Applying Operations Research Techniques to Financial Markets

John Board • Charles Sutcliffe • William T. Ziemba

Department of Accounting and Finance, London School of Economics and Political Science, London WC2A 2AE, United Kingdom

Accounting and Finance Division, School of Management, University of Southampton, Southampton SO17 1BJ, United Kingdom

Operations and Logistics Division, Faculty of Commerce and Business Administration, University of British Columbia, 2053 Main Mall, Vancouver, British Columbia, Canada V6T 1Z2

j.board@lse.ac.uk • cms@socsci.soton.ac.uk • ziemba@interchange.ubc.ca

This paper was refereed.

OR techniques are applied to nonportfolio problems in financial markets, such as the equity, debt, and foreign exchange markets and the corresponding derivatives markets. Finance problems are an excellent application area for OR researchers. OR techniques are used to value financial instruments, identify market imperfections, design securities, regulate markets, evaluate and control risks, model strategic problems, and understand the functioning of financial markets. Mathematical programming is probably the most widely applied OR technique, but Monte Carlo simulation methods are of increasing importance. With the improvements in the real-time availability of data and the power of computers, the role of OR techniques in financial markets can only increase.

(*Finance. Professional: OR/MS implementation.*)

Operations research (OR) has been extensively applied to problems in finance during the last half century. The extent of such applications is evident in the INFORMS database of academic papers in OR journals since 1982. This database classifies nearly three percent of the entries as concerned with finance. For *Management Science* over the same period, this proportion is over 10 percent. An even larger number of papers on the application of OR techniques to finance have been published in the finance, mathematics, engineering, and other literatures; so that, in total, several thousand papers concerning the application of OR techniques to finance have appeared in academic journals. In the professional world, OR has influenced financial markets to adopt new finance theories. For example, in the 1960s and 1970s, the management science group at Wells Fargo Bank in San Francisco pioneered the application of new finance theories and introduced the first index-tracking fund in July 1971 (Bernstein 1992). As they increased their use of mathematical models in finance (Merton 1995, 2002), investment banks recruited staff skilled in quantitative techniques, including OR, to devise pricing equations, control risks, and analyze market data—the so-called quants.

The relationship between finance and OR is bidirectional. Various OR techniques have been applied to finance problems, and finance theories have motivated the development and improvement of OR solution techniques. OR research was part of the prize-winning work of the three Nobel prize winners in economics in 1990. Harry Markowitz, who worked at the Rand Corporation in the 1950s and 1960s, was honored in 1989 by ORSA/TIMS for his work on sparse matrices and for developing the computer simulation language SIMSCRIPT. Both Markowitz and William Sharpe conducted research and produced computer algorithms

INTERFACES, © 2003 INFORMS
Vol. 33, No. 2, March–April 2003, pp. 12–24

0092-2102/03/3302/0012$05.00
1526-551X electronic ISSN

for solving and analyzing portfolio problems. William Sharpe was a member of the logistics department at the Rand Corporation where he wrote a paper on optimizing the design of military transport aircraft, and he was an associate professor of operations research at the University of Washington. Merton Miller wrote five papers on linear programming, of which one was an application to finance, and two papers on applying inventory models to finance (Grundy 2001).

We survey the application of OR techniques to financial markets (for example, the equity, debt, and foreign exchange markets and the corresponding derivatives markets). This is a growing area for the application of OR techniques. Because Ashford et al. (1988) have reviewed the more traditional applications of OR to the management of firms' finances, such as the management of working capital (which can be subdivided into cash, receivables, and liabilities), capital investment (including the appraisal of sets of interdependent investments), multinational taxation, and financial-planning models (such as those developed for banks), we excluded these applications from our survey. We also excluded models concerned with choosing the debt-equity ratio and determining the cost of capital, and forecasting models. Campbell et al. (1997) and Mills (1999) covered the use of forecasting models in financial markets. Board et al. (2002) survey the use of OR techniques to solve the portfolio problem (that is, the problem of finding the optimal allocation of an investment budget between a set of assets).

We have not given full descriptions of the relevant concepts from financial markets and OR, but we have tried to define jargon finance words. Paxon and Wood (1997) provide fuller definitions of the finance concepts we mention, and Copeland and Weston (1988) and Campbell et al. (1997) provide detailed treatments of the theory and application of the issues. Although the definition of what constitutes an OR technique is arbitrary, we consider the techniques listed in the contents of OR textbooks, such as Hillier and Lieberman (2001). We summarize the OR techniques used to analyze and suggest improved solutions to the various problems from financial markets considered in this paper in Table 1.

Attractiveness of Finance Problems

In most applications of OR to financial markets (excluding portfolio theory), the aim is not necessarily to optimize some objective function, but to produce feasible solutions to highly constrained problems (for example, when designing financial securities) or to estimate some number (for example, valuing financial instruments). Financial institutions also use OR techniques to estimate market and credit risks, not to devise strategies that minimize these risks. Because very large sums of money can be involved and many problems are repetitive in nature, applying OR techniques can result in making (or saving) large amounts of money, at least until competitors begin using the same techniques.

Problems in financial markets are generally more separable and well defined than most OR problems. Although contagion between markets may reduce the separability of problems (for example, when one financial market crashes, others may crash too), contagion is unimportant for many of the applications we consider. If such interaction is important, the OR solution will be applicable only to normal trading conditions, and more complicated models will be needed for abnormal circumstances.

Often the relevant objective function, constraints, and variables are amenable to quantification in monetary terms. However, in some cases, the decision requires risk preferences. In such cases the OR researcher can provide decision makers with the efficient set of risk-return choices and allow them to select their preferred solution, use the risk-return trade-off implicit in the market, or assume a specific utility function (Kallberg and Ziemba 1983). The OR applications we describe do not require such risk-return trade-offs.

