by 0.0012 percentage points.

Interpreting the coefficients: tuition

Let's interpret the tuition coefficient of 0.0012:

- **Holding SAT constant**, each additional dollar of in-state tuition increases our predicted graduation rate by 0.0012 percentage points.
- **Among colleges that have the same average SAT scores**, an increase in tuition of \$1 would result in a predicted graduation rate that is 0.0012 percentage points higher.

Interpreting the coefficients: tuition

Let's interpret the tuition coefficient of 0.0012:

- **Holding SAT constant**, each additional dollar of in-state tuition increases our predicted graduation rate by 0.0012 percentage points.
- **Among colleges that have the same average SAT scores**, an increase in tuition of \$1 would result in a predicted graduation rate that is 0.0012 percentage points higher.
- **If we compared two colleges that have the same average SAT scores but differ in their tuition by \$1**, the college with the higher tuition would be predicted to have a graduation rate that is 0.0012 percentage points higher.

What's the difference?!

- The difference between “the predicted effect of a 1-point increase in SAT score” and “the predicted effect of a 1-point increase in SAT score, holding tuition constant” really are **two different things**.

What's the difference?!

- The difference between “the predicted effect of a 1-point increase in SAT score” and “the predicted effect of a 1-point increase in SAT score, holding tuition constant” really are **two different things**.
- The relationship between X_1 and Y may change when we **control for** (i.e., add to the model) another predictor X_2 .

R^2

- R^2 has a similar meaning as in simple regression: how much of the variation in the response variable (Y) are explained by the predictor variables (X 's) together?

R^2

- R^2 has a similar meaning as in simple regression: how much of the variation in the response variable (Y) are explained by the predictor variables (X 's) together?
- Another way to think about R^2 is that

$$R^2 = \frac{\text{Var}(\hat{Y})}{\text{Var}(Y)},$$

i.e., it represents how much variance in Y the model predicts.

R^2

- R^2 has a similar meaning as in simple regression: how much of the variation in the response variable (Y) are explained by the predictor variables (X 's) together?
- Another way to think about R^2 is that

$$R^2 = \frac{\text{Var}(\hat{Y})}{\text{Var}(Y)},$$

i.e., it represents how much variance in Y the model predicts.

- R^2 always increases when you add more variables, **even if you add variables that have no real relationship with Y .**

```
> model1 <- lm(Graduation.rate ~ Average.combined.SAT + In.state.tuition,
+             data=my.sample)
> summary(model1)
```

Call:

```
lm(formula = Graduation.rate ~ Average.combined.SAT + In.state.tuition,
    data = my.sample)
```

Residuals:

	Min	1Q	Median	3Q	Max
	-45.52572	-9.18156	0.05085	8.70420	43.66097

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	-8.324645625	4.370827909	-1.90459	0.057238 .
Average.combined.SAT	0.061122082	0.004887825	12.50496	< 2e-16 ***
In.state.tuition	0.001248638	0.000111119	11.23692	< 2e-16 ***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 13.7466 on 709 degrees of freedom
(19 observations deleted due to missingness)

Multiple R-squared: 0.44686, Adjusted R-squared: 0.4453

F-statistic: 286.387 on 2 and 709 DF, p-value: < 2.22e-16

```
> Random.numbers <- rnorm(nrow(my.sample))
> model2 <- lm(Graduation.rate ~ Average.combined.SAT + In.state.tuition
+               + Random.numbers, data=my.sample)
> summary(model2)
```

Call:

```
lm(formula = Graduation.rate ~ Average.combined.SAT + In.state.tuition +
    Random.numbers, data = my.sample)
```

Residuals:

Min	1Q	Median	3Q	Max
-45.59477	-9.13473	0.06836	8.75583	43.74968

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)	
(Intercept)	-8.43359857	4.378188630	-1.92627	0.054471	.
Average.combined.SAT	0.061244215	0.004896088	12.50881	< 2e-16	***
In.state.tuition	0.001248531	0.000111177	11.23012	< 2e-16	***
Random.numbers	0.277098090	0.537299499	0.51572	0.606208	

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 13.7537 on 708 degrees of freedom
(19 observations deleted due to missingness)

