Sudesh Rajbhar 3067_①

Cost-Unit 3 Assignment

Chapter 9: Estimation Theory.

Confidence Interval for Means:

Confidence Level (%)	99.73	99	98	96	95.45	95	90	80	68.27	50
Z_c	3	2.58	2.33	2.05	2	1.96	1.64	1.28	1.00	0.6745

If the Statistic S is the sample mean, then 95% and 99% confidence limits for estimating the population mean μ, are given by and respectively.

Confidence Limits for population is given by:
1. If the sampling is from an infinite population or with replacement from a finite population.
2. If the sampling is without replacement from a population of finite size N.

1) $\bar{X} \pm z_c \frac{\sigma}{\sqrt{N}} \sqrt{\frac{N_p - N}{N_p - 1}}$

2) $\bar{X} \pm z_c \frac{\sigma}{\sqrt{N}}$

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1. In Measuring User Reaction time to the mouse movement, a physcologist estimates that the standard deviation is 0.05 second. How large a sample measurement must he take inorder to be.

A): (a) 95%.

The 95% confidence limits are $\bar{X} \pm \frac{1.96\sigma}{\sqrt{N}}$, the error of the estimate being $1.96\sigma/\sqrt{N}$. Taking $\sigma = s = 0.05s$, we see that this error will be equal to 0.01s if $(1.96)(0.05)/\sqrt{N} = 0.01$; that is $\sqrt{N} = (1.96)(0.05)/0.01 = 9.8$ or $N = 96.04$. Thus we can be 95% confident that error of the estimate will be less than 0.01s if N is 97 or larger.

(b) 99% confident that the error of his estimate will not exceed 0.01s?

The 99% confidence limits are $\bar{X} \pm 2.58\sigma/\sqrt{N}$. Then $(2.58)(0.05)/\sqrt{N} = 0.01$, or $N = 166.4$. Thus we can be 99% confident that the error of estimate will be less than 0.01s only if N is 167 or larger.

2. A sample of 12 measurements of the breaking strengths of cotton threads gave a mean of 7.38 oz and a standard deviation of s=1.24 oz. Find bounds of the 99% confidence interval for actual mean breaking strength.

A):- $n = 12$

$\bar{x} = 7.38$

$s = 1.24$

(a) 99% : $\bar{x} \pm z_c \frac{\sigma}{\sqrt{N}}$

: $7.38 \pm z_c \frac{(1.24)}{\sqrt{12}}$

($\because z_c = 3.106$)

: $7.38 \pm \frac{(3.106)(1.24)}{3.4641}$

: $7.38 \pm \frac{3.85144}{3.4641}$

: 7.38 ± 1.1118

Bound : 6.2682 to 8.4918

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3. In a Survey, 11 people were asked how much they spent on their child's last birthday gift. The results were roughly shaped as a normal curve with a mean of \$44 and standard deviation of \$7. Find margin of error at a 80% confidence level. Give your answer to two decimal places.

A) = $n = 11$

$\bar{x} = 44$

$\sigma = 7$

80% confidence level $\therefore Z_c = 1.28$

$: \bar{x} \pm Z_c \times \frac{\sigma}{\sqrt{N}}$

$: 44 \pm 11 \times \frac{7}{\sqrt{11}}$

$: 44 \pm \frac{11 \times 7}{3.3166}$

$: 44 \pm \frac{77}{3.3166}$

$: 44 \pm 23.2165$

$: 20.7835$ to 67.2165

Margin of error $: Z \times \frac{S}{\sqrt{n}}$

$: 11 \times \frac{7}{\sqrt{11}}$

$: \sqrt{11} \times 7$

$: 3.3166 \times 7$

Margin of error : 23.2165.

4. A random Sample of 50 mathematics grades out of total of 200 showed a mean of 75 and a Standard deviation of $s = \cancel{1.24\ oz}\ 10$.

A) (a) what are the 95% confidence limits for estimates of mean of 200 grades?

→

Since the population size is not very large compared with sample size, we must adjust for it. Then the 95% confidence limits are.

$$\bar{x} \pm 1.96\sigma_{\bar{x}} = \bar{x} \pm 1.96 \frac{\sigma}{\sqrt{N}} \sqrt{\frac{N_p - N}{N_p - 1}} = 75 \pm 1.96 \times \frac{10}{\sqrt{50}} \sqrt{\frac{200-50}{200-1}} = 75 \pm 2.4.$$

(b) The confidence limits can be represented by

$$\bar{x} \pm z_c \sigma_{\bar{x}} = \bar{x} \pm z_c \frac{\sigma}{\sqrt{N}} \sqrt{\frac{N_p - N}{N_p - 1}} = 75 \pm z_c \frac{10}{\sqrt{50}} \sqrt{\frac{200-50}{200-1}} = 75 \pm 1.23 z_c$$

Since this must equal 75 ± 1, we have $1.23 z_c = 1$, or $z_c = 0.81$. The area under the normal curve from $z = 0$ to $z = 0.81$ is 0.2910; hence required degree of confidence is $2(0.2910) = 0.582$ or 58.2%.