In contrast to investigators in some other OR applications, investigators in these cases have few worries about ensuring that they have identified the correct question (for example, they do not have to consider whether the problem is to reschedule the company's vehicle fleet to meet customer needs or to decide whether the company needs to operate a fleet of vehicles at all). In finance problems, the relationships between the variables are usually well defined, so that, for example, the way in which a change in the volatility

	Monte Carlo Simulation	Linear Programming	Dynamic Programming	Quadratic Programming	Integer Programming	Nonlinear Programming	Data-Envelopment Analysis	Goal Programming	Game Theory	Network Model	Markov Chain	Neural Network	Simulated Annealing
Valuation													
Options	✓	✓	✓	✓		✓						✓	
MBS-CMO	✓												
Loans	✓										✓		
Imperfections													
Weak efficiency						✓					✓		
Arbitrage		✓								✓			
Trading	✓											✓	
Design													
Municipal bonds		✓	✓		✓	✓		✓					
Callable bonds	✓	✓	✓			✓							✓
MBS-CMO					✓								
Leases		✓			✓								
Regulatory													
Capital reserves	✓												
Credit risk	✓												
Banks							✓						
Margins		✓											
Lawsuits		✓											
Kuwait		✓											
Strategy													
Market making			✓						✓				
Takeovers									✓				
Understanding													
Innovation		✓											
Arbitrage pricing												✓	
Over-the-counter							✓						
Mutual funds							✓						

Table 1: OR techniques have been applied to many financial market problems. (MBS-CMO indicates mortgage backed securities—collateralized mortgage obligations.)

of the underlying asset affects the price of an option is clear. These relationships are generally fairly stable over time. Thus, the resulting OR model is a good representation of reality, particularly because the role of nonquantitative factors is often small.

Solutions to finance problems can usually be implemented. In other areas, unspecified restrictions concerned with human behavior and preferences may prevent the implementation of some solutions. Furthermore, finance practitioners are accustomed to the quantitative investigation of problems and have adopted practices that incorporate the results of numerical analyses. This quantitative orientation and the availability of real-time data mean that solutions can be implemented very quickly.

OR researchers are likely to find that much of the historical data they need has already been collected and is available from company records or recorded market transactions, and that large amounts of data are available on individual trades and quotes in financial markets. For example, Gopikrishnan et al. (1998) used 40 million observations on equity markets, while Nath (2002) used 110 million observations on the Indian stock market. Because this data is mostly captured electronically, it is accurate and available in a machine readable form.

As trading in financial markets often involves very large sums of money, even a small improvement in the quality of the solution (under 0.5 percent) is beneficial. Also, any automation that saves time for highly paid staff will be attractive. Amenable problems tend to recur, possibly many times per day, spreading the costs of developing an OR solution over many transactions. This scale and repetition makes developing OR models more attractive for such purposes than for small or one-time decisions. In addition, decisions in financial markets must often be made very quickly (for example, while a potential customer waits on the phone for a quote). Appropriately designed OR models, in conjunction with powerful computers, permit quick decisions. For other problems, solutions must comply with regulations, and OR techniques can find feasible solutions where they exist.

Because problems in finance (especially in financial markets) are largely numerical, with well-defined boundaries and objectives, clear and stable relationships between variables, large benefits from very small improvements in the quality of decision making, and excellent data, they are well suited to OR analysis.

The Valuation of Financial Instruments

Traders in financial markets need good models for valuing assets. OR techniques have been used in valuing options, mortgage-backed securities (MBSs), collateralized mortgage obligations (CMOs) and loan portfolios.

The Valuation of Options

Although the Black-Scholes model provides good closed-form solutions to valuing European-style call and put options, OR techniques can be used to price more complex derivatives for which no analytical solutions exist. For example, Boyle (1977) proposed using Monte Carlo simulation as an alternative to the binomial model for pricing options. It has the advantage over the binomial model that its convergence rate is independent of the number of state variables (for example, the number of underlying asset prices and interest rates), while that of the binomial model is exponential in the number of state variables.

Analysts use Monte Carlo simulation to generate paths for the price of the underlying asset until maturity. They can then discount the cash flows from the option for each path, weighted by their risk-neutral probabilities (inferred from prices by assuming that investors have risk-neutral linear utility functions) back to the present using the risk-free rate, to compute the average present value across all the sample paths to obtain the current price of the option (Boyle et al. 1997). Analysts have used a range of variance-reduction methods in the Monte Carlo pricing of options (for example, control variates, antithetic variates, stratified sampling, Latin hypercube sampling, importance sampling, moment matching, and conditional Monte Carlo). They have also applied quasi-Monte Carlo methods to finance problems to speed up the simulation (Joy et al. 1996). Monte Carlo simulation can also be used to compute the various sensitivities, which are called the Greeks and include delta (the hedge ratio) and are essential for many trading strategies (Broadie and Glasserman 1996).

No closed-form solutions exist for American-style options (options that can be exercised at any time before expiration), and until recently people thought Monte Carlo simulation could not be used to price such options. This is a major problem, because most options are American style. However, researchers are developing Monte Carlo simulation techniques for pricing American-style options (Broadie and Glasserman 1997, Grant et al. 1997). Analysts have also used finite difference approximations to price options. Dempster and Hutton (1999) and Dempster et al. (1998) proposed using linear programming to solve the finite difference approximations to the price of American-style put options. American-style options can also be priced using dynamic programming (Dixit and Pindyck 1994).

If a closed-form pricing equation cannot be derived for an option or other derivative, and a price history is available, a neural network can be trained to produce prices using a specified set of inputs. The model can then be used for out-of-sample pricing (Hutchinson et al. 1994). This approach outperformed the Black-Scholes formula in pricing options on S&P500 futures, and it has potential for generating prices for hard-to-price derivatives that are already traded on competitive markets.

Empirical research shows that, although the Black-Scholes pricing model provides accurate prices for at-the-money options (the current price of the underlying asset is close to the price at which the option can be exercised), some patterns occur in options prices, such as the "volatility smile." A smile occurs when the implied volatility for out-the-money options (especially puts) exceeds that for at-the-money options. However, OR techniques can be used to allow for the smile when pricing options. Given a contemporaneous set of prices for European-style put and call options on the same underlying asset, Rubinstein (1994) showed how to compute the implied probability distribution using quadratic programming. Analysts can use these probabilities to infer a recombining binomial tree that is consistent with the observed options prices in a way that allows for the presence of the smile. Jackwerth and Rubinstein (1996) generalized this approach using nonlinear programming to minimize four other objective functions. (For recent research in this area, see Brown and Toft (1999), Gemmill and Saflekos (2000), Rosenberg (2000), and the survey of Jackwerth (1999).) Gemmill and Saflekos (2000) argue that the stability of the estimates is questionable.

