An Asset and Liability Management System for Towers Perrin-Tillinghast

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Towers Perrin-Tillinghast employs a stochastic asset-and-liability management system for helping its pension plan and insurance clients understand the risks and opportunities related to capital market investments and other major decisions. The system has three major components: (1) a stochastic scenario generator (CAP:Link); (2) a nonlinear optimization simulation model (OPT:Link); and (3) a flexible liability- and financial-reporting module (FIN:Link). Each part improves over existing technology as compared with traditional actuarial approaches. The integrated investment system links asset risks to liabilities so that company goals are best achieved. For example, US WEST saved $450 to $1,000 million in opportunity costs in its pension plan by following the advice of the asset-and-liability system.

Towers Perrin is the largest employer of actuaries in the world (850 actuaries) with over 8,000 total employees, offices in 69 countries, and well over $1 billion in annual revenues. The company provides information and consulting services concerning benefits to 10,000 national and international clients, including 379 of the world's largest companies, and 799 of the Fortune 1,000 companies. A primary emphasis is pension plans. In addition, the Tillinghast subsidiary supports the insurance industry throughout the world. The Tillinghast actuarial system

(TAS) is used by over 40 percent of life-insurance companies in the US.

Towers Perrin-Tillinghast employs a suite of stochastic financial-planning models to provide guidance to managers of pension plans and insurance companies. The system simulates asset-and-liability decisions across a long-term, multiperiod planning horizon—typically five to 15 years. The system has been under development since 1991 and is now employed in over 16 countries, mostly European and English-speaking countries. It is the most extensive global investment system for actuarial studies; its distinguishing features result from a collaborative eight-year effort between an operations researcher (Mulvey) and Towers Perrin's worldwide consulting staff.

The asset-and-liability management (ALM) system applies to two major domains: (1) designing and managing pension plans, and (2) integrated financial risk management for insurance companies. In both cases, the system simulates an asset policy with liability decisions to maximize the company's wealth or pension-plan surplus, while maintaining a safe level of operations. The optimization module selects the best dynamic policies for the insurance company or the pension plan.

For defined-benefit pension plans, the primary goals are to design the plan and to manage the pension surplus so that the company will meet its obligations to its employees, to minimize the company's contribution over time, and to maximize the growth in the plan's surplus.

For insurance companies, the three primary goals are to help the insurance companies to provide insurance products at a fair price to individuals and corporations, to generate reasonable profit for shareholders (or dividends for mutual companies), and to maintain capital adequacy and risk management so that the company remains solvent in the long run.

Evaluating pension plans and insurance companies is complicated because of the long time horizons, the number of parties with diverse interests who are affected, and extensive government regulations. In most developed countries, regulations require an annual actuarial analysis. Actuaries evaluate the soundness of a pension plan or an insurance company and can require cash contributions from the company for a pension plan in deficit or from stockholders or outside investors for an undercapitalized insurance company. Such contributions (or their absence) can have a major impact on a company's profits or losses. For example, several aerospace companies have improved their profits by 40 percent over the past five years through improved pension management. The actuarial profession closely monitors its members and certifies them through periodic tests, educational requirements, and other means.

The Towers Perrin stochastic investment system is employed widely. Several hundred large US and international companies run the system for pension plans, approximately 40 percent of US life insurance companies use the related Tillinghast TAS system, and Tillinghast has a major presence in property-casualty insurance. Multinational companies, such as Unilever and IBM-Europe, employ the global system for modeling their worldwide pension plans on a consistent basis.

In this paper, we discuss the nature of pension planning and integrated financial planning. We focus on pension-plan applications in order to limit the discussion. Mulvey, Correnti, and Lummis [1997] discuss a related model in the insurance industry. Insurance models possess a structure similar to pension plans, but they include many additional variables and constraints due to the complexity of integrating a large financial organization.

Modeling Framework

The Towers Perrin investment system simulates asset-allocation policy in conjunction with liability decisions over a multiyear planning horizon. Integrating assets and liability decisions helps the company achieve its goals. The system has three major elements: a stochastic scenario generator, a decision-rule or policy simulator, and an optimization module (Figure 1). The first two elements form the corporate (or pension-plan) simulation module and are deployed before the optimization algorithm searches for the best compromise policy given the relevant business, policy, and regulatory constraints. In effect, the optimization runs the simulation by identifying decision strategies that best fit the proposed objective function over a multiperiod planning horizon. For example, we will make recommendations about the optimal dynamic investment rules for stocks, bonds, cash, and pension contribution over differing economic environments.

Three features distinguish the Towers Perrin approach, as compared with such alternatives as the Frank Russell system [Cariño et al. 1994]: (1) its reliance on a core set of structural economic factors for
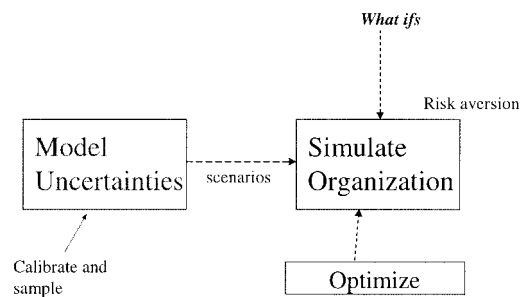
Figure 1: The three elements of asset-and-liability modeling must be consistently linked to a set of core economic factors. The stochastic model determines values for the economic factors prior to analyzing the decision policies.

driving both asset returns and liability movements, (2) a set of policy rules that underlie the decision-making processes, and (3) a full actuarial analysis of pension design and cash-flow liabilities for each economic scenario.

The core economic factors link assets and liabilities. For example, interest-rate changes affect asset returns and the present value of liability cash flows. We use a set of structural stochastic differential equations to model the economic factors. While this approach complicates calibrating the model parameters [Mulvey, Morin, and Pauling 1999], we believe that a structural model depicts the relationships between assets and liabilities better than simpler methods, such as those relying on a covariance or vector autocorrelation matrix for asset returns.

We apply a set of policy rules. The model recommendations are seen by pension-plan administrators, beneficiaries, employees, regulators, and other interested parties. Policy rules provide intuition regarding the recommendations and allow for extensive sensitivity analysis.

