Deep Generative Models — VAEs and GANs
(part 1)

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(University of Oxford)

<u>Presented at</u>

Machine Learning Summer School
IIIT Hyderabad, India
(9th July 2019)

Roadmap

- Objective
- Why?
- How?
- Classical Bayes' to Deep Generative Models
- VAEs — derive Elbo
- GANs
- GAN regularisations
- BigGAN

Generative Models — Objective

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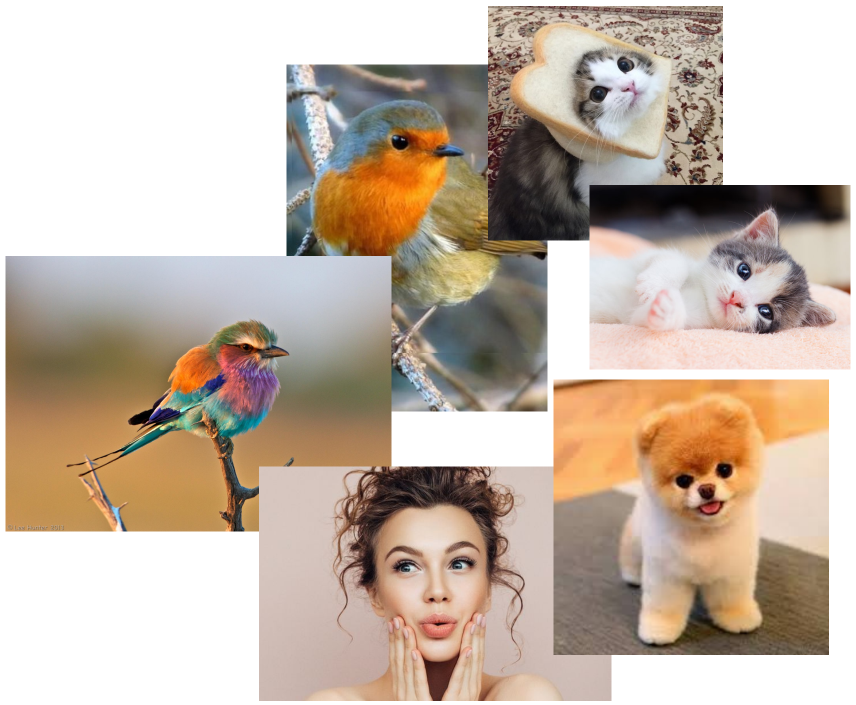

Given Dataset

$$\mathcal{D} = \{\mathbf{x}_i\}_{i=1}^{n}$$

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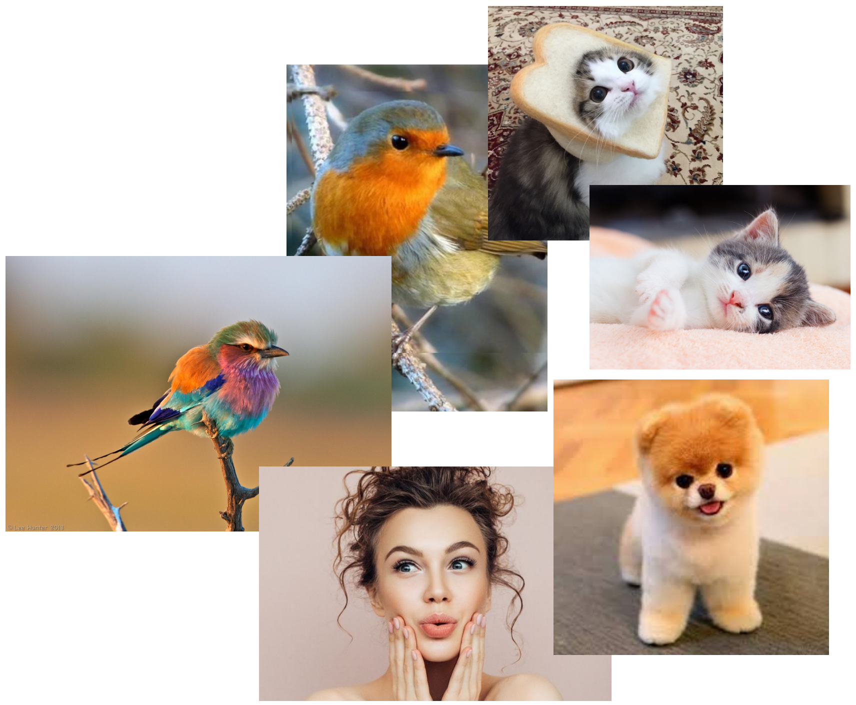

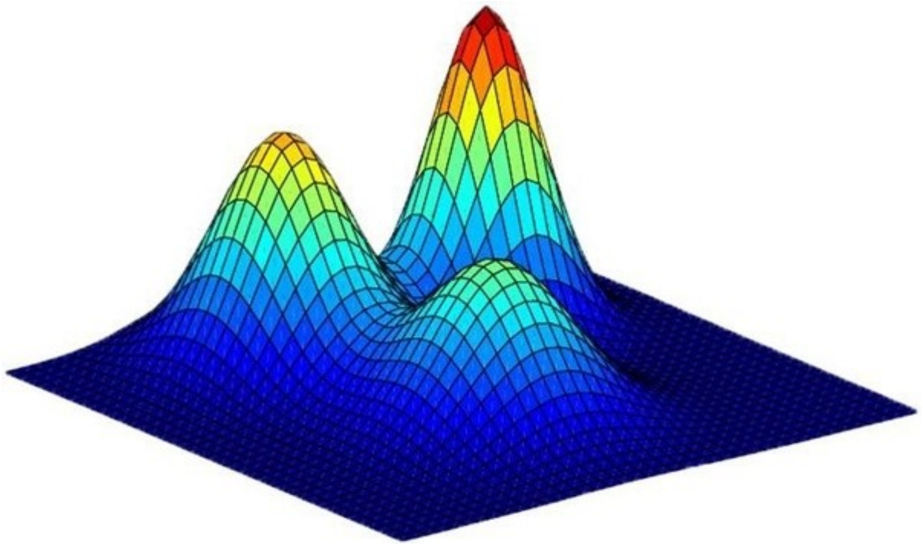

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Learn

$$p_\theta(\mathbf{x})$$

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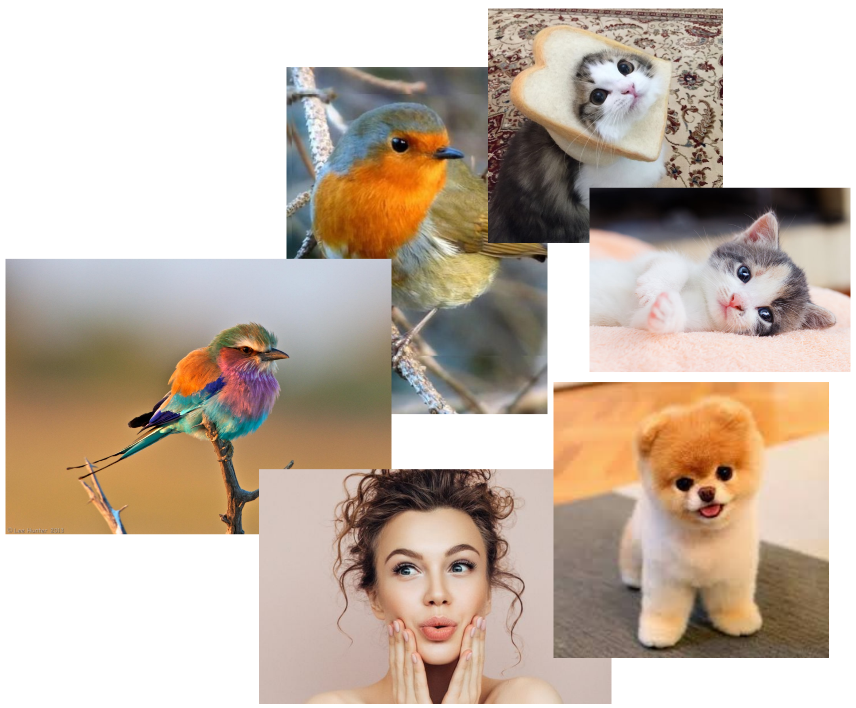

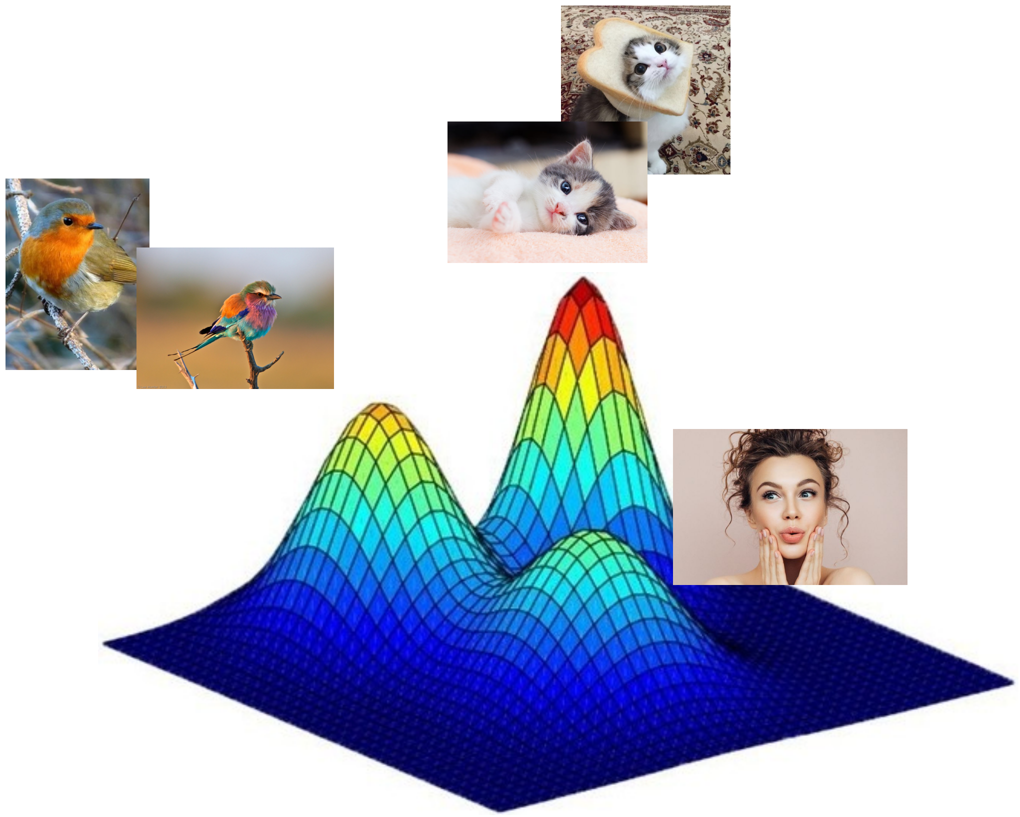

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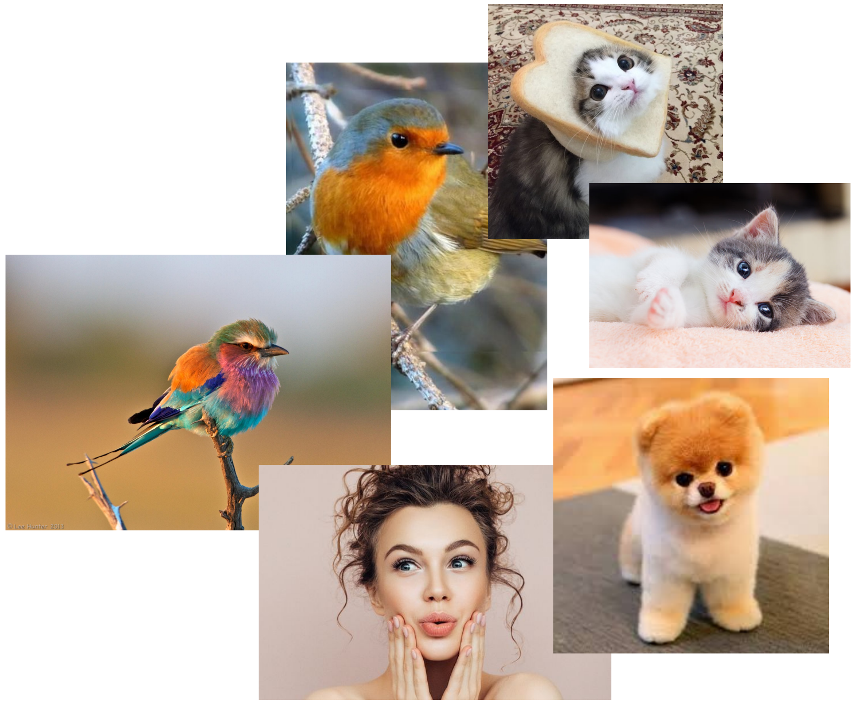

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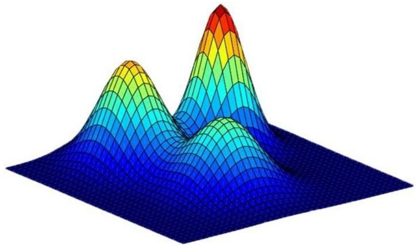

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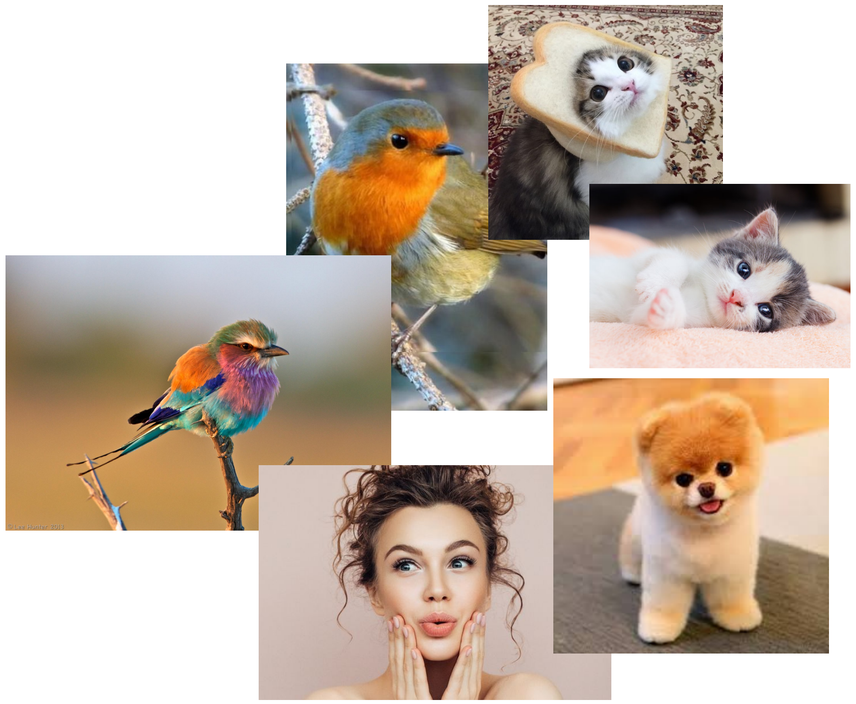

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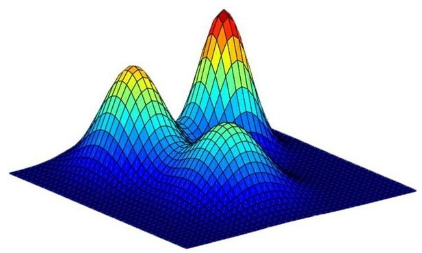

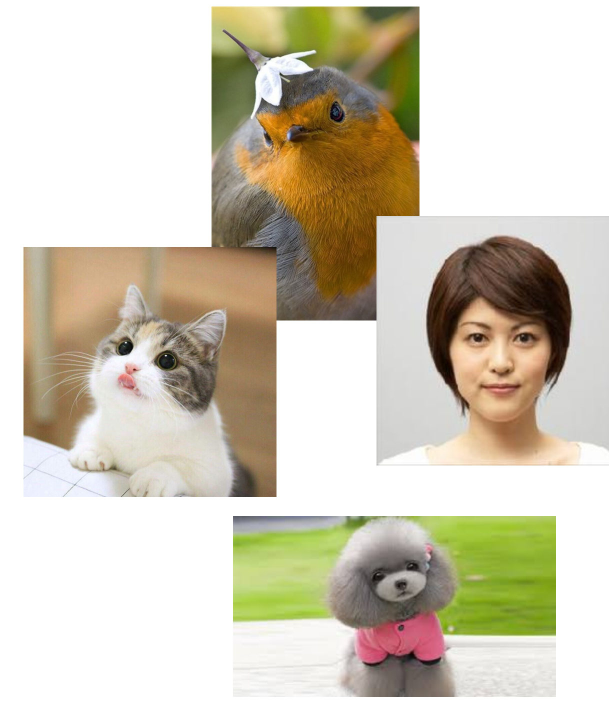

Given Dataset

New Data

Generative Models — Why?

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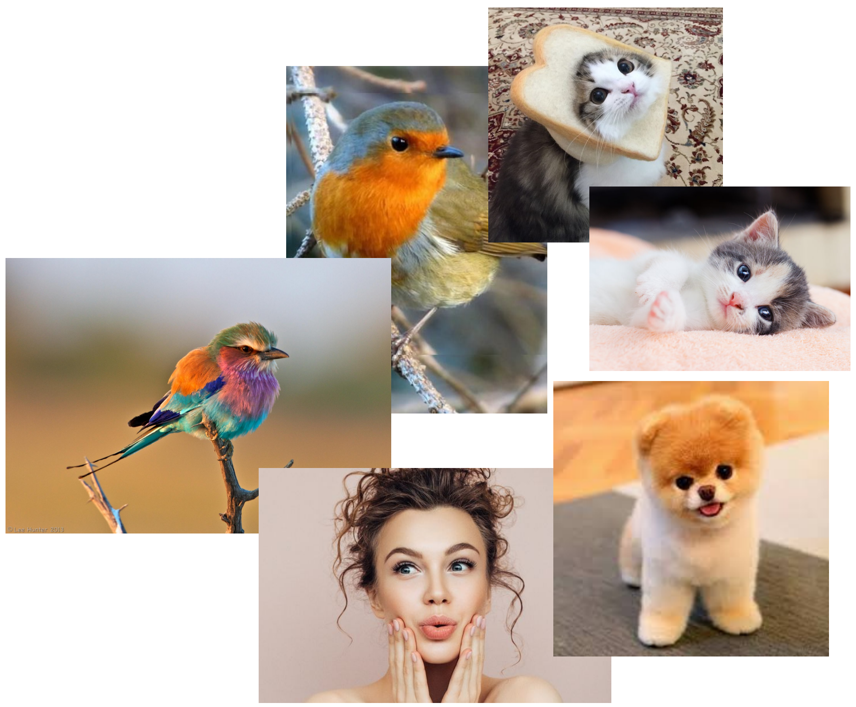

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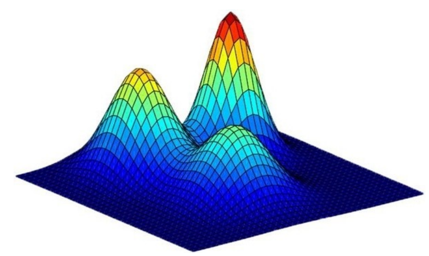

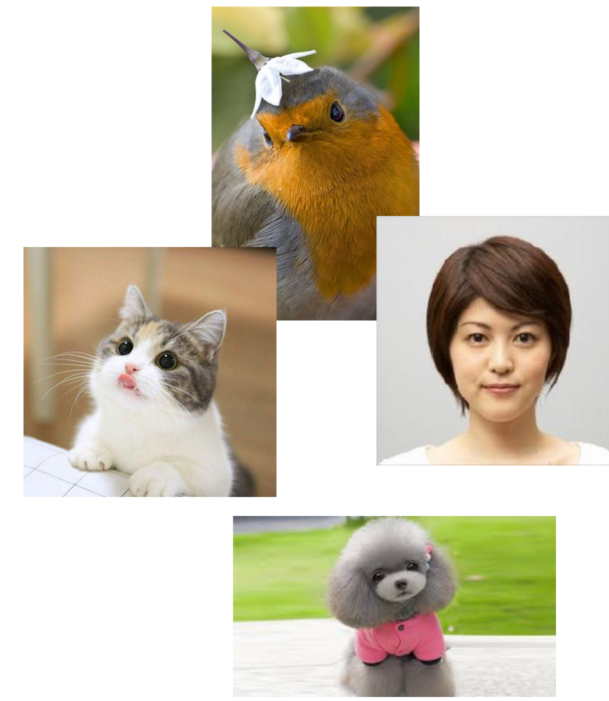

Given Dataset **New Data**

- **Density and Uncertainty Estimation**

Generative Models — Why?

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- **Many right answers** — Sample from the conditional distribution

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Sketch to Real Images

$$\mathbf{x} \sim p_\theta(\mathbf{x}|y = sketch)$$

Generative Models — Why?

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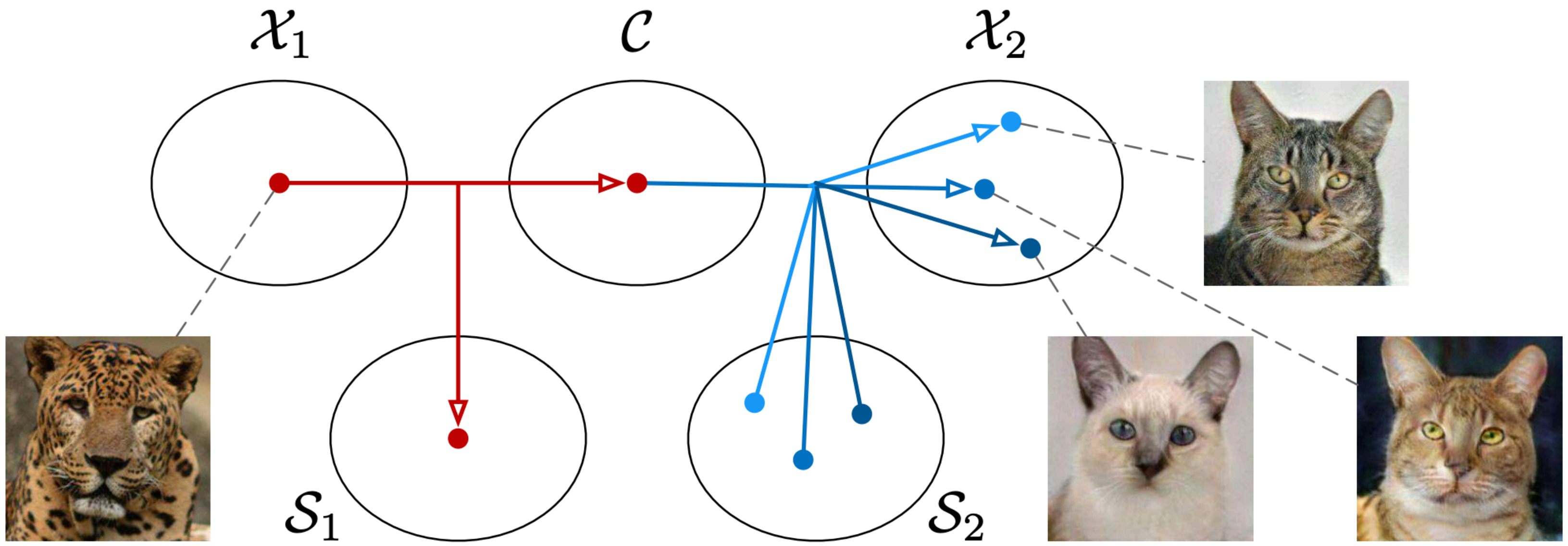
Visual Q&A

Q: How old is the girl
A1: about 7 or 8
A2: looks young

$$\mathbf{x} \sim p_\theta(\mathbf{x}|y = I,\ Q)$$

Generative Models — Why?

- **Learning intrinsic properties of the data**
  - composition, style etc.



Generative Models — Why?

Many more

Generative Model — Basics

- **Use Bayes' theorem with some standard approximations**

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$$p_\theta(\mathbf{x})$$

High Dim Complex Space

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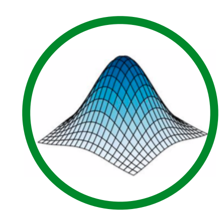

$p(z)$

Low Dim
Interpretable Space

- May contain information about
  - Number of objects
  - Type/Appearance of objects
  - Background and lighting condition

Generative Model — Basics

- **Use Bayes' theorem with some standard approximations**

$p_\theta(\mathbf{x}, z)$

$p_\theta(\mathbf{x})$

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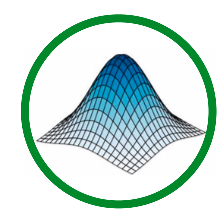

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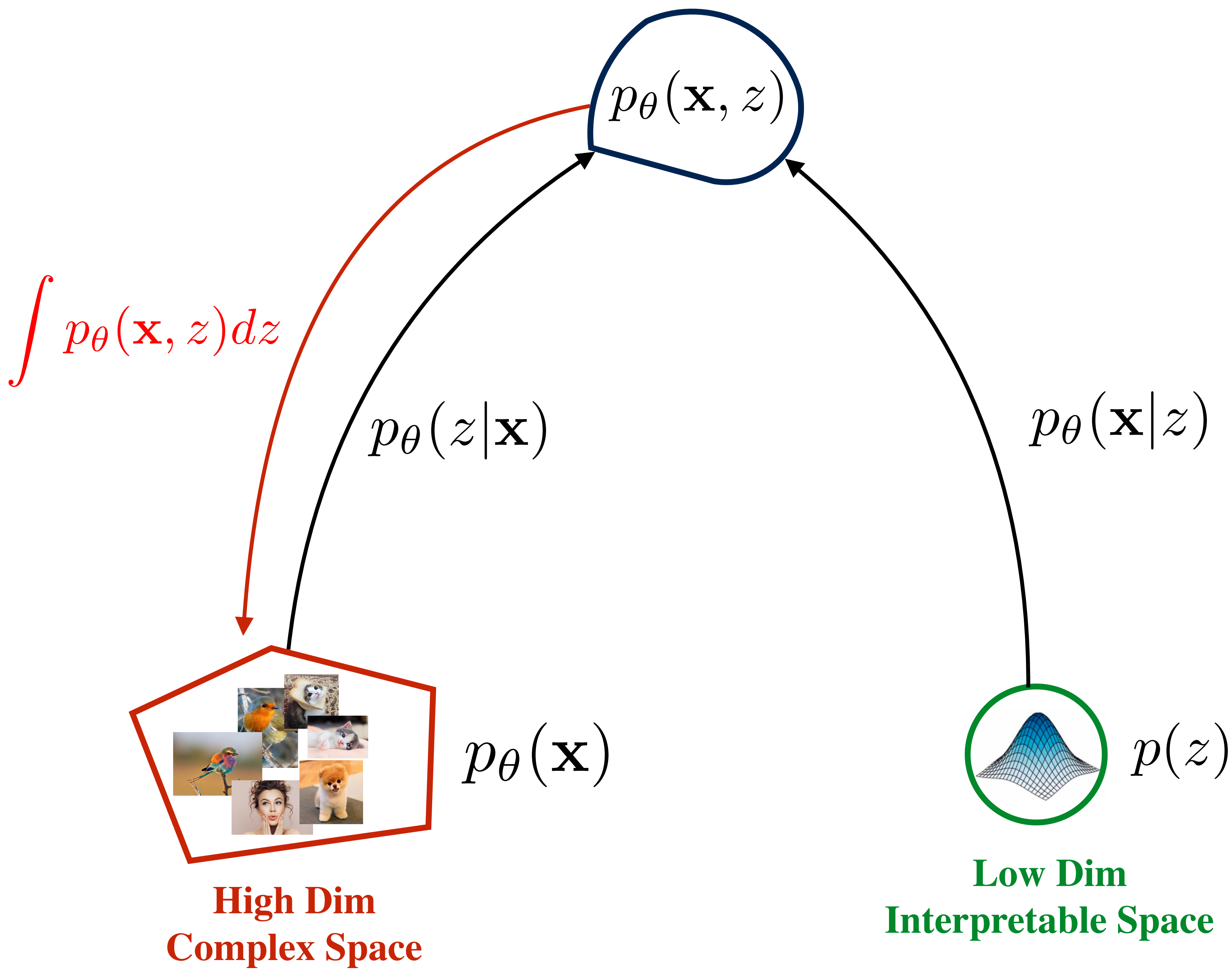

