HW3

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Q1) 1. Show that the following languages are context free by constructing a CFG G that generates the language or a PDA M that accepts the language. If you want more challenge, give both G and M.

(a) $L = \{a^i b^j c^k \mid i, j, k \geq 0 \text{ and } i + j = k\}$
(b) $L = \{a^i b^j c^k \mid i, j, k \geq 0 \text{ and } i + k = j\}$
(c) $L = \{a^n b^m c^p d^q \mid n, m, p, q \geq 0 \text{ and } n > m \text{ and } p < q\}$.
(d) $L = \{a^n b^m \mid 2n = 3m, n \geq 0, m \geq 0\}$.
(e) $L = \{uawb : u, w \in \{a, b\}^*, |u| = |w|\}$.
(f) $L = \{a^m b^n c^p d^q \mid m + n = p + q\}$
(g) $L = \{w \in \{a, b\}^* \mid n_a(w) > n_b(w)\}$
(h) $L = \{w \in \{a, b\}^* \mid n_a(w) = 2 * n_b(w)\}$

a) $L = \{a^i b^j c^k \mid i, j, k \geq 0 \text{ and } i + j = k\}$

b) $L = \{a^i b^j c^k \mid i, j, k \geq 0 \text{ and } i + k = j\}$

b) $L = \{a^i b^j c^k \mid i,j,k \geq 0 \text{ and } i+k = j\}$

$S \to AC$

$A \to aAb \mid \epsilon$

$C \to bCc \mid \epsilon$

c) $L = \{a^n b^m c^p d^q \mid n,m,p,q \geq 0 \text{ and } n > m \text{ and } p < q\}$

$S \to \epsilon \mid aAB \mid CDd \mid aABCDd$

$B \to \epsilon \mid ABb$

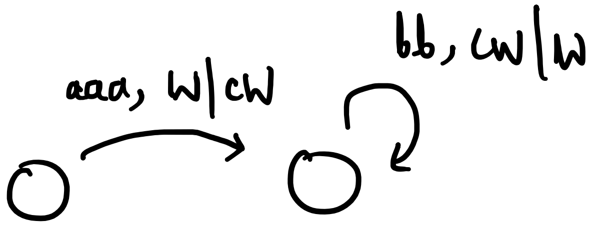
$C \to \epsilon \mid cCD$

$A \to a \mid aA$

$D \to d \mid Dd$

d) $L = \{a^n b^m \mid 2n = 3m,\ n \geq 0 \text{ and } m \geq 0\}$

$S \to aaaSbb \mid \epsilon$



e) $L = \{uawb \mid u,w \in \{a,b\}^*;\ |u| = |w|\}$

$S \to Gb$

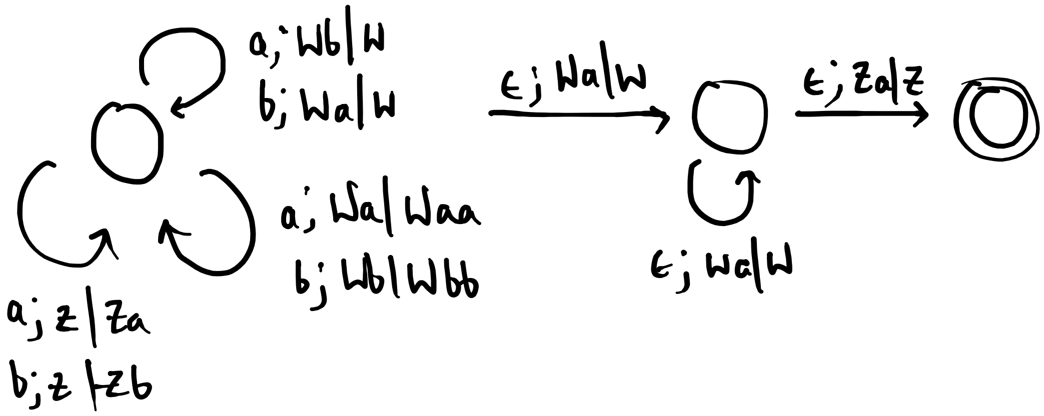
$G \to UGW \mid a$

$U \to a \mid b$

$W \to a \mid b$

f) $L = \{a^m b^n c^p d^q \mid m + n = p + q\}$

g) $L = \{w \in \{a, b\}^* \mid n_a(w) > n_b(w)\}$



h) $L = \{w \in \{a, b\}^* \mid n_a(w) = 2n_b(w)\}$

$S \to aaSb \mid bSaa \mid PbP \mid \epsilon \mid SS$

$P \to aS \mid PS$

2. Convert the following CFG into an equivalent CFG in Chomsky normal form

S → PSP | P | ε

P → 00 | ε

→ Chomsky form:

$A \to BC$

$A \to a$

Now, $S \to PSP \mid P$ —— $4n+2$ 0's
$P \to 00$

Chomsky form is - $S \to P_1P_2 \mid PP$
$P_1 \to P_3P_1 \mid P_5P_5$
$P_2 \to P_2P_4 \mid PP$
$P_3 \to P_5P_5$
$P_4 \to PP$
$P_5 \to PP$
$P \to 0$

3. Convert to Greibach normal form

$S \to AA \mid 0$
$A \to SS \mid 1$

Greibach form : $C \to a\alpha$
Given : $S \to AA \mid 0$
$A \to SS \mid 1$

Palindromic strings with odd characters with 0 at center

Converted form -
$S \to 0S0 \mid 1S1 \mid 0$

4. Show that L = { ww | w ∈ (a+b)*} is not context-free

Is the complement of L (strings not of the form ww) context-free? Prove or disprove.

→ $L = \{ww \mid w \in (a+b)^*\}$

For a given 'n' consider $w' \in L$ of the form

$$w' = a^n b^n a^n b^n$$

Now, for u, v, w, x, y from pumping lemma

vwx will be of the form a^i, b^i, $a^j b^k$, $b^j a^k$ for $n \geq i \geq 1$, $0 \leq j, k$ and $j + k \leq n$

We then have the following cases -

- $v = a^i$ and $x = a^j b^k$ for $0 \leq i, j, k$ and $1 \leq i + j + k \leq n$

According to the lemma, $a^{n + (i+j)(t-1)} b^{n + k(t-1)} a^n b^n$

or $a^n b^n a^{n + (i+j)(t-1)} b^{n + k(t-1)}$

belong to L for all $t \geq 0$

This is a contradiction for any $t > 0$

- $v = b^i$ and $x = b^j a^k$

We have similar reasoning as above.

Therefore, L is not CFG //

For L^c we shall make a CFG

for L, we shall propose a CFG

$$S \to A \mid B \mid AB \mid BA$$

$$A \to a \mid aAa \mid bAb \mid aAb \mid bAa$$

$$B \to b \mid bBb \mid aBa \mid bBa \mid aBb$$