5. The quality control manager of a tyre company has sampled 100 tyres and has found the mean life time to be 30214 kms. The population S.D. is 860. Construct a 95% confidence interval for mean life time for this particular brand of tyres.

A):- $n = 100$

95% confidence interval : $Z_c = 1.96$

$\mu = 30,214$

$\sigma = 860$

Confidence Interval: $\bar{x} \pm Z_c \frac{(\sigma)}{\sqrt{N}}$

$: 30,214 \pm \frac{860}{\sqrt{100}}$

$: 30,214 \pm 86(1.96)$

~~CI : 30,128 to 30,210~~.

$: 30,214 \pm 168.56$

CI : 30045 to 30382.56.

6. In a random selection of 64 of 600 raad acident in a town, the mean number of automobile accident per year was found to be 4.2 and sample s.d. wa 0.8. Construct 95% confidence interval for mean number of accidents crossing per year.

A):- 95% confidence $\therefore z_c = 1.96$

$\sigma = 0.8$

$\bar{x} = 4.2$

$N_p = 600$

$N = 64$

$\therefore CI : \bar{x} \pm z_c \frac{\sigma}{\sqrt{N}} \sqrt{\frac{N_p - N}{N_p - 1}}$

$: 4.2 \pm \frac{(1.96)(0.8)}{\sqrt{64}} \sqrt{\frac{600-64}{600-1}}$

$: 4.2 \pm \frac{(1.96)(0.8)\sqrt{536}}{\sqrt{599} \times 8}$

$: 4.2 \pm \frac{(1.96)(0.8)(23.15)}{24.4744 \times 8}$

$: 4.2 \pm \frac{36.2992}{24.4744 \times 8}$

$: 4.2 \pm \frac{1.4829}{8}$

$: 4.2 \pm 0.1853625$

$CI : 4.0146$ to 4.3853.

7) A sample random of size 100 has mean 15, the population variance is 25. Find the interval estimate of population means with a confidence level of

a. 99% and (b) 95%.

A) = $n = 100$

$\bar{x} = 15$

$\sigma = 5$

95% confidence interval : Z_c : 1.96.

$\therefore$ Interval : $15 \pm (1.96) \times \frac{5}{\sqrt{100}}$

$= 15 \pm 1.96 \times \frac{5}{10}$

$= 15 \pm 1.96 \times 0.5$

$= 15 \pm 0.98$

$\therefore$ Interval = 14.02 and 15.98

99% : $Z_c = 2.58$

$\therefore$ Interval : $15 \pm 2.58 \times \frac{5}{\sqrt{100}}$

: $15 \pm 2.58 \times \frac{5}{10}$

: $15 \pm 2.58 \times 0.5$

$\therefore$ Interval : 13.71 and 16.29

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(8) The mean and Standard deviation of diameter of a sample of 250 rivet heads manufactured by a company of are 0.72642 inch and 0.00058 inch.

Find (a) 99% (c) 95%
(b) 98% (d) 90% Confidence limits.

A):- $n = 250$

$\bar{x} = 0.72642$.

$\sigma = 0.00058$.

(a) 99% ($Z_c = 2.58$)

$\therefore$ Conf. limit: $\bar{x} \pm Z_c \times \frac{S}{\sqrt{N}}$

$: 0.72642 \pm 2.58 \times \frac{0.00058}{\sqrt{250}}$

$: 0.72642 \pm 2.58 \times \frac{0.00058}{15.8114}$

$: 0.72642 \pm \frac{0.0014964}{15.8114}$

$: 0.72642 \pm 0.00009464057$.

C Limit $: 0.726326$ and 0.726514

(b) 98% ($Z_c = 2.33$)

$\therefore$ Conf. limit: $\bar{x} \pm Z_c \times \frac{S}{\sqrt{N}}$

$: 0.72642 \pm 2.33 \times \frac{0.00058}{15.8114}$

$: 0.72642 \pm \frac{0.0013514}{15.8114}$ $: 0.72642 \pm 0.00008546$

CL $: 0.72633454$ and 0.72650546

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(c) 95% ($z_c = 1.96$)

$\therefore$ confidence Limit: $\bar{x} \pm z_c \dfrac{s}{\sqrt{N}}$

$: 0.72642 \pm \dfrac{1.96 \times 0.00058}{\sqrt{250}}$

$: 0.72642 \pm \dfrac{1.96 \times 0.00058}{15.8114}$.