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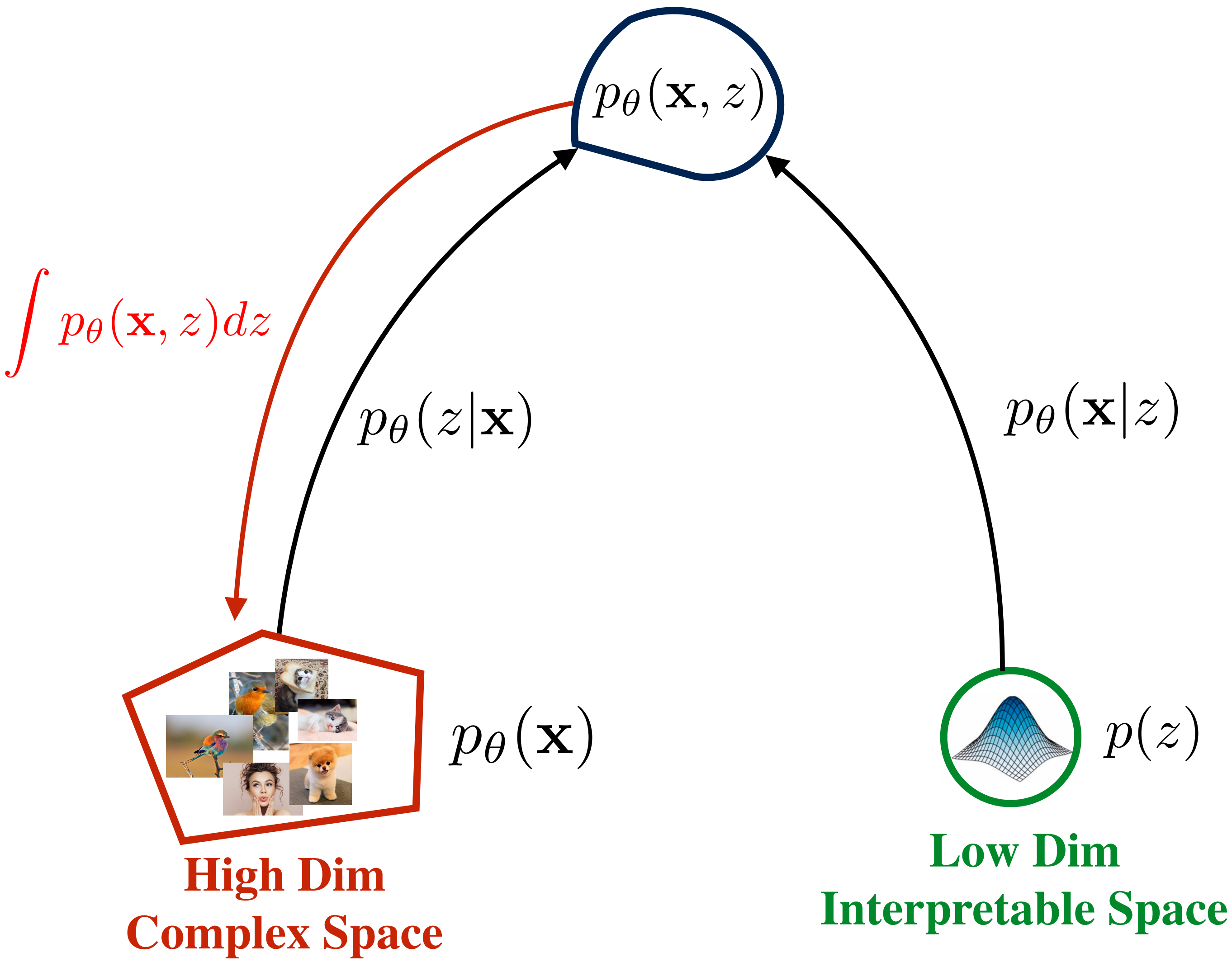

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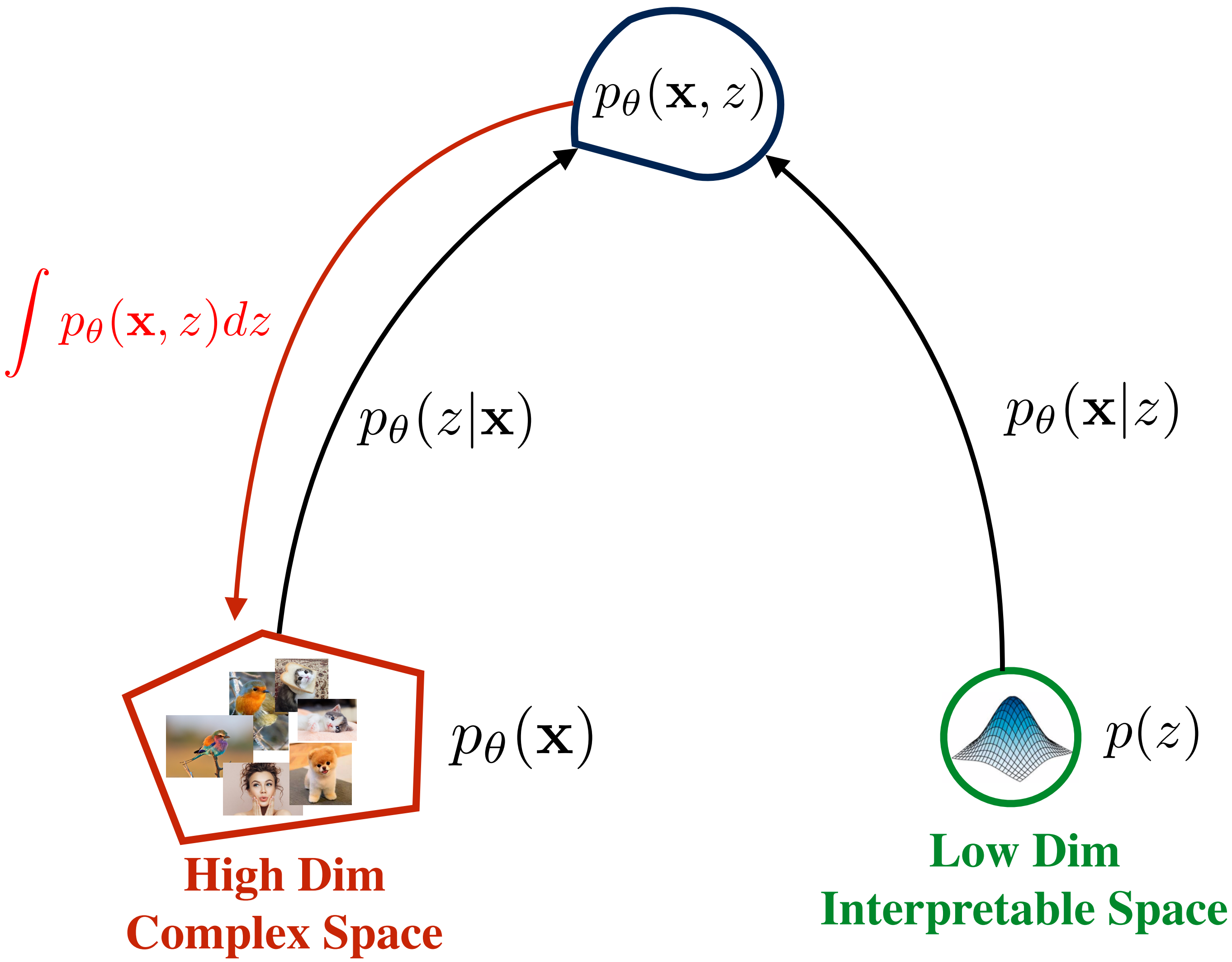
Marginal? — Infinite mixture model

$$p(\mathbf{x}) = \int p(\mathbf{x}, z)dz = \int p_\theta(\mathbf{x}|z)p(z)dz$$



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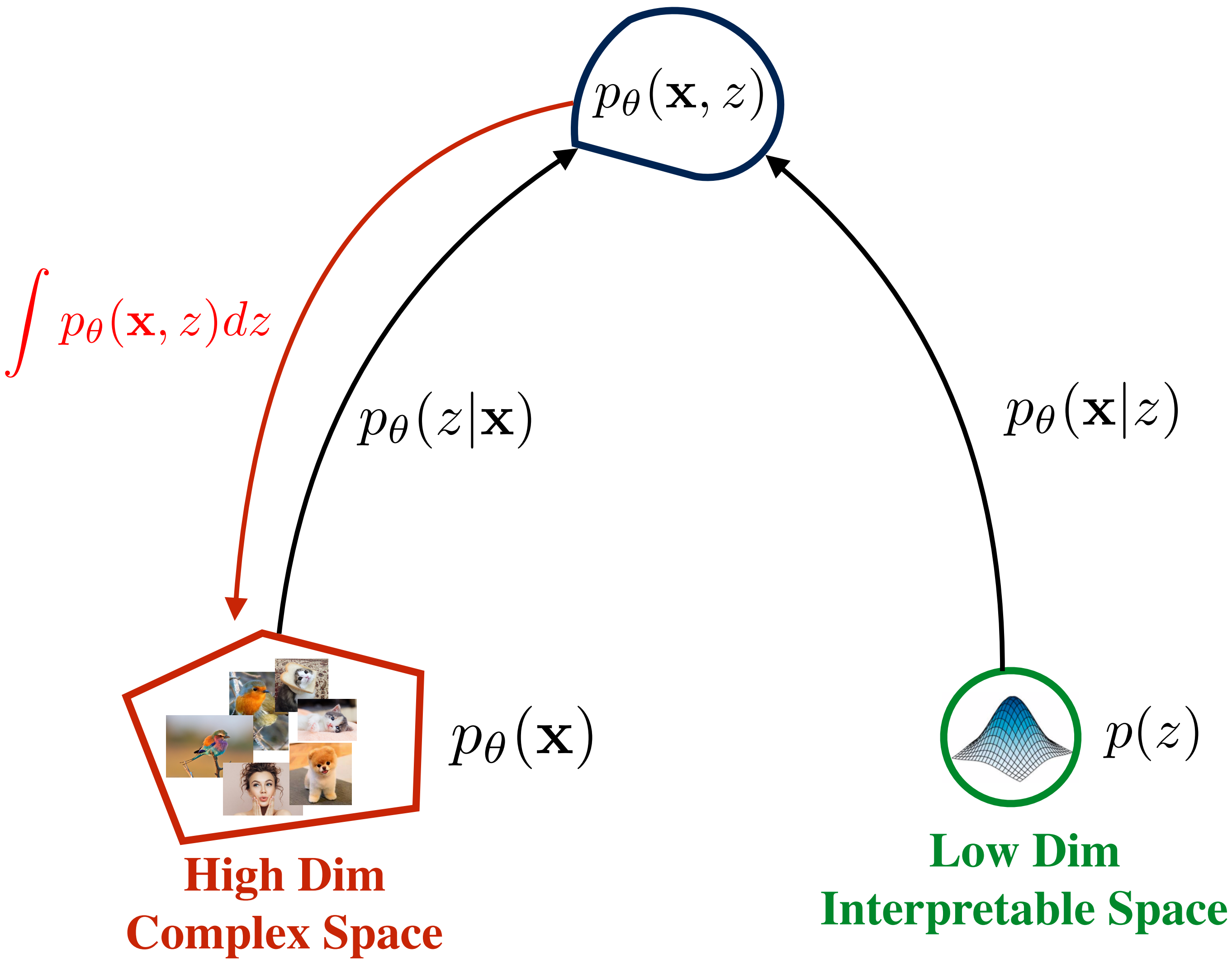

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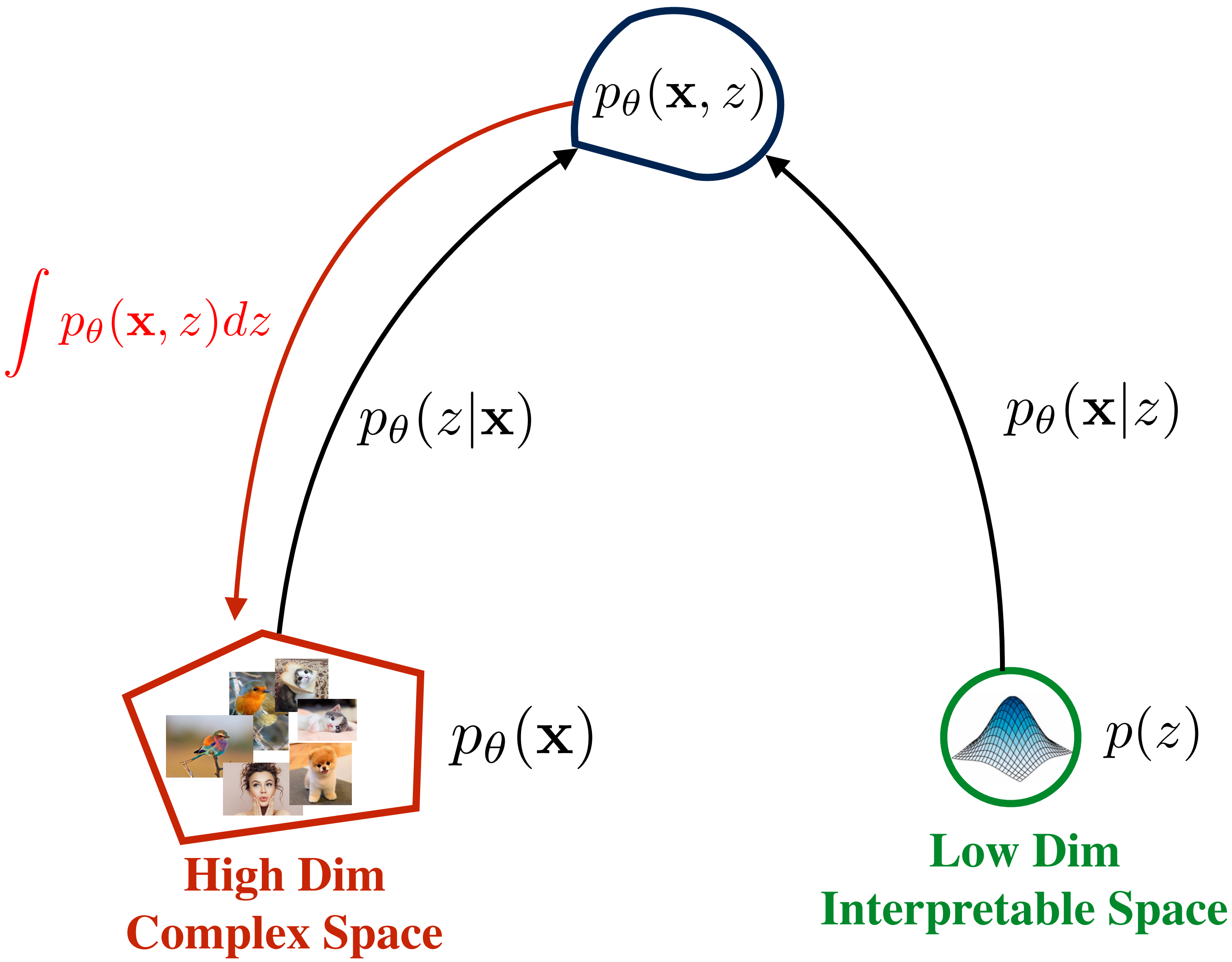

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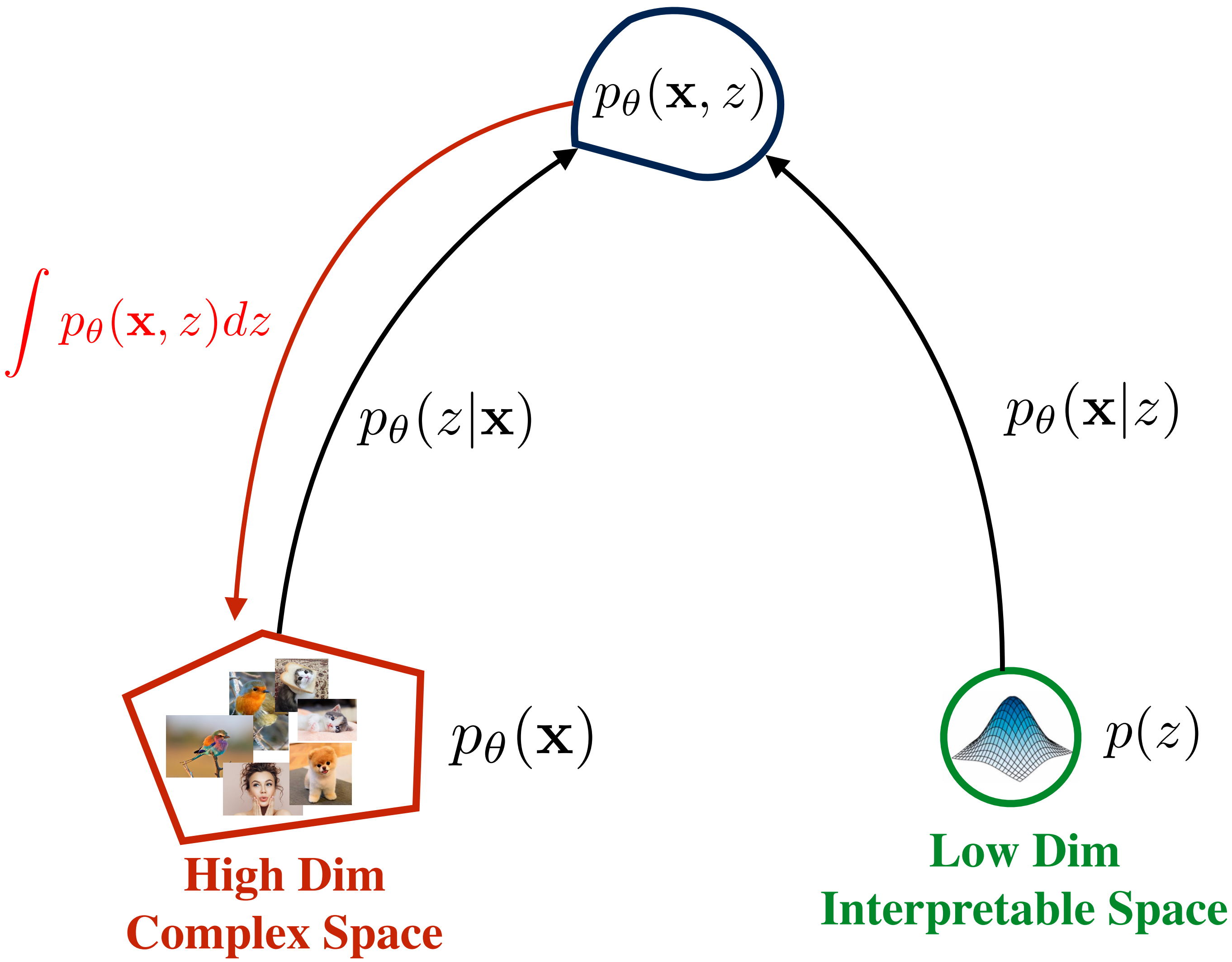

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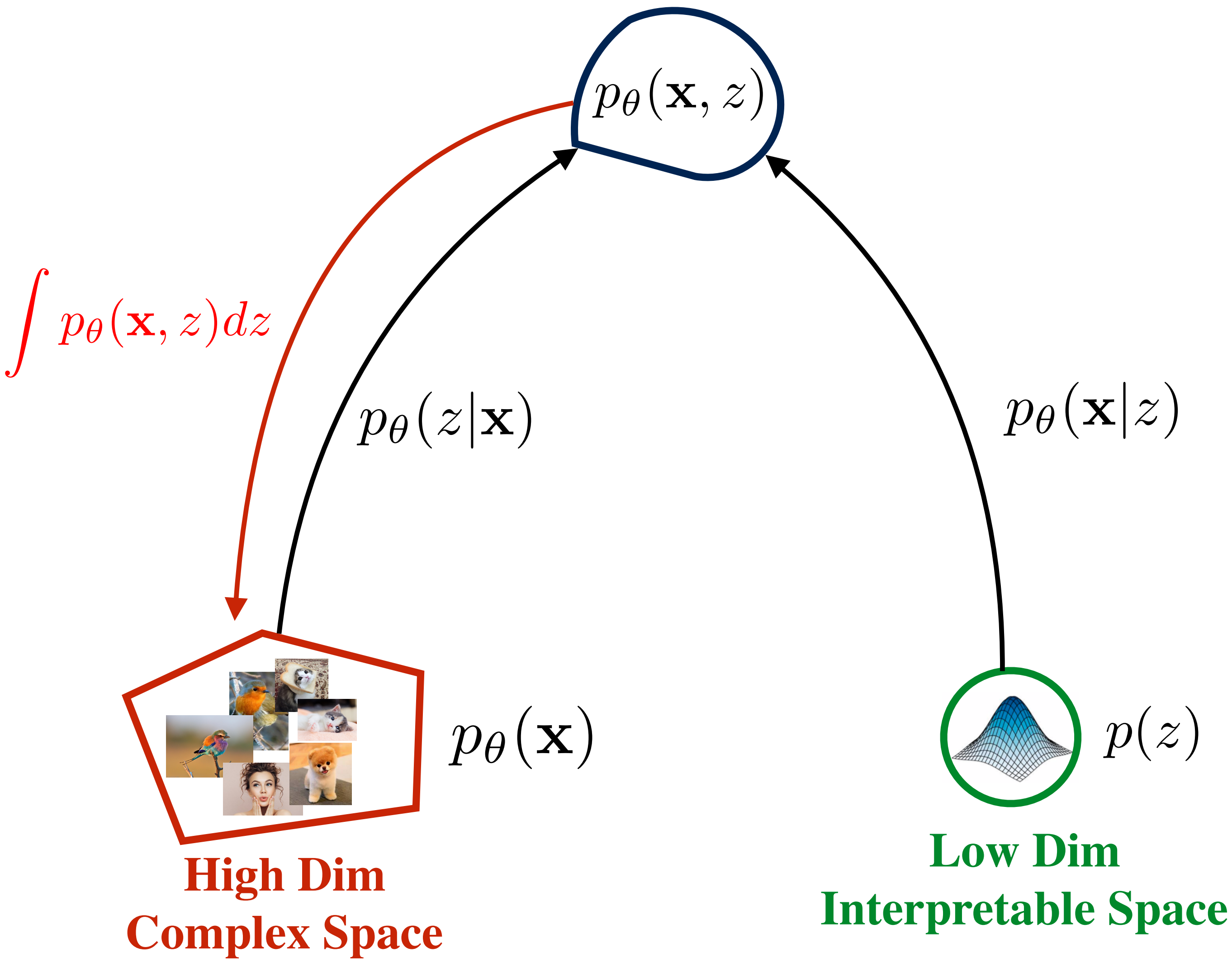

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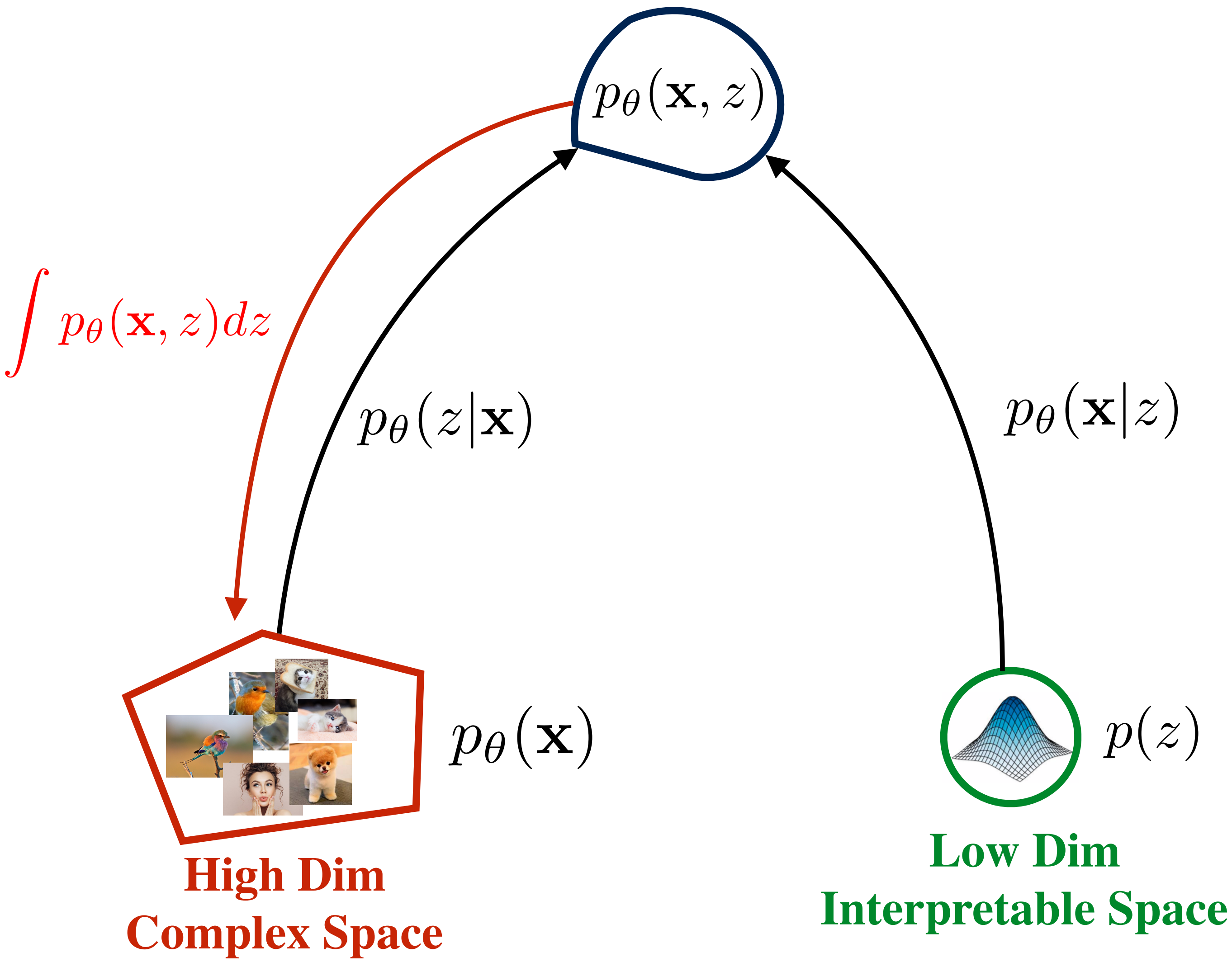

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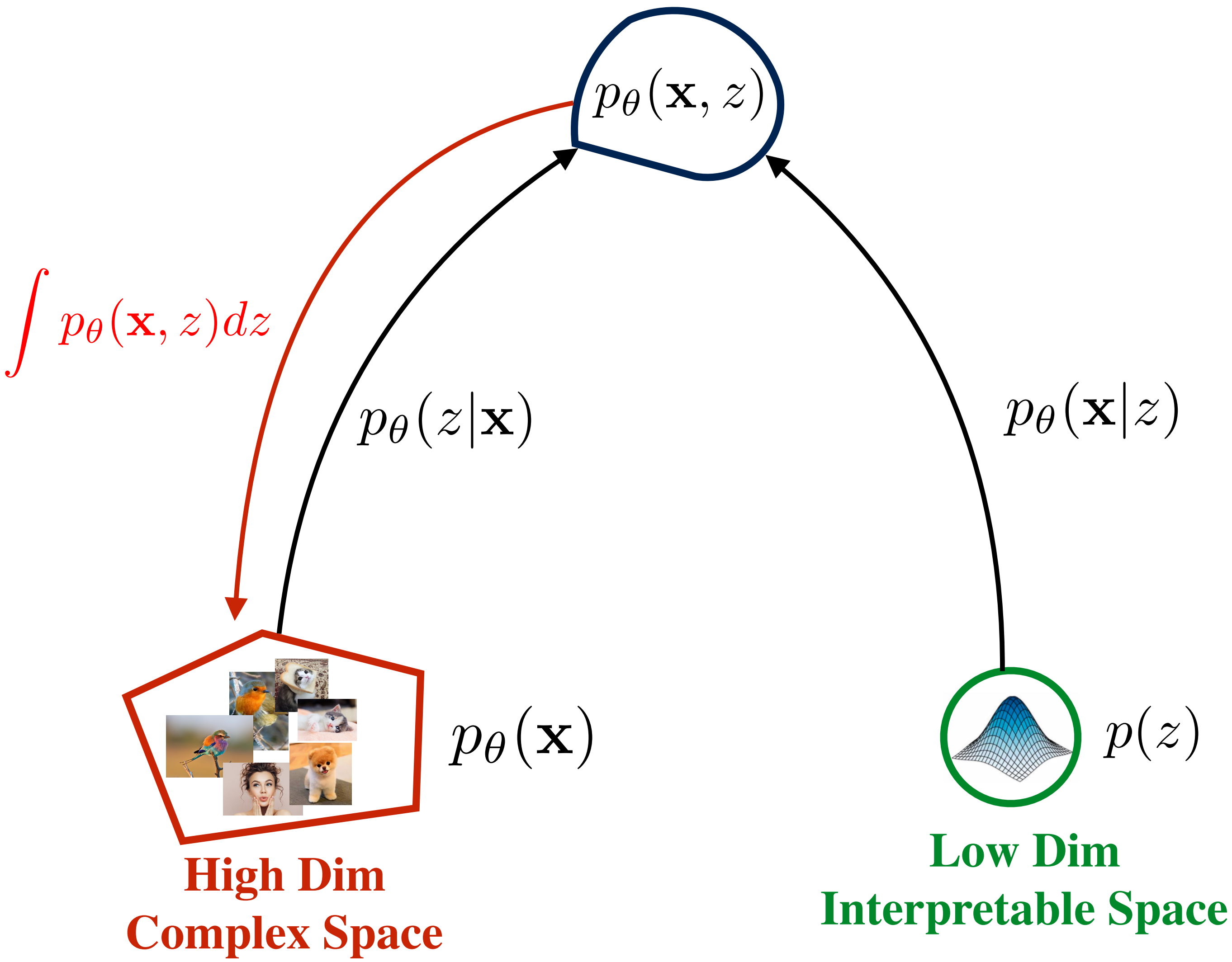