Pension plans have grown in complexity. There are many choices in plan design. For example, traditional defined-benefit and defined-contribution plans are now supplemented by cash-balance and related hybrid plans. A cash-balance plan is a portable pension (meaning that employees can transfer their balance to another company) that accumulates each year by a credited amount. Each employee's plan gains in value by a certain percentage of salary plus a credit for money already in the plan. The company sets its own asset allocation but is required to fund the cash balances of each employee upon termination. The credited amounts can be constrained by caps and floors (minimum and maximum). Actuaries are particularly suited to assisting companies to choose a benefit program that fits its needs.

ALM System Variables and Equations

The ALM investment system consists of $t = \{0,1,2,3,\ldots,T\}$ time stages. The first stage represents the current date. The end of the planning period, T, is called the planning horizon. Typically, for a pension plan, we aim for a long-term horizon—five to 15 years in the future with quarterly or annual time steps. We generate a set of $s = \{1,2,3,\ldots,S\}$ scenarios for the future course of the economy, the capital markets, and other uncertainties. Stochastic scenarios can be depicted by means of a scenario tree diagram (Figure 2). In such a tree, at each node, the system renders decisions regarding the asset mix, the amount of payments to pension beneficiaries, and possible cash contribution. A complete path in the scenario tree equates with a single scenario $s \in \{S\}$ (Appendix).

At the beginning of each period and for
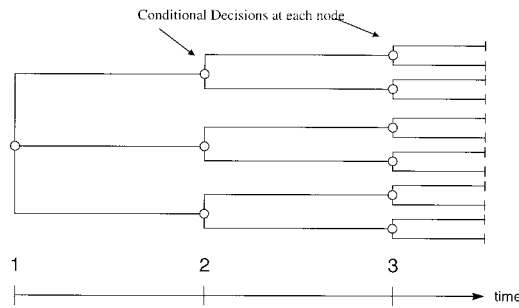
Figure 2: Within a scenario tree for modeling uncertainties, the decisions are made at each node of the tree depending upon current information.

each scenario, the investment system renders policy decisions regarding the asset mix, liability values, and establishing and achieving goals. Between time steps, uncertainties take over. For example, interest rates might increase because of an increase in inflation, and the bond and stock markets might react in a negative manner. We use a system of stochastic differential equations to model all stochastic parameters over time. These relate a set of key economic factors to the other components, such as asset-and-liability returns [Mulvey 1996, and Mulvey and Thorlacius 1998].

For a pension plan, the primary decision variables designate asset proportions, liability-related decisions, and pension payments:

$x_{j,t}^s$: investment in asset j, time t, scenario s.

$y_{k,t}^s$: liability payment k, time t, scenario s.

$u_{l,t}^s$: contribution l, time t, scenario s.

The pension-plan surplus is calculated in two ways. For determining regulatory contribution, we assume that current and retired employees define the target population. For this group, we calculate the amount of money needed to satisfy a legal definition of a pension surplus. Cash flows

are calculated by actuaries who estimate the life spans of employees, their salaries at retirement dates, and myriad other factors. We omit these details. This approach goes by the title—accumulated benefit obligations (ABO). A second approach for determining a surplus is to assume that the company will continue to exist over time, adding new employees while others resign, retire, and so forth. This method requires additional assumptions regarding the company's future; the resulting surplus is determined by the projected benefit obligations (PBO) method. These calculations provide a more robust definition of the company's ability to meet its future goals. However, the ABO is appropriate for determining company contribution.

In either case, the estimated cash flows (for future pensioners) are discounted at an appropriate rate to derive the plan's surplus: The surplus is the market value of assets minus the market value of liability cash flows. For most assets, we can readily discover market value, say, by checking the current price in a newspaper. However, for pension liabilities and private market assets, we must estimate the fair value. Typically, this requires discounting cash flows by an appropriate discount rate. For pension benefits, typical US regulations require discounting at an investment-grade corporate bond rate. Thus, interest rates are critical; the full yield curve must be modeled in a consistent fashion (Appendix).

At each time period, t, the ALM model maximizes an objective function, $f(x)$, by moving money between asset categories, making distributions to the beneficiaries, and contributing cash to the pension plan.

There are numerous candidates for the objective function. In addition, we impose constraints on the process, for example, by limiting the stock-asset ratio, addressing transactions costs whenever assets are bought or sold, and taking advantage of investment opportunities. Our goal is to find a feasible decision policy that maximizes a temporal objective function. Since we are dealing with temporal uncertainty, the optimal solution, like all feasible solutions, encompasses a set of paths or trajectories for the pension-plan surplus across each of the scenarios. To give an idea of the model's structure, we list two basic equations for the flow of funds: Equation (1) for the jth asset category:

$$x_{j,t+1} = (x_{j,t}^s \times r_{j,t}^s) - p_{j,t}^s$$
$$+ q_{j,t}^s (1 - t_j) \tag{1}$$

for asset j, time t, scenario s,
where $r_{j,t}^s$ = return for asset j, time t,
   scenario s,
$p_{j,t}^s$ = *sales of asset* j, time t,
   scenario s,
$q_{j,t}^s$ = purchase of asset j, time t,
   scenario s,
 t^j = transaction costs for asset j, and
Equation (2) for the cash flows:

$$x_{l,t+1} = (x_{l,t}^s \times r_{l,s}^s) - \sum_j q_{j,t}^s$$
$$+ \sum_j p_{j,t}^s (1 - t_j) - \sum_k y_{k,t}^s$$
$$+ \sum_l u_{l,t}^s. \tag{2}$$

The model avoids looking into the future in an inappropriate fashion. To prevent this, we add special constraints called nonanticipatory conditions. The general form of the constraints is $x_{j,t}^{s1} = x_{j,t}^{s2}$ for

all scenarios $s1$ and $s2$ that inherent a common past up to time t. In words, scenarios that share a common path will have non-anticipatory constraints for variables occurring on the shared path. Any financial-planning system must address these conditions, either explicitly or implicitly. Special purpose algorithms are available for solving the resulting convex or non-convex optimization model [Ziemba and Mulvey 1998].

Pension-Planning Objectives

A major element of asset-and-liability management is to trade off risks and rewards at various temporal junctures. It is natural to expect that investments possessing volatility will generate greater expected returns over time than assets with lower levels of volatility. The temporal issue complicates the decision process since long-term horizons provide a cushion to recoup losses; thus volatile assets may be, in fact, safer in terms of contextual risks than less volatile assets. An example is the stock-cash comparison: stocks provide higher expected returns but are more volatile than cash. We must consider the time horizon when measuring contextual risks, that is, the chances of meeting the investor's goals. An integrated ALM system provides an ideal method for understanding the temporal risk-reward trade-offs and for evaluating the probability of meeting company goals.

