Multi-scale Fractal Dimension Analysis of Vein Network, Margin and Colour Texture in Mangrove Species Identification

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Abstract. In this study, the colour texture, margin, and vein networks of three different mangrove species—*Avicennia marina*, *Avicennia officinalis*, and *Rhizophora apiculata* are analysed comprehensively for leaf morphology and health using Multi-Scale Fractal Dimension (MFD) analysis. The main goal is to develop an accurate and quantitative method for distinguishing between patterns unique to a species and subtleties in leaf structures. Using Multi-scale Fractal Dimension, we have simulated the leaf venation topologies of three kinds of mangrove plants in this study. To study the interconnections and complexities of each colour channel across scales, we also constructed each channel as a surface using MFD analysis. This strategy outperforms existing methods in providing a detailed multi-dimensional evaluation of leaf texture, margin, and venation. The outcome demonstrates that, even though the differences might not be very noticeable, there are anomalies that can be brought to light more clearly using multi-scale research, enabling a more precise perception of these traits for better identification at the species level and ecological adaptations. Our results demonstrate that our methodology guarantees high accuracy when assessing leaf morphology for purposes of environmental monitoring or botanical research. Cross Validation using a raw sample of *Avicennia marina* proves the superiority of MFD analysis of leaf vein networks in Mangrove species identification.

Keywords: Fractal, Mangroves, Remote Sensing, Texture Analysis, Leaf Pattern.

1 Introduction

A mathematical method called multi-scale fractal analysis (MSFA) looks at how fractal features, such as fractal size, vary between scales to analyze intricate, asymmetrical patterns and structures. Fractal dimensions are a measure of complexity that varies depending on the measurement scale and are a defining characteristic of self-similar fractals. MSFA uses multi-fractal analysis, wavelet transform, and box-counting approach among other techniques to capture these fluctuations. Applications in biology, ecology, geophysics, medicine, and finance provide insights into the hierarchical structure of structures. Studies that have compared the Bouligand-Minkowski Fractal Dimension method to the conventional box-counting method have investigated the impact of this approach on visual pattern detection. [12] proposed an approach to increase the box-counting method's accuracy that is based on intervals and mathematical definitions. [8] presented an enhanced differential technique that tackles problems like excessive and insufficient box counting for determining the fractal dimension of gray-scale images. [11] demonstrated the method's competitive performance by combining local binary patterns with fractal measurements for texture categorization. A generalized box-counting method was created by [7], and it performed better than previous approaches in terms of accuracy and adaptability to different image sizes and shapes. A gray-level shift-invariant differential box-counting technique was presented by [8], and it performed better than previous techniques. This was further improved by [8], who also introduced a new xy-plane shifting mechanism, a changed formula for determining nr, and three improved approaches based on the eigenvalue, kurtosis, and skewness of a picture. The probabilistic box-counting approach was improved by [5] using an umbra, which boosted the accuracy of fractal dimension estimation. [12] proposed a technique centered on intervals and mathematical definitions to remove each box scale's variance and produce more accurate findings. An improved method for determining surface roughness was presented by [7] using a generalized box-counting technique. Similar to this, [11] produced competitive results for texture classification by combining local binary patterns with fractal measurements.

