THE UNIVERSITY OF TEXAS AT AUSTIN
McCOMBS SCHOOL OF BUSINESS

Simulation 2

Lecture 23

STA 371G

Announcements

- HW 8 (last one!) is posted and is due Tuesday, May 7.
- Peer evaluations for Part 3 of the project are due this Friday.
- Part 4 of the project is due this Friday.

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- But how do you know that the money you have saved will last you from retirement age until death?
- Particularly with expected lifespans growing, how do you know that you won’t outlive your money?

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- Each year, at the beginning of the year, I put $10,000 into a retirement account.
- The entire account will be invested in a single asset that has normally distributed annual returns, with mean 12% and SD 18%.

Will you have enough money for retirement?

- Let's start by building a simple model of a portfolio.
- Each year, at the beginning of the year, I put $10,000 into a retirement account.
- The entire account will be invested in a single asset that has normally distributed annual returns, with mean 12% and SD 18%.
- What will the portfolio look like in 30 years?

Let's start by writing R code for the first year:

```r
# Start with nothing
account.value <- 0
# Add a $10,000 investment
account.value <- account.value + 10000
# Simulate this year's return
this.years.return <- rnorm(1, mean=.12, sd=.18)
# Apply the return
account.value <- account.value * (1 + this.years.return)
# Examine the account value
account.value

[1] 10072.38
```

Repeating a process within a run

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Repeating a process within a run

- Within a single run of our simulation, we need to simulate 30 years of saving.
- To do this, we can use a `for` loop, which lets us run a block of code repeatedly; for example, we can print out the first 5 square numbers:

```
for (n in 1:5) {
  print(n^2)
}
```

```
[1] 1
[1] 4
[1] 9
[1] 16
[1] 25
```

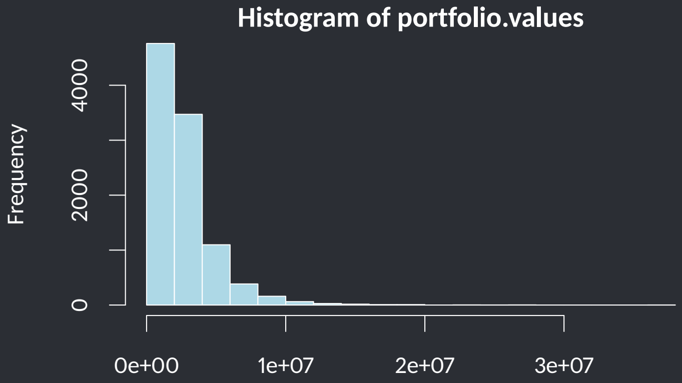
The value of n is set to a different value on each iteration of the code block `{ ... }`.

Returning to our portfolio simulation, let's repeat the process for 30 years:

```
account.value <- 0
for (year in 1:30) {
  account.value <- account.value + 10000
  this.years.return <- rnorm(1, mean=.12, sd=.18)
  account.value <- account.value * (1 + this.years.return)
}
account.value

[1] 3330261
```

```
portfolio.values <- replicate(10000, {
  account.value <- 0
  for (year in 1:30) {
    account.value <- account.value + 10000
    this.years.return <- rnorm(1, mean=.12, sd=.18)
    account.value <- account.value * (1 + this.years.return)
  }
  return(account.value)
})
hist(portfolio.values, col="lightblue")
```



This looks pretty good at first, but there is a wide range of outcomes—let's look at the percentiles:

```
quantile(portfolio.values,
  probs=c(0.05, 0.25, 0.5, 0.75, 0.95))

       5%       25%       50%       75%       95%
 674417.4 1300215.9 2087403.8 3345929.9 6745048.2
```

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That's a 10x spread between the 95th percentile outcome and the 5th percentile outcome!

The expected (average) value of my portfolio is:

```
mean(portfolio.values)

[1] 2683627
```

What if I only contribute $5,000 per year?

```
portfolio.values <- replicate(10000, {
  account.value <- 0
  for (year in 1:30) {
    account.value <- account.value + 5000
    this.years.return <- rnorm(1, mean=.12, sd=.18)
    account.value <- account.value * (1 + this.years.return)
  }
  return(account.value)
})
```

Let's look at the percentiles, under this new scenario where I only contribute $5,000 per year:

```
quantile(portfolio.values,
  probs=c(0.05, 0.25, 0.5, 0.75, 0.95))

       5%       25%       50%       75%       95%
 331238.9  661227.0 1047610.8 1697519.4 3368771.5
```

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[1] 1344782
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```
mean(portfolio.values)

[1] 1344782
```

Not contributing that extra $5,000 per year is expected to cost me $1.3M in the value of my retirement account!

How likely is it we'll retire a millionaire?

```r
results <- replicate(10000, {
  account.value <- 0
  for (year in 1:30) {
    account.value <- account.value + 10000
    this.years.return <- rnorm(1, mean=.12, sd=.18)
    account.value <- account.value * (1 + this.years.return)
  }
  return(account.value >= 1000000)
})
sum(results) / 10000
```

```
[1] 0.8591
```

How likely is it that I'll outlive my money?

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- Let's consider a random 37-year old male, nonsmoker, moderately active, always wear seat belt, statistics fanatic, etc.—his life expectancy is

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How likely is it that I'll outlive my money?

- Suppose that after I turn 67 (i.e., year 30), I start withdrawing $100,000 each year to live on, and I stop making annual contributions.
- Let's consider a random 37-year old male, nonsmoker, moderately active, always wear seat belt, statistics fanatic, etc.—his life expectancy is 88.
- So let's simulate this process, and see how often my money outlives me.

```r
results <- replicate(10000, {
  account.value <- 0
  for (age in 37:66) {
    account.value <- account.value + 10000
    this.years.return <- rnorm(1, mean=.12, sd=.18)
    account.value <- account.value * (1 + this.years.return)
  }
  for (age in 67:88) {
    account.value <- account.value - 100000
    if (account.value < 0) {
      account.value <- 0
    } else {
      this.years.return <- rnorm(1, mean=.12, sd=.18)
      account.value <- account.value * (1 + this.years.return)
    }
  }
  return(account.value > 0)
})
sum(results) / 10000

[1] 0.843
```

```r
results <- replicate(10000, {
  account.value <- 0
  for (age in 37:66) {
    account.value <- account.value + 10000
    this.years.return <- rnorm(1, mean=.12, sd=.18)
    account.value <- account.value * (1 + this.years.return)
  }
  for (age in 67:88) {
    account.value <- account.value - 100000
    if (account.value < 0) {
      account.value <- 0
    } else {
      this.years.return <- rnorm(1, mean=.12, sd=.18)
      account.value <- account.value * (1 + this.years.return)
    }
  }
  return(account.value > 0)
})
sum(results) / 10000

[1] 0.843
```


There is a 84.3% chance that I don’t run out of money during retirement.