Unit-3

Constraint Satisfaction Problems
Chapter-6

Constraint Satisfaction problem (CSP)

CSP consists of three components (X,D,C)

-> X is a set of Variable(x_1, x_2, ..., x_n)

-> D is a set of Domains (D_1, D_2, ..., D_n) one for each variable

-> C is a set of constraints that satisfies allowable combination of values.

C_i= (Scope, Rel)

Where **Scope** is set of variables that participates in constraint, **Rel** is relation that defines the values that variable can take.

Variables

The simplest kind of CSP involves variables that have **discrete, finite domains.**

Constraint satisfaction problems with **continuous** domains are common in the real world and are widely studied in the field of operations research.

For example, the scheduling of experiments on the Hubble Space Telescope requires very precise timing of observations;

The best-known category of continuous-domain CSPs is that of linear programming problems, where constraints must be linear equalities or inequalities.

Constraints

In addition to examining the types of variables that can appear in CSPs, it is useful to look at the types of constraints.

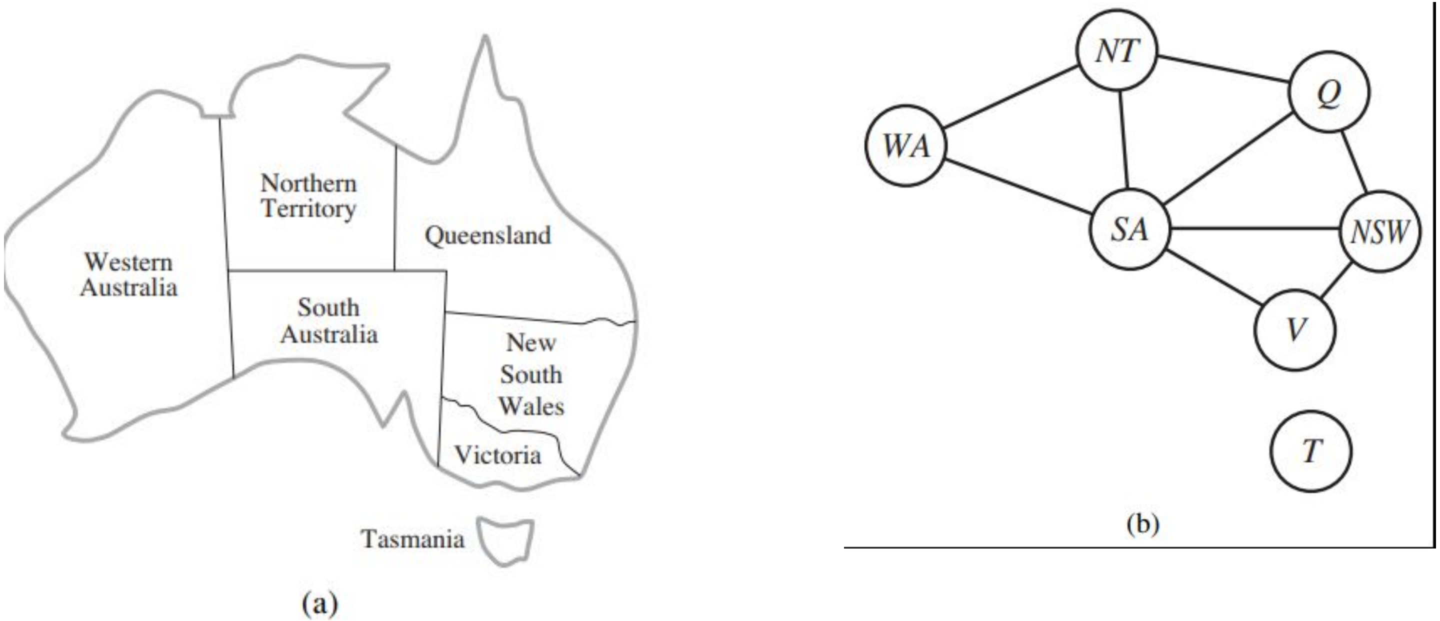
The simplest type is the **unary constraint**, which restricts the value of a single variable.

