Logistic Regression

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- $\ln\left(\frac{p}{1-p}\right) = \beta_0 + \beta_1 X$, where $p = P\{Y\ occurs\} = P\{Y = 1\}$
- Now we have restricted the odds to the interval $[0, +\infty)$, and the probability of *Y* occurring to values in $[0,1]$.

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- $p = \frac{odds}{odds+1} = \frac{e^{\beta_0+\beta_1 X}}{1+e^{\beta_0+\beta_1 X}}$ (probability)

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- The odds get multiplied by $e^{\beta_1 \delta}$.
- The percentage <u>change</u> is $\left(1 - e^{\beta_1 \delta}\right) \cdot 100\%$.