Overview

The aim of the assignment was to write code to perform portfolio analysis on 4 assets. Mean-Variance analysis was performed on the portfolio and to select the weights and both unconstrained as well as constrained portfolios were considered. The risk and return of the portfolios were calculated and compared to a benchmark. The adequacy of the risk measures were also assessed by performing backtesting.

Data

I decided to use the Dow Jones Index to perform the data analysis on. I selected four random individual stocks from the index being IBM, CISCO, JOHNSON and JOHNSON and GENERAL MOTORS. A portfolio of these stocks was formed and compared to the performance of the index. We downloaded four years daily price data adjusted for dividends from DataStream

Strategy for Portfolio Optimization

Through the use of Markowitz mean/variance optimization technique (MV from now) we tried to find an efficient portfolio solution that minimized the variance. We did not try to find an optimal portfolio which minimized variance (risk) and maximized returns as at many times during rebalancing of portfolio the MV gave results that expected to short sell our portfolio completely and invest in risk free rate. Such type of methodology would incur huge transaction costs hence is infeasible. MV has many limitations typically the weights are highly sensitivity to small changes in the input data proving that by the adoption of a different forecast for variance and covariances (EWMA method) of the assets the results could be significantly different. Hence cannot rely completely on MV and have to add some constraints based on market outlook.

I analysed the portfolio composed of the above stocks mainly by using *frontcon* command. From all the portfolios returned by the *frontcon* command each with a unique expected return and variance on the efficient frontier, chose the portfolio that has minimum variance. This is a quadratic constraint and is computed by the *quadprog* when it is implicitly called from the *frontcon* by default. I analysed few strategies for adding constraints on weights which are typically used in portfolio optimization. The various strategies are linear constraints and are described as follows

1) Unconstrained Strategy– In this strategy the short selling is not allowed and we require that weights allocated to different stocks constituting the portfolio is $\mathbf{w} \geq 0$ and $\sum \mathbf{w}=1$.These constraints are used frequently by many funds and institutional investors as they are prohibited to short sell. These constraints are taken care of in the command *frontcon* by default. In this strategy the weights of individual stocks can vary from 0% to 100% .
2) Constrained Strategy-In this strategy the weights of the different stocks are constrained to some upper and lower bounds based upon the outlook of the market and upon the fact that well diversified portfolio should not exhibit large concentrations in any specific assets. The constraints on assets I have used are

Asset	Minimum Exposure	Maximum Exposure
IBM(w1)	-0.10	0.75
CSCO(w2)	-0.10	0.50

JOHNSON(w3)	-0.10	0.50
GM(w4)	-0.10	0.50

Group	IBM	CSCO	JOHNSON	GM
TECHNOLOGY	X	X	-	-

Here I also want to constrain the assets belonging to the technology sector to a maximum of 75% and minimum of 50%.Also the sum of total asset should equal 1.In this strategy short selling is allowed upto -10% of asset value. The constraints can be written in the form:

1. w1+ w2 >= 0.50
2. w1 + w2 <=0.75
3. w1+w2+w3+w4=1

Converting these constraints into group form we get:
Groups = [1 1 0 0; 1 1 1 1];
Groupbounds = [0.5 .75; 1 1];

The upper and lower bound constraints on the asset can be written in the form
Asset Bounds LUB
LowerBound -0.1 -0.1 -0.1 -0.1
UpperBoound 0.75 0.5 0.5 0.5

3) ShortSell Strategy- In this strategy the short selling is allowed upto a maximum of -30% of the asset value and we apply the usual constraint that $\sum \mathbf{w}=1$.
4) Constrained Turnover-In this strategy the weights are constrained relative to average daily volume of a stock. Adding constraints to the variability of the asset weights (in this case in a range of +/- 5%) has the advantage to reduce extreme positions on assets as it is not efficient for financial institutions to move relevant amount of capitals according to the “unstable” solutions coming from the mean/ variance optimization. Such decisions could heavily penalize the asset managers of a bank because of the transaction costs involved in the operations and liquidity problems when significant volumes of a single asset have to be sold/bought in the market. We apply other constraints like no short selling and $\sum \mathbf{w}=1$ to this strategy

