THE UNIVERSITY OF TEXAS AT AUSTIN
McCOMBS SCHOOL OF BUSINESS

Residuals and autocorrelation 1

Lecture 5

STA 371G

Announcements

- Homework 1 due Thursday at 11:59 PM, in MyStatLab
- Submit an R script in Canvas that contains the commands you used for each problem

1. Transformations

2. Extrapolation

Residuals

Recall that the residual for the ith case in the data is $Y_i - \hat{Y}_i$.

- When the residual is *positive*, the actual Y-value is *higher* than our predicted Y-value.
- When the residual is *negative*, the actual Y-value is *lower* than our predicted Y-value.

Looking at residuals can tell us a lot about how well a model is working, and give us ideas for how to improve it.

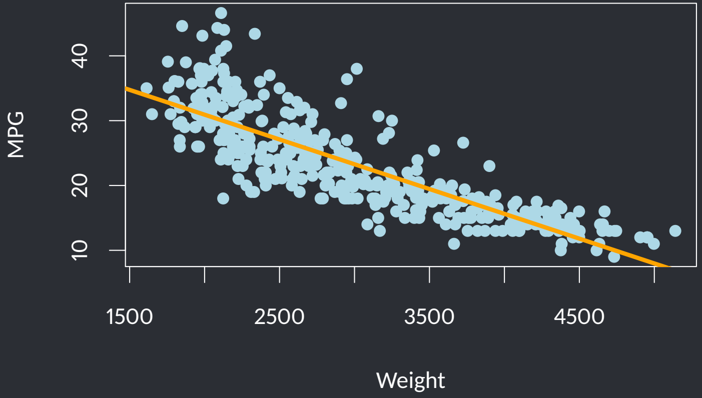
Mileage efficiency data set

The data set `cars` contains specs for 392 different cars. We'll focus on two variables:

- **MPG** is fuel efficiency, measured as miles per gallon
- **Weight** is the weight of the car, in pounds

What problems do you see here?

```
> plot(MPG ~ Weight, data=cars, pch=16, col="lightblue")
> model <- lm(MPG ~ Weight, data=cars)
> abline(model, col="orange", lwd=4)
```



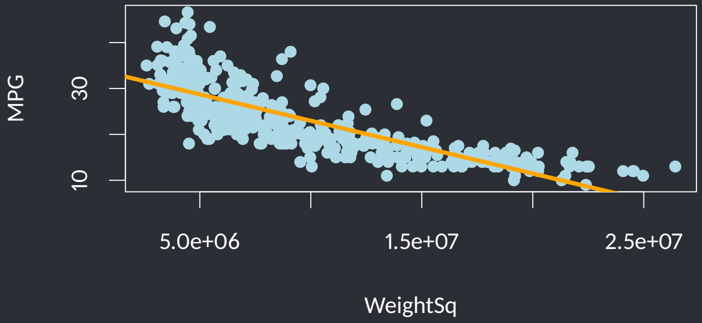
Using transformations to fix problems

- Sometimes, a violation of regression assumptions can be fixed by transforming one or the other of the variables (or both).
- When we transform a variable, we have to also transform our interpretation of the equation.

A bad example

What if we predict MPG from squared weight?

```
> cars$WeightSq <- cars$Weight^2
> plot(MPG ~ WeightSq, data=cars, pch=16, col="lightblue")
> sq.model <- lm(MPG ~ WeightSq, data=cars)
> abline(sq.model, col="orange", lwd=4)
```



The log transformation

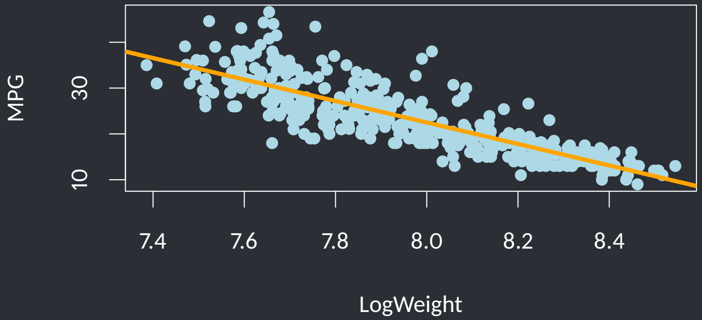
The log transformation is frequently useful in regression, because many nonlinear relationships are naturally exponential.

- $\log_b x = y$ when $b^y = x$.
- For example, $\log_{10} 1000 = 3$, $\log_{10} 100 = 2$, and $\log_{10} 10 = 1$.
- The natural log is $\log_e$, where $e \approx 2.72$ — when we say "log" we will usually mean "natural log."


LOG!

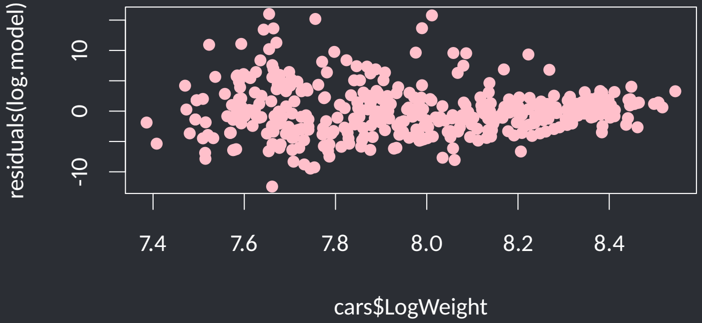
Applying a log transformation

```
> cars$LogWeight <- log(cars$Weight)
> plot(MPG ~ LogWeight, data=cars, pch=16, col="lightblue")
> log.model <- lm(MPG ~ LogWeight, data=cars)
> abline(log.model, col="orange", lwd=4)
```



