Newsvendors Tackle the Newsvendor Problem

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In 1998, the retail sales and marketing division of Time Inc., the largest publisher of consumer magazines in the US, reviewed its newsstand distribution principles and procedures for its magazines. This review affected the three major distribution decisions: the evaluation of each magazine's national print order, the wholesaler allotment procedure, and the store distribution process. For this three-echelon distribution problem, we had to adapt well-known formal solutions so that they could be implemented within the constraints of the magazine distribution channel. The revised process, referred to as Time Inc.'s Draw Management Program, has generated incremental profits in excess of $3.5 million annually.
(*Inventory/production: applications. Industries: communications/journalism.*)

In the US, the distribution channel for consumer magazines sold at retail consists of publishers, wholesalers, and retailers. Currently, several thousand consumer magazines are distributed to about 200,000 stores. These numbers, however, belie the market concentration in this industry. In many retail outlets, the leading 100 consumer magazines, by and large published by a handful of major publishing houses, account for over 80 percent of all unit sales volume. Most of the 200 magazine wholesalers servicing retailers are organized into four groups. The leading 200 retail chains account for over 80 percent of all magazine sales.

Distribution levels in the channel are negotiated by all partners. As publishers take unsold product back for a full refund of cost, they are mostly responsible for determining the national print order as well as each wholesaler's allocation. Wholesalers usually determine each store's allocation.

All channel members should be concerned with efficiently distributing magazines. Publishers, because they take unsold product back for a refund, stand to lose most from inefficient distribution. This financial exposure motivated the retail sales and marketing division of Time Inc. in 1998 to review its distribution principles and procedures. As a result, it made changes in three major areas: how it evaluated each magazine's national print order, how it allocated magazines to wholesalers, and how it recommended wholesalers distribute magazines to stores.

The year 1998 was also the 75th anniversary of *Time* magazine, Time Inc.'s flagship publication. This coincidence makes one question why it took 75 years to resolve the distribution problem. The optimal allocation, after all, is prescribed by the solutions to the

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newsvendor problem and its many cousins (Porteus 1990). Such formal solutions, however, require timely and accurate information that decision makers in the magazine retail channel often lack.

Imperfect knowledge makes allocation decisions dependent on managerial judgment. In our review of product allocation in the magazine distribution channel, we realized that a process based on judgment is inherently unstable. The boundaries between decisions that should be based on managerial judgment and those that should be based on formal reasoning blur easily, and the outcome is often a process largely driven by heuristics and unsubstantiated beliefs with little attention paid to formal accounting. Human decision makers left to their own judgment tend to make biased allocation decisions (Schweitzer and Cachon 2000). This observation was confirmed in our review.

Our review included interviews with distribution managers at Time Inc. and at wholesalers. These managers often provided conflicting reasons for their allocation decisions. Further, for titles in the Time Inc. portfolio, we reviewed inventory and sales data from wholesalers as well as stores. There we found tangible evidence of biased allocations, encountering stores that either sold out almost every issue of a magazine and stores that consistently sold only a small fraction of their allocations.

An organization that wants to draw on research in inventory management, thus benefiting from formal allocation methods, must through periodic reviews exercise constant vigilance and commit to such methods at all levels of the organization. After the initial review, which was performed with the assistance of an outside consultant, Time Inc., under the guidance of the chief operating officer of its magazine retail division, assembled a group of experienced retail managers and dedicated research analysts who were charged with developing distribution guidelines and monitoring the implementation of these guidelines for Time Inc. magazines.

The Problem

The allocation problem has a managerial side and an analytical side. The managerial concerns stem from the business realities in the channel and the associated limitations to optimally allocate product in a formal manner. To consider such limitations in a formal analysis of distribution, one needs to account for the managerial concerns and constraints by looking for optimal or near-optimal solutions within the set of feasible allocation alternatives.

For each issue of a magazine, the allocation decision breaks down into three components that together result in an allocation for each store that sells magazines. The first is determining the national print order $\mathbf{D}$, the total number of copies printed and shipped. Because the cost and revenue structure of each member in the channel is quite complicated, this decision does not lend itself easily to formal optimization, and it is based on managerial judgment as much as on formal economic reasoning. This decision is critically dependent on ongoing and successful negotiations among all channel members to determine a print order they all accept and that each member actually handles and processes. Such negotiations typically take place between publishers and wholesalers on one side and between wholesalers and retailers on the other side.

Once all agree on $\mathbf{D}$, the publisher decides on the allocations $D_1, \ldots, D_N$ for the N wholesalers who must agree to distribute their allotments. The publisher's allocation task is complicated by timing and information constraints. Publishers, in general, lack timely access to store-level data, but they have to make wholesale allocation decisions three to four weeks before a magazine's issue goes on sale.

Each wholesaler needs to distribute its allotment D_j to the n_j stores it services; that is, it needs to determine the store allocations $d_{i,j}$, $i = 1, \ldots, n_j$ to retailers who must consent to accept and display those allocations. Making these decisions is sometimes challenging because the decision window is narrow, only two to three days before an issue goes on sale.

The outcome of this three-stage process is an allocation for each of the $\sum_j n_j$ stores that sell the magazine:

$$A_{\mathbf{D}} = (d_{1,1}, d_{1,2}, \ldots, d_{2,1}, d_{2,2}, \ldots, d_{N,1}, d_{N,2}, \ldots).$$

With an agreed-on national print order $\mathbf{D}$, the overall objective is to find the allocation that maximizes expected overall sales and hence profits. The publisher clearly identifies with this objective, and its channel partners' objectives are easily aligned with it. The leading groups of wholesalers share the publisher's interest in spreading a given print order optimally across different wholesalers. Also, the typical retailer operates several hundred stores, sharing the wholesaler's and publisher's interests in spreading print orders optimally across different stores.