Multiple R-squared: 0.447068, Adjusted R-squared: 0.444725

F-statistic: 190.816 on 3 and 708 DF, p-value: < 2.22e-16

```
> Random.numbers <- rnorm(nrow(my.sample))
> model2 <- lm(Graduation.rate ~ Average.combined.SAT + In.state.tuition
+             + Average.math.SAT, data=my.sample)
> summary(model2)
```

Call:

```
lm(formula = Graduation.rate ~ Average.combined.SAT + In.state.tuition +
    Average.math.SAT, data = my.sample)
```

Residuals:

Min	1Q	Median	3Q	Max
-45.27189	-9.06503	0.03009	8.64981	43.89591

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)	
(Intercept)	-8.144252350	4.434188943	-1.83669	0.066675	.
Average.combined.SAT	0.054229967	0.023519872	2.30571	0.021416	*
In.state.tuition	0.001256312	0.000115918	10.83790	< 2e-16	***
Average.math.SAT	0.012667133	0.041953872	0.30193	0.762794	

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 13.7492 on 706 degrees of freedom
(21 observations deleted due to missingness)

Multiple R-squared: 0.447693, Adjusted R-squared: 0.445346

F-statistic: 190.758 on 3 and 706 DF, p-value: < 2.22e-16

Adjusted R^2

- There are many, many possible models (think of how many combinations of predictors there are!) so we need some criterion to determine which model is best.

Adjusted R^2

- There are many, many possible models (think of how many combinations of predictors there are!) so we need some criterion to determine which model is best.
- R^2 is not good because adding even a variable of random numbers increases R^2 .

Adjusted R^2

- There are many, many possible models (think of how many combinations of predictors there are!) so we need some criterion to determine which model is best.
- R^2 is not good because adding even a variable of random numbers increases R^2 .
- **Adjusted R^2** makes an adjustment to R^2 by adding a penalty for each variable added (in this example, adjusted R^2 went down even though R^2 increased).

```
> model1 <- lm(Graduation.rate ~ Average.combined.SAT + In.state.tuition,
+             data=my.sample)
> summary(model1)
```

Call:

```
lm(formula = Graduation.rate ~ Average.combined.SAT + In.state.tuition,
    data = my.sample)
```

Residuals:

	Min	1Q	Median	3Q	Max
	-45.52572	-9.18156	0.05085	8.70420	43.66097

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	-8.324645625	4.370827909	-1.90459	0.057238 .
Average.combined.SAT	0.061122082	0.004887825	12.50496	< 2e-16 ***
In.state.tuition	0.001248638	0.000111119	11.23692	< 2e-16 ***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 13.7466 on 709 degrees of freedom
(19 observations deleted due to missingness)

Multiple R-squared: 0.44686, Adjusted R-squared: 0.4453

F-statistic: 286.387 on 2 and 709 DF, p-value: < 2.22e-16


```
> Random.numbers <- rnorm(nrow(my.sample))
> model2 <- lm(Graduation.rate ~ Average.combined.SAT + In.state.tuition
+             + Random.numbers, data=my.sample)
> summary(model2)
```

Call:

```
lm(formula = Graduation.rate ~ Average.combined.SAT + In.state.tuition +
    Random.numbers, data = my.sample)
```

Residuals:

Min	1Q	Median	3Q	Max
-45.59477	-9.13473	0.06836	8.75583	43.74968

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)	
(Intercept)	-8.43359857	4.378188630	-1.92627	0.054471	.
Average.combined.SAT	0.061244215	0.004896088	12.50881	< 2e-16	***
In.state.tuition	0.001248531	0.000111177	11.23012	< 2e-16	***
Random.numbers	0.277098090	0.537299499	0.51572	0.606208	

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 13.7537 on 708 degrees of freedom
(19 observations deleted due to missingness)

Multiple R-squared: 0.447068, Adjusted R-squared: 0.444725

F-statistic: 190.816 on 3 and 708 DF, p-value: < 2.22e-16

Which model is the best?

- In general, we want to select the model that is the most **parsimonious**, that is, the model that has the best combination of being simple with a high R^2 .

Which model is the best?

- In general, we want to select the model that is the most **parsimonious**, that is, the model that has the best combination of being simple with a high R^2 .
- This is easier said than done—using Adjusted R^2 is not enough. We'll come back to this next week!