Chapter 5: Moment Assignment

5.1) Find first, second, third and fourth moments of set 2, 3, 7, 8, 10.

A) (a) First Moment: (Arithmetic Mean):

$$X = \frac{\sum X}{N} = \frac{2+3+7+8+10}{5} = \frac{30}{5} = 6.$$

(b) Second Moment:

$$\overline{X^2} = \frac{\sum X^2}{N} = \frac{2^2+3^2+7^2+8^2+10^2}{5} = \frac{226}{5} = 45.2.$$

(c) third Moment.

$$\overline{X^3} = \frac{\sum X^3}{N} = \frac{2^3+3^3+7^3+8^3+10^3}{5} = \frac{1890}{5} = 378.$$

(d) fourth Moment.

$$\overline{X^4} = \frac{\sum X^4}{N} = \frac{2^4+3^4+7^4+8^4+10^4}{5} = \frac{16,594}{5} = 3318.8.$$

3067_(2)

5.2) Find (a) first (b) second (c) third and (d) fourth moments about mean for set of numbers 2, 3, 7, 8 & 10.

A) a) $m_1 = \overline{(x - \bar{x})} = \frac{\Sigma(x-\bar{x})}{N} = \frac{(2-6)+(3-6)+(7-6)+(8-6)+(10-6)}{5} = \frac{0}{5} = 0$

m_1 is always equal to zero since $\overline{x-\bar{x}} = \bar{x} \cdot \bar{x} = 0$.

(b) $m_2 = \overline{(x-\bar{x})^2} = \frac{\Sigma(x-\bar{x})^2}{N} = \frac{(2-6)^2+(3-6)^2+(7-6)^2+(8-6)^2+(10-6)^2}{6} = \frac{46}{5} = 9.2$

(c) $m_3 = \overline{(x-\bar{x})^3} = \frac{\Sigma(x-\bar{x})^3}{N} = \frac{(2-6)^3+(3-6)^3+(7-6)^3+(8-6)^3+(10-6)^3}{6} = \frac{-18}{5} = -3.6$

(d) $m_4 = \overline{(x-\bar{x})^4} = \frac{\Sigma(x-\bar{x})^4}{N} = \frac{(2-6)^4+(3-6)^4+(7-6)^4+(8-6)^4+(10-6)^4}{6} = \frac{610}{5} = 122$

5.3) Find (a) first (b) second, (c) third and (d) fourth moments about origin 4 for set of numbers.

(a) $m_1' = \overline{(x-4)} = \frac{\Sigma(x-4)}{N} = \frac{(2-4)+(3-4)+(7-4)+(8-4)+(10-4)}{5} = 2.$

(b) $m_2' = (\overline{x-4})^2 = \frac{(\Sigma(x-4)^2}{N} = \frac{(2-4)^2+(3-4)^2+(7-4)^2+(8-4)^2+(10-4)^2}{5} = \frac{66}{5}$

$= 13.2$

(c) $m_3' = (\overline{x-4})^3 = \frac{\Sigma(x-4)^3}{N} = \frac{(2-4)^3+(3-4)^3+(7-4)^3+(8-4)^3+(10-4)^3}{5} = \frac{298}{5}$

$= 59.6$

(d) $m_4' = (\overline{x-4})^4 = \frac{\Sigma(x-4)^4}{N} = \frac{(2-4)^4+(3-4)^4+(7-4)^4+(8-4)^4+(10-4)^4}{5} = \frac{1650}{5}$

$= 330.$

Page No:

Date: / /

3067 - 3

5.9) Apply Sheppard corrections to determine the moments about the mean for data.

$m_2 = 8.5275$, $m_4 = 199.3759$.

Hint:-

Sheppard's correction for moments are as follows:

Corrected $m_2 = m_2 - \frac{1}{12} c^2$, $m_4 = m_4 - \frac{1}{2} c^2 m_2 + \frac{7}{240} c^4$.

(a) Corrected $m_2 = m_2 - \frac{c^2}{12} = 8.5275 - \frac{3^2}{12} = 7.7775$.

Corrected $m_4 = m_4 - \frac{1}{2} c^2 m_2 + \frac{7}{240} c^4$

$= 199.3579 - \frac{1}{2}(3)^2(8.5275) + \frac{7}{240}(3)^4$

$= 163.3646$

m_1 & m_2 need no correction.

