4

Mathematical models for decision making

In the previous chapters we have emphasized the critical role played by mathematical models in the development of business intelligence environments and decision support systems aimed at providing *active* support for knowledge workers. In this chapter we will focus on the main characteristics shared by different mathematical models embedded into business intelligence systems. We will also develop a taxonomy of the most common classes of models, identifying for each of them the prevailing application domain.

4.1 Structure of mathematical models

Mathematical models have been developed and used in many application domains, ranging from physics to architecture, from engineering to economics. The models adopted in the various contexts differ substantially in terms of their mathematical structure. However, it is possible to identify a few fundamental features shared by most models.

Generally speaking, a model is a selective abstraction of a real system. In other words, a model is designed to analyze and understand from an abstract point of view the operating behavior of a real system, regarding which it only includes those elements deemed relevant for the purpose of the investigation carried out. In this respect it is worth quoting the words of Einstein on the development of a model: 'Everything should be made as simple as possible, but not simpler.' Figure 4.1 expresses in graphical terms the definition of a model.

Business Intelligence: *Data Mining and Optimization for Decision Making* *Carlo Vercellis*
 ISBN: 978-0-470-51138-1

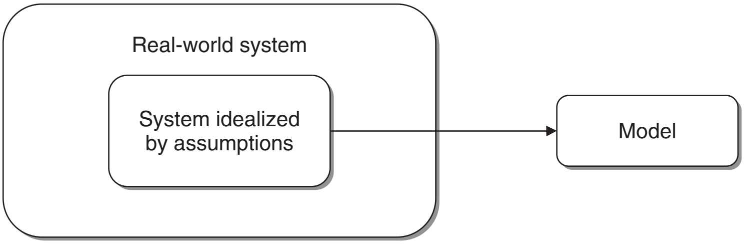

Figure 4.1 A model is a selective abstraction of reality

Scientific and technological development has turned to mathematical models of various types for the abstract representation of real systems. As an example, consider the *thought experiment* (*Gendankenexperiment*) popularized in physics at the beginning of the twentieth century, which involved building a mental model of a given phenomenon and verifying its validity by imagining the consequences caused by hypothetical modifications in the model itself. The analogy is well apparent between this conceptual paradigm and *what-if analyses* that can be easily performed using a simple spreadsheet to find an answer to questions such as: given a model for calculating the budget of a company, how are cash flows affected by a change in the payment terms, such as 90 days vs. 60 days, of invoices issued in favor of the main customers?

According to their characteristics, models can be divided into *iconic*, *analogical* and *symbolic*.

Iconic. An *iconic* model is a material representation of a real system, whose behavior is imitated for the purpose of the analysis. A miniaturized model of a new city neighborhood is an example of iconic model.

Analogical. An *analogical* model is also a material representation, although it imitates the real behavior by analogy rather than by replication. A wind tunnel built to investigate the aerodynamic properties of a motor vehicle is an example of an analogical model intended to represent the actual progression of a vehicle on the road.

Symbolic. A *symbolic* model, such as a mathematical model, is an abstract representation of a real system. It is intended to describe the behavior of the system through a series of symbolic variables, numerical parameters and mathematical relationships.

Business intelligence systems, and consequently the models presented in this book, are exclusively based on symbolic models.

A further relevant distinction concerns the probabilistic nature of models, which can be either *stochastic* or *deterministic*.

Stochastic. In a *stochastic* model some input information represents random events and is therefore characterized by a probability distribution, which in turn can be assigned or unknown. Predictive models, which will be thoroughly described in the following chapters, as well as waiting line models, briefly mentioned below in this chapter, are examples of stochastic models.

Deterministic. A model is called *deterministic* when all input data are supposed to be known a priori and with certainty. Since this assumption is rarely fulfilled in real systems, one resorts to deterministic models when the problem at hand is sufficiently complex and any stochastic elements are of limited relevance. Notice, however, that even for deterministic models the hypothesis of knowing the data with certainty may be relaxed. Sensitivity and scenario analyses, as well as what-if analysis, allow one to assess the robustness of optimal decisions to variations in the input parameters.

A further distinction concerns the temporal dimension in a mathematical model, which can be either *static* or *dynamic*.

Static. *Static* models consider a given system and the related decision-making process within one single temporal stage. For instance, the optimization model described in Section 14.2.7 determines an optimal plan for the distribution of goods in a specific time frame.

Dynamic. *Dynamic* models consider a given system through several temporal stages, corresponding to a sequence of decisions. In many instances the temporal dimension is subdivided into discrete intervals of a previously fixed span: minutes, hours, days, weeks, months and years are examples of discrete subdivisions of the time axis. *Discrete-time* dynamic models, which largely prevail in business intelligence applications, observe the status of a system only at the beginning or at the end of discrete intervals. *Continuous-time* dynamic models consider a continuous sequence of periods on the time axis.