Determining an optimal allocation is difficult because demand at the store level is uncertain. This demand uncertainty is captured by probability distributions of demand that figure prominently in the development and analysis of optimal-allocation algorithms. Denote by $f_{i,j}(k)$ the probability that demand for an issue for *Store i* serviced by *Wholesaler j* equals k copies, and denote the corresponding cumulative probability distribution function by $F_{i,j}(k)$. With a product allocation of $d_{i,j}$ copies, expected sales in this store equal

$$E(s_{i,j} \mid d_{i,j}) = \sum_{k=0}^{d_{i,j}-1} k f_{i,j}(k) + d_{i,j}(1 - F_{i,j}(d_{i,j}-1)), \quad \text{(1a)}$$

and, correspondingly, for an allocation $A_{\mathbf{D}}$, expected sales for the issue in question and all the stores distributing it are

$$ES(A_{\mathbf{D}}) = \sum_i \sum_j E(s_{i,j} \mid d_{i,j}). \quad \text{(1b)}$$

The process of finding the allocation $A_{\mathbf{D}}$ that maximizes $ES(A_{\mathbf{D}})$ has three parts.

The first task is to estimate store-level probability distributions of demand, $f_{i,j}$, for all the stores selling the magazine. We performed a detailed analysis of historical sales data made available by a major wholesaler group for the Time Inc. title portfolio, and we identified a flexible class of parametric probability distributions that provide a parsimonious summary of store-level demand variation. We also found a simple way of coping with the problem of estimating demand. The problem stems from the fact that, in general, we observe sales as the lesser of demand and the amount of product distributed; in instances of sellout, the unobserved demand may be larger than observed sales (Appendix).

The second task is to find a process that generates the best possible $A_{\mathbf{D}}$ that can actually be implemented. In inventory management, this problem is called a constrained multiechelon problem (Silver et al. 1998) for which optimal solutions under idealized conditions are readily available. Idealized conditions do not prevail in magazine distribution. For example, the principal decision makers for the first allocation decision, the publishers, have incomplete information. Further, the principal decision makers for the second allocation decision, the wholesalers, generally have IT systems that do not permit a direct implementation of formal optimal solutions.

The third task is to choose among competing feasible allocation procedures, essentially by estimating and evaluating expected sales under different procedures.

Allocation Procedure

The top-down sequence of allocation decisions determining a national print order and allocating it to wholesalers and then to retailers is dictated by timing constraints and channel arrangements. The analysis, on the other hand, is easily described from the bottom up.

Store-Level Allocation

The algorithm for allocating product across stores served that maximizes unit sales has a well-known form. Assume that a particular wholesaler's allotment for a given issue of a magazine has been determined to be D_j. If the wholesaler serves n_j stores, we will denote the draw allocation that maximizes expected sales by $d_{1,j}, d_{2,j}, \ldots, d_{nj,j}$. Assume that the cumulative distribution of demand $F_{i,j}$ can be modeled in a parametric family; that is, $F_{i,j}(k) = F(k \mid \boldsymbol{\mu}_{i,j})$, where $\boldsymbol{\mu}_{i,j}$ is a vector of store specific parameters. Then, the optimal allocation for *Store i* is

$$d_{i,j} = F^{-1}(\lambda \mid \boldsymbol{\mu}_{i,j}), \quad \text{(2)}$$

where λ has to be chosen such that $\sum_i d_{i,j} = D_j$. The sellout probability for each of the stores served is $1 - \lambda$.

The practical application of this formula depends on two critical steps. The first is analytical and requires store-level data on sales or, preferably, on demand. With such data and careful statistical analysis, an analyst can determine a class of suitable probability distributions and estimate the vector of parameters $\boldsymbol{\mu}_{i,j}$ for each store. The second step is to implement the allocation formula in wholesalers' product-allocation systems. If a system does not permit an exact implementation of Formula (2), one must determine the best possible approximation to (2) that can be implemented.

For magazines, sales data are readily available. The practice of returning unsold product for refund has over time resulted in extensive store-level databases that wholesalers maintain to track the associated financial transactions. In particular, for each of the stores that they supply, wholesalers record the amount of product shipped (in industry parlance: *draw*), the amount returned (returns) and, by calculating the difference between draw and returns, the amount sold (net sales). By collecting these data, wholesalers establish at the store level a sales history for each magazine that they can use to make future product allocations.

Using statistical analysis (Appendix) we found the following: At the store level, the distribution of demand is near normal or Gaussian on a power-transformed scale. It depends on only one parameter, the location parameter. Estimating the location parameter of demand by the sample median of sale resolves the problem that sale, in general, is less than demand. For making future allocations, using sales data from the most recent four months usually yields the best results.

These empirical findings, in conjunction with Formula (2), have an immediate implication: the optimal draw level is a function of the median of demand (Figure 1a). An actual implementation of the resulting allocation algorithm would require a wholesaler's store-allocation system to capture the relation defined by Formula (2). Wholesalers' systems generally lack this capability, and wholesalers are generally reluctant to change their systems.

