The Québec Ministry of Natural Resources Uses Linear Programming to Understand the Wood-Fiber Market

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In spring 1996, Québec's Ministry of Natural Resources began using a descriptive mathematical programming model to support various negotiations in the wood-fiber markets. The model, which uses linear programming to solve an economic-equilibrium program, allows the representative of the ministry to come to industry roundtables with accurate scenario analyses for the wood-fiber market. The tool we developed and implemented uses the large amounts of data available to government agencies to foresee and explain the general economic trends facing both lumber and paper producers. During its development, our team of operations-research experts, economists, engineers, and civil servants developed an unprecedented understanding of the wood-fiber market. The ministry incorporated these insights in subsequent government policy aimed at improving sawmill yield and stabilizing market behavior.

The fiber market where wood chips and wood shavings are traded constitutes an important link between the lumber industry and the pulp-and-paper industry. Two to three times a year, Québec's provincial government organizes a round table on the wood-fiber market. The gathering provides an opportunity for various actors in the lumber industry and the pulp-and-paper industry to meet with

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representatives of the Ministry of Natural Resources (MNR) and discuss the state of the fiber market. Wood fiber is a free market, in which prices are set by demand, availability, and the willingness of various actors to trade. Therefore, the round table is not a mechanism to set prices, but industry representatives have regularly attended in the hopes of smoothing demand and avoiding long periods of overproduction or shortage [*The Paper Tree Letter* 1984] as well as roller-coaster prices. Discussion topics include prices, stocks, availability, and long-term trading contracts between paper producers and sawmills, as well as acquisitions of sawmills by paper producers.

In recent years, the ministry has been under pressure to provide help and understanding in this erratic market, one in which stocks went from high to zero and back in a few months. Data was not a problem: the MNR regularly commissions provincewide comprehensive studies on the state of forests, sawmills, paper mills, technologies, lumber, paper prices, and so forth. Still, these did not turn into reliable insights into the price of fiber. Meanwhile, linear programming (LP) has gained a reputation among governmental agencies throughout the western world as an efficient tool to deal with economic-equilibrium models [Braier, Nautiyal, and Kant 1997; Gilles and Buongiorno 1987], public-sector policy analysis [Pollock and Maltz 1989], and general forest planning [Garcia 1990; Gautier and Granot 1995; Golden and Wasil 1989]. The Department of Economic Studies at the MNR knew that several models were being developed, in particular, in the forest management areas. In early 1995, the department asked us to develop an LP-based economic model for the fiber market, and one year later it was using the model's results at the round table [*Info-Forêt* 1996].

Most of the analysis and conceptual model development took place between the middle of January and the end of April 1995. Every week the team members

. . . chip rates nearly halve with the introduction of optimized sawmill cutting.

held a half-day meeting in which they exchanged information, discussed ideas, and confronted modeling assumptions with government data and industrial practice. Further tasks were also assigned to the team members for report at the next meeting, and the MNR arranged an industrial visit to a local paper plant (a sawmill visit was not needed because all team members were already familiar with sawmill operations).

The project team consisted of the four authors together with four technical experts, overseen by two high-ranking MNR executives. All modeling assumptions had to be discussed and justified with the technical experts. Specifically, the project team was formed of

—two university professors, who specialized in operations research and mathematical programming and who acted as consultants;

—a junior civil servant, who specialized in economics and statistics and who perfected the model, prepared economic scenarios, and acted as the model's main user;

—a senior civil servant, who oversaw sawmill and paper-plant operation in the province, permits, and licences, and roundtable discussions with the industry and who acted as the client;
—the four technical experts, who were a forest economist, a data analyst from the forest registry, a forest engineer, and a pulp-and-paper engineer.

A model prototype was delivered at the end of April 1995, allowing initial scenarios to be solved for a small number of sawmills and paper plants. At that point, the linear programs had to be coded by hand. The computer programs for automated matrix generation of the LP models were delivered in December 1995. Further testing and validation with complete scenarios were performed during the winter 1996.

The model correctly anticipated a major chip surplus that actually occurred in 1996.

A Brief History of Fiber

Traditionally, two methods were widely used in the production of paper. Chemical pulping uses wood chips as its raw material. It produces high-quality pulp (for example, Kraft) but has a low efficiency and therefore a high cost of production. Mechanical pulping on the other hand is a very efficient method that yields lower-quality pulp, such as that used for newspaper, but it requires costly logs as opposed to just chips.

More recently the dominant technology for high-volume, lower-grade papers has been thermo-mechanical pulping (TMP), which yields good-quality pulp using wood chips. Chips are steam heated before being squashed between fast rotating metal disks. Nowadays, newsprint is usually made from close to 100 percent TMP pulp to which a very small percentage of Kraft pulp and recycled fibre may be added. In a country such as Canada, where electrical energy is fairly cheap and environmental concerns are high, TMP has become the preferred method.

In this study, we considered only softwood chips that come from coniferous trees, representing most of the chip consumption in Québec. We did not consider hardwood chips because few paper plants have the technology to process them. For similar reasons, we did not account for jack-pine chips.

The Fiber Market

Wood chips are the primary raw ingredient for pulp and paper mills. An increase in pulp and paper production, combined with tighter timber supplies and more efficient sawmills, has created an especially strong market for chips. Unlike lumber and paper products, for which large volumes of imports and exports are reported each year, wood chips are considered a purely domestic commodity. Indeed, about 80 percent of Québec's timber production is harvested from public forests. As a rule, wood chips from public forest timber are never imported nor exported through provincial boundaries (exceptions are scarce and require a governmental decree). Imports and exports of wood chips harvested from private forests were considered negligible.