4.2 Development of a model

It is possible to break down the development of a mathematical model for decision making into four primary phases, shown in Figure 4.2. The figure also includes a *feedback* mechanism which takes into account the possibility of changes and revisions of the model.

Problem identification

First of all, the problem at hand must be correctly identified. The observed critical symptoms must be analyzed and interpreted in order to formulate hypotheses

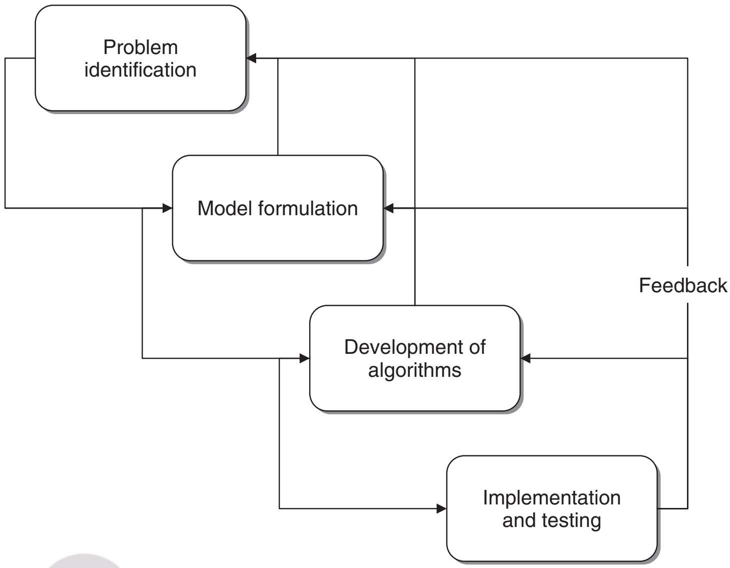

Figure 4.2 Phases in the development of mathematical models for decision making

for investigation. For example, too high a stock level, corresponding to an excessive stock turnover rate, may possibly represent a symptom for a company manufacturing consumable goods. It is therefore necessary to understand what caused the problem, based on the opinion of the production managers. In this case, an ineffective production plan may be the cause of the stock accumulation.

Model formulation

Once the problem to be analyzed has been properly identified, effort should be directed toward defining an appropriate mathematical model to represent the system. A number of factors affect and influence the choice of model, such as the *time horizon*, the *decision variables*, the *evaluation criteria*, the *numerical parameters* and the *mathematical relationships*.

Time horizon. Usually a model includes a temporal dimension. For example, to formulate a tactical production plan over the medium term it is necessary to specify the production rate for each week in a year, whereas to derive an operational schedule it is required to assign the tasks to each production line for each day of the week. As we can see, the time span considered in a model, as well as the length of the base intervals, may vary depending on the specific problem considered.

Evaluation criteria. Appropriate measurable performance indicators should be defined in order to establish a criterion for the evaluation and comparison of the alternative decisions. These indicators may assume various forms in each different application, and may include the following factors:

- monetary costs and payoffs;
- effectiveness and level of service;
- quality of products and services;
- flexibility of the operating conditions;
- reliability in achieving the objectives.

Decision variables. Symbolic variables representing alternative decisions should then be defined. For example, if a problem consists of the formulation of a tactical production plan over the medium term, decision variables should express production volumes for each product, for each process and for each period of the planning horizon.

Numerical parameters. It is also necessary to accurately identify and estimate all numerical parameters required by the model. In the production planning example, the available capacity should be known in advance for each process, as well as the capacity absorption coefficients for each combination of products and processes.

Mathematical relationships. The final step in the formulation of a model is the identification of mathematical relationships among the decision variables, the numerical parameters and the performance indicators defined during the previous phases. Sometimes these relationships may be exclusively deterministic, while in other instances it is necessary to introduce probabilistic relationships. In this phase, the trade-off between the accuracy of the representation achieved through the model and its solution complexity should be carefully considered. It may turn out more helpful at a practical level to adopt a model that sacrifices some marginal aspects of reality in the representation of the system but allows an efficient solution and greater flexibility in view of possible future developments.

Development of algorithms

Once a mathematical model has been defined, one will naturally wish to proceed with its solution to assess decisions and to select the best alternative. In other words, a solution algorithm should be identified and a software tool that incorporates the solution method should be developed or acquired. An analyst in charge of model formulation should possess a thorough knowledge of current solution methods and their characteristics.

Implementation and test

When a model is fully developed, then it is finally implemented, tested and utilized in the application domain. It is also necessary that the correctness of the data and the numerical parameters entered in the model be preliminarily assessed. These data usually come from a data warehouse or a data mart previously set up. Once the first numerical results have been obtained using the solution procedure devised, the model must be validated by submitting its conclusions to the opinion of decision makers and other experts in the application domain. A number of factors should be taken into account at this stage:

- the plausibility and likelihood of the conclusions achieved;
- the consistency of the results at extreme values of the numerical parameters;
- the stability of the results when minor changes in the input parameters are introduced.

