

# Blind Modulation Classification for OFDM in the Presence of Timing, Frequency, and Phase Offsets

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**Abstract**—This paper proposes a blind modulation classification (MC) algorithm for linearly modulated signals of orthogonal frequency division multiplexing (OFDM) system. The proposed MC algorithm works with unknown frequency, timing, and phase offsets and without the prior requirement of channel statistics. In this research, a larger pool of modulation formats, i.e., binary phase-shift keying (BPSK), quadrature PSK (QPSK), offset QPSK (OQPSK), minimum shift keying (MSK), and 16-quadrature amplitude modulation (16-QAM) for OFDM signal has been classified. Classification takes place in two stages. First, we compute the discrete Fourier transform (DFT) of the received OFDM signal and then a normalized fourth-order cumulant is used in frequency domain to classify OQPSK, MSK, and 16-QAM modulation formats. At the second stage, the normalized fourth-order cumulant is used on the DFT of the square of the received OFDM signal to classify BPSK and QPSK modulation formats. The success rate and computation of the proposed MC algorithm are evaluated and compared with the previous methods.

**Index Terms**—Blind modulation classification, discrete Fourier transform, fourth-order statistics, orthogonal frequency division multiplexing.

## I. INTRODUCTION

**B**LIND modulation classification (MC) recognizes the received signal modulation format and guarantees proper demodulation [1], [2]. It plays an important role in many commercial and military applications, including spectrum surveillance, cognitive radio, signal intelligence [3]–[7]. The MC for single carrier systems was studied extensively in [7]–[12] and references therein. For future wireless communication systems, orthogonal frequency division multiplexing (OFDM) is considered as a popular multi-carrier modulation technology. OFDM has a major advantage in dealing with severe channel conditions [13]. Thus, the OFDM technology is used as the primary transmission scheme over multipath fading channels.

Classification of OFDM signal modulation formats is an ongoing research problem. There are very few MC algorithms available in the literature for OFDM systems [14]–[18]. The MC algorithms discussed in [14], [15] based on maximum a-posteriori and maximum likelihood-based, are considered for both known and unknown channel statistics with a perfect synchronization condition to classify the Mary-phase-shift keying (M-PSK)/Mary-quadrature amplitude modulation (M-QAM) modulation formats. The method based on the Kolmogorov-Smirnov (KS) test discussed in [16] is a non-parametric approach for M-PSK/M-QAM modulation formats and works for different channels with unknown frequency and phase offset, although it requires perfect timing synchronization.

The method given in [17] extracts the features by employing statistical properties of the received OFDM signal, i.e., mean, skewness, and kurtosis, to classify quadrature PSK (QPSK), 16-QAM, and 64-QAM modulation formats. However, this algorithm requires works with known timing and frequency offsets. The MC algorithm based on the correlation between any two subcarriers is studied in [18]. This method uses amplitude moments to classify only 16-QAM and 64-QAM modulation formats. The MC algorithms studied in [14]–[18] are limited to known channel and/or perfect synchronization conditions. To the best knowledge of authors, there is no MC algorithm exists in the literature for OFDM systems, which works in the presence of timing, frequency, and phase offsets and without the prior information about the channel statistics. Moreover, classifying QPSK, offset QPSK (OQPSK), and MSK (minimum shift keying) together is an another major problem.

To deal with the above issues, we propose a blind MC algorithm for linearly modulated signals of the OFDM system by using the discrete Fourier transform (DFT) and normalized fourth-order cumulant of the received OFDM signal. The proposed MC algorithm is able to classify a larger pool of modulation formats, i.e., binary PSK (BPSK), QPSK, MSK, OQPSK, and 16-QAM over frequency-selective fading channel. It works without prior knowledge of the channel statistics with unknown timing, frequency, and phase offsets.