The main characteristic of wood fiber as a commodity is that it is both a raw material in the production of paper and a by-product of the lumber industry. Sawmills have two kinds of residue: sawdust, which has little commercial value, and chips, which are now the prime ingredient for the production of paper. But chips are not always residue, and sawmills have grown to rely on revenues from their sales to cover part of their operating expenses. Tree tops that once were left on the logging grounds are now brought back to the chipper or used to produce short-length lumber. Moreover, when fiber prices are high enough, or lumber prices low enough, lumber producers will deliberately sacrifice low-grade lumber and chip it.

We considered three groups of actors in this market: independent sawmills, paper plants, and sawmills owned by paper producers. Since wood chips are not imported or exported through provincial boundaries, we included only sawmills and paper plants located in the Québec province in the model.

The business of independent sawmills is quite simple. Each has a Forest Logging Right contracted with the government that allows for yearly logging volumes in a determined area of publicly owned forests. These rights are contracted over a period of 25 years with revisions every five years, thus ensuring a steady inflow of trees to the mill. They may obtain some additional timber from private forests. Out of the trees, the sawmill will produce high-grade construction lumber, such as 2 × 4s in standard lengths, which come from the cores of the logs. It obtains lower grades of lumber from outer parts of the log. The remainder of the log, including the narrow top of the tree, is usually sent to the chipper (Figure 1).

The proportions of grades of lumber that mills get out of logs vary from one mill to another depending on the type and quality of lumber grown in the area and technological factors: more sophisticated mills are able to better optimize the cutting patterns and produce higher percentages of the higher grades. The impact on the chip market of the current trend towards computer-assisted board cutting needs to be investigated since case studies already report that chip rates nearly halve with the introduction of optimized sawmill cutting [Lavertu 1995].

Sawmills follow the market very closely. The production mix may change almost daily, including the percentage of chips that are drawn from logs. Also, since mills are not investment intensive and enjoy very low fixed costs, they are able to slow

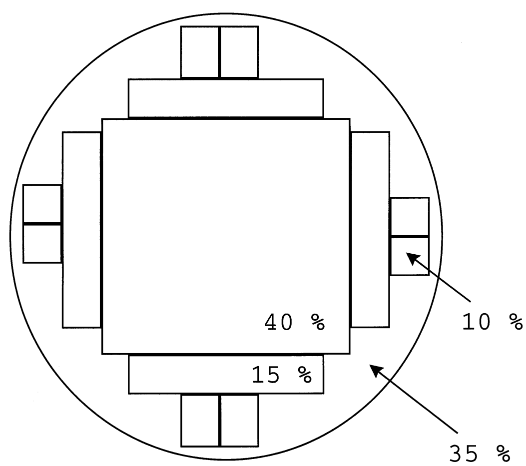

Figure 1: In this typical cutting pattern of a log, the central part will be further divided into higher quality 2 × 4s and represents about 40 percent of the log. 15 percent goes to second grade boards, 10 percent into lower grades, and 35 percent cannot be used to produce lumber.

or even shut down when lumber prices fall.

Sawmills that are owned by paper producers are known as *integrated mills*. Although their business is very similar to that of independent mills, government studies clearly indicate that they produce higher volumes of chips (60 percent versus 40 percent). It was suggested that this systematic difference originated from the fact that their owners were less interested in investing to improve log-cutting efficiency. Our economic model offers a different, more insightful explanation.

Scenario Exploration

One goal of the project was to make possible the exploration of a number of scenarios. More than predicting the price of wood chips accurately (a goal that we achieved with approximately 15-percent accuracy, an adequate performance since the round tables do not regulate prices), MNR project members wanted to investigate the effects of the prices of construction lumber, newsprint paper, and other well-understood prices, as well as the effects of paper corporations acquiring sawmills. We wanted to be able to present the results of five to 20 scenario analyses at the round table.

Given the absence of big players that could act as price leaders, our model assumes that the Québec fiber market exhibits perfect competition. This is justified by the large number of both paper plants and sawmills in the province. In other parts of the world, the chip market has a different competitive structure; for example, in the Argentinian model of Braier, Nautiyal, and Kant [1997], a single, dominant paper plant determines prices oligopsonistically. Moreover, we designed our model to simulate the fiber market over a short period of time, say a quarter, with all data other than chip prices given as exogenous (timber supplies, lumber prices, paper prices, and so forth). A simple economic-equilibrium model seemed adequate for the ministry's purpose.

A Simple Economic Equilibrium Model

One can model simple economic equilibria for one commodity with linear programming as follows: A market is composed of a number of actors (buyers and sellers). An actor exhibits to the market a demand or offer curve, thus expressing marginal prices or willingness to pay as a function of the quantities traded by the actor.

At first, each actor (seller or buyer) is modeled in a separate linear program, in which the quantity of commodity the actor trades is fixed arbitrarily through a constraint. For each such problem, the right-hand side (rhs) of this constraint is changed in a parametric analysis to obtain dual prices, which are interpreted as the price at which the seller (buyer) would sell (buy) the rhs quantity. Then, dual prices and corresponding quantities are fed into a global linear program that matches buyers and sellers, taking into account the amounts bought and sold as well as transportation costs. The optimal solution provides the exchange prices as well as traded quantities [Schrage 1986].

An actor in the economic model may represent a single physical actor, such as a production plant, as well as a grouping of several business units belonging to the same corporation. Because we were asked to explain the difference in chip rates between independent and integrated sawmills, we could not use other modeling approaches, such as aggregating plants within subregions. Instead, we had to adopt a more detailed approach to account for the lumber and chip production of all major sawmills. In our model, we allow for two types of sellers and one type of buyer:

—Independent sawmills that are in the business of selling chips,

—Paper plants that own local sawmills and that sell a chip surplus, and

—Paper plants that may own local sawmills and that have a chip deficit.

Independent Sawmills

Sawmills are easily modeled, at least when one is concerned with average production over a period of time. Given quantities of available timber, wood prices for various categories of boards, production costs, and an imposed quantity of wood chips to produce, one can write a short product-mix program that represents the average production (Appendix). We did not take into account the hourly production-level constraints (scheduling, optimal log-cutting, setups, and so forth). Figure 2 shows a typical dual-price graph for a sawmill.

