Mount Sinai Hospital Uses Integer Programming to Allocate Operating Room Time

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An integer-programming model and a post-solution heuristic allocates operating room time to the five surgical divisions at Toronto's Mount Sinai Hospital. The hospital has used this approach for several years and credits it with both administrative savings and the ability to produce quickly an equitable master surgical schedule.
(*Health care: hospitals. Programming: integer, applications*)

I n most developed countries, except the United States, strategies for providing universal access to health care while containing costs involve formal or informal caps on health-care spending (Pierskalla and Brailer 1994, p. 470). As a result of centrally constrained funds, resource-allocation problems in which a central authority must plan for and allocate limited resources to competing groups or services are common issues in health-care planning at the national, regional, and institutional levels.

Canadian experience suggests that a centrally imposed limit on expenditures is an effective method for controlling the size of a national health-care system. While the percentage of gross domestic product (GDP) spent on health care in Canada in 1997 was 9.0 per cent, the proportion of GDP spent on health care in the United States was 13.6 per cent (Canadian Institute for Health Information 1999, p. 26). Centrally limited budgets do, however, create a problem of scarcity. Under such systems, demand for resources outstrips supply, necessitating a complex and often contentious process for determining how to best allocate resources between competing demands.

In a resource-allocation problem at Mount Sinai

Hospital, the hospital must allocate a limited amount of operating room (OR) time to its surgical divisions. After a decrease in provincial funding resulted in a reduction in the total amount of operating-room time available, we devised a method for equitably dividing operating-room time using integer-programming. Other authors have employed a similar approach to problems in the public sector. For example, Keown and Martin (1976) describe the application of integer goal programming to capital-budgeting decisions in hospitals; Tingley and Liebman (1984) present an integer-programming model that allocates federal funds for social welfare programs to local provider organizations; and Diminnie and Kwak (1986) and Kwak and Diminnie (1987) describe the use of integer goal programming to select program alternatives under a shrinking budget in a university environment.

Background

As in the United States, health care in Canada is delivered by independent physician-entrepreneurs, who are paid a fixed fee for each service they provide. Hospitals in Canada are private, not-for-profit corporations. Neither physicians nor hospitals are formally tied to the federal or provincial governments. Each

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province in Canada runs its own health-insurance program. Participation in the public insurance scheme is mandatory and universal.

The Canadian health-care system is funded through a combination of premiums and taxes, which varies from province to province (Lave et al. 1992). Each provincial insurance program acts as a single-source payor for both hospital and physician services. The programs pay physicians retrospectively for the services they provide, while hospitals are funded in advance, each receiving a fixed annual budget to cover the anticipated costs for the coming year.

The difference in funding methods for hospitals and physicians is designed to balance the competing objectives of promoting cost containment and protecting the physician-patient relationship. When an individual

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After weeks of negotiation, an acceptable schedule would emerge.

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seeks health-care services, his or her knowledge of need, the treatment options available, and the efficacy of those options is limited. Since purchasers are at an obvious information disadvantage with respect to health-care providers, the concept of physician as patient agent has been widely adopted (Dranove and White 1987). Under an agency relationship, a patient does not purchase services from a physician. Rather, the patient "hires" the physician to act as his or her agent, to determine the services that are required, to identify the costs and effectiveness of the various options, and to select the treatment that is best for the patient. Agency is a contractual mechanism that provides physicians with an economic incentive to act in the patient's best interest (van Ackere 1993). Historically, fee-for-service payment mechanisms have been used to promote agency in health care; the more services the physician provides, the more money he or she earns.

The fee-for-service funding mechanism is not without problems. Since physicians are paid for each service they deliver, they have an economic incentive to provide any and all services that provide some benefit to a patient, regardless of how expensive the service or how small the benefit. There is a real danger under

fee-for-service payment mechanisms that physicians will provide too many services (Hay and Leahy 1982). Too much health care can be as dangerous as too little (Evans 1982).

To protect the agency relationship but to counter the tendencies toward over-servicing, governments in Canada have chosen to control hospital spending. Since the agency relationship is sacrosanct, governments cannot directly control physician behavior. Physicians, because they control the number, type, and intensity of treatment provided, have a significant impact on the overall cost of the health-care system. Governments in Canada have historically controlled the size of the health-care system by placing strict limits on hospital spending. Because physicians use hospital resources to provide treatment, controlling the size and number of hospital resources through a centrally limited budget acts as a surrogate mechanism for physician control and provides a natural limit on system growth.