Checking assumptions of our new model

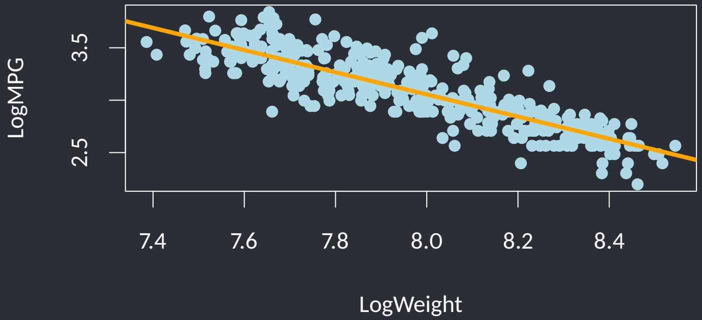
```
> plot(cars$LogWeight, residuals(log.model), pch=16, col="pink")
```



Linearity looks good, but homoscedasticity is still not satisfied!

Applying a second log transformation

```
> cars$LogMPG <- log(cars$MPG)
> plot(LogMPG ~ LogWeight, data=cars, pch=16, col="lightblue")
> log.log.model <- lm(LogMPG ~ LogWeight, data=cars)
> abline(log.log.model, col="orange", lwd=4)
```



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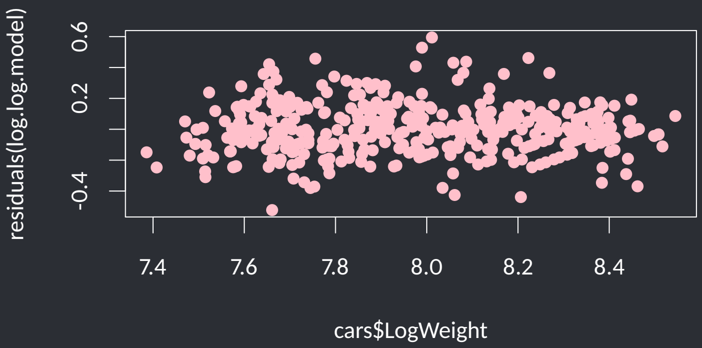
```
> plot(cars$LogWeight, residuals(log.log.model), pch=16, col="pink")
```



Much better—transforming MPG to log(MPG) gives us both linearity and homoscedasticity!

Another example

Last class, we looked at predicting the *returns* of AMZN based on the *returns* of W5000. What if we just predicted the weekly closing *price* of AMZN based on the price of W5000?

```
Call:
lm(formula = AMZN ~ W5000, data = prices)

Residuals:
     Min       1Q   Median       3Q      Max
-237.747 -100.865    2.552   72.583  260.783

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) -510.293     48.869  -10.44   <2e-16 ***
W5000         57.333      2.996   19.14   <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 120.4 on 259 degrees of freedom
Multiple R-squared:  0.5858,Adjusted R-squared:  0.5842
F-statistic: 366.2 on 1 and 259 DF,  p-value: < 2.2e-16
```

Making a transformation to address the issues

The natural transformation of closing prices $\longrightarrow$ returns address the issues with this model, as we saw last time—all assumptions are satisfied when using % returns instead of absolute prices!

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Key takeaway: examine diagnostic plots of residuals to ensure regression assumptions are met; a high R^2 doesn't necessarily mean that model is appropriate!

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- Thinking about whether you want to stretch or squeeze one of the axes, and apply a transformation accordingly (e.g., $\sqrt{x}$ or $\log x$ to squeeze; x^2 or e^x to stretch).

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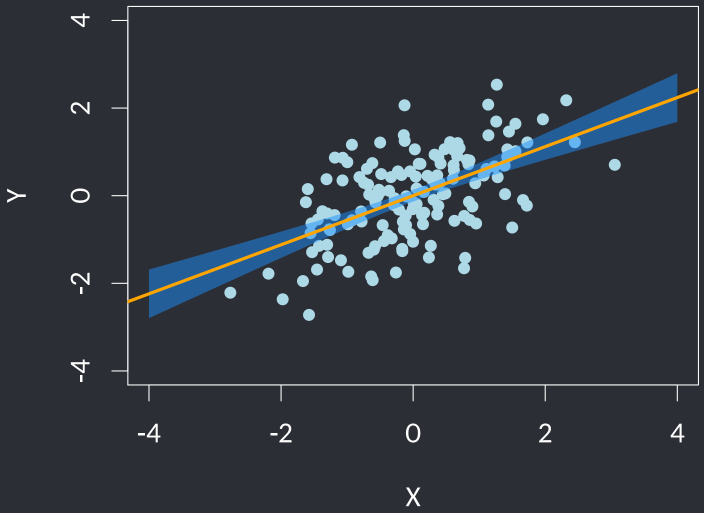
- You might need to transform both X and Y; if so, start by transforming Y to address the heteroscedasticity, and then transform X to address nonlinearity if necessary.
- It's OK to do a little trial and error!

1. Transformations
2. Extrapolation

Going beyond the data

- It's natural to want to predict Y beyond the X values that we have in the data set (otherwise, why build a model in the first place?).
- But things get dicey when trying to predict Y for an X value that is far from the other X values in the data set: how can we be so sure that the observed trend continues?

The shaded area shows the 95% confidence interval for predicting the mean. As X moves away from $\overline{X}$, the CI becomes wider since we know our estimates are less precise.



There's nothing wrong with extrapolating a little bit beyond the data, but when you move beyond the data even the confidence intervals may *underestimate* the degree of uncertainty.