Paper Plants

Paper plants are also amenable to simple linear-programming blending models [Thompson and Thore 1992]. Various types of papers have their own recipes, which may include TMP, chemical, or mechanical pulps from logs or chips; market pulp; possibly a small proportion of Kraft pulp for strength; and recycled paper. One difference with sawmills though is that paper plants cannot easily slow down:

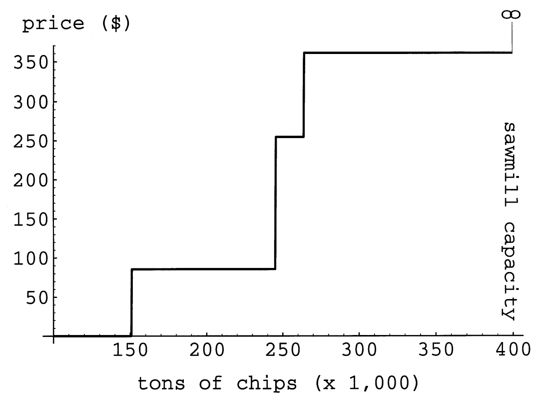

Figure 2: Here a sawmill is forced to produce to satisfy at least a certain demand for chips. This typical dual-price (or marginal price) graph starts at zero since residue chips are sufficient to cover low levels of demand. The price then rises sharply as more and more valuable parts of the logs are chipped in order to cover chip demand.

either they run at 80- to 100-percent capacity or shut down for several weeks, which constitutes a major business decision. Since accomodating shutdowns could have led to binary variables and a harder derivation of dual prices, we decided, in view of the small number of paper plants and the knowledge of MNR experts as to which plants would remain in operation, not to let the model decide but rather to let the user slow down, add, or remove paper plants from the global model by hand.

Several paper corporations own sawmills in addition to paper plants. In practice, since paper plants represent much higher capital investments and volume capacity, they tend to be quite sparse geographically. The sawmills that are close to a given paper plant and that belong to the same corporation are said to be integrated with that plant. Practically speaking, this integration involves coordinating production levels and sharing profit goals. Indeed, integrated sawmills usually have long-term supply contracts with their associated paper plants. While sawmills obviously sell wood chips and paper plants are buyers, one paper plant together with its integrated sawmills may act as a single selling or buying actor in the economic model.

Paper Plants with Integrated Sawmills

Although paper companies often own several sawmills and several paper plants, each integrated sawmill usually supplies only one associated paper plant for reasons of geographical proximity and relative production capacity. Paper plants, on the other hand, may have purchase contracts with more than one sawmill. In our model, therefore, we treat each paper plant, together with its integrated sawmills, as a single business entity (actor). The model we developed had to account for these two situations: paper plants as buyers and paper plants as sellers.

Integrated Chip Sellers

A paper plant with one or more large integrated sawmills that generate a wood-chip supply exceeding the plant's demand is modeled as a net seller on the market. In this situation, experience has shown that one sawmill's production usually is predominant (Figure 3). The fact that all trade between this actor and the market originates from the predominant sawmill has two consequences. First, the paper plant is only present implicitly on the market, since it makes no purchases, and sales originate from one of its integrated sawmills. Second, for the global model, all transportation costs between this actor and potential buyers are computed using the sawmill's location.

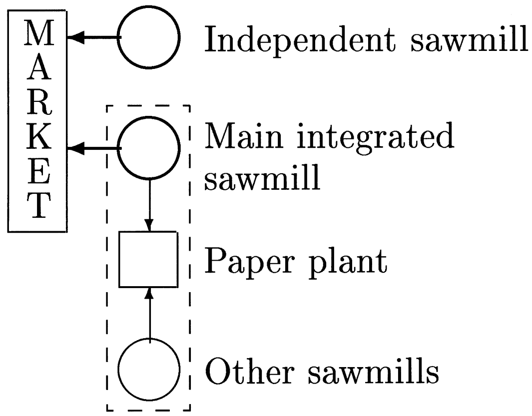

Figure 3: In the wood chip market, typical sellers are independent sawmills and the main sawmill integrated with a paper plant selling surplus chips. The arrows indicate the flow of wood chips.

The dual prices for this actor are obtained from a linear program in which the objective function reflects the net, combined profit for the whole group (paper plant and integrated sawmills). This includes revenues from sales of wood and paper products, minus variable production costs and wood-chip transportation costs from the integrated sawmills to the paper plant.

Integrated Chip Buyers

When the combined residual wood-chip production of a group of integrated sawmills is insufficient to meet the demand of their associated paper plant, the economic-equilibrium model treats the paper plant as a buyer on the market (Figure 4). Here again, the dual prices are obtained from the net combined profit of the paper plant and its integrated sawmills.

We model such a paper plant as one actor, a net buyer of chips. All trade between this actor and the market originates from the paper plant. Therefore, the plant's inte-

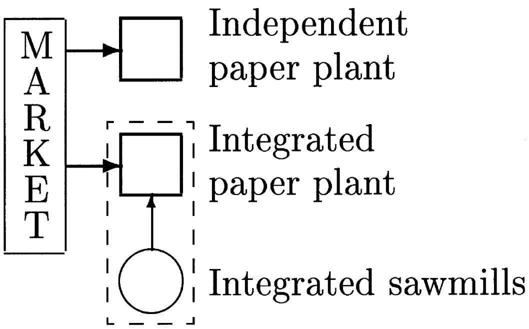

Figure 4: In the wood-chip market, typical buyers are paper plants that may own local sawmills and that have a chip deficit. Arrows indicate flow of wood chips.