The remainder of the paper is arranged as follows: Section II provides the OFDM signal model. Then we will discuss the proposed MC algorithm based on the combined properties of DFT and normalized fourth-order cumulant in Section III. It includes feature extraction and selection analysis. The performance analysis through Monte Carlo simulations is illustrated in Section IV. Finally, in Section V, conclusions are drawn.

## II. OFDM SIGNAL MODEL

In this study, we deal with a wireless OFDM system having  $N_0$  subcarriers. The  $m$ th OFDM symbol is generated by using  $N$ -point inverse DFT (IDFT) and can be obtained as

$$x_m[k] = \sum_{n=0}^{N-1} X_m[n] e^{j2\pi nk/N}, \quad 0 \leq k \leq N-1, \quad (1)$$

where  $N = \rho \times N_0$ ,  $\rho$  is the oversampling factor,  $X_m[n]$  is the modulated oversampled data and can be written as

$$X_m[n] = \begin{cases} \bar{X}_m[n] & 0 \leq n \leq N_0/2 - 1 \\ \bar{U}_0 & N_0/2 \leq n \leq N_0(\rho - 1/2) - 1 \\ \bar{X}_m[n] & N_0(\rho - 1/2) \leq n \leq N - 1, \end{cases} \quad (2)$$

where  $\bar{X}_m[n]$  is the modulated information of  $m$ th OFDM symbol and  $\bar{U}_0$  is the zeros padding vectors of length  $N_0(\rho - 1)$ . To mitigate the effect of intersymbol interference, in front of  $x[k]$ , the cyclic prefix (CP) of length  $N_{cp}$  is added as follows:

$$\bar{x}_m[k] = \begin{cases} x_m[k + N], & -N_{cp} \leq k \leq -1 \\ x_m[k], & 0 \leq k \leq N - 1. \end{cases} \quad (3)$$

The received OFDM signal over frequency-selective fading channel can be expressed as

$$y_m[k] = e^{j(2\pi f_o k/N + \phi)} \sum_{l=0}^{L-1} h[l] \bar{x}_m[k - l - \tau]_{\text{mod } K} + w[k], \quad 0 \leq k \leq K - 1, \quad (4)$$

where  $f_o$  is the normalized frequency offset,  $\phi$  is the phase offset,  $\tau$  is the timing offset,  $K$  is the length of received signal,  $K = N + N_{cp}$ ,  $w[k]$  is the additive white Gaussian noise (AWGN) with zero mean, and variance  $\sigma_w^2$ . The frequency-selective fading channel response is represented as  $h[l]$ , where  $l = 0, 1, \dots, L - 1$  and  $L$  is the channel response length, providing  $N_{cp} \geq L$ .

### III. PROPOSED BLIND MODULATION CLASSIFICATION

In this work, we propose a blind MC algorithm for linearly modulated signals of the OFDM system. The modulated information  $X_m[n]$  of the OFDM signal is in the frequency domain. Hence, the feature extraction process of the proposed MC algorithm also uses a frequency domain signal. To nullify the effect of timing offset in the feature extraction process, we introduce uniform timing offsets,  $\theta_u$ , in each OFDM symbol, which is taken from a uniform distribution with parameter  $-K/2$  and  $K/2$ , i.e.,  $\mathcal{U}(-K/2, K/2)$  as shown in Fig. 1. The overall OFDM signal length  $K$  can be easily estimated by using the cyclic correlation function in [19]. Now, we can write the modified expression for the  $m$ th OFDM symbol as:

$$\bar{y}_m[k] = y_m[k - \tau - \theta_u], \quad 0 \leq k \leq K - 1.$$

1) *First-Stage Classification*: At the first-stage, the normalized fourth-order cumulant is used on the DFT of the received OFDM signal to classify OQPSK, MSK, and 16-QAM modulation formats. The DFT of the  $m$ th OFDM symbol,  $\bar{y}_m[k]$ , is obtained as

$$\begin{aligned} \bar{Y}_m[v] &= \sum_{k=0}^{K-1} \bar{y}_m[k] e^{-j2\pi kv/K}, \quad 0 \leq v \leq K - 1, \\ &= \sum_{k=0}^{K-1} y_m[k - \tau - \theta_u] e^{-j2\pi kv/K}, \\ &= e^{-j2\pi v/K(\tau + \theta_u)} Y_m[v], \end{aligned} \quad (5)$$

where  $Y_m[v]$  is the DFT of the received OFDM signal  $y_m[k]$ . For complex-valued random processes  $\bar{Y}_m[v]$ , mixed moments

