Novel Blind Modulation Classification of Circular and Linearly Modulated Signals Using Cyclic Cumulants

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***Abstract*—This paper presents blind modulation classification algorithm of circular linearly modulated signals using cyclic cumulant. The classification uses positions of non-zero cyclic frequencies (symbol rate frequency or carrier frequency) of the received signals. The method works over flat and frequency selective fading channels without having any knowledge of the signal parameters and channel statistics. The proposed method is used for classifying circular constellation modulation format which has not been addressed before. It requires lower complexity than the maximum likelihood approach. The performance of this novel algorithm is compared with the existing methods. In this paper, we have considered a four-class problem including quadrature phase-shift keying (QPSK), offset-QPSK, π/4-QPSK, minimum shift keying.**

***Index Terms*—Blind modulation classification, elementary cumulant, cyclic cumulants**

I. INTRODUCTION

BLIND modulation classification (BMC) is a rapidly evolving area with applications in monitoring, spectrum management in cognitive radio, and cooperative communications systems [1], [2]. The blind classification process are bandwidth efficient as it is non-data aided and does not use the predefined training sequence or pilot symbols in the classification process. As a result, the time complexity of the algorithm reduces. Several BMC algorithms such as ML, non-parametric likelihood function, 2-D histograms, and probability distance approach have been published in the literature [3]–[8].

Recently, feature based statistical methods have been found to be more useful. The BMC algorithm based on a higher order statistical moment, elementary cumulant (EC), cyclic cumulant and expectation-maximization have been studied in detail in [9]–[11]. But the ECs are not able to differentiate between circular constellation modulation formats, i.e., quadrature phase-shift keying (QPSK), offset-QPSK (OQPSK), π/4-QPSK and minimum shift keying (MSK). The cyclic cumulant approach and their combined methods [12] are robust to various impairments, such as synchronization errors like timing offset, carrier frequency offset, phase jitters and have a satisfactory performance. To the best of our knowledge, no modulation classifier incorporates these circular constellations modulation formats.

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To address above problems, a new BMC algorithm is proposed for circular constellation modulation signal. The proposed BMC algorithm is based on the fourth and the second order cyclic cumulants. Monte Carlo simulations were carried out based on a four-class problem which includes QPSK, OQPSK, π/4-QPSK, and MSK modulation formats to access the performance. It is observed from the theoretical and simulation studies that the proposed feature based BMC algorithm adds robustness against various impairments and works well even in a flat and frequency selective fading channels.

The rest of the paper is organized as follows. Section II discusses a signal model. Section III presents BMC algorithm based on the fourth and the second order cyclic cumulant. Section IV illustrates simulation results and Section V concludes the paper.

II. SIGNAL MODEL

In this study, we assume that signal is a cyclostationary process (CP). We use an intermediate frequency (IF) signal and complex baseband signal at the receiver. A digitized IF signal is obtained by a down conversion of a received RF signal and baseband signal is obtained by an additional down conversion and filtering of the digitized IF signal. The IF frequency is determined by a coarse and fine carrier frequency estimations. The coarse carrier frequency estimate is obtained by smoothing power spectral density (PSD) of the received IF signal. The fine carrier frequency estimate is obtained by using fourth order cyclic cumulant proposed in [2], [13].

In this context, we consider a real discrete-time cyclostationary signal. Let us start with a continuous-time domain, where the transmitted signal is given by

$$x(t) = Re\bigg(\sum_{l=0}^{L-1} a[l]\tilde{g}(t-lT-\tau)e^{-j(w_c t+\phi)}\bigg), \quad (1)$$

In (1) L is the number of symbols, $a[l]$ is the lth symbol drawn from any of four-class modulation schemes, T is the symbol period, τ is the timing offset, w_c is the carrier frequency of a IF signal, and ϕ the carrier phase noise. The transmitted pulse shape is $\tilde{g}(t)$ representing the root raised cosine (RRC) function. Eq (1) can be deduced as

$$x(t) = s_r(t)cos(w_c t + \phi) + s_i(t)sin(w_c t + \phi), \quad (2)$$

where $a[l] = a_r[l] + ja_i[l]$, $s_r(t) = \sum_{l=0}^{L-1} a_r[l]\tilde{g}(t - lT - \tau)$, $s_i(t) = \sum_{l=0}^{L-1} a_i(l)\tilde{g}(t - lT - \tau + \tau_1)$ and,

$$\tau_1 = \begin{cases} T/2, & \text{OQPSK, MSK} \\ 0, & \text{otherwise} \end{cases} \quad (3)$$