$: 0.72642 \pm \dfrac{0.0011368}{15.8114}$

$\begin{array}{r} 38 \\ 56 \\ \hline 94 \end{array}$

$: 0.72642 \pm 0.00007189$

$\therefore 0.72642 - 0.00007189$ to

$0.72642 + 0.00007189$

C.L.: 0.72634811 to 0.72649189.

(d) 90% ($z_c = 1.64$)

$\therefore$ confidence Limit : $\bar{x} \pm z_c \times \dfrac{s}{\sqrt{N}}$

$: 0.72642 \pm \dfrac{1.64 \times 0.00058}{\sqrt{250}}$

$: 0.72642 \pm \dfrac{1.64 \times 0.00058}{15.8114}$

$: 0.72642 \pm 0.103722 \times 0.00058$

$: 0.72642 \pm 0.00006015876$

C.L. : 0.726359 to 0.72648015 9.

(9) If the standard deviation of the lifetime of television tubes is estimated to be 100h, how large a sample must we take in order to be a
(a) 95% (b) 90% and (c) 99.73% confident that the error in estimated mean lifetime will not exceed 20h?

A):- $C.I. = \bar{x} \pm z_c \times \frac{\sigma}{\sqrt{n}}$

(a) 95%.

$\therefore C.I. = \bar{x} \pm z_c \times \frac{\sigma}{\sqrt{n}}$

$\because$ 95%, $z_c = 1.96$

Now, $z_c \times \frac{\sigma}{\sqrt{n}} \le 20$

$\therefore \frac{1.96 \times 100}{\sqrt{n}} \le 20$

$\therefore \frac{1.96 \times 100}{20} \le \sqrt{n}$

$\therefore \frac{196}{20} \le \sqrt{n}$

$\therefore (9.8)^2 \le n$

$\therefore n \ge 96.04$

$\therefore$ To be 95% confident that estimated mean lifetime will not exceed 20h we need a sample of atleast 96.04.

(b) 90

$\because C.I. = \bar{x} + Z_c \times \frac{\sigma}{\sqrt{n}}$

$\because 90\%$, $\therefore Z_c = 1.64$

Now, $Z_c \times \frac{\sigma}{\sqrt{n}} \leq 20$

$\therefore 1.64 \times \frac{\sigma}{\sqrt{n}} \leq 20$

$\therefore \frac{1.64 \times 100}{20} \leq \sqrt{n}$

$\therefore \frac{164}{20} \leq \sqrt{n}$

$\therefore 8.2 \leq \sqrt{n}$

$\therefore 67.24 \leq n$

$\therefore$ To be 90% sure that the estimated mean lifetime will not exceed in error, is 67.24 in sample number.

(c) 99.73.

$\because C.I. = \bar{x} + Z_c \times \frac{\sigma}{\sqrt{n}}$

$\because 99.73$, $\therefore Z_c = 3$

Now, $Z_c \times \frac{\sigma}{\sqrt{n}} \leq 20$

$\therefore \frac{3 \times 100}{\sqrt{n}} \leq 20$

$\therefore \frac{3 \times 100}{20} \leq \sqrt{n}$

$\therefore \frac{300}{20} \leq \sqrt{n}$

$\therefore 15 \leq \sqrt{n}$

$\therefore 225 \leq n$

$\therefore$ To be 99.73% that the error in estimated mean lifetime will not exceed 20h, we need to have a sample of 225.

(10) The Mean and Standard deviation of Maximum load supported by 60 cables are given by 11.09 tons and 0.73 ton, respectively, Find (a) 95% & (b) 99% confidence limits for the mean of the maximum loads of all cables produced by the company.

A):= $\bar{x} = 11.09$ At 95% $Z_c = 1.96$

$\sigma = 0.73$ At 99% $Z_c = 2.58$

$n = 60$

(a) $C.I. = \bar{x} \pm Z_c \times \frac{\sigma}{\sqrt{n}}$

$C.I. = 11.09 \pm \frac{(1.96) \times 0.73}{\sqrt{60}}$

$C.I. = 11.09 \pm \frac{1.4308}{7.746}$

$C.I. = 11.09 \pm 0.1847$

$C.I. : 10.9053$ to 11.2747

(b) $C.I. = \bar{x} \pm Z_c \times \frac{\sigma}{\sqrt{n}}$

$C.I. = 11.09 \pm \frac{(2.58) \times 0.73}{\sqrt{60}}$

$C.I. = 11.09 \pm \frac{1.8834}{7.746}$

$C.I. = 11.09 \pm 0.24314$

$C.I. : 10.84686$ to 11.33314.