In the early 1990s, financial pressures led to policies designed to restrict the size of Canada's health-care system. Federal and provincial governments each reduced health-care funding. While in the period from 1987 to 1992, real (adjusted for inflation) per capita hospital expenditures in Ontario grew by 3.2 percent per annum, real expenditures in the period from 1992 to 1996 fell by 0.3 percent per annum (Health Canada 1997). Total real funds made available to hospitals declined by almost $850 million during this period. In response to these reductions, hospitals in Ontario were forced to restructure their programs. Mount Sinai restructured by streamlining the types of services it delivered. As a result, the number of operating room hours declined by 6.1 percent from 423.5 hours per week in 1994 to 397.5 hours per week in 1999. As available operating room time declined, the hospital was forced to make a number of changes to the master surgical schedule.

Problem Description

The master surgical schedule is a cyclic timetable that defines the number and type of operating rooms available in an institution, the hours that rooms will be open, and the service or surgeons (if any) who are to

be given priority for operating room time. Surgical schedules generally follow one of two major booking strategies: block and nonblock (or first-come, first-served). The master surgical schedule, under a block-scheduling methodology, assigns a fixed amount of time at a given day and time to a particular surgeon or service. For example, every Monday the Department of Ophthalmology at Mount Sinai is assigned 7.5 hours in Main–8 from 08:00 to 15:30 (Table 1). Master schedules developed under nonblock methodologies do not assign a priority for operating room time to any particular service or surgeon; under such systems all surgeons compete for operating room time on a first-come, first-served basis. While common in the 1960s and 70s, first-come, first-served scheduling systems are now rarely seen in practice.

Mount Sinai Hospital has 14 operating rooms. Twelve main operating rooms are located on the hospital's fifth floor, although only 10 are staffed on a scheduled basis. Historically these rooms were used for major inpatient procedures. The hospital also has two elective operating rooms, located on the main floor of the hospital, which are used for elective outpatient procedures. This area consists of a stand-alone outpatient surgical unit where patients are admitted, walked to the operating room, held while recovering in a first- and second-stage post-anesthetic care unit (PACU), and discharged. This area is known as the elective outpatient surgery (EOPS) unit. With the increased use of outpatient surgery in the past decade, the designation of main and elective rooms is now somewhat anachronistic; many outpatients are now treated in the main operating rooms. In general, however, the hospital reserves the use of the elective rooms for shorter, less complex, outpatient cases.

Mount Sinai has five surgical departments: Surgery, Gynecology, Ophthalmology, Otolaryngology, and Oral Surgery. The largest of these departments, Surgery, consists of five subspecialties: Orthopaedics, General Surgery, Plastic Surgery, Vascular Surgery, and Urology. The Department of Surgery is by far the largest user of perioperative services, consuming 189.0 hours of surgical time per week, or 47.5 percent of the 397.5 hours available each week. The smallest, Oral Surgery, consumes only 19.7 hours per week or 5.0 percent of the total. In addition, the hospital maintains a small reserve of time (7.5 hours per week) for emergency surgery above and beyond the time allocated to each of the five departments. For the purposes of planning, emergency surgery is treated like a sixth department, although any department may book cases into the emergency block. At Mount Sinai, a committee comprised of surgeons, anaesthetists, and the nurse manager allocates surgical time to the six departments. The individual department chiefs then allocate the department time to subspecialties or individual surgeons.

In some senses, the master surgical schedule (MSS) can be thought of as being equivalent to the aggregate production plan in a manufacturing environment. Because it defines the number and types of procedures that will be performed by a hospital over the medium term, the MSS defines aggregate resource requirements, such as the demand for nurses, drugs, diagnostic procedures, laboratory tests, and perioperative nurses. The daily operating room schedule, which lists the actual cases to be performed, start times, end times, attending physician, and anaesthetist, is an operational document that functions in a manner similar to the

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production schedule in a manufacturing plant. The daily operating list (Table 2) defines a series of tasks (cases) to be completed and assigns these to production resources (rooms).

A substantial and mature operations research literature describes techniques for manipulating the master surgical schedule, or the order of cases on the daily operating list, to maximize institutional goals or objectives. For example, Hancock and associates developed a simulation-based tool to maximize inpatient surgical occupancy (Hancock and Walter 1983, Hancock and Isken 1992, Magerlein 1978).

As is the case in manufacturing planning, the master surgical schedule is used as a tool to respond to short-term and medium-term capacity constraints. In Canadian hospitals, the overarching production constraint is typically operating funds. The master surgical

Perioperative Services OR Schedule-8 + 2 Rooms (Option 1)