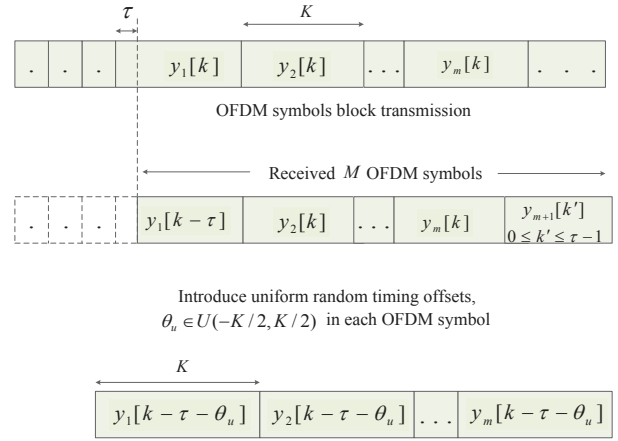


Fig. 1. Timing offset analysis in the proposed blind MC algorithm.

are defined according to the conjugation placements. The mixed moment of order  $p$  with  $q$  conjugations for  $\bar{Y}_m[v]$  is defined as in [9]

$$\bar{M}_{pq} = E \left[ \bar{Y}_m[v]^{p-q} \bar{Y}_m^*[v]^q \right]. \quad (6)$$

In this research, the cumulants of most interest are fourth-order. The sample average of the fourth-order cumulant of (5) with two conjugations is obtained as [9]

$$\begin{aligned} \hat{C}_{42\bar{Y}_m} &= \frac{1}{K} \sum_{v=0}^{K-1} \left| e^{-j2\pi v/K(\tau + \theta_u)} Y_m[v] \right|^4 \\ &\quad - \left| \frac{1}{K} \sum_{v=0}^{K-1} e^{-j4\pi v/K(\tau + \theta_u)} Y_m^2[v] \right|^2 \\ &\quad - 2 \left( \frac{1}{K} \sum_{v=0}^{K-1} \left| e^{-j2\pi v/K(\tau + \theta_u)} Y_m[v] \right|^2 \right)^2, \\ &= \bar{C}_{42Y_m} - \left| \frac{1}{K} \sum_{v=0}^{K-1} e^{-j4\pi v/K(\tau + \theta_u)} Y_m^2[v] \right|^2, \end{aligned} \quad (7)$$

where  $\bar{C}_{42Y_m} = \frac{1}{K} \sum_{v=0}^{K-1} |Y_m[v]|^4 - 2 \left( \frac{1}{K} \sum_{v=0}^{K-1} |Y_m[v]|^2 \right)^2$ . It is noted that the above cumulant has the form of the sum of signal and noise. In practice, we estimate the normalized fourth-order cumulant by

$$\tilde{C}_{42\bar{Y}_m} = \frac{\bar{C}_{42Y_m} - \left| \frac{1}{K} \sum_{v=0}^{K-1} e^{-j4\pi v/K(\tau + \theta_u)} Y_m^2[v] \right|^2}{\frac{1}{K} \sum_{v=0}^{K-1} |Y_m[v]|^2 - C_{21,W}}, \quad (8)$$

where  $C_{21,W} = \sigma_W^2$  is an estimated variance of AWGN. Now we take the average of all cumulant values, i.e., equal to the total number of OFDM symbols  $M$ , given as

$$\tilde{C}_{42Y} = \frac{1}{M} \sum_{m=1}^M \frac{\bar{C}_{42Y_m} - \left| \frac{1}{K} \sum_{v=0}^{K-1} e^{-j4\pi v/K(\tau + \theta_u)} Y_m^2[v] \right|^2}{\frac{1}{K} \sum_{v=0}^{K-1} |Y_m[v]|^2 - C_{21,W}}. \quad (9)$$