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11) A survey of 40 retired women revealed that mean age at which their income was maximum to be 45 years with a standard deviation of 6.3 years. Find 95% confidence limits for the mean age if max earning of women who survive till they retire.

A):- $n = 40$

$\bar{x} = 45$

$\sigma = 6.3$

At 95% C.L., $Z_c = 1.96$

$$C.I. = \bar{x} \pm Z_c \times \frac{\sigma}{\sqrt{n}}$$

$$= 45 \pm 1.96 \times \frac{6.3}{\sqrt{40}}$$

$$= 45 \pm \frac{1.96 \times 6.3}{6.324}$$

$$= 45 \pm \frac{12.348}{6.324}$$

$$= 45 \pm 1.9525$$

C.I. = 43.0475 to

~~C.I~~ 46.9525.

* Confidence Interval for Proportions.

Confidence Limits for the population proportion is given by

1. $P \pm Z_c\sqrt{\frac{pq}{N}}$ if the sampling is from an infinite population or with replacement from a finite population.

2. $P \pm Z_c\sqrt{\frac{pq}{N}}\sqrt{\frac{N_p - N}{N_p - 1}}$ if the sampling is without replacement from a population of finite set N_p,

where p is the Probability of success and $q = (1-p)$. P is the proportion of success in the sample of size N.

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1. A random sample of 100 balls selected from a large consignment of cricket balls gave 10% defective balls. Find 99% confidence limits for percentage of defective balls in the consignment.

A) = $N = 100$.

$p = \frac{10}{100} = 0.1$.

C.L. = 99% ∴ $Z_c = 2.58$

A) = C.I : $P \pm Z_c\sqrt{\frac{pq}{N}}$

: $0.1 \pm (2.58)\sqrt{\frac{(0.1)(0.9)}{100}}$

: $0.1 \pm (2.58) \times \frac{0.3}{10}$

: $0.1 \pm 2.58 \times 0.03$

: $0.1 \pm (2.58) \times 0.03$

: 0.1 ± 0.0774

1. A random sample of 100 balls selected from a large consignment of cricket balls gave 10% defective balls. Find 99% confidence limits for the percentage of defective balls in the consignment [0.4173 and 0.4716].

A) : $N_p = 100$

$N = 10$

$p = 0.1, q = 0.9$

$CL = 99\%$ $\therefore Z_c = 2.58$

$\therefore CI : p \pm Z_c \sqrt{\frac{pq}{n}} \sqrt{\frac{N_p - n}{N_p - 1}}$

$: 0.1 \pm 2.58 \sqrt{\frac{(0.1)(0.9)}{10}} \times \sqrt{\frac{100-10}{100-1}}$

$: 0.1 \pm 2.58 \times \sqrt{\frac{0.09}{10}} \sqrt{\frac{90}{99}}$

$: 0.1 \pm (2.58) \times \frac{0.3}{3.1622} \times 0.953462$

$: 0.1 \pm (2.58) \times (0.09487) \times (0.953462)$

$: 0.1 \pm 0.23337$

:

0.0259859

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2. A random sample of 800 units from a large consignment showed that 200 were damaged. Find 95% confidence limits for the population proportion of damaged units in the consignment [0.19 & 0.31].

A):

$N = 800$

$p = \frac{200}{800} = \frac{1}{4} = 0.25$

$q = \frac{3}{4} = 0.75$

$\because CL = 95\%,\ Z_c = 1.96.$

$CI : p \pm Z_c \sqrt{\frac{pq}{n}}$

$: 0.25 \pm (1.96)\sqrt{\frac{0.25 \times 0.75}{800}}$

$: 0.25 \pm (1.96)\sqrt{\frac{0.1875}{800}}$

$\therefore 0.25 \pm (1.96) \times \frac{0.4330}{28.284}$

$: 0.25 \pm (1.96) \times (0.01530)$

$: 0.25 \pm 0.029988$

$CI : 0.22$ and 0.279988

$Np = 800,\ N = 200$

$p = 0.25,\ q = 0.75$

$CL = 95 \therefore Z_c = 1.96$

$C.I. = p \pm Z_c \sqrt{\frac{pq}{n}} \sqrt{\frac{Np - N}{Np - 1}}$

$: 0.25 \pm (1.96)\sqrt{\frac{0.25 \times 0.75}{200}} \times \sqrt{\frac{800 - 200}{800 - 1}}$

$: 0.25 \pm (1.96)\sqrt{\frac{(0.1875)}{200}}\sqrt{\frac{600}{799}}$

$: 0.25 \pm 1.96 \times \frac{0.4330}{14.14} \times 0.8665$

$: 0.25 \pm (1.96) \times (0.03062) \times (0.8665)$

$: 0.25 \pm 0.052003$

$: 0.197997$ and $0.302003.$

3. Out of 300 households in a town 123 have T.V. Sets. Find 95% confidence limits to the true values of proportion of households with TV sets in whole town. [0.355 & 0.465].