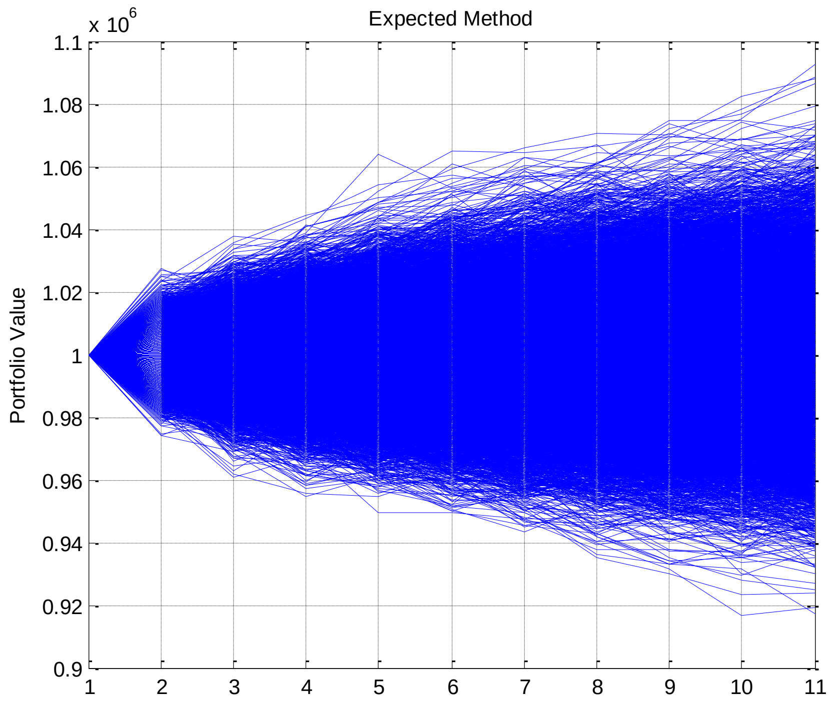
Description of Code

By entering *GUI_Portfolio_Analysis* on the command line a GUI is generated. The program first loads in the data from the Downloaded excel file called AssetPrices.xls. The 4 individual stock prices and the benchmark stock prices are saved in matrices.
A number of variables are initialised including the size of the data.
The daily prices are converted to daily returns using the matlab function *tick2ret* and saved in a matrix.

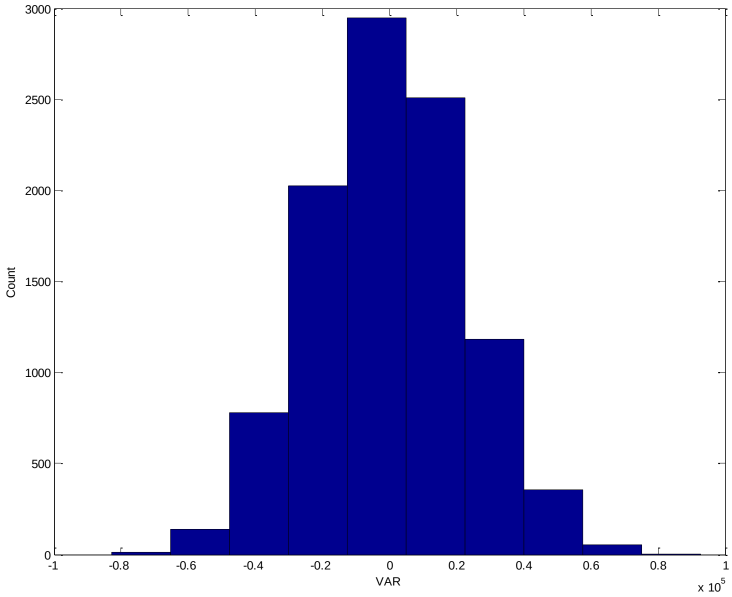
The 4 years of Price and returns data is then run through a loop. This loop is a 2 year window that begins from the start of the data and on each iteration moves the 2 year window forward 10 days until the data is exhausted. This results in 50 iterations over the 2 year period.