For example, in the map-coloring problem it could be the case that South Australians won't tolerate the color green; we can express that with the unary constraint **<(SA), SA ≠ green>**

A binary constraint relates two variables. For example, SA ≠ NSW is a binary constraint.

A binary CSP is one with only binary constraints; it can be represented as a constraint graph,

Figure 6.1



The nodes of the graph correspond to variables of the problem, and a link connects any two variables that participate in a constraint.

Constraints

A constraint involving an arbitrary number of variables is **called a global constraint. One of the most common global constraints is Alldiff, which says that all of the variables involved in the constraint must have different values.**

Example-

1. In Sudoku problems (see Section 6.2.6), all variables in a row or column must satisfy an Alldiff constraint.
2. An another example is provided by cryptarithmetic puzzles.

Constraint Propogation: Inference in CSPs

In regular state-space search, an algorithm can do only one thing: **search**.

In CSPs there is a choice: an algorithm can search (choose a new variable assignment from several possibilities) or do a specific type of **inference called constraint propagation: using the constraints to reduce the number of legal values for a variable, which in turn can reduce the legal values for another variable, and so on.**

The key idea is local consistency. Types of Consistency - Node , Arc, Path, K- consistency

Types of Consistency

6.2.1 Node consistency -

A single variable (corresponding to a node in the CSP network) is node-consistent if **all the values in the variable's domain** satisfy the **variable's unary constraints**.

For example, in the variant of the Australia map-coloring problem (Figure 6.1) where **South Australians** dislike **green**, the variable SA starts with domain **{red, green, blue}**, and we can make it node consistent by **eliminating green**, leaving SA with the reduced domain **{red, blue}.**

We say that a network is node-consistent if **every variable in** the network is node-consistent.

6.2.2 Arc consistency

A variable in a CSP is arc-consistent if every value in its domain satisfies the variable’s binary constraints.

Backtracking Search

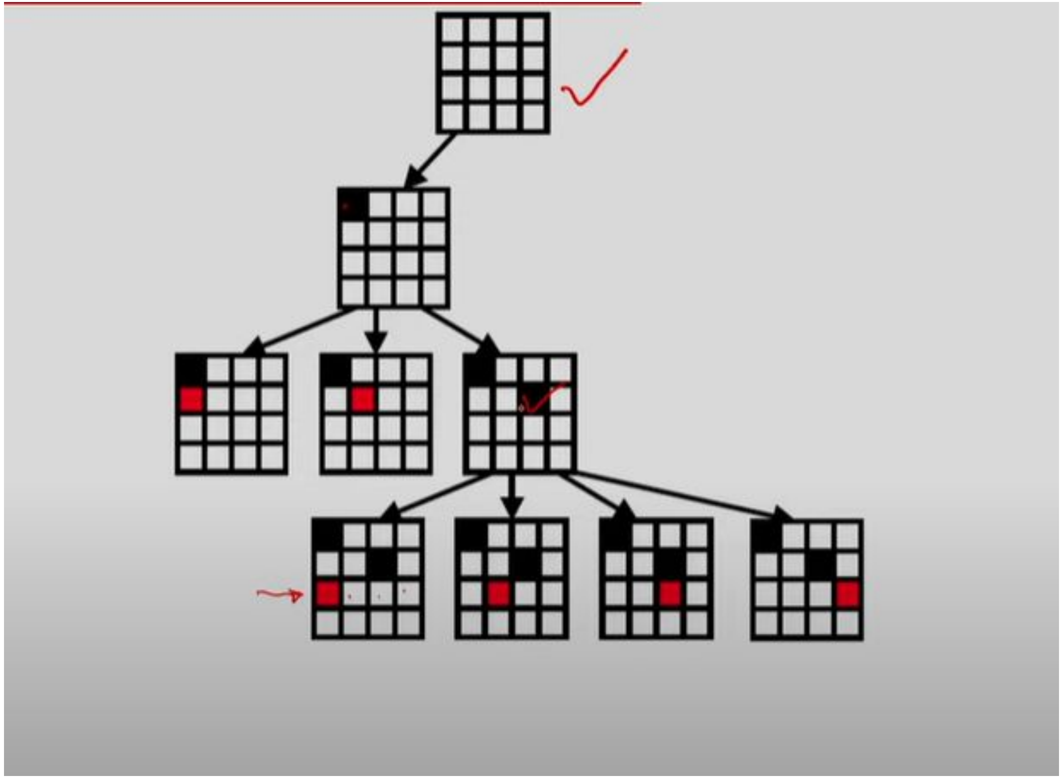
CSP can be **solved by a specialized version of depth first search.**