A) $N_p = 300 \quad N = 123$

$p = \frac{123}{300} = 0.41, \quad q = 0.59$

C.L. = 95, $\therefore Z_c = 1.96$.

$\therefore$ C.I. $= p \pm Z_c \times \sqrt{\frac{pq}{n}} \times \sqrt{\frac{N_p - n}{N_p - 1}}$

$= 0.41 \pm 1.96 \times \sqrt{\frac{(0.41)(0.59)}{123}} \times \sqrt{\frac{300-123}{300-1}}$

$= 0.41 \pm 1.96 \times \sqrt{\frac{0.2419}{123}} \times \sqrt{\frac{177}{299}}$

$= 0.41 \pm 1.96 \times \frac{0.4918}{11.0905} \times \frac{13.3041}{17.2916}$

$= 0.41 \pm \frac{1.96 \times 0.4918 \times 13.3041}{191.77249}$

$= 0.41 \pm \frac{12.8241945}{191.77249}$

$= 0.41 \pm 0.06687$

C.I. = 0.34313 and 0.47687

(4) A factory is producing 50,000 pairs of shoes daily. From a sample of 500 pairs, 2% were found to be of substandard quality. Estimate the number of pairs that can be reasonably expected to be spoiled in daily production and assign limits at 95% level of confidence.

A):- $N_p = 50{,}000$

$n = 500$

$P = \frac{2}{100}, \; q = \frac{98}{100}$

∴ The estimated percentage of spoiled pairs $= \frac{2}{100} \times 50000 = 1000.$

∴ The limits for number of spoiled pairs at 95% level of confidence is;

$$C.I = P \pm 1.96\sqrt{\frac{PQ}{n}} \times (50000)$$

$$CI = \frac{2}{100} \pm 1.96 \times \sqrt{\frac{2}{100} \times \frac{98}{100} \times \frac{1}{500}} \times (50000)$$

$$CI = \frac{2}{100} \pm 1.96 \times \sqrt{\frac{196}{500}} \times (50000)$$

$$= \frac{2}{\cancel{50}} \pm 1.96\sqrt{0.392} \times (500\cancel{00})$$

$$= \frac{2}{\cancel{50}} \pm 1.227 \times (50{,}0\cancel{00})$$

$$= 500 \times (2 - 1.227) \text{ and } 500 \times (2 + 1.227)$$

$$= 500 \times 0.773 \text{ and } 500 \times 3.227$$

$C.I. = 387$ and 1614.

∴ 0.773 and 0.3227 are estimate errors & Quantity expected to be spoiled are 387 and 1614.

$\frac{65}{500} \times 100$

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(5) A random sample of 500 pineapples was taken from a large consignment and 65 of them were found to be bad. Show that the standard deviation of population of bad one in a sample of this size is 0.015 and deduce that percentage of bad pineapples in the consignment lie between 8.5 & 17.5.

A):- $n = 500$

$P = \frac{65}{500} = 0.13$ Prove = S.D = 0.015.

$Q = 0.87$

$\therefore SD = \sqrt{\frac{PQ}{N}}$

$= \sqrt{\frac{(0.13)(0.87)}{500}}$

$= \sqrt{\frac{0.1131}{500}}$

$= \sqrt{0.0002262}$

$= 0.01504$

$S.D = 0.015$ — (1)

$\therefore$ Probable limits $= P \pm Z_c \times \sqrt{\frac{pq}{n}}$

$= 0.13 \pm 3 \times 0.015$ ... (From (1))

$= 0.13 \pm 0.045$

Limits. $= 0.085$ and 0.175

(6) A sample poll of 100 voters chosen at random from all voters in a given district indicated that 55% of them were in favour of a particular candidate.
Find 95%, 99% & 99.73 conf. limits.

Sol: (a) $n = 100$ (95%), $p = 0.55$, $q = 0.45$.
$z_c = 1.96$

$\therefore C.I. = P \pm 1.96\sqrt{\frac{pq}{N}}$

$= 0.55 \pm 1.96 \times \sqrt{\frac{0.55 \times 0.45}{100}}$

$= 0.55 \pm 1.96 \times \sqrt{\frac{0.2475}{100}}$

$= 0.55 \pm 1.96 \times \frac{0.49749}{10}$

$= 0.55 \pm 1.96 \times 0.049749$

$= 0.55 \pm 0.097508$

$= 0.55 - 0.097508$ and $0.55 + 0.097508$

$C.I = 0.452492$ and 0.647508

$\therefore C.I. = P \pm 2.58\sqrt{\frac{pq}{n}}$ [99%]

$C.I. = 0.55 \pm 2.58\sqrt{\frac{0.55 \times 0.45}{100}}$

$= 0.55 \pm 2.58 \times \sqrt{\frac{0.2475}{100}}$

$= 0.55 \pm 2.58 \times \frac{0.49749}{10}$

$= 0.55 \pm 2.58 \times 0.049749$

$C.I. = 0.55 \pm 0.12835338$

$C.I. = 0.42164662$ and 0.67835338

6. From give sample of 100, 35 are working as professor. Conduct a 95% confidence interval for the probability that almost of education people from sample are working as a professor.

