

**Subject Name: Theory of Computation** 

Subject Code: IT-5001

Semester: 5<sup>th</sup>





# IT-5001 Theory of computation Unit-IV: Pushdown Automata (PDA)

#### 4.1 Introduction

The class of languages that can be accepted by PDA is exactly the class of context-free languages. A PDA consists of the following, see Figure 5.1

a a b b a b a

Finite State Control

Figure :5.1

Z0

- 1. There is a tape which is divided into cells. Each cell stores a symbol belonging to a finite set  $\Sigma$ , called the tape alphabet. There is a special Symbol B not in  $\Sigma$  is called blank symbol.
- 2. There is a tape head which can move along the tape, one cell to the right per move. This tape head can also read the cell it currently scans.
- 3. There is a stack containing symbols from a finite set  $\Gamma$ , called the stack symbol/alphabet. This set contains a special symbol ZO called initial symbol in top of stack.
- 4. There is a stack head which can read the top symbol of the stack. This head can also pop the top symbol, and it can push symbols of  $\Gamma$  onto the stack.
- 5. There is a state control, which can be in any one of a finite number of states. The set of states is denoted by Q. The set Q contains one special state q0, called the start state.

The input for a pushdown automaton is a string in  $\Sigma^*$ . This input string is stored on the tape of the pushdown automaton and, initially, the tape head is on the leftmost symbol of the input string. Initially, the stack only contains the special symbol ZO, and the pushdown automaton is in the start state qO. In one computation step, the pushdown automaton does the following:

- 1. Assume that the pushdown automaton is currently in state q0. Let a be the symbol of  $\Sigma$  that is read by the tape head, and let Z0 be the symbol of  $\Gamma$  that is on top of the stack.
- 2. Depending on the current state q0, the tape symbol a, and the stack symbol Z0,
- (a) the pushdown automaton switches to a state r' of Q (which may be equal to r),
- (b) the tape head either moves one cell to the right or stays at the current cell, and
- (c) the top symbol z0 is replaced by a string w that belongs to  $\Gamma^*$ .

To be more precise,

i. if w = Q, then A is popped from the stack, whereas ii. if w = B 1 B 2 ... B k, with  $k \ge 1$  and  $B 1, B 2, ..., B k \in \Gamma$ , then A is replaced by w, and B k becomes the new top symbol of the stack.



# 4.2 Definition of PDA

A deterministic pushdown automaton is a 7-tuple

 $M = (Q, \Sigma, \Gamma, \delta, q0, Z0, F)$ , where

- 1. Q is a finite set of states,
- 2. Σ is a finite set of tape alphabet or input symbole
- 3. Γ is a finite set of stack alphabet
- 4. q0 is initial state, q0 is an element of Q
- 5. Z0 is Initial symbol on top of stack
- 6. F set of final state which is sub set of Q.
- 7.  $\delta$  is transition function which maps ( Q x  $\Sigma$  x  $\Gamma$  ) into Q x  $\Gamma^*$

## 4.3 Transition of PDA

Consider current state of PDA is  $q \in Q$ , tape symbol under scanning is  $a \in \Sigma$  and symbol at top of stack is  $X \in \Sigma$  then applicable transition function will be

$$\delta(q, a, X) = (p,Y)$$

After transition

New state is p

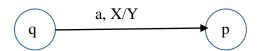
and if  $Y = \varepsilon$ , symbol from top of stack is popped

if Y = X, symbol at top of stack is unchanged

if Y = ZX, symbol Z pushed on to the top of stack.

## 4.4 Representation of PDA

Corresponding to transition function  $\delta(q, a, X) = (p,Y)$  transition diagram will have



## 4.5 Instantaneous Description (ID) of PDA

Instantaneous Description (ID) of PDA represent current description of PDA. It is used to representation of precessing of string. Two IDs are connected by symbol |--

The ID is represented by

where g is current state of PDA

w is string of remaining input symbol

Y is the content of stack in which left most symbol is on top of stack and right most symbol is at bottom of stack.

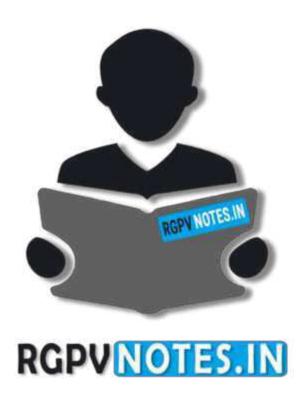
#### Example:

(q0, 000011, Z) |-- (q1, 00011, Z) This is corresponding to  $\delta(q0, a0, Z) = (q1,Z)$ 

### 4.6 Deterministic PDA

A PDA is said to be deterministic if and only if following conditions are met

- 1.  $\delta(q, a, X)$  has at most one transition.
- 2.  $\delta(q, \epsilon, X) = \Phi$



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