Computer Graphics Assignment 1

Sudesh Rajbhar - 3067 1

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* Two Dimensional Transformations *

* 2D - Transformations:

- Transformations are the operations applied to geometrical description of an object to change it's position, orientation, or size are called geometric transformation.

(1) TRANSLATION.

- 2D Translation means moving or shifting an object from one position to another, parallel to itself.

- The path is represented by a vector, called translation or shift vector.

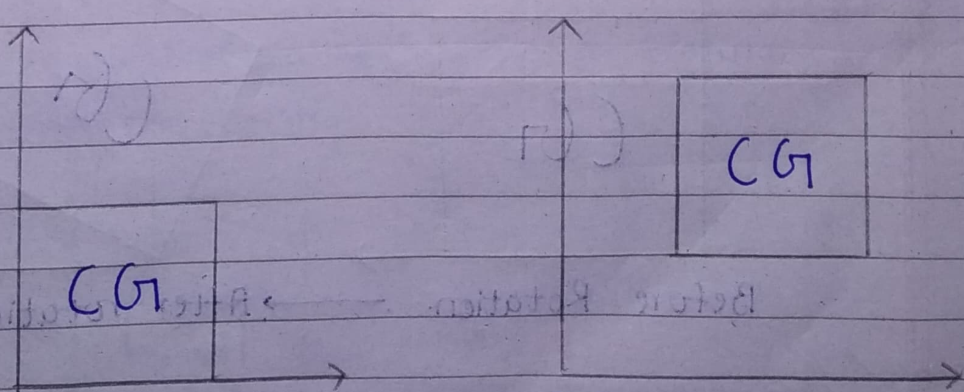
We can write components:

$$P'_x = P_x + t_x$$
$$P'_y = P_y + t_y$$

or in Matrix form:

$$P' = P + T$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \end{bmatrix}$$



Before Translation ⟶ After Translation.

(2) Rotation:

- A 2D trotation is applied to an object by repositioning it along circular path in the xy plane centred at pivot point.

We can write the components:

$$P'x = Px \cos\theta - Py \sin\theta.$$

$$P'y = Px \sin\theta + Py \cos\theta.$$

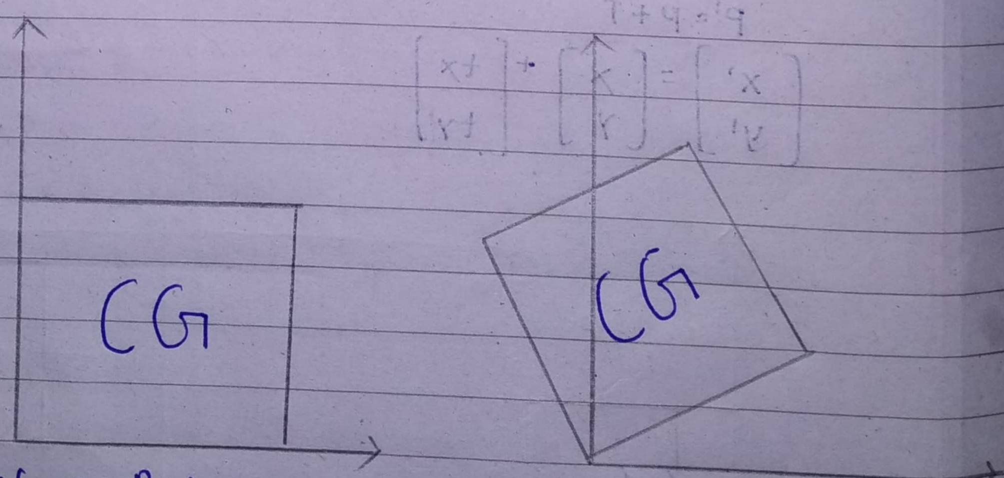
or in matrix form,

$$P' = R \cdot P$$

θ can be clockwise (-ve) or counterclockwise (+ve)

Rotation Matrix

$$R = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$



Before Rotation ⟶ After rotation.