There are many metrics for financial risks, just as there are alternative measures of profitability or return. For an insurance company, we might consider the chance of a loss over the next year. Or we might set a profitability target and evaluate the probability of missing the target. In both cases, risk increases as a function of probability. An improved alternative for evaluating risks for an insurance company is to estimate the entire probability distribution of shareholders equity at each time period, along with other measures of financial well-being. In the case of a pension plan, we measure the discounted contribution and the plan's surplus at the end of the planning horizon.

Typically, we equate reward with expected value. We have particular interest in the pension plan's surplus at the horizon:

$$Expected\ surplus\ =\ \sum_{s \in S} p_s * z^s,$$

where p_s is the probability of scenario s, z^s is the pension surplus under scenario s, time T, and S is a set of representative scenarios. We are interested in risk at the horizon and include a number of alternative measures, including semi-variance, downside risks, and surplus variance. In addition, we can show the trajectory paths for the pension plan under any of the generated scenarios.

Policy Rules

As a distinguishing feature, the Towers Perrin system establishes a set of policy rules for managing investments, liability payments, and pension contributions. We define a policy rule as a specified formula or set of rules for all the decisions—as a function of the state of the world—at each time period and scenario. As a simple example, we might put a fixed proportion (say 70 percent) of the pension *surplus* into an S&P500 index fund with the remaining assets assigned to a fixed-income bond portfolio that mimics the pension liabili-

ties. Thus at each time period, the system must calculate pension surplus based on discounted cash-flow projections, determine the excess of assets over market value of liabilities, and rebalance the investment portfolio to achieve the desired target ratios. We call this particular policy rule fixed proportional surplus (FPS). Numerous alternative policy rules are available for testing.

The optimization module compares policy rules and determines the best combination of rules and parameter settings. For example, in the FPS rule the parameter defining equity proportion will be found via optimization. Policy optimization provides a natural approach for analyzing long-term investment problems. The rules can be as complex as the investor likes. Policy rules satisfy the nonanticipatory constraints in an implicit sense. In addition, the approach simplifies the stochastic programming model since it reduces the number of decision variables over tree-based stochastic programming methods and it can readily parallelize the optimization problem. In addition, senior managers can readily grasp policy rules. As a disadvantage, the resulting model is likely to be nonconvex.

Pension Planning Examples

A defined-benefit pension plan for a large US company encompasses issues that affect a range of stakeholders. The company must set its benefit policy so that both retired and working employees are properly compensated and fairly treated. The terms properly and fairly are subject to alternative interpretations. The investment policy must consider the long-term nature of the planning horizon. At the

same time, pension planners must satisfy the prudent-man rule. The company must follow numerous regulations scrupulously—government, tax, accounting, and so forth. Clearly, there are a variety of perspectives on the risk-reward trade-offs.

To address these issues, Towers Perrin developed the Retirement Plan Financial Management approach. It combines an assessment of the plan liabilities and assets to help determine benefit, funding, investment, and accounting policies that are consistent with a company's overall goals. This methodology applies to all components of a company's retirement program—including pension, savings, and retiree benefits. Some of the key financial measures that are typically included in a company's statement of objectives are:
—Future cash-contribution requirements;
—Long-term portfolio return relative to interest-crediting rates (for cash-balance plans);
—Possible reductions to shareholder equity;
—Recovery of plan costs in rates (for regulated companies);
—Future cash-flow needs for benefit payments;
—Funded status; and
—Financial expense.

A pension-plan design should address the four areas—benefit design, funding policy, investment strategy, and accounting policy—that affect the company's financial health as measured by the statement of objectives. These financial "levers" link together and link to the company's broader business objectives (Figure 3).

Benefit design includes the types of plans utilized and the level of benefits each plan
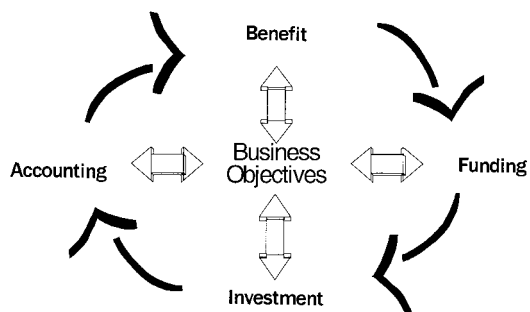
Figure 3: The four elements of pension-plan design should together fit the company's goals. Each element should assist in achieving the company's business objectives.

provides.

The *funding policy* covers the amount of capital the company contributes in advance to support promised benefits.

The *investment strategy* controls the investment of the capital allocated by the funding policy.

The *accounting policy* recognizes the annual corporate expenses and balance-sheet reserves.

A fundamental objective is to minimize or at least manage cash contributions, financial expense, or otherwise-defined financial risk using these levers. Adopting a systematic process helps the company fulfill its financial duties to shareholders and its fiduciary duties under its regulatory guidelines, such as the Employee Retirement Income Stability Act (ERISA). Recently, our clients have used the financial-management process to:

—Shape funding requirements to reduce excessive funding requirements in any single year;

—Evaluate the need to cap the interest crediting rate in a new cash balance plan;

—Determine the optimal asset allocation aimed at maximizing return on the plan's

surplus;

—Develop funding, accounting, and investment policies to minimize the probability of future reductions in shareholder equity;

—Determine the financial and cash-flow impact of offering lump sums to pension-plan participants at retirement;

—Shape the financial-expense patterns to compress spikes and reduce year-to-year volatility; and

—Analyze the probable impacts on the financial statement of actuarial assumptions.

The following three real-world case studies contain details that are similar to many pension plans.

Example A

Company A's retirement plan has been fully funded for many years. As a result, it has not made cash contributions in recent memory. Because of several acquisitions, the retirement plan will likely no longer be fully funding in a few years. In fact, a simple deterministic forecast projected that a cash contribution in excess of $60 million would be required in the year 2000. While the company accepted the fact that it would have to contribute cash, it wanted to minimize the probability that it would have to contribute more than $60 million in any single year of the next five.