Most wholesalers' systems determine store-level allocations using formula files. A formula file is essentially a small table of multiplication factors, consisting typically of four to eight numbers. Depending on the magnitude of a store's median sale, the system applies one of these factors to the median to determine the store's draw. In general, the larger a store's sales volume, the smaller the multiplication factor. The result-

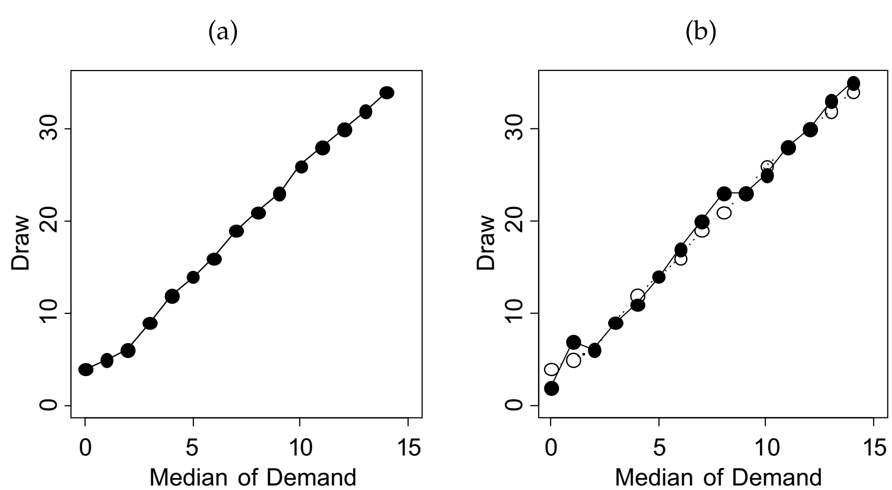

Figure 1: The left graph (Figure 1a) shows the optimal draw allocation based on Formula (2) as a function of the median of demand, which for most practical purposes is also the median of sales. The right graph (Figure 1b) shows the best approximation using a formula file superposed.

ing relation between a store's median sales level and the draw allocation is then piecewise linear, although in general this relation is nonmonotonic.

The optimal formula files are those in which the implied piecewise linear relation between median sales and draw closely matches the relation determined by Formula (2) (Appendix). The overall match between the optimal allocation and the one based on the formula file is quite good, although the lack of monotonicity of the formula-file allocation as well as the fairly pronounced discrepancy for stores with very small sales volume is worth noting (Figure 1b).

At a wholesaler, the allocation based on formula files works as follows: There is usually one formula file per magazine that is implemented across all wholesalers and that stays unchanged for at least a year, often longer. For each issue and each store served by a wholesaler the formula file is applied and generates the store's base draw. The difference between the wholesaler's base draw, the sum of its stores' base draw, and the wholesaler allotment, called *float*, is then allocated proportionately.

Wholesaler Allocation

The publisher or its designated agent typically allocates a national print order to wholesalers prior to the store-level allocation. Short-term adjustments to this production and distribution schedule are costly, and publishers are loath to make them. Publishers in general also lack easy and timely access to all of the store-level data required for a comprehensive wholesaler allocation. Hence, they need a procedure that preserves the essence of the newsvendor problem but that uses available data.

The data readily available to publishers are each wholesaler's historical draw, sales volume for each magazine issue, and the respective ratios of sale over draw. This ratio is referred to as *gross efficiency*, or efficiency for short. Related to gross efficiency, but not readily available to publishers, is *marginal efficiency*. For a magazine and *Wholesaler* j, the marginal efficiency at draw D_j, denoted by $m_j(D_j)$, is defined as the efficiency at the margin, that is, as the probability of selling the last copy of a magazine's issue shipped to that wholesaler. A wholesaler's marginal efficiency depends on the composition of the stores that it services, the distributions of demand at the stores serviced, and the rules it uses to allocate draw across stores. In terms of the respective marginal efficiencies, the algorithm for optimally allocating a national print order $\mathbf{D}$ across N wholesalers has a form similar to the prescription for store-level allocation spelled out by Formula (2). Specifically, the optimal draw allocation across the wholesalers, denoted by $D_1, D_2, \ldots, D_N$, has to satisfy $m_j(D_j) = m$, for a suitable m, and $\sum D_j = \mathbf{D}$.

For practical purposes, a wholesaler's marginal efficiency can be estimated as $m_j(D_j) = \Delta S_j/\Delta D_j$, where ΔD_j denotes the last of D_j units shipped to *Wholesaler* j and ΔS_j are the unit sales lost would these last ΔD_j units not have been shipped. One can estimate wholesalers' marginal efficiencies in market experiments by varying draw across wholesalers and over time and recording changes in sales and relating them to the changes in draw. If store-level data are available for a wholesaler, one can perform a pseudo market experiment at a fraction of the effort required for a full-fledged market experiment with similar results (Appendix).

Determining marginal efficiencies through experimentation is not a trivial undertaking because it requires either time or store-level data. Hence, it is desirable to find a more readily available proxy for a wholesaler's marginal efficiency. The relationship between the gross efficiencies of wholesalers and their corresponding marginal efficiencies is usually pronounced and monotonic (Figure 2). In view of this relationship, wholesalers that are regulated to the same gross efficiency will also have very similar marginal efficiencies. Therefore, regulating wholesalers to the same gross efficiency results in a near optimal allocation.

The practical allocation for a particular wholesaler is the outcome of a pragmatic decision process that takes into account a magazine's historical sales performance, including seasonal sales patterns, the wholesaler's store composition, expected market conditions, and factors related to a magazine's editorial offering. These considerations must be reconciled with the overall objective of regulating all wholesalers to the same efficiency. This process is not entirely formulaic but informed by market intelligence.