If **during the process of building up a solution**, we **falsify a constraint**, we can immediately **reject all possible ways of extending the current partial assignment.**

The simplest kind of CSP involves variables that have discrete, finite domains

Backtracking Search - Example- 4 Queen

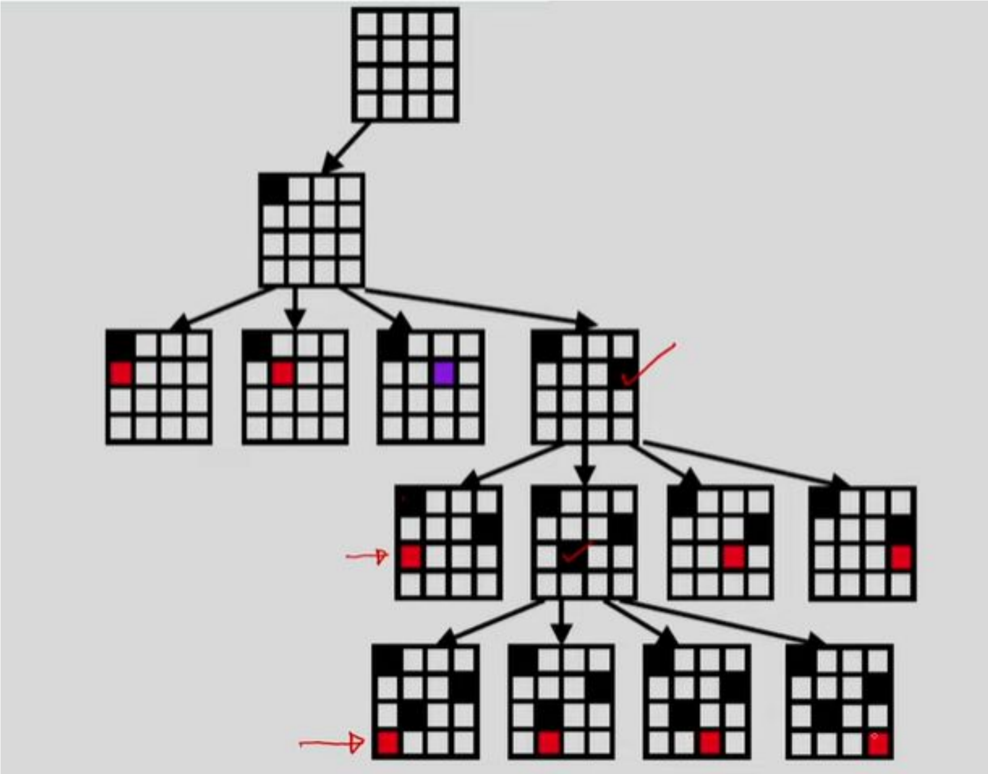
We have realized at this point is - that after we have assigned the second queen, we are not left with any legal position for the third queen. So we have to **backtrack**.