Following our analysis, the company adopted a new funding policy to shape its future cash contributions. The new policy reduced the probability that the cash contribution would exceed $60 million in any of the next five years from 58 percent to 18 percent. The new funding policy included two key changes from the current policy: the asset-smoothing method was reset so

that investment gains during the past few years could be recognized more quickly, and the company committed to an annual contribution of three percent of payroll. We considered changing the investment strategy. However, our analysis showed that while this offers a long-term solution, changing the allocation to a more aggressive portfolio mix would not significantly affect results during the company's five-year time horizon. Thus, the accounting change fit the company's needs (Figure 4).

Example B

Company B adopted a cash-balance plan with a contribution credit of five percent of pay, plus interest credits on the account balance equal to the return on one-year Treasury bills plus one percent. Company B was concerned that during a period of high inflation, the interest-crediting rate would exceed the actual return on the trust assets, causing the plan's funded status (surplus) to decline. Thus, the company wanted to estimate the implications of applying a cap on the annual interest-crediting rate.

After our review, the company decided not to adopt a cap for the following reasons: Over the long-term there was only a small risk that the interest-crediting rate would exceed the actual return on the trust assets. A cap would do little to further reduce this risk. A cap would detract from the employee-perceived value of the cash-balance plan, especially in years that the cap applied (Figure 5).

Example C

Company C converted its traditional pension plan to a cash-balance plan and wished to reevaluate its asset allocation. The company's two key financial objec-

Distribution of Annual Cash Contribution under Recommended Funding Policy
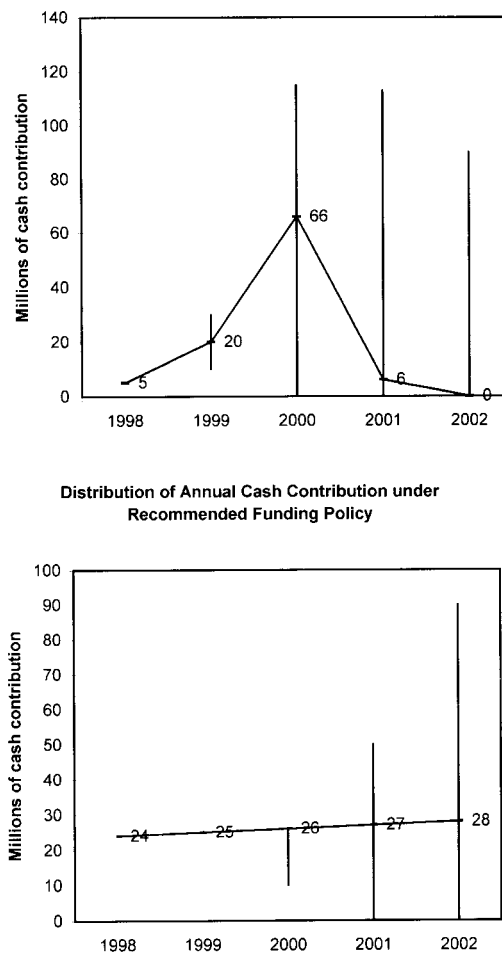
Figure 4: The range of cash contribution depends upon the resolution of the uncertainties and the investment policy over the planning horizon. The current policies for Company A provide a wider range of outcomes than the recommended policies.

tives were to maximize the plan's surplus and to minimize the risk that the funded status would fall below 100 percent over the next five years.

A common approach would be to determine the asset-allocation strategy by optimizing the expected return on the plan's portfolio of assets relative to the standard deviation of portfolio return (Figure 6).
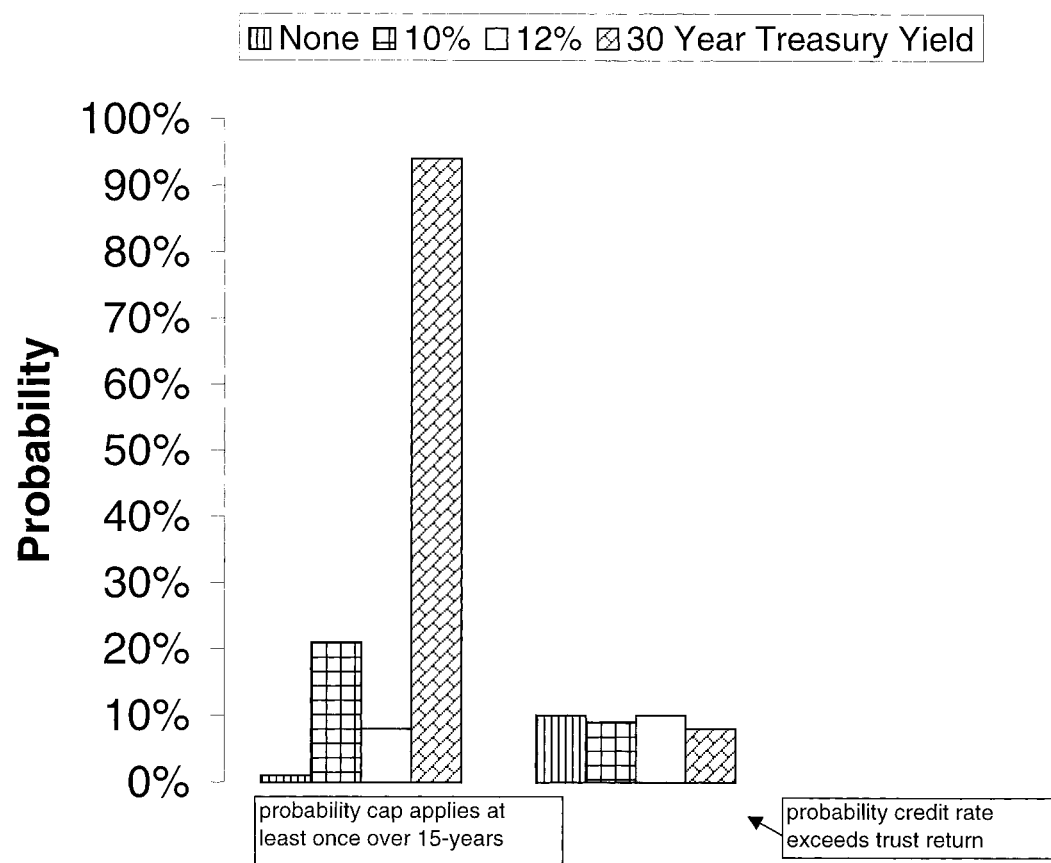
Figure 5: Adding a cap to the pension plan will have little impact on Company B's financial health. It will detract from the perceived value of the plan to the participants.