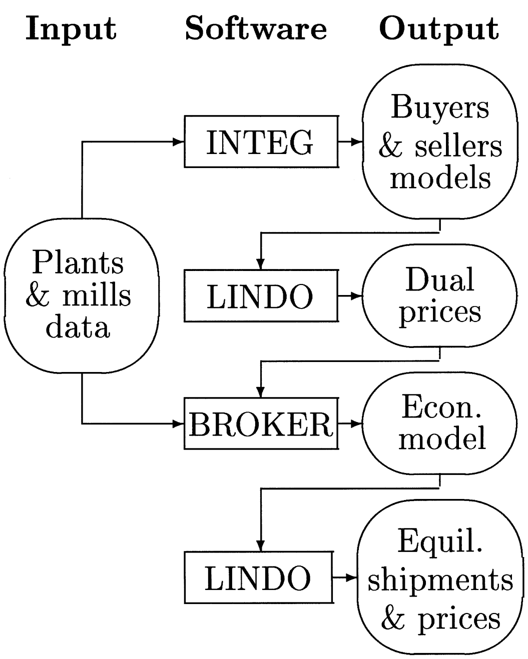

Figure 5: In the INOFIB model implementation on the PC-386, the user prepares the data files describing the sawmills and the paper plants and then runs DOS batch files for the computations. The INTEG program builds the parametric LPs of the buyers and sellers and LINDO solves the parametric LPs to obtain the dual prices. Next, the BROKER program constructs the economic equilibrium LP and LINDO solves it, giving the equilibrium prices and shipments.

grated sawmills are absent from the market (all their wood-chip production is bought by their associated paper plant) and all transportation costs between this actor and potential sellers are computed using the location of the paper plant.

Model Implementation

We implemented the various model components in a software package called INOFIB, designed to run on a PC-386 under DOS. The package contains two programs (INTEG and BROKER) coded in Borland Turbo Pascal that produce the LP models. The linear programs were solved using LINDO by calling DOS batch files. We retained this choice because the ministry's staff was already familiar with LINDO and Turbo Pascal; using more sophisticated LP software would have increased training and installation costs.

A given economic scenario is resolved in five steps: (1) The user prepares data files containing technical and economic data about all paper plants and sawmills, transportation costs, and integration information. Next, he or she runs DOS batch files to perform the other four steps. (2) The program INTEG reads the data, creates the actors based on integration information, and then produces the parametric linear programs for all actors, buyers and sellers. (3) A DOS batch file calls LINDO to solve the parametric LPs, yielding the dual prices and their corresponding intervals. (4) The program BROKER reads the transportation costs and the dual prices of all buyers and sellers and constructs the economic-equilibrium model. (5) Finally, a DOS batch file calls LINDO to solve the economic equilibrium model LP, giving the equilibrium shipments and prices.

The INTEG and BROKER programs can handle up to 90 paper plants and 90 sawmills. This is sufficient because there are about 30 (newsprint) paper plants in Québec and, although there are many more sawmills, the many smaller sawmills of a region can be grouped into a single aggregate sawmill model. The parametric LPs are fairly small (less than 200 variables). The global economic-equilibrium-model LP is larger. For example, with 50 buyers and 50 sellers and with 10 dual prices per site, there are 2,500 shipment variables and 1,000 interval variables.

Applying the INOFIB Model to Scenarios

After a few test runs, the first real job for the INOFIB model was to analyze a number of scenarios concerning the chip market. We set up a *base* scenario, reflecting the market situation of the third quarter of 1995. During this period, lumber prices were high and paper-product prices were along the long-term trend value. Such a situation is expected to favor high levels of lumber production, limited mainly by the physical availability of trees.

The whole domestic production is included in INOFIB; at the time, there were 68 large sawmills, and the remaining smaller units were clustered into eight regional groups. On the demand side, we included all 27 Québec paper plants that use coniferous chips. Taking into account the ownership of sawmills by paper corporations left us with 53 independent sawmills and 23 integrated units.

The focus of the study was to evaluate the impact of exogenous changes on the chip market. In addition to the base scenario, we presented six variations at the round table.

—In Timber96, we increased the overall forest logging rights by 10 percent because of the opening of five new sawmills—a situation that actually occurred in 1996.

—Timpap96 was the same as Timber96 except that the paper plants operated on reduced capacities while two new paper plants entered the market; overall, the province's paper production capacity was the same as for the base.

—In Lumber + 20, the price that sawmills got for lumber increased by 20 percent.

—In Lumber − 20, the price that sawmills got for lumber decreased by 20 percent.

—In Paper + 10, the price of paper increased by 10 percent.

—In Timber + 5, the cost of logged trees increased by five percent.

We presented selected results from the INOFIB runs at the round table (Table 1). While we obtained actual prices from INOFIB runs, these may vary according to the physical location of the buyer or seller due to transportation costs.

We validated the model by comparing the solution obtained for the base scenario with the actual market activity reported for the third quarter of 1995. We found that the model predicted the production levels correctly, and the exchanges were plausible according to historic data, although the actual exchanges were somewhat more diversified than the model's predictions. The regional variations predicted in the model have the correct sign, but they are more pronounced than those observed. The forecast of chip surplus, or absence thereof, was reasonably good. The equilibrium prices obtained by the model

Scenario	Number of sawmills	Number of paper mills	Equilibrium price variation from base (in %)	Coniferous chip trade volume (in tons)	Independent sawmills that chip lumber-quality wood
Base	76	27	=	2,416,000	23 out of 53
Timber96	81	27	−67.3	2,743,000	none
Timpap96	81	29	−63.5	2,548,000	none
Lumber+20	76	27	+3.8	2,332,000	11 out of 53
Lumber−20	76	27	−.6	2,507,000	24 out of 53
Paper+10	76	27	+39.0	5,524,000	32 out of 53
Timber+5	76	27	=	2,416,000	23 out of 53

Table 1: This summary of INOFIB results for the seven scenarios show substantial chip price variations in response to realistic, exogenous changes. Only variations in average price over the Province of Québec are shown; actual levels are withheld to preserve the confidential nature of the round-table discussions.

were accurate to within 15 percent of actual prices. The model's prediction for prices reflected particularly well the 1996 situation represented in the second scenario.