The above equation (9), gives distinct values for OQPSK, MSK, and 16-QAM modulations, as shown in Fig. 2, where

$(\mu_1, \sigma_1^2)$ ,  $(\mu_2, \sigma_2^2)$ ,  $(\mu_3, \sigma_3^2)$ ,  $(\mu_4, \sigma_4^2)$ , and  $(\mu_5, \sigma_5^2)$  are the mean and variance value for MSK, QPSK, BPSK, 16-QAM, and OQPSK modulations, respectively. The decision rule used to classify these modulations is

$$\begin{aligned} \tilde{C}_{42Y} < \eta_1 &\Rightarrow \text{MSK} \\ \eta_1 \leq \tilde{C}_{42Y} < \eta_2 &\Rightarrow \text{BPSK and QPSK} \\ \eta_2 \leq \tilde{C}_{42Y} < \eta_3 &\Rightarrow \text{16-QAM} \\ \tilde{C}_{42Y} \geq \eta_3 &\Rightarrow \text{OQPSK} \end{aligned} \quad (10)$$

where  $\eta_1$ ,  $\eta_2$ , and  $\eta_3$  are the thresholds, discussed in Appendix.

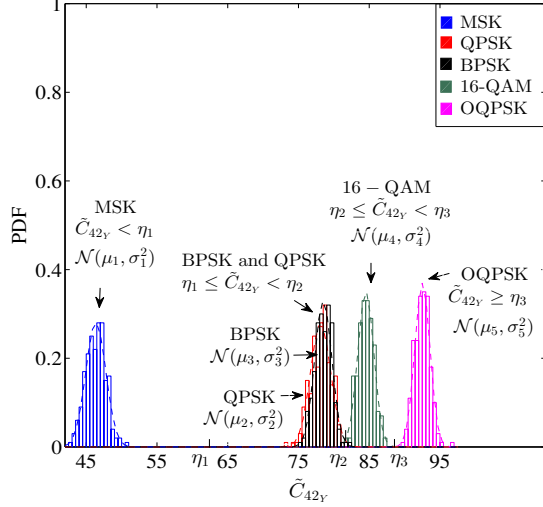


Fig. 2. Probability density function (PDF) of  $\tilde{C}_{42Y}$  for BPSK, QPSK, OQPSK, MSK, and 16-QAM modulation formats at SNR = 20 dB.

2) *Second-Stage Classification*: To classify BPSK and QPSK modulation formats, we use the DFT of the square of the received OFDM signal and then apply the normalized fourth-order cumulant. The DFT of the square of the OFDM symbol,  $\bar{y}_m^2[k]$ , can be expressed as

$$\begin{aligned} \bar{Z}_m[v] &= \sum_{k=0}^{K-1} y_m^2[k - \tau - \theta_u] e^{-j2\pi kv/K}, \\ &= e^{-j2\pi v/K(\tau + \theta_u)} Z_m[v], \end{aligned} \quad (11)$$

where  $Z_m[v] = Y_m[v] \otimes Y_m[v]$ ,  $\otimes$  is the linear convolution operation. Similar to (7), we find the sampled estimates of the fourth-order cumulant with two-conjugations of  $\bar{Z}_m[v]$  as

$$\hat{C}_{42Z_m} = \bar{C}_{42Z_m} - \left| \frac{1}{K} \sum_{v=0}^{K-1} e^{-j4\pi v/K(\tau + \theta_u)} Z_m^2[v] \right|^2, \quad (12)$$

where  $\bar{C}_{42Z_m} = \frac{1}{K} \sum_{v=0}^{K-1} |Z_m[v]|^4 - 2 \left( \frac{1}{K} \sum_{v=0}^{K-1} |Z_m[v]|^2 \right)^2$ . Now, the normalized fourth-order cumulant is obtained as

$$\tilde{C}_{42Z_m} = \frac{\bar{C}_{42Z_m} - \left| \frac{1}{K} \sum_{v=0}^{K-1} e^{-j4\pi v/K(\tau + \theta_u)} Z_m^2[v] \right|^2}{\frac{1}{K} \sum_{v=0}^{K-1} |Z_m[v]|^2 - C_{21,W}}, \quad (13)$$