A) $N_p = 100$ $\quad p = 0.35 \quad q = 0.65$

$N = 35$

$CL = 95$, $z_c = 1.96$.

$$C.I. = 0.35 \pm (1.96)\sqrt{\frac{0.35 \times 0.65}{35}} \times \sqrt{\frac{100-35}{100-1}}$$

$$= 0.35 \pm 1.96 \times \frac{0.2275}{35} \times \frac{65}{99}$$

$$= 0.35 \pm 1.96 \times 0.0065 \times 0.656565$$

$$= 0.35 \pm 0.007084$$

$C.I = 0.342916$ and 0.357084

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PROBABLE ERROR:

1. A measurement was recorded as 216.480 gms with a probable error of 0.272 gms. what are 95% confidence limits for measurements.

A) The probable error is $0.272 = 0.6745\, \sigma \bar{x}$,

or $\sigma \bar{x} = \frac{0.272}{0.6745}$.

Thus 95% confidence limits are,

$\bar{x} \pm 1.96 \sigma \bar{x} = 216.480 \pm 1.96(0.272/0.6745)$

$= 216.480 \pm 0.790\,g.$

$= 215.689\,l$ and $217.270\,g$.

2. A voltage of 50 batteries of the same type have a mean of 18.2 volts and a standard deviation of 0.5 volts. Find (a) Probable error of mean and (b) 50% conf. limits.

[Zc of 50% is 0.6745]

A) (a) Probable error of mean $= 0.674\, \sigma \bar{x} = 0.6745 \times \frac{\sigma}{\sqrt{N}}$

$= 0.6745 \times \frac{0.5}{\sqrt{49}}$

$= 0.048\,V$

Note that if standard deviation of 0.5 V is computed as $\bar{x}$, the probable error is $0.6745(0.5/\sqrt{50}) = 0.048$, also, so that either estimate can be used if N is large enough.

(b) The 50% confidence limits are $18 \pm 0.048\,V$

i.e. 17.952 and 18.048.

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Chapter 10: Hypothetical testing: Test of Hypotheses & Significance

Level of Significance, α	0.10	0.05	0.01	0.005	0.002
Critical (1-tailed)	-1.28 or 1.28	-1.645 - 1.645	-2.33 or 2.33	-2.58 or 2.58	-2.88 or 2.88
Critical values of z for two-tailed tests	-1.645 - 1.645	-1.96 and 1.96	-2.58 and 2.58	-2.81 & 2.81	-3.08 & 3.08

Formula: Normal Distribution.

1. Mean: Here $s = \bar{x}$, the sample mean, $\mu_s = \mu_{\bar{x}} = \mu$ the population mean, and $\sigma_s = \sigma_{\bar{x}} = \sigma/\sqrt{N}$, where σ is population deviation & N is the sample size. The z score is given by

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{N}}$$

when necessary, the sample deviation s or $\hat{s}$ is used to estimate σ.

2. Proportions: Here $s = P$, the proportions of "successes" in a sample: $\mu_s = \mu_P = p$, where p is the population proportion of successes and N is sample size; and $\sigma_s = \sigma_P = \sqrt{pq/N}$, where $q = 1-p$.

The z score is given by $z = \dfrac{P - p}{\sqrt{pq/N}}$

2. The claim is made that 40% of tax filer use computer software to file their taxes. In a Sample of 50, 14 used computer software to file their taxes. Test $H_0: p = 0.4$ versus $H_1: p < 0.4$ at $\alpha = 0.05$ where p is population proportion who use computer software to file their taxes.