All scenarios agree with the economic intuitions of the experts. For instance, in Timber96 and Timpap96, the high wood supply clearly causes high volumes of chips that are pure residues, as opposed to chipped lumber-quality wood. This leads to the paper producers depending less on independent sawmills, and thus to a much lower market price for chips. Moreover, independent sawmills almost never chip lumber-quality wood. In this instance, the model correctly anticipated a major chip surplus that actually occured in 1996.

Price variations among the scenarios also agree with intuition but bring the added insight that price sensitivity is far from symmetrical. Indeed, increases in the variable operating costs of sawmills led to larger price variations than decreases of the same magnitude. Scenario Paper+10 shows very clearly how an increase in paper prices will naturally improve the paper mills' marginal profits, thereby giving them greater buying power on the chip market, so that they share the additional profit from paper sales with independent sawmills by paying higher chip prices.

Finally, the cost of wood has no effect on the base situation, because it has no bearing on the advantages that are compared in deciding whether or not to chip lumber-quality wood. A very large increase in the price of wood would be expected to cause shutdowns in less efficient sawmills, thereby driving chip prices up.

In prior studies, conducted in the mid-'80s, NMR researchers attempted to understand the structure of the wood-fiber market from supply-and-demand data, but they lacked the conceptual framework that our LP model provided. Our model was the first tool available to the MNR that could simulate and predict the effect of several exogenous factors on the behavior of the chip market. Based on these results, the MNR found the model useful, in spite of the relative coarseness of its predictions, because it provided many valid insights. For instance, it revealed the extremely low elasticity between supply and demand on the chip market, and it explained the impact of integrated sawmills. At this point, the MNR judged the INOFIB model as sufficiently sophisticated for its intended purpose, and it considered the project completed.

Exploring Market Dynamics with a Simplified Model

About half way into the study, we decided we needed a deeper understanding of the chip market. Some team members had legitimate doubts about the overall direction of investigation, and we needed some way to prevalidate our approach. After all, a chip price deduced from plant technologies, production capacities, and macroeconomic data may not be appropriate to fit a market known for its erratic, moody behaviour. The prospect of developing a rather large model without some basis for confidence did not appeal to some. Although we built the simple model we are about to discuss early in the study, we chose to present it late in the paper because it brings insights that were later confirmed in scenario analysis in rather striking ways.

The goals of the reduced model were to use all the information gathered from individual actors, especially dual prices, and obtain an estimated market price that would take into account the "competition" between independent sawmills and paper producers. To that end we formed two aggregate actors, one (the aggregate seller) composed of all independent sawmills and the other (the aggregate buyer) of all paper producers together with their integrated sawmills. The aggregate buyer has its own demand for wood chips that can be satisfied by a mix of purchases (from the aggregate seller) and internal production (from the integrated sawmills). The demand for chips, a quantity closely related to the paper-production level, turned out to be the most interesting parameter. For the sake of simplification, we did not take transportation costs into account—we did not see them as an important factor in understanding the market behavior.

On the one hand, the aggregate seller's unit-price (say, p_1) curve is a plain sum of individual sawmill curves (Figure 2). Using recent data, we obtained the curve for the aggregate seller (Figure 6).

Since the aggregate seller sells chips at the marginal cost, the purchase (put) price of a given quantity x, say $P_1(x)$, would be the price for that quantity multiplied by that quantity (that is, $P_1(x) = p_1(x) \times x$) (Figure 7).

On the other hand, the aggregate buyer may acquire part of its chips from its own

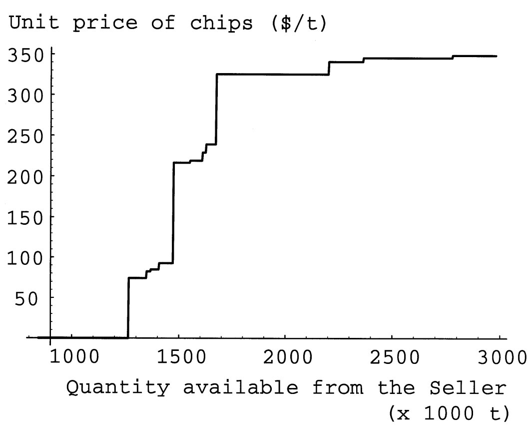

Figure 6: The dual-price (or marginal price) p_1 for the aggregate seller in the simplified model rises at every demand level at which one of the sawmills begins to chip higher quality wood.