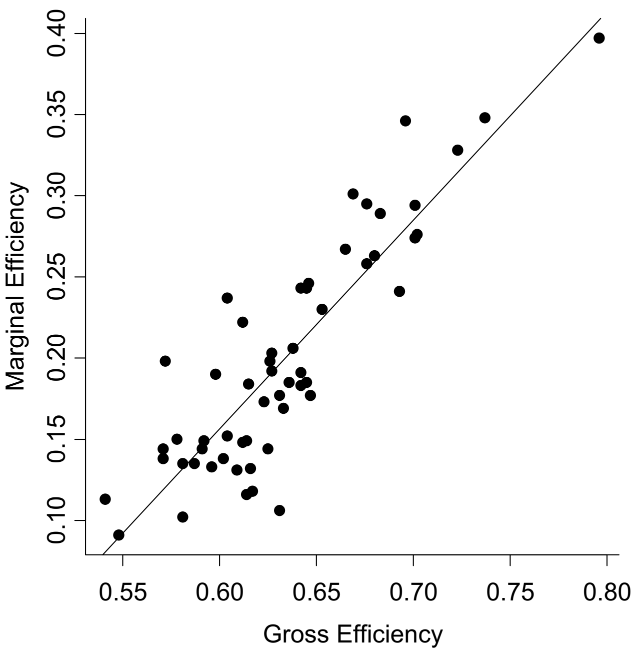

Figure 2: This relationship between gross efficiency and marginal efficiency by wholesaler is very pronounced, and it is captured well by the line resulting from an OLS fit to the data.

National Print Order

The optimal national print order is determined by the profitability of selling at the margin. Extending the concept of marginal efficiency that we introduced earlier, we denote the channel's marginal efficiency for a national print order of $\mathbf{D}$ by $M(\mathbf{D})$ and define it as the probability of selling the last copy printed and distributed. Were the marginal cost of producing and distributing one copy of the magazine, c, and the revenue per copy sold, r, unambiguously known, then the national print order $\mathbf{D}_o$ that maximizes newsstand profits has to satisfy $M(\mathbf{D}_o) = c/r$. Further, the channel partners would have to share costs and revenues to make their cost/revenue ratios equal (Pasternak 1985). Unfortunately, the concept of a known constant marginal cost and a fixed revenue figure per copy sold for each member of the channel does not accurately reflect business reality.

The wholesaler and the retailer each keep a fixed percentage of the newsstand cover price for every copy sold, and there is little ambiguity in the revenue structure. The cost structure is a different matter. The wholesaler returns unsold product for a full refund but incurs the cost of handling sold as well as unsold product. This cost increases as the product allotment increases, although not necessarily at a constant rate. Adding an extra 100 copies to the distribution may for all practical purposes add no cost; adding an extra 100,000 copies to the distribution, on the other hand, may require a costly change to the wholesaler's delivery infrastructure. The retailer's cost structure has additional layers of complexity. The retailer, too, incurs handling costs for both sold and unsold product. In addition, the retailer has inventory holding costs. Other and less tangible costs are related to loss of consumers' goodwill that may result from stockouts (Fitzsimmons 2001). Retailers recognize such costs even though they may be very hard to quantify. They are also concerned with opportunity costs; for example, putting an extra 100 copies in a store may take space they could use to sell another product more profitably. Hence, for both wholesalers and retailers, the assumed known and constant marginal unit costs are rough approximations at best.

The publishers' problem is reversed. The marginal cost per copy printed and shipped is indeed known and may be assumed to be constant; the revenue structure is complicated. Publishers receive a certain percentage of the cover price of each copy of their magazines sold. Newsstand profits, however, are not the only profit contribution; publishers also derive revenues from advertising and subscriptions. Both of these other revenue streams affect their newsstand allocation decisions.

Determining the contribution to profits of incremental advertising revenues is not easy. Advertising revenues generally increase with the number of copies sold, albeit at a decreasing rate (Koschat and Putsis 2000, 2002). The complication arises from the manner in which advertising space in magazines is sold. Publishers guarantee advertisers that a minimum number of copies of each issue will be sold; this is called a magazine's rate base. As long as a magazine's sales are below the rate base, an incremental sale of one copy is highly valued. Once the sales volume exceeds the rate base, the incremental value of an additional copy sold is small in terms of its impact on advertising revenues.

Apart from the tangible contribution of advertising revenues, less tangible benefits come with printing and distributing one copy of a magazine even though this copy might never be sold. In general, consumer magazines do very little advertising. Instead, each displayed copy of the magazine serves as its own advertisement, informing its customers—readers and advertisers—of its presence and communicating its major attributes through its cover page. Hence, a particular copy of a magazine may not be bought but may increase the probability of future purchases of the magazine. Similarly, the value of a copy sold may exceed the marginal newsstand revenue if that copy motivates its buyer to subscribe to the magazine. Such considerations make determining a revenue number difficult.

Publishers often take newsstand economics as a starting point for determining their national print order targets for retail. Again, denote by $M(\mathbf{D})$ the channel's marginal efficiency, which can be estimated in a manner similar to the estimation of wholesalers' marginal efficiencies. Denote by $\mathbf{D}_o$ the solution to the equation $M(\mathbf{D}) = c/r$, where c is the marginal cost of production and distribution, and r is the publisher's newsstand net revenue per copy sold. $\mathbf{D}_o$ would be the optimal print order level if newsstand revenues were the only concern.