A) : $H_0 = 0.4$

$H_a: p < 0.4$

$\alpha = 0.05$

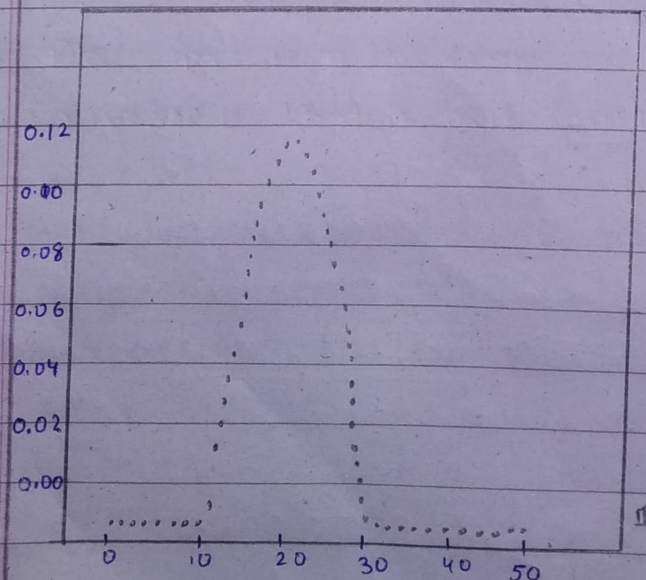
Null is rejected if $X \leq 15$, rejection region.

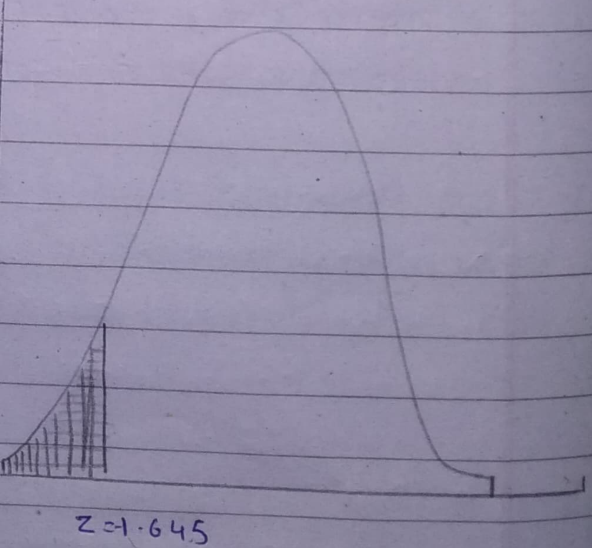
Test based on normal approx. the null is rejected if $Z < -1.645$ this is rejection region.

$X = 14$ is test statistic

Using Normal approximation; test statistic :- $Z = \frac{14-20}{3.46} = -1.73$.

Actual value $\rightarrow \alpha - 0.054$ and rejection region $X \leq 15$ and the $(P)(X<15)$ is used. $Z < -1.645$.





3. A device was thrown 9000 times and of these 3220 yielded a 3 or 4. Is this consistent with the hypothesis that the dice was unbiased? Level of significance is 0.05.

A):- Given

$n = 9000$

p = proportion of getting 3 (or) 4 in 9000 throws

$= \frac{3220}{9000} = 0.3578$

P = population proportion of success.

$= p$ (getting 3 (or) 4)

$= 1/6 + 1/6 = 2/6 = 0.3333$

$Q = 1 - P = 0.66$

Null Hypothesis (H_0) = The die is unbiased.

Alternative Hypothesis (H_1) = The die is biased.

Test statistic: $z = \frac{p - P}{\sqrt{\frac{PQ}{n}}} = 4.94$

Table value: Table value z at 1% level is 2.58

Conclusion: calculated value > table value

Therefore, we reject the null hypothesis H_0.

i.e. The die is biased.

3067 - (4)

(4) In a sample of 400 burners there were 12 whose internal diameter were not within tolerances. Is this sufficient evidence for concluding that the manufacturing process is turning out more than 2% defective burners. Let $\alpha = 0.05$

A):- Ho = 0.02

Ha = p: NE 0.02

alpha = 0.05

p {reject Ho|Ho true}

test is a 1 sample proportion, test stat is z, reject if $|z| > 1.96$

z = (0.03 - 0.02) / sqrt (0.02*0.98/400);

the 0.03 is point estimate of 12/400 = 0.01/0.007 = 1.43

fail to reject Ho because of insufficient evidence to show a difference.

p-value = 0.15.

(5) A manufacturer claimed that atleast 95% of equipment which he supplied to a factory confronted to specification. An examination of sample of 200 pieces of equipment revealed that 18 were faulty. Test this claim at a significance level of 0.05.

A):-

3067 _ 5

⇒ Let p be the probability of success, which bring the probability of equipment supplied to the factory conformed to the specifications.

∴ data : $p = 0.95$ $q = 0.05$

$H_0 : p = 0.95$ and claim is correct.
$H_1 : p < 0.95$ and claim is false.

we choose one tailed test to determine whether the supply is conformal to specification.

$\mu = np = 200 \times 0.95 = 190$
$\sigma = \sqrt{npq} = \sqrt{200 \times 0.95 \times 0.95} = 3.082$.

Expected number of equipments according to it's specification $= 190$.