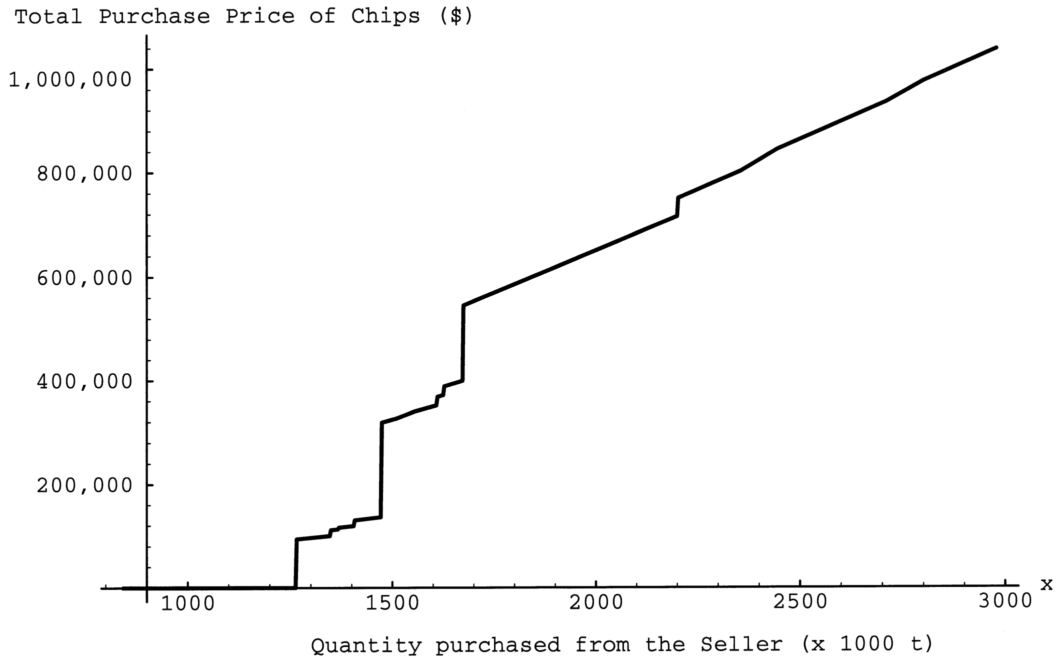

Figure 7: The total purchase price $P_1(x)$ of a quantity x of chips on the market is both discontinuous and piecewise linear, the slope being the marginal price p_1 of Figure 6. Abrupt changes in marginal prices clearly cause sharp, global increases in total cost for chip purchasers.

integrated sawmills. The marginal cost, say p_2, follows a pattern similar to that of the independent sawmills (Figure 6). However, since the transactions are intrafirm, each unit is priced at its *marginal* cost and the total cost for the aggregate buyer of purchasing a given quantity x from such sources is $P_2(x) = \int_0^x p_2(y)\, dy$ (Figure 8).

Putting it all together, we assume that the aggregate buyer needs a certain quantity D of chips (in tons). The demand D will be met by a mix of chips, say a quantity Q acquired internally and the remainder $D - Q$ from the aggregate seller. The resulting total cost is given by

$$P(D, Q) = P_1(D - Q) + P_2(Q).$$

Given the demand D, the aggregate buyer faces the sole decision of fixing the volume to be acquired internally (Q) to minimize its total cost $P(D, Q)$ (Figure 9).

There is a clear global minimum (marked [1] in Figure 9). The market price at the minimum can be traced in earlier graphs to a value around Cdn$ 75, a reasonable estimate. In addition, a number of local minima exist (for example, [2] in Figure 9). If the decision makers use something like a local search to improve costs, the industry may get stuck in nonoptimal local minima. Moreover, most minima correspond to quantities in the 3,500 to 3,700 range, where the aggregate buyer is nearly indifferent.

The corresponding quantity $D - Q$ of chips bought from independent sawmills (1,300 to 1,500) involves no extra chipping or chipping from all lower-grade timber (the first step of the dual-price function in

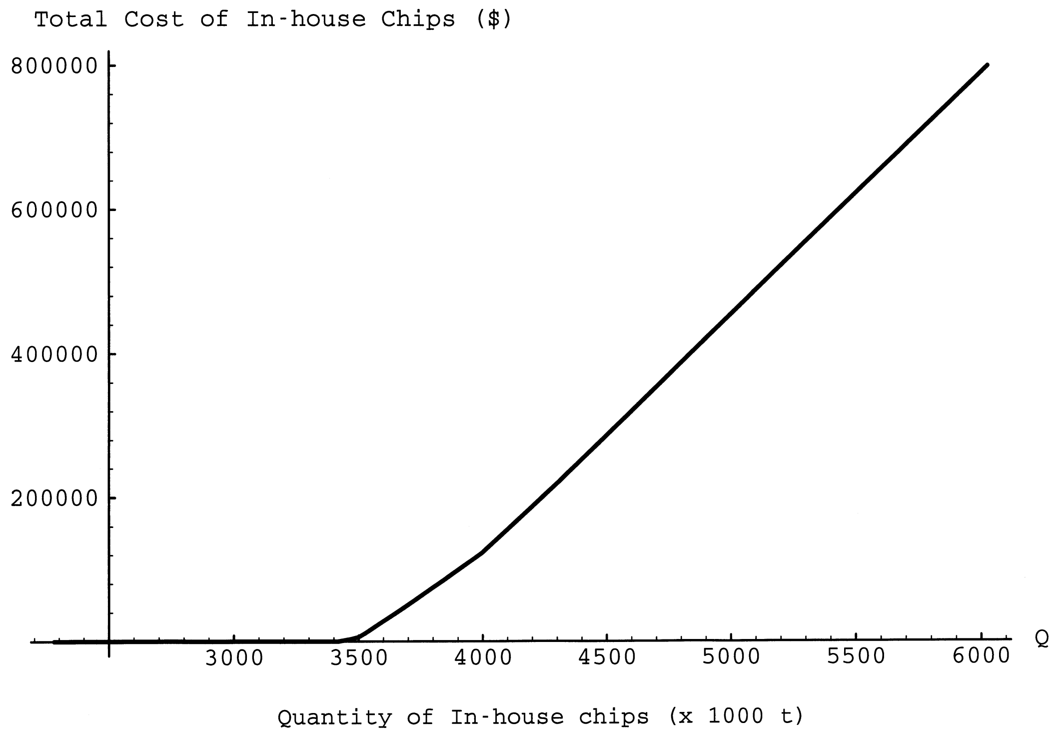

Figure 8: The total cost of a quantity *Q* of in-house chips for the buyer in the simplified model retains the same marginal price as purchases made on the market (Figure 7) but avoids sharp increases.

Figure 6). Therefore, independent sawmills may chip any amount of low-grade lumber into chips with some effect on market prices (which vary between 80 and 100) but hardly any on the aggregate buyer's total cost. Such a phenomenon may lead to increased influence of less rational price-point decisions, favoring the appearance of a very volatile market.