In view of its other revenue sources and as the result of intense internal negotiations where newsstand, subscription, and advertising concerns are taken into account, however, the publisher aims for a national print order larger than $\mathbf{D}_o$. In this decision process, the retail marketing managers find the relation $M(\mathbf{D})$ to be an extremely valuable tool because it provides a top-level view of sales at the margin. $M(\mathbf{D})$ depends on the probability distributions of demand at the stores, the procedure wholesalers use to allocate product across stores, and the procedure publishers use to allocate product across wholesalers. The retail marketing manager can use $M(\mathbf{D})$ to evaluate the impact of different national-print-order levels on her bottom line easily, thus determining acceptable draw levels.

This use of business research results is a valuable example of how business research and managerial practice interact. Business research does not provide foregone conclusions, in this case "optimal product levels"; instead, it provides tools for managers with fiscal responsibility that help them find acceptable solutions, even optimal solutions, within the constraints of their business realities. Brody (2001) made a similar point on fruitful exchanges between business researchers and managers.

The other channel members, wholesalers and retailers, have their own reasons for establishing inventory targets. Wholesalers make inventory decisions for a multitude of titles, and retailers make stocking decisions for numerous product categories. Therefore, they undergo possibly elaborate internal negotiations also. In determining the actual national print-order, the publisher must reconcile its print-order targets with those of wholesalers and retailers. This reconciliation is part of an ongoing dialog among these three channel partners.

Implementation

The success of our initiative ultimately depends on how effectively we implement the formal solutions developed by our researchers. Even though we devised solutions for each link of the distribution channel, we depend on the cooperation of the other channel partners in the implementation. We call the implementation of these solutions in the channel the Draw Management Program. For each magazine, its retail manager is the gatekeeper of this program.

Store Allocation

The retail marketing manager's task consists of ensuring that the formula files our research group developed are implemented in wholesalers' systems and allowed to work as intended. Wholesalers traditionally used a small handful of ad hoc formula files that often differed significantly from the optimal formula files. Further, distribution managers at wholesalers often preferred to allocate magazines to stores based on their market knowledge and felt uncomfortable letting a computer algorithm perform this function.

As consolidation in the distribution channel forced wholesalers to improve their economic performance, the major wholesalers have become familiar with the theoretical underpinnings of optimal allocation and are, in principle, open to implementing store-level allocations based on the concepts we outline. Time Inc. wholesale representatives negotiate with wholesaler personnel to get them to implement the formula files that we propose. In the past, most of our formula files have been implemented.

Time Inc.'s retail marketing managers guide and monitor the implementation through written communications called field assignments. These field assignments are initiated by the marketing manager and are completed and returned to the retail marketing managers by the Time Inc. wholesale representatives. With regard to the Draw Management Program, these field assignments concern (1) the wholesaler's implementation of the formula files, and (2) the amount of draw they allocate using the formula files.

In completing the field assignments, the wholesale representatives confirm that the wholesalers have put the formula in place exactly as written. When wholesaler systems have limitations, Time Inc. researchers create appropriately modified formula files. In the process, we have compiled a history of system idiosyncrasies to facilitate future implementations.

The retail marketing managers also regularly originate the make-order-and-freeze assignment to monitor and remove unwarranted make-orders and freezes. Make-orders and freezes are draw allocations made by a person, usually a wholesale employee, rather than the system. A freeze keeps the draw at a constant level that can only be altered manually. Make-orders are short-term freezes, normally lasting for four to eight issues, after which the formula file again regulates draw. Make-orders and freezes are acceptable in some instances. For example, stores that receive a magazine for the first time have no sales history and therefore require a manual initial draw allocation.

The first make-order-and-freeze assignment distributed as part of the Draw Management Program resulted in a reduction in frozen draw of 12 to 43 percent (Table 1). This is important: A store that is frozen cannot benefit from optimal draw allocation. Overall, many stores can now benefit from optimal store-level draw allocation based on formula files. The frozen draw as a fraction of total draw, after this assignment was completed, ranged from about 20 to 30 percent, a level Time Inc. currently considers acceptable.

Magazine	Before	After	Reduction
People	33%	23%	30%
Sports Illustrated	42	24	43
Entertainment Weekly	40	31	23
Time	41	31	24
Money	25	20	20
Fortune	41	30	28
Parenting	32	26	19
InStyle	30	22	26
Life	22	17	23
Sunset	26	22	15
Teen People	25	22	12

Table 1: We compared frozen and make-ordered draw prior to executing the make-order-and-freeze protocol and after its execution. This execution resulted in a significant reduction of frozen draw.

Wholesaler Allocation

Time Inc.'s retail marketing managers also determine the allocations to wholesalers. Once the publisher decides the national print order, the retail marketing managers review historical draw and sales allocations, consider the local market knowledge provided by Time Inc. field personnel, and allocate magazines to wholesalers (Figure 3).

Because publishers must make draw decisions to meet future demand, they must anticipate market changes. Time Inc. relies on its field personnel to spot changes that could affect a wholesaler's optimal draw allocation. For example, frequent changes that can be known in advance occur in the number of stores served by a wholesaler. Editorial decisions may also affect demand for a wholesaler. For example, the cover subject of an upcoming issue may be of special interest in the wholesaler's market. In such cases, the publisher must modify the wholesaler's allocation.