Traders in financial markets need good models for valuing assets.

For incomplete markets, when Black-Scholes is inapplicable, Ritchken (1985) used linear programming to compute upper and lower bonds on option prices. Zmeškal (2001) recognized the stochastic and fuzzy nature of the input data for real options problems and used nonlinear programming to value European-style call options.

The Valuation of MBS and CMOs

Mortgage-backed securities (MBSs) are shares or certificates representing an interest in a pool of mortgages. For any specific mortgage, a borrower may repay the loan early—the prepayment option—or may default on the payments of capital and interest. These risks feed through to the owners of MBSs, as do the risks of fluctuations in the rate of interest payable on flexible-rate mortgages (Zipkin 1993). As they have variable interest rates and early exercise options, MBSs are hybrid securities. Analysts can use Monte Carlo simulation to produce interest-rate paths for future years. Using forecasts of the mortgage prepayment rates, they can compute the cash flows from each interest rate path and then use these sequences of cash flows to value the MBS (Zenios 1993, Ben-Dov et al. 1992, Boyle 1989). This procedure, which can be used to identify mispriced MBSs in real time, is computationally demanding, and analysts use parallel and distributed processing to solve such problems.

Collateralized mortgage obligations (CMOs) represent a mortgage pool structured into a series of bonds (or tranches), each with a different maturity and risks. Analysts also use simulation to price CMOs (Paskov 1997). Valuing other hybrid securities, such as callable and putable bonds and convertible bonds, presents problems similar to valuing MBSs and requires similarly intensive solution methods. (The issuer of a callable bond has the right to redeem the bond at a time of their choice, the owner of a putable bond has the right to sell the bond back to the issuer at par value on designated dates, while convertible bonds can be converted into shares in the company at some future date.)

The Valuation of Loan Portfolios

An active secondary market exists for portfolios of loans that carry a high default risk. Del Angel et al. (1998) used a Markov chain analysis with 14 loan-performance states and Monte Carlo simulation to derive the probability distribution of the present value of loan portfolios.

The Valuation of Bonds and Bond Stripping

After computing the yield curve (which shows the interest rates for different maturities), a trader can use it to compute the bond price for any chosen maturity. Because most bonds traded in liquid markets have coupons (or interest payments), one must strip the bonds of their coupons when using these bond prices to compute the yield curve. Allen et al. (2000) proposed formulating this problem as a linear programming problem. For risky bonds, this approach, unlike other methods, guarantees a set of bond prices that is arbitrage free (that is, if two bonds have the same maturity, the bond with a lower credit risk should have the higher price).

Imperfections in Financial Markets

In addition to pricing financial securities accurately, traders want to find mispricings and imperfections that they can exploit to make profits (Keim and Ziemba 2000, Ziemba 1994a, 1994b).

Imperfections and Weak Inefficiency

In looking for profitable opportunities, traders may search for weak form inefficiency (this is, they may try to use an asset's past prices as the basis for a profitable trading rule). Early attempts to find such exploitable regularities in stock prices include Dryden's (1968, 1969) application of Markov chains. Clark and Ziemba (1987) applied fractional Kelly strategies (that is, mixing cash and the optimal capital-growth portfolio to form a portfolio to maximize the expected rate of growth of wealth) to January turn-of-the-year spreads (small minus large capitalization stocks) on US index futures and managed to beat the market. Ziemba (1994b) and Hensel and Ziemba (2000) describe more recent work in this area.

Hausch et al. (1981) and Hausch and Ziemba (1985) used the odds on the current race and a capital-growth model to derive a strategy for racetrack betting to find bets that have positive expected values. This nonlinear programming model was profitable, indicating weak inefficiency in this market. Hausch et al. (1994) survey later research in this area in their book, which is the standard reference for professional racetrack betting teams in Hong Kong.

Imperfections and Trading Strategies

Interest has been growing in the use of artificial-intelligence-based techniques (for example, expert systems, neural networks, genetic algorithms, fuzzy logic, and inductive learning) to develop trading strategies for financial markets (Goonatilake and Treleaven 1995, Refenes 1995, Trippi and Turban 1993, Wong and Selvi 1998). Such approaches pick up nonlinear dynamics and require little prior specification of the relationships involved. For example, Firer et al. (1992) simulated the returns from a stock market timing strategy for a range of levels of forecasting skill, quantifying the likely benefits from various levels of forecasting ability. Taylor (1989) used Monte Carlo simulation to produce a long time series of data to use in back-testing the performance of trading rules for a variety of financial assets.

Imperfections and Arbitrage Opportunities

A fundamental feature of financial markets is the absence of arbitrage opportunities. An arbitrage opportunity is a situation in which one can combine mispriced securities with other securities to create a portfolio that offers riskless profits in all future scenarios. Because traders compete to exploit such opportunities, they usually eliminate the profit opportunity. If traders are willing to accept some risk, they can take advantage of additional profitable opportunities involving arbitrage-trading strategies. Shaw et al. (1995) describe an example of such risk arbitrage. Analysts have used network models to find arbitrage opportunities between sets of currencies (Christofides et al. 1979, Kornbluth and Salkin 1987, Mulvey 1987, Mulvey and Vladimirou 1992). This problem can be specified as a maximal-flow network, where the aim is to maximize the flow of funds out of the network, or as a shortest-path network. Some network formulations are linear and could be formulated and solved as linear-programming models. Interpreting the problem as a network, however, enables the analysts to use computationally faster algorithms.

Garman (1976) used linear programming to search for arbitrage opportunities across a large number of options, while Hausch and Ziemba (1990a, 1990b) used linear programming to devise arbitrage strategies for racetrack markets.

Hodges and Schaefer (1977) devised a linear programming model that minimizes the cost of a given pattern of cash flows, enabling them to trade underpriced bonds. Chandy and Kharabe (1986) developed a model for identifying underpriced bonds by solving a linear-programming model to form a bond portfolio with maximum yield. The solution gives the break-even yield, which is the minimum yield the bond must provide to be included in the portfolio. The bonds identified in this way can then be used in an arbitrage strategy.