2 Background Study

The examination of multi-scale Fractal Dimensions (MFD) has become a viable technique for evaluating leaf morphology and health. Research has demonstrated that MFD can enhance plant identification based on leaf texture [1] and successfully distinguish leaf venation topological patterns in medicinal plants [10]. High accuracy in species differentiation has been demonstrated by the technique's successful application to the analysis of both interior (veins) and external (outline) leaf shapes [9]. The application of Multi-Scale Fractal Dimension (MFD) analysis for leaf morphology and plant identification has been studied in the past. MFD has been used to analyze leaf texture for plant species classification [6] and leaf venation patterns of medicinal plants [10]. Research has demonstrated that MFD is superior to conventional texture analysis techniques in its ability to distinguish between distinct plant species [1]. Accurate plant identification has been further enhanced by the use of color information with MFD [1]. Furthermore, when paired with other biometric measurements, fractal dimension-based features are effective descriptors of leaf shape and complexity, achieving over 90% accuracy in species recognition [9]. Although these studies demonstrate the promise of MFD in leaf analysis and plant identification, further research is required to enhance and broaden its usage across diverse plant species and environmental conditions. Particularly for intricate leaf features, fractal analysis techniques have demonstrated the potential to differentiate between plant species. Over 93% accuracy in recognizing Brachiaria species has been attained by the use of fractal descriptors in conjunction with machine learning approaches [4]. According to [9], the multi-scale Minkowski fractal dimension has also shown effective in distinguishing between Passiflora species according to their leaf morphology. Scale constraints, quantization mistakes, and the inability to capture lacunarity are among the drawbacks of the conventional box-counting fractal analysis in leaf complexity investigations [6]. Although fractal dimensions for pattern identification can be obtained by Fourier analysis, various patterns may have comparable dimensions, requiring additional techniques for separation [2]. Fractal-based methods are useful for classifying and identifying plants despite these difficulties because they have benefits including immunity to viewing angle and range [4] [2]. There is a growing interest in employing Gate-based (GTE) fractal techniques in mangrove dynamics research. In [3] the unique approach seeks to offer information on the ecological stability and diversity of mangrove ecosystems by examining their complicated spatial patterns. The research [3] employs a GTE fractal dimension evaluation methodology, incorporating sophisticated techniques such as Exponential Dilation and Shadow Removal to enhance edge recognition and minimize noise in images, thus augmenting the precision of habitat complexity evaluations. To comprehend the spatial heterogeneity, diversity, and long-term durability of the mangrove habitats, fractal dimensions, and Hurst exponent values are determined. The enhanced stability and dependability of the modified gate-based fractal analysis method over conventional box-counting methods yields a stronger comprehension of the intricacy and heterogeneity of these environments. Two main focuses of this research are: (1) the development of a quantitative method for identifying species-specific patterns and subtleties in leaf structures; and (2) the thorough analysis of leaf morphology and health using Multi-Scale Fractal Dimension (MFD) analysis, which looks at the colour texture, margin, and vein networks of three mangrove species: Avicennia marina, Avicennia officinalis, and Rhizophora apiculata. A thorough multi-dimensional assessment of leaf texture, margin, and venation is provided by the study's simulation and evaluation of these species' leaf venation topologies, which are achieved by building each colour channel as a surface using MFD analysis. The aim is to seek the performance of these approaches over the current approaches in identifying features peculiar to a species and its ecological adaptations.

3 Dataset Description

This research utilized an internet-sourced mangrove dataset containing numerous images of *Avicennia officinalis* and *Rhizophora apiculata*. Five images from each species were analyzed for vein network and leaf margin patterns, alongside color texture analysis. Additionally, a raw sample of *Avicennia marina*, collected from the Sundarbans region in Kaikhali, Gopalganj, West Bengal, India, was used for color texture analysis and cross-validation of the leaf vein network analysis. The authenticity of this *Avicennia marina* sample was confirmed through a geotagged image of the collection site.The dataset consists of several leaf photos that have metadata, including GPS positions, and are tagged with the names of the corresponding species. This data offers a useful but trustworthy resource for ecological assessments and comparative analyses, as well as support for more research in mangrove conservation, sustainable development, and occupancy.

(a) Avicennia marina

(b) Location from where the dataset was collected

Fig. 1: Comparison of Avicennia marina and Avicennia officinalis leaves

Fig. 2: Leaves of *Avicennia officinalis*

Fig. 3: Leaves of *Rhizophora apiculata*

4 Methodology

This paper proposes modelling of leaf color texture, vein networks and outline, using the Bouligand-Minkowski fractal dimension through multiscale analysis. The results of the experiment are presented with plot analysis, demonstrating the effectiveness of this method.