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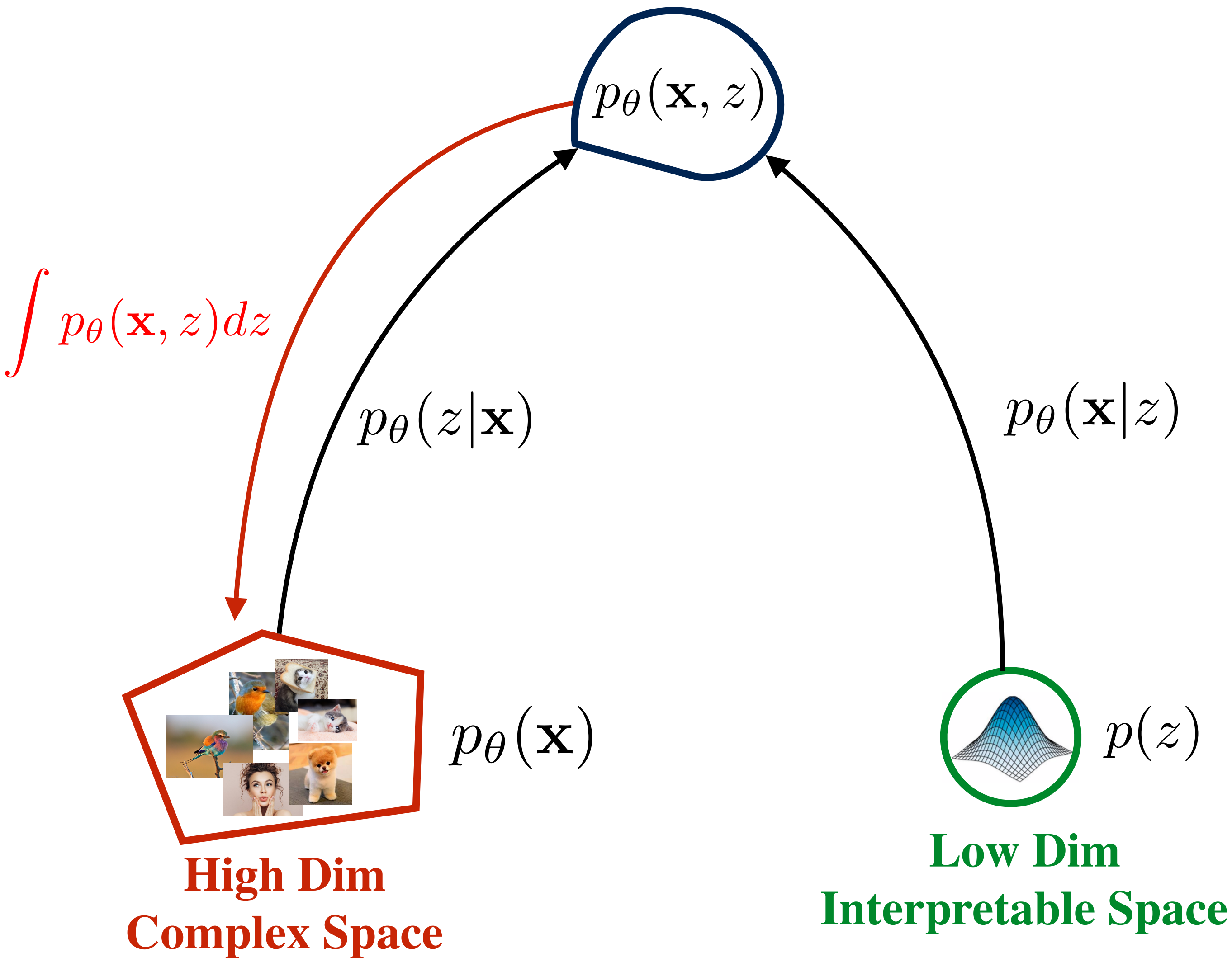

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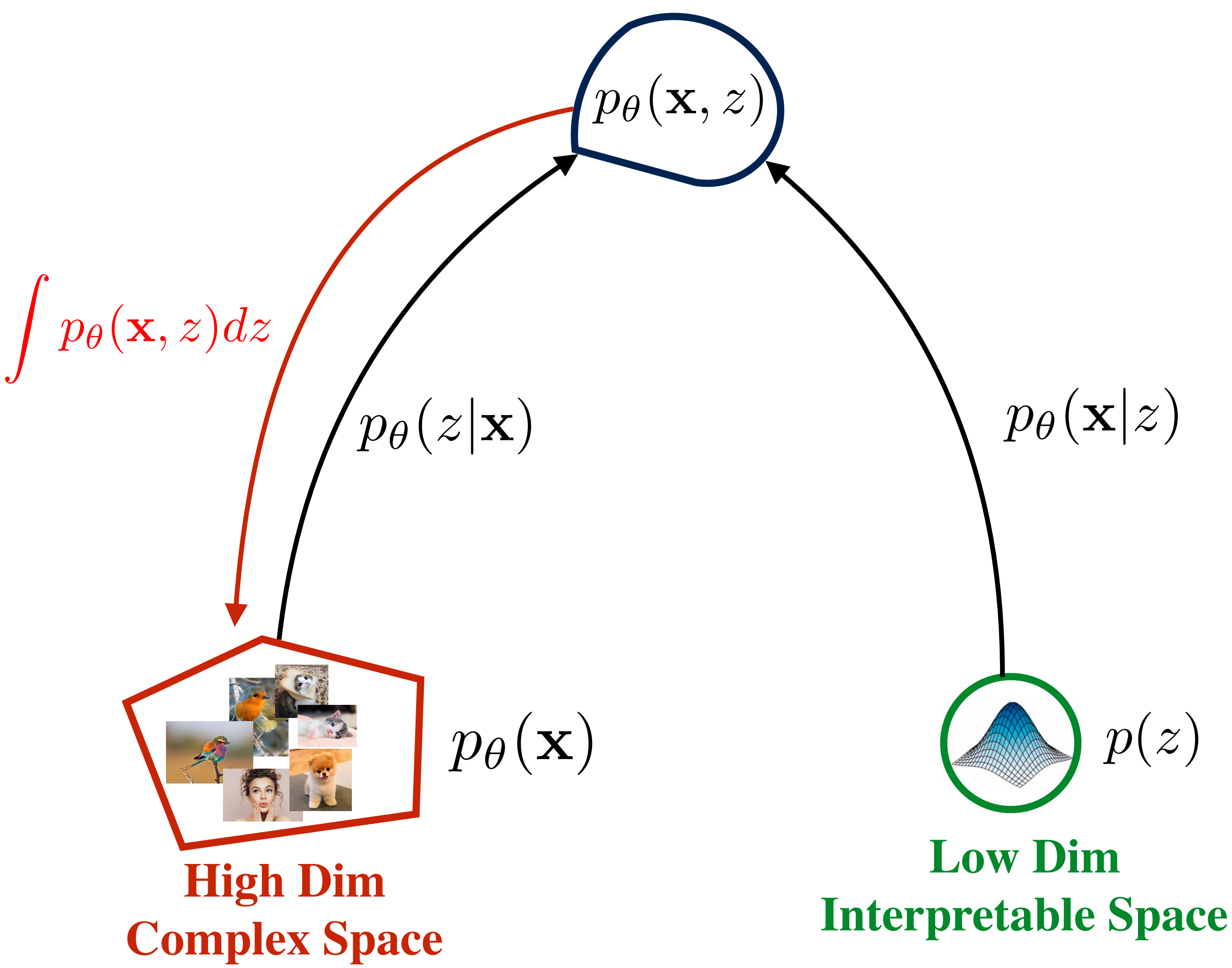

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proposal distribution

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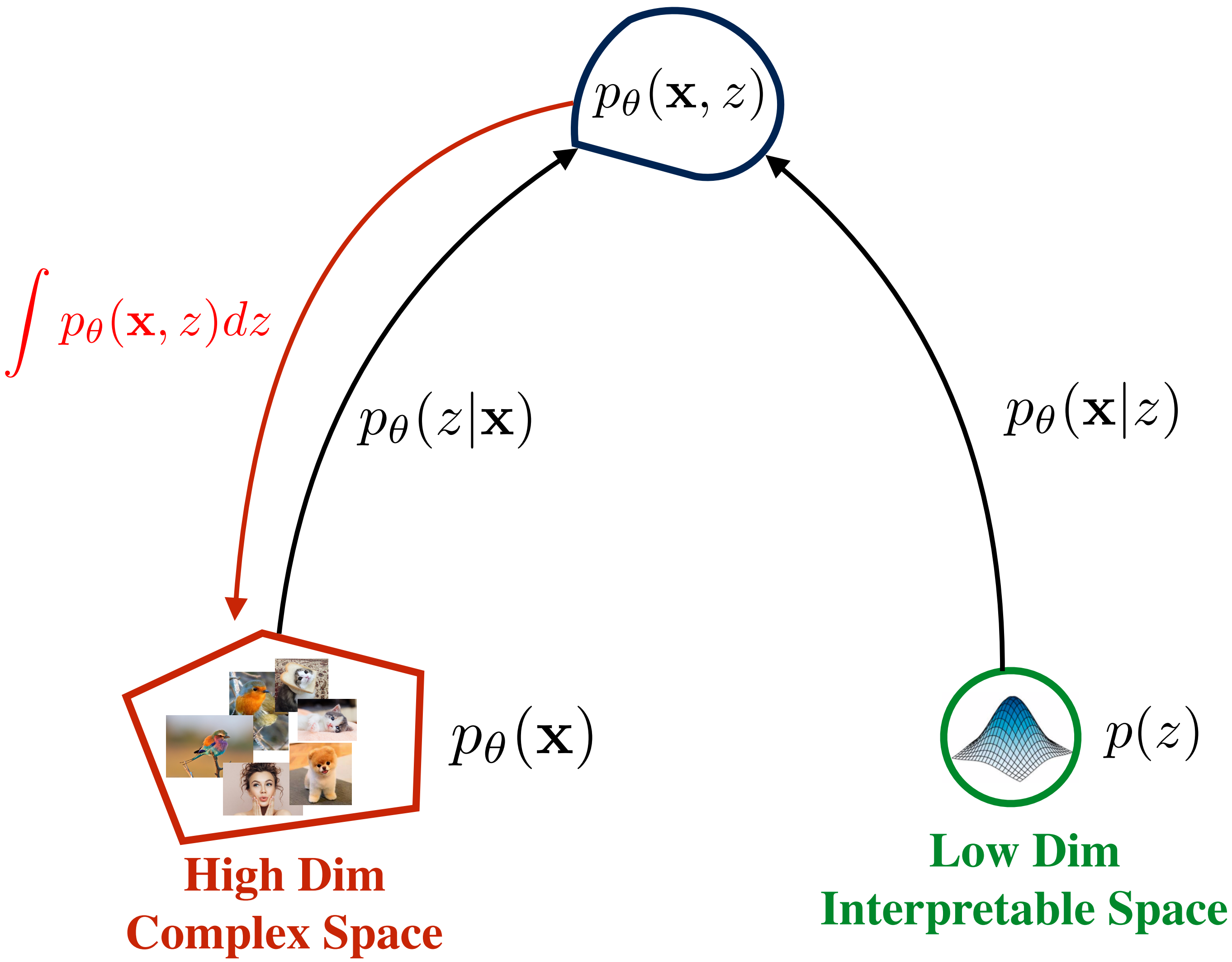

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proposal distribution

- **Use NN to define relationships**
- **Learn parameters to minimise some notion of distance (KL) — VAE**

Deep Generative Models — Overview of VAE and GAN

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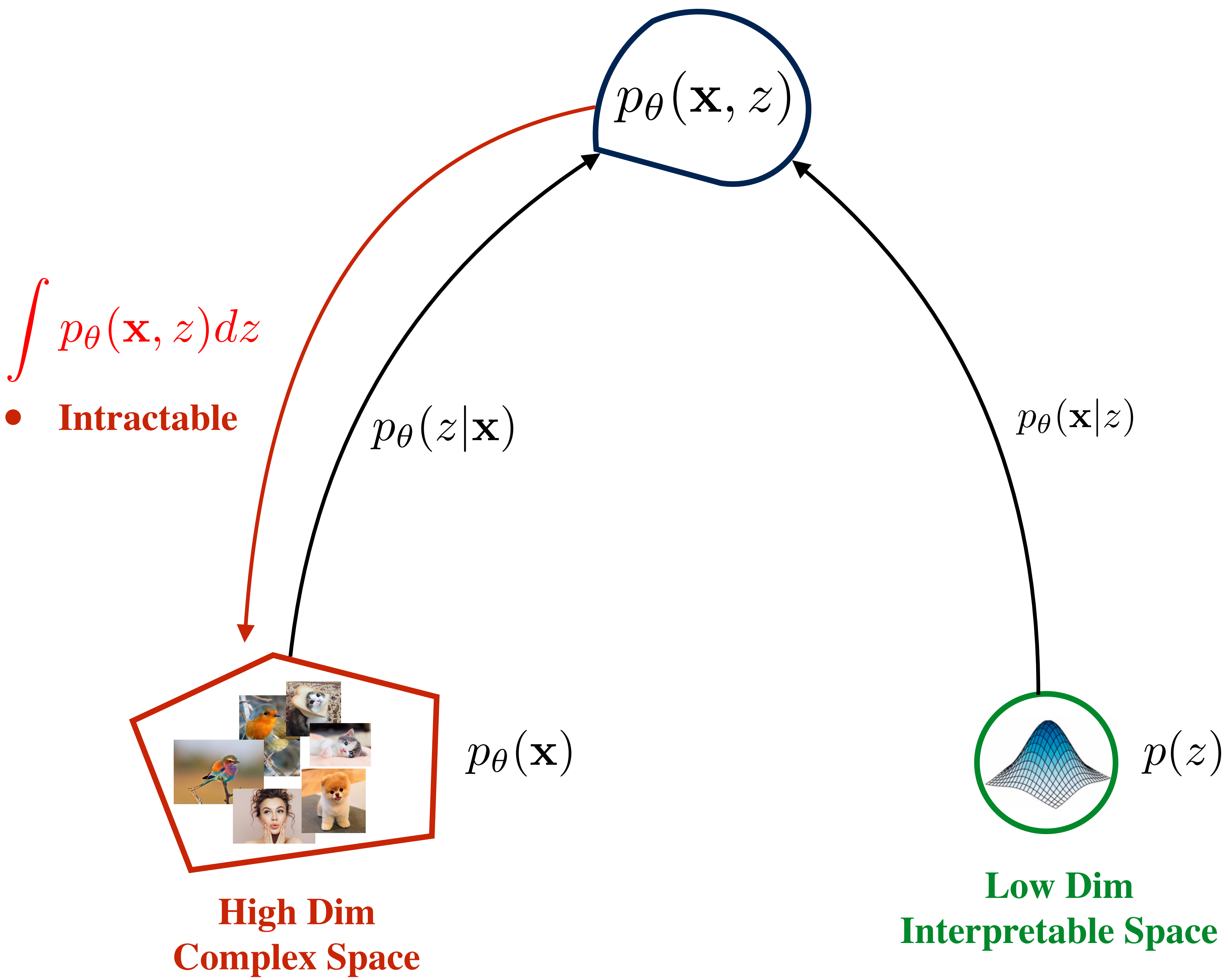

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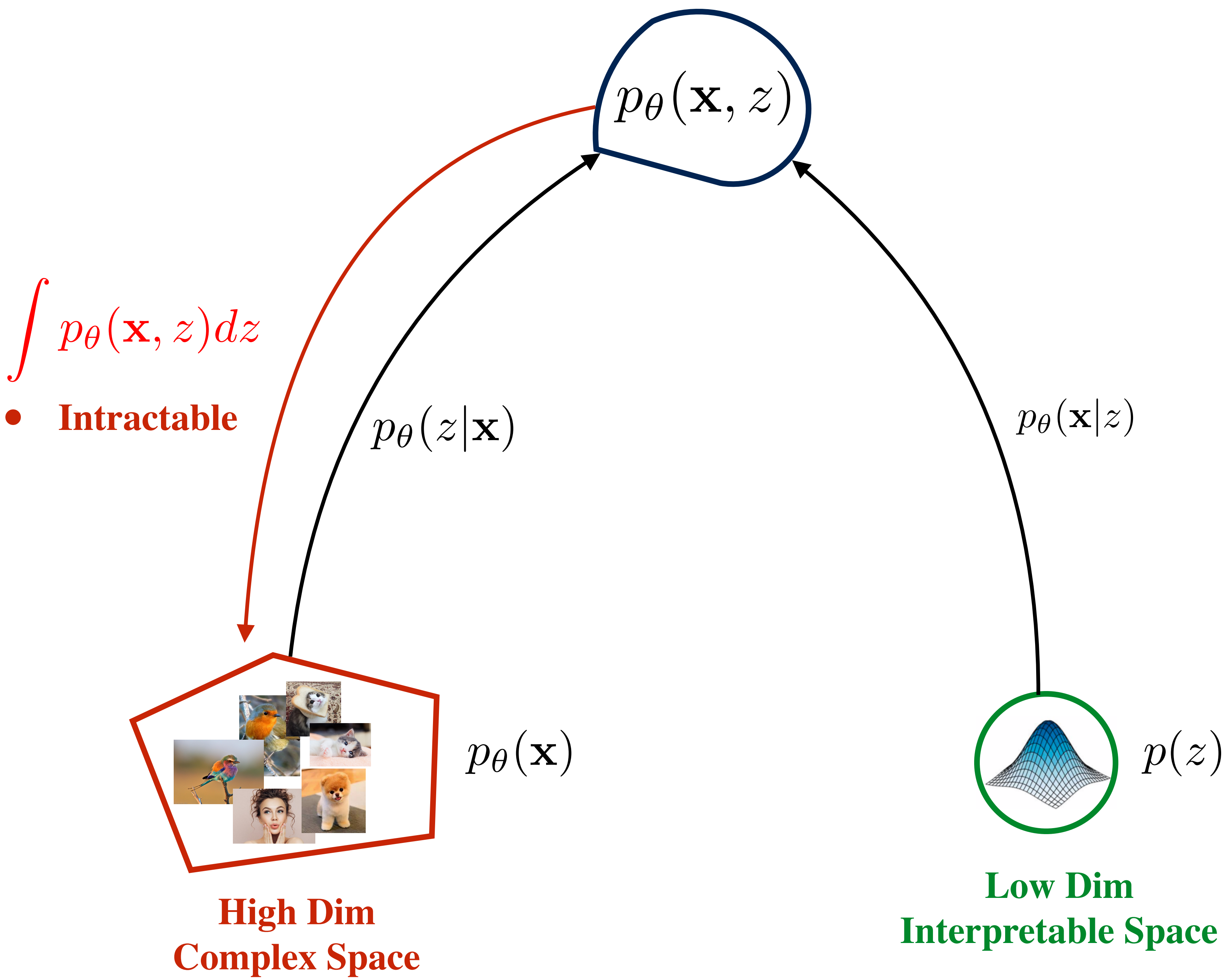

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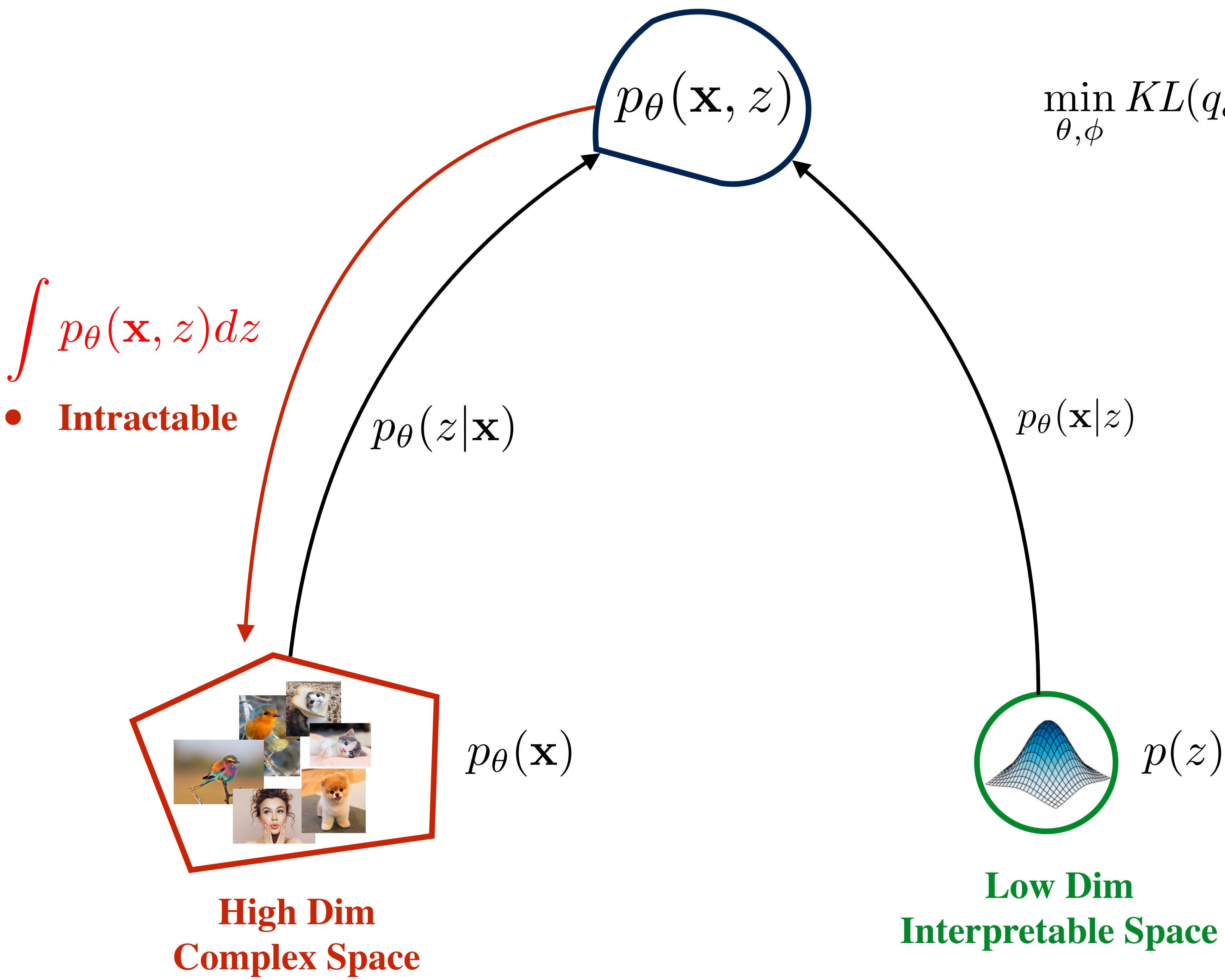
$\boxed{\text{VAEs}}$



$$\min_{\theta,\phi} KL(q_\phi(z|\mathbf{x})||p_\theta(z|\mathbf{x})) \approx$$

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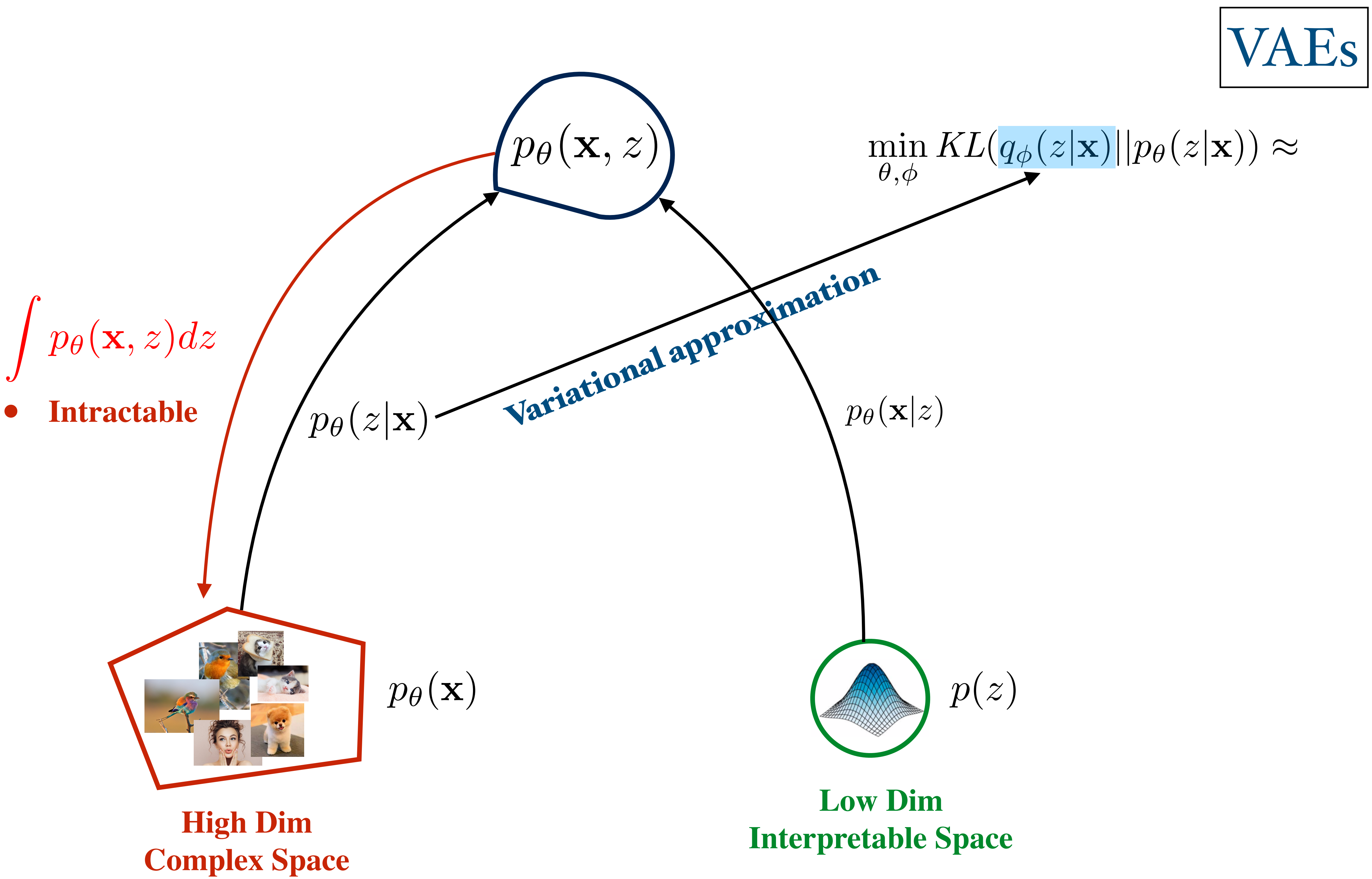

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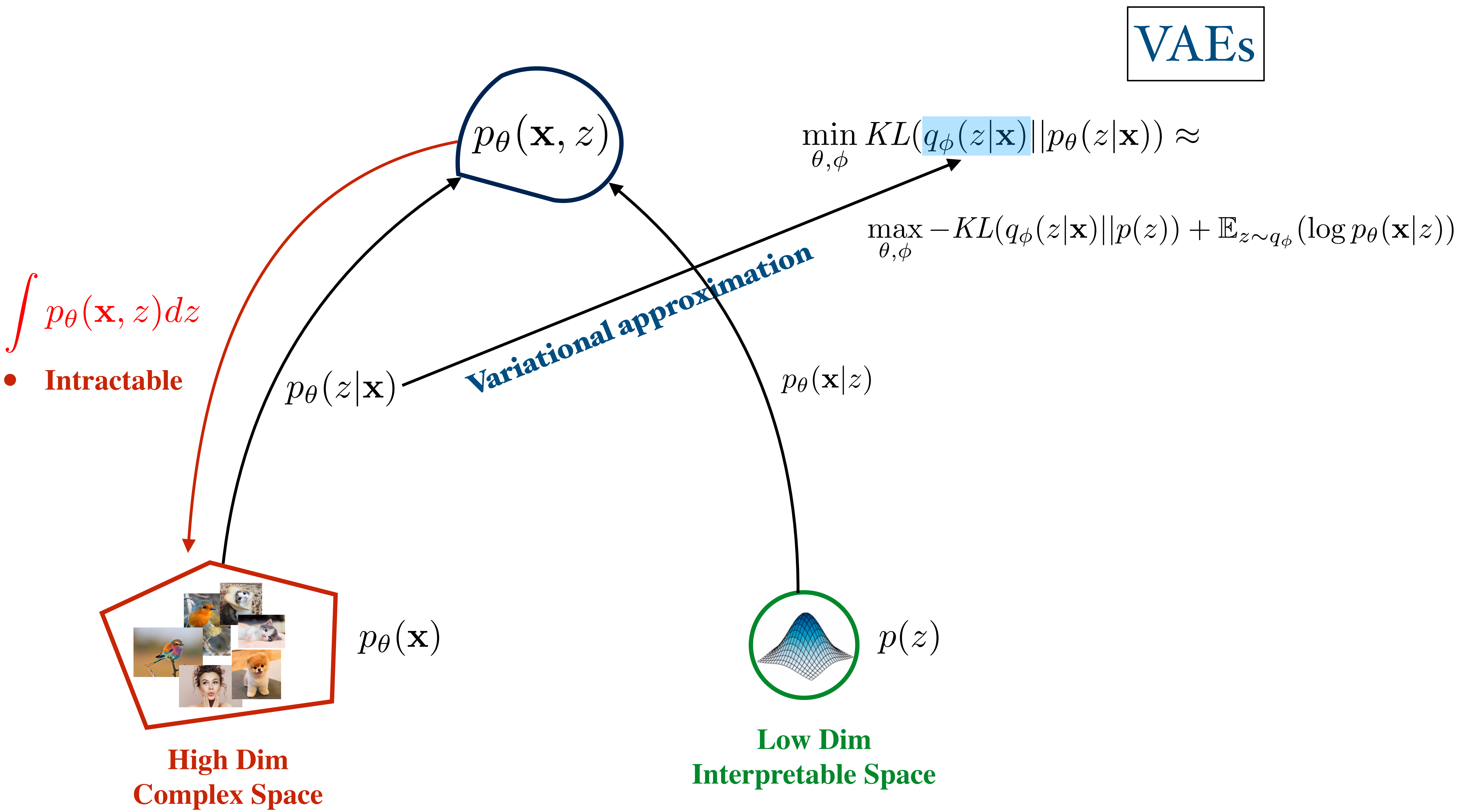