4.3 Classes of models

There are several classes of mathematical models for decision making, which in turn can be solved by a number of alternative solution techniques. Each model class is better suited to represent certain types of decision-making processes. In this section we will cover the main categories of mathematical models for decision making, including:

- predictive models;
- pattern recognition and learning models;
- optimization models;
- project management models;
- risk analysis models;
- waiting line models.

Predictive models

A significant proportion of the models used in business intelligence systems, such as optimization models, require input data concerned with future events.

For example, the results of random events determine the future demand for a product or service, the development of new scenarios of technological innovation and the level of prices and costs. As a consequence, *predictive* models play a primary role in business intelligence systems, since they are logically placed upstream with respect to other mathematical models and, more generally, to the whole decision-making process.

Predictions allow input information to be fed into different decision-making processes, arising in strategy, research and development, administration and control, marketing, production and logistics. Basically, all departmental functions of an enterprise make some use of predictive information to develop decision making, even though they pursue different objectives.

Predictive models can be subdivided into two main categories. The purpose of *explanatory* models is to functionally identify a possible relationship between a dependent variable and a set of independent attributes. *Regression* models, described in Chapter 8, belong to this category as well as *classification* models, described in Chapter 10, even though it would be more appropriate to place these latter together with pattern recognition and learning models, covered in the following paragraph. The purpose of *time series* models, described in Chapter 9, is to functionally identify any temporal pattern expressed by a time series of observations referred to the same numerical variable.

Pattern recognition and machine learning models

In a broad sense, the purpose of pattern recognition and learning theory is to understand the mechanisms that regulate the development of intelligence, understood as the ability to extract knowledge from past experience in order to apply it in the future. Mathematical models for learning can be used to develop efficient algorithms that can perform such task. This has led to intelligent machines capable of learning from past observations and deriving new rules for the future, just like the human mind is able to do with great effectiveness due to the sophisticated mechanisms developed and fine-tuned in the course of evolution.

Besides an intrinsic theoretical interest, mathematical methods for learning are applied in several domains, such as recognition of images, sounds and texts; biogenetic and medical diagnosis; relational marketing, for segmenting and profiling customers; manufacturing process control; identification of anomalies and fraud detection.

Mathematical models for learning have two primary objectives. The purpose of *interpretation* models is to identify regular patterns in the data and to express them through easily understandable rules and criteria. *Prediction* models help to forecast the value that a given random variable will assume in the future, based on the values of some variables associated with the entities of a database, as for explanatory models. Based on the existence or not of a *target* attribute, the learning process may be *supervised* or *unsupervised*. In the first case, the target attribute expresses for each record either the membership class or a measurable quantity. *Classification* and *regression* models belong to this category. In the second case, no target attribute exists and consequently the purpose of the analysis is to identify regularities, similarities and differences in the data. It is also possible to derive *association rules*, as described in Chapter 11. Alternatively one can determine groups of records, called *clusters*, characterized by similarity within each cluster and by dissimilarity among the elements of distinct clusters, as described in Chapter 12.

Optimization models

Many decision-making processes faced by companies or complex organizations can be cast according to the following framework: given the problem at hand, the decision maker defines a set of *feasible* decisions and establishes a criterion for the evaluation and comparison of alternative choices, such as monetary costs or payoffs. At this point, the decision maker must identify the *optimal* decision according to the evaluation criterion defined, that is, the choice corresponding to the minimum cost or to the maximum payoff. The conceptual paradigm outlined determines a wide and popular class of mathematical models for decision making, represented by *optimization* models.

In general, optimization models arise naturally in decision-making processes where a set of limited resources must be allocated in the most effective way to different entities. These resources may be personnel, production processes, raw materials, components or financial factors. Among the main application domains requiring an optimal allocation of the resources we find:

- logistics and production planning;
- financial planning;
- work shift planning;
- marketing campaign planning;
- price determination.

Mathematical optimization models represent a fairly substantial class of optimization problems that are derived when the objective of the decision-making process is a function of the decision variables, and the criteria describing feasible decisions can be expressed by a set of mathematical equalities and inequalities in the decision variables. Mathematical optimization models offer ample opportunities for application, as we will see in the following chapters, due to the high flexibility of their formulations and the availability of efficient solution methods.

In light of the structure of the objective function and of the constraints, optimization models may assume different forms:

- linear optimization;
- integer optimization;
- convex optimization;
- network optimization;
- multiple-objective optimization.