National Print Order

The number of copies of a magazine that a publisher prints for the retail market is the subject of great inter-

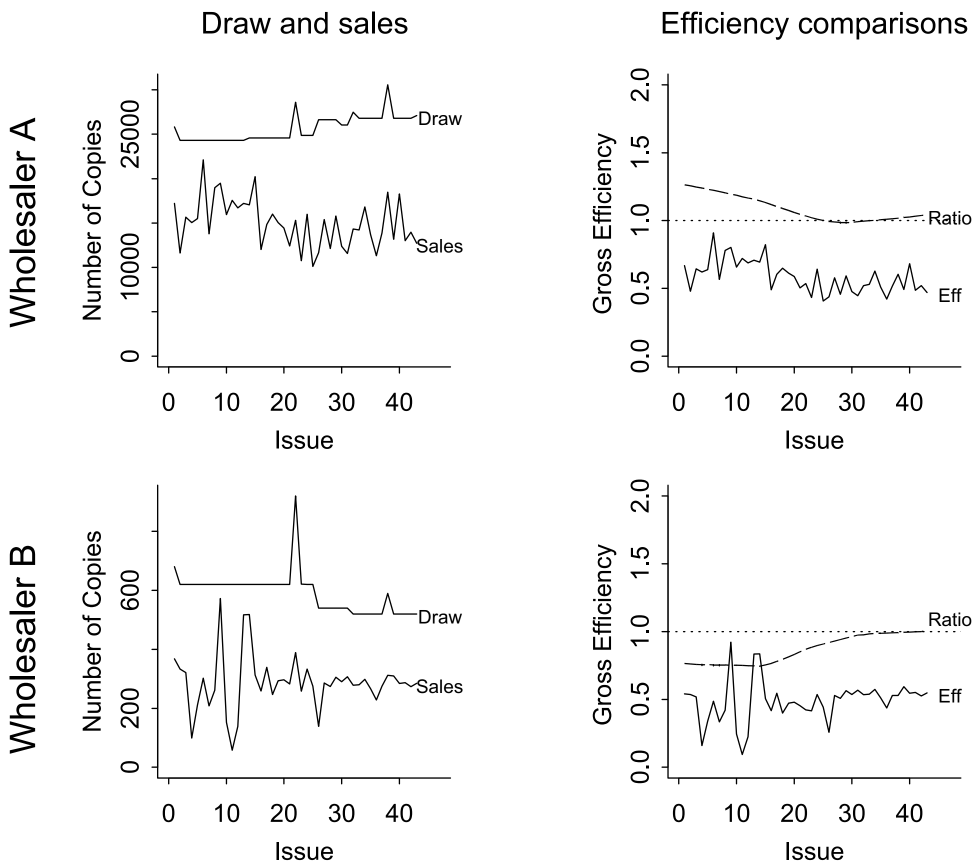

Figure 3: These graphs illustrate how historical sales performance enters into consideration by comparing draw and sales for *People* across two different wholesalers. The graphs on the right plot the wholesaler's efficiency as well as a smoothed plot of the ratio of the wholesaler's efficiency to national efficiency (estimated using LOWESS, Cleveland 1979). In the middle of this 43-issue time period, we reviewed wholesaler allocations. Prior to the review, Wholesaler A consistently had a gross efficiency above the national average while Wholesaler B consistently fell below the national average. While Wholesaler A received too much product, Wholesaler B received too little. We corrected both wholesalers' allocations. The extent to which the smoothed lines move toward 1.0 indicates the success of this reallocation.

nal debate. Editors often seek to maximize this number, because their compensation is usually based on absolute not marginal newsstand sales. Publishers are judged on financial results, and thus the profitability of a marginal sale is important to them, but not at the expense of ruining their working relationships with them.

After the publisher resolves its internal debates, it must obtain wholesalers' and retailers' consent to the proposed print-order levels. The retail marketing manager must mediate among all parties—the editor, the publisher, the wholesaler, and the retailer—and this role in a system of such diverse interests is far from easy.

Impact

The Draw Management Program has given our organization a clear understanding and acceptance of the role of formal reasoning and managerial judgment. The formal reasoning is largely driven by careful analysis of demand patterns at the various levels of the distribution channel. The objective of this analysis is to capture the uncertainty in future demand by estimating marginal efficiencies at the national level and at the wholesaler level and by estimating probability distributions of demand at the store level. These estimates can guide all channel members in making proper allocations and help them to determine the economic impact of specific allocations.

The availability of store-level demand distributions lets us calculate expected sales for different allocations. We evaluated a particular allocation by applying Formula (1) to the estimated demand distributions from a statistical sample of stores and a set of consecutive issues of a given magazine. Once we had chosen the stores and issues, we determined each store's and each issue's draw as prescribed by the procedure we describe, estimated the parameter that governs a store's probability distribution of demand, and calculated expected sales for each store and issue. We evaluated the allocation procedure by considering the sum of expected sales across all stores and issues.

There are three store-level draw-allocation procedures that we have evaluated. The first procedure, referred to as the ad hoc procedure, is the process that was in place prior to the implementation of the Draw Management Program. It is a loose amalgam of heuristics and opinions that varied from case to case. There is no need to describe these in any detail because what matters for our purpose is the actual allocation resulting from this process. We acquired historical examples of such allocations when we acquired the store-level sales data used in our demand analysis.

The second procedure is the formula file procedure. We assumed that the national print order and each wholesaler's allocation (and their equivalents in our analytical universe) were the same as in the ad hoc procedure. For each magazine, we used the best formula file to determine the allocation. Specifically, for each issue, we calculated a historical sales average, applied the formula file, and allocated each wholesaler's float.

In the third procedure, we combined the formula file and the wholesaler reallocation procedure. Again, we assumed that the national print order and its equivalent in our analytical universe were the same as in the ad hoc procedure. However, we also optimized wholesaler allocation by allocating wholesalers to a common efficiency in addition to using formula files to optimize store-level allocation.