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(3) Scaling

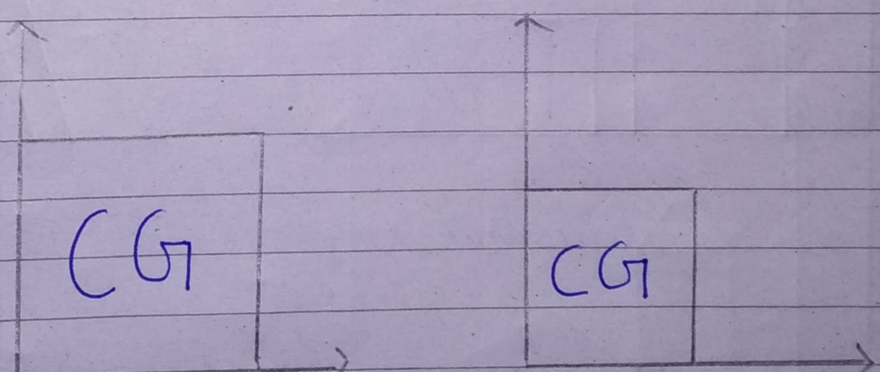
Scaling changes the size of an object and involves two scale factors, S_x and S_y for the x-axis and y-axis respectively.

$P'_x = S_x * P_x$
$P'_y = S_y * P_y$

or in Matrix form, $P' = S * P$

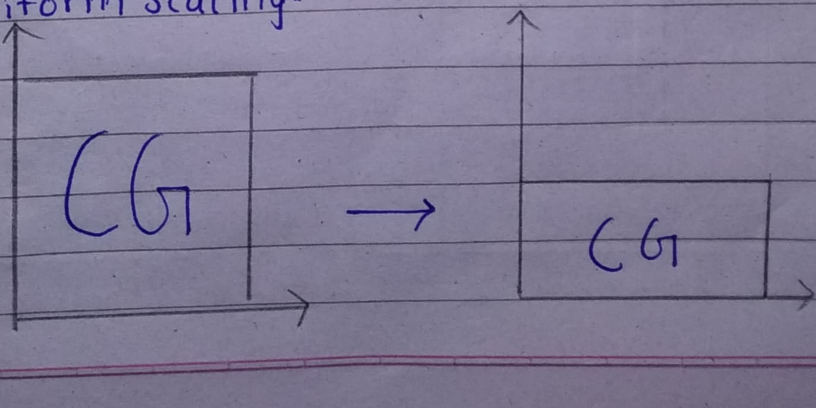
Scale Matrix, $S = \begin{bmatrix} S_x & 0 \\ 0 & S_y \end{bmatrix}$

***Uniform Scaling*:**
Scaling where Horizontal & Vertical factors are same is uniform Scaling.



Before Scaling ⟶ After Scaling.

***Non Uniform Scaling:**



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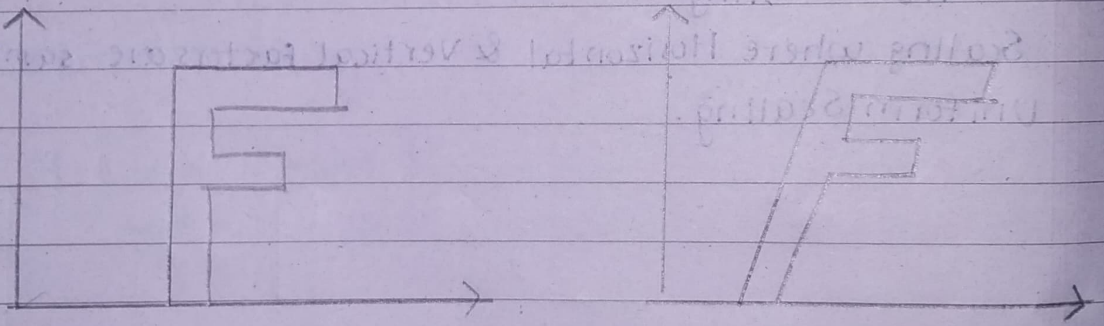
(4) Shearing.

The process of applying tangential force to any object which distorts the shape of an object such that the transformed shape appears as if the object is slide over or bended over is called as shearing.
"Shear is occassionally called skew".