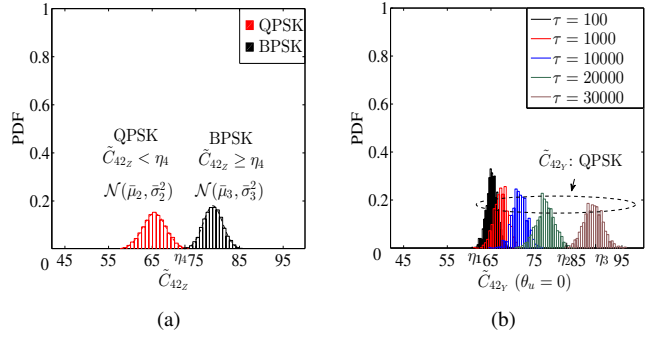


Fig. 3. (a) Probability density function (PDF) of  $\tilde{C}_{42Z}$  for BPSK and QPSK modulation formats at SNR = 20 dB. (b) PDF of the normalized fourth-order cumulant of QPSK modulated signal at the first-stage classification, without introducing uniformly distributed timing offsets, i.e.,  $\theta_u = 0$ .

Similarly, we take the average of  $M$  cumulant values as

$$\tilde{C}_{42Z} = \frac{1}{M} \sum_{m=1}^M \frac{\bar{C}_{42Z_m} - \left| \frac{1}{K} \sum_{v=0}^{K-1} e^{-j4\pi v/K(\tau + \theta_u)} Z_m^2[v] \right|^2}{\frac{1}{K} \sum_{v=0}^{K-1} |Z_m[v]|^2 - C_{21,W}}. \quad (14)$$

The above equation (14), gives distinct values for BPSK and QPSK modulation formats, as shown in Fig. 3a, where  $(\bar{\mu}_2, \bar{\sigma}_2^2)$  and  $(\bar{\mu}_3, \bar{\sigma}_3^2)$  are the mean and variance value for QPSK and BPSK modulations, respectively. The decision rule used is

$$\begin{aligned} \tilde{C}_{42Z} < \eta_4 &\Rightarrow \text{QPSK} \\ \tilde{C}_{42Z} \geq \eta_4 &\Rightarrow \text{BPSK} \end{aligned} \quad (15)$$

where  $\eta_4$  is the threshold, discussed in Appendix.

*Remark*: By substituting  $\theta_u = 0$  in (9) and (14), we can observe that, in the exponential term only timing offset ( $\tau$ ) is left as dependent term. For an OFDM block  $\tau$  is fixed, if  $\tau$  is small, the cumulant values lies between  $\eta_1$  and  $\eta_2$  and when  $\tau$  is large then the cumulant values fall outside the threshold region, i.e., greater than  $\eta_2$  as shown in Fig. 3b for the QPSK modulated signal. Similarly, for the other modulation formats, the threshold cannot be estimated properly. However, in the proposed algorithm to nullify the effect of timing offset, we have introduced a uniformly distributed random timing offsets,  $\theta_u$ , in each OFDM symbol as shown in Fig. 1. In this case  $\tau + \theta_u$  will be a new uniform random variable whose mean and variance at the first stage and second stage is given by  $\{(\mu_1, \sigma_1^2), (\mu_2, \sigma_2^2), (\mu_3, \sigma_3^2), (\mu_4, \sigma_4^2), (\mu_5, \sigma_5^2)\}$  and  $\{(\bar{\mu}_2, \bar{\sigma}_2^2), (\bar{\mu}_3, \bar{\sigma}_3^2)\}$ , respectively. After taking the average over OFDM symbols, cumulant values  $\tilde{C}_{42Y}$  and  $\tilde{C}_{42Z}$ , lies within the thresholds, i.e., we will get the unique features for different modulation formats as shown in Fig. 2 and Fig. 3a.