Backtracking Search - Example- 4 Queen

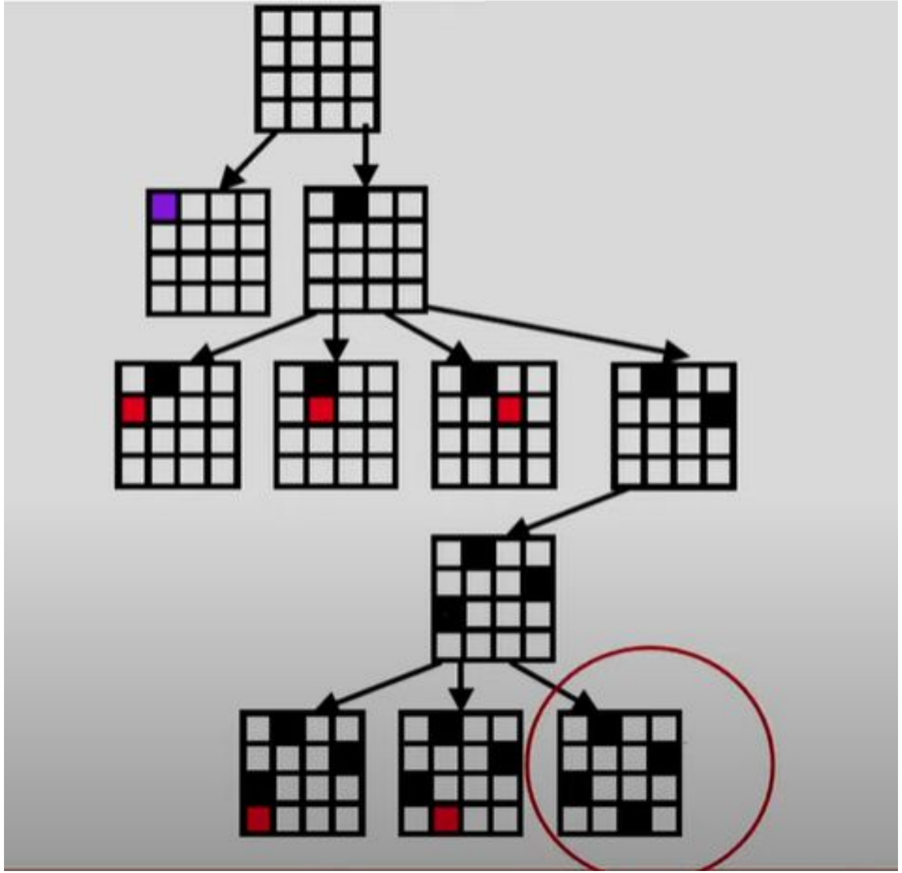
We remember what we started with-

Again we backtracked.



Backtracking Search - Example- 4 Queen

- We backtrack to the first position for **first quent.**
- Plain **backtracking** is **uninformed algorithm**.
- **Not very effective** for large problems.



Improving Backtracking Search

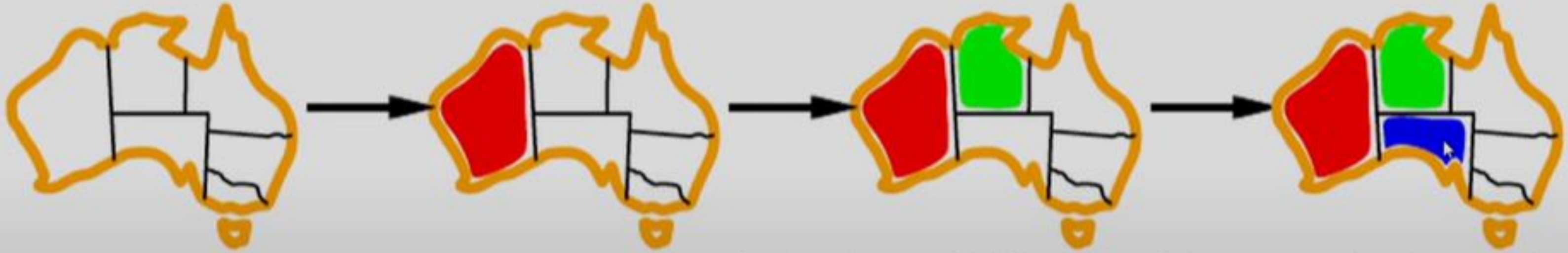
- We can solve CSPs efficiently without such domain-specific knowledge; general-purpose methods that address the following questions:
  1. Which variable should be assigned next, and in what order should its values be tried?
  2. What are the implications of the current variable assignments for the other unassigned variables?
  3. When a path fails i.e., a state is reached in which a variable has no legal values; can the search avoid repeating this failure in subsequent paths?