The second and third procedures yielded greater expected sales than the ad hoc procedure (Table 2). Our evaluation of the second procedure captured the effect of using optimal formula files and removing freezes. In practice, not all freezes can be removed, and the numbers reported may overstate the sales lift by up to 30 percent. We can infer the effect of optimizing wholesaler allocations alone by taking the difference in the sales stimulated by the second procedure and the third procedure.

The median sales lift is 1.05 percent. This number is somewhat misleading because the titles that show the greatest increase in sales are also those with the greatest circulation. Hence, the overall sales stimulation effect is solidly above one percent; about half of the increase comes from improved wholesaler allocation and the remaining half is from improved store allocation. While the improvement seems small, it comes simply from improving allocation and without any additional production or distribution costs. Hence, the incremental revenue from the added sales goes straight to the bottom line, resulting in a handsome profit to Time Inc.

The increased profits are the result of careful statistical analysis, judicious optimization that can be implemented, and, most important, the institution's

Magazine	Percent Sales Lift	Due to Improved Formula File	Due to Improved Wholesaler Allocation
People	1.38%	0.58%	0.80%
Sports Illustrated	1.26	0.56	0.70
Entertainment Weekly	1.22	0.62	0.60
Time	1.10	0.41	0.40
Money	1.05	0.79	0.26
Fortune	0.94	0.30	0.64
Parenting	0.82	0.26	0.56
InStyle	0.73	0.44	0.29
Life	0.69	0.39	0.30
Sunset	0.49	0.26	0.23
Teen People	0.47	0.30	0.17

Table 2: We estimated sales lift from using improved formula files and from improving wholesaler allocations.

commitment to improvement. The importance of this commitment at all levels of Time Inc. cannot be overemphasized. The commitment we needed was not a commitment to save a dollar here or there. It was a commitment to rational decision making, to discussion, to debate, and to reason, and most important, a commitment to staff with experts who are respected by diverse constituencies, internal and external. At Time Inc., our process has yielded more than \$3.5 million in incremental annual profits.

Appendix
Estimating Demand Distribution

Denote the cumulative probability distribution of sales for a particular magazine and for *Store i* by $G_i(k), k = 0, 1, 2, 3, \ldots,$ and denote the corresponding distribution of demand by $F_i(k)$. If the draw for *Store i* is d_i, the relation between G_i and F_i is as follows: $G_i(k) = F_i(k)$ for $0 \le k \le d_i$; also, $G_i(k) = 1$ for $k \ge d_i$. Note that if the sellout probability, which is found as $1 - F_i(d_i - 1)$, is less than 0.5, then $G_i^{-1}(0.5) = F_i^{-1}(0.5)$; that is, the median of the demand distribution equals the median of the sales distribution. Generally, the larger d_i and hence the smaller the sellout probability, the closer the two distributions G_i and F_i are. For very small sellout probabilities $F_i \approx G_i$.

From an analytical point of view, inefficient distribution can have benefits. In particular, if stores receive too much product and the sellout probability is small, then for all practical purposes, the probability distribution of sales equals the distribution of demand. This was the case for several thousand of the stores in our analytical database, which allowed us to perform a comprehensive analysis of the shape of the demand distribution. The two major insights are the following: (1) For the demand distribution, a simple parametric model generally provides an excellent fit. A power transformation (also known as a Box-Cox transformation (Hinkley and Runger 1984)) transforms the distribution of demand from a skewed distribution to a distribution that is well approximated by a normal or Gaussian distribution. The exponent in this transformation typically varies from 0.25 (equivalent to a fourth root transformation) to 0.33 (equivalent to a third root transformation). (2) The demand distribution, on the transformed scale, is characterized by a location and a scale parameter. The location parameter changes with a store's volume of demand. With regard to the scale parameter, we found that the variation in demand is a function of expected demand; that is, two stores that have the same expected demand also have, for all practical purposes, the same level of variation.

The distribution of demand can therefore be modeled as

$$F_i(k) = F(k \mid \mu_i) = \Phi((k^\alpha - \mu_i^\alpha)/s(\mu_i)). \qquad \text{(A1)}$$

Here, Φ is the cumulative distribution function of a standard normal distribution; α and the function $s(\cdot)$ are magazine specific and must be estimated for each magazine. For a given store, this distribution then depends on a single parameter, namely μ_i, which is also the median of the demand distribution. Again, if the sellout probability is small, the median of the demand distribution equals that of the sales distribution, and therefore, sales data can be used to estimate the parameter of the demand distribution.

The practical problem is to allocate draw for a future issue, which requires predicting the median demand for that issue. If the distribution of demand for a particular magazine in a particular store is stationary, then we should use all of the magazine's sales history in the store to estimate the demand distribution. If this distribution changes over time, we need to strike a trade-off between bias and variance by considering the mean-square error of the prediction. The mean-square error is a function of the number of consecutive issues we use in making the prediction (Figure 4). The mean-square error is highest when we use only one issue, and it drops sharply as the number of issues we use increases. It reaches its minimum for four issues before slightly increasing. Across numerous magazines we found the optimal number of issues to span a time period of about four months, which for weekly titles corresponds to 17 issues and for monthly titles corresponds to four issues.