(b) Corrected $m_2 = m_2 - \frac{c^2}{12} = 109.5988 - \frac{4^2}{12} = 108.2655$.

Corrected $m_4 = m_4 - \frac{c^2 m_2}{2} + \frac{7}{240} c^4$

$= 35627.2853 - \frac{1}{2}(4)^2(109.5988) + \frac{7}{240}(4)^4$

Corrected $m_4 = 34757.9616$.

5.10) Find Pearson's (a) first and (b) second coefficients of skewness for wage distribution of 65 employee at P&R company.

A):= Mean = 279.76.

Median = 279.06.

Mode = 277.50.

S.D. = 15.60.

First Coefficient of skewness = $\frac{\text{Mean-Mode}}{S} = \frac{279.76 - 277.5}{15.60} = 0.1448 \sim 0.14$

Second Coefficient of skewness = $\frac{3(\text{Mean- median})}{S} = \frac{3(279.76 - 279.06)}{15.60} = 0.13.$

∴ Correct Coefficient of First = $\frac{\text{Mean-Mode}}{15.33} = \frac{279.76 - 277.5}{15.33} = \underline{0.15}.$

∴ Correct Coefficient of Second = $\frac{3(\text{mean-median})}{15.33} = \frac{3(279.76 - 279.06)}{15.33} = \underline{0.14}.$

Above data went towards positive i.e. 0.14 to 0.15 & 0.13 to 0.14

Hence skewed positively

3067 5

5.11) Find (a) quartile (b) percentile coefficients of skewness

$Q_1 = 286.25$, $Q_2 = P_{50} = 279.06$, $Q_3 = 290.75$, $P_{10} = D_1 = 258.12$, $P_{90} = 301.00$

(a) Quartile coefficient of skewness $= \dfrac{Q_3 - 2Q_2 + Q_1}{Q_3 - Q_1}$

$= \dfrac{290.75 - 2(279.06) + 268.25}{290.75 - 268.25} = 0.0391$

(b) Percentile coefficient of skewness $= \dfrac{P_{90} - 2P_{50} + P_{10}}{P_{90} - P_{10}}$

$= \dfrac{301.00 - 2(279.06) + 258.12}{301.00 - 258.12}$

$= 0.0233$

5.12) Find moment coefficient of skewness, a_3, for
(a) heigh of students at XYZ University.
(b) IQ's of elementary school children.

A) = (a) $m_2 = s^2 = 8.5275$, and $m_3 = -2.6932$.

$a_3 = \dfrac{m_3}{s^3} = \dfrac{m_3}{(\sqrt{m_2})^3} = \dfrac{-2.6932}{(\sqrt{8.5275})^3} = -0.1081$

Using Sheppard's correction;

corrected $a_3 = \dfrac{m_3}{(\sqrt{\text{corrected } m_2})^3} = \dfrac{-2.6932}{(\sqrt{7.7775})^3} = -0.1242$

Page No:
Date: / /

(b) $a_3 = \frac{m_3}{s^3} = \frac{m_3}{(\sqrt{m_2})^3} = \frac{202.8158}{(\sqrt{109.5988})^3} = 0.1768.$

Corrected $a_3 = \frac{m_3}{(\sqrt{\text{corrected } m_2})^3} = \frac{202.8158}{(\sqrt{108.2655})^3} = 0.1800.$

5.13) Find moment coefficient of Kurtosis, a_4, for data of (a) & (b).

(a) $a_4 = \frac{m_4}{s^4} = \frac{m_4}{(m_2)^2} = \frac{199.3759}{(8.5275)^2} = 2.74$

Using Sheppard's correction,

Corrected $a_4 = \frac{\text{corrected } m_4}{(\text{corrected } m_2)^2} = \frac{163.36346}{(7.7775)^5} = 2.7007$

(b) $a_4 = \frac{m_4}{s^4} = \frac{m_4}{m_2^2} = \frac{35627.2853}{(109.5988)^2} = 2.9660.$

Using Sheppard's correction.

Corrected $a_4 = \frac{\text{corrected } m_4}{(\text{corrected } m_2)^2} = \frac{34757.9616}{(108.2655)^2} = 2.9653.$

COST Assignment : Chp 6

Chp 6 : Elementary Probability Theory.