Actual number $= 182$, Since ~~out~~ 18 out of 200 were faulty to the specification. ~~= 190~~
∴ Difference $= 190 - 182 = 8$

$$\therefore z = \frac{x - np}{\sqrt{npq}} = \frac{8}{3.082} = 2.6$$

3067-⑥

6) The breaking strengths of cables produced by a manufacturer have a mean of 1800 pounds and a standard deviation of 100 pounds. By a new technique in the manufacturing process, it is claimed that the breaking strength can be increased. To test this claim, a sample of 50 cables is tested and it is found that mean breaking strength is 1850 pounds. Can we support claim at 0.01 S.Level?

A):- We have to decide between 2 hypotheses:

$H_0 = \mu = 1800 lb$, and there is really no change in breaking through.

$H_1 = \mu > 1800 lb$, and there is change in breaking through.

A one-tailed test should be used here, the diagram associated with this test is identical with;

$$z = \frac{\hat{x} - \mu}{\sigma / \sqrt{N}} = \frac{1850 - 1800}{100 / \sqrt{50}} = 3.55$$

3.55 is greater than 2.33. Hence we conclude that results are highly significant & claim should be supported.

3064-(7)

Q8) A Stenographer claims that she can write 120 wpm. Can we reject her claim on basis of 100 trials in which she demonstrated a mean of 116 words with S.D. of 15 words?

$H_0 = 120$ (m) $\bar{x} = 116$, $s = 15$, $n = 100$

$H_a = 120$ (m)

H_0: She can type 120 wpm.

$$|z| = \left| \frac{\bar{x} - u}{6\sqrt{n}} \right|$$

$$|z| = \left| \frac{116 - 120}{15 / \sqrt{100}} \right|$$

$|2.67| > 1.96$

The difference is not significant at both 5% & 1% level of significance i.e. the value of z-score is 2.67 is highly significant. Hence H_0 is rejected, Her claim is rejected.

Q9) A sample of 900 members has a mean 3.4 cms and S.D. 2.61 cms. Can the sample be regarded as one drawn from population with 3.25 cm mean? Using L.O.S. a 0.005 is claim acceptable?

A) =

3067 - 8

Given:

Sample size $n=900$, $\bar{x}=3.4$.

$S.D. = \sigma = 2.61cm$

Pop. Mean $= \mu = 3.25cm$.

Pop. S.D $= \sigma = 2.61$ cm.

Null Hypothesis $(H_0) = \mu = 3.25cm$
(sample drawn from pop. mean)

Alternate (H_1): sample not drawn from pop. mean : $\mu \neq 3.25cm$.

$\alpha = 5\% = 0.05$

Test statistic

$$z = \frac{3.4-3.25}{\frac{2.61}{\sqrt{9000}}} = \frac{0.15}{0.087} = 1.724$$

$z = 1.96$

$\because 1.724 < 1.96$

Hence we conclude that 2 data doesn't provide us any evidence against null hypothesis. Therefore, sampe has been drawn from pop. mean $\mu = 3.25$ cm & SD, $\sigma = 2.61cm$.

3067 - 9

Q10) The mean life of a sample of 400 flourescent bulbs produced by a company is found to be 1570 hours with S.D. of 150 hours. Test hypothesis that mean life time of bulbs produced by company is 1600 hours at 1% significance level.

$n = 400$

$\bar{x} = 1570$

$\sigma = 150$

$\mu_0 = 1600$

$H_0 = \mu = 1600$

$H_a = \mu \neq 1600$

$n = 400 > 30$, σ is known $\Rightarrow$ z-test.

Two-tailed test is $z_c = 2.58$

$$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{1570-1600}{150/\sqrt{400}} = -4$$

$|z| = 4 > 2.58 = z_c$

RC.

Test statistic (s.e.) of xbar =

$s/\sqrt{n} = 150/\sqrt{400} = 150/20 = 7.5$

level of significance = 1%

$\alpha = 0.1$

$\therefore$ Level of confidence = 0.99 or 99%

$\therefore z = 2.33$

Since Mod value is more than critical value Null is rejected & Alternate hypothesis is accepted.

Hence bulb lifetime is more than 1600 hours.

3067 - 10

(11) A random sample of 100 students gave a mean weight of 58 kg with s.d. of 4 kg. Test the hypothesis that mean weight in population is 60 kg.

A):

$n = 100$

$\bar{x} = 58$ kg

$\sigma = 4$ kg

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{|58 - 60|}{4 \times \sqrt{100}} = \frac{|-2|}{40} = \frac{|-1|}{20} = \frac{1}{20} = 0.05$$

Since calculated value is less than 1.96 it is ~~sig. accepted~~ is rejected at levels of significance that mean weight of population is not 60 kg.