The computational complexity of the proposed algorithm is  $\mathcal{O}(M^2 \log M)$ . The computational complexity of cumulant [9], [16], KS-test [16], and ML-based algorithm [14] is of  $\mathcal{O}(M)$ ,  $\mathcal{O}(M \log M)$ , and  $\mathcal{O}(M \times A_c \times J)$ , respectively, where  $A_c$  is the constellation size of the  $c$ th modulation format and  $J$  is the total number of modulations considered for classification. The complexity order of the proposed MC algorithm is greater than the algorithms based on the KS-test and higher-order

cumulant, but less than that of the ML algorithm. However, the algorithms based on the ML, KS-test, and higher-order cumulant require known channel and/or perfect synchronization conditions. Moreover, these methods cannot classify the variants of QPSK, i.e., OQPSK and MSK modulation formats.

#### IV. SIMULATION RESULTS

In this section, using Monte Carlo simulations, the simulations performance of the proposed MC algorithm is evaluated. In the simulations, the number of subcarriers is set to 1024, symbol rate is set to 1 MHz, and Rayleigh multipath fading channel with  $L = 4$  channel taps is implemented. However, the number of subcarriers can be varied between 256 to 2048 and symbol rate from 1 MHz to 4 MHz. The impairments, i.e., timing, phase, and frequency offset are introduced in the simulation as:  $\tau \in [-K/2, K/2]$ ,  $\phi \in [-\pi, \pi)$ , and  $f_o \in [0, 1]$ , respectively. The number of OFDM symbols is set at 50 and the performance is measured for 10000 iterations.

Figs. 4 presents the percentage of correct classification ( $P_{cc}$ ) versus received signal-to-noise ratio (SNR) of the proposed MC algorithm for five modulation schemes, i.e., BPSK, QPSK, OQPSK, MSK, and 16-QAM. It has been observed that up to 10 dB SNR, performance increases exponentially and after that, it is almost steady.

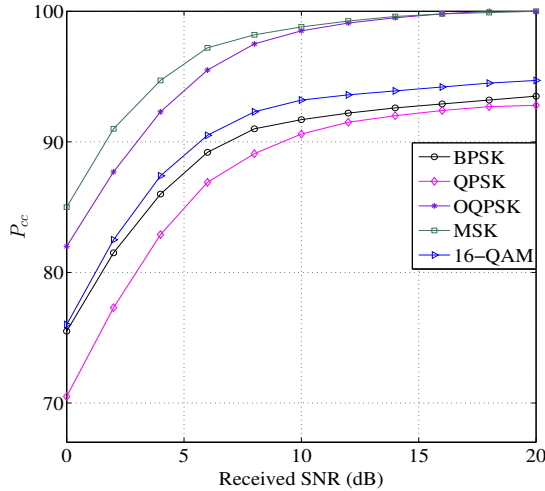


Fig. 4. Simulation results:  $P_{cc}$  vs. SNR for {BPSK, QPSK, OQPSK, MSK, 16-QAM} modulation formats.

Fig. 5 presents the simulation result of the proposed algorithm and compares with previous methods. It is noted that the proposed MC exceeds the KS test [16] and cumulant-based algorithms [9], [16]. Moreover, existing algorithms require perfect timing synchronization condition as compared to the proposed one which works for unknown timing, frequency, and phase offset. Furthermore, these methods can able to classify BPSK, 16-QAM, and QPSK modulation formats, whereas the proposed MC algorithm is suitable for a more extensive pool of modulations, i.e., BPSK, QPSK, OQPSK, MSK, and 16-QAM modulation formats. Additionally, the

other existing algorithms for OFDM systems [14], [15], [17], [18], cannot work in the presence of synchronization errors and also requires knowledge of CSI.