4.1 Multi-Scale Fractal Dimension

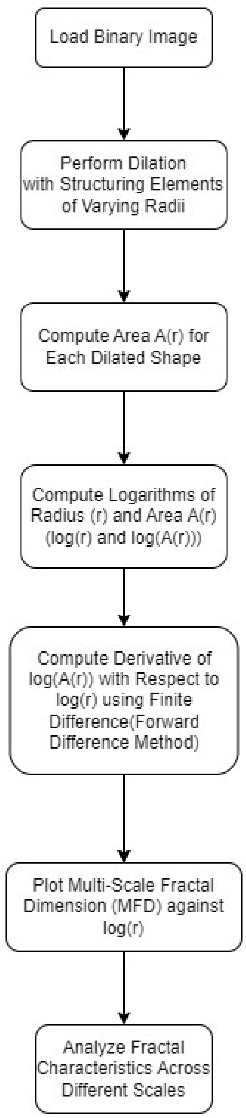


Fig. 4: Process of Generating MFD Curves

The Bouligand-Minkowski Fractal Dimension method depends on studying the influence area created by shape dilation using a disc of radius(r). Small modifications in the shape result in changes to the computed influence area A(r).

Consider $A \in R^2$ as the shape under analysis. The dilation of A(r), is defined as the set of points in R^2 whose distance from A is smaller than or equal to r.

$$A(r) = \{x \in R^2 | \exists y \in A : |x - y| \leq r\} \quad (1)$$

Hence, the Bouligand-Minkowski fractal dimension D can be given as

$$D = 2 - \lim_{x \to 0} \frac{log\ A(r)}{log\ r} \quad (2)$$

Some shapes allow their points to be freely dilated, while in other points this dilation becomes saturated at some radius. This behaviour provides the Bouligand-Minkowski log-log curve with a richness of details that cannot be captured by a single numeric value, as is done with line regression. By using the derivative, it is possible to find a function that relates changes in the Fractal Dimension to changes in the dilation radius. This function is called the Multi-Scale Fractal Dimension (MFD), and it is defined as follows:

$$D(r) = 2 - \frac{d\ log\ A(r)}{d\ log\ r} \quad (3)$$

where $D(r)$ represents the complexity of the object at scale r.

The derivative of a function $u(t)$ with respect to a variable t using forward difference approximation is given as follows:

$$\frac{du}{dt} \approx \frac{\Delta u}{\Delta t} = \frac{u(t + \Delta t) - u(t)}{\Delta t} \tag{4}$$

Here $u(t)$ corresponds to $log\ A(r)$ and (t) corresponds to $log\ r$.

In the research, a multi-scale approach is employed. First, the image is uploaded and converted to a binary image. Then it is followed by the dilation of the binary image using a structuring element of varying radii. Finite Difference method, specifically Forward difference approximation is used to calculate the derivative of the logarithms for areas and their corresponding logarithmic radius, where the ratio of the successive-log transformed Area values and their corresponding log-transformed radius is calculated for the derivation. Now the Multi-Scale Fractal Dimension (MFD) is plotted against the logarithm of the radius, which provides a clear view of the fractal features across different scales. This methodology integrates dilation operations, log-log analysis, and finite difference methods to robustly assess the fractal properties of binary images.

4.2 Colour Texture Analysis

The technique for colour texture analysis of leaves processes the image's colour channels through several steps. Firstly, we load the input image and separate it into its red (R), green (G), and blue (B) components. For each colour channel, Gaussian blur is applied in order to reduce noise and smooth out the image.

$$g_\sigma(t) = \frac{1}{\sigma\sqrt{2\pi}} \exp \frac{-t^2}{2\sigma^2} \tag{5}$$

Here, $g_\sigma(t)$ is the Gaussian filter applied to t which refers to $log\ (r)$ and σ here represents standard deviation. Then, depending on a preset threshold value, the blurred image is converted into a binary image via the thresholding process. The binary image is then subjected to dilatation to emphasize its most notable features. This meticulous process is applied independently to the red, blue, and green channels, allowing for a comprehensive analysis of the leaf's colour and texture.