However, because a plan's assets and liabilities are both affected by common economic variables, an asset-only approach is inappropriate. Instead, we employ the surplus efficient-frontier optimization tool. It defines risk and reward in terms of the plan's surplus value. Company C defined risk as the likelihood of falling below 100-percent funded, and it defined reward as the expected funded status. A comparison of the asset mixes identified on the asset-only efficient frontier with the surplus frontier for Company C showed that the asset-only optimal solution lies off the efficient frontier and is dominated by other solutions on the surplus-efficient frontier (Figure 7). Although the company's current mix is efficient relative to the asset-only frontier, it is inefficient relative to Company C's stated objectives (Table 1). The company adopted the recommended asset allocation.

Operations Research Contributions

The Towers Perrin project's success can be attributed to innovations based on operations research methods. Four areas have benefited from interactions among the team of operations researchers, econometricians, and financial actuaries.

Solution of Nonconvex Optimization

The policy-optimization framework gives rise to nonconvex programs [Ziemba and Mulvey 1998]. This property hinders the search for optimal solutions to stochastic financial-planning models over multi-period horizons. Most multiperiod financial-decision and policy rules lead to nonconvex nonlinear programs. To address this issue, the integrated system starts the search with a carefully chosen set of points, near to the top of the efficient frontier. It finds subsequent points by moving down the efficient frontier while satisfying the linear constraints. These heuristic methods have proven effective in practice for many of Towers Perrin's clients.

Calibration of Stochastic Equations via Nonlinear Optimal Fitting

An essential element of any planning system is parameter settings. For instance, the mean reversion parameter for interest rates has an impact on the average level of returns for fixed-income assets (Appendix). We address the issue from several standpoints. First, there should be a connection between historical results and the overall pattern of the generated scenarios. The distribution of equity returns, for instance, should be compared with the cumulative distribution over history. Major discrepancies should be reviewed and accepted or rejected based on the actuaries' judgment. In general, the Towers Perrin model fits summary statistics, such as variances, covariances, autocorrelations, cross-correlations, and the interquartile ranges. These values provide indicators of the overall volatility and interrelationship of markets over the planning period.

The actuaries and economists input criti-
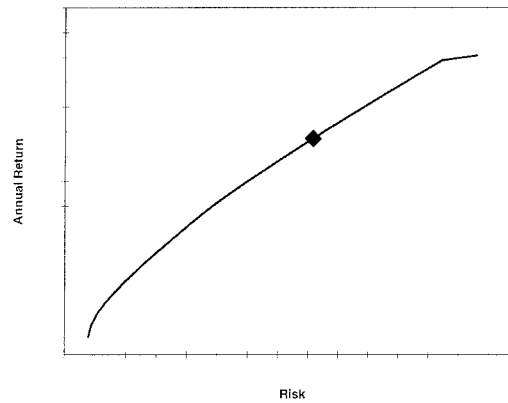
Figure 6: The asset-only efficient frontier shows the optimal combination of risk and reward for the pension-plan assets. Company C's current policy (diamond) lies on the asset efficient frontier.

cal sets of statistics directly into the model. These parameters set general trends and averages. For instance, the growth in equity earnings is determined by expert judgment in each country. Certainly, the actuaries must account for past performance when they fix these values. But they must be able to take a stand when they are convinced that historical averages must be modified.

The total parameter-setting exercise combines the judgment of the actuaries and economists with a formal calibration tool. The tool fits historical patterns of summary statistics, while meeting the targets set by the econometric staff as closely as possible. The optimal fitting program requires a nonconvex optimization solver and a routine for running CAP:Link [Mulvey, Morin, and Pauling 1999]. Operations researchers have much to offer in the calibration of structural stochastic forecasting models.

Constructing Representative Scenarios

The scenario set must represent a rea-

Surplus Efficient Frontier
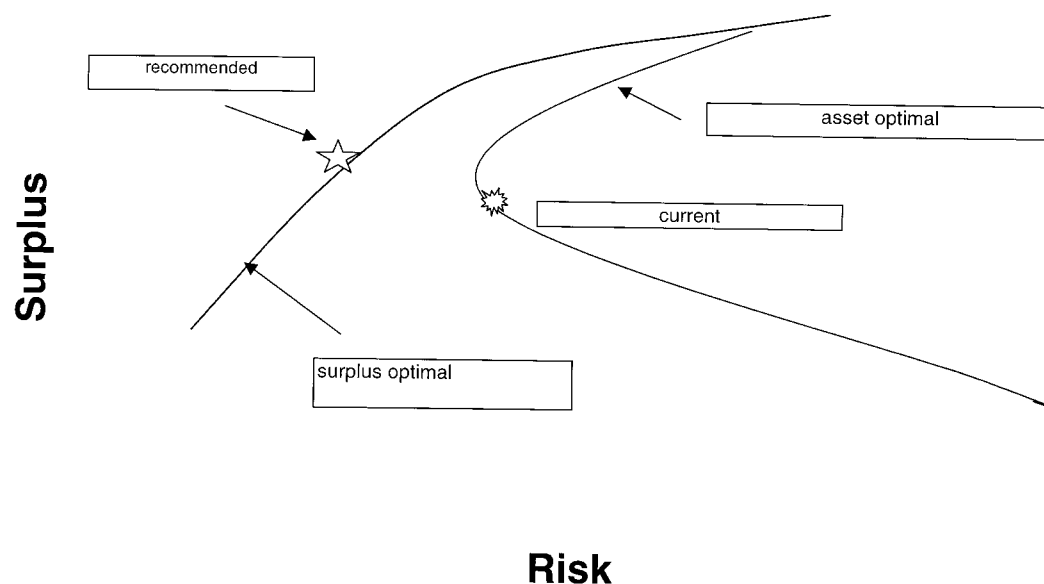
Figure 7: The surplus efficient frontier shows the optimal combination of risk and reward for the pension-plan surplus. Company C's current policies are inefficient with respect to its surplus.

sonable range of outcomes, depending upon the investor's needs. We minimize the number of scenarios by employing variance-reduction methods including antithetic variables. In addition, we can evaluate the model's recommendations by running these through a process that we call stratified filtered search [Ziemba and Mulvey 1998]. Here, the goal is to generate a series of scenarios that minimizes the sampling errors and maximizes confidence in the resulting efficient frontier and asset allocation.