1> If E_1 is the event "drawing an ace from a deck of cards" and E_2 is the event "drawing a king" then

$Pr(E_1) = \frac{4}{52} = \frac{1}{13}$ and $Pr\{E_2\} = \frac{4}{52} = \frac{1}{13}$. The probability of

drawing either an ace or a king in a single draw is

$$Pr\{E_1 + E_2\} = Pr\{E_1\} + Pr\{E_2\} = \frac{1}{13} + \frac{1}{13} = \frac{2}{13}$$

2> If E_1 is the event "drawing an ace" from a deck of cards and E_2 is the event "drawing a spade", then E_1 & E_2 are not mutually exclusive since the ace of spades can be drawn. Thus probability of drawing either an ace or a spade or both is

$$Pr\{E_1 + E_2\} = Pr\{E_1\} + Pr\{E_2\} - Pr\{E_1 E_2\}$$
$$= \frac{4}{52} + \frac{13}{52} - \frac{1}{52} = \frac{16}{52} = \frac{4}{13}$$

6.1> 3> Determine the probability p, or an estimate of it, for each of following events

(a) an odd number appears in a single toss of a fair dice.

(b) At least one head appears in two tosses of a fair coin.

(c) An ace, 10 of Diamonds, 2 of spades in a single card from a well-shuffled 52 card deck.

(d) The sum 7 appears.

(e) A tail appears in next toss if out of 100 tosses 56 were heads

Chp 6: 3067 - 2

A) (a) Out of six possible equally likely cases, three cases are favourable to event, Thus $p = \frac{3}{6} = \frac{1}{2}$.

(b) If H denotes "head" and T denotes "tail" the two tosses can lead to four cases:

HH, HT, TH, and TT, all equal likely. Only the first three cases are favourable to event. $p = \frac{3}{4}$.

(c) The event can occur in six ways out of 52 equally likely cases. Thus $p = \frac{6}{52} = \frac{3}{26}$.

(d) Each of six faces of one side can be associated with each of the six faces of other die, so that total cases are: $6 \times 6 = 36$.

There are 6 ways of obtaining sum 7,
$(1,6)(2,5)(3,4)(4,3)(5,2)(6,1)$ $\therefore p = \frac{6}{36} = \frac{1}{6}$.

(e) Since $100 - 56 = 44$ tails were obtained in 100 tosses, est. prob. of a tail is relative freq. $\frac{44}{100} = 0.44$.

Chp 6 : 3067 - 3

6.2) An experiment consists of tossing a coin and a die. If E_1 is the event that "head" comes up in tossing coin and E_2 is the event that "3 or 6" comes up in tossing dice. state in words meaning, for following.

(a) $\bar{E_1}$: Tails on the coin and any thing on the dice.
(b) $E_1 E_2$: ~~t, 2,~~ Heads on the coin and 3 or 5 on dice.
(c) E_2 : 1, 2, 4 or 5 on the die and any thing on coin.
(d) $Pr\{E_1 E_2\}$: Probability of heads on coin and 1, 2, 4 or 5 on side.
(e) $Pr\{E_1 | E_2\}$: Probability of heads on coin, given that a 3 or 5 has to come up on die.
(f) $Pr\{E_1 + E_2\}$: Prob. of tails on coin or 1, 2, 4 or 5 on dice.

6.5) Two cards are drawn from a well-shuffled ordinary deck of 52 cards. Find probability that they are both aces if first card is (a) replaced and (b) not replaced.

A) Let E_1 = event "ace" on first draw and let E_2 = event "ace" on second draw.

(a) If the first card is replaced. E_1 and E_2 are independent events. Thus $Pr\{$both cards drawn are aces$\} = Pr\{E_1 E_2\} = Pr\{E_1\}$ $Pr\{E_2\} = \left(\frac{4}{52}\right)\left(\frac{4}{52}\right) = \frac{1}{169}$

(b) The first card can be drawn in any one of 52 ways, and second card can be drawn in rest 51 ways. Thus both cards are drawn in 52.51 ways.
There are 4 ways that E_1 can occur and 3 ways in E_2 can occur, so that E_1 & E_2, or $E_1 E_2$ occur in 4.3 ways.
Thus $Pr\{E_1 E_2\} = (4.3)/(52.51) = 1/221$.

Chp6:3067 - (4)

6.8) One bag contains 4 white balls and 2 black balls, another contains 3 white balls and 5 black balls. If one ball is drawn from each bag, find probability that (a) both are white (b) both are black, (c) one is white & one is black.