Improving Backtracking Search

- General-purpose methods can give huge gains in speed:
- Ordering:
  - Which variable should be assigned next?
  - In what order should its values be tried?
- Filtering:
  - Can we detect inevitable failure early?
- Structure:
  - Can we take advantage of problem structure?

Variable and Value Ordering

- Minimum remaining values (MRV):
  - choose variable with the fewest legal values.



After the assignments for WA=red and NT =green, there is only one possible value for SA, so it makes sense to assign SA=blue next rather than assigning Q.

It also has been called the "most constrained variable" or "fail-first" heuristic, the latter because it picks a variable that is most likely to cause a failure soon, thereby pruning the search tree.

Variable and Value Ordering

- Degree Heuristics:
  - choose the variable with the most constraints on remaining vars.

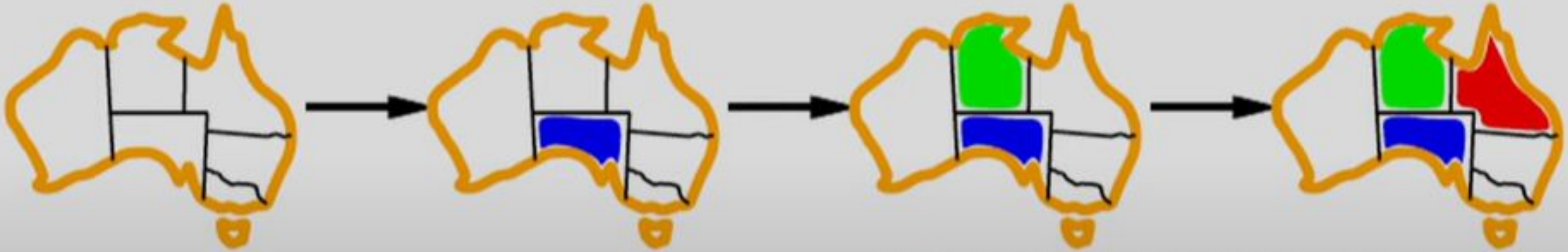
The MRV heuristic does not help at all in choosing the first region to colour in the above map, because initially every region has three legal colours.

The degree heuristic comes in handy!

It attempts to reduce the branching factor on future choices by selecting the variable that is involved in the largest number of constraints on other unassigned variables.

Variable and Value Ordering

- Degree Heuristics:
  - choose the variable with the most constraints on remaining vars.

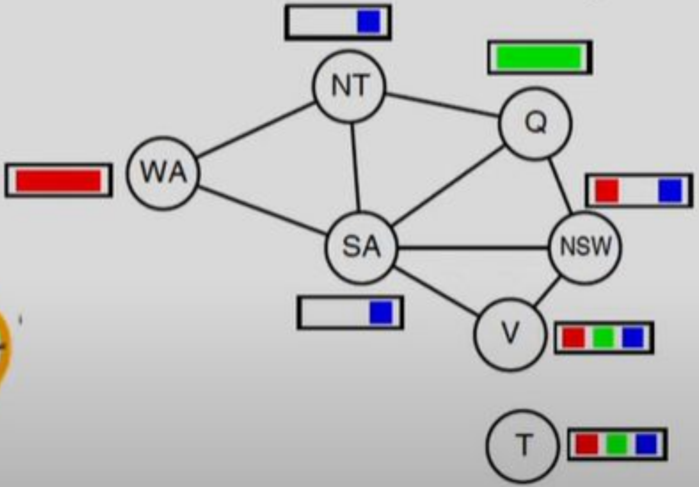


SA is the variable with highest degree, 5; the other variables have degree 2 or 3, except for T, which has 0.