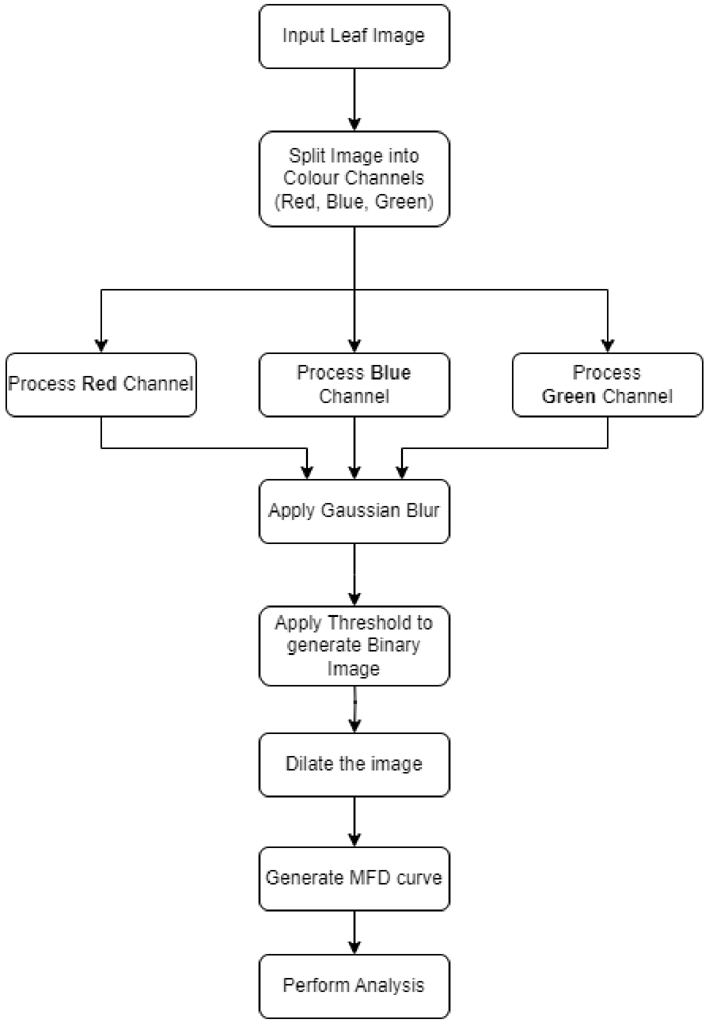


Fig. 5: Steps for Colour Texture Analysis

4.3 Leaf Vein Network and Leaf Margin Analysis

The input image is initially loaded in grayscale, whose conversion formula is:

$$Gray = 0.299\ R + 0.587\ G + 0.114\ B \tag{6}$$

Here, R, G and B refer to red, green and blue respectively.