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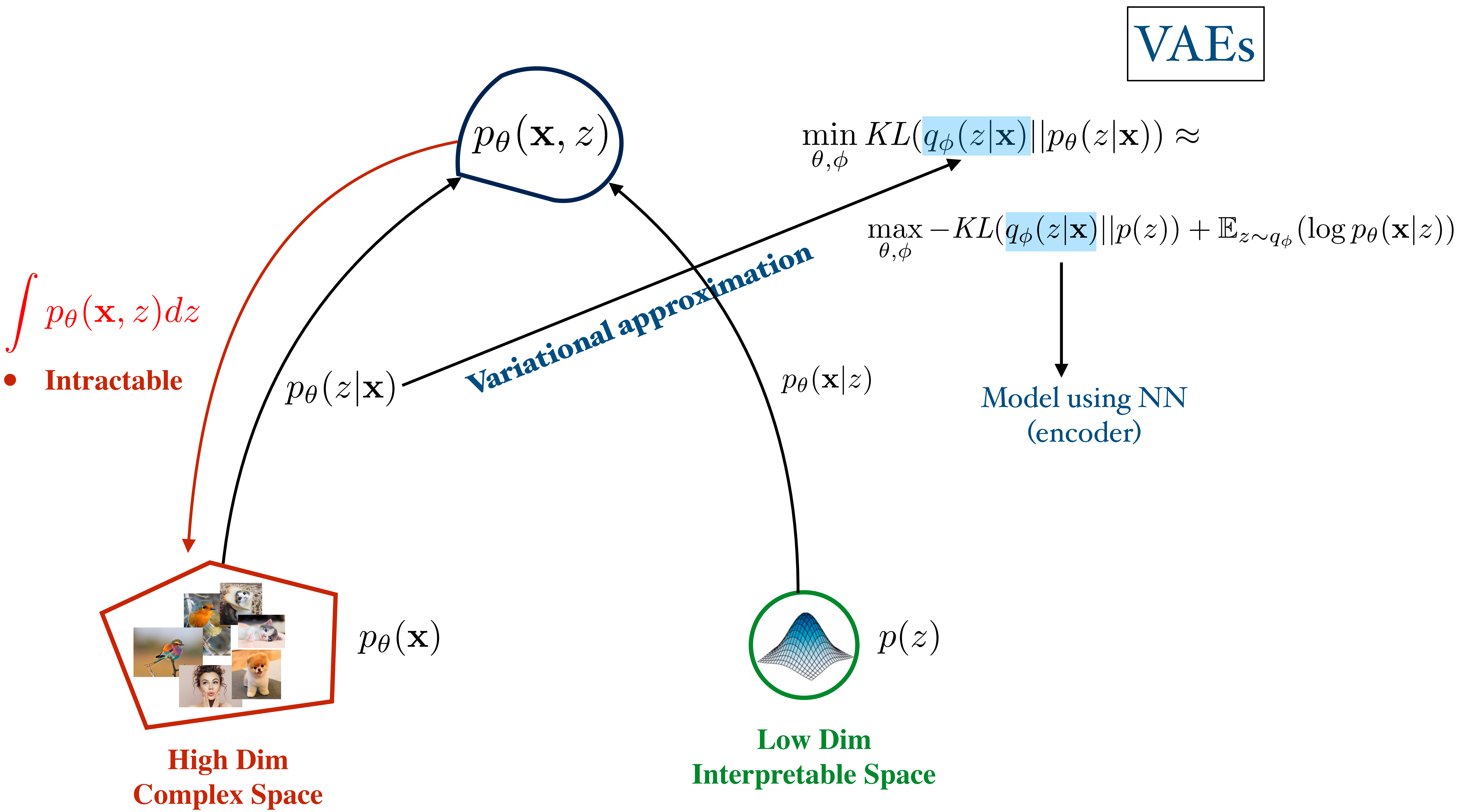

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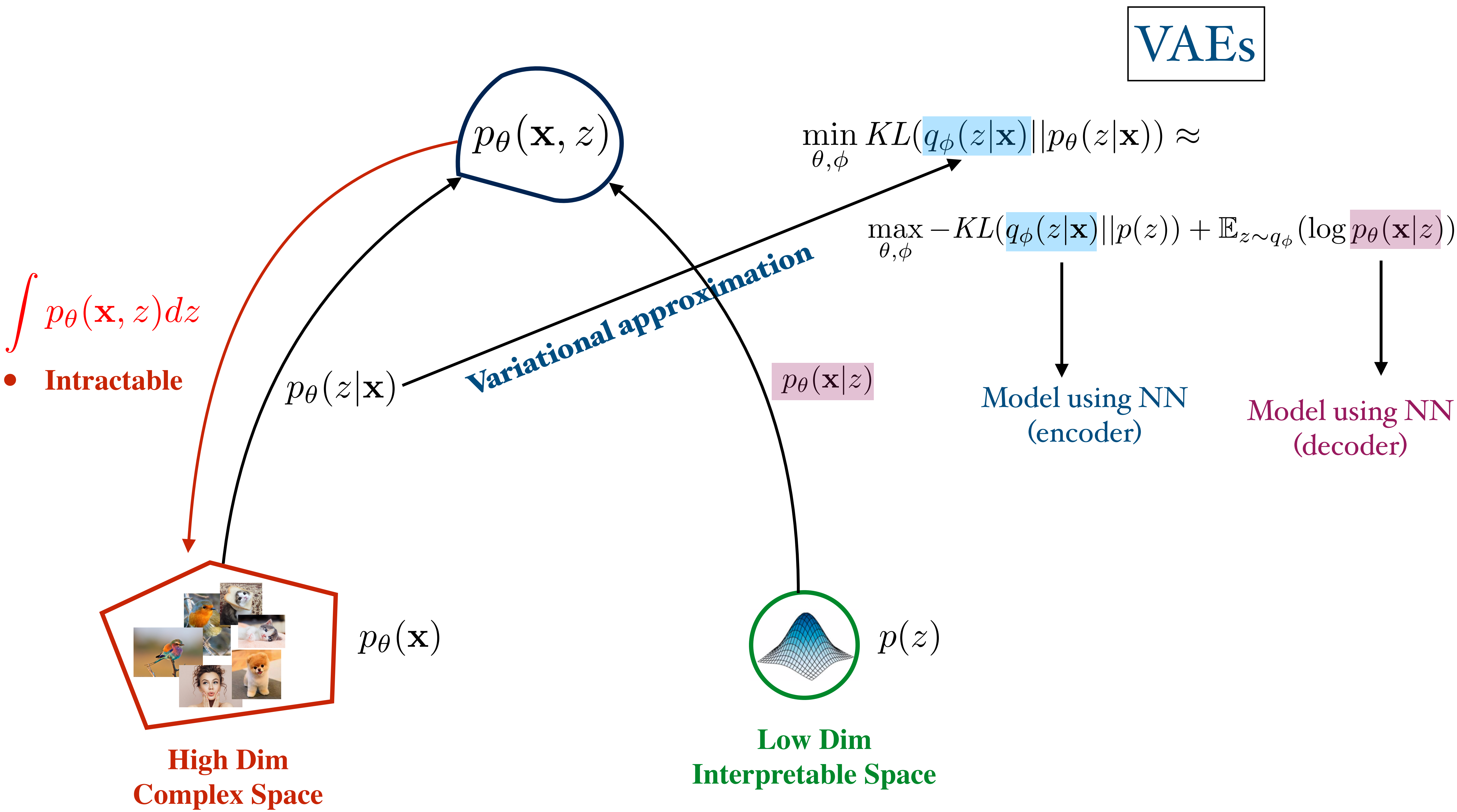

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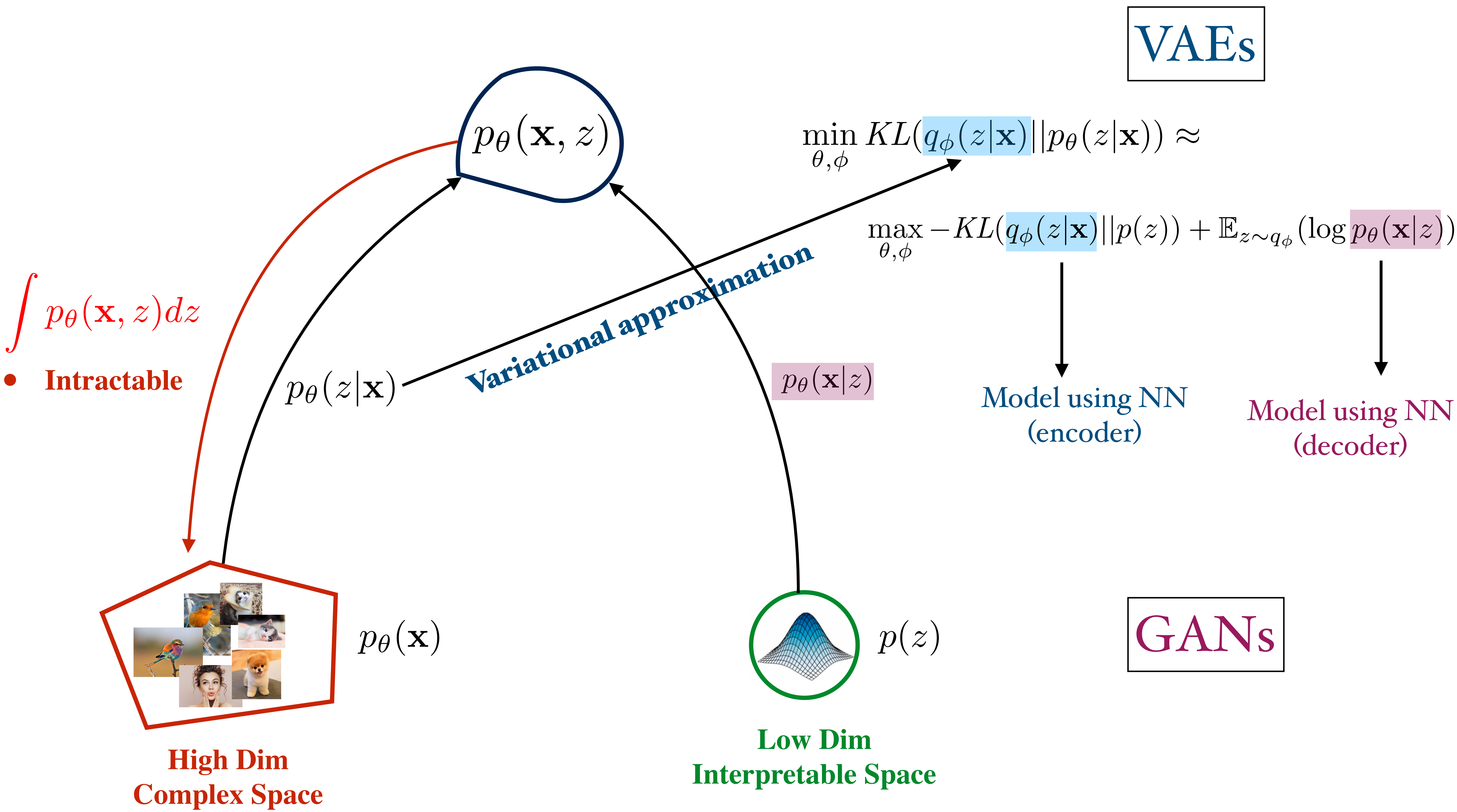

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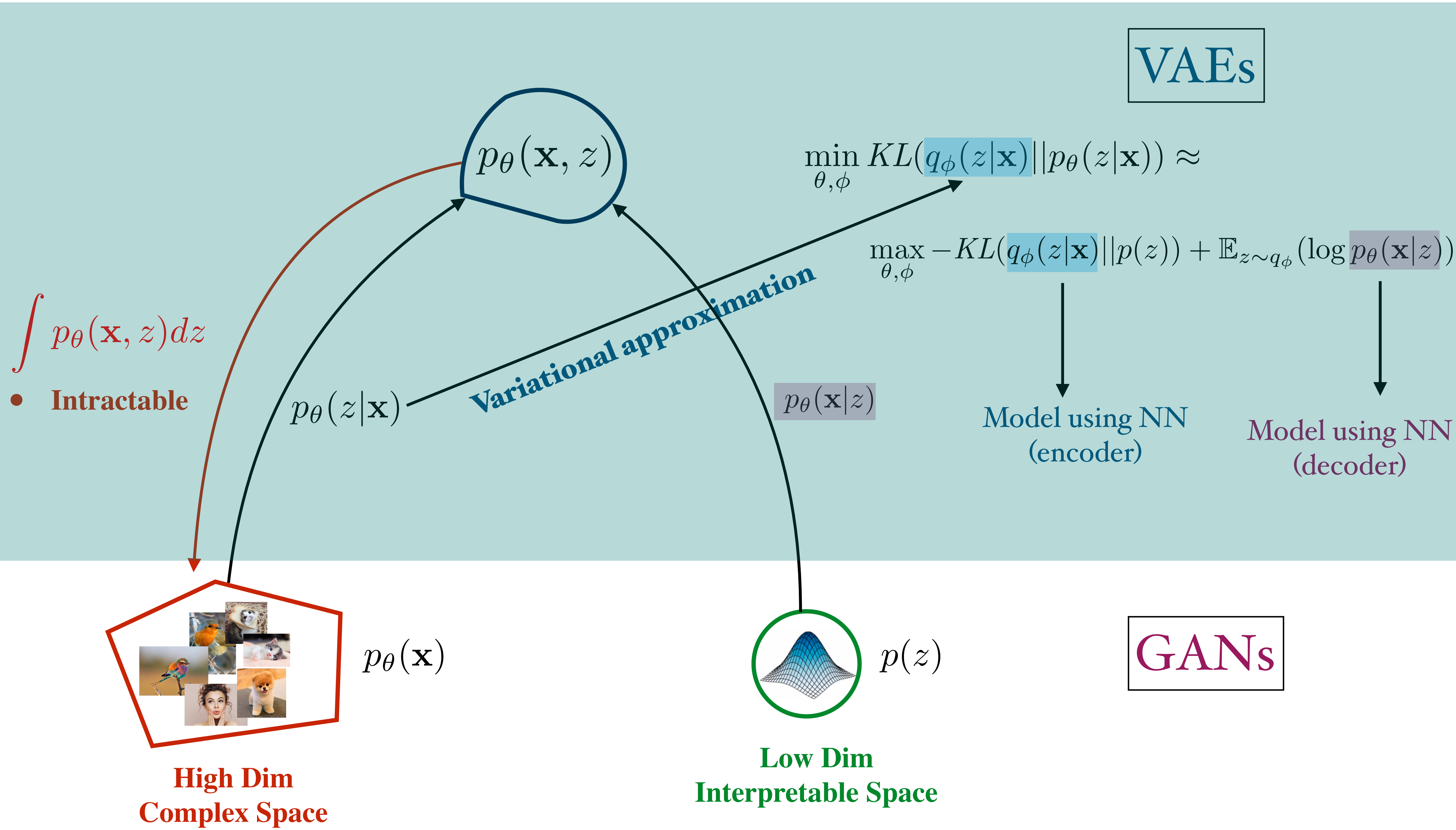

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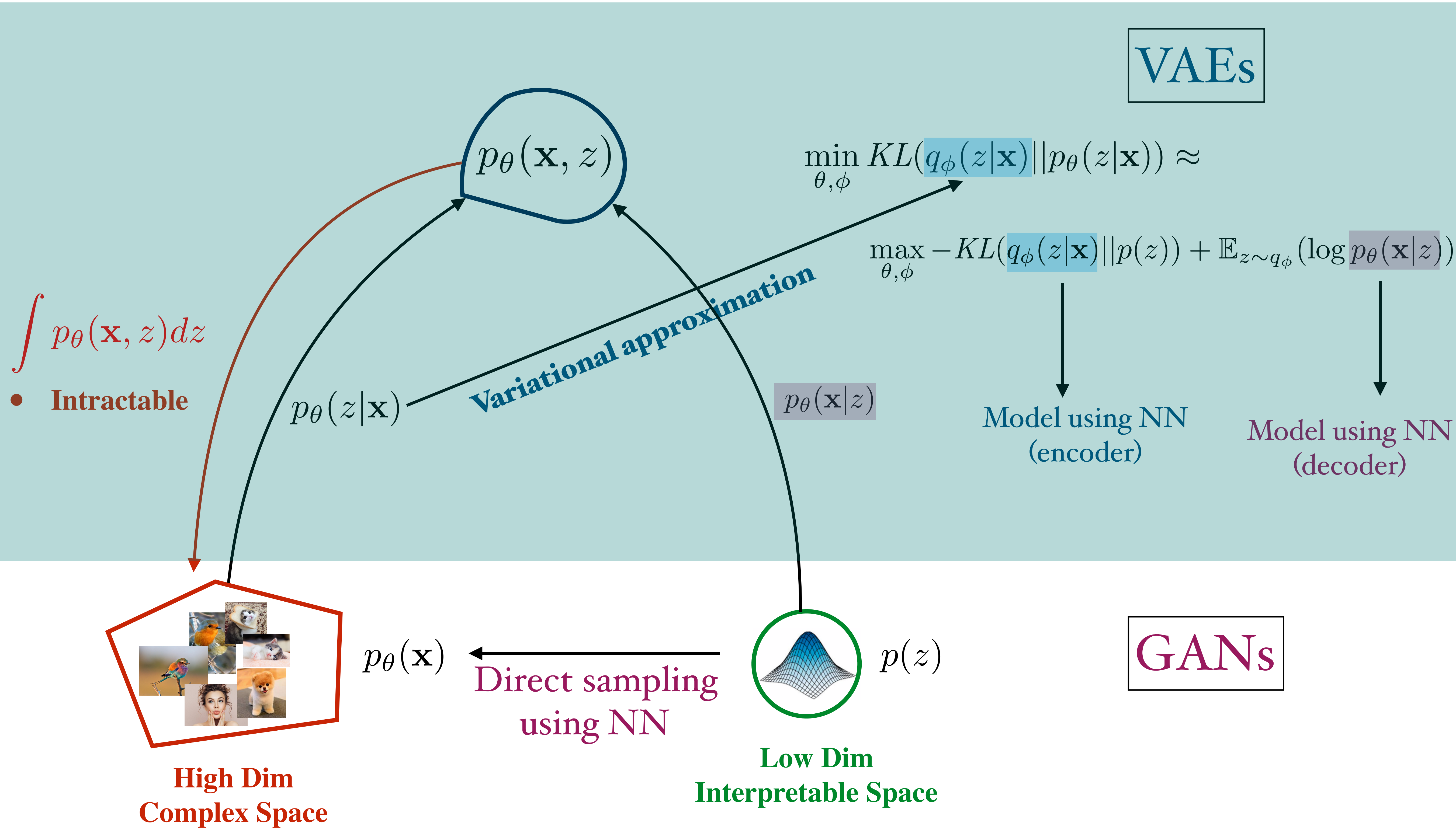

Variational Auto-encoders (VAEs)

(Kingma et al., 2013)

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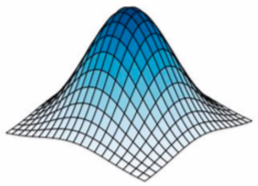

$p(z) = N(0, 1)$

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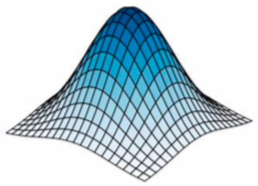

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Test

$z \sim p(z)$

(low dim)

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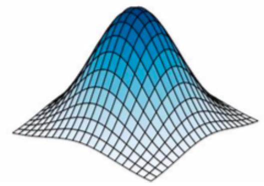

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Decoder

NN with parameters

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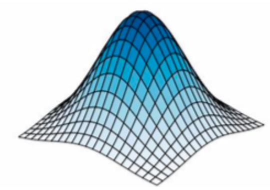

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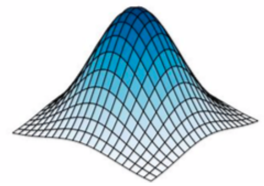

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NN with parameters

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(high dim)

$\mathbf{x} \sim p_\theta(\mathbf{x}|z)$

(high dim)

$\mathbf{x} \sim \mathcal{D}$

Encoder/Inference

NN with parameters

ϕ

Variational Auto-encoders (VAEs)

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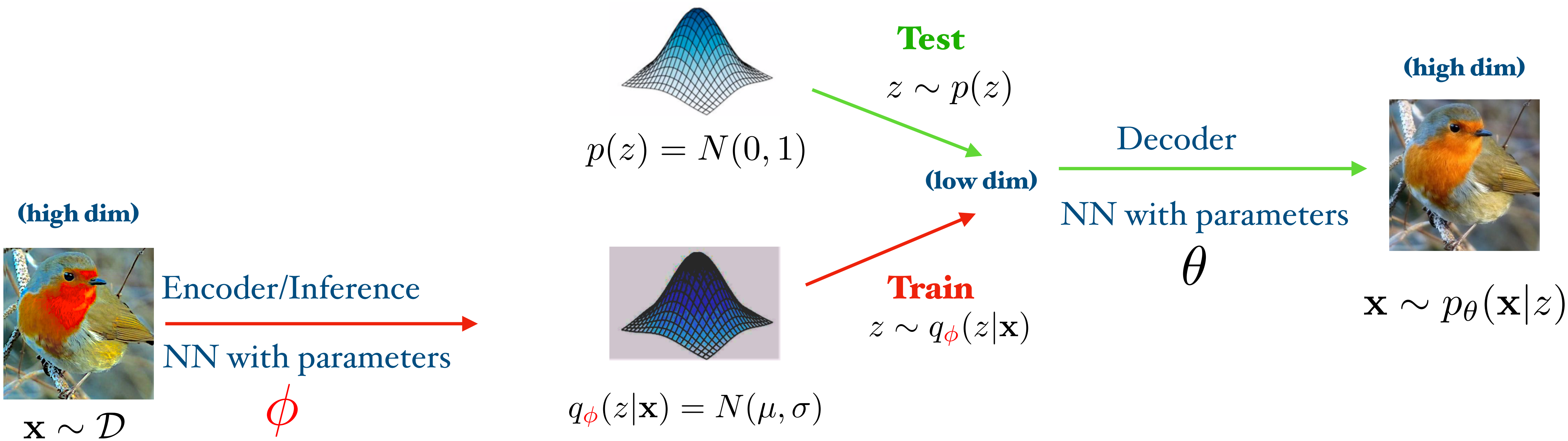

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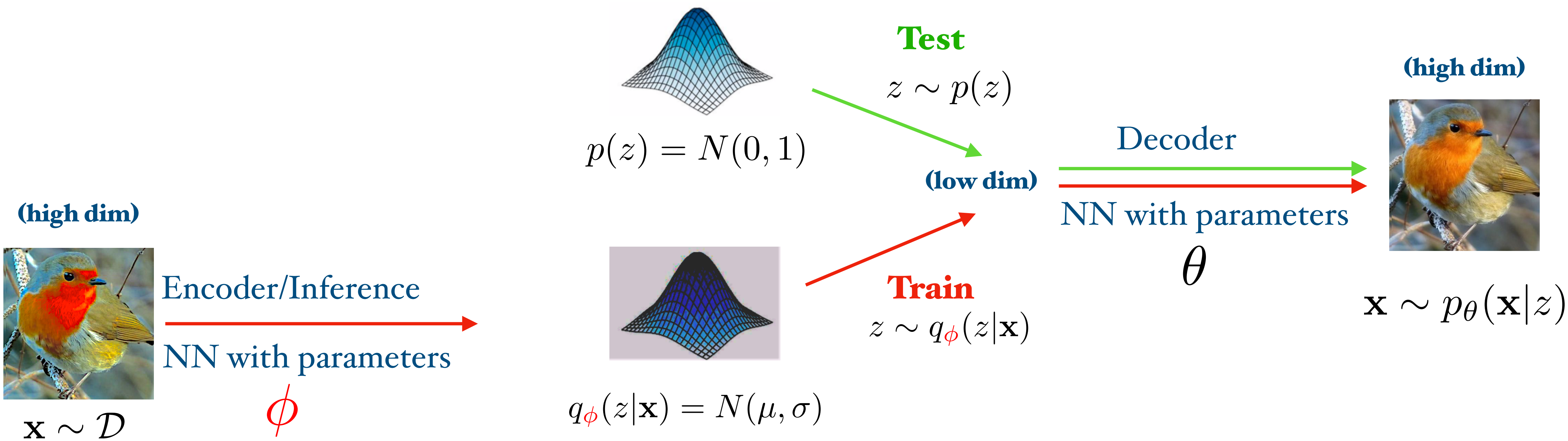

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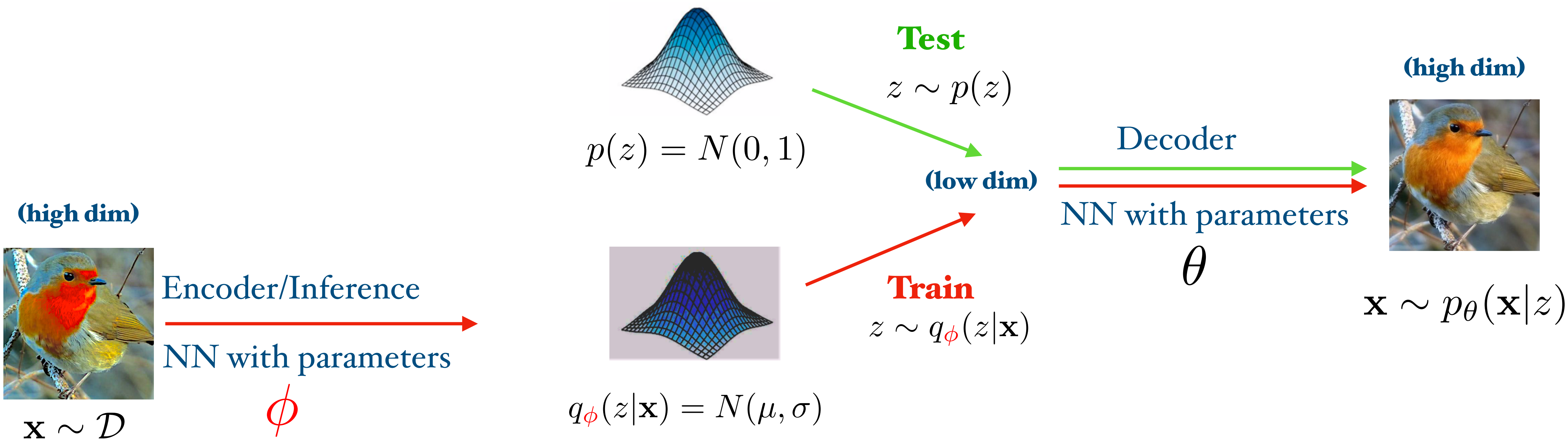

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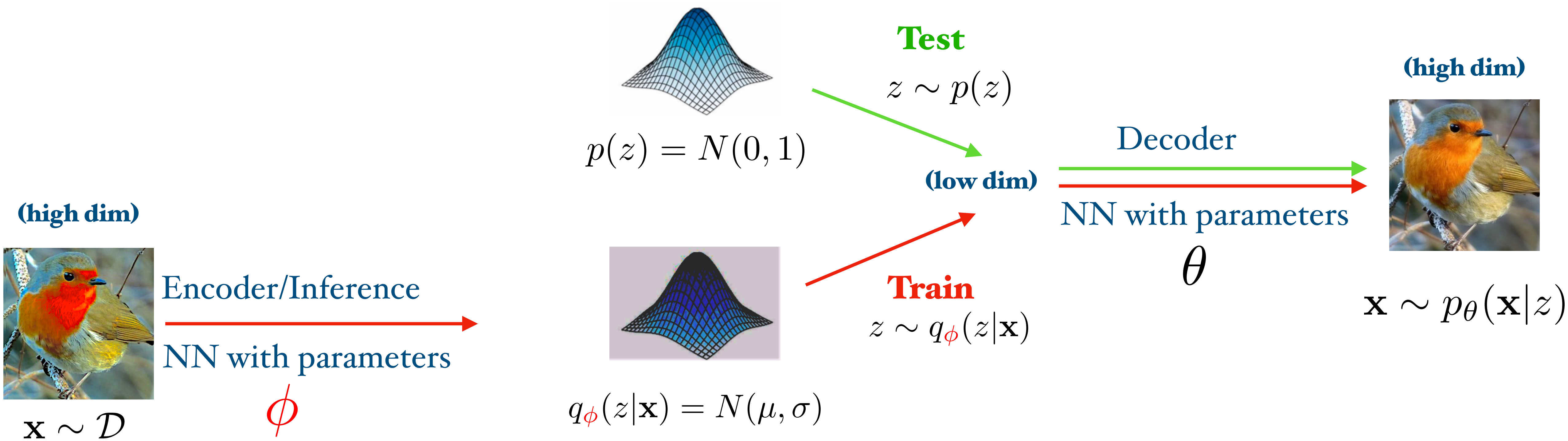

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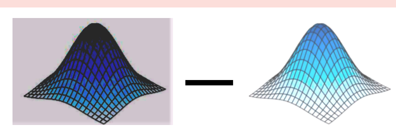

(Regularization)

