Roll no - 2020900003

Q2.

2 Subjective Question [2 markss]

Prove that there exist a relationship between PCA and SVD for a given data matrix X. Explain in one line how will you use the the SVD of the data matrix X to perform dimensionality reduction?

(a)

Proof: Consider decomposition of a covariance matrix C

$$C = \frac{X^T X}{n-1}$$ (decomposition of X^T & X seperately)

$$= \frac{V \Sigma U^T \cdot U \Sigma V^T}{n-1}$$

$$= \frac{V \Sigma^2 V^T}{n-1}$$

$$= V \frac{\Sigma^2}{n-1} V^{-1}$$

This result is same as eigen decomposition of C. This shows the relationship between singular values (Σ) and eigenvalues v

$$v = \frac{\Sigma^2}{n-1}$$

(b)

SVD on data matrix X can be used to perform dimensionality reduction by choosing the first K eigen values of Σ diagonal matrix (Σ diagonal matrix represents important factors in descending order).