The signal received over the channel with Rayleigh flat/frequency selective fading and AWGN can be written as $\tilde{y}(t) = x(t)\otimes h(l_1)+v(t)$, where $h(l_1)$ is the Rayleigh flat/frequency selective fading channel coefficients and is modeled as $h(l_1)=[h(0), ..., h(L_1 - 1)]$, L_1 denotes the maximum number of channel taps, $v(t)$ the complex AWGN with two sided PSD with variance, $\sigma_v^2 = N_0/2$ where, N_0 is PSD of noise. The signal $\tilde{y}(t)$ is over sampled at rate of P/T to yield the discrete-time version

$$\tilde{y}[n]=Re\left(\sum_{l=0}^{L-1} a[l]\tilde{g}[n-lP-N_1]e^{-jw_c n+\phi}+v[n]\right), \quad (4)$$

where N_1 is the number of samples corresponding to the delay τ. The oversampling factor is defined as $P= F_s/f_s=T/T_s=N/L$ where F_s denotes the sampling rate, $f_s=1/T$ is the symbol rate, N is the received signal length, and $T_s=1/F_s$ is the sampling period. The above signal can be represented in terms of complex signal as

$$\begin{aligned} y[n] &= \tilde{y}[n] + j\hat{\tilde{y}}[n] \\ &= s[n]e^{j(w_c n+\phi)} + v[n], \end{aligned} \quad (5)$$

where $\hat{\tilde{y}}[n]$ is a Hilbert transform of $\tilde{y}[n]$ and $s[n]$ is the baseband signal.

The problem statement can be formulated as follows: *Given N samples, $\{y[n]\}_1^N$, modulation format need to be estimated for four circular linearly modulated signals without having any prior knowledge of the signal parameters and channel statistics such as symbol rate, IF carrier, $\{a[l], g[n], \beta, N_1, f_o, \phi, L, h(l_1)\}$ over flat and frequency selective fading channels*. The IF carrier, symbol rate and bandwidth of the signal are estimated by using our previous work given in [1], [2].

Once the carrier frequency of IF signal is retrieved a baseband signal can be obtained as

$$s[n] = \sum_l a[l]\tilde{g}[n - lP - N_1]e^{-j(2\pi f_o n)} + w[n], \quad (6)$$

where $w[n]$ is the low-pass complex baseband noise and f_o is the frequency offset. It has been observed that after the estimation of IF carrier, a negligible frequency offset, $|f_o|\leq\pm100$ Hz, present in the received baseband signals, does not affect the modulation classification.

III. Modulation Classification

The existing methods are unable to classify modulation formats, if the signals are from circular constellation modulation formats. For example, the modulation formats QPSK, OQPSK, π/4-QPSK and MSK have the same EC values illustrated in Table I thus the method given in [9]–[11] cannot be applied to classify these modulation formats.

TABLE I
Theoritical value of fourth order elementary cumulant

QPSK	OQPSK	π/4QPSK	MSK
-1	-1	-1	-1

A. *Proposed Blind Modulation Classification*

In this section, we use a novel approach for BMC which is based on the fourth and the second order cyclic cumulant of the IF and baseband signal, respectively. First, the features of the cyclic cumulants for different modulation formats are studied and then we use these features to classify the modulation formats.

1) The fourth Order Cyclic Cumulant: The fourth order time-varying correlation function of $y[n]$ at time lag $\mathbf{0} = [0, 0, 0]$ can be expressed as [13]

$$c_{r[y,4,0]}[n; \mathbf{0}] = \sum_{l=1}^{L} \sigma_{s_l}^4 e^{4j(w_c n+\phi)} + \sigma_v^4, \quad (7)$$

where $\sigma_{s_l}^4 = E\{s_l^4[n]\}$, and $\sigma_v^4 = E\{v^4[n]\}$. The corresponding Fourier series (FS) coefficient of $c_{r[y,4,0]}[n; \mathbf{0}]$, known as cyclic cumulant, can be expressed as

$$C_{[y,4,0]}[\alpha; \mathbf{0}] = \sum_{l=1}^{L} \sigma_{s_l}^4 e^{j\phi}\delta[\alpha - 4w_c] + \sigma_v^4\delta[\alpha], \quad (8)$$

where α is cyclic frequency. In practice, (8) can be estimated as [13]

$$\hat{C}_{[y,4,0]}[\alpha; \mathbf{0}] = \frac{1}{N}\sum_{n=0}^{N-1} y^4[n]e^{-jn\alpha}, \quad (9)$$