Developing Formula Files

The optimal draw allocation for *Store i* is given by $d(\mu_i) = F^{-1}(\lambda \mid \mu_i)$, where $1 - \lambda$ equals each store's

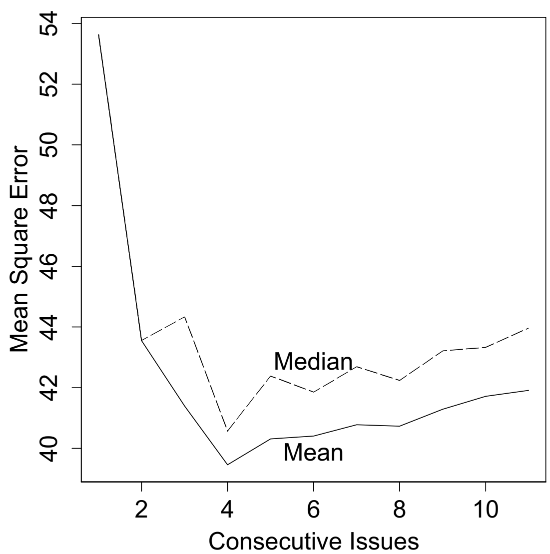

Figure 4: This graph is based on two years' worth of data from 1,200 stores for *InStyle* magazine (23 issues). We calculated the mean square error as the sum of squares of the difference between the historical mean (median) and sales across the 1,200 stores and the 14 final issues for consecutive issues ranging from one issue to 11. The sample mean is a slightly better predictor than the sample median. Sample means and sample medians calculated from four consecutive issues produce the best prediction.

probability of selling out. Under optimal allocation, $1-\lambda$ also equals each wholesaler's marginal efficiency. Gross efficiency, marginal efficiency, and sellout probability are closely related. Hence, if the channel partners agree to aim for a particular gross efficiency, this target can easily be translated into targets for marginal efficiency and sellout probability targets.

Denote a store's historical sales median by m and the optimal allocation by $d(m) = F^{-1}(\lambda \mid m)$. Denote the allocation defined by a formula file by $l(m \mid \mathbf{b}, \mathbf{r})$; here $\mathbf{b} = (b_0, b_1, \ldots, b_8)$ is the vector of integer valued bucket boundaries such that $0 = b_0 \leq b_1 \leq \cdots \leq b_8 = \infty$, and $\mathbf{r} = (r_1, \ldots, r_8)$ is the vector of multiplication factors corresponding to the eight buckets. For $b_{i-1} \leq m < b_i$, $l(m \mid \mathbf{b}, \mathbf{r}) = r_i m$.

To determine the optimal formula files, we consider a sample of stores—with historical sales medians m_i—that reflect the retail environment we operate in, and we define the objector function $T(\mathbf{b}, \mathbf{r})$ as the weighted average of the absolute differences between the optimal draw allocation and the allocation defined by $l(\cdot \mid \mathbf{b}, \mathbf{r})$, that is,

$$T(\mathbf{b}, \mathbf{r}) = \sum_{m_i} \left| l(m_i \mid \mathbf{b}, \mathbf{r}) - d(m_i) \right| / m_i.$$

We want to determine the formula file, $\mathbf{b}$ and $\mathbf{r}$, that minimizes the objector function. Note that

$$\begin{aligned} T(\mathbf{b}, \mathbf{r}) &= \sum_j \sum_{b_{j-1} \leq m_i < b_j} \mid l(m_i \mid \mathbf{b}, \mathbf{r}) - d(m_i) \mid / m_i \\ &= \sum_j \sum_{b_{j-1} \leq m_i < b_j} \mid r_j - d(m_i)/m_i \mid . \end{aligned}$$

It is well known that, for a given $\mathbf{b}$, each minimizing r_j is the median of the $d(m_i)/m_i$ corresponding to the stores that fall into the jth bucket. Hence, it remains to determine the bucket boundaries that minimize $T(\mathbf{b}, \mathbf{r})$. Because the bucket boundaries have to be integer valued, this is a fairly straightforward numerical exercise. We omit the details.

Estimating Marginal Efficiencies

For *Wholesaler j*, marginal efficiency is defined as the probability of selling the last copy shipped. This probability can be estimated as $m_j = \Delta S_j / \Delta D_j$, where ΔD_j denotes the last of D_j units shipped to *Wholesaler j* and ΔS_j are the unit sales that would not have been made had these last ΔD_j units not been shipped. We can easily estimate the marginal efficiency for wholesalers for whom we know store-level draw and sales data for enough issues by taking the following steps: (1) Denote the total draw across the set of issues considered for *Wholesaler j* by D_j and consider a small draw reduction of ΔD_j that may amount to five to 10 percent of D_j. (2) Allocate the draw reduction ΔD_j across the stores served by the wholesaler in a manner consistent with that wholesaler's practice, that is, allocate a negative float of ΔD_j. (3) For each store and each issue, compare the (hypothetically) reduced draw with actual sales. If actual sales were below the reduced draw, the store would have lost no sales. If actual sales were above the reduced draw, the difference between the two are sales the store would have lost. (4) Add the sales losses across stores and across issues and denote this sum by ΔS_j. ΔS_j is the sales the wholesaler would have lost had its draw been reduced by ΔD_j. (5) Estimate the wholesaler's marginal efficiency as $m_j = \Delta S_j / \Delta D_j$.

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Diana DeFrate, Vice President of Finance and Strategic Planning, Time Distribution Services Inc., Time & Life Building, Rockefeller Center, New York, New York 10020-1393, writes: "This letter is to confirm that the Time Inc. Draw Management Program described in 'Newsvendors Tackle the Newsvendor Problem' has resulted in budgeted annual savings in excess of \$3.5 million to Time Inc."