X shearing: $[S']_{2\times2} = \begin{bmatrix} 1 & 0 \\ a & 1 \end{bmatrix}$

Y shearing $[S']_{2\times2} = \begin{bmatrix} 1 & b \\ 0 & 1 \end{bmatrix}$



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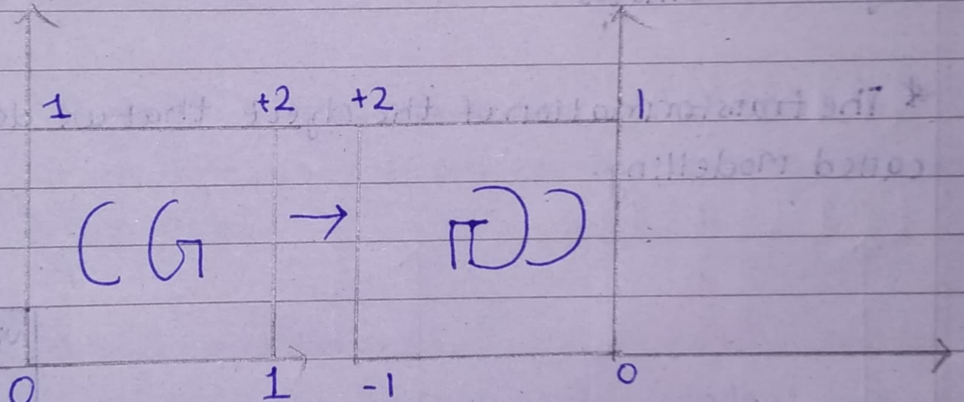
(5) Reflection.

· Reflection is the mirror image of original Object. Reflection on both x and y co-ordinates can be represented as;

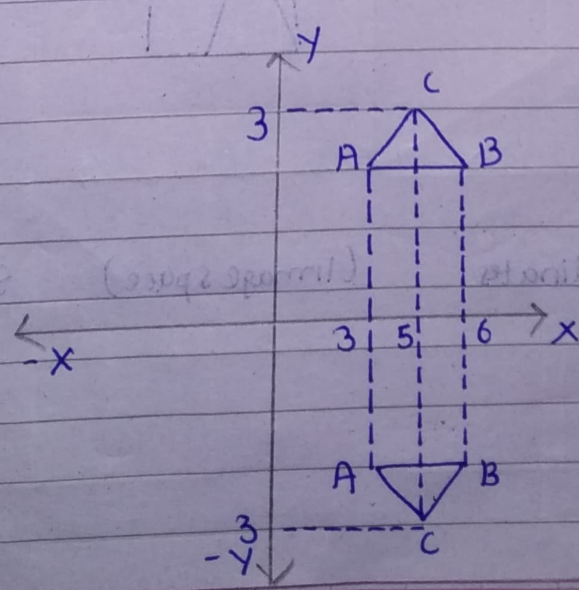
· [R'] → Matrix for Reflection.

$$\therefore [R']_{2\times 2} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

Let: Initial co-ordinates of O: (x, y)
Final co-ordinates of O: (x', y')



Before Reflection → After Reflection.