Security Design

Instead of trading financial securities designed by others, some issuers allow purchasers to choose the details of the instrument to suit themselves, subject to complying with various legal requirements. These securities are mainly debt instruments (for example, municipal and callable bonds, MBSs, and CMOs).

Designing Municipal Bonds

Municipal authorities in the USA who want to borrow money by issuing bonds usually invite bids from underwriting syndicates. In these bids, the syndicate must specify a schedule of bond coupons (interest payments) that will allow it to market the bonds to the public, subject to restrictions imposed by the municipality. The winning bid is generally that with the lowest net interest cost to the municipality. The underwriting syndicates typically have only 15 to 30 minutes to prepare bids, and so they need computerized solution procedures. In an early application of OR to a finance problem, Percus and Quinto (1956) and Cohen and Hammer (1965, 1966) formulated this problem as a linear-programming problem, while Weingartner (1972) respecified it as a dynamic-programming problem. If the municipality places an upper bound on the number of different coupon rates, the problem becomes an integer-programming problem that can also be solved as a zero-one dynamic-programming problem (Friemer et al. 1972, Weingartner 1972). Nauss and Keeler (1981) added the constraint that the coupon rates be set to integers times a specified multiplier and proposed an integer-programming formulation. If the municipality chooses to specify the true interest cost (which is the internal rate of return, IRR, on the bond), rather than the net interest cost as the selection criterion to be used, the problem becomes nonlinear. However, Bierwag (1976) suggested that this problem could be solved using a linear-programming algorithm. Nauss (1986) added some additional restrictions that make the problem integer and suggested an approximate solution using integer linear programming. Finally, Puelz and Lee (1992) used goal programming to solve this problem with integer variables and linearized goal constraints.

Designing Mortgage-Backed Securities and Collateralized Mortgage Obligations

Some MBSs are traded on a to-be-announced basis with forward delivery. In these cases, the originators have mortgages they have not yet pooled, giving them flexibility in structuring the securitization to benefit themselves. In the US, extensive rules govern how to-be-announced MBSs can be structured, making the problem of devising a feasible solution complex. Maximizing the originator's profit can be specified as a complicated integer-programming problem (Dahl et al. 1993). However, the problem is computationally demanding, and heuristics are generally used in practice. Various restrictions also apply to the structuring of CMOs, and finding feasible solutions may be difficult. Dahl et al. (1993) also proposed a complex zero-one programming model for solving this problem with the objective of maximizing the proceeds from the issue. Solving this formulation is also computationally demanding.

Designing Callable Bonds

Firms, governmental organizations, and others can issue callable bonds, retaining the option to repay the bond before its maturity date. The issuer must choose various parameters of the callable bond, and Consiglio and Zenios (1997a, 1997b) used nonlinear programming to design such securities to be most beneficial to the issuer. Holmer et al. (1998) used a simulated annealing algorithm to address this problem. A related problem is the bond-scheduling problem. Firms that have issued callable bonds must decide when to call (repay) the existing bonds and refinance the debt with a new issue, presumably at a lower cost. This is a dynamic-programming problem and has been modeled as such by Elton and Gruber (1971), Kraus (1973), and Weingartner (1967). Baker and Van Der Weide

(1982) extended the dynamic-programming model to cover a multisubsidiary company with debt requirements for each subsidiary. Dempster and Ireland (1988) developed a model in which they applied a range of OR techniques in a complementary fashion to the bond-scheduling problem. They began by using stochastic linear programming to devise a multiperiod plan for both issuing and calling bonds. They then refined the plan using heuristics, possibly producing multiple plans, and then they derived the probability distributions of these revised plans using simulation. Finally, they used an expert system to help them to decide between alternative plans.

Designing Leveraged Leases

To receive US tax benefits, firms must design leveraged leases to satisfy the rules of the Internal Revenue Service. Capettini and Toole (1981) proposed an integer-programming model to structure leveraged leases to meet the IRS rules, with the objective of maximizing the net present value of the lessor's cash flow. Litty (1994) developed an approach to this problem using linear-programming heuristics that provided fast solutions for untrained users.

Regulatory and Legal Problems

Financial regulators have become increasingly concerned about financial markets with their very large and rapid international flows of money. OR techniques have proved useful in regulating the capital reserves held by banks and other financial institutions to cover their risk exposure and in solving other legal problems relating to financial markets.

Regulation and Capital Reserves

A key regulatory issue is determining the capital financial institutions need to underpin their activities in financial markets. A commonly used approach to this problem is to quantify the value at risk (VAR). If the specified period and probability are one day and one percent, the VAR is the lowest possible value so that the probability of losses less than VAR exceeds 99 percent. Thus, determining VAR involves quantifying the lower tail of the probability distribution of outcomes from the firm's portfolio. A particular problem with measuring risk exposure is that portfolios usually include options (or financial securities with option-like characteristics), and options have asymmetric payoffs. For such securities, analytical solutions to finding the probabilities in the lower tail of the payoff distribution are unreliable. RiskMetrics, a model J. P. Morgan developed in 1994 for quantifying VAR, uses approximations based on the Greeks for options that are at or near the money, and Monte Carlo simulation for other option positions (Morgan and Reuters 1996). Monte Carlo simulation can either make distributional assumptions or use bootstrapping on the distribution of historical realizations (Pritsker 1997). A related application of Monte Carlo simulation is stress testing, which quantifies the sensitivity of a portfolio to specified, often adverse, market scenarios. Lucas and Klaassen (1998) point out that meeting some specified VAR is equivalent to imposing a chance constraint (that is, a probabilistic constraint on the maximum loss), and demonstrate that unrecognized nonnormality can have a substantial effect on the resulting portfolios. The RiskMetrics approach to risk measurement (with its employment of OR techniques) has proved very useful in assessing capital adequacy for banks and similar institutions.

Although useful, VAR has serious theoretical difficulties. For example, it does not satisfy the coherency axioms of Artzner et al. (1999). The aggregate VAR for two identical and independent risks does not equal the VAR for a portfolio of the two risks. When there are losses, the penalty is not proportional in a linear or convex fashion to these losses. Geyer et al. (2001) utilize a convex penalty function in their asset-liability planning model for Siemens, Austria, which seems more appropriate.

