THE UNIVERSITY OF TEXAS AT AUSTIN
McCOMBS SCHOOL OF BUSINESS

Multiple regression 2

Lecture 8

STA 371G

Attend STA 371G PLUS Study Groups

Sundays	Mondays	Tuesdays	Wednesdays	Thursdays
5-6:30 PM BEN 1.122	5:30-7 PM BEN 1.124	7-8:30 PM BEN 1.122	5:30-7 PM BEN 1.126	5:30-7 PM BEN 1.122

Benefits

- Keep up with your work
- Practice your understanding of the content in a low-stakes environment
- Explain your thinking on how to approach problems and hear how others understand the material
- Ask and answer questions in small, inclusive teams

The colleges data set

Today's data set is a sample of 1302 colleges with various factors about the colleges, including SAT scores, student/faculty ratios, tuition rates, acceptance rates, etc.

Multiple regression assumptions

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1. The errors are independent.
2. Y is a linear function of the X's (except for the errors).
3. The errors are normally distributed.
4. The variance of Y is the same for any value of X ("homoscedasticity").

Assumption 1: Independence of errors

Independence means that knowing the error (over-/under-prediction by the regression line) for one case doesn't tell you anything about the error for another case.

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Since each college is completely separate, there is no reason to think the errors are not independent.

Multiple regression assumptions

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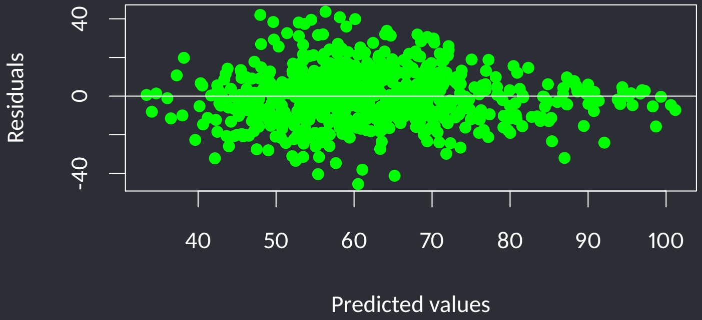
Assumption 2: Linearity

Look at the residual plot:

```
> plot(predict(model), residuals(model), col="green",
+   xlab="Predicted values", ylab="Residuals", pch=16)
> abline(h=0)
```



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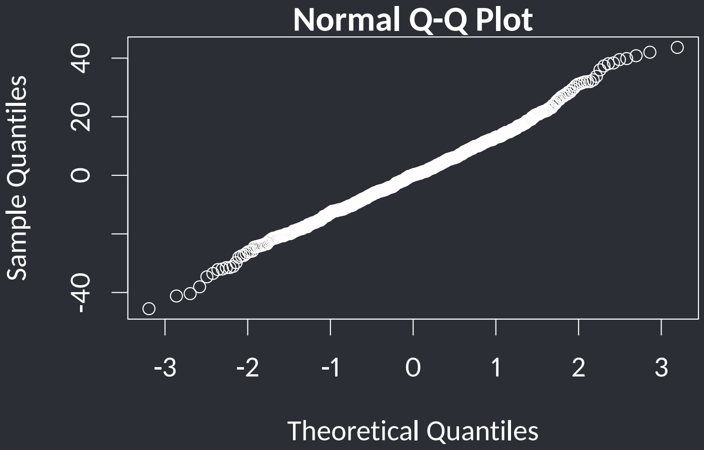
Assumption 3: Normality of residuals

```
> qqnorm(residuals(model))
```



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Assumption 4: Homoscedasticity