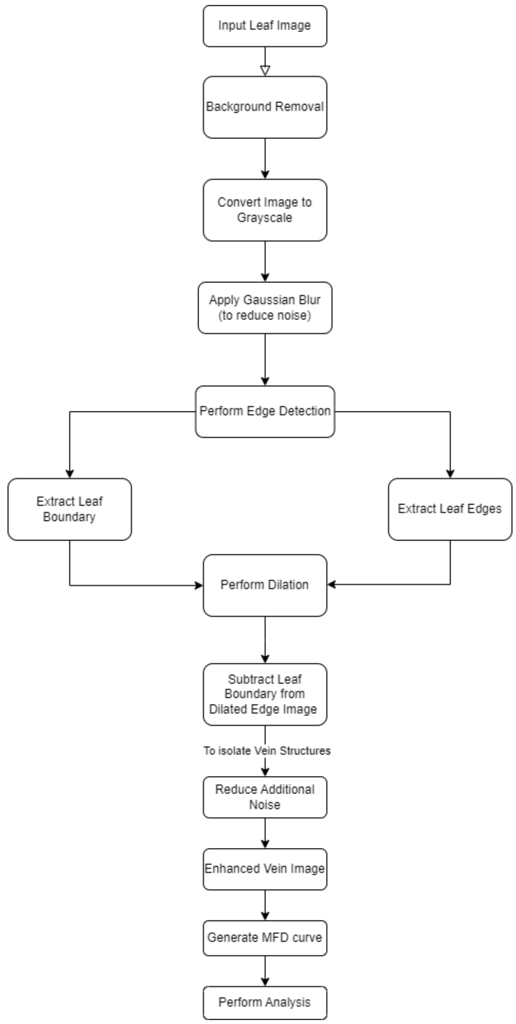


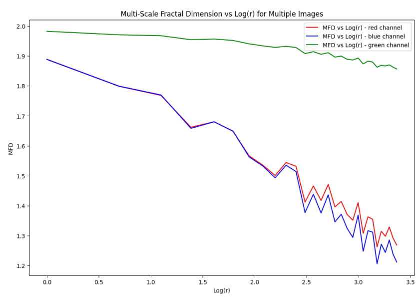
Fig. 6: Steps for Vein Network and Margin Analysis

Noise is then reduced by applying Gaussian blur using formula 5. The leaf's border and veins are then identified by applying the edge detection algorithm with different values for the parameters. The detected edges are dilated to thicken the vein structures. The leaf boundary is eliminated from the dilated edge image to isolate the veins, which are then reinforced by dilation. The vein-only image that is produced has contours that have been recognized and filtered based on their area to eliminate small, irrelevant details. A mask for the filtered contours is created and applied to the vein-only image to isolate significant vein structures. This method effectively highlights the vein structures of the leaf, facilitating further analysis or feature extraction in research focused on plant morphology and health.

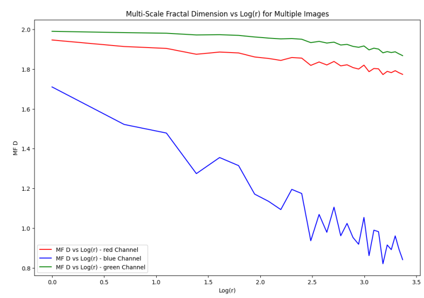
5 Results and Discussion

5.1 MFD analysis of Leaf Colour Texture

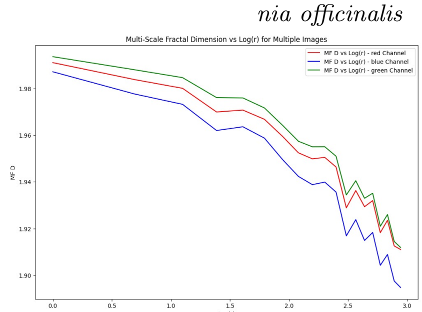
Each color channel in a leaf texture has been modeled as a surface, so the complexity in each channel, as well as the interaction among these three color channels, can be analyzed using the Multi-scale fractal dimension.



(a) Color Texture Analysis of *Avicennia marina*



(b) Color Texture Analysis of *Avicennia officinalis*



(c) Color Texture Analysis of *Rhizophora apiculata*

Fig. 7: Colour Texture Analysis of Avicennia and Rhizophora species

Graph 7a provides the textural analysis for each of the channels of the leaf from *Avicennia marina*. It indicates that red and blue channels behave similarly, with MFD reducing as scale increases. Red and blue channels have moderate complexity with mean MFD values of 1.2905 and 1.2781 respectively, and high variability with standard deviations of 0.1960 and 0.2031 respectively; they also exhibit significant fluctuations, with average absolute rates of change being 3.7373 and 4.5248 respectively meaning a decline in complexity at larger scales.

