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Q3.

3 Subjective Question [2 marks]

What is the relation between Linear Discriminant Analysis and Bayes Rule?

Sol:- LDA classification by following Bayes approach.

We already know,

The posterior probability of class K given x

$P(K|x)$ is given by

$$P(K|x) = \frac{P(K)\ P(x|K)}{P(x)}$$

where $P(K)$ is the unconditional probability of class K and $P(x)$ is the unconditional probability of point x. Observing currently point x, it is equated to 1

$P(x) = 1$ and,

$P(x|K)$ - is the probability density function $PDF(x|K)$ for point x in class K.

In that LDA approaches the problem by assuming the coditional probability density function $P(x \mid k=0)$ and $p(x \mid k=1)$ are both normally distributed with mean and covariance parameters $(\vec{\mu}_0, \Sigma_0)$ and $(\vec{\mu}_1, \Sigma_1)$

Under this assumption, the Bayes optimal solution predict points as being from the second class if log of likelihood ratio is bigger than some threshold T.

$$(\vec{x} - \mu_0)^T \Sigma_0^{-1} (\vec{x} - \vec{\mu}_0) + \ln|\Sigma_0| - (\vec{x}_1 - \vec{\mu}_1)^T \Sigma_1^{-1} (\vec{x}_1 - \mu_1)$$
$$- \ln|\Sigma_1| > T$$

where μ_0, μ_1 are the means of each class $y \in \{0, 1\}$

$\Sigma_{y=0}$ and $\Sigma_{y=1}$ are covariance then likelihood ratio is

$$= \frac{\sqrt{|2\pi\Sigma_{y=1}|}^{-1} \exp\left(-\frac{1}{2}(x-\mu_{y=1})^T \Sigma_{y=1}^{-1} (x-\mu_{y=1})\right)}{\sqrt{|2\pi\Sigma_{y=0}|}^{-1} \exp\left(-\frac{1}{2}(x-\mu_{y=0})^T \Sigma_{y=0}^{-1} (x-\mu_{y=0})\right)} < t$$