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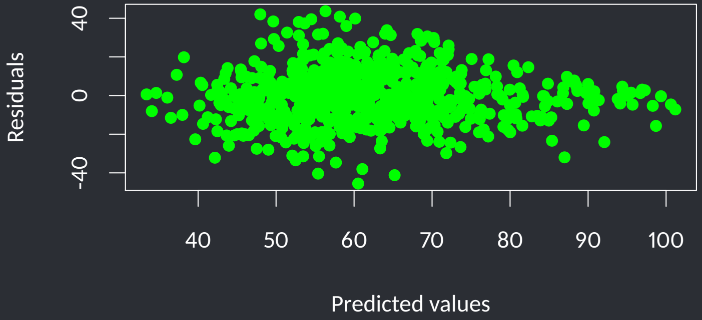

Multiple regression assumptions

1. The errors are independent. ✓
2. *Y* is a linear function of the *X*'s (except for the errors). ✓
3. The errors are normally distributed. ✓
4. The variance of *Y* is the same for any value of *X* ("homoscedasticity"). ¯_(ツ)_/¯

Since one of the assumptions is not completely satisfied, we'll proceed with caution—i.e., take the p-values and confidence intervals with a grain of salt. (We could try and fix the problem with a transformation, or by building different models for different subsets of the data, but let's just live with it for now.)

The overall null hypothesis for a regression model

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- $\beta_1 = \beta_2 = \cdots = \beta_k = 0$ (i.e., all coefficients are 0 except the intercept)
- The model has no predictive power
- Predictions from this model are no better than predicting $\overline{Y}$ for every case

We should always test the overall null hypothesis for a model first. If we can't reject the overall null hypothesis, there's no reason to interpret the model further.

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In this model, the overall p-value is very small, so we reject the overall null hypothesis and conclude that yes, we have statistical significance and that this model does have some predictive power.

Statistical vs practical significance

- As in simple regression, once we determine that there is statistical significance, we want to then assess whether there is also practical significance.
- For the test of the overall null hypothesis, we look to the value of R^2 in the sample to assess practical significance.

Testing individual coefficients

- Next, we want to test $H_0 : \beta_i = 0$ for each of the predictors X_i, for each $i = 1, 2, \ldots, k$.

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- The test statistic for testing the null hypothesis $\beta_i = S$ follows a t-distribution with $n - k - 1$ degrees of freedom:

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- If we reject this null hypothesis for a coefficient, we say that X_i is a (statistically) significant predictor of Y in the model.

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If a predictor is not statistically significant, we should:

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1. Interpret it as if it were zero.
2. Remove it from the model (unless there are other reasons to keep it), as it does not contribute to predicting Y above and beyond the other predictors.

Residual standard error

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- Since one of the assumptions of regression is that the residuals are approximately normal, we can conclude that approximately 95% of the residuals will be less than $\pm 2 s_e$.

Confidence intervals for coefficents

Confidence intervals for the individual coefficients are found the same way as in simple regression, and interpreted the same way:

```
> confint(model)

                           2.5 %     97.5 %
(Intercept)         -16.905960009 0.25666876
Average.combined.SAT  0.051525739 0.07071843
In.state.tuition      0.001030476 0.00146680
```

Confidence intervals for predictions

We can also put confidence intervals on our predictions for Y.

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A 95% CI for the graduation rate at the University of California, Merced, which is not in the data set and has an average SAT score of 1100 and in-state tuition of $11,502:

```
> predict(model, list(Average.combined.SAT=1100,
+                     In.state.tuition=11502),
+                interval="prediction")

       fit      lwr   upr
1 73.27148 46.24296 100.3
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```

Our best guess for UC Merced is 73.27%, with a 95% CI of (46.24%, 100.3%). (It turns out that the actual graduation rate at UC Merced is 64%.)

Confidence intervals for predictions

A 95% CI for average graduation rate among all colleges with an average SAT score of 1100 and in-state tuition of $11,502:

```
> predict(model, list(Average.combined.SAT=1100,
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1 73.2715 71.8091 74.7338
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1 73.2715 71.8091 74.7338
```

As with simple regression, our point estimate is the same, but the confidence interval is much narrower, because it's easier to estimate a mean than a prediction for a single new case.