For each iteration the following is performed.

1. Based on the 2 year window, the Average Return and Covariance matrix of the window is calculated.
2. The average return is used as the Expected Return, and this and the covariance are fed into the matlab function *frontcon* which calculates the efficient frontier. A vector of Groups, GroupBounds and ACLUGroups is also input into the *frontcon* function to add constraints to the portfolio. These are detailed above.
3. The first point on the efficient frontier returned by *frontcon* is the minimum variance portfolio. The weights of this portfolio are selected as the weights to invest for the next 10 days.
4. The VAR and CVAR are then calculated on the selected portfolio. The VARs are calculated by performing Monte Carlo Simulation on portfolio. The matlab function *portsim* is used with the statistics calculated in Step 1. *Portsim* simulates the returns while maintaining the correlations between the returns. Ten thousand simulations are performed to generate a distribution. An example of one of the simulations is as follows:

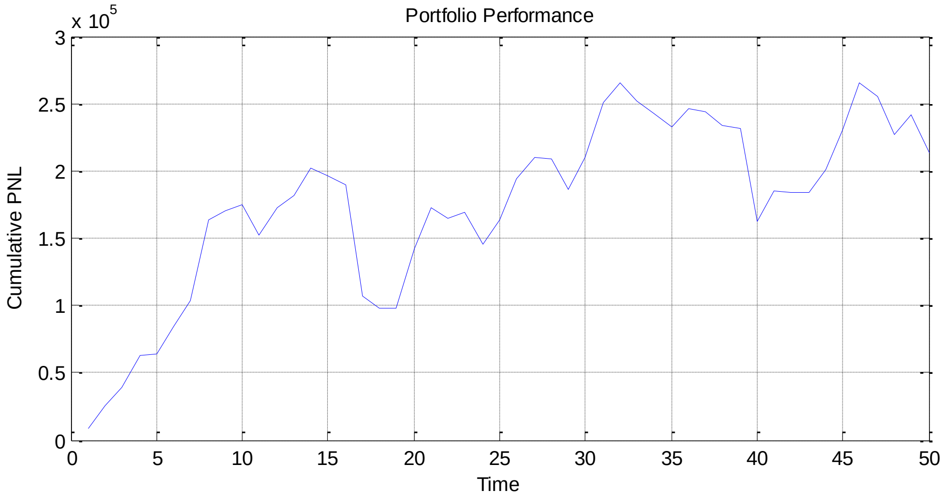


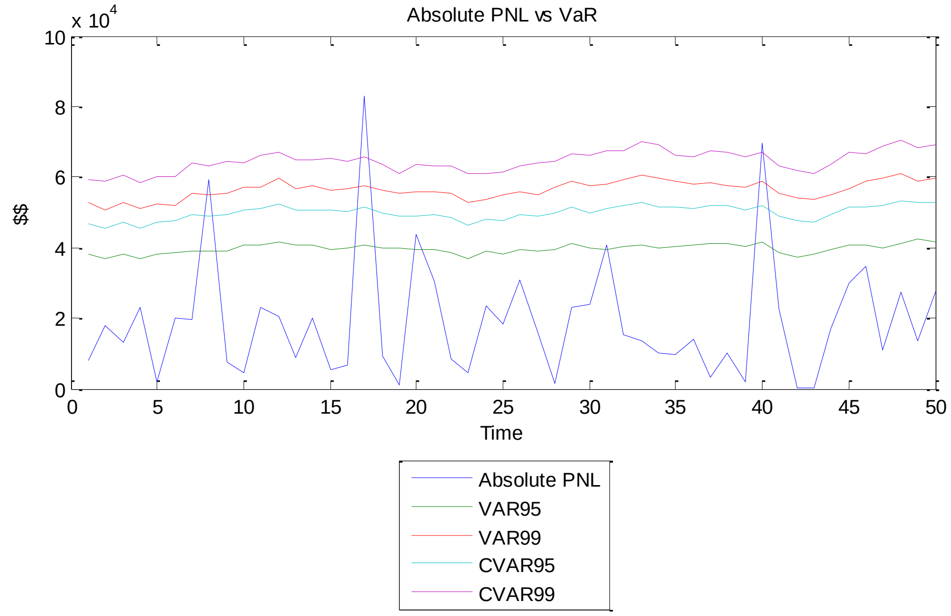
From the resulting distribution the 1st percentile and 5th percentile are selected using the matlab function *prctile*, and they represent the VAR using a 99% confidence interval and the VAR using a 95% confidence interval respectively. The mean of the distribution in the percentile ranges 0-1% and 0-5% are also calculated and they represent the CVAR using a 99% confidence interval and the CVAR using a 95% confidence interval respectively. An example of a histogram generated from Matlab using the *hist* function is below.



5. The actual 10 day PNL is then calculated using the weights calculated in step 3 and the opening prices and closing prices, 10 days forward.
6. The amount of the PNL is then compared to each VAR measure to see if the VAR was breached. A VAR breach is defined as a loss on the portfolio that exceeds the VAR level. Any breaches are recorded in a matrix.
7. The portfolio turnover is calculated which is the Absolute value of the change in weights.
8. Transaction costs are also calculated as the transaction rate multiplied by the Portfolio End Value
9. Steps 1 to 8 are repeated until the data is exhausted.
10. The risk and return of the Portfolio are calculated and the VAR analysed. The PNL is compared to the DOW Jones Index to see if the portfolio outperformed the index. These results are presented in detail below.