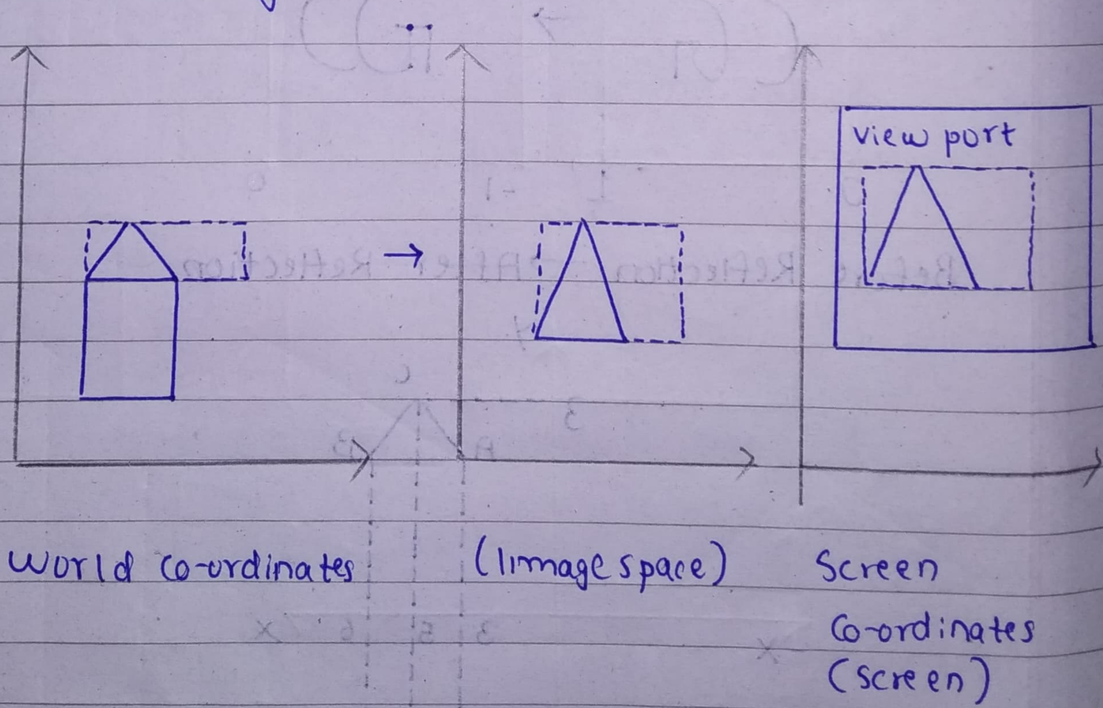
* Viewing and Modelling.

* Viewport: In a typical application, we have a rectangle made of pixels, with it's natural pixel co-ordinates, where an image will be displayed. This rectangle will be called Viewport.

* World co-ordinates: These objects make up the scene of world we want to view, and the co-ordinates used to define that scene is 'World co-ordinates'.

* The co-ordinates that we used, to define an object is object co-ordinates.

* The transformation of the object that we desire is called Modelling.



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* World-co-ordinate System (object Space)
- Space in which the application model is defined.
- Representation of an object is measured in some physical or abstract units.
- Spaces in which object geometry is defined.

* World window (object subspace)
- Rectangle defining part of world we wish to display.

* Screen co-ordinate System (Image Space)
- Space in which image is displayed.
- Usually measured in pixels but could use any units.
- Space in which object's raster image is defined.

* Interface Window (Image Subspace)
- Visual representation of screen co-ordinate system for windowed displays.

* Viewing Transformations.
- The process of mapping from a world window to a viewport

* Viewport.
- A rectangle on raster graphics screen (or window) defining where the image will appear, usually entire screen or window.

(6) Three - Dimensional Scaling.

- Scaling means changing the size of an object.
- Scaling in 3D can be represented by a scaling vectors in scaling matrix.

Scaling vector V is defined as $[S_x \; S_y \; S_z]$.

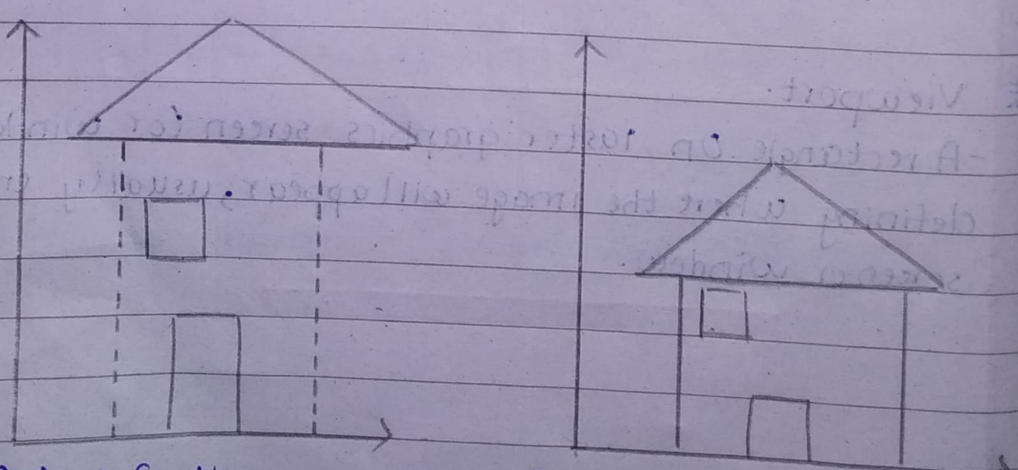
Scaling matrix(sv) in 3D-space with scaling vectors V is given by,