Applying the degree heuristic solves the problem without any false steps —choose any consistent colour at each choice point and still arrive at a solution with no backtracking.

Constraint Propogation

Propagate the implications of a constraint on one variable onto other variables to detect inconsistency.



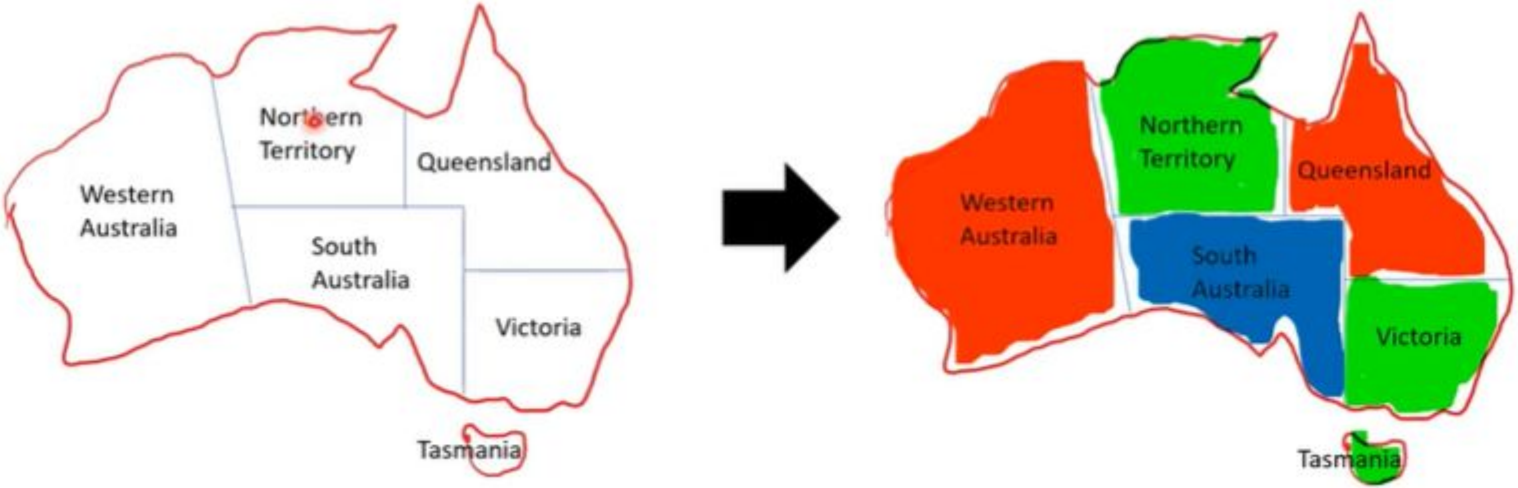
FC propagates information, but doesn't provide early detection for all failures:

NT and SA cannot both be blue!

Constraint propagation repeatedly enforces constraints locally.

Map coloring problem using Constraint Satisfaction

The goal is to assign colors to each region so that no neighbouring regions have the same color



Map coloring problem using Constraint Satisfaction

- Color the following map using red, green and blue such that adjacent regions have different colors.
- Variables: { WA, NT, Q, V, SA, T}
- Domains: {red, green, blue}
- Constraints: adjacent regions must have different colors.
- Eg, WA ≠ NT, WA ≠ SA

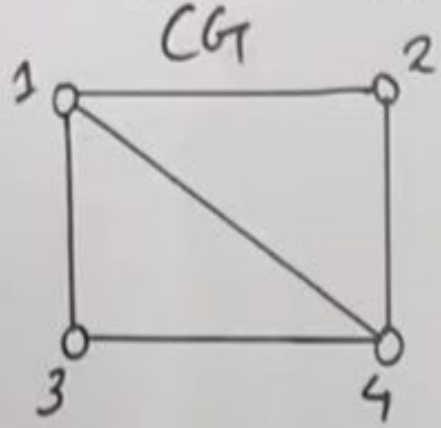
Map coloring problem using Constraint Satisfaction

	WA	NT	SA	Q	V	T
Initial Domain	R, G, B	R, G, B	R, G, B	R, G, B	R, G, B	R, G, B
WA=R	R	G, B	G, B	R, G, B	R, G, B	R, G, B
NT=G	R	G	B	R	G	R, G, B
T=R	R	G	B	R	G	R

Example-

CSP

'Constraint Satisfaction Problem (CSP)' Backtracking



Intell.