Note that in fact (9) is nothing but the normalized discrete Fourier transform (DFT) of $y^4[n]$, and can be implemented efficiently using the fast Fourier transform (FFT) algorithm. Once $\hat{C}_{[y,4,0]}[\alpha; \mathbf{0}]$ is obtained, frequency estimation is straight-forward, given in [2], [13].

$$\hat{w} = \arg\max_{\alpha\in[l_5,u_5]} \left|\hat{C}_{[y,4,0]}[\alpha; \mathbf{0}]\right|, \quad (10)$$

where $[l_5, u_5]$ is the interval discussed at the end of section III-A. The non-zero cyclic cumulant peak for QPSK and OQPSK modulated signal appears at $4w_c$ as shown in Fig. 1. The fourth order cyclic cumulant for π/4-QPSK and MSK modulated signals are described below:

a) Cyclic Cumulant Peak for π/4-QPSK : The transmitted signal for π/4-QPSK given as [14].

$$\begin{aligned} S_{\pi/4-QPSK}(t) &= u_k cos(w_c t) - v_k sin(w_c t) \\ &= A cos(w_c t + \phi_k), \end{aligned} \tag{11}$$

where u_k and v_k are the amplitudes in the kth symbol interval of coded I and Q signal component respectively, the signal amplitude is $A{=}\sqrt{u_k^2+v_k^2}$ and phase $\phi_k = tan^{-1}\frac{v_k}{u_k}$. The baseband representation of π/4-QPSK modulated signal is $s_p(t) = Ae^{j\phi_k}$. The phase ϕ_k is a differentially encoded and can be expressed as $\phi_k{=}\phi_{k-1} + \Delta\phi_k$, where phase, $\Delta\phi_k{=}\pm\pi/4, \pm\, 3\pi/4$, and in general it is written as

$$\begin{aligned} \Delta\phi_k &= \Delta w_k.\Delta t_k \\ \pm\pi/4 &= 2\pi f_k.T_s \\ f_k &= \pm f_s/8, \end{aligned} \tag{12}$$

In (12) Δw_k is the change of angular frequency, f_k is the corresponding frequency and Δt_k is the change of time in the kth symbol interval. The baseband signal for π/4-QPSK can further be expressed as

$$s_p(t) = Ae^{j\phi_k} = Ae^{j2\pi f_k t} = Ae^{j\frac{\pm w_s}{8}t}, \tag{13}$$

where, w_s is angular symbol rate frequency, and the received IF signal of π/4-QPSK can be given as

$$y_p(t) = \sum_{l=1}^{L} A_l e^{j\frac{\pm w_s}{8}t} e^{j(w_c t+\phi)} + v(t), \tag{14}$$

where A_l is amplitude of the lth symbol, $s_{p_l}(t)$. After converting to the discrete-time version $y_p[n]$, the fourth order time-varying correlation function of $y_p[n]$ at time lag $\mathbf{0} = [0, 0, 0]$ can be expressed as [13]

$$c_{r[y_p,4,0]}[n;\mathbf{0}] = \sum_{l=1}^{L} \sigma_l^4 e^{j\frac{\pm w_s}{2}n} e^{4j(w_c n+\phi)} + \sigma_v^4, \tag{15}$$

where $\sigma_l^4{=}E[A_l^4]$ and $\sigma_v^4 = E[v^4(n)]$, and the generalized FS coefficient of $c_{r[y_p,4,0]}[n;\mathbf{0}]$ is given by [13]

$$C_{[y_p,4,0]}[\alpha;\mathbf{0}] = \sum_{l=1}^{L} \sigma_l^4 e^{4j\phi} \delta(\alpha - (4w_c \pm \frac{w_s}{2})) + \sigma_v^4 \delta(\alpha). \tag{16}$$

In practice, (16) can be estimated as

$$\begin{aligned} \hat{C}_{[y_p,4,0]}[\alpha;\mathbf{0}] &= \frac{1}{N}\sum_{n=0}^{N-1} y^4[n] e^{-jn\alpha} \\ \hat{w} &= \underset{\alpha\in[l_3,u_3] \text{ or } [l_4,u_4]}{\arg\max} \left|\hat{C}_{[y_p,4,0]}[\alpha;\mathbf{0}]\right| \end{aligned} \tag{17}$$

where $[l_3, u_3], [l_4, u_4]$ are the intervals discussed at the end of section III-A. It can be seen that the non-zero cyclic cumulant frequency of IF signal of π/4-QPSK appears at $4w_c \pm w_s/2$ which is shown in Fig. 1.