$$S_v = \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & S_z \end{bmatrix}$$

$$[P'] = [P][S_v]$$

$$[P'] = [x \; y \; z] \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & S_z \end{bmatrix}$$

where P' is transformed.

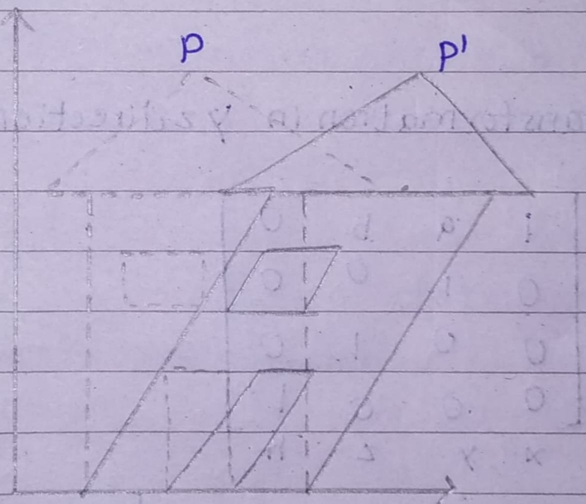


(7) Three-dimensional Shearing.

- When a tangential force is applied on an object the the shape of an object gets distorted in direction of force applied.
- This slant or tilt of object is called as shear and such transformation is called shearing transformation.

$$[T_s] = \begin{bmatrix} a & b & c & 0 \\ d & e & f & 0 \\ g & h & i & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Above Matrix is generalized shear transformation.



3-D Shearing.

Shear transformation in xy direction.

- Shearing factors 'a' & 'b' of x & y respec.

$$[T_s] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ a & b & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$[P'] = [P][T_s]$

$[P'] = [x' \ y' \ z' \ 1]$

$[P] = [x \ y \ z \ 1]$

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$$[x' \ y' \ z' \ 1] = [x \ y \ z \ 1] \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ a & b & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Shear transform in xz direction,
'Shearing factor a & b for x & z

$$[T_s] = \begin{array}{c} \begin{bmatrix} 1 & 0 & 0 & 0 \\ a & 1 & b & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\ \begin{array}{cccc} x & y & z & h \end{array} \end{array}$$

$$[x' \ y' \ z'] = [x + ay \quad y \quad by + z \quad 1]$$

Shear transformation in yz direction.

$$[T_s] = \begin{array}{c} \begin{bmatrix} 1 & a & b & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\ \begin{array}{cccc} x & y & z & h \end{array} \end{array}$$

$$[x' \ y' \ z' \ 1] = [x \ y \ z \ 1] \begin{bmatrix} 1 & a & b & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

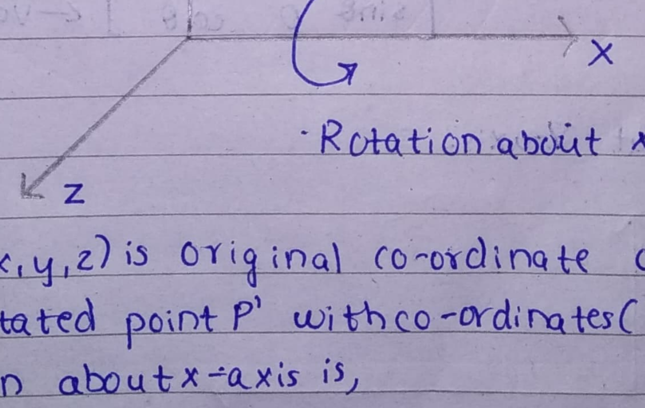
(8) Three Dimensional Rotation.