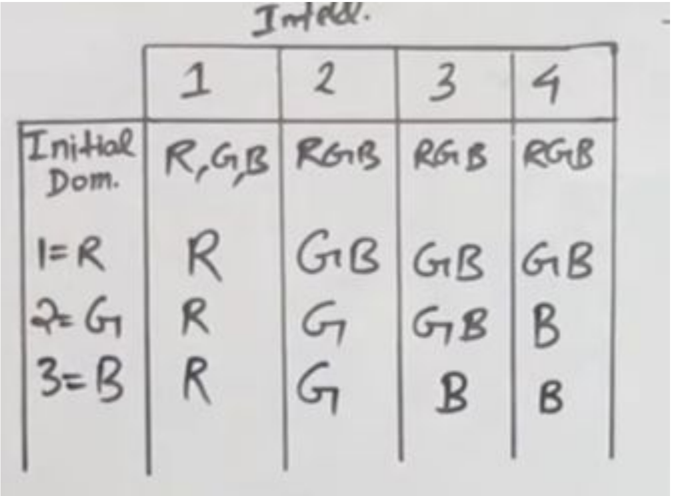
	1	2	3	4
Initial Dom.	R,G,B	RGB	RGB	RGB
1=R	R	GB	GB	GB

$V = \{1, 2, 3, 4\}$

$D = \{\text{Red, Green, Blue}\}$

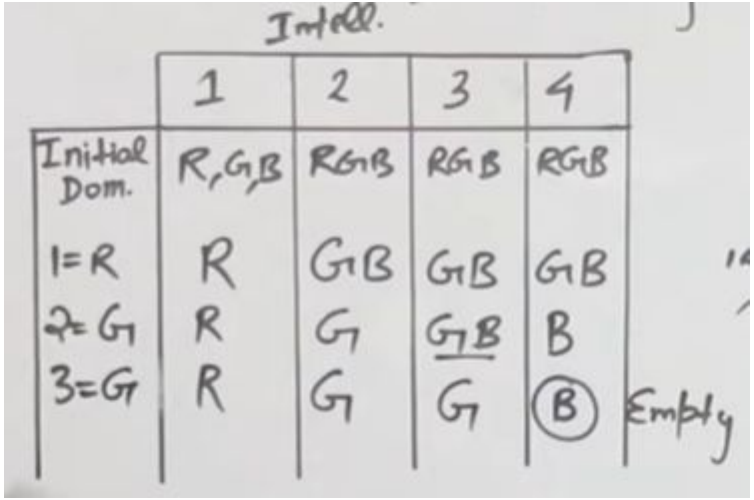
$C = \{1 \neq 2, 1 \neq 3, 1 \neq 4, 2 \neq 4, 3 \neq 4\}$

Example - CSP



Intell.

	1	2	3	4
Initial Dom.	R,G,B	RGB	RGB	RGB
1=R	R	GB	GB	GB
2=G	R	G	GB	B
3=B	R	G	B	B



Intell.