On the other hand, green channel with a mean MFD value of 1.8324 demonstrates higher complexity, it has low variability (standard deviation=0.0680) and minimal fluctuations with a mean absolute rate of change of 0.5968, suggesting that its texture retains complexity better and exhibits a more uniform pattern across scales. Analysis of the graph 7b on Multi-Scale Fractal Dimension (MFD) curve for leaf of *Avicennia officinalis* shows that green channel has the highest and most stable complexity with a mean MFD of 1.95, low standard deviation of 0.023 implying more consistent complexity and minimal fluctuations (0.181), implying consistent texture across scales. The red channel also demonstrates high complexity with a mean MFD of 1.80, moderate variability with standard deviation of 0.053 and slight fluctuations (0.688), suggesting some texture simplification with scale. In contrast, blue channel has a low mean MFD value as 1.17, the highest standard deviation of 0.205 showing greater variation in texture patterns and substantial fluctuations (5.281), indicating significant texture complexity reduction at larger scales. According to the Multi-Scale Fractal Dimension (MFD) analysis of graph 7c, all color channels of the leaf of *Rhizophora apiculata* begin with high initial complexity. The green channel shows the highest complexity (mean: 1.9513) and the most consistent texture (std: 0.0249) with the lowest rate of change (fluctuation: 0.1677) suggesting the lowest loss of texture. The red channel has a high complexity (mean: 1.9329) and slightly more variability (std: 0.0282), with a moderate rate of change (fluctuation: 0.2068) implying a comparatively higher loss of texture. The blue channel, starting with the lowest complexity (mean: 1.8970), exhibits the greatest variability (std: 0.0375) and the highest rate of change (fluctuation: 0.3092), indicating a rapid loss of texture complexity at larger scales.

Table 1: Comparison of statistics of Colour Channels

Species Names	Red Channel			Blue Channel			Green Channel		
	μ	σ	Fluctuations	μ	σ	Fluctuations	μ	σ	Fluctuations
Avicennia marina	1.29	0.20	3.74	1.28	0.20	4.52	1.83	0.07	0.60
Avicennia officinalis	1.80	0.05	0.69	1.17	0.20	5.28	1.95	0.02	0.18
Rhizophora apiculata	1.93	0.03	0.21	1.90	0.04	0.31	1.95	0.02	0.17

Here, μ refers to the mean of MFD values and σ indicates the standard deviation of MFD values for each colour channel.

5.2 MFD analysis of Leaf Margin

Unique species-specific patterns can be captured using the MFD curves, which provide a detailed and quantifiable method making it possible to analyze the leaf outline complexity across scales. Machine learning techniques, using the characteristic features of these curves, can be used to develop a robust system that can be used for the identification of leaf species based on leaf morphology.

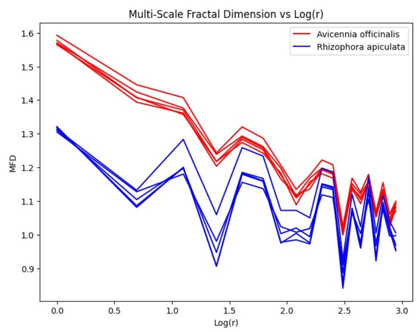


Fig. 8: Leaf Margin analysis of *Avicennia officinalis* and *Rhizophora apiculata*

Higher MFD values for *Avicennia officinalis* leaves are shown in graph 8, with mean values ranging from 1.186 to 1.217. These high figures indicate that the leaves are more complex and possess more detailed outlines at different scales. The standard deviation for *Avicennia officinalis* (0.134 - 0.139) reflects a rather consistent complexity of its margin. Also, lower fluctuation values (3.219 - 3.722) imply uniformity of its margin across scales as it retains complexity better. On the other hand, *Rhizophora apiculata* leaves have lower MFD values, with mean values between 1.055 and 1.119, suggesting that they are less complex when compared to *Avicennia officinalis*. The higher standard deviation (0.090 - 0.118) and greater fluctuation values (5.451 - 8.413) for *Rhizophora apiculata* leaves indicate more variation and considerable fluctuations in their leaf margin patterns respectively. These qualities cause sudden changes in outline complexity due to certain traits or irregularities resulting into higher peaks and deeper troughs. Thus, *Rhizophora apiculata* leaves are characterized by high irregularity and diversity among leaf outline patterns. This technique can help researchers identify an unknown plant by plotting similar MFD curves of its leaves with those belonging to known *Avicennia officinalis* and *Rhizophora apiculatas* profiles. This method employs leaf outline fractal complexity as a valid metric for species identification, providing an objective means of distinguishing between morphologically similar species within a population.