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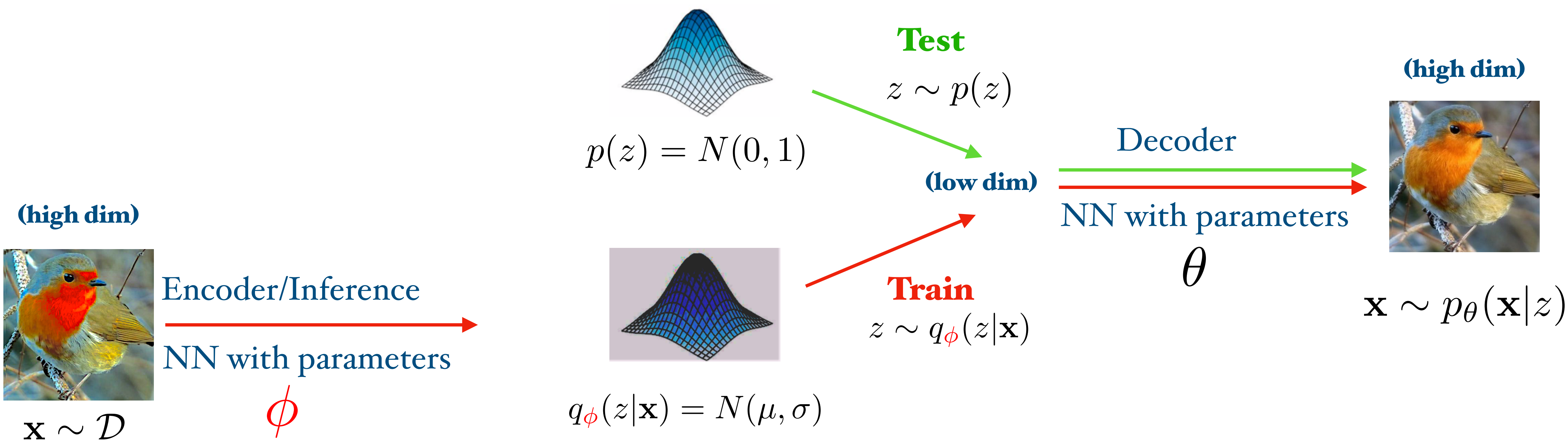

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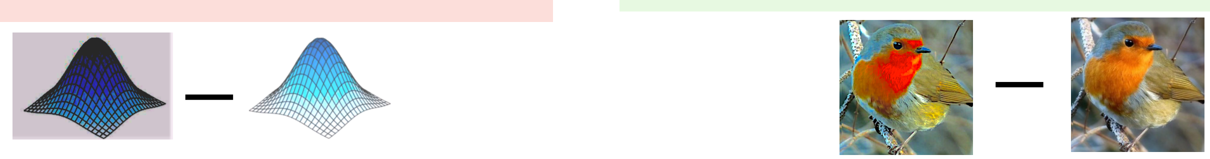

(Regularization) (Reconstruction)

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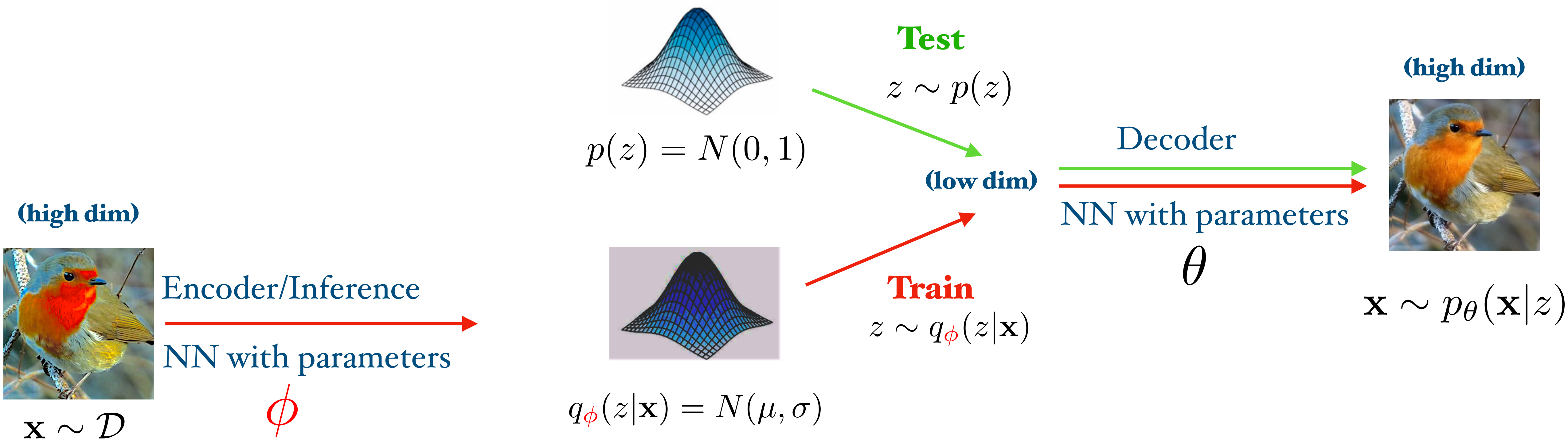

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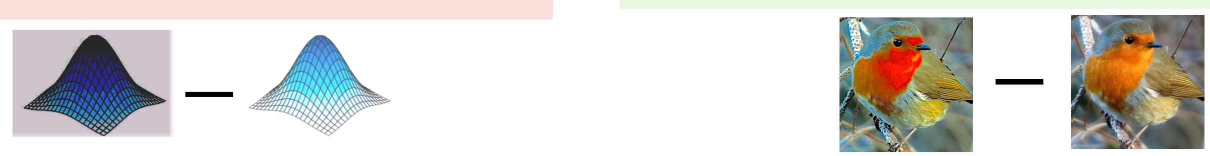

(Regularization) (Reconstruction)

- Why does the above objective make sense?
  - It maximizes the lower-bound on the data-loglikelihood $\log p_\theta(\mathbf{x}) \geq L_{elbo}$

VAE — Elbo (quick derivation) $\log p_\theta(\mathbf{x}) \geq L_{elbo}$

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$$\log p_\theta(\mathbf{x}) = \underbrace{\mathbb{E}_{q_\phi(z|\mathbf{x})} \log \frac{p_\theta(\mathbf{x}, z)}{q_\phi(z|\mathbf{x})}}_{L_{elbo}} + \underbrace{KL(q_\phi(z|\mathbf{x})||p_\theta(z|\mathbf{x}))}_{\text{Non-negative}}$$

L_{elbo}

$(-KL(q_\phi(z|\mathbf{x})||p(z)) + \mathbb{E}_{z\sim q_\phi}(\log p_\theta(\mathbf{x}|z)))$

Non-negative

Variational Auto-encoders (VAEs) — Training

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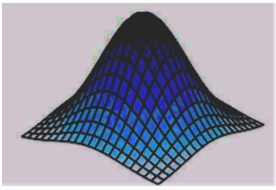
(high dim)

$\mathbf{x} \sim \mathcal{D}$

Encoder/Inference

NN with parameters

ϕ



$q_\phi(z|\mathbf{x}) = N(\mu, \sigma)$

Train

$z \sim q_\phi(z|\mathbf{x})$

(low dim)

Decoder

NN with parameters

θ

(high dim)

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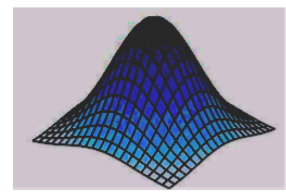

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Backpropagate?

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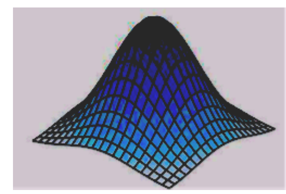

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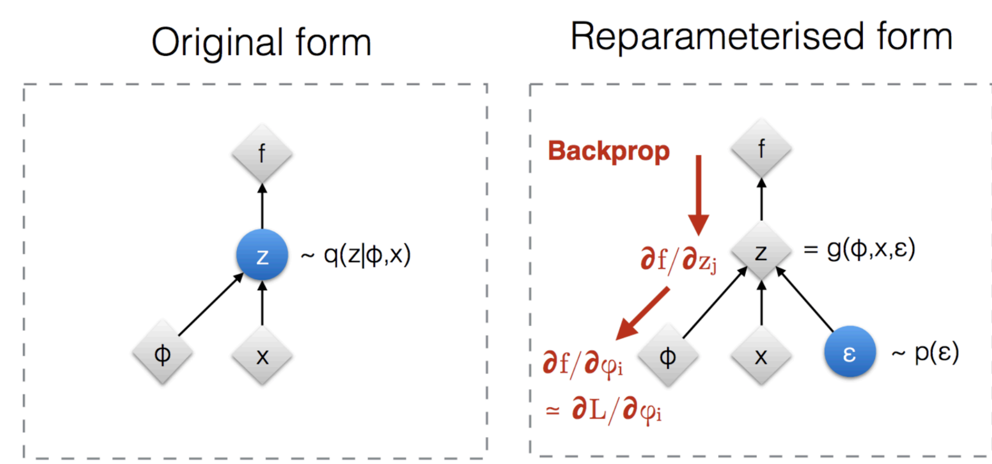
Reparametrization Trick

$z \sim N(\mu, \sigma)$ equivalent to $\mu + \epsilon\, \sigma, \epsilon \sim N(0, 1)$



Puneet Dokania

Variational Auto-encoders (VAEs) — Training

(high dim)

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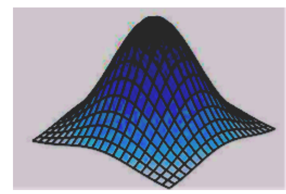
Encoder/Inference

NN with parameters ϕ



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Decoder

NN with parameters θ

(high dim)

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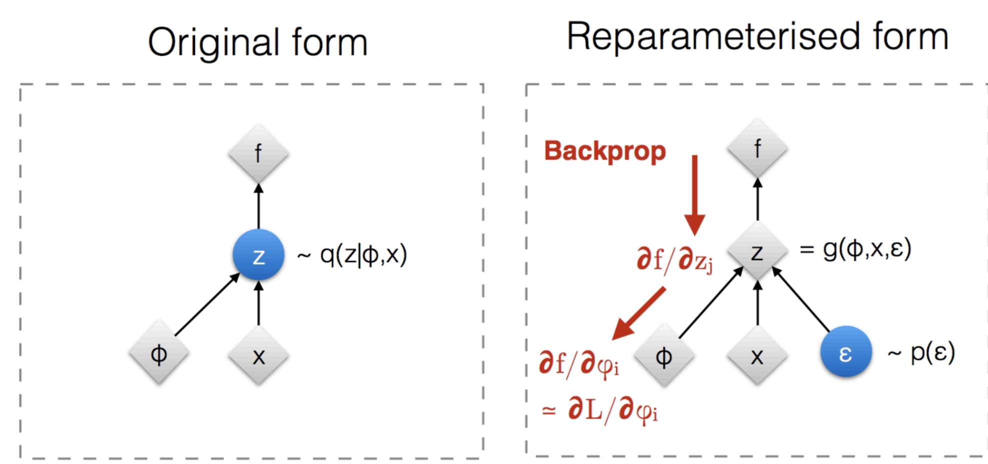
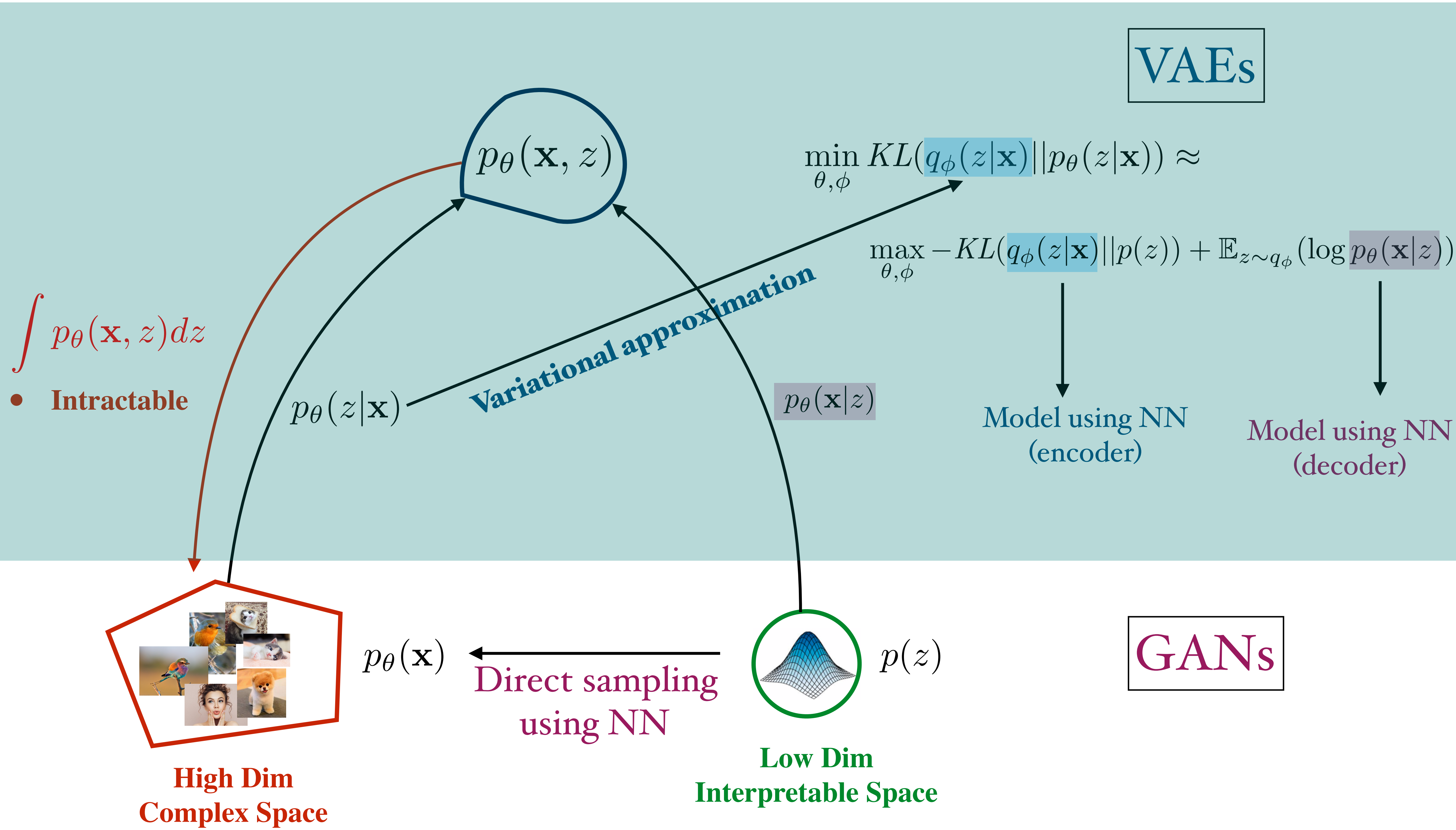


Training

- Random mini batch of M training samples
- Sample M noise $\epsilon \sim N(0,1)$
- Perform forward and backward pass

Generative Adversarial Networks

Generative Adversarial Networks



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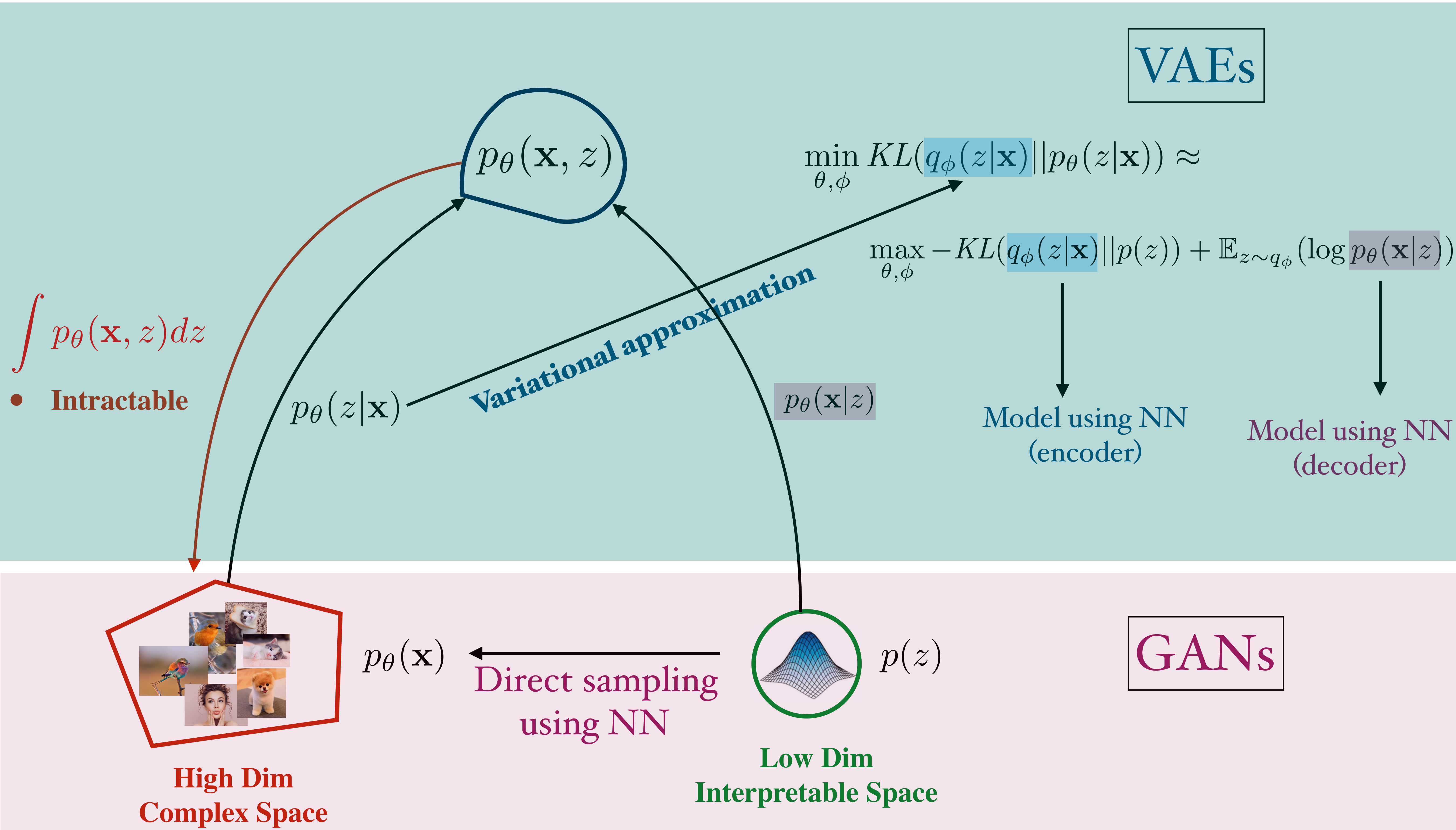
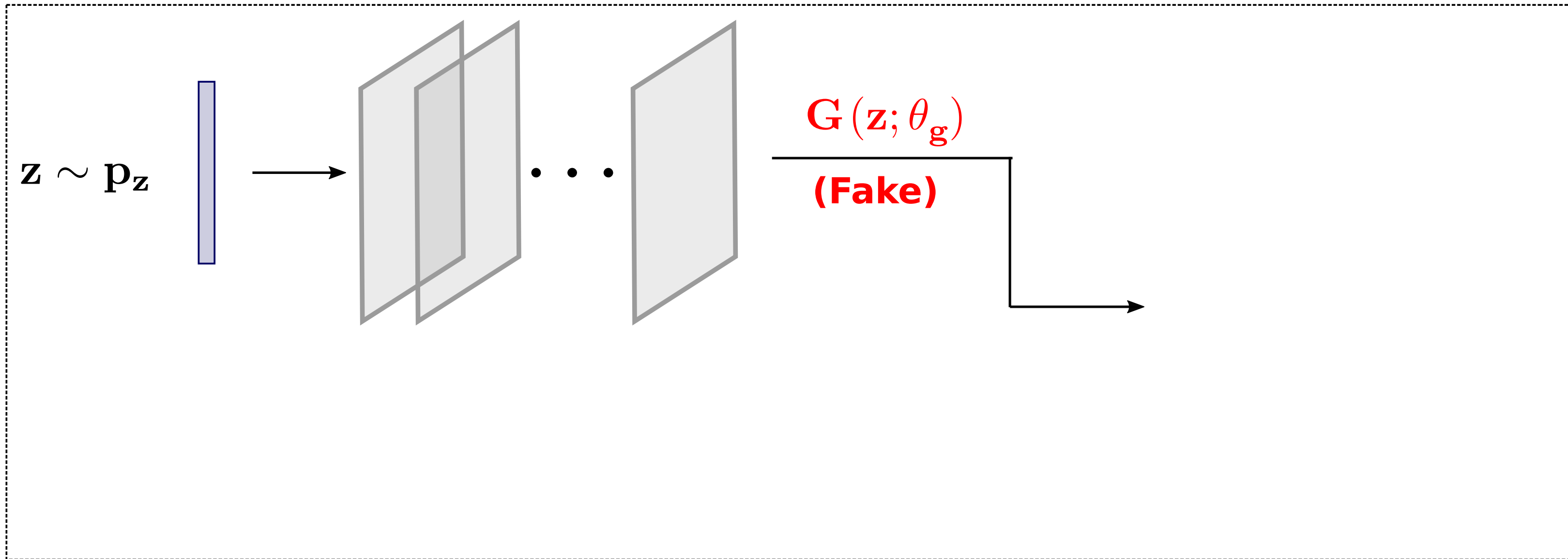


GAN — Objective function (Goodfellow et al., 2014)



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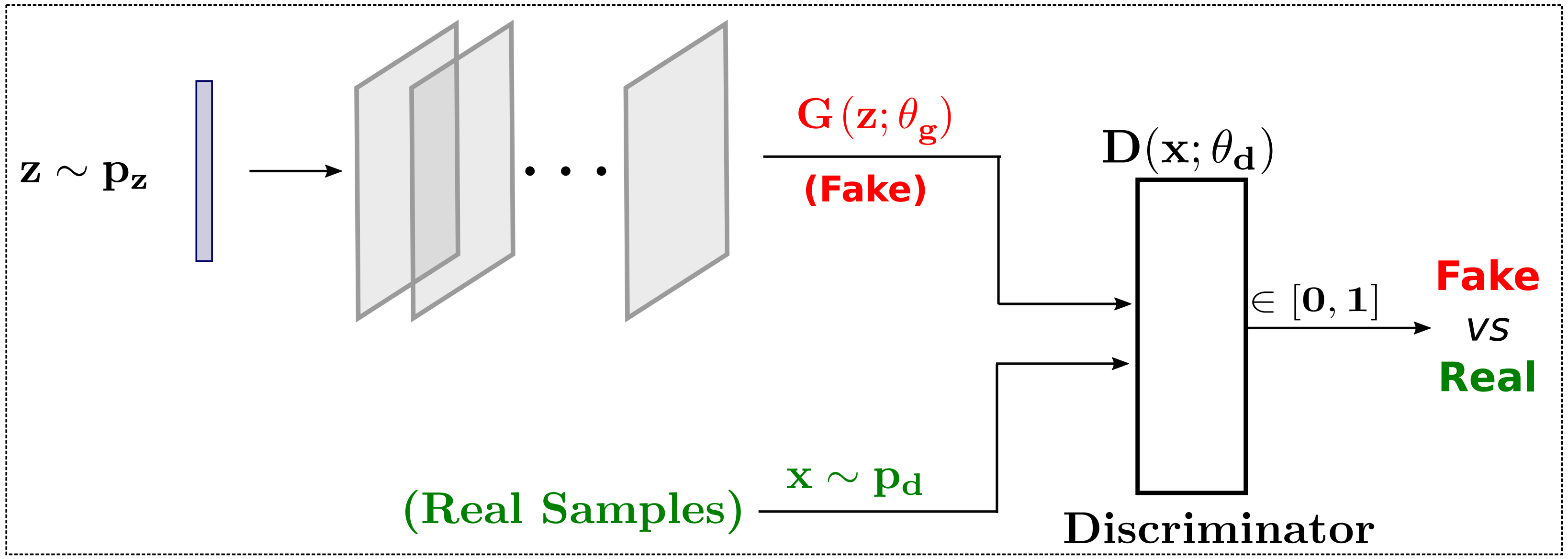


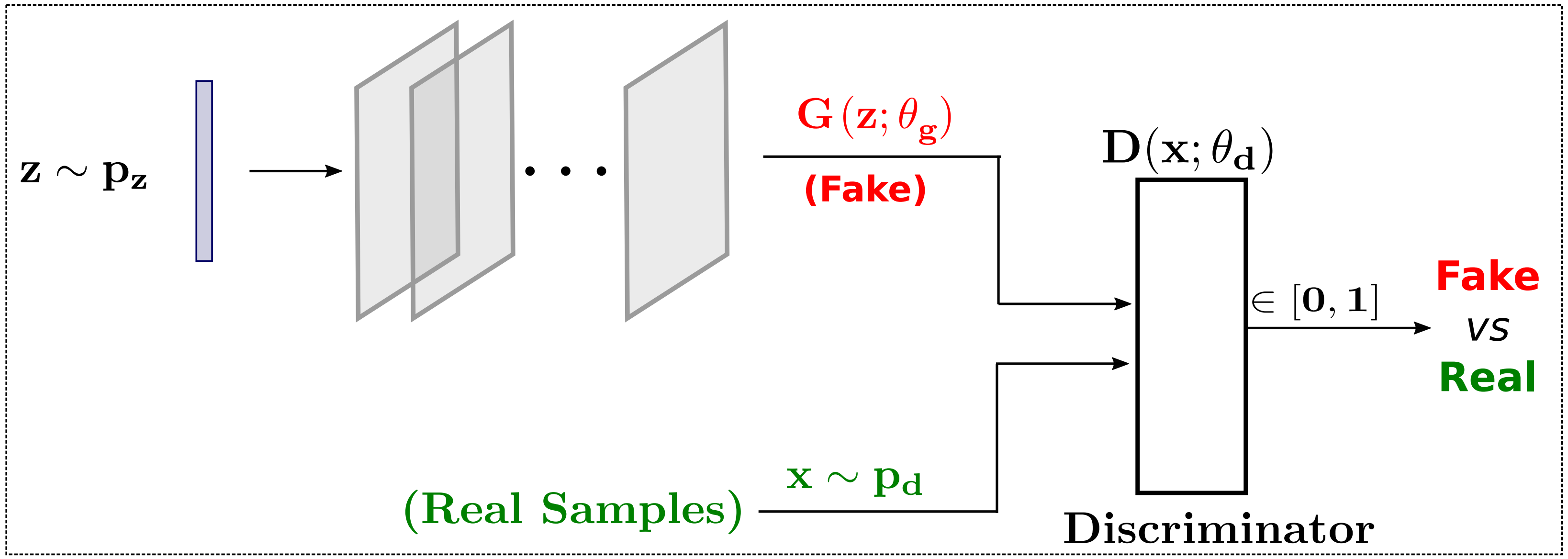
The minimax game

$$\min_{\theta_g} \max_{\theta_d} \mathbb{E}_{x \sim p_d} \log D(x; \theta_d) + \mathbb{E}_{z \sim p_z} \log \left(1 - D(G(z; \theta_g); \theta_d)\right)$$

Real

Fake

GAN — Objective function (Goodfellow et al., 2014)



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Real **Fake**

at equilibrium

$$p_g = p_d$$

GAN — A closer look at the generator

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Vanishing gradient

- if the discriminator is confident $D(x_g) \approx 0$

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Vanishing gradient

- if the discriminator is confident $D(x_g) \approx 0$

Modify generator objective, keep discriminator intact

$$\min_{\theta_g} \mathbb{E}_{z \sim p_z} - \log D(G(z; \theta_g); \theta_d)$$

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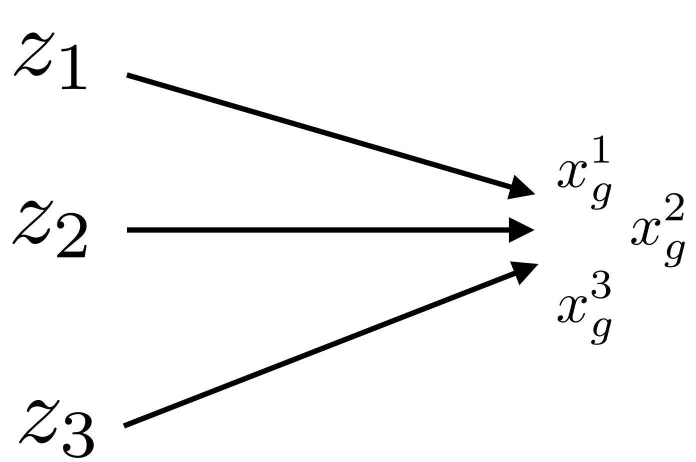

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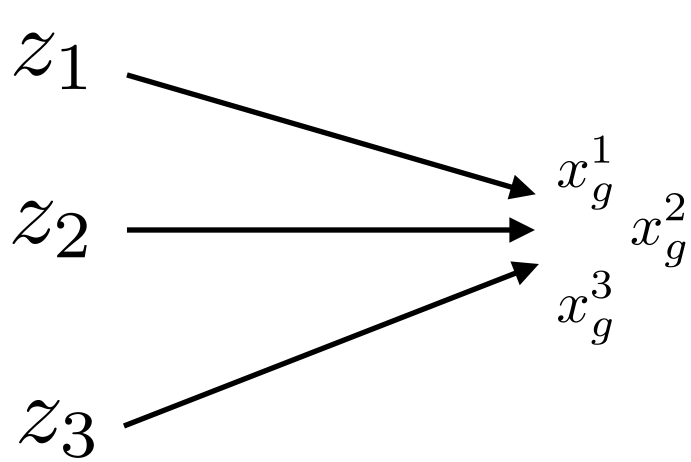