Project management models

A *project* is a complex set of interrelated activities carried out in pursuit of a specific goal, which may represent an industrial plant, a building, an information system, a new product or a new organizational structure, depending on the different application domains. The execution of the project requires a planning and control process for the interdependent activities as well as the human, technical and financial resources necessary to achieve the final goal. *Project management* methods are based on the contributions of various disciplines, such as business organization, behavioral psychology and operations research.

Mathematical models for decision making play an important role in project management methods. In particular, network models are used to represent the component activities of a project and the precedence relationships among them. These models allow the overall project execution time to be determined, assuming a deterministic knowledge of the duration of each activity.

Stochastic models, on the other hand, usually referred to as *project evaluation and review techniques* (*PERT*), are used to derive the execution times when stochastic assumptions are made regarding the duration of the activities, represented by random variables.

Finally, different classes of optimization models allow the analysis to be extended to the complex problem of optimally allocating a set of limited resources among the project activities in view of execution costs and times.

Risk analysis models

Some decision problems can be described according to the following conceptual paradigm: the decision maker is required to choose among a number of available alternatives, having uncertain information regarding the effects that these options may have in the future.

For example, assume that senior management wishes to evaluate different alternatives in order to increase the company's production capacity. On the one hand, the company may build a new plant providing a high operating efficiency and requiring a high investment cost. On the other hand, it may expand an existing plant with a lower investment but with higher operating costs. Finally, it may subcontract to external third parties part of its production: in this case, the investment cost is minimized but the operating costs are the highest among the available alternatives.

Clearly, in this situation the effects of the different options are strongly influenced by future stochastic events. In particular, a high level of future demand makes the construction of a new plant advantageous, while low demand levels tend to favor the subcontracting option. At an intermediate level of demand the expansion of an existing plant may be convenient.

However, the decision maker is forced to make a choice *before* knowing with absolute certainty the level of future demand. At best, she may obtain some stochastic information regarding the likelihood of occurrence of future events by carrying out some market research.

In situations of this type, the methodological support offered by risk analysis models, primarily based on Bayesian and utility theories, may prove quite helpful. Indeed, this class of models is used successfully in several application domains, such as technology investment, design of new products, research and development, and financial and real estate investment.

Waiting line models

The purpose of *waiting line* theory is to investigate congestion phenomena occurring when the demand for and provision of a service are stochastic in nature. If the arrival times of the customers and the duration of the service are not known beforehand in a deterministic way, conflicts may arise between customers in the use of limited shared resources. As a consequence, some customers are forced to wait in a line.

Schematically, a waiting line system is made up of three main components: a *source* generating a stochastic process in which entities, also referred to as customers, arrive at a given location to obtain a service; a set of *resources* providing the service; a *waiting area* able to receive the entities whose requests cannot immediately be satisfied.

Waiting line models allow the performance of a system to be evaluated once its structure has been defined, and therefore are mostly useful within the system design phase. Indeed, in order to determine the appropriate values for the parameters that characterize a new system, relevant economic factors are considered, which depend on the service level that the system should guarantee when operating in optimal conditions. More precisely, a model takes into account the cost of meeting the requests, which increases as the service level increases, and the cost generated by customer waiting times, which decreases as the level of service provided decreases. Since customers hope for shorter waiting times while the provider is interested in holding down the cost of service provision, the structure of the system is defined in such a way as to obtain an optimal trade-off between service costs and waiting line costs. In other words, the optimal service level, which in turn determines the ideal structure of the system, can be found at the minimum point of the curve expressing the sum of the two types of cost.

The main components of a waiting line system are the *population*, the *arrivals process*, the *service process*, the *number of stations*, and the *waiting line rules*. The population, which can be finite or infinite, represents the source from which potential customers are drawn and to which they return once the requested service has been received. The arrivals process describes how customers arrive at the system entry point. In general this is a stochastic process described by the probability distribution of the inter-arrival times, that is, the time intervals between the arrival of two consecutive customers. The service process describes how the providers meet the requests of the customers waiting in line. This in turn is a stochastic process, defined by the probability distribution of the service time, that is, the amount of time that customers spend with the resources providing the service. The number of existing stations and the number of providers assigned to each station are additional relevant parameters of the waiting line system. The waiting rules describe the order in which customers are extracted from the line to be admitted to the service. A primary role is finally played by priority schemes in which a level of priority is assigned to each customer. The customer with the highest priority is then served before all the other customers waiting in line.

4.4 Notes and readings

For the topics that will be discussed in the following chapters, such as pattern recognition and predictive models, see the suggested reading section in the respective chapters. For more in-depth information on specific classes of models, see: Winston and Venkataramanan (2002), Bertsekas (2003) and Vercellis (2008) on mathematical optimization, and Nemhauser *et al*. (1994) on optimization in general; Elmaghraby (1978) on network models for project management; Clemen (1997) and Skinner (1999) on risk analysis; Kleinrock (1975) on waiting line theory. Keys (1995) deals with the epistemological aspects involved in operations research.