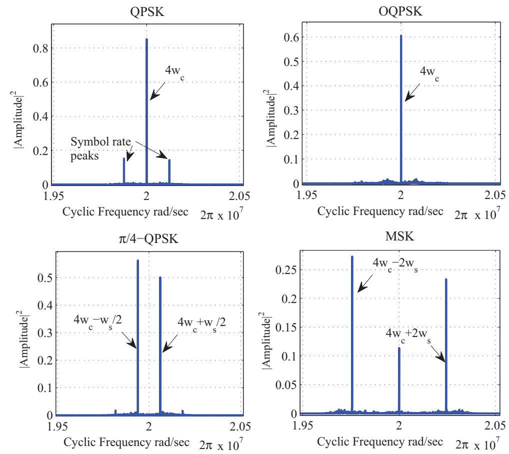

Fig. 1. Non-zero cyclic cumulant peak for QPSK, OQPSK, $\pi/4$-QPSK and MSK modulated signals.

b) Cyclic Cumulant Peak for MSK: The transmitted signal for MSK can be expressed as

$$S_{MSK}(t) = \sum_{l=1}^{L} s_{m_l}(t) e^{\frac{j\pi t}{2T_b}}.e^{j(w_c t+\phi)}, \tag{18}$$

where symbol interval is $T{=}2T_b$, bit rate $f_b{=}2f_s$, T_b bit duration, and f_s the symbol rate. Baseband representation of MSK modulated signal is $s_m(t) = s(t)e^{\frac{jw_s t}{2}}$ where $s(t)$ is the magnitude of I and Q phase components. The corresponding received signal for MSK is

$$y_m(t) = \sum_{l=1}^{L} s_l(t) e^{\frac{jw_s t}{2}} e^{j(w_c t+\phi)} + v(t), \tag{19}$$

where $s_l(t)$ is the magnitude of the lth symbol. After converting to the discrete-time version $y_m[n]$, the fourth order time-varying correlation function of $y_m[n]$ at time lag $\mathbf{0} = [0, 0, 0]$ can be expressed as [13]

$$c_{r[y_m,4,0]}[n;\mathbf{0}] = \sum_{l=1}^{L} \sigma_m^4 e^{j2w_s n} e^{4j(w_c n+\phi)} + \sigma_v^4, \tag{20}$$

where $\sigma_m^4 = E[s_l^4(t)]$ and $\sigma_v^4 = E[v^4(n)]$, and the generalized FS coefficient of $c_{r[y_m,4,0]}[n;\mathbf{0}]$ is given by

$$C_{[y_m,4,0]}[\alpha;\mathbf{0}] = \sum_{l=1}^{L} \sigma_s^4 e^{4j\phi} \delta(\alpha - (4w_c \pm 2w_s) + \sigma_v^4 \delta(\alpha). \tag{21}$$

In practice, (21) can be estimated as

$$\begin{aligned} \hat{C}_{[y_m,4,0]}[\alpha;\mathbf{0}] &= \frac{1}{N}\sum_{n=0}^{N-1} y^4[n] e^{-jn\alpha} \\ \hat{w} &= \underset{\alpha\in[l_1,u_1] \text{ or } [l_2,u_2]}{\arg\max} \left|\hat{C}_{[y_m,4,0]}[\alpha;\mathbf{0}]\right| \end{aligned} \tag{22}$$

where $[l_1, u_1], [l_2, u_2]$ are the intervals discussed below. It can be seen that the non-zero cyclic cumulant frequency of IF signal for MSK appears at $4w_c{\pm}2w_s$ which is shown in Fig. 1. Thus, $\pi/4$-QPSK, and MSK have the distinct feature for their fourth order cyclic cumulant frequency. Since, QPSK and OQPSK provide non-zero cyclic cumulant frequency at the same position, so these cannot be classified by using this feature.