A)= - Rotation is nothing but a movement of an object in a circular motion.

- Rotation is 3D is defined by an angle of rotation θ.
- In 3D it is necessary to specify the axis of rotation as well.
- Rotation in 3D do not only depend up on the angle of rotation but it also requires the specification of axis of rotation.

Case 1: Positive Rotation about x-axis.

- When rotation occurs in x-axis it keeps x-coordinate constant and rotation occurs in a plane which is perpendicular to x-axis.



Positive rotation perpendicular to x-axis

· Rotation about x-axis.

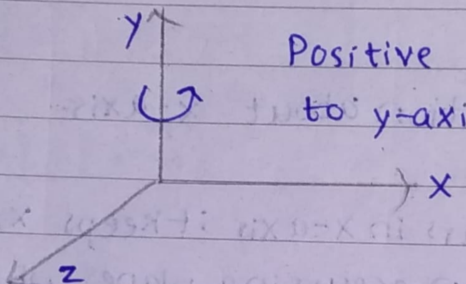
If (x, y, z) is original co-ordinate of Point P, then for rotated point P' with co-ordinates (x' y' z') if rotation about x-axis is,

$$x' = x$$
$$y' = y\cos\theta - z\sin\theta.$$
$$z' = y\sin\theta + z\cos\theta$$

* Rotation about x-axis:

$$R_{\theta x} = \begin{array}{l} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{bmatrix} \end{array} \begin{array}{l} \leftarrow \text{value of } x \text{ coefficient.} \\ \leftarrow \text{value of } y \text{ coefficient.} \\ \leftarrow \text{value of } z \text{ coefficient.} \end{array}$$

Case II: Positive Rotation of y-axis.



Positive rotation perpendicular to y-axis.

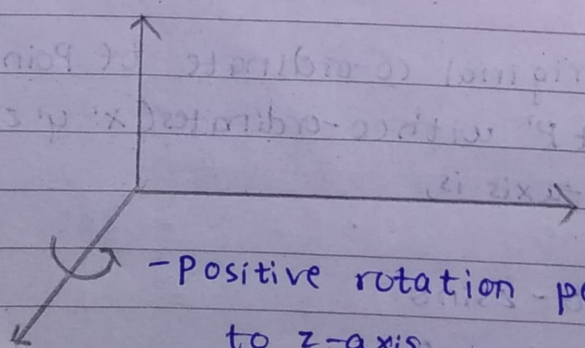
$x' = x\cos\theta + z\sin\theta$

$y' = y.$

$z' = -x\sin\theta + z\cos\theta.$

Rotation about y-axis: $\begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix} \begin{array}{l} \leftarrow \text{value of } x \text{ coeff.} \\ \leftarrow \text{value of } y \text{ coeff.} \\ \leftarrow \text{value of } z \text{ coeff.} \end{array}$

* Rotation about Z-axis.



-Positive rotation perpendicular to z-axis

$x' = x\cos\theta - y\sin\theta$

$y' = x\sin\theta + y\cos\theta$

$y' = z.$

Rotation about z-axis

$$R_{\theta z} = \begin{array}{cl} \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} & \begin{array}{l} \leftarrow x \text{ coeff.} \\ \leftarrow y \text{ coeff.} \\ \leftarrow z \text{ coeff.} \end{array} \end{array}$$

$R_{\theta x}$, $R_{\theta y}$ & $R_{\theta z}$ are matrices of transformation for rotation about x, y & z.

(9) Three Dimensional Reflection.

1→ Reflection can be achieved as follows:

To reflect a point P with co-ordinates of system $[x\ y\ z\ 1]$ as $[x'\ y'\ z'\ 1]$.