Table 2: Comparison of statistics of Leaf Margin

Species Names	μ	σ	Fluctuations
Avicennia officinalis	1.201	0.134	3.219
	1.191	0.138	3.722
	1.186	0.135	3.430
	1.186	0.134	3.644
	1.216	0.139	3.648
Rhizophora apiculata	1.071	0.105	7.230
	1.068	0.090	5.451
	1.119	0.103	6.626
	1.054	0.117	8.413
	1.058	0.116	8.375

Here, μ refers to the mean of MFD values and σ indicates the standard deviation of MFD values for the leaves of each species.

5.3 MFD analysis of Leaf Vein Network

The vein networks of *Avicennia officinalis* and *Rhizophora apiculata* can be compared by performing a Multi-Scale Fractal Dimension (MFD) analysis. These species show differences in their fractal dimensions, which reveal very important insights into their internal vascular systems and how they adapt to different ecological conditions.

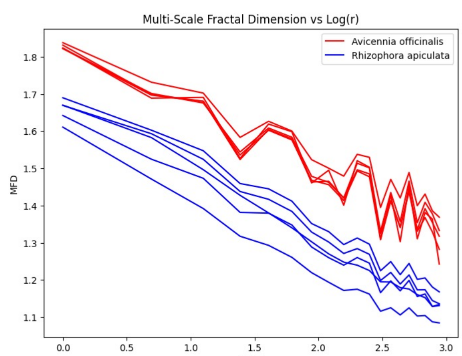


Fig. 9: Vein Network Analysis of *Avicennia officinalis* and *Rhizophora apiculata*

It is through analysis of the Multi-Scale Fractal Dimension (MFD) curve in the graph 9, that complexities of veins are revealed for *Avicennia officinalis* and *Rhizophora apiculata*. As a result, higher mean MFD values of between 1.4791 and 1.5268, indicate more complex venation for *Avicennia officinalis* and standard deviation between 0.1245 and 0.1492, indicates moderate variation in complexity. It is evident from this that the fluctuations in vein topology for *Avicennia officinalis* range from 2.1889 to 3.8904 showing significant fluctuations and decreasing complexity at greater scales. Conversely, this shows that *Rhizophora apiculata* has lower mean MFD values which range from 1.2167 to 1.3383 giving lower vein complexity, hence a standard deviation among its veins varying from 0.1394 to 0.1522 demonstrating lower increments in variability. Consequently, the fluctuation in vein topology for *Rhizophora apiculata* ranges between 1.3953 and 1.5845 suggesting a consistent complexity across scales as opposed to *Avicennia officinalis*. These differences highlight the more intricate but less stable vein network of *Avicennia officinalis* versus the more consistent vein complexity of *Rhizophora apiculata*.

Table 3: Comparison of statistics of Leaf Vein Network

Species Names	μ	σ	Fluctuations
Avicennia officinalis	1.492	0.133	3.033
	1.480	0.138	2.845
	1.479	0.149	3.890
	1.479	0.143	2.738
	1.527	0.125	2.189
Rhizophora apiculata	1.217	0.140	1.395
	1.338	0.146	1.425
	1.312	0.151	1.422
	1.282	0.139	1.584
	1.289	0.152	1.027

Here, μ refers to the mean of MFD values and σ indicates the standard deviation of MFD values for the leaves of each species.