Fig. 6 presents the  $P_{cc}$  versus SNR, for distinct values of  $M$ , i.e., 10, 30, 50, and 70. It has been observed that a substantial improvement in the performance is attained as  $M$  increases. For higher classification accuracy, the value of  $M$  is kept greater than 30.

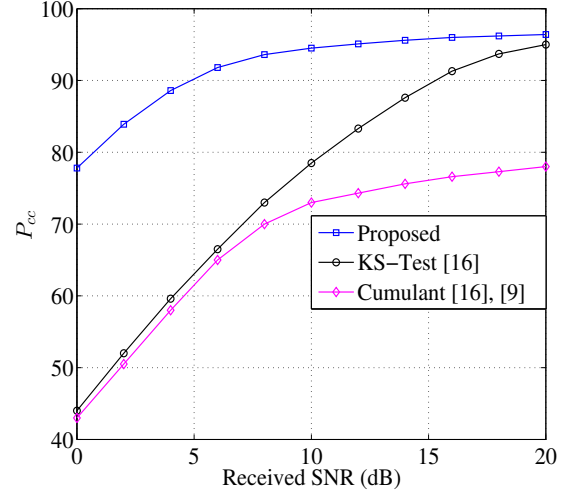


Fig. 5. Simulation results comparison:  $P_{cc}$  vs. SNR for {BPSK, QPSK, 16-QAM} modulation formats.

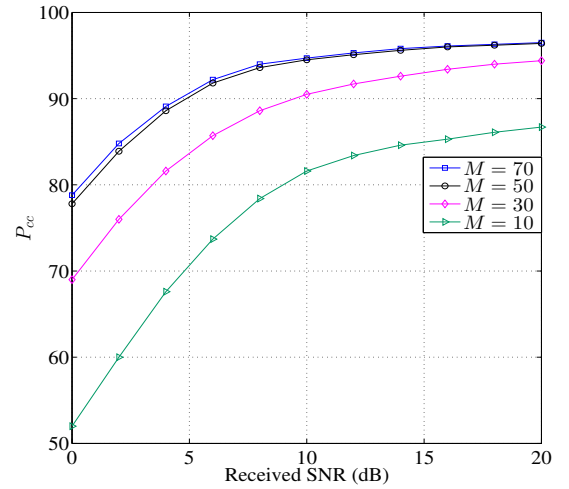


Fig. 6. Simulation results: Effect of  $M$  on  $P_{cc}$  for five-class of modulation formats.

#### V. CONCLUSION

In this paper, we introduced a feature-based blind MC algorithm for an OFDM system in the presence of synchronization errors, i.e. frequency, timing, and phase offsets and without the prior information about the channel statistics. The

proposed algorithm utilizes the properties of the DFT and normalized fourth-order cumulant of the received OFDM signal. The results show that the proposed algorithm can achieve better classification performance as compared to the existing algorithms for OFDM systems, and it can also recognize a different variants of QPSKs, i.e., QPSK, OQPSK, and MSK modulation formats.

#### APPENDIX

In general, we can calculate mean and variance at the first-stage classification as:  $\mu_{\mathcal{M}} = E[\tilde{C}_{42Y}]_{\mathcal{M}}$  and  $\sigma_{\mathcal{M}}^2 = Var[\tilde{C}_{42Y}]_{\mathcal{M}}$ , respectively, where  $\mathcal{M} = \{1, 2, 3, 4, 5\}$  belongs to MSK, QPSK, BPSK, 16-QAM, and OQPSK modulation formats, respectively. Similarly, at the second-stage, mean is represented by  $\bar{\mu}_{\bar{\mathcal{M}}}$  and variance  $\bar{\sigma}_{\bar{\mathcal{M}}}^2$ , where  $\bar{\mathcal{M}} = \{2, 3\}$  belongs to QPSK and BPSK modulation formats, respectively.