We made further interesting observations by studying the sensitivity of the total cost function $P(D, Q)$ to the demand D (which is very closely linked to the demand for paper). For example, a modest decrease of 10 percent in demand reduces the market price to zero (Figure 10), since only pure residues are purchased (one should add the cost of transportation). The prices are therefore very sensitive to the demand for chips, which depends on the demand for paper.

The simplified model doesn't faithfully represent actual market behavior. After all, it is monopolistic and neglects transportation costs. Nonetheless, it turned out to be a useful conceptual tool that dramatically revealed strong market forces that could induce paper producers to integrate sawmill operations to keep fiber prices down. A strong wave of sawmill integration was indeed happening in the industry. Moreover, the simplified model also revealed a situation in which it might be beneficial for integrated sawmills to produce less lumber than technologically allowed.

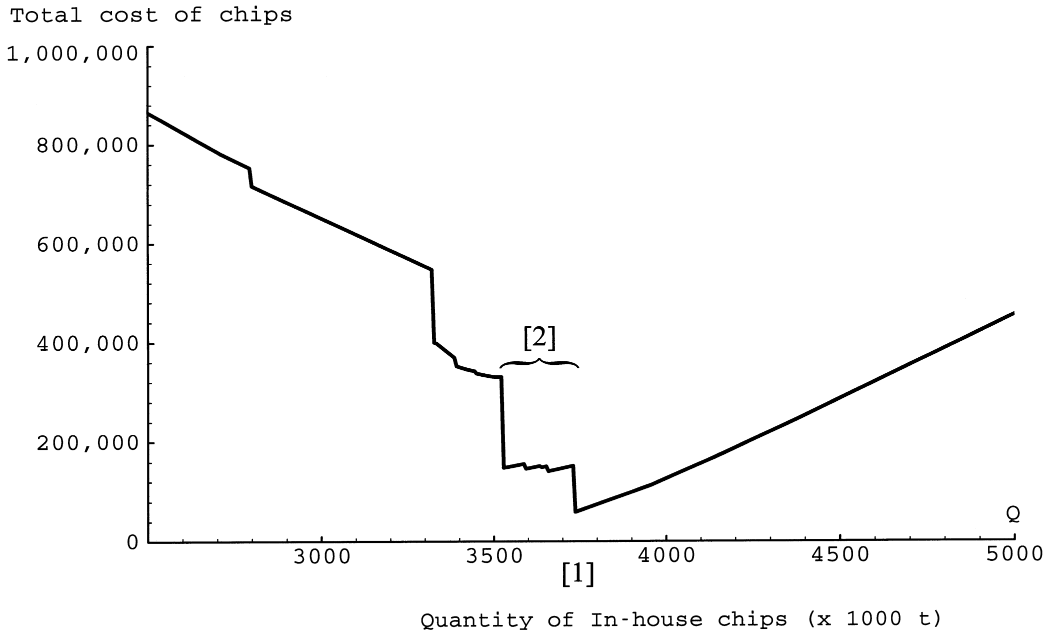

Figure 9: The total cost *P*(*D*, *Q*) of a given volume of chips (*D*) for the aggregate buyer depends on how it is split between in-house chips (*Q*) and chips purchased on the market ($D - Q$). The curve clearly shows the importance for the aggregate buyer of the decision to chip some lumber-quality wood in order to increase *Q* and thus reduce costs. Here $D = 5{,}000{,}000$ tons, a reasonable value.

Conclusion

While forest-management problems are known in the operations-research community to be difficult [Brumelle, Carley, Vertinsky, and Wehrung 1991; Weintraub and Bare 1996], this work presented no theoretical challenges. We encountered the most difficulties in pinning down the problem, making a number of simplifying assumptions, determining important factors in the chip price, and understanding the dynamics of that peculiar market.

Epilogue

Shortly after we completed this study, the MNR was confronted with an industrywide crisis due to record-breaking surpluses in wood-chip supplies in 1996. Industry representatives asked the government to take prompt action to smooth out the crisis. While the model predicted the chip surplus for one of the scenarios, the MNR had to choose its course of action carefully. Prices were excellent for lumber products, and export quotas were in effect (as a result of recent negotiations with the US government). It would have been unwise to decrease sawmill production because the export quotas on lumber could have been lost. Meanwhile, it needed to maintain the price of wood chips. The MNR decided to take a series of measures to improve the overall effectiveness of Québec's sawmill operations. It took three main steps to increase supply elasticity: (1) The province modified the law governing forest management in 1997 in such a way that the Forest Logging Rights will