2) Second Order Cyclic Cumulant: The second order cyclic spectrum of the baseband signal $s[n]$ given in (6) can be obtained as [1]

$$C_{[s,2,1]}[\alpha;\mathbf{0}]=\frac{\sigma_a^2}{P}G\left[\frac{1}{P}\right]\delta(\alpha-1/P)e^{-j2\pi N_1/P}+\sigma_w^2\delta[\alpha], \tag{23}$$

In (23) $\sigma_a^2=\sigma_r^2+\sigma_i^2$, $\sigma_r^2=E\big[(a_r[l])^2\big]$, $\sigma_i^2=E\big[(a_i[l])^2\big]$ and $G[.]$ is the DFT of $\tilde{g}^2[.]$. The non-zero cyclic cumulant frequency of baseband QPSK signal appears at w_s as shown in Fig. 2. In practice, (23) can simply be estimated as

$$\hat{C}_{[s,2,1]}[\alpha;\mathbf{0}]=\frac{1}{N}\sum_{n=0}^{N-1}s^2[n]e^{-jn\alpha}$$
$$\hat{w}=\underset{\alpha\in[l_1',u_1']}{\arg\max}\left|\hat{C}_{[s,2,1]}[\alpha;\mathbf{0}]\right|, \tag{24}$$

where $[l_1',u_1']$ is the interval discussed below. However, for OQPSK, the second order cyclic cumulant does not provide non-zero cyclic peak at symbol rate frequency due to $P/2$ shift between I and Q components of OQPSK signal appearing in $s_i[n]=\sum_{l=0}^{L-1}a_i[l]g[n-lP-P/2-N_1]$. The detailed derivation of second order cyclic cumulant of OQPSK is given in [1] as

$$C_{[s,2,1]}[\alpha;\mathbf{0}]=\frac{1}{P}G\left[\frac{1}{P}\right]e^{-j2\pi N_1/P}\left(\sigma_r^2\delta\big[\alpha-\frac{1}{P}\big]-\sigma_i^2\delta\big[\alpha-\frac{1}{P}\big]\right)+\sigma_w^2\delta[\alpha], \tag{25}$$

In (25), cyclic spectrum function at $\alpha=1/P$ is being canceled as I and Q component powers are same, i.e., $C_{[s,2,1]}[\alpha;\mathbf{0}]|_{\alpha=1/P}=\sigma_w^2\delta(\alpha)$, and only noise term is left as shown in Fig. 2. Therefore, a distinct feature between QPSK and OQPSK can be observed from their second order cyclic cumulant. In this algorithm, five pre-estimated intervals are used to detect the cyclic spectrum of MSK, $\pi/4$-QPSK, QPSK, OQPSK given as follows: The first and second search intervals for classifying MSK modulated signal is given as

$$[l_1,u_1]\equiv[4\hat{w}_c-2w_{bw},4\hat{w}_c-0.8w_{bw}],$$
$$[l_2,u_2]\equiv[4\hat{w}_c+0.8w_{bw},4\hat{w}_c+2w_{bw}],$$

where w_{bw} is the 3dB signal bandwidth, roughly estimated between $1.2\hat{w}_s\le w_{bw}\le 2\hat{w}_s$, $\hat{w}_s$ is the estimated symbol rate frequency, and $\hat{w}_c$. The third and fourth search intervals for classifying $\pi/4$-QPSK modulated signal are given as

$$[l_3,u_3]\equiv[4\hat{w}_c-1.2w_{bw},4\hat{w}_c-0.2w_{bw}],$$

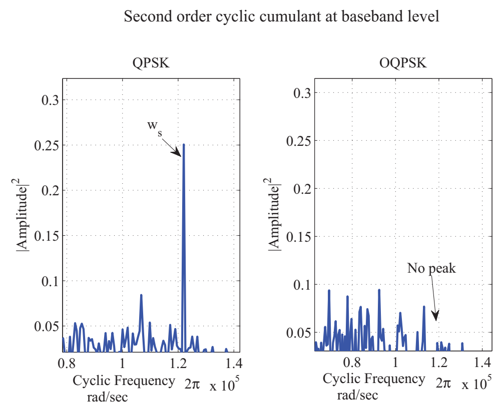

Fig. 2. Non-zero cyclic cumulant peak for QPSK and OQPSK modulated signal.

$$[l_4,u_4]\equiv[4\hat{w}_c+0.2w_{bw},4\hat{w}_c+1.2w_{bw}],$$

The fifth search interval for classifying QPSK and OQPSK modulated signal is given as

$$[l_5,u_5]\equiv[4\hat{w}_c-0.05\hat{w}_c,4\hat{w}_c+0.05\hat{w}_c],$$

where $\hat{w}_c$ is the estimated carrier frequency. Now for classifying QPSK and OQPSK one search interval is required, i.e.,

$$[l_1',u_1']\equiv[w_{bw}/2,w_{bw}/1.2].$$

For classifying QPSK and OQPSK, the second order cyclic cumulant of a baseband signal is obtained and if the cyclic cumulant peak is present at the symbol rate frequency, we classify the signal as QPSK, modulated signal otherwise as OQPSK modulated signal as shown in Fig. 2. The computational complexity of the proposed algorithm is of $\mathcal{O}\left(NlogN\right)$. The computational complexity of EC and maximum likelihood (ML) algorithm is of $\mathcal{O}\left(N\right)$ and $\mathcal{O}\left(N\times M_i\times I\right)$ respectively, where N is the number of data samples, M_i is the constellation size of the ith modulation scheme and I is the total number of modulation schemes used for classification.