Regulation and Credit Risk

While RiskMetrics quantifies market risk, some securities are also subject to credit risk. Credit risk differs from market risk in that default is rare, but if it occurs, either in whole or in part, the loss is substantial. As a result, although the market risk of financial instruments (apart from options) tends to produce returns with an approximately normal distribution, credit risk produces returns that are highly nonnormal for all instruments. Therefore, Monte Carlo simulation is relevant to modeling the credit risk of portfolios of financial instruments (for example, loans, letters of credit, bonds, trade credit, swaps, forwards) as in CreditMetrics (Morgan 1997). Currently, the trading of securities based on credit risk is growing rapidly, along with interest in their valuation. As explained earlier, Monte Carlo simulation can be used to price such securities.

Regulation and Banks

Barr et al. (1993) and Bauer et al. (1998) used data-envelopment analysis (DEA) in regulating banks by measuring bank efficiency, which they then used to predict bank failure.

Regulation and Margins

Traders are required to put up margin when they trade options, and complicated rules determine the total margin required on a portfolio of options and shares. Traders try to structure their positions to minimize their margin payments, and Rudd and Schroeder (1982) developed a linear-programming model in which they modeled the problem as a transportation problem.

Legal Problems and Lawsuits

Sharda (1987) proposed a linear-programming formulation to establish the maximum loss that investors could have sustained from trading in a company's shares. The company's lawyers can then use this figure when fighting a lawsuit claiming damages from a misleading statement by the company.

Legal Problems and the Kuwait Stock Exchange Collapse

In August 1982, the Kuwait stock market collapsed, leaving $94 billion of debt to be resolved. The problem was to devise a fair method for distributing the assets seized from insolvent brokers among the other brokers and private investors. Analysts used linear programming to reduce the total unresolved debt to $20 billion, saving an estimated $10.34 billion in lawyer's fees (Elimam et al. 1996, 1997, Taha 1991).

Strategic Problems

In financial markets, strategic issues often arise, including ways to minimize market impact when trading large blocks of shares and modeling the behavior of two large shareholders who are competing for corporate control.

Strategic Problems and Market Makers

In recent years, researchers have used game theory to analyze some of the decisions traders and market makers in financial markets face (Dutta and Madhavan 1997, O'Hara 1995). These models typically concern one or more market makers and traders who may be informed or uninformed and discretionary or nondiscretionary. Traders in stock markets seek to trade at the most attractive prices, and they often break large trades up into a sequence of smaller trades in an effort to minimize the impact on the stock price. Because the initial trades influence the price of subsequent trades, executing the large trade at the lowest cost is a dynamic problem. It can be viewed as a strategic problem with the aim of devising a strategy for trading the block of shares. Bertsimas and Lo (1998) use stochastic dynamic programming to define best execution and to compute an optimal trading strategy. So far theorists have been the main users of game-theoretic models, but if the requisite data becomes available, analysts could apply them in real situations.

Strategic Problems and Corporate Control

Powers (1987) applied game theory to the situation in which a company has two major shareholders and many very small shareholders. Powers modeled this situation as an oceanic game in which the two large players behave strategically while the many small shareholders (the ocean) do not. This approach can be used to derive the highest price a large shareholder will pay in the market for corporate control. Again, the main limitation on the empirical application of game theory is access to the data.

Economic Insight

In addition to improving the quality of decision making, OR can also help those trying to understand the economic forces shaping the finance sector.

Economic Insight and Financial Innovation

Financial innovation may occur when an exogenous factor changes the constraints or the costs of meeting existing constraints. Using a linear-programming model of a bank, Ben-Horim and Silber (1977) employed annual data to compute movements in the shadow prices of the various constraints. They suggested that a rise in the shadow price of the deposits constraint led to the financial innovation of negotiable certificates of deposit (CDs).

Economic Insight and Testing the APT

Arbitrage-pricing theory (APT), which can be viewed as a generalization of the capital-asset-pricing model (CAPM), seeks to identify the factors that affect asset returns. Most researchers testing the APT use factor analysis and have difficulty in determining the number and definition of the factors that influence asset returns. To overcome these problems, Ahmadi (1993) suggested using a neural network to test the APT. This also has the advantage that the results are distribution free.

Economic Insight and the Efficiency of OTC Traders

The volume of trading in Australian financial securities, particularly trading in over-the-counter (OTC) markets, has dropped in recent years. In an attempt to discover a cause for this decrease, Mahama et al. (2001) used DEA to examine the efficiency of 19 OTC market makers operating in Sydney. They found that 16 of the 19 market makers had costs that were twice those of the three efficient firms. The market makers passed these inefficiencies on in the form of higher trading costs, which may account for the decline in OTC trading volume. The inefficiencies were concentrated in settlement costs, front office management costs, and general trading costs.

Economic Insight and Mutual Fund Performance

Basso and Funari (2001) and Murthi et al. (1997) employed DEA to study the factors responsible for the performance of US and Italian mutual funds.

Conclusions

Mathematical programming is the OR technique most widely applied in financial markets, and analysts have used many types of mathematical programming (linear, quadratic, nonlinear, integer, goal, DEA, and dynamic) (Table 1). Monte Carlo simulation is also widely used in financial markets, mainly to value exotic derivatives and securities containing embedded options (for example, MBSs and CMOs) and to estimate the VAR for various financial institutions. Simulation has also been useful in testing trading rules and for examining the risks of loan portfolios. In some cases, the employment of OR techniques has influenced the way financial markets function because they permit traders to make better decisions in less time. For example, some exotic options would trade with much wider bid-ask spreads, if they traded at all, in the absence of the accurate prices computed using Monte Carlo simulation.

Other OR techniques are less frequently used in financial markets. Network models, Markov chains, neural networks, and game theory have been used or proposed for particular problems, as has simulated annealing. Two important OR techniques, queuing theory and PERT-CPM, have not been applied to financial markets.

OR techniques play an important role in financial markets. Holmer and Zenios (1995) advocated that OR techniques be used to improve the productivity of financial intermediaries by integrating product design, pricing, asset allocation, and asset management. Models to price securities are usually applied in real time, with speed being essential. The computation of other numerical solutions and the design of financial instruments is less time-critical. As the real-time availability of data and computing power continue to improve, the role of OR techniques in financial markets can only increase. The application of OR to risk control and credit risk is likely to increase. The availability of faster computers using better algorithms enables the solution of more complex problems, and this in turn facilitates the trading of new complex securities, creating the opportunity for OR techniques to play an even greater role in financial markets.

