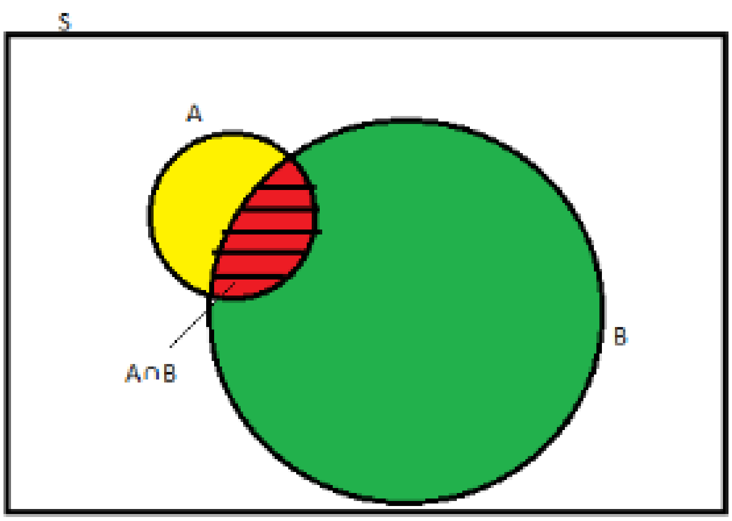
Uncertain Knowledge and Probabilistic reasoning: Quantifying uncertainty: Acting under uncertainty Basic probability notation, Inference using full joint distributions, independence, Bayes' rule and its use, fuzzy logic.

Conditional Probability Definition

The probability of occurrence of any event A when another event B in relation to A has already occurred is known as conditional probability. **It is depicted by P(A|B).**



- As depicted by the above diagram, sample space is given by S and there are two events A and B.
- In a situation where event B has already occurred, then our sample space S naturally **gets reduced to B** because now the chances of occurrence of an event will lie inside B.
- As we have to figure out the chances of occurrence of event A, only portion common to both A and B is enough to represent the probability of occurrence of A, when B has already occurred.

- Common portion of the events is depicted by the intersection of both the events A and B i.e. A ∩ B.

- **The probability of an event A, given B occurred, is called a conditional probability and indicated by**
- The conditional probability is defined as

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}, \text{ for } P(B) \neq 0.$$

Bay's theorem - $p(A|B) = p(B|A).\ p(A)/p(B)$

Ex You've been planning a picnic for your family. You're trying to decide whether to postpone due to rain. The chance of rain on any day is 15%. The morning of the picnic, it's cloudy. The prob. of it being cloudy is 25% and on days where it rains, it's cloudy in the morning 80% of the time. Should you postpone the picnic?

$P(\text{rain}) = 0.15$

$P(\text{cloudy}) = 0.25$

$P(\text{cloudy}|\text{rain}) = 0.80$

$$P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B)}$$

$$P(\text{rain}|\text{cloudy}) = \frac{P(\text{cloudy}|\text{rain}) \cdot P(\text{rain})}{P(\text{cloudy})} = \frac{0.8 \cdot 0.15}{0.25}$$

$$\boxed{P(\text{rain}|\text{cloudy}) = 0.48}$$

Conditional probability Example:

1. Two dies are thrown simultaneously and the sum of the numbers obtained is found to be 7. What is the probability that the number 3 has appeared at least once?

   Solution:

   The sample space S would consist of all the numbers possible by the combination of two dies. Therefore S consists of 6 × 6 i.e. 36 events.

   Find the probability of A - Event A indicates the combination in which 3 has appeared at least once.
   Already occurred - Event B indicates the combination of the numbers which sum up to 7.

   A = {(3, 1), (3, 2), (3, 3)(3, 4)(3, 5)(3, 6)(1, 3)(2, 3)(4, 3)(5, 3)(6, 3)}

   B = {(1, 6)(2, 5)(3, 4)(4, 3)(5, 2)(6, 1)}

   P(B) = 6/36

   A ∩ B = 2

   P(A ∩ B) = 2/36

   Applying the conditional probability formula we get,

$$p(A|B) = p(B|A).\ p(A)/p(B)$$
$$= (2/36\ .\ 11/36\ )\ /\ \ (6/36)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{2}{36}}{\frac{6}{36}} = \frac{1}{3}$$

Example: In a class, there are 70% of the students who like English and 40% of the students who likes English and mathematics, and then what is the percent of students those who like English also like mathematics?

Solution: Let, A is an event that a student likes Mathematics.And B is an event that a student likes English.

$$P(A|B) = \frac{P(A \wedge B)}{P(B)} = \frac{0.4}{0.7} = 57\%$$

Hence, 57% are the students who like English also like Mathematics.