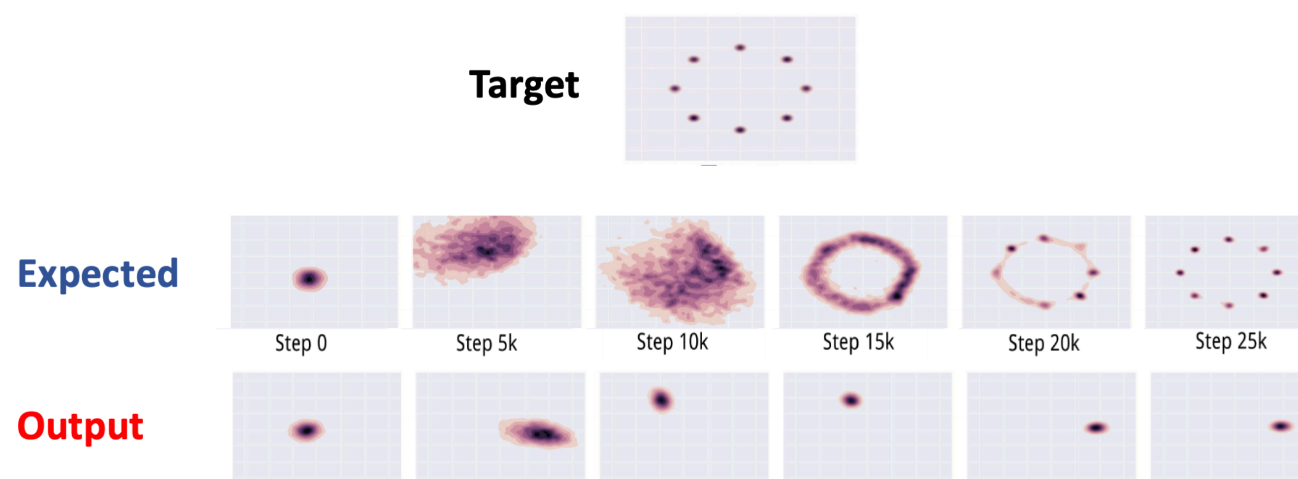

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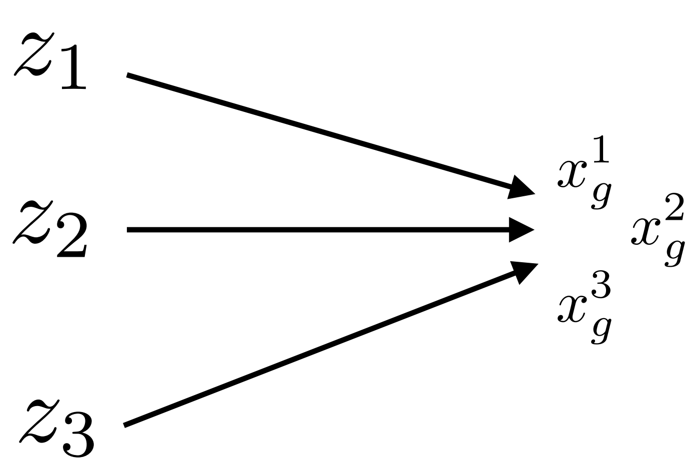

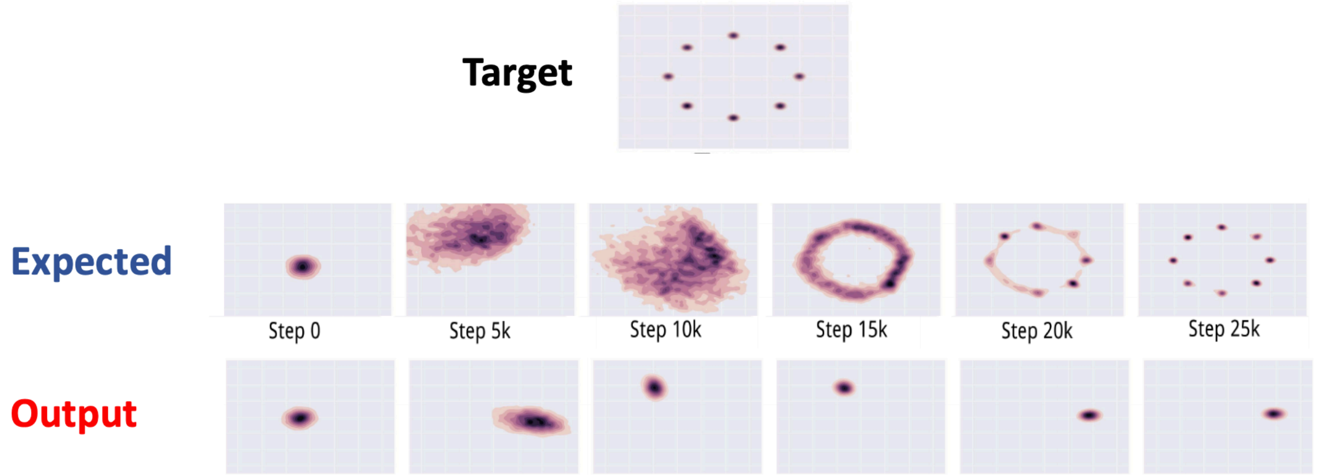

- Tons of papers in 2-3 years trying to resolve these issues

GAN Variants — Discriminator focused

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Tension between Capacity and Generalizability

GAN Variants — Discriminator focused

Tension between Capacity and Generalizability

- Most of the GAN variants focus on the
  - objective/generalizability of the discriminator
    - low Lipschitz constant — objective modification
    - 1-Lipschitz discriminator
      - WGAN — objective modification + weight clipping
      - WGAN (Gradient penalty) — objective modification
  - training stability of the discriminator
    - Spectral norm GAN — acts on the linear mappings
    - Orthonormal GAN — acts on the linear weights
    - Stable rank GAN — acts on the linear weights

GAN Variants — Discriminator focused

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$$\mathbf{x} \in \mathbb{R}^m \xrightarrow{f_W} f_W(\mathbf{x}) \in \mathbb{R}^n$$

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Global Lipschitz Constant $$L_g = \max_{\mathbf{x}_i, \mathbf{x}_j} \frac{||f(\mathbf{x}_i) - f(\mathbf{x}_j)||_2}{||\mathbf{x}_i - \mathbf{x}_j||_2}$$ **Sensitivity**

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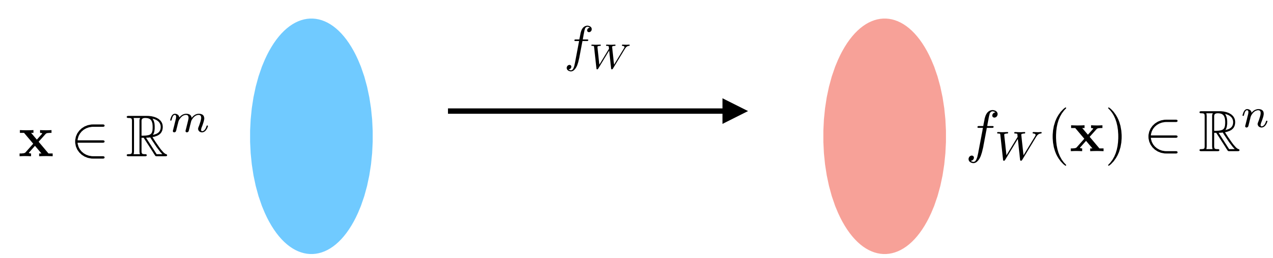

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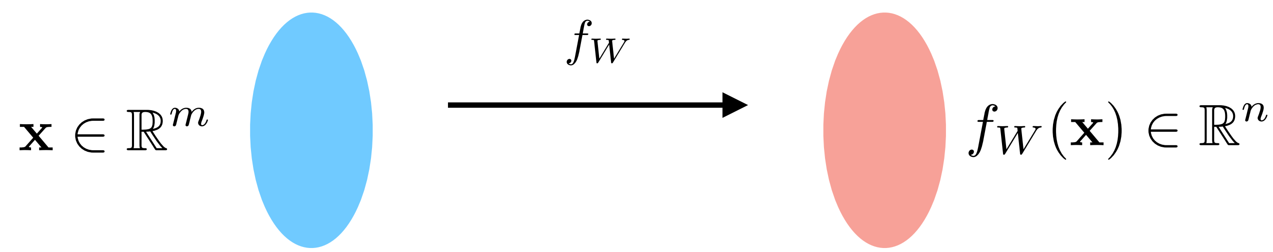

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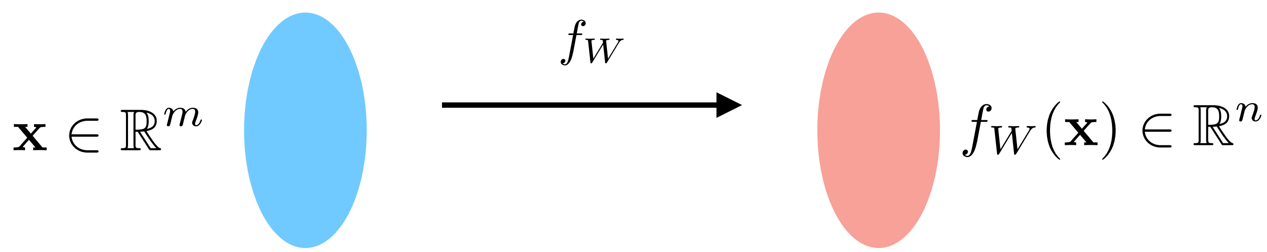
Spectral Norm GAN (Miyato et al., ICLR18) $\quad \bar{W}_{SN} = \frac{W}{\sigma_{max}(W)}$ Optimal?

GAN Variants — Discriminator focused



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WGAN-GP (Guljarani et al., NeurIPS17) $(||J_f(\mathbf{x})||_2 - 1)^2$ Additional penalty — interpolated samples

GAN Variants — Generator focused

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- Mode collapse GANs — once again

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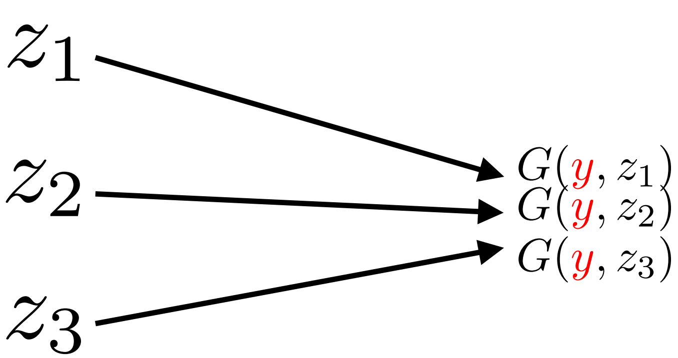

condition y

GAN Variants — Generator focused

- Mode collapse GANs — once again

$$z_1 \rightarrow G(y, z_1)$$
$$z_2 \rightarrow G(y, z_2)$$
$$z_3 \rightarrow G(y, z_3)$$

$$\max_{\theta_g} \mathbb{E}_{z_i, z_j} \frac{||G(y, z_i) - G(y, z_j)||}{||z_i - z_j||}$$

(Yang et al., ICLR19)

(hinge type version)

condition y

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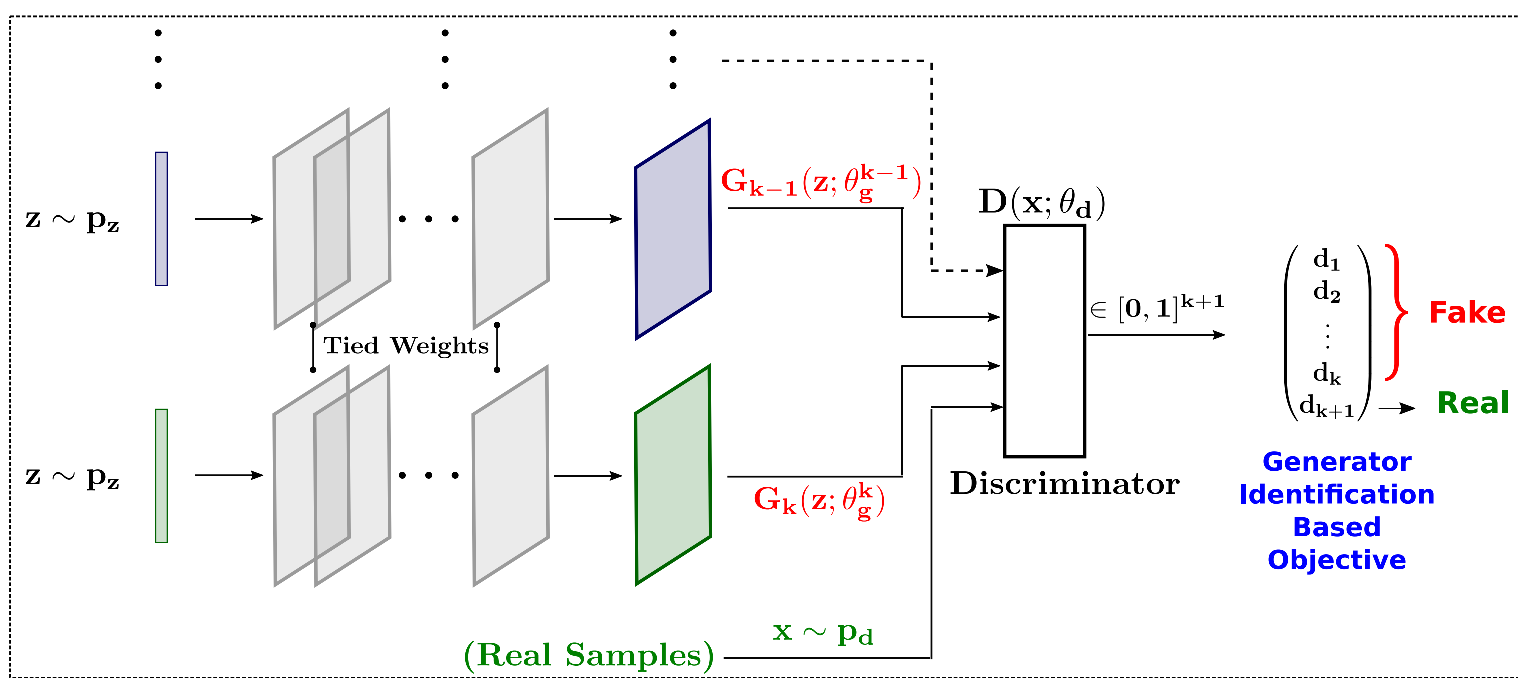
MAD-GAN — Use multiple generators

(Ghosh et al., CVPR18)



Big-GAN (Brock et al., ICLR19)

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Focus on both — Discriminator and Generator

Big-GAN (Brock et al., ICLR19)

Focus on both — Discriminator and Generator

Among other tricks —

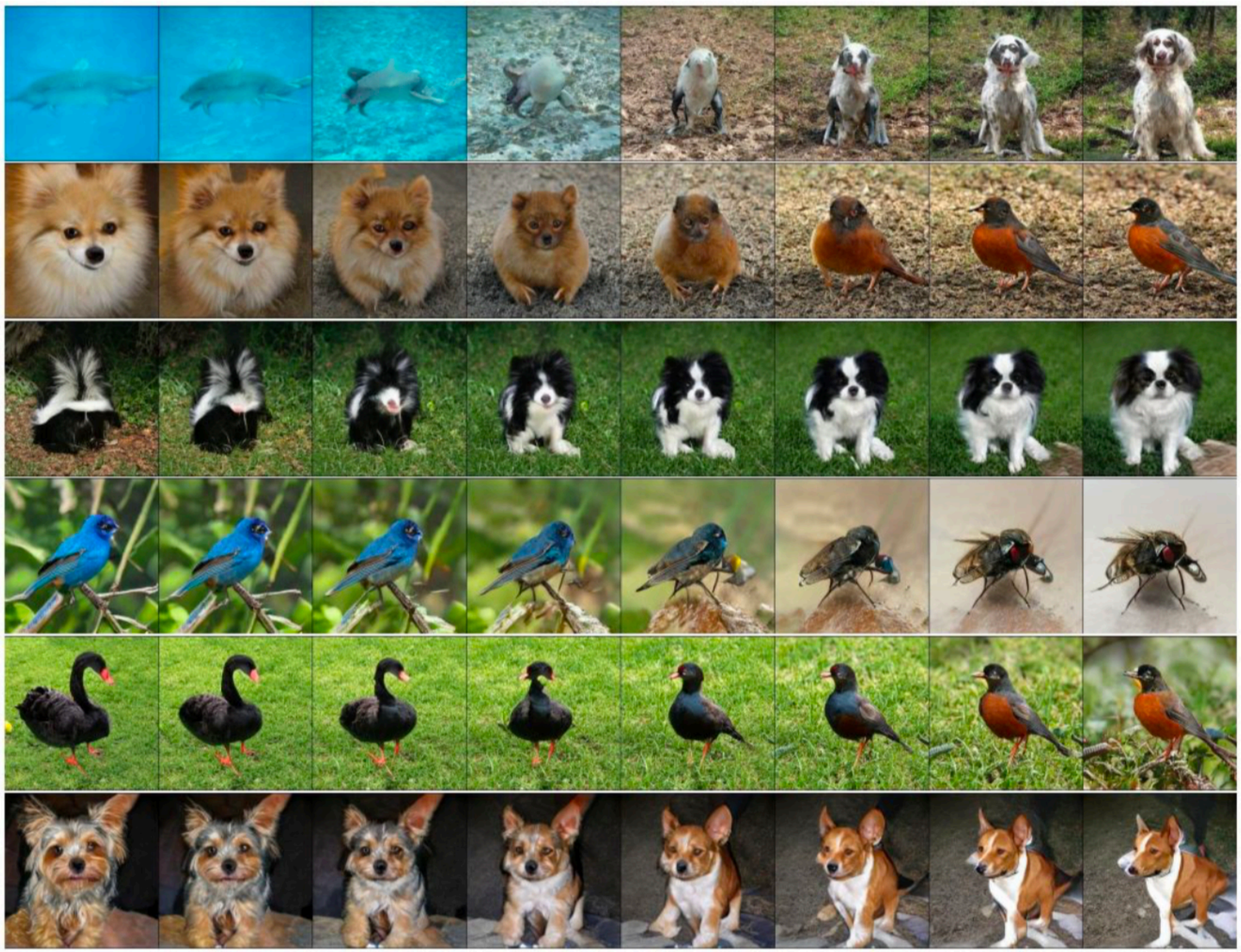
- Spectral norm on both Discriminator and Generator
- Orthogonal initalization of parameters — training speed improves
- A new variant of orthonormal regularisation on the Generator
  - all singular values nearly one
- Increased batch size (8x), increase width (50%), etc.

Big-GAN (Brock et al., ICLR19)

512x512 generations

Big-GAN (Brock et al., ICLR19)

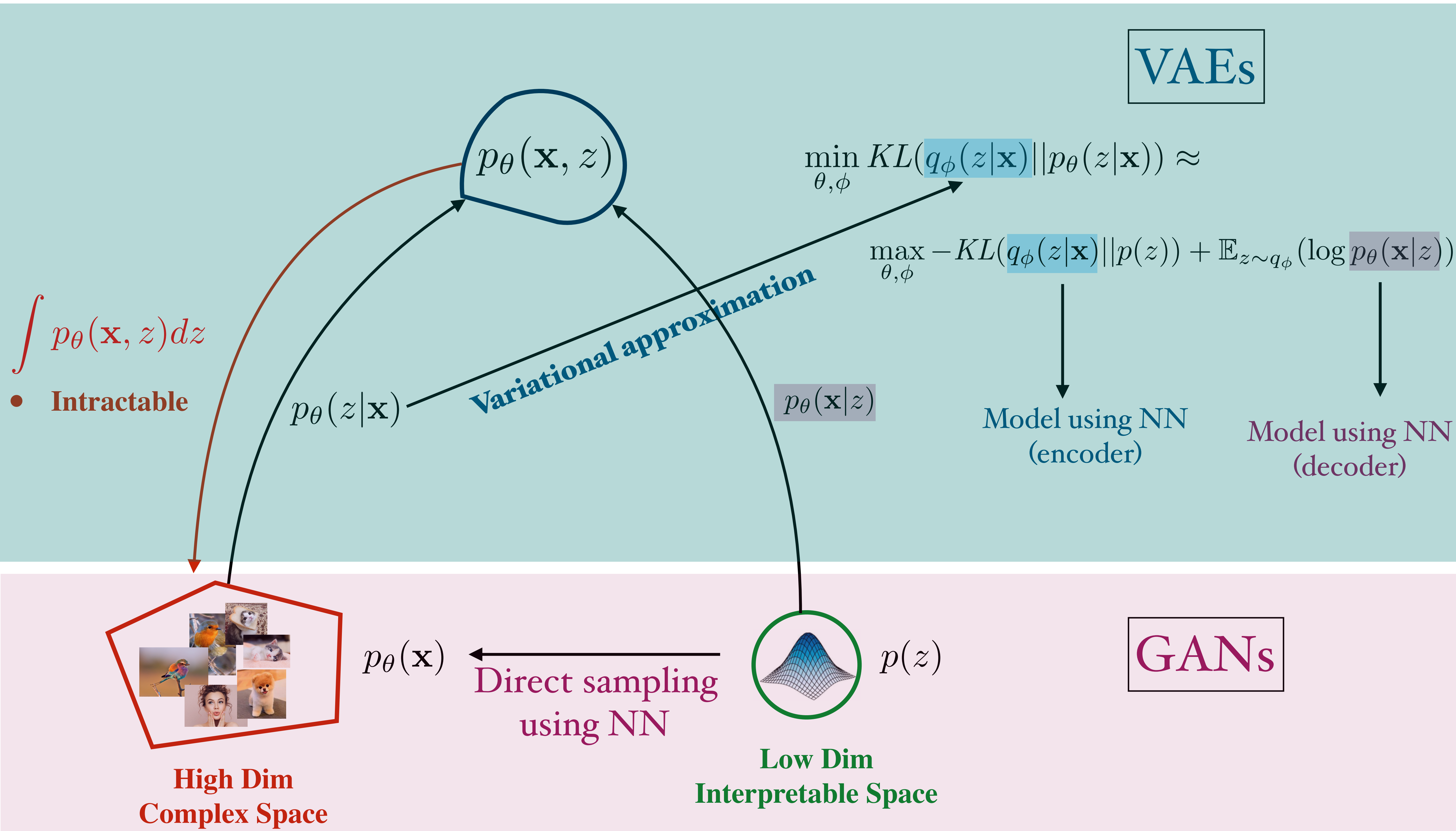
Interpolation



GAN — few insights

- Focus on both — generator and discriminator
- Trade-off between the capacity and the generalizability of discriminator
- Think about the invertibility of the generator
- Invertibility might not be enough, think about improving the condition number

VAEs and GANs



Thank You

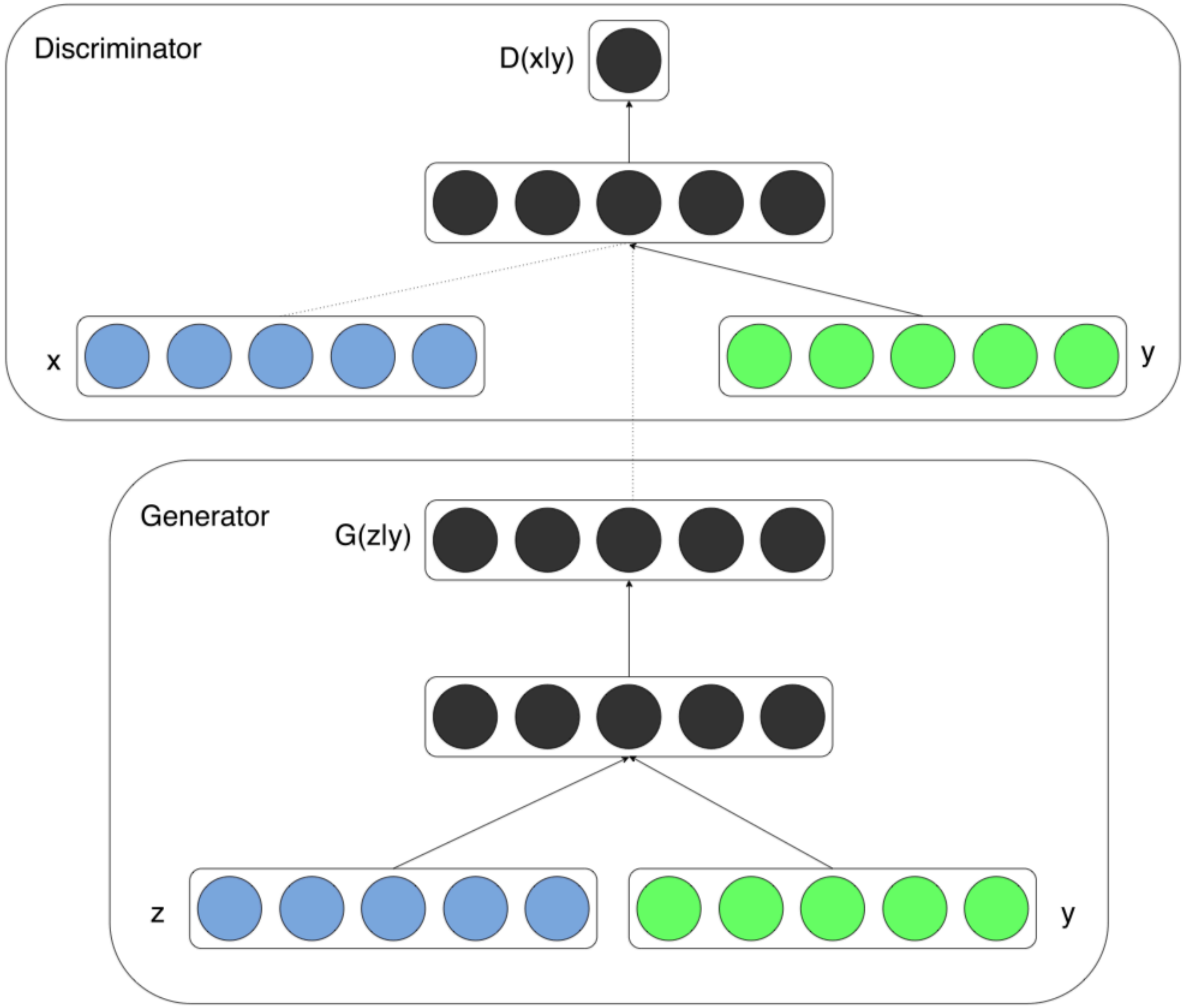
Deep Generative Models — Applications of GANs
(part 2)

Puneet K. Dokania
(University of Oxford)

<u>Presented at</u>

Machine Learning Summer School
IIIT Hyderabad, India
(9th July 2019)

Conditional GAN (Mirza et al., 2014)



$$\min_{\theta_g} \max_{\theta_d} \mathbb{E}_{x \sim p_d} \log D(x|y; \theta_d) + \mathbb{E}_{z \sim p_z} \log \big(1 - D(G(z|y; \theta_g); \theta_d)\big)$$

Image to Image Translation — Paired
(Isola et al., CVPR17)

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(Isola et al., CVPR17)

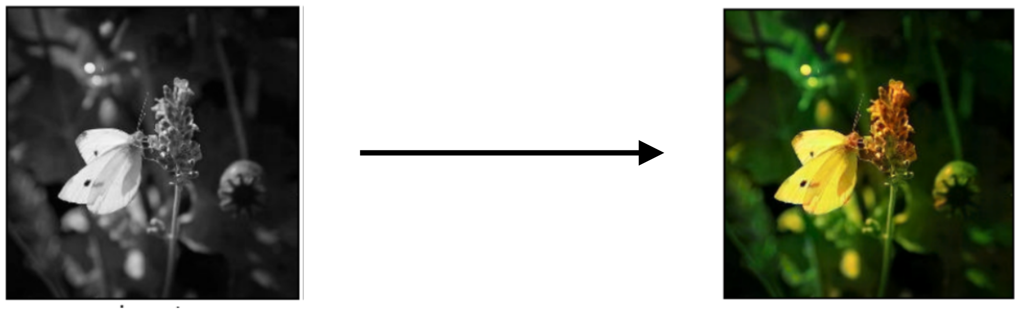
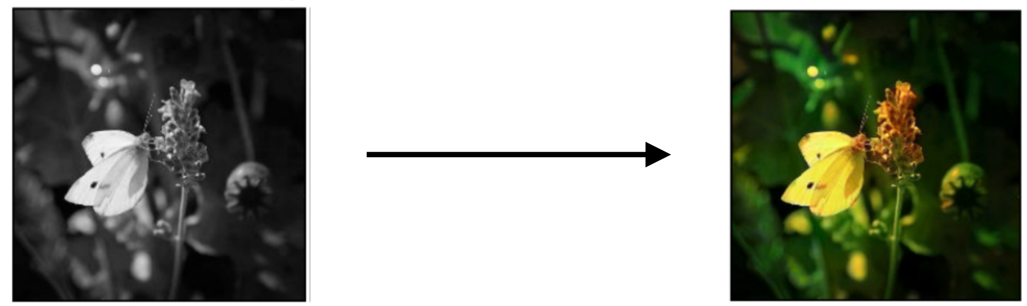


Image to Image Translation — Paired
(Isola et al., CVPR17)