Graphical Results – Unconstrained Portfolio





Portfolio Performance – Unconstrained Portfolio

The PNL for the Unconstrained Portfolio over the 2 year period was $217,357 which is equivalent to a 10.42% annual return compounded over the 2 years. Using a 1% transaction rate we calculated the Transaction Costs to be $15,659. Taking transaction costs into account reduces the annual return to 9.7% per year.

	Portfolio	**Pf-TxnCost**	**Dow Jones**
Annualized return	10.42%	9.7%	2.80%
Standard Deviation	33.64%	33.67%	12.93%
Sharpe Ratio	0.16	0.14	-0.17
Portfolio PNL	217,357	201,698	
Transaction Costs	15,659		

Comparing the Portfolio to the Dow Jones index we can see that the Portfolio outperformed the Dow Jones on an absolute basis with the Dow Jones only earning 2.8% per annum. In addition the Portfolio outperformed on a risk adjusted basis, based on the Sharpe ratio. Although the volatility of the portfolio was higher than the Dow Jones volatility, the Dow Jones return was less than the risk free rate of 5% which resulted in negative Sharpe Ratio.

VAR Results – Unconstrained Portfolio

The VAR Results slightly change for every simulation. We set the number of simulations to 10,000 and obtained the following average VAR results.

VAR Type	Average VAR	Breaches	%Percent
VAR 95	40,491	2	4
VAR 99	57,763	1	2

As can be seen from the table the VAR measures seem reasonable. As we only have 50 periods the percentages recorded on the VAR95 and VAR99 are as expected, indicating that analytical VAR is an adequate risk measure for this portfolio.

CVAR is short for Conditional VAR. It is the expected loss conditional on the VAR being breached. For comparison purposes it is best to compare the CVAR number to the mean of the actual breaches of the VAR as follows:

VAR Type	Average VAR	Mean of Breaches of VAR
CVAR95	51,111	67,689
CVAR 99	66,240	84,882

In both percentiles of CVAR the amount calculated was aggressive, with the actual Loss amount greater than the expected loss amount. This is caused by the following:

1. Small sample size with only 50 actual periods and 2 breaches of VAR.
2. Lognormal Assumption of stock prices movements were used in the VAR calculation, where actual stock prices have been empirically found to be only approximately lognormal and actually have fatter tails, thus experiencing more extreme losses.