Case 1: Reflection of P about xy.

$$\therefore R'_{xy} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The reflected points P' with co-ordinates $[x'\ y'\ z'\ 1]$ is,

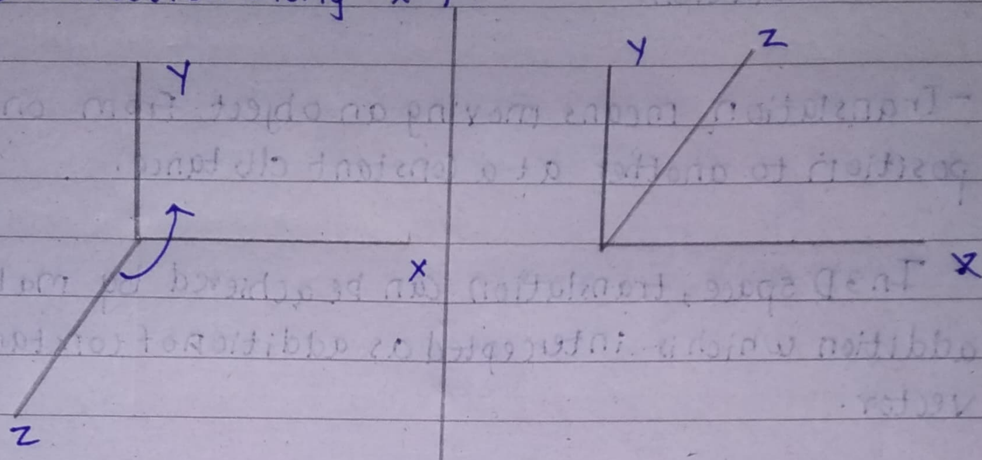
$$[x'\ y'\ z'\ 1] = [x\ y\ z\ 1] \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\therefore [x'\ y'\ z'\ 1] = [x\ y\ -z\ 1]$$

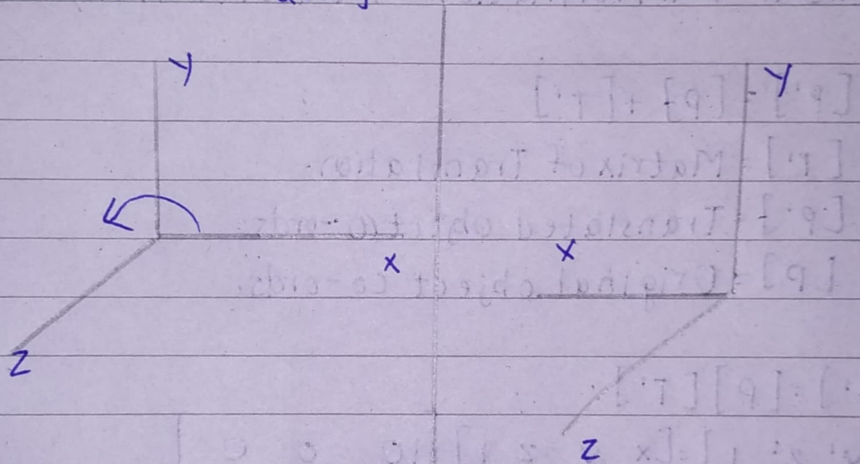
Similarly for xz plane.
$[x'\ y'\ z'\ 1] = [x\ -y\ z\ 1]$
Similarly for yz plane.
$[x'\ y'\ z'\ 1] = [-x\ y\ z\ 1]$.
Reflection about origin,