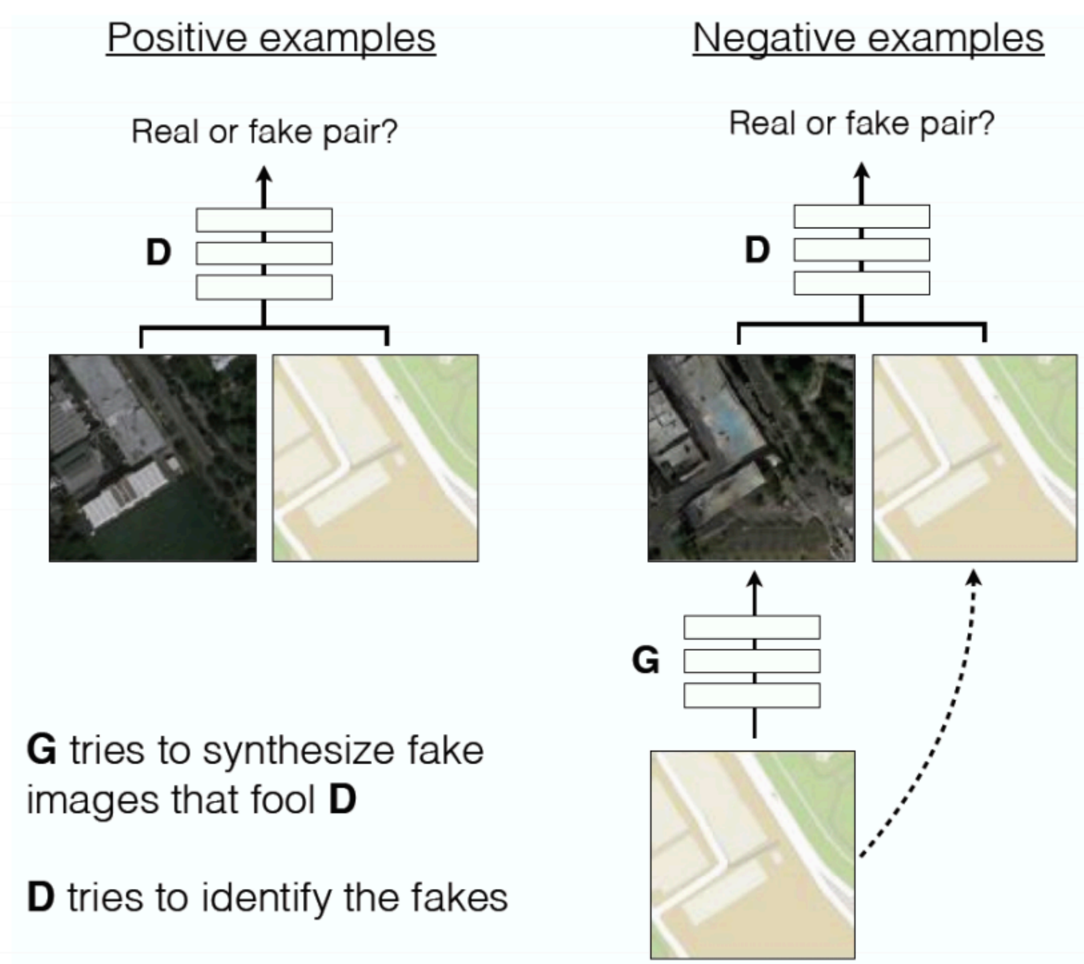


Figure 2 in the original paper.

$$\mathcal{L}_{L1}(G) = \mathbb{E}_{x,y,z}[\|y - G(x,z)\|_1].$$

$$G^* = \arg\min_G \max_D \mathcal{L}_{cGAN}(G,D) + \lambda\mathcal{L}_{L1}(G).$$

Image to Image Translation — Paired
(Isola et al., CVPR17)

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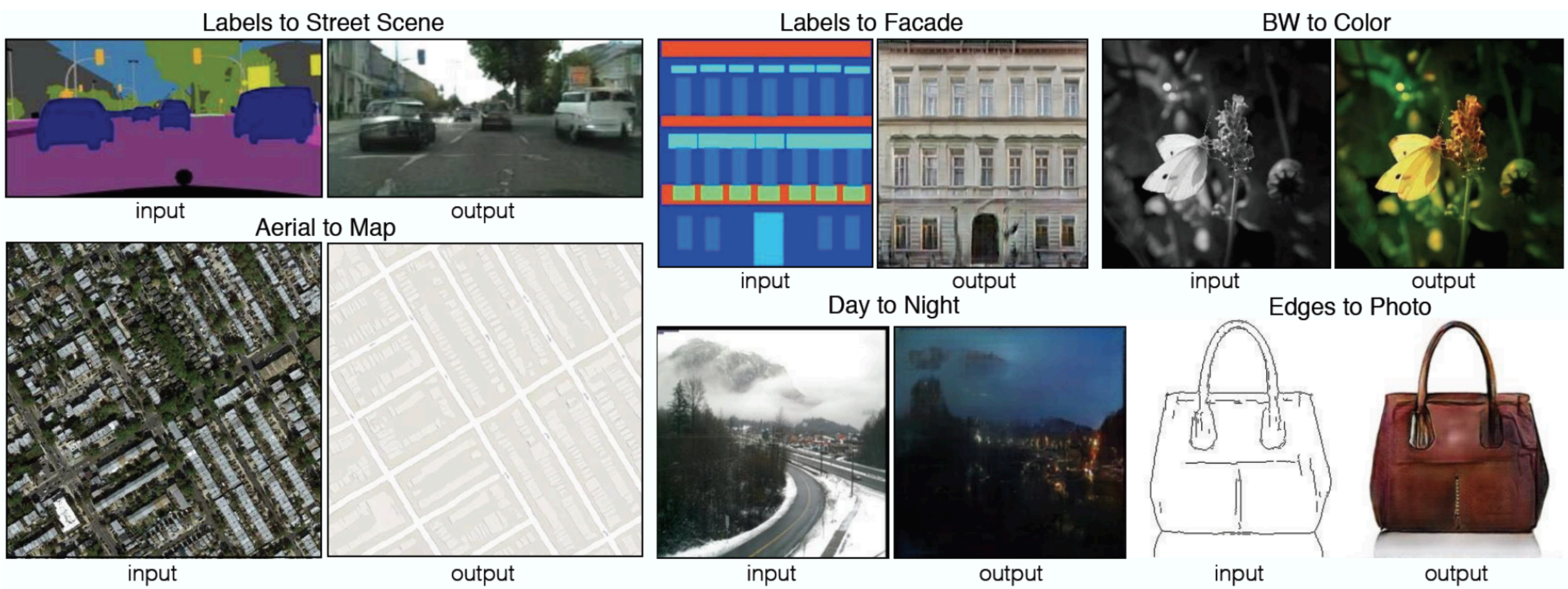


Figure 1 in the original paper.

Image to Image Translation — Paired
(Isola et al., CVPR17)

- **No diversity — Why?**
  - **Noise gets absorbed**

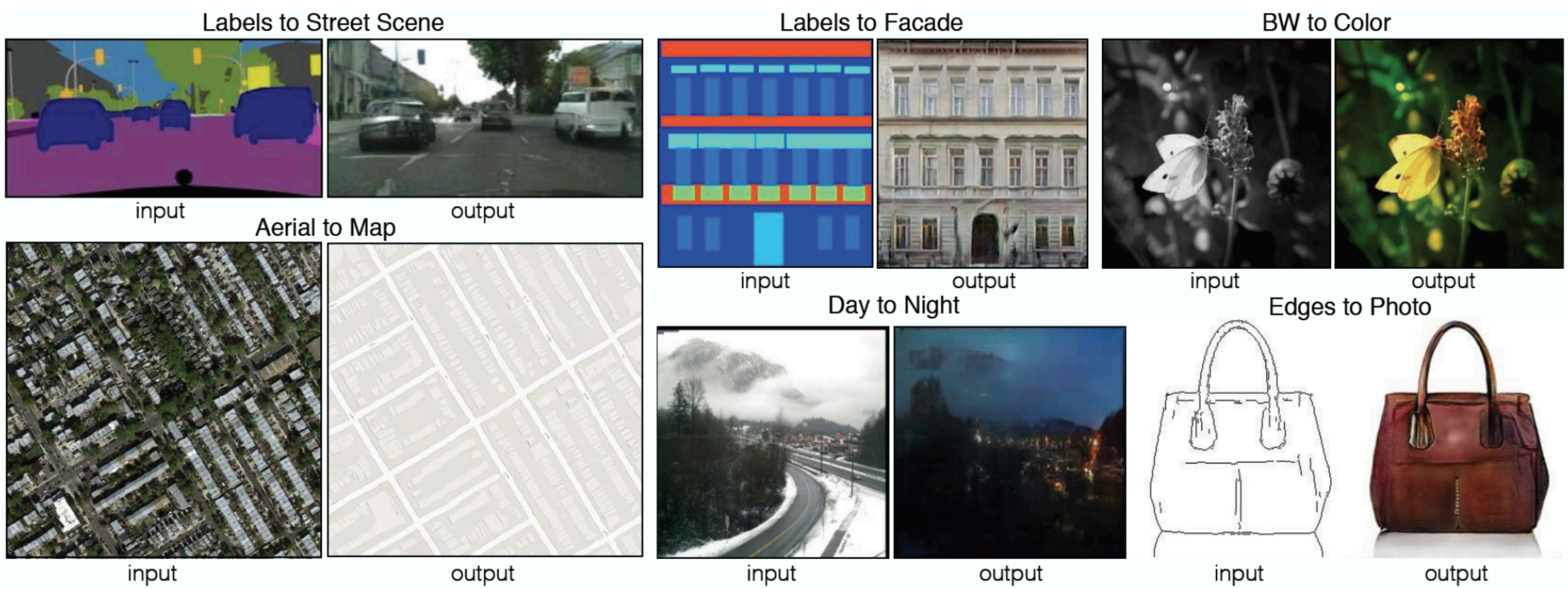
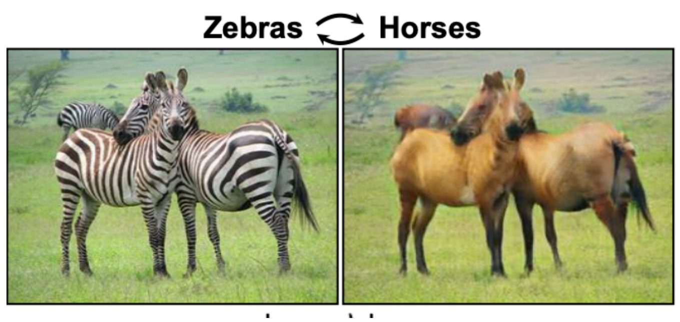


Figure 1 in the original paper.

Image to Image Translation — Unpaired (CycleGAN) Zhu et al., ICCV17

Image to Image Translation — Unpaired (CycleGAN) Zhu et al., ICCV17



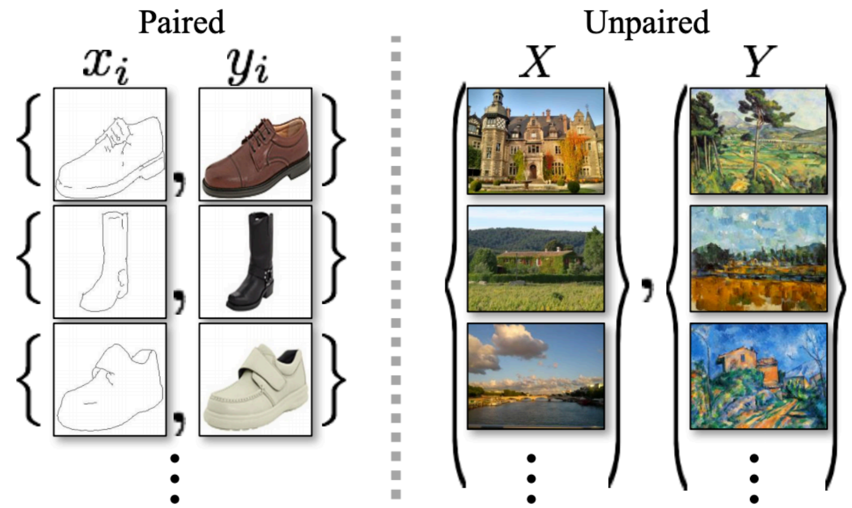
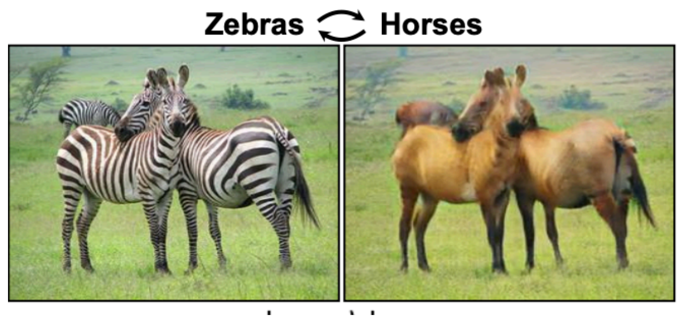
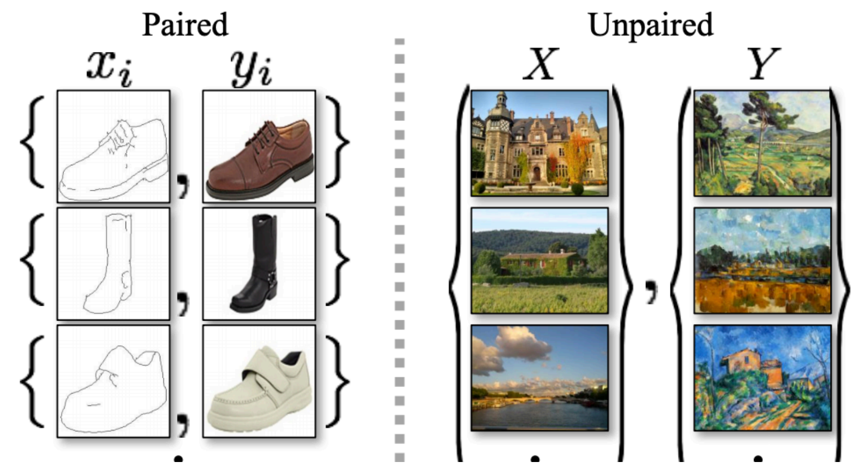
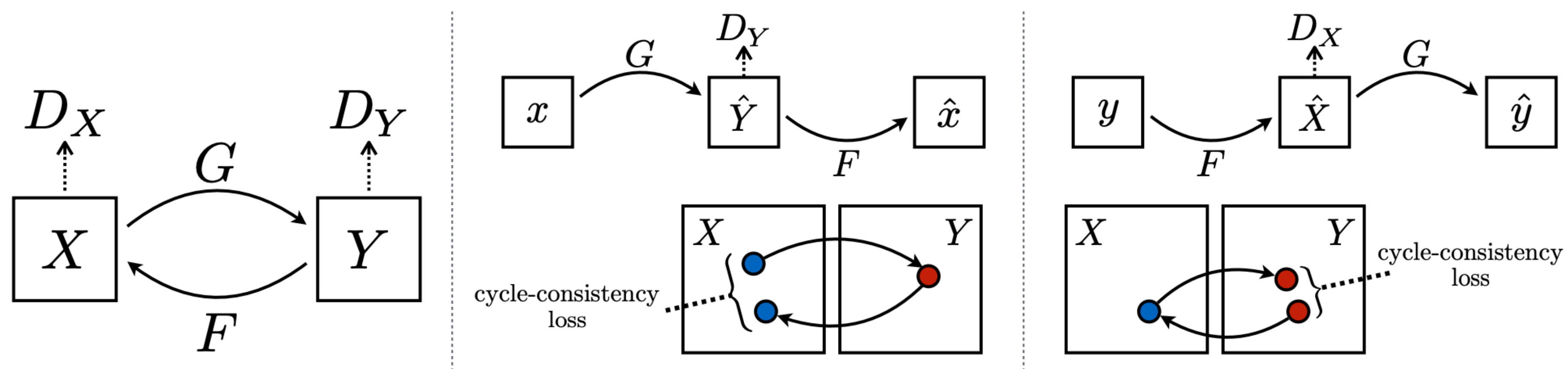


Image to Image Translation — Unpaired (CycleGAN) Zhu et al., ICCV17







(a) (b) (c)

Forward cycle consistency

$$x \rightarrow G(x) \rightarrow F(G(x)) \approx x$$

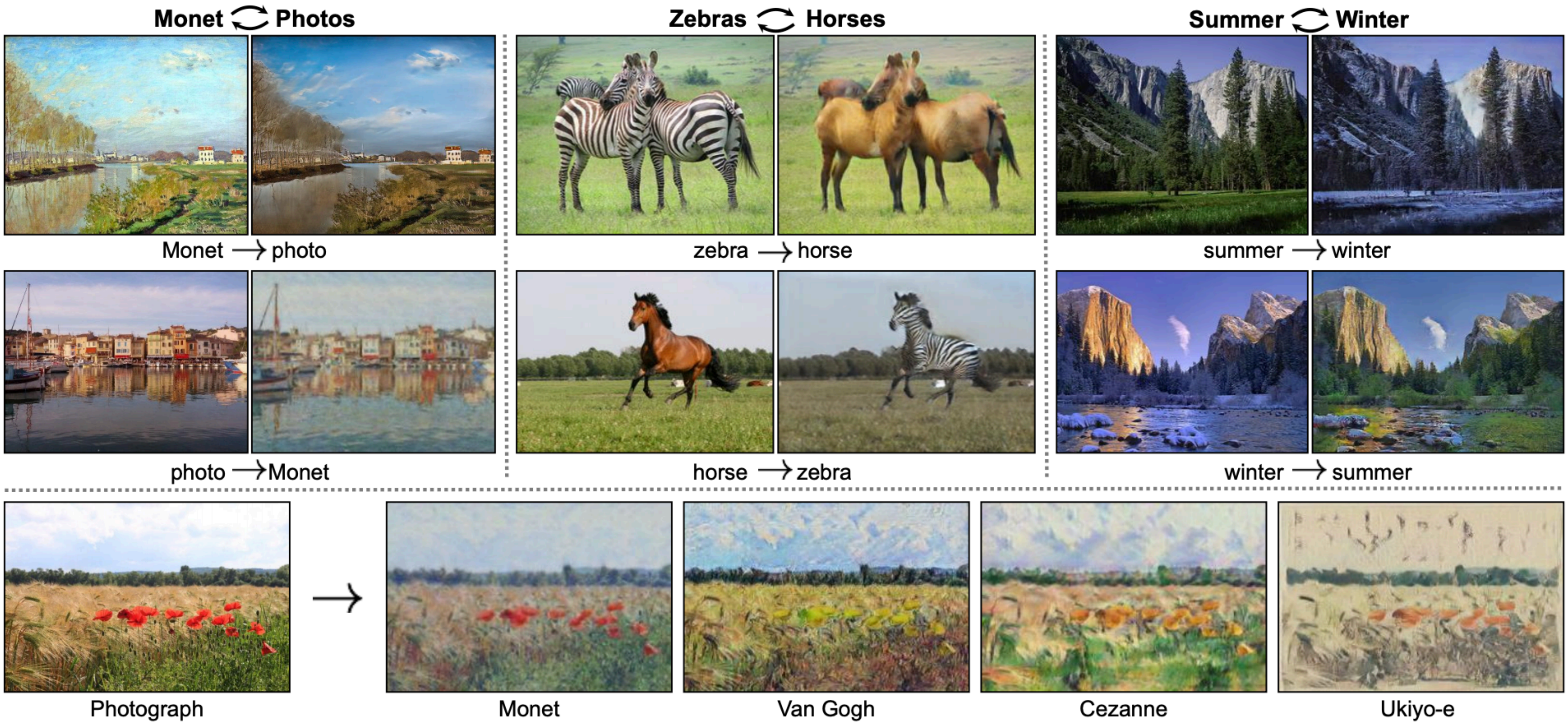
$$||F(G(x)) - x||_1$$

Backward cycle consistency

$$y \rightarrow F(y) \rightarrow G(F(y)) \approx y$$

$$||G(F(y)) - y||_1$$

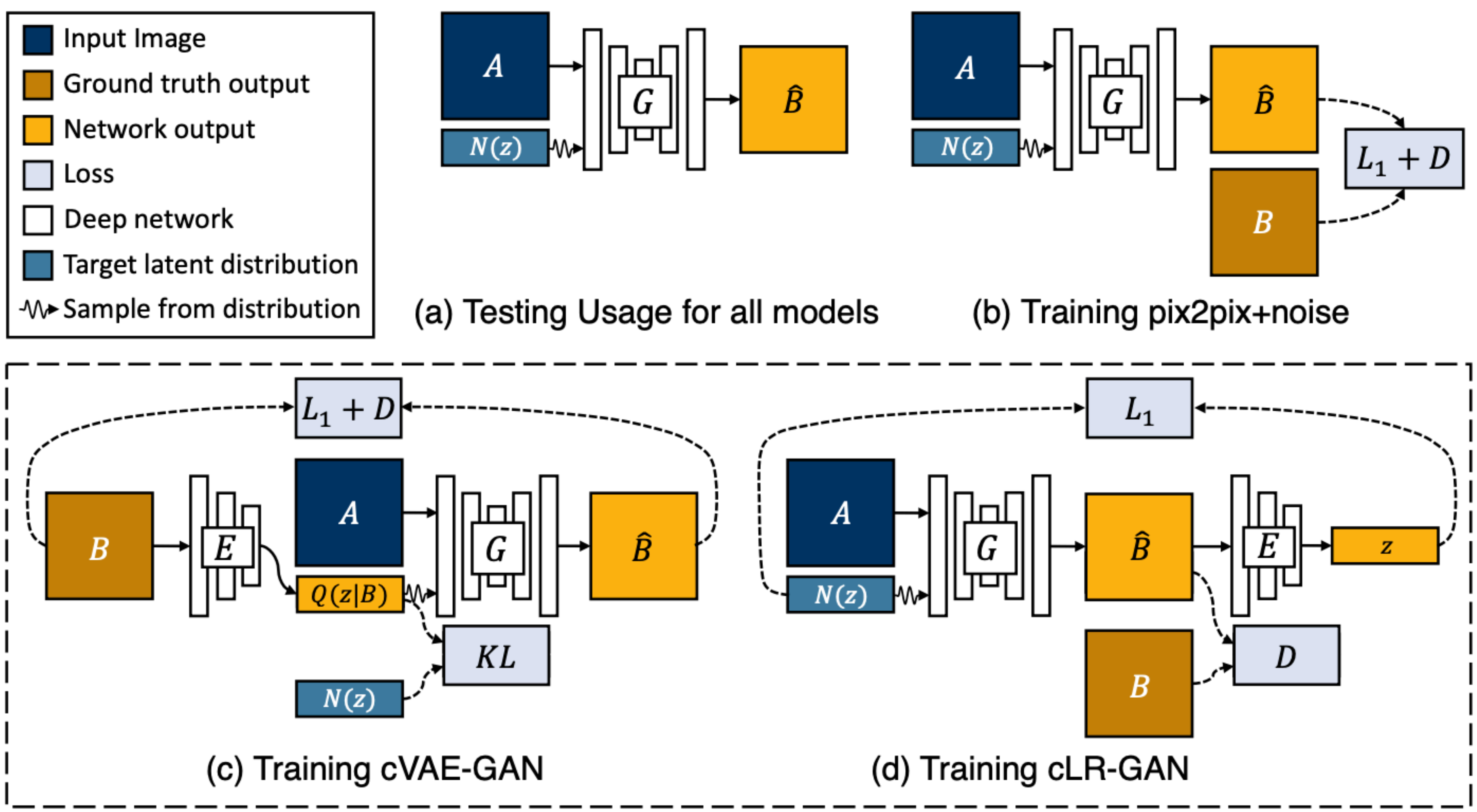
Image to Image Translation — Unpaired (CycleGAN) Zhu et al., ICCV17



Bicycle GAN — Hybrid GAN + VAE

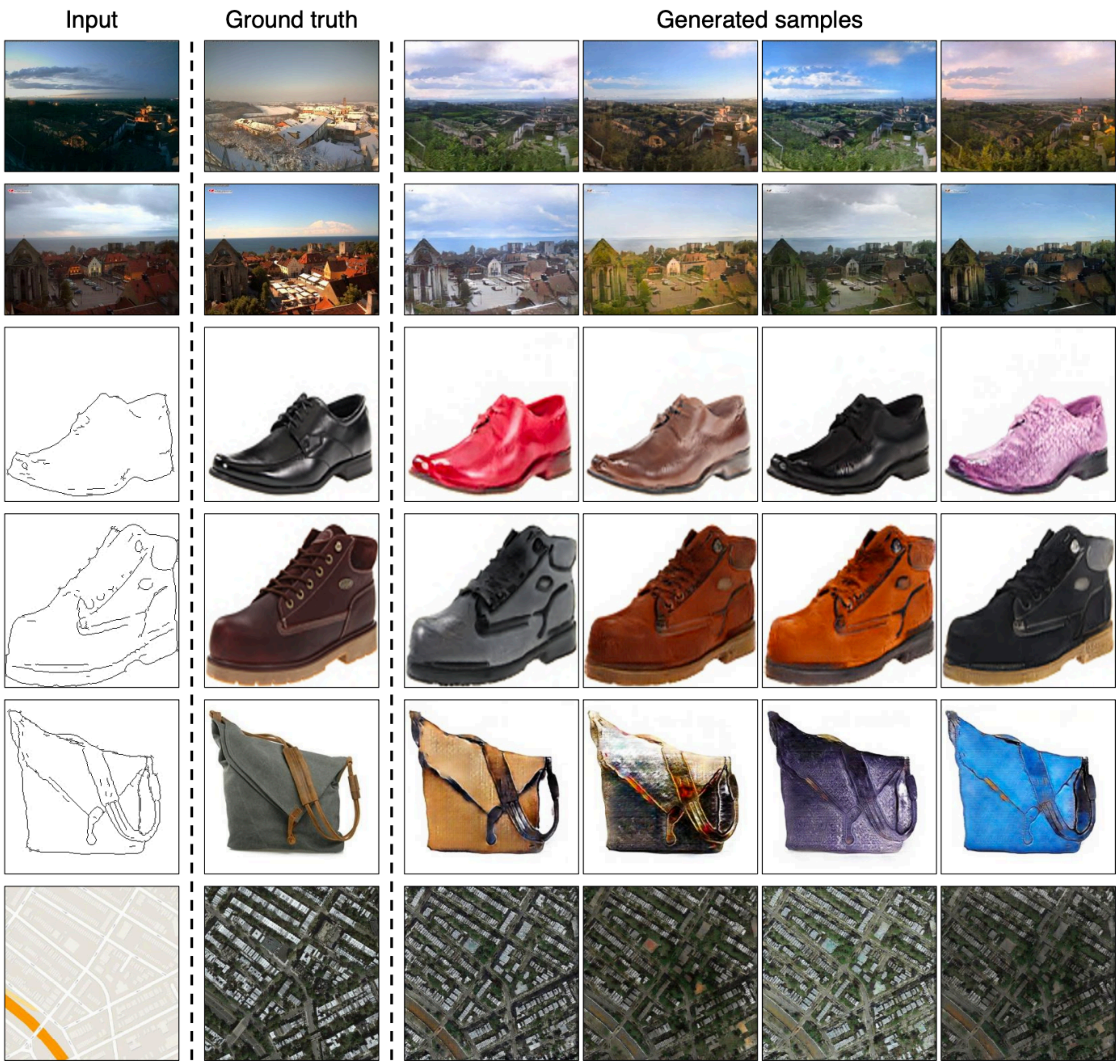
Zhu et al., NeurIPS17

- Handles diversity issue with image to image translation
- Reconstruct the desired output — cVAE type — Use encoder and generator
- Reconstruct the noise