Therefore CVAR does not appear to be an accurate risk measure for this portfolio.

Results of Different Constrained Strategies

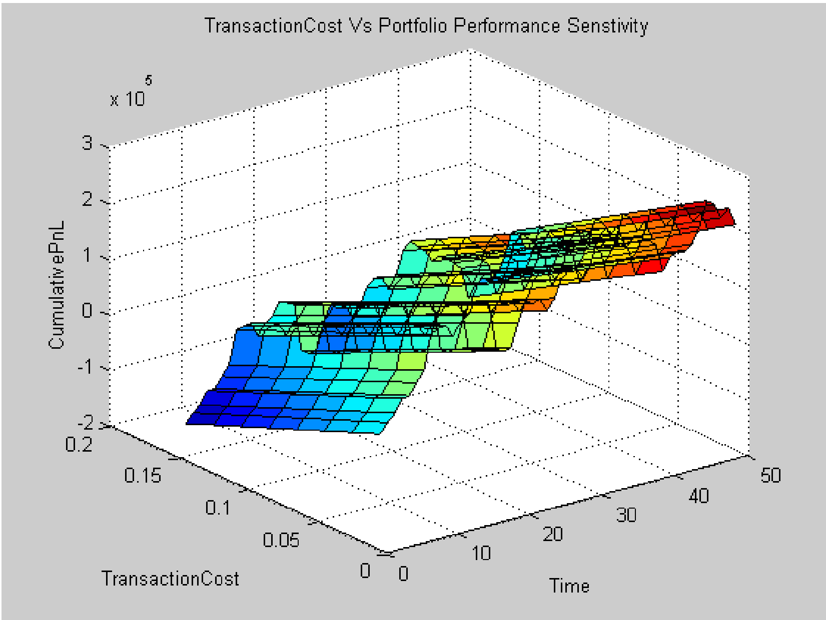
	Constrained	Short Sell	Constrained Turn
Annualized return	14.87%	10.42%	10.38%
Standard Deviation	35.04%	33.64%	33.65%
Sharpe Ratio	0.28	0.16	0.16
Portfolio PNL	316,677	217,357	216,437
Transaction Costs	13,587	15,659	15,259
VAR95	43,205	40,624	40,531
VAR99	61,508	57,877	57,837

CVAR95	54,372	51,208	51,140
CVAR99	70,477	66,313	66,160
VAR95 Breaches	1	2	2
VAR99 Breaches	1	1	1

Comparing the constrained portfolios to the unconstrained portfolio, the results that changed the most were the constrained (sector) portfolio. Constraining the portfolio to the IT sector turned out to be a good strategy with the portfolio earning a higher return of 14.87% p.a.. The portfolio was riskier at 35% standard deviation but had a superior sharpe ratio. Although the calculated VAR was higher as expected, the actual VAR breaches were lower. The reason for this is that in our sample of the 2 stocks in the IT sector, they did not experience large falls in the period.
Other notable results were:

1. Allowing short selling did not change the return or risk of the portfolio. This is because the minimum variance portfolio was always selected, and any portfolios with short selling only increased the variance.
2. Constraining turnover resulted in a lower return but lower transaction costs.

We also observed by increasing the transaction costs the cumulative PnL steadily decreased. The graph below is generated by running the script *create_3Dgraph_PNL_sens.m*.



Conclusion

As all minimum variance portfolios (subject to constraints) outperformed the index it can be concluded from our results that mean-variance portfolio analysis is a useful tool that can result in enhanced risk-adjusted returns.
In addition Value at Risk was found to be a very suitable risk measure, adequately measuring the risk of the portfolio, while our CVAR risk measures were found to be aggressive.

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