Each species has its own distinct fractal signatures that can be used by researchers to build up a database for the identification of species using complexity and structure of vein network. The variations in MFD values have scale dependence and give insights into how organisms adapt to different ecological situations. This method enhances identification accuracy through correlating MFD values of unknown samples with those of known species. Moreover, MFD analysis provides insights into plant functional and structural adaptations which contribute to understanding their ecological roles and evolutionary history.

Cross Validation of Multi-Scale Fractal Dimension Analysis of Leaf Vein Networks

To validate the study of the vein networks of *Avicennia officinalis* and *Rhizophora apiculata*, the Multi-Scale Fractal Dimension (MFD) analysis was cross-validated using the MFD curve of the raw sample of *Avicennia marina* as a reference. The red curves of *Avicennia officinalis* and the blue curves of *Rhizophora* *apiculata* were compared to the green MFD curve of *Avicennia marina*. The MFD curves for both *Avicennia marina* and *Avicennia officinalis* were closely aligned, indicating that these species have similar fractal dimensions with *Avicennia marina* having mean MFD values in the range from 1.4791 to 1.5268 and Avicennia marina has a mean MFD of 1.493, which justifies their belonging to one genus even though they are different species. However, totally separate MFD profiles in *Rhizophora apiculata* exhibit consistently lower values for fractal dimension with mean MFD values in the range from 1.2167 to 1.3383, which indicates conspicuous differences in structure among veins hence a separate genus.

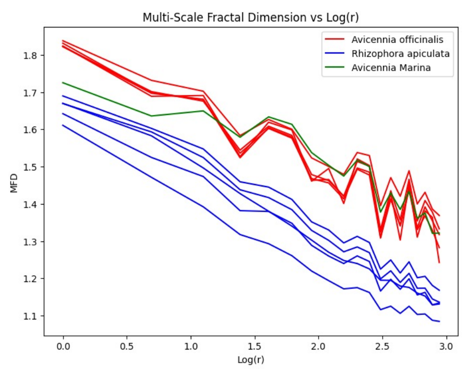


Fig. 10: Cross Validation of MFD analysis of Leaf Vein Networks

From fig.10, it can inferred that the MFD analysis of the leaf vein network can be effectively used to differentiate plant species through their unique and intricate leaf vein structures. Therefore, for different species belonging to the same genus, it proves to be a more reliable method for plant species identification compared to MFD analysis of Leaf Margin and Colour Texture.

Novelty of the approach

- Species-specific unique leaf margin patterns can be captured using the MFD curves, which provide a detailed and quantifiable method making it possible to analyze the leaf outline complexity across scales. The characteristic features of these curves can be used to identify leaf species based on leaf morphology.
- The MFD Analysis of Colour Texture is very sensitive to changes in textural behaviour, which allows the study of the organization of pixels across different colour channels at varying scales. The results prove the potential of this approach for colour texture analysis applications.
- The structure of vein networks of each species has its own distinct fractal signatures that can be used by researchers for the identification of species. The variations in MFD values reveal the complexities of veins which provide

important insights into their internal vascular systems and how they adapt to different ecological conditions.

6 Conclusion

The use of Multi-Scale Fractal Dimension (MFD) analysis in determining the species of mangroves through their leaf vein networks as well as margins and color textures is highlighted by this research. The MFD technique was able to distinguish between subtle structural differences in a way that improves the accuracy of identification beyond traditional ones. Our results confirmed the fact that *Avicennia marina* and *Avicennia officinalis* have similar curves of MDF, thus supporting their generic classification; on the other hand, *Rhizophora apiculata* differed greatly from all other genera. Moreover, cross-validation with an independent sample of *Avicennia marina* confirms its reliability. In sum, MFD analysis is a very accurate and robust tool for botanical research and environmental monitoring allowing deep insights into plant morphology, ecological functions, evolutionary adaptations etc. This study opens up several avenues for future research which includes expanding the dataset to include more mangrove species and images captured under varying environmental conditions, developing a real-time identification system using the proposed method for in-field applications and combining fractal dimension analysis with machine learning algorithms to further improve species identification accuracy.

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