Acknowledgments

Without implicating them, we thank Julia Bennell, Sally Brailsford, Con Connell, Ruth Davies, and Jonathan Klein (all from Southampton University), Susan Powell (London School of Economics), Bob Berry (Nottingham University), participants in a seminar at the Nottingham University Business School, the referees, associate editor, and editor of this journal for their comments on earlier drafts of this paper, and Mary Haight, managing editor of *Interfaces*.

References

Ahmadi, H. 1993. Testability of the arbitrage pricing theory by neural networks. R. R. Trippi, E. Turban, eds. *Neural Networks in Finance and Investing: Using Artificial Intelligence to Improve Real World Performance*. Probus Publishing Co., Chicago, IL, 421–432.

Allen, D. E., L. C. Thomas, H. Zheng. 2000. Stripping coupons with linear programming. *J. Fixed Income* **10**(2) 80–87.

Artzner, P., F. Delbaen, J. M. Eber, D. Heath. 1999. Coherent measures of risk. *Math. Finance* **9**(3) 203–228.

Ashford, R. W., R. H. Berry, R. G. Dyson. 1988. Operational research and financial management. *Eur. J. Oper. Res.* **36**(2) 143–152.

Baker, K. R., J. H. Van Der Weide. 1982. The bond scheduling problem of the multi-subsidiary holding company. *Management Sci.* **28**(7) 738–748.

Barr, R. S., L. M. Seiford, T. F. Siems. 1993. An envelopment analysis approach to measuring the managerial efficiency of banks. *Ann. Oper. Res.* **45**(1/4) 1–19.

Basso, A., S. Funari. 2001. A data envelopment analysis approach to measure the mutual fund performance. *Eur. J. Oper. Res.* **135**(3) 477–492.

Bauer, P. W., A. N. Berger, G. D. Ferrier, D. B. Humphrey. 1998. Consistency conditions for regulatory analysis of financial institutions: A comparison of frontier efficiency methods. *J. Econom. Bus.* **50**(2) 85–114.

Ben-Dov, Y., L. Hayre, V. Pica. 1992. Mortgage valuation models at Prudential Securities. *Interfaces* **22**(1) 55–71.

Ben-Horim, M., W. L. Silber. 1977. Financial innovation: A linear programming approach. *J. Banking Finance* **1**(3) 277–296.

Bernstein, P. L. 1992. *Capital Ideas: The Improbable Origins of Modern Wall Street*. Free Press, Macmillan Inc., New York.

Bertsimas, D., A. W. Lo. 1998. Optimal control of execution costs. *J. Financial Markets* **1**(1) 1–50.

Bierwag, G. O. 1976. Optimal TIC bids on serial bond issues. *Management Sci.* **22**(11) 1175–1185.

Board, J. L. G., C. M. S. Sutcliffe, W. T. Ziemba. 2002. The application of operations research to portfolio theory and investment management. Mimeo. London School of Economics, London, U.K.

Boyle, P. P. 1977. Options: A Monte Carlo approach. *J. Financial Econom.* **4**(3) 323–338.

——. 1989. Valuing Canadian mortgage backed securities. *Financial Analysts J.* **45**(3) 55–60.

——, M. Broadie, P. Glasserman. 1997. Monte Carlo methods for security pricing. *J. Econom. Dynamics Control* **21**(8–9) 1267–1321.

Broadie, M., P. Glasserman. 1996. Estimating security price derivatives using simulation. *Management Sci.* **42**(2) 269–285.

——, ——. 1997. Pricing American style securities using simulation. *J. Econom. Dynamics Control* **21**(8–9) 1323–1352.

Brown, G., K. B. Toft. 1999. Constructing binomial trees from multiple implied probability distributions. *J. Derivatives* **7**(2) 83–100.

Campbell, J. Y., A. W. Lo, A. C. MacKinlay. 1997. *The Econometrics of Financial Markets*. Princeton University Press, Princeton, NJ.

Capettini, R., H. Toole. 1981. Designing leveraged leases: A mixed integer linear programming approach. *Financial Management* **10**(4) 15–23.

Chandy, P. R., P. Kharabe. 1986. Pricing in the government bond market. *Interfaces* **16**(5) 65–71.

Christofides, N., R. D. Hewins, G. R. Salkin. 1979. Graph theoretic approaches to foreign exchange. *J. Financial Quant. Anal.* **14**(3) 481–500.

Clark, R., W. T. Ziemba. 1987. Playing the turn-of-the-year effect with index futures. *Oper. Res.* **35**(6) 799–813.

Cohen, K. J., F. S. Hammer. 1965. Optimal coupon schedules for municipal bonds. *Management Sci.* **12**(1) 68–82.

——, ——. 1966. Optimal level debt schedules for municipal bonds. *Management Sci.* **13**(3) 161–166.

Consiglio, A., S. A. Zenios. 1997a. A model for designing callable bonds and its solution using tabu search. *J. Econom. Dynamics Control* **21**(8–9) 1445–1470.

——, ——. 1997b. High performance computing for the computer aided design of financial products. L. Grandinetti, J. Kowalik, M. Vajtersic, eds. *Advances in High Performance Computing*. NATO Advanced Science Institute Series, Vol. 30. Kluwer Academic Publishers, Dordrecht, The Netherlands, 273–302.

Copeland, T. E., J. F. Weston. 1988. *Financial Theory and Corporate Policy*, 3rd ed. Addison Wesley, Reading, MA.

Dahl, H., A. Meeraus, S. A. Zenios. 1993. Some financial optimization models: II. Financial engineering. S. A. Zenios, ed. *Financial Optimization*, Cambridge University Press, Cambridge, U.K. 37–71.

Del Angel, G. F., J. M. Diez-Canedo, E. P. Patiño. 1998. A discrete Markov chain model for valuing loan portfolios: The case of Mexican loan sales. *J. Banking Finance* **22**(10–11) 1457–1480.

Dempster, M. A. H., J. P. Hutton. 1999. Pricing American stock options by linear programming. *Math. Finance* **9**(3) 229–254.