A second issue is the errors that arise in sampling scenarios within the optimization module. We address this issue by adjusting the number of scenarios sampled depending upon the investor's needs. In many cases, a set of 500 scenarios has proven adequate for investors interested in standard risk-reward analysis, such as surplus efficient frontiers. Additional scenarios can be generated as needed when there is particular interest in rare events. For instance, the Tillinghast property casualty system requires a large number of scenarios (over 10,000) due to its emphasis on rare catastrophes arising from large hurricanes and earthquakes.

To validate the model, we spend a great deal of time evaluating scenarios for reasonableness. We often look at individual scenarios to understand the connection between variables, for example, the level of interest rates and the accompanying level of inflation (Figure 8). Although interest

rates increase over the period 1997 to 2005, the real rate of interest—interest rate minus inflation—decreases during this period, until eventually, long interest rates spike in 2005, causing real interest rates to increase. The high level of real rates then pushes down inflation and interest rates follow. Patterns of this type are judged reasonable by the economic staff. The ALM system possesses interactive features that allow users to easily see connections between any sets of variables.

Systematic Stress Testing

While economic theory can help actuaries in setting valid assumptions, no guiding laws govern the future state of the world's economy. Thus, unlike engineers who depend upon physical laws, financial engineers must take extra care when following the recommendations of financial-planning systems. In this environment, we employ a proprietary stress-testing procedure. The method filters scenarios that fit specified patterns.

US WEST Application

US WEST has increased its pension surplus by $450 to $1,000 million over alternative strategies based on running the financial-planning system. This client has a large pension portfolio, over $12 billion (1997), with a ratio of assets to liabilities equal to 150 percent, and a plan covering 106,000 participants. The company employs the ALM system regularly to understand the impact of possible changes in the investment mix and to explore modifications to the pension-plan design.

During the fall 1997, Towers Perrin, in conjunction with US WEST, employed its stress-testing procedure. In addition to standard analyses, US WEST pays close attention to factors that could decrease its current ample surplus. It wants to continue participating in the upside of the strong US equity markets but is concerned about protecting its surplus from unpleasant surprises.

We developed a systematic approach for identifying scenarios that lead to poor performance across the planning horizon of 10 years. Once we locate the cause of the scenarios, we set up the CAP:Link system to filter scenarios that fit a set of descriptors. For example, a combination of dropping equity markets and falling interest rates is the most devastating for a pension plan. We create a set of scenarios that fulfils this requirement. In 1997, we generated four sets of 500 scenarios: (1) normal conditions; (2) equity market crash, that is, a two-year equity dropping at least 10 percent per year; (3) an equity bear market, that is, compound equity returns less than three percent over the next 10 years; and

	Asset Only	Recommended
Large cap US equity	35%	55%
Small cap US equity	15	10
International equity	15	15
Corporate bonds	35	0
20-year Treasuries	0	20
Total	100%	100%

Table 1. The surplus-optimal investment mix (recommended) is better than the asset-only solution since it achieves the company's goals with less risk.
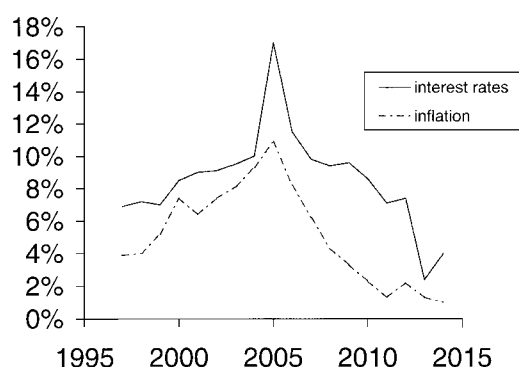
Figure 8: Over the planning for a single scenario, the economic factors should display sensible relationships to each other. Future inflation will often depend upon the real rate of interest. Negative real rates will cause inflation to increase, whereas positive real rates will push future inflation lower as shown.

(4) disinflation, that is, long-bond yields average less than 3.5 percent over the next 10 years. For simplicity, we compared three investment strategies—the current equity/bond mix, an equity hedged mix, and a long-bond strategy (Figure 9). The current solution was more robust over the four sets of scenarios. Based on these and similar results, US WEST decided to maintain the existing asset allocation. The cost to hedge the equity exposure would have caused an expected surplus decline of over $2.5 billion. The long-bond strategy should be avoided as well. While contributions were less likely with improved asset-liability matching, the expected surplus would also have declined by over $2.5 billion over the next 10 years. The resulting cost savings was significant. The cost to hedge the US equity portion would have been a $450 million reduction in surplus over the past 18 months. The opportunity gain realized by not moving $4.8 billion of equity into bonds is $1.01 billion, representing an equity return of 40 percent versus bond returns of 19 percent over 18 months. The subsequent risk-reward analysis showed the benefits of these decisions to the financial health of the US WEST pension plan.

In summary, we reran the investment system under filtered sets of stressful scenarios to develop contingency plans for the pension plan. US WEST has successfully employed this dynamic stress analysis and has maintained a substantial equity position over the past few years. The ALM system gives it confidence that its pension plan will be able to maintain a substantial surplus. A stochastic framework is essential for understanding the temporal risk/reward issues for a large, complex pension plan.

Conclusions

Towers Perrin's stochastic planning system has had a major impact on the actuarial profession and on many pension plans. Actuarial consultants throughout the world use the system in evaluating pension plans and insurance companies. Linking assets and liability decisions is difficult for most investors. The integrated ALM system provides an intuitive approach for coordinating these elements. The investor can concentrate on the primary task—to balance the goals of the various groups, for example, maximizing the surplus of the pension plan, while maintaining retirement benefits, and protecting the company's balance sheet.