	1	2	3	4	
Initial Dom.	R,G,B	RGB	RGB	RGB	
1=R	R	GB	GB	GB	
2=G	R	G	GB	B	
3=G	R	G	G	B	Empty

Cryptarithmetic problem

https://www.youtube.com/watch?v=HC6Y49iTg1k

https://www.youtube.com/watch?v=dmxNbDDxCeE

Cryptarithmeticproblemhttps://www.youtube.com/watch?v=HC6Y49iTg1khttps://www.youtube.com/watch?v=dmxNbDDxCeE

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Crypt arithmetic questions: INFOSYS and ELITMUS (easy and medi

Crypt-Arithmetic Problem

In simpler words, the crypt-arithmetic problem deals with the converting of the message from the readable plain text to the non-readable ciphertext. The constraints which this problem follows during the conversion is as follows:

1. A number 0-9 is assigned to a particular alphabet.
2. Each different alphabet has a unique number.
3. All the same, alphabets have the same numbers.
4. The numbers should satisfy all the operations that any normal number does.

Let us take an example of the message: SEND MORE MONEY.

```
      S E N D
      M O R E
  -------------
    M O N E Y
```

Cryptarithmetic problem

TO + GO =OUT, find the value of O+U+T

O = 1
T = 2
G = 8
U = 0

$$
\begin{array}{ccc}
 & T & O \\
+ & G & O \\
\hline
O & U & T
\end{array}
\qquad
\begin{array}{ccc}
 & 2 & 1 \\
+ & 8 & 1 \\
\hline
1 & 0 & 2
\end{array}
$$

$T + G = U + 10$

$2 + G = U + 10$

$1 + 0 + 2 = 3$

$$
\begin{array}{ccc}
 & 2 & 1 \\
 & 9 & 1 \\
\hline
1 & 1 & 2
\end{array}
$$

$9 + 2 = 1 + 10$

T G O U

$2 + 8 = 10$

$2 + 9 = \cancel{11}$

Cryptarithmetic problem

IS + THIS =HERE, find the value of T*E+I*R*H-S

$$\begin{array}{cccc} \overset{1}{T} & \overset{1}{H} & I & S \\ & & I & S \\ \hline H & E & R & E \end{array}$$

T = 8, H = 9, E = 0

8+1 = ✗

7+1 = ✗

9+1 = ✓

~~E = 0, 2, 4, 6, 8~~

T+1 = H

$$\begin{array}{cccc} \overset{1}{8} & \overset{1}{9} & \overset{1}{I}\,6 & S\,5 \\ & & I\,6 & S\,5 \\ \hline 9 & 0 & R\,3 & E\,0 \end{array}$$

H = 9

T = 8

E = 0

S = 5

$$\begin{array}{c} 7 \\ 7 \\ \hline 14 \end{array} \qquad \begin{array}{c} 8 \\ 8 \\ \hline \end{array}$$

Cryptarithmetic problem

Constraint Satisfaction- Cryptographic

THIS+IS=HERE

```
   FAT
+  CAB
------
  SCBT
```

```
   FAT
+  CAB
------
  SCBT
```

```
    A
    A
+   A
-----
   BA
```

```
   AB
+  CB
-----
  BBA
```

Questions Unit 3

1. Consider a problem of map coloring. Choose a CSP formulation. In your formulation, what are the variables?
2. How many solutions are there for the map-coloring problem in Figure 6.1? How many solutions if four colors are allowed? Two colors?
3. Solve the cryptarithmetic problem
4. What is constraint propagation, constraint propagation, and   different types of local consistency - **Types of Consistency - Node , Arc, Path, K-consistency.**
5. Define in your own words the terms constraint, backtracking search, arc consistency,,

6.7 Consider the following logic puzzle: In five houses, each with a different color, live five persons of different nationalities, each of whom prefers a different brand of candy, a different drink, and a different pet. Given the following facts, the questions to answer are “Where does the zebra live, and in which house do they drink water?

House	Color	Nationality	Candy	Drink	Pet
1	Yellow	Norwegian	Kit Kats		
2	Blue				
3	Red	Englishman			
4	Ivory				
5	Green				

From clue 1, we know the Englishman lives in the red house, so he must be in house 3. From clue 6, we know Kit Kats are eaten in the yellow house, so they must be in house 1.

We can continue this process of deduction to fill in the rest of the table. The final solution will depend on the rest of the clues given in the puzzle.