Multiplicative Law of probability (*The multiplication rule of probability explains the condition between two events)* is then defined as -

1. If A and B are **<u>dependent</u>** events, then the probability of both events occurring simultaneously is given by:

    $$P(A \cap B) = P(A \mid B)\, P(B)$$

    which is equivalent to the following

    $$P(A \cap B) = P(B \mid A)\, P(A)$$

2. If A and B are two **<u>independent</u>** events in an experiment, then the probability of both events occurring simultaneously is given by:

$$\mathbf{P(A \cap B) = P(A)\,.\,P(B)}$$

- **Addition law**
- If events A and B are mutually exclusive/independent , then

$$P(A \cup B) = P(A) + P(B)$$

- If events A and B are not mutually exclusive/dependent , then

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

This is also called **Addition law**.

<u>Bayes' theorem:</u>

Bayes' theorem is also known as **Bayes' rule, Bayes' law**, or **Bayesian reasoning**, which determines the probability of an event with uncertain knowledge.

In probability theory, it relates the conditional probability and marginal probabilities of two random events.

Bayes' theorem was named after the British mathematician **Thomas Bayes**. The **Bayesian inference** is an application of Bayes' theorem, which is fundamental to Bayesian statistics.

It is a way to calculate the value of P(B|A) with the knowledge of P(A|B).

Bayes' theorem allows updating the probability prediction of an event by observing new information of the real world.

Bayes' theorem can be derived using *product rule and conditional probability* of event A with known event B:

Example: If cancer corresponds to one's age then by using Bayes' theorem, we can determine the probability of cancer more accurately with the help of age.

As from product rule we can write:

1. $P(A \wedge B) = P(A|B) P(B)$ or

Similarly, the probability of event B with known event A:

1. $P(A \wedge B) = P(B|A) P(A)$

Equating right hand side of both the equations, we will get:

$$\boxed{P(A|B) = \frac{P(B|A)\, P(A)}{P(B)}} \quad \text{....(a)}$$

The above equation (a) is called as **Bayes' rule or Bayes' theorem.** This equation is basic of most modern AI systems for probabilistic inference.

****P(A|B) is known as posterior, which we need to calculate, and it will be read as Probability of hypothesis A when we have occurred an evidence B.

****P(B|A) is called the likelihood, in which we consider that hypothesis is true, then we calculate the probability of evidence.

****P(A) is called the prior probability, probability of hypothesis before considering the evidence.

**** P(B) is called marginal probability, pure probability of an evidence.

<u>Applying Bayes' rule:</u>

Bayes' rule allows us to compute the single term P(B|A) in terms of P(A|B), P(B), and P(A). This is very useful in cases where we have a good probability of these three terms and want to determine the fourth one. Suppose we want to perceive the effect of some unknown cause, and want to compute that cause, then the Bayes' rule becomes:

$$\textbf{P(cause | effect)} = \frac{P(\text{effect}|\text{cause})\, P(\text{cause})}{P(\text{effect})}$$

Example-1:
Question: what is the probability that a patient has diseases meningitis with a stiff neck?

Given Data:
A doctor is aware that disease meningitis causes a patient to have a stiff neck, and it occurs 80% of the time. He is also aware of some more facts, which are given as follows:

- The Known probability that a patient has meningitis disease is 1/30,000.
- The Known probability that a patient has a stiff neck is 2%.

- Let a be the proposition that patient has stiff neck and b be the proposition that patient has meningitis. , so we can calculate the following as:

P(a|b) = 0.8
P(b) = 1/30000
P(a)= .02

$$P(b|a) = \frac{P(a|b)P(b)}{P(a)} = \frac{0.8*(\frac{1}{30000})}{0.02} = 0.001333333.$$

Hence, we can assume that 1 patient out of 750 patients has meningitis disease with a stiff neck.

Example-2:
Question: From a standard deck of playing cards, a single card is drawn. The probability that the card is king is 4/52, then calculate posterior probability P(King|Face), which means the drawn face card is a king card.

Solution:

$$P(king|face) = \frac{P(Face|king)*P(King)}{P(Face)} \quad ....(i)$$

P(king): probability that the card is King= 4/52= 1/13

P(face): probability that a card is a face card= 3/13

P(Face|King): probability of face card when we assume it is a king = 1

Putting all values in equation (i) we will get:

$$P(king|face) = \frac{1 * (\frac{1}{13})}{(\frac{3}{13})}$$ **= 1/3, it is a probability that a face card is a king card.**

Probability and Baye's Theoram

https://www.javatpoint.com/bayes-theorem-in-artifical-intelligence

https://medium.com/swlh/uncertain-knowledge-and-reasoning-831ee709930d