IV. Simulation Results and Discussion

In this section, the performance of BMC is evaluated through simulation results over flat and frequency selective fading channels. For frequency selective fading, $L_1=4$ multipath channel coefficients was considered. In the simulation process, we use 5 MHz IF carrier frequency and 4 MHz symbol rate. However, the IF carrier can be varied between 1 MHz to 5 MHz and symbol rate can be varied between 100 kHz to 5 MHz. The number of symbols for each iteration is set to 500 and the performance is measured for 1000 iterations.

Fig. 3 shows the success rate of simulation results for a four-class of modulation classification problem over

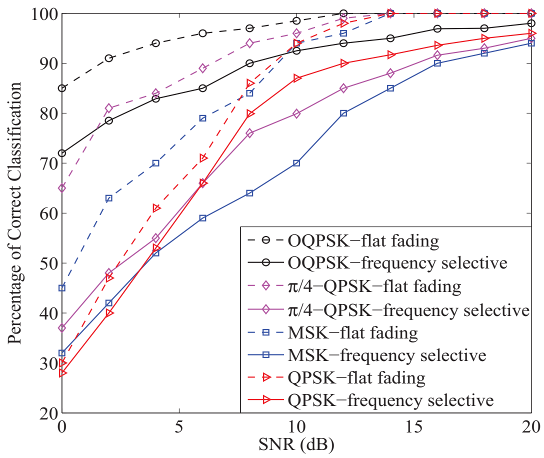

Fig. 3. Success rate for simulation results for a four-class of modulation classification problem of proposed algorithm.

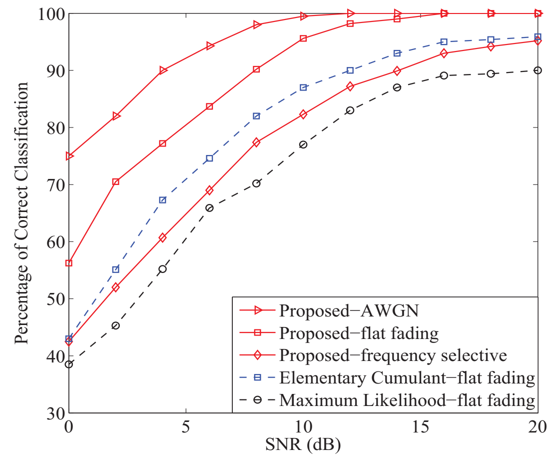

Fig. 4. Success rate for simulation results comparison of proposed, elementary cumulant and maximum likelihood algorithms.

flat and frequency selective fading channels. It has been observed that the 100% accuracy of QPSK, OQPSK, $\pi/4$-QPSK and MSK is around 14 dB SNR for flat fading channel. For frequency selective fading channel each modulation formats still provides some error even beyond 20 dB SNR. Fig. 4 shows the simulation result comparison of the proposed classifier with EC and ML methods for AWGN, flat and frequency selective fading channels. The proposed BMC method shows 100% accuracy beyond 10 dB SNR for AWGN channel. The new BMC method shows 100% accuracy beyond 14 dB SNR for flat fading channel. For frequency selective fading channel proposed algorithm still provides some error beyond 20 dB SNR. To our best knowledge no modulation classifier incorporates OQPSK, $\pi/4$-QPSK, and MSK modulation formats for flat/frequency selective fading channels. However, EC and ML method still provides some error even after 20 dB SNR for flat fading channel. Moreover, EC and ML methods cannot work for all four-class modulation schemes.

V. Conclusion

A new BMC has been proposed for classifying four different modulated signals namely QPSK, OQPSK, $\pi/4$-QPSK, MSK in the presence of flat and frequency selective fading channels. The proposed method is based on the fourth and the second order cyclic cumulant. It has been observed that the proposed method is robust compared to EC and ML methods in the presence of synchronization errors.

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