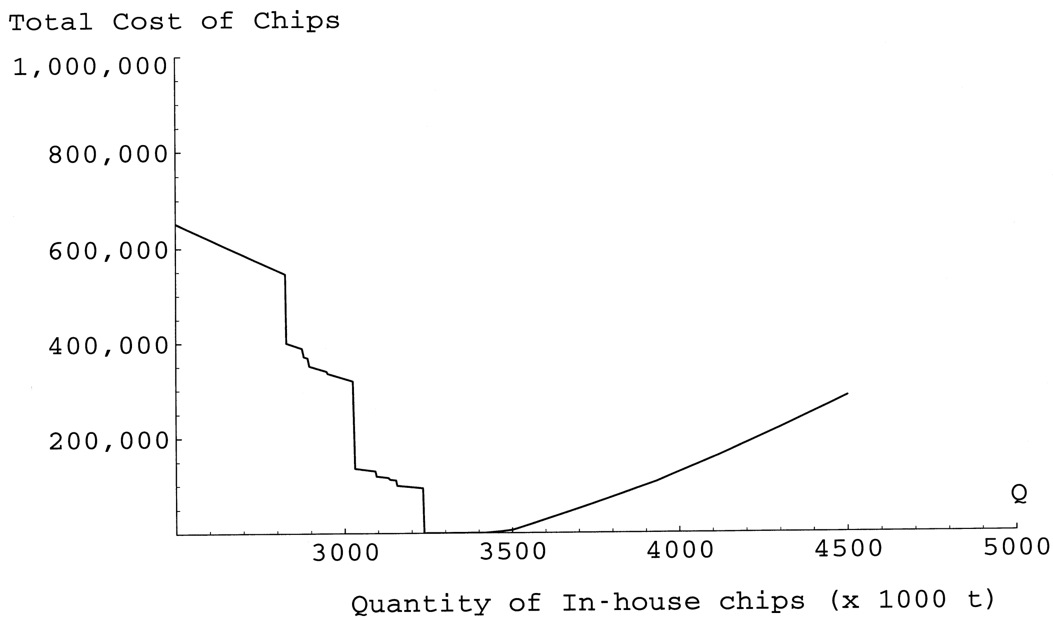

Figure 10: A slight 10 percent reduction in chip demand (*D*) reveals the possibility for the aggregate buyer to set its in-house chip production level ($Q \approx 3.3$ M tons) to drive both its total chip cost and the market price to zero.

henceforth be attributed according to lumber yield, so that the more-efficient sawmills will be allowed to harvest more timber than the least-efficient ones.

(2) The MNR ranked all major Québec sawmills according to yield. The ministry invited sawmills in the bottom quartile (the least-efficient ones) to submit plans explaining how they intended to improve performance.

(3) The ministry produced and distributed a technical leaflet [Forintek and MRN 1998] to sawmill operators explaining ways to increase yield.

The ministry also commissioned other economic studies to seek ways of improving market mechanisms for wood-chip exchanges and a technical study for better quality control of wood chips.

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APPENDIX

LP models

First, we present a linear-programming model for the optimal production of a typical independent sawmill during a planning period. The objective is to maximize profits obtained by subtracting variable production costs from sales revenues (for lumber and sawdust, but excluding wood chips).

All decision variables are in thousands of tons. C and S are wood chips and sawdust (including shavings), respectively. T, R, and X are raw timber (total, regular, and extra supply). T_j is the raw timber used for producing a quantity Q_j of finished lumber of grade j (1: 2×4, 2: utility, 3: short). All variables are nonnegative.

The first constraint represents market demand (for parametric dual-price analysis):

$C \geq 100$ (say).

The second group of constraints restricts timber supply, allowing up to 10 percent from extra sources:

$R \leq 343,$

$X \leq 0.10\ R,$

$T = R + X.$

The third group insures mass conservation of raw material and finished products, including an extra six percent for treetops:

$C + T_1 + T_2 + T_3 = 1.06\ T,$

$S + C + Q_1 + Q_2 + Q_3 = 1.06\ T.$

The fourth group of constraints simulates sawmill technology:

$C \geq 0.40\ T,$

$T_1 \leq 0.36\ T,$

$T_1 + T_2 \leq 0.41\ T.$

The fifth group accounts for shavings removed when finishing lumber:

$Q_1 = 0.97\ T_1,$

$Q_2 = 0.90\ T_2,$

$Q_3 = 0.90\ T_3.$

The LP model for a paper plant is slightly more involved and contains blending constraints. (The details are not crucial for understanding the problem under consideration.) We built the LP model for a paper plant with integrated sawmills by combining the paper plant and sawmill constraints, adding the paper, lumber, and sawdust revenues, and subtracting variable production and transportation costs.

Parametric analysis yields the piecewise linear functions $F_j(y_j)$ and $G_i(z_i)$, where the first derivative of $F_j(y_j)$ gives the marginal price the jth buyer would pay for the y_jth unit of wood chips it uses, and the first derivative of $G_i(z_i)$ gives the marginal price the ith seller will want for the z_ith unit of wood chips it produces.

The economic equilibrium model can be written as follows. The decision variables are x_{ij}, the quantity of wood chips exchanged between the ith producer and the jth consumer. Let c_{ij} be the unitary transportation cost.

$$\text{Max} \sum_j F_j(y_j) - \sum_i G_i(z_i) - \sum_i \sum_j c_{ij} x_{ij}$$

$$\text{subject to } z_i = \sum_j x_{ij} \text{ (for all } i)$$

$$y_j = \sum_i x_{ij} \text{ (for all } j).$$

This mathematical program can easily be transformed into a linear-programming model if one replaces z_i and y_j by a sum of bounded variables and uses the dual prices of parametric analysis as coefficients in the objective function.

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Jean-Paul Gilbert, director, Direction du développement de l'industrie des produits forestiers, Ministère des Ressources naturelles, Gouvernement du Québec, 880, Chemin Sainte-Foy, Québec, G1S 4X4, Canada, writes: "Le projet INOFIB a permis l'élaboration d'un modèle de programmation linéaire reproduisant le fonctionnement du marché des copeaux de bois résineux au Québec.

"Ce projet a eu des impacts positifs importants au ministère des Ressources naturelles du Québec. En effet, l'élaboration du modèle a fourni le cadre conceptuel permettant de regrouper l'information stratégique dont nous disposions sur les usines de bois de sciage et de papier journal. La spécification du modèle et l'analyse des résultats de simulation nous a permis de mieux comprendre le fonctionnement du marché des copeaux ainsi que l'interaction des variables et a fait ressortir des aspects qui n'étaient pas pris en compte auparavant dans nos travaux. Le modèle a également permis de démontrer qu'en dépit de sa caractéristique d'oligopsone, ce marché pouvait générer de véritables prix de marché si les agents économiques optimisaient leur comportement économique.

"L'utilisation d'outils quantitatifs comme INOFIB permet d'informer les agents économiques d'envisager des approches novatrices dans la gestion de la Loi sur les forêts afin de permettre aux forces du marché de jouer un rôle véritable sur le marché des copeaux au Québec."