$$\therefore R'_{xyz} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

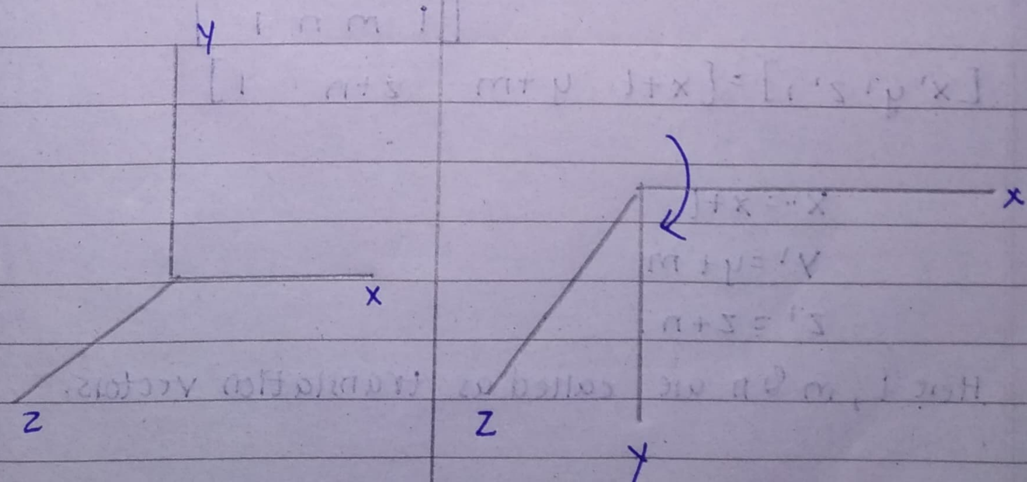
① Reflection along X-Y :-



② Reflection along Y-Z :



③ Reflection along X-Z :



10. 3D - Translation.

- Translation means moving an object from one position to another at a constant distance.

- In 3D space, translation can be achieved by matrix addition which is intercepted as addition of constant vector.

The general transformation of translation is,

$[P'] = [P] + [T']$

$[T']$ = Matrix of Translation.

$[P']$ = Translated object co-ords.

$[P]$ = Original object co-ords.

$\therefore [P'] = [P][T']$.

$$[x' \ y' \ z' \ 1] = [x \ y \ z \ 1] \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ l & m & n & 1 \end{bmatrix}$$

$$[x' \ y' \ z' \ 1] = [x+l \quad y+m \quad z+n \quad 1]$$

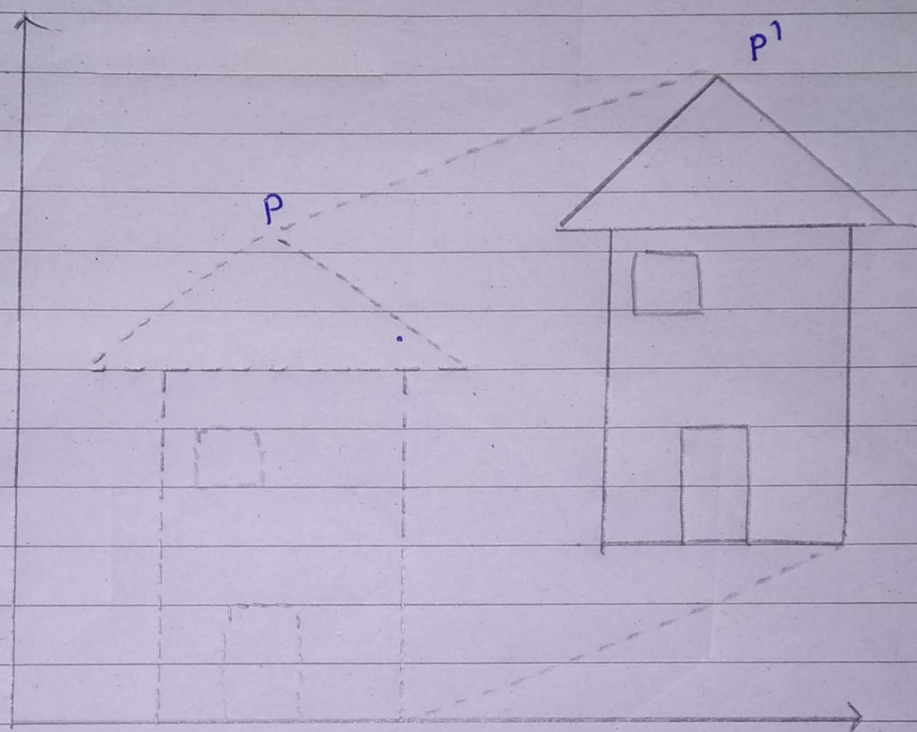
$x' = x + l$

$y' = y + m$

$z' = z + n$.

Here l, m & n are called as translation vectors.

Hence foll, is the result:



3D Translation.