The threshold values, i.e.,  $\eta_1, \eta_2, \eta_3$ , and  $\eta_4$  are calculate by using likelihood ratio test. We have probability density function for MSK, QPSK, BPSK, 16-QAM, and OQPSK modulation formats at the first-stage as

$$p(\tilde{C}_{42Y}|\mathcal{M}) = \frac{1}{\sqrt{2\pi\sigma_{\mathcal{M}}^2}} \exp\left[-\frac{(\tilde{C}_{42Y} - \mu_{\mathcal{M}})^2}{2\sigma_{\mathcal{M}}^2}\right]. \quad (16)$$

In an asymptotic threshold value analysis the variance of the different modulation formats is considered almost alike, equals  $\sigma^2$ . At the first-stage classification, as shown in Fig. 2 the mean value obtained for QPSK and BPSK are almost same, i.e.,  $\mu_2 \approx \mu_3$ . The likelihood ratio of MSK and QPSK/BPSK, given by

$$\begin{aligned} \Lambda(\tilde{C}_{42Y}) &= \exp\left[\frac{(\tilde{C}_{42Y} - \mu_1)^2 - (\tilde{C}_{42Y} - \mu_2)^2}{2\sigma^2}\right] \\ &= \exp\left[\left(\frac{\mu_2 - \mu_1}{\sigma^2}\right)\left(\tilde{C}_{42Y} - \frac{\mu_2 + \mu_1}{2}\right)\right]. \end{aligned} \quad (17)$$

To calculate  $\eta_1$ , we have

$$\exp\left[\left(\frac{\mu_2 - \mu_1}{\sigma^2}\right)\left(\tilde{C}_{42Y} - \frac{\mu_2 + \mu_1}{2}\right)\right] \underset{\text{MSK}}{\overset{\text{QPSK, BPSK}}{\gtrless}} \Psi_1, \quad (18)$$

where  $\Psi_1$  is the ratio of the prior probabilities of MSK and QPSK/BPSK modulation formats. This can be further simplified as

$$\tilde{C}_{42Y} \underset{\text{MSK}}{\overset{\text{QPSK, BPSK}}{\gtrless}} \frac{\sigma^2 \ln(\Psi_1)}{\mu_2 - \mu_1} + \frac{\mu_2 + \mu_1}{2} = \eta_1. \quad (19)$$

Similarly, we can calculate the threshold values  $\eta_2, \eta_3$ , and  $\eta_4$  as

$$\begin{aligned} \tilde{C}_{42Y} &\underset{\text{QPSK, BPSK}}{\overset{16\text{-QAM}}{\gtrless}} \frac{\sigma^2 \ln(\Psi_2)}{\mu_4 - \mu_2} + \frac{\mu_4 + \mu_2}{2} = \eta_2, \\ \tilde{C}_{42Y} &\underset{16\text{-QAM}}{\overset{\text{OQPSK}}{\gtrless}} \frac{\sigma^2 \ln(\Psi_3)}{\mu_5 - \mu_4} + \frac{\mu_5 + \mu_4}{2} = \eta_3, \\ \tilde{C}_{42Z} &\underset{\text{QPSK}}{\overset{\text{BPSK}}{\gtrless}} \frac{\sigma^2 \ln(\Psi_4)}{\bar{\mu}_3 - \bar{\mu}_2} + \frac{\bar{\mu}_3 + \bar{\mu}_2}{2} = \eta_4, \end{aligned} \quad (20)$$

where  $\Psi_2$  is the ratio of the prior probabilities of QPSK/BPSK and 16-QAM and  $\Psi_3$  is the ratio of the prior probabilities of 16-QAM and OQPSK modulation formats at the first-stage classification.  $\Psi_4$  is the ratio of the prior probabilities of QPSK and BPSK modulations at the second-stage classification.

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