Over the past years, the financial-services industry has been moving towards consolidation and merging of disparate organizations. A prominent example is Citigroup, formed by combin-
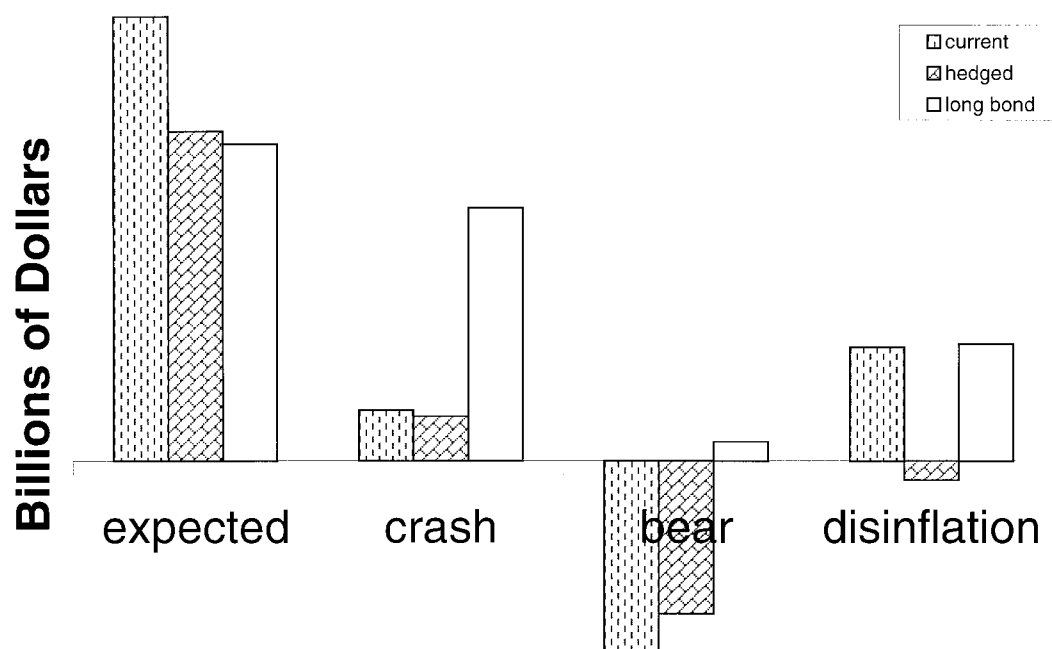
Figure 9: The outcomes of three strategies over the standard 500 scenarios and three stressful scenarios sets. The current policy has the best risk-reward relationship.

ing Travelers and Citicorp (with Smith Barney and Salomon Brothers as divisions). European companies are well along the same path. This trend will accelerate as Congress replaces depression-era regulations, such as the 1933 Glass-Steagall Act. These conglomerate financial organizations need to integrate their decisions. Companies must understand the risk-adjusted returns for each operation. A financial company must also be able to identify risk factors that span the organization, giving rise to correlated risks. The Towers Perrin ALM system can be extended to cover business-related activities. As a consequence, the property casualty (PC) division of Tillinghast—Towers Perrin has developed an integrated risk-management system for the PC industry. Also, Renaissance Reinsurance, in conjunc-

tion with Tillinghast, has developed an integrated risk-management system [Lowe and Stanard 1996].

Financial companies have traditionally been managed as decentralized businesses, with occasional meetings between divisional executives to set policy and targets. The situation is reminiscent of manufacturers and retailers in the 1960s and 1970s with their production, inventory, and transportation subsystems. Integrated logistics is the norm for these companies today. A similar change will take place in the management of financial organizations. The Towers Perrin system provides a prominent example of the advantages of integrating asset and liability decisions within a common framework.

APPENDIX

The scenarios generated by CAP:Link

contain key economic variables, such as price and wage inflation, interest rates for different maturities (real and nominal), stock dividend yields and growth rates, and exchange rates through each year for a period of up to 40 years. We model returns on asset classes and liability projections consistent with the underlying economic factors, especially interest rates and inflation. The model simultaneously determines economic variables for multiple economies within a common global framework. Long-term asset-and-liability management is the primary application.

The global CAP:Link system forms a linked network of single-country modules. The three major economic powers—the United States, Germany (now the EU), and Japan—occupy a central role, with the remaining countries designated as home or other countries. We assume that the other countries are affected by but do not affect the economies of the three major countries. The basic stochastic differential equations are identical in each country, although the parameters reflect unique characteristics of each particular economy. We can readily include additional countries in the framework by increasing the number of other countries.

Within each country, the basic economic structure is such that variables at the top of the structure influence those below, but not vice-versa (Figure 10). This approach eases the task of calibrating the model's parameters. The ordering does not reflect causality between economic variables but rather captures significant comovements. Linkages across countries occur at various levels of the model—for example, interest rates and stock returns. These connections are discussed by Mulvey and Thorlacius [1998]. Roughly, the economic conditions in a single country are more or less affected by those of its neighboring countries and by its trading partners. The degree of interaction depends upon the country. The structure is based on a cas-

cade format. Modules above and equal to that level can affect each submodule within the system. Briefly, the first level consists of short and long interest rates, and price inflation. Interest rates are a key attribute in modeling asset returns and especially in coordinating the linkages between asset returns and liability investments. To calculate a pension plan or an insurance company's surplus, we must be able to discount the projected liability cash flows at a discount rate that is consistent with bond returns, under each scenario. Also, since dynamic relationships are essential in risk analysis, the interest-rate model forms a critical element.

The second level entails real yields, currency-exchange rates, and wages. At the third level, we focus on the components of equity returns: dividend yields and dividend growth. Returns for the remaining asset classes form the next level, with fixed-income assets reflecting the term structure of interest rates and other mechanisms. We project each economic variable by means of a stochastic differential equation, relating the variable through time and with the stochastic elements of the equation and to other variables and factors at the same or higher levels in the cascade.