(e) Training BicycleGAN

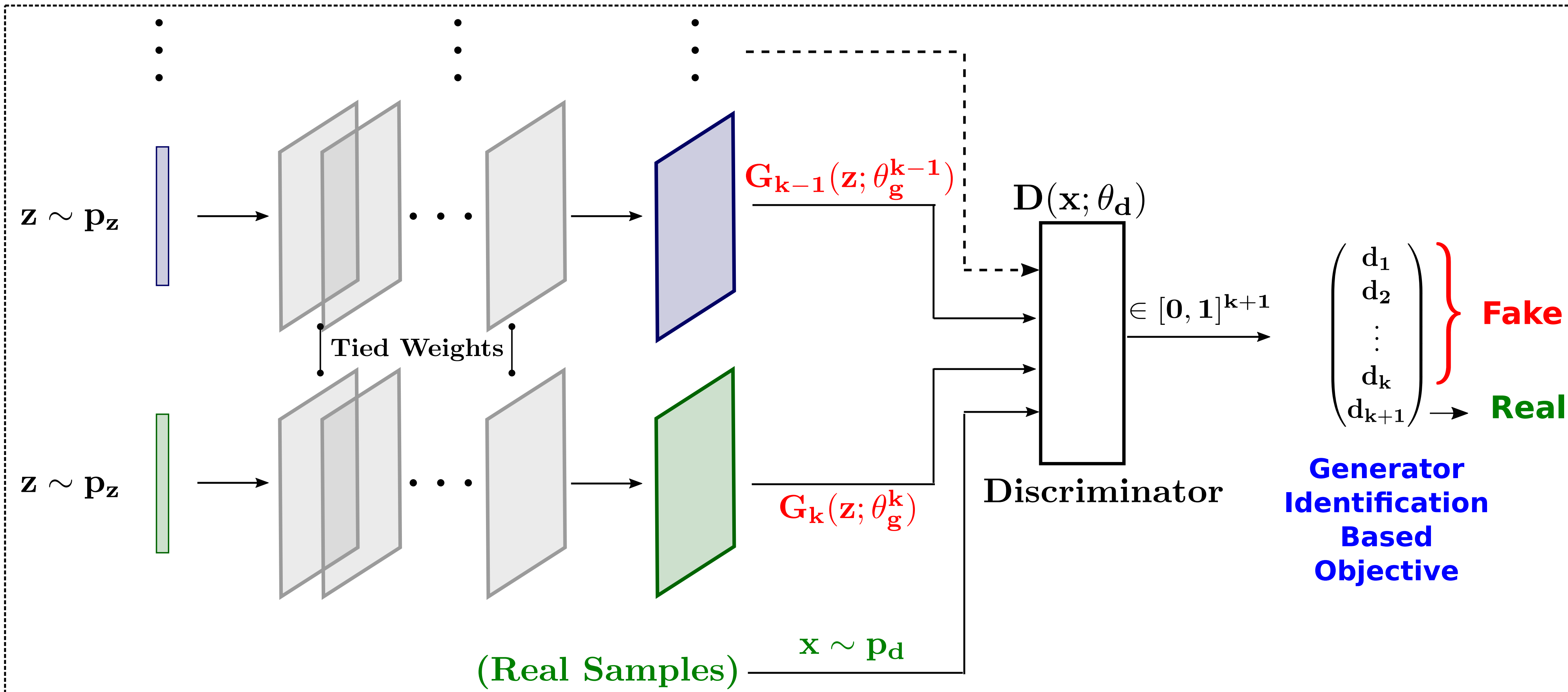
Bicycle GAN
Zhu et al., NeurIPS17



MAD-GAN

Ghosh et al., CVPR18

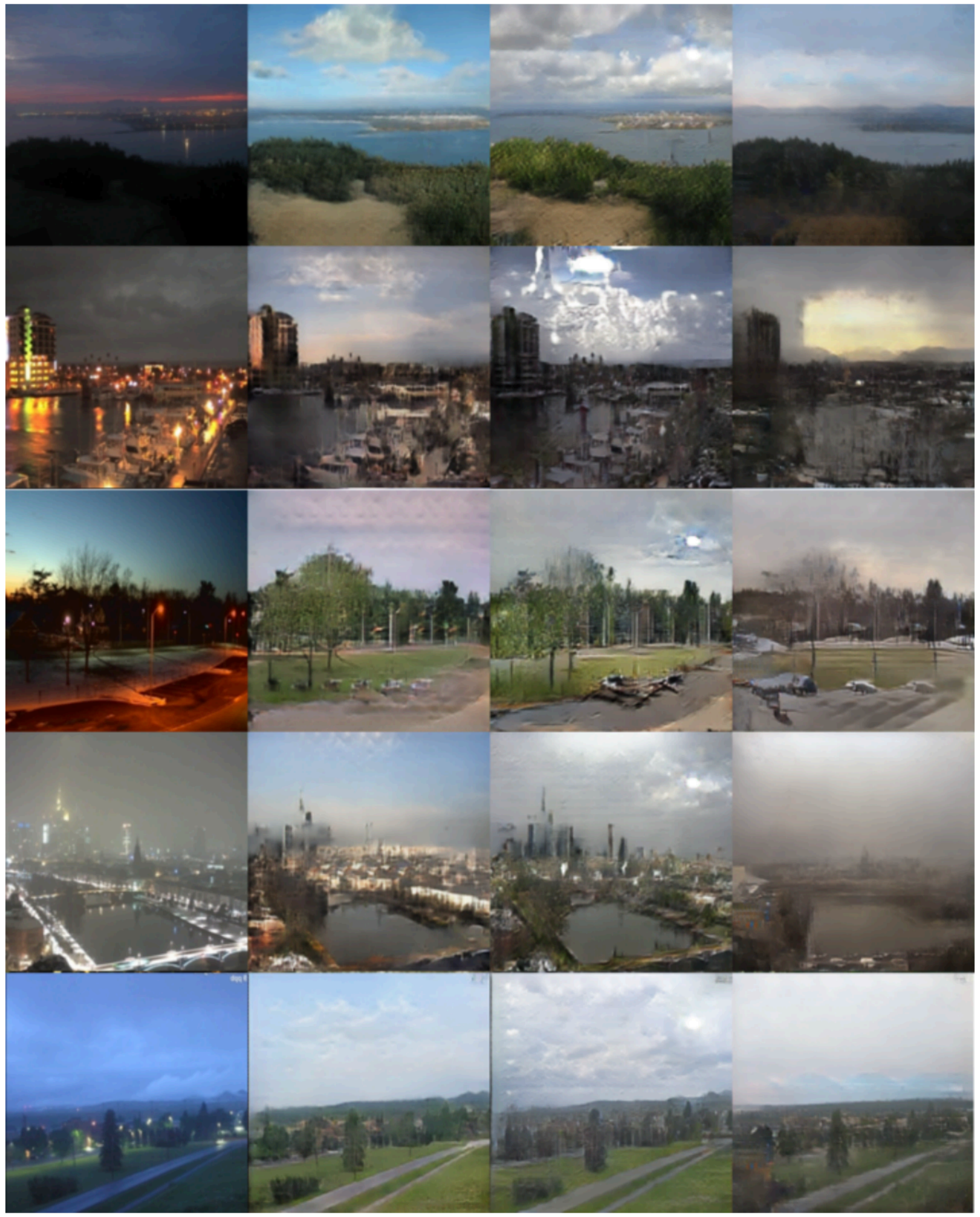
- Handles diversity issue with image to image translation



- Find which generator generated the given `fake' sample
- Disentangles to achieve this task

MAD-GAN

Ghosh et al., CVPR18

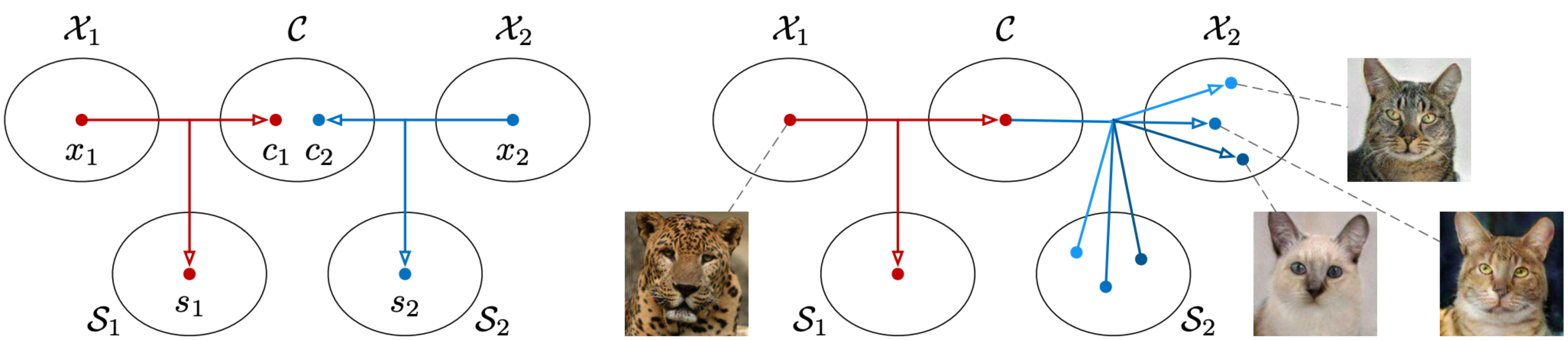


Multimodal Unsupervised Im2Im (MUNIT)

Huang et al., ECCV18

- Image representation is decomposed into two factors
  - Content code — domain invariant
  - Style code — domain specific

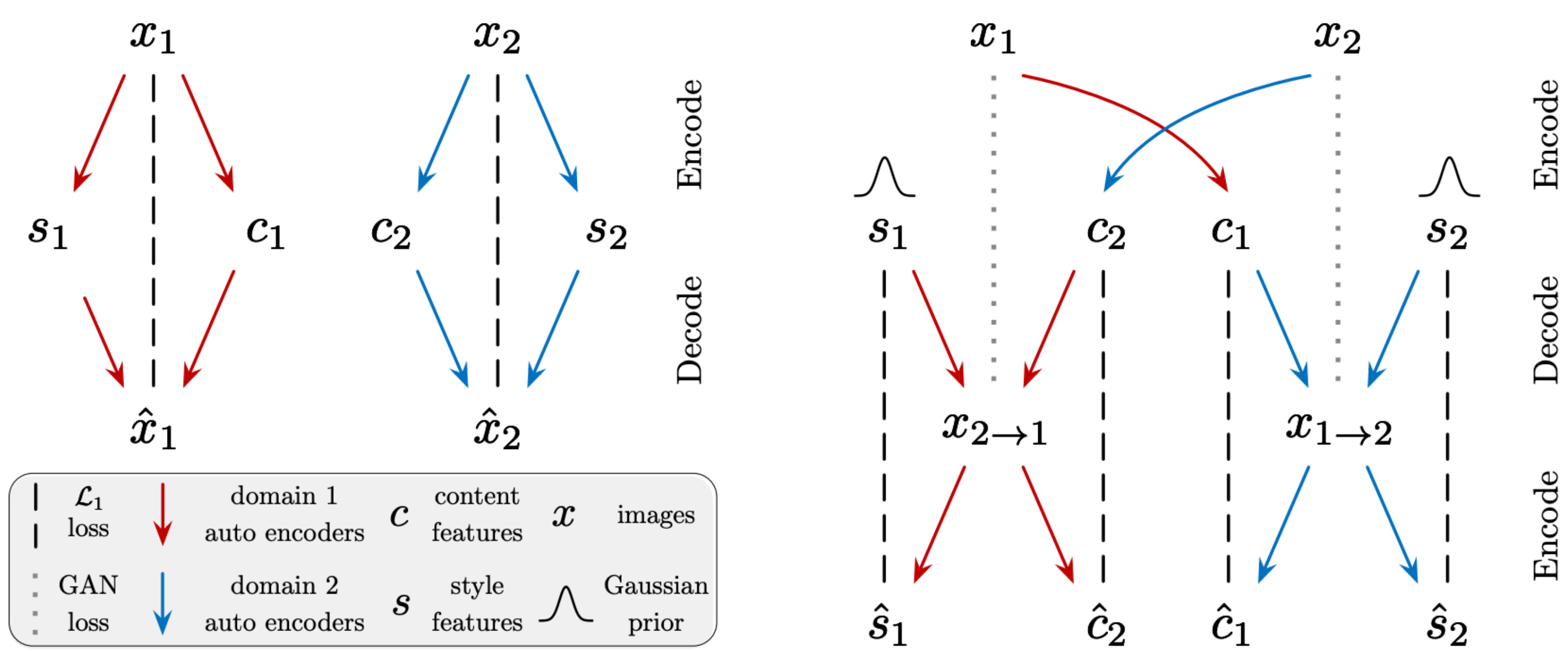
- Domain X1 (Leopard) to Domain X2 (Cat)
  - Combine content code of X1 with randomly sampled style code of X2



(a) Auto-encoding (b) Translation

Multimodal Unsupervised Im2Im (MUNIT)

Huang et al., ECCV18



(a) Within-domain reconstruction (b) Cross-domain translation

- **Image reconstruction.** Given an image sampled from the data distribution, we should be able to reconstruct it after encoding and decoding.

$$\mathcal{L}_{\text{recon}}^{x_1} = \mathbb{E}_{x_1 \sim p(x_1)}[||G_1(E_1^c(x_1), E_1^s(x_1)) - x_1||_1] \tag{1}$$

- **Latent reconstruction.** Given a latent code (style and content) sampled from the latent distribution at translation time, we should be able to reconstruct it after decoding and encoding.

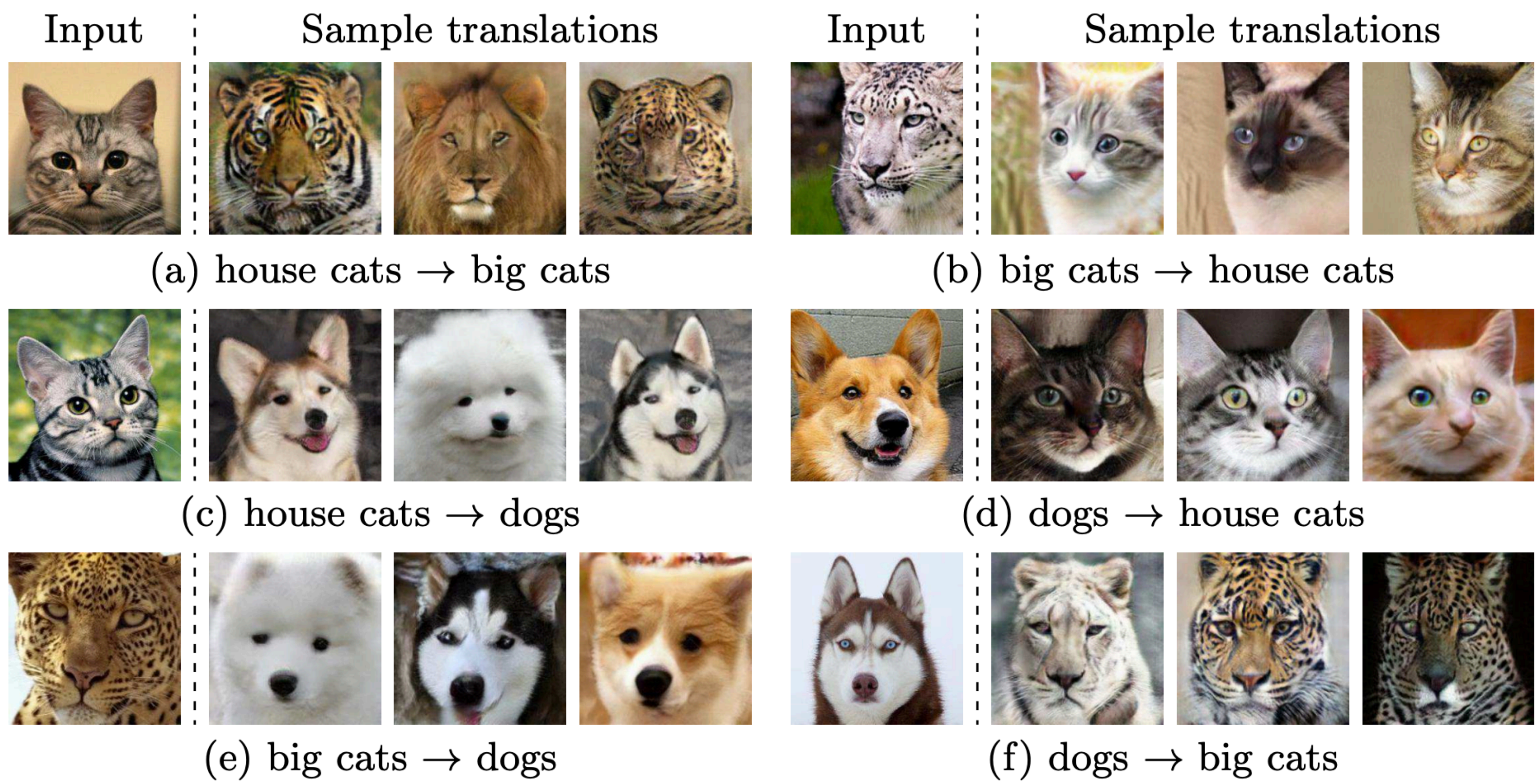
$$\mathcal{L}_{\text{recon}}^{c_1} = \mathbb{E}_{c_1 \sim p(c_1), s_2 \sim q(s_2)}[||E_2^c(G_2(c_1, s_2)) - c_1||_1] \tag{2}$$

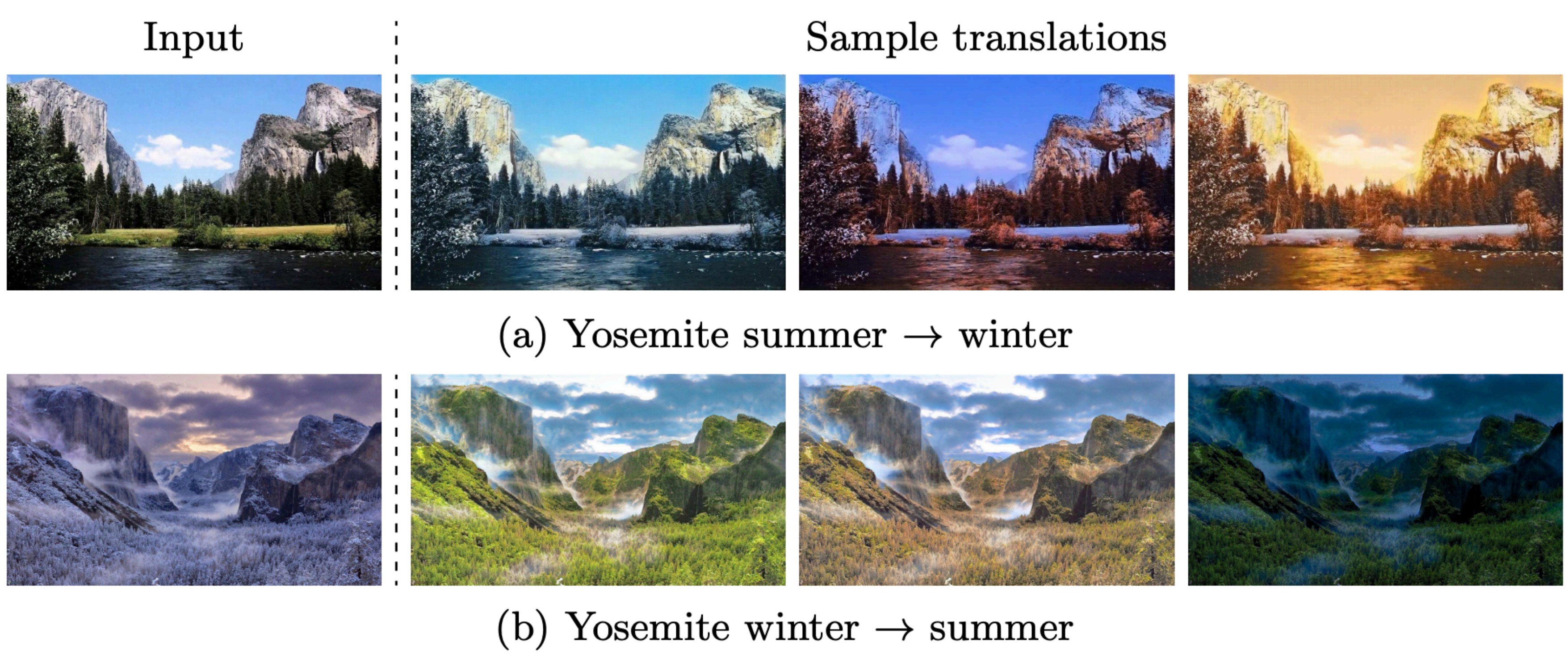
$$\mathcal{L}_{\text{recon}}^{s_2} = \mathbb{E}_{c_1 \sim p(c_1), s_2 \sim q(s_2)}[||E_2^s(G_2(c_1, s_2)) - s_2||_1] \tag{3}$$

where $q(s_2)$ is the prior $\mathcal{N}(0, \mathbf{I})$, $p(c_1)$ is given by $c_1 = E_1^c(x_1)$ and $x_1 \sim p(x_1)$.

Multimodal Unsupervised Im2Im (MUNIT)

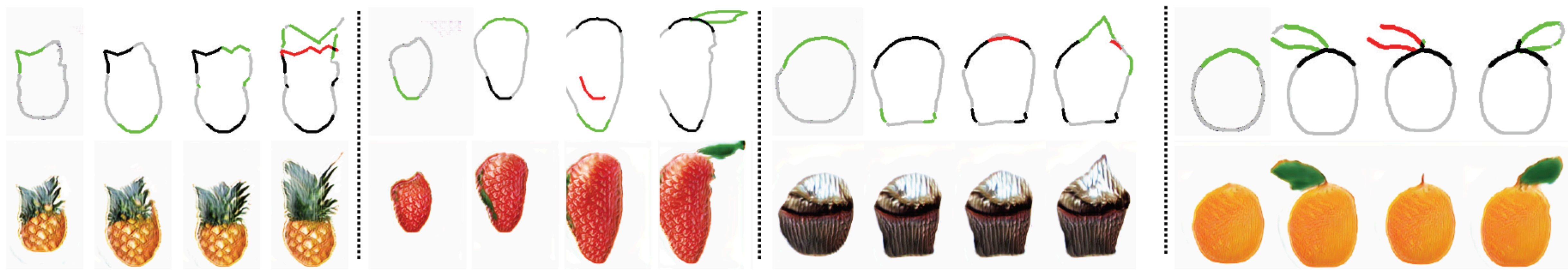
Huang et al., ECCV18





Interactive Sketch & Fill — Work under progress

(A. Ghosh, R. Zhang, P. K. Dokania, A. Efrso, P. H. S. Torr, O. Wang, E. Shechtman)

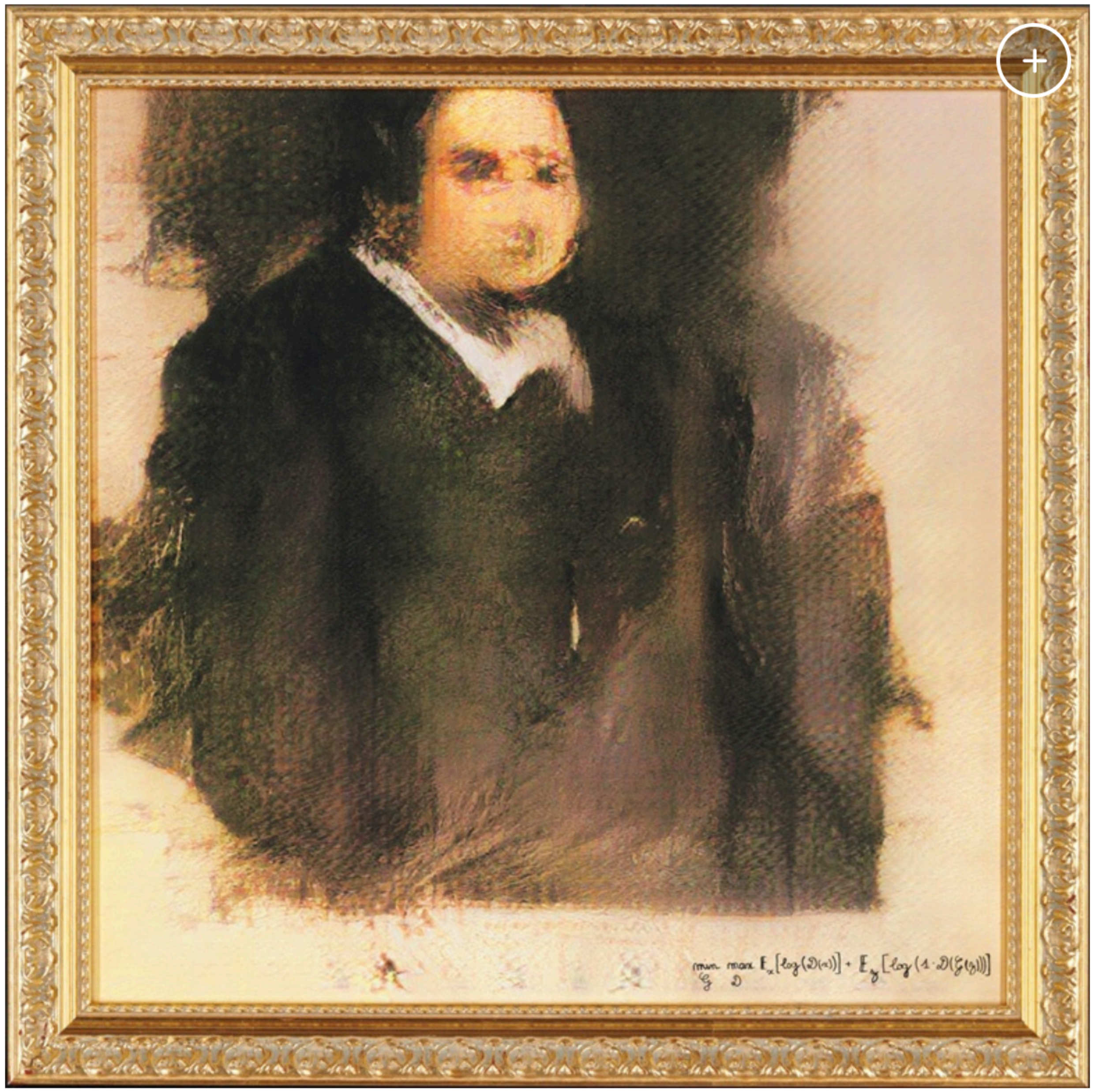


Interactive Sketch & Fill

Class-conditioned Outline-to-Image Translation

GAN Art — Portrait of Edmond de Belamy

GAN Art — Portrait of Edmond de Belamy



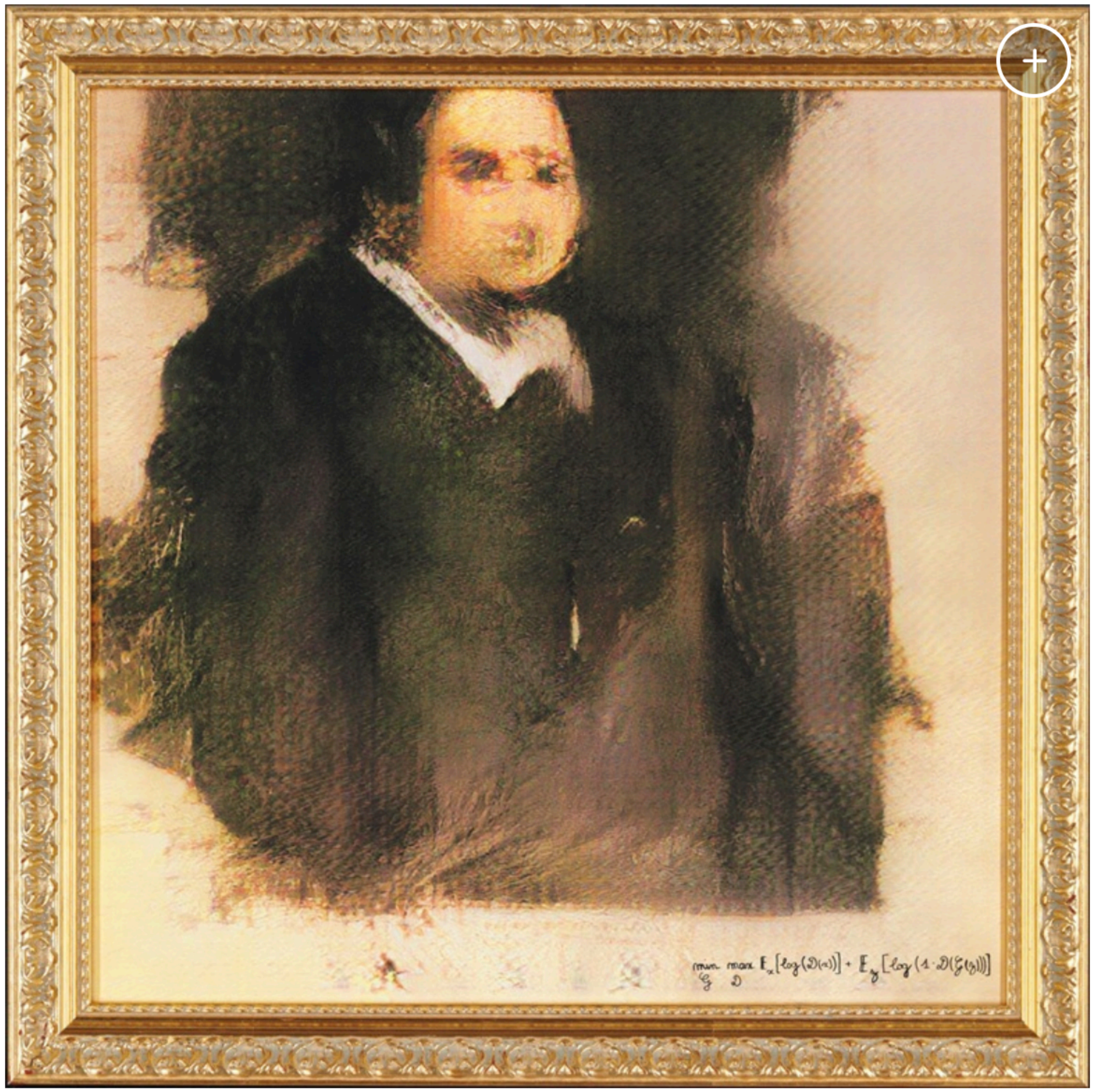
- First AI (GAN) generated art
- Sold (Oct 2018, not sure) for almost 432,500 USD

- Christie's New York

The portrait in its gilt frame depicts a portly gentleman, possibly French and — to judge by his dark frockcoat and plain white collar — a man of the church. The work appears unfinished: the facial features are somewhat indistinct and there are blank areas of canvas. Oddly, the whole composition is displaced slightly to the north-west. A label on the wall states that the sitter is a man named Edmond Belamy, but the giveaway clue as to the origins of the work is the artist's signature at the bottom right. In cursive Gallic script it reads:

$$\min_G \max_D \mathbb{E}_x[\log(D(x))] + \mathbb{E}_z[\log(1 - D(G(z)))]$$

GAN Art — Portrait of Edmond de Belamy



- First AI (GAN) generated art
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rejection?

- Christie's New York

The portrait in its gilt frame depicts a portly gentleman, possibly French and — to judge by his dark frockcoat and plain white collar — a man of the church. The work appears unfinished: the facial features are somewhat indistinct and there are blank areas of canvas. Oddly, the whole composition is displaced slightly to the north-west. A label on the wall states that the sitter is a man named Edmond Belamy, but the giveaway clue as to the origins of the work is the artist's signature at the bottom right. In cursive Gallic script it reads:

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Thank You