——, A. M. Ireland. 1988. A financial expert decision support system. G. Mitra, ed. *Mathematical Models for Decision Support.* NATO Advanced Science Institute Series, Vol. F48. Springer-Verlag, Berlin, Germany, 415–440.

——, ——, D. G. Richards. 1998. LP valuation of exotic American options exploiting structure. *J. Computational Finance* **2**(1) 61–84.

Dixit, A. K., R. S. Pindyck. 1994. *Investment Under Uncertainty.* Princeton University Press, Princeton, NJ.

Dryden, M. M. 1968. Short-term forecasting of share prices: An information theory approach. *Scottish J. Political Econom.* **15**(November) 227–249.

——. 1969. Share price movements: A Markovian approach. *J. Finance* **24**(1) 49–60.

Dutta, P. K., A. Madhavan. 1997. Competition and collusion in dealer markets. *J. Finance* **52**(1) 245–276.

Elimam, A. A., M. Girgis, S. Kotob. 1996. The use of linear programming in disentangling the bankruptcies of Al-Manakh stock market crash. *Oper. Res.* **44**(5) 665–676.

——, ——, ——. 1997. A solution to post crash debt entanglements in Kuwait's Al-Manakh stock market. *Interfaces* **27**(1) 89–106.

Elton, E. J., M. J. Gruber. 1971. Dynamic programming applications in finance. *J. Finance* **26**(2) 473–506.

Firer, C., M. Sandler, M. Ward. 1992. Market timing: A worthwhile strategy? *Omega* **20**(3) 313–322.

Friemer, M., M. R. Rao, H. M. Weingartner. 1972. Note on municipal bond coupon schedules with limitations on the number of coupons. *Management Sci.* **19**(4) 379–380.

Garman, M. B. 1976. An algebra for evaluating hedge portfolios. *J. Financial Econom.* **3**(4) 403–427.

Gemmill, G., A. Saflekos. 2000. How useful are implied distributions? Evidence from stock index options. *J. Derivatives* **7**(3) 83–98.

Geyer, A., W. Herold, K. Kontriner, W. T. Ziemba. 2001. The Innovest Austrian Pension Fund financial planning model (InnoALM). Working paper, University of Economics, Vienna, Austria.

Goonatilake, S., P. Treleaven, eds. 1995. *Intelligent Systems for Finance and Business.* John Wiley and Sons, Chichester, U.K.

Gopikrishnan, P., M. Meyer, L. A. N. Amaral, H. E. Stanley. 1998. Inverse cubic law for the distribution of stock price variations. *Eur. Physical J. B* **3**(2) 139–140.

Grant, D., G. Vora, D. Weeks. 1997. Path dependent options: Extending the Monte Carlo simulation approach. *Management Sci.* **43**(11) 1589–1602.

Grundy, B. D. 2001. Merton H. Miller: His contribution to financial economics. *J. Finance* **56**(4) 1183–1206.

Hausch, D. B., W. T. Ziemba. 1985. Transactions costs, extent of inefficiencies, entries and multiple wagers in a racetrack betting model. *Management Sci.* **31**(4) 381–394. Reprinted in Hausch, Lo, and Ziemba (1994).

——, ——. 1990a. Arbitrage strategies for cross track betting on major horse races. *J. Bus.* **63**(1) 61–78. Reprinted in Hausch, Lo, and Ziemba (1994).

——, ——. 1990b. Locks at the racetrack. *Interfaces* **20**(3) 41–48. Reprinted in Hausch, Lo, and Ziemba (1994).

——, V. Lo, M. Rubinstein. 1981. Efficiency of the market for racetrack betting. *Management Sci.* **27**(12) 1435–1452. Reprinted in Hausch, Lo, and Ziemba (1994).

——, ——, W. T. Ziemba, eds. 1994. *The Efficiency of Racetrack Betting Markets.* Academic Press, San Diego, CA.

Hensel, C. R., W. T. Ziemba. 2000. Anticipation of the January small firm effect in the US futures markets. D. Keim, W. T. Ziemba, eds. *Security Market Imperfections in World Wide Equity Markets.* Cambridge University Press, Cambridge, U.K., 179–202.

Hillier, F. S., G. J. Lieberman. 2001. *Introduction to Operations Research,* 7th ed. McGraw-Hill, Boston, MA.

Hodges, S. D., S. M. Schaefer. 1977. A model for bond portfolio improvement. *J. Financial Quant. Anal.* **12**(2) 243–260.

Holmer, M. R., S. A. Zenios. 1995. The productivity of financial intermediation and the technology of financial product management. *Oper. Res.* **43**(6) 970–982.

——, D. Yang, S. A. Zenios. 1998. Designing callable bonds using simulated annealing. C. Zopounidis, ed. *Operational Tools in the Management of Financial Risks.* Kluwer Academic Publishers, Dordrecht, The Netherlands, 177–196.

Hutchinson, J. M., A. W. Lo, T. Poggio. 1994. A non-parametric approach to pricing and hedging derivative securities via learning networks. *J. Finance* **49**(3) 851–889.

Jackwerth, J. C. 1999. Option-implied risk-neutral distributions and implied binomial trees: A literature review. *J. Derivatives* **7**(2) 66–82.

——, M. Rubinstein. 1996. Recovering probability distributions from options prices. *J. Finance* **51**(5) 1611–1631.

Joy, C., P. P. Boyle, K. S. Tan. 1996. Quasi-Monte Carlo methods in numerical finance. *Management Sci.* **42**(6) 926–938.

Kallberg, J. G., W. T. Ziemba. 1983. Comparison of alternative utility functions in portfolio selection problems. *Management Sci.* **29**(11) 1257–1276.

Keim, D. B., W. T. Ziemba, eds. 2000. *Security Market Imperfections in Worldwide Equity Markets.* Cambridge University Press, Cambridge, U.K.

Kornbluth, J. S. H., G. R. Salkin. 1987. *The Management of Corporate Financial Assets: Applications of Mathematical Programming Models.* Academic Press, London, U.K.

Kraus, A. 1973. The bond refunding decision in an efficient market. *J. Financial Quant. Anal.* **8**(5) 793–806.