A critical feature for a global scenario generator is the currency module. Several issues complicate modeling currency-exchange rates. First, currencies must enforce the arbitrage free condition among spot exchange rates and among forward rates with differential interest rates. The second issue concerns symmetry and numeraire independence. We must create a structure in which the distribution of currency returns from country A to country B has the same distribution as returns from B to A. Both issues limit the form of the currency-exchange models, especially when integrating three or more currencies. To avoid these problems, we focus on the strength of each country's currency. Ex-
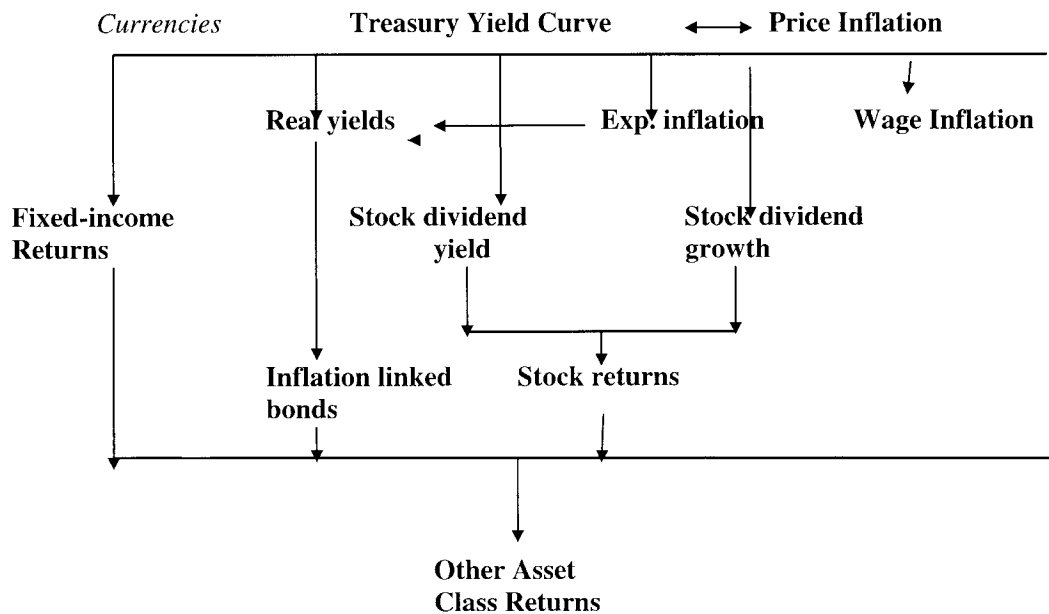
Figure 10: The stochastic differential equations are arranged in a cascade structure. Factors at the higher levels of the cascade affect those below, but not vice versa. Each country has the identical structure with its unique parameters.

change rates follow as the ratio between the strengths of any two countries. The absolute strength of any currency is a notional concept; the relative levels reflect the difference in the exchange rates [Mulvey and Thorlacius 1998].

We list two of the key differential equations (for short and long government spot rates) to give examples of the types of equations that we employ in the CAP:Link system. Modeling economic factors is critical. The two equations define the path of interest rates over the planning horizon. There are seven parameters: mean reversion levels (l_μ and u_μ), drift terms (a_r and a_1), volatility terms (σ_r and σ_1), and a correlation value for the Wiener terms (Z_r and Z_l). While the stochastic model is more complex than one based on a simple regression structure, an ALM system must possess consistent linkages between assets and liabilities. A factor approach seems to be the most appropriate method for providing this consistent linkage.

Short interest rates:

$$dr_t \ = \ a_r \ (r_\mu \ - \ r_t) \ dt \ + \ r_t \ \sigma_r \ dZ_r$$

Long interest rates:

$$di_t \ = \ a_q \ (l_\mu \ - \ l_t) \ dt \ + \ l_t \ \sigma_l \ dZ_l$$

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Stephen P. Lowe, FCAS, MAAA Principal, Chief Actuary Tillinghast-Towers Perrin, Forestal Centre, 175 Powder Forest Drive, Weatogue, Connecticut 06069-9658, writes: "Insurance companies are increasingly recognizing the need to build dynamic financial models to support the management of risk and capital. A cornerstone of this process is the generation of the economic scenarios. We are using Global CAP:Link for this purpose, with great success. Global CAP:Link provides scenarios for the key economic variables that drive the behavior of the assets and liabilities. Equally important is that the scenarios extend across multiple periods, which is critical when modeling long-term, non-tradable insurance contracts.

"We are using Global CAP:Link, in conjunction with our TAS financial modeling software to assist insurers in looking at a number of critical issues.

1. We have analyzed alternative investment strategies, given an insurer's liabilities. These projects entail the construction of asset/liability efficient frontiers.

2. We have analyzed alternative portfolio hedging strategies, in the form of different reinsurance treaties that the insurer might buy to protect itself from adverse claim experience.

3. We have analyzed the impact of alternative debt/equity ratios on risk and return for an insurer interested in the efficiency of its capital structure.

"We expect the use of tools such as Global CAP: Link to explode in the next few years, and to fundamentally alter the way insurers manage risk and capital. Our clients are excited by the results we have obtained; most importantly, they are acting on the recommendations stemming from them."

J. Stanford Willie, PhD, Vice President, US WEST Investment, Management Company, 7800 East Orchard Road, Suite 290, Englewood, Colorado 80111, writes: "One of the most unique features of the Towers Perrin System is the linkages between the asset and liability components of the models underlying the system. Many systems available in the marketplace focus only on the asset side of the problem and simply determine a mix of assets that attempts to maximize expected return and/or minimize volatility of return. They ignore the importance of the liability performance in the evaluation of total risk. Even in the cases where these systems take the liabilities into account, the forecasted performance is often inconsistent with the simulated asset performance and is, we believe, unreliable, especially over multi-periods. With the Towers Perrin System, there are direct linkages between the various model components underlying the system. These linkages give us confidence that the outcomes are reasonable and truly reflect the risk/reward trade-offs we are making in

our investment decisions.

"Another important aspect of the Towers Perrin System is its flexibility relative to the analysis of various pension fund objectives, a topic of particular interest to senior management. At US WEST, we want to increase our surplus position, but we also want to minimize corporate contributions and avoid large losses. With the Towers Perrin System, we can analyze the impact of changing our asset allocation on each of these objective functions. We can also look at a particular asset allocation and analyze the impact of various extreme economic conditions. We have found this scenario generation capability to be a very powerful tool for asset allocation. And, because it allows us to specifically quantify risks relative to achieving our objectives, it is a powerful tool for communicating to senior management.

"Also, traditional methods of asset allocation based on mean-variance optimizations are sensitive to the assumed parameters, such as returns, risks, and correlations of individual asset classes. Often, asset mixes that are unpalatable, or essentially uninvestable, are found to be "optimal." Furthermore, since we cannot know these parameters with much precision, and given the sensitivity of the analysis to these parameters, the output of any mean-variance optimization must be viewed with considerable caution. We have found the Towers Perrin simulation methodology to be far less sensitive to the assumed parameters of the individual asset classes than the traditional mean-variance approach and, therefore, more useful.

"In summary, because of the flexibility and reliability of the Towers Perrin System, US WEST Investment Management Company has significantly improved its decision-making ability relative to total fund management. We have been able to conduct more thorough, sophisticated, and useful analysis, which has led to more thoughtful decisions. The system has also given our senior management a better understanding of, and more confidence in, the nature of our investment strategies. We believe the system is worthy of the Edelman Award."