A):- Let W_1 = event "white" ball from first bag, and let W_2 = event "white" ball from 2nd.

(a) $Pr\{W_1W_2\} = Pr\{W_1\}\ Pr\{W_2\} = \left(\frac{4}{4+2}\right)\left(\frac{3}{3+5}\right) = \frac{1}{4}$

$Pr\{\bar{W_1}\bar{W_2}\} = Pr\{\bar{W_1}\} \cdot Pr\{\bar{W_2}\} = \left(\frac{2}{4+2}\right)\left(\frac{5}{3+5}\right) = \frac{5}{24}$

(c) $Pr\{W_1\bar{W_2} + \bar{W_1}W_2\} = Pr\{W_1W_2\} + Pr\{W_1W_2\}$

$= Pr\{W_1\} \cdot Pr\{\bar{W_2}\} + Pr\{\bar{W_1}\}\ Pr\{W_2\}$

$= \left(\frac{4}{6}\right)\left(\frac{3}{8}\right) + \left(\frac{2}{6}\right)\left(\frac{3}{8}\right)$

$= \frac{13}{24}$

* Consider the sample of finding probability of select a black card or a 6 from 52 cards.

A):- We need to find out P(B or 6)

Prob. of Black = 26/52

Prob of 6 = 4/52

Prob. of selecting both = 2/52

$\therefore$ P(B or 6) = P(B) + P(6) - P(B and 6)

= 26/52 + 4/52) - 2/52

= 28/52 = 7/13

When we throw 2 dice what is prob of sum 9: Ans = 1/9.

Chp 6: 3067 - 5

Page No:
Date:

* A card is drawn from a pack of 52 cards and then a second card is drawn. What is prob. that both cards are queen.

A) = First - 4/52
Second - 3/51
Both - 4/52 * 3/51 = 1/13 * 1/17 = 1/221.

* A bag contains 5 white and 3 black balls. Two balls are drawn one after other without replacement. Find prob. that both balls black are drawn.

A) = First draw = 3/8.
Second draw = 2/7.
Both balls drawn are black = 3/8 * 2/7 = 3/28.

Chp 8 : Elementary Sampling Theory

1. Sampling distribution of Mean.

Suppose that all possible samples of size N are drawn without replacement from a finite population of size $N_p > N$. If we denote mean & S.D. of sampling distribution of means by $\mu_{\bar{x}}$ and $\sigma_{\bar{x}}$ and population mean and standard deviation by μ and σ respectively then,

$$\mu_{\bar{x}} = \mu \text{ and } \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{N}}\sqrt{\frac{N_p - N}{N_p - 1}}$$

If population is infinite or if sampling is with replacement, the above results reduce to

$$\mu_{\bar{x}} = \mu \text{ and } \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{N}}$$

2. Sampling distribution of Proportions.

$$\mu_p = p \text{ and } \sigma_p = \sqrt{\frac{pq}{N}} = \sqrt{\frac{p(1-p)}{N}}$$

Sampling Distribution	Standard Error
Mean	$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{N}}$
Proportion	$\sigma_p = \sqrt{\frac{p(1-p)}{N}} = \sqrt{\frac{pq}{N}}$

8.3) Assume heights of 3000 male students at a university are normally distributed with mean 68.0 inches (in) and standard deviation 3.0 inches. If 80 samples consisting of 25 ~~Stu~~ students each are obtained, what would be expected mean and S.D. of resulting sampling dist. of mean (a) With replacement (b) without replacement.

(a) $\mu_{\bar{x}} = \mu = 68$ in. and $\sigma_x = \frac{\sigma}{\sqrt{N}} = \frac{3}{\sqrt{25}} = 0.6$ in.

(b) $\mu_{\bar{x}} = 68.0$ in and $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{N}}\sqrt{\frac{N_p - N}{N_p - 1}} = \frac{3}{\sqrt{25}}\sqrt{\frac{3000-25}{3000-1}}$

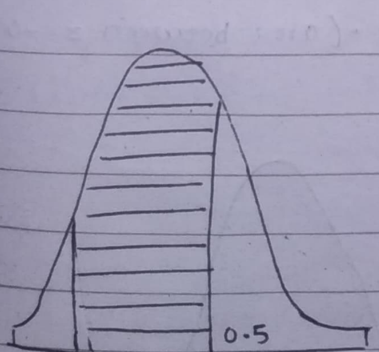
8.4) In how many samples of Above would you expect to find mean (a) between 66.8 & 68.3 in and (b) less than 66.4 in?