Litty, C. J. 1994. Optimal lease structuring at GE. *Interfaces* **24**(3) 34–45.

Lucas, A., P. Klaassen. 1998. Extreme returns, downside risk and optimal asset allocation. *J. Portfolio Management* **25**(1) 71–79.

Mahama, H., M. Briers, S. Cuganesan. 2001. Cost efficiency in the Australian over-the-counter financial markets: An empirical examination. Working paper, University of New South Wales, Australia.

Merton, R. C. 1995. Influence of mathematical models in finance on practice: Past, present and future. S. D. Howison, F. P. Kelly, P. Wilmott, eds. *Mathematical Models in Finance.* Chapman and Hall, London, U.K., 1–13.

——. 2002. Future possibilities in finance theory and finance practice. H. Geman, D. Madan, S. Pliska, T. Vorst, eds. *Mathematical Finance—Bachelier Congress 2000: Selected Papers from the First World Congress of the Bachelier Finance Society*. Springer Finance, Heidelberg, Germany.

Mills, T. C. 1999. *The Econometric Modelling of Financial Time Series*, 2nd ed. Cambridge University Press, Cambridge, U.K.

Morgan, J. P. 1997. *CreditMetrics*™ technical document. Morgan Guarantee Trust Company, New York.

——, Reuters. 1996. *RiskMetrics*™ technical document. Morgan Guarantee Trust Company, New York.

Mulvey, J. M. 1987. Nonlinear network models in finance. K. D. Lawrence, J. B. Guerard, G. R. Reeves, eds. *Advances in Mathematical Programming and Financial Planning*, Vol. 1. JAI Press, Greenwich, CT, 253–271.

——, H. Vladimirou. 1992. Stochastic network programming for financial planning problems. *Management Sci.* **38**(11) 1642–1664.

Murthi, B. P. S., Y. K. Choi, P. Desai. 1997. Efficiency of mutual funds and portfolio performance measurement: A non-parametric approach. *Eur. J. Oper. Res.* **98**(2) 408–418.

Nath, P. 2002. Do price limits behave like magnets? Working paper, London Business School, London, U.K.

Nauss, R. M. 1986. True interest cost in municipal bond bidding: An integer programming approach. *Management Sci.* **32**(7) 870–877.

——, B. R. Keeler. 1981. Minimizing net interest cost in municipal bond bidding. *Management Sci.* **27**(4) 365–376.

O'Hara, M. 1995. *Market Microstructure Theory.* Blackwell Business, Cambridge, MA.

Paskov, S. H. 1997. New methodologies for valuing derivatives. Michael A. H. Dempster, S. R. Pliska, eds. *Mathematics of Derivative Securities.* Cambridge University Press, Cambridge, U.K., 545–582.

Paxon, D., D. Wood, eds. 1997. *The Blackwell Encyclopaedic Dictionary of Finance*, Vol. 8. C. L. Cooper, C. Argyris, eds. *The Blackwell Encyclopaedia of Management*. Blackwell Publishers, Oxford, U.K.

Percus, J., L. Quinto. 1956. The application of linear programming to competitive bond bidding. *Econometrica* **24**(4) 413–428.

Powers, I. Y. 1987. A game theoretic model of corporate takeovers by major stockholders. *Management Sci.* **33**(4) 467–483.

Pritsker, M. 1997. Evaluating value at risk methodologies: Accuracy versus computational time. *J. Financial Services Res.* **12**(2–3) 201–242.

Puelz, A. V., S. M. Lee. 1992. A multi-objective programming technique for structuring tax-exempt serial revenue debt issues. *Management Sci.* **38**(8) 1186–1200.

Refenes, A. P., ed. 1995. *Neural Networks in the Capital Markets*. John Wiley and Sons, Chichester, U.K.

Ritchken, P. H. 1985. On option pricing bounds. *J. Finance* **40**(4) 1219–1233.

Rosenberg, J. V. 2000. Implied volatility functions: A reprise. *J. Derivatives* **7**(3) 51–64.

Rubinstein, M. 1994. Implied binomial trees. *J. Finance* **49**(3) 771–818.

Rudd, A., M. Schroeder. 1982. The calculation of minimum margin. *Management Sci.* **28**(12) 1368–1379.

Sharda, R. 1987. A simple model to estimate bounds on total market gains and losses for a particular stock. *Interfaces* **17**(5) 43–50.

Shaw, J., E. O. Thorp, W. T. Ziemba. 1995. Risk arbitrage in the Nikkei put warrant market of 1989–1990. *Appl. Math. Finance* **2**(4) 243–271.

Taha, H. A. 1991. Operations research analysis of a stock market problem. *Comput. Oper. Res.* **18**(7) 597–602.

Taylor, S. J. 1989. Simulating financial prices. *J. Oper. Res. Soc.* **40**(6) 567–569.

Trippi, R. R., E. Turban, eds. 1993. *Neural Networks in Finance and Investing: Using Artificial Intelligence to Improve Real World Performance.* Probus Publishing Co., Chicago, IL.

Weingartner, H. M. 1967. Optimal timing of bond refunding. *Management Sci.* **13**(7) 511–524.

——. 1972. Municipal bond coupon schedules with limitations on the number of coupons. *Management Sci.* **19**(4) 369–378.

Wong, K., Y. Selvi. 1998. Neural network applications in finance: A review and analysis of literature (1990–1996). *Inform. Management* **34**(3) 129–139.

Zenios, S. A. 1993. Parallel Monte Carlo simulation of mortgage backed securities. S. A. Zenios, ed. *Financial Optimization.* Cambridge University Press, Cambridge, U.K., 325–343.

Ziemba, W. T. 1994a. World wide security market regularities. *Eur. J. Oper. Res.* 74(2) 198–229.

——. 1994b. Investing in the turn of the year effect in the United States futures markets. *Interfaces* 24(3) 46–61. (A story about this paper is in the December 14, 1990 issue of the *Wall Street Journal*.)

Zipkin, P. 1993. Mortgages and Markov chains: A simplified evaluation model. *Management Sci.* **39**(6) 683–691.

Zmeškal, Z. 2001. Application of the fuzzy-stochastic methodology to appraising the firm value as a European call option. *Eur. J. Oper. Res.* **135**(2) 303–310.