A):- The mean $\bar{x}$ of a sample in standard units is here given by

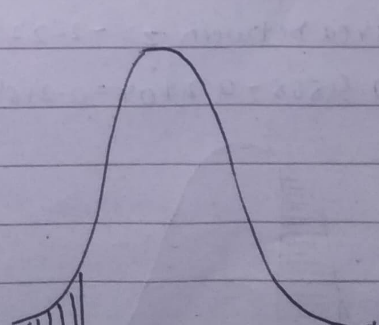
$$z = \frac{\bar{x} - \mu_{\bar{x}}}{\sigma_{\bar{x}}} = \frac{\bar{x} - 68}{0.6}$$

66.8 in standard units $= \frac{66.8 - 68.0}{0.6} = -2.0$

68.3 in standard units $= \frac{68.3 - 68.0}{0.6} = 0.5$



(a)



(b)

8.5) Five hundred ball bearings have a mean weight of 5.02 grams (g) and a standard deviation of 0.3g. Find prob. that random sample of 100 ball bearings chosen from this group will have a combined weight of
(a) between 496 and 500 g and
(b) more than 510 g.

Sol: For sample distribution of means, $\mu_{\bar{x}} = \mu = 5.02g$, and

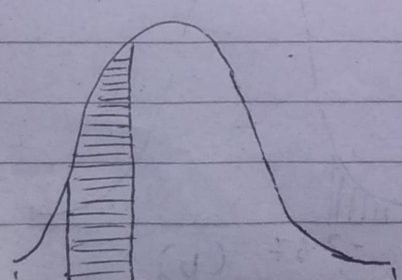
$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{N}}\sqrt{\frac{N_p - N}{N_p - 1}} = \frac{0.30}{\sqrt{100}}\sqrt{\frac{500-100}{500-1}} = 0.027\,g.$$

(a) The combined weight will lie between 496 & 500g if the mean weight of 100 ball bearings lies in 4.96 & 5.00g.

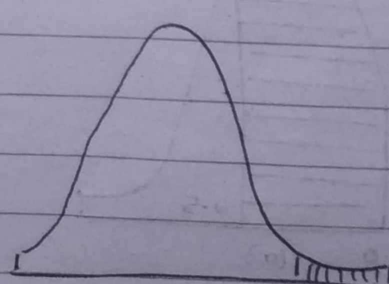
4.96 in standard units $= \frac{4.96 - 5.02}{0.0027} = -2.22$

5.00 in standard units $= \frac{5.00 - 5.02}{0.027} = -0.74$.

Req. prob =
area between $z = -2.22$ and $z = -0.74$
= (area between $z = -2.22$ and $z = 0$) - (area between $z = -0.74$ & $z = 0$)
$= 0.4868 - 0.2704 = 0.2164$



(a)



(b)

8·9) It has been found that 2% of tools produced by a certain number machine are defective. What is the probability that in a shipment of 400 such tools (a) 3% of more (b) 2% or less.

A):- $\mu_p = p = 0.02$ and $\sigma_p = \sqrt{\frac{pq}{N}} = \sqrt{\frac{(0.02)(0.98)}{400}} = \frac{0.14}{20} = 0.007$

(a) First Method.

Using correction for discrete variables,

$\frac{1}{2N} = \frac{1}{800} = 0.00125,$

$(0.03 - 0.00125)$ in standard Units $= \frac{0.03 - 0.00125 - 0.02}{0.007} = 1.25$

Required prob = (area under normal curve to right $z = 1.25$) $= \underline{\underline{0.1056}}$.

If we would have not used correction, we would have obtained 0.0764.

(b) Another method.

(3% of 400) = 12 defective tools. On a continous basis 12 or more tools means 11.5 or more.

$\bar{x} = (2\% \text{ of } 400) = 8$ and $\sigma = \sqrt{Npq} = \sqrt{400 \times 0.02 \times 0.98} = 2.8$

$(0.02 + 0.00125)$ in stand. units $= \frac{0.02 + 0.00125 - 0.02}{0.007} = 0.18$

$\therefore$ Req. prob $= 0.500 + 0.0714 = \underline{0.5714}$.