	Main-1	Main-2	Main-3	Main-4	Main-5	Main-6	Main-7	Main-8	Main-9	Main-10	EOPS-1	EOPS-2
Mon	Surgery	Surgery	Surgery	Gynecology	Surgery	Otolaryngology* Surgery	Gynecology	Ophthalmology			Oral Surgery	Gynecology
	08:00–17:00	08:00–17:00	08:00–17:00	08:00–17:00	08:00–15:30	08:00–15:30	08:00–15:30	08:00–15:30			08:00–16:00	08:00–15:30
Tue	Surgery	Surgery	Surgery	Surgery	Surgery	Otolaryngology	Gynecology	Oral Surgery** Ophthalmology			Gynecology	Gynecology
	08:00–17:00	08:00–17:00	08:00–17:00	08:00–17:00	08:00–15:30	08:00–15:30	08:00–15:30	08:00–15:30			08:00–15:30	08:00–16:00
Wed	Surgery	Surgery	Surgery	Surgery	Surgery	Otolaryngology	Gynecology	Gynecology			Gynecology	Ophthalmology
	08:00–17:00	08:00–17:00	08:00–17:00	08:00–17:00	08:00–15:30	08:00–15:30	08:00–15:30	08:00–15:30			08:00–16:00	08:00–15:30
Thur	Surgery	Surgery	Gynecology	Gynecology	Surgery	Gynecology	Emergency	Ophthalmology			Gynecology	Ophthalmology
	08:00–17:00	08:00–17:00	08:00–17:00	08:00–17:00	08:00–15:30	08:00–15:30	08:00–15:30	08:00–15:30			08:00–16:00	08:00–15:30
Fri	Surgery	Surgery	Surgery	Surgery	Surgery	Otolaryngology	Gynecology	Ophthalmology			Oral Surgery	Gynecology
	08:00–17:00	09:00–17:00	09:00–17:00	09:00–17:00	09:00–15:30	09:00–15:30	09:00–15:30	09:00–15:30			09:00–15:30	09:00–16:00

Notes: *Surgery 1, 2; Otolaryngology 3, 4, 5.
**Oral Surgery 1, 2, 3; Ophthalmology 4, 5.

Version Summary	Surgery	Emergency	Gynecology	Ophthalmology	Oral Surgery	Otolaryngology	Total
Original Hours	208.5	6.0	129.5	43.5	22.0	29.0	438.5
Original %	47.5%	1.4%	29.5%	9.9%	5.0%	6.6%	100.0%
Target Hours	189.0	5.4	117.4	39.4	19.9	26.3	397.5
Assigned Hours	189.0	7.5	117.0	38.8	19.7	25.5	397.5
Assigned %	47.5%	1.9%	29.4%	9.8%	5.0%	6.4%	100.0%
Difference − Hours	0.0	2.1	−0.4	−0.6	−0.3	−0.7	0.0
Difference − %	0.0%	0.5%	−0.1%	−0.2%	−0.1%	−0.2%	0.0%

Table 1: This example master schedule uses a time horizon of one week with exceptions based on the number of calendar days in a month, for example, Main–6 is shared by Otolaryngology and Surgery on Mondays. The note indicates that the room is assigned to Surgery on the first and second Monday of every month and to Otolaryngology on the third, fourth, and fifth Monday of every month.

The version summary indicates the total amount of operating room time and percentage share of total time each department has before and after time has been reallocated. In this example, Surgery originally held 208.5 hours of operating room time (47.5 percent of the 438.5 hours available). The target time of 189.0 indicates the amount of time Surgery should be allotted to maintain a 47.5 percent share of 397.5 hours of available operating room time. In the actual allocation given in the table, the Department of Surgery received 189.0 hours or 47.5 percent of the total hours available.

schedule is also used as a tool to respond to short-term shortages of surgeons or anaesthetists, seasonal fluctuations in demand during the summer and at Christmas time, or long-term changes in program emphasis resulting from staffing changes or strategic management-decisions. Thus, in any given year, the manager may produce three or four distinct schedules. Whenever the schedule is revised, the nurse manager must ensure that a consistent number of rooms run each day of the week with the same opening hours from day to day. The nurse manager must also ensure that the master schedule makes efficient use of the perioperative nursing staff and does not violate the collective agreement between the hospital and the nurses' union.

Schedule development is a three-stage process. Hospital management, working in conjunction with the operating room subcommittee, determines the gross number of operating room hours that will be made

OR Schedule		Wednesday-2000/10/03		Anaesthesia
Room: 01		ANAES: Davis, L.		ANAES RES: Student, P.
Time	Patient	Admission	Surgeon	Procedure
8:00	Smith, J.	OP	Internist, A.	Gall Bladr: Cholecystectomy
9:45	Jones, P.	SDA	Internist, A.	Groin: LT Hernia Repair Inguinal
11:15	Henry, K.	SDA	Internist, A.	Groin: LT Hernia Repair Inguinal
12:45	Green, C.	SDA	Internist, A.	Groin: LT Hernia Repair Inguinal
Room: 02		ANAES: Ballard, H.		ANAES RES: Richard, M.
8:00	Horner, R.	7B	Smythe, C.	Uterus: Hyst ABD; BSO; Pelvic Lymph
9:45	Patrick, L.	SDA	Smythe, C.	Uterus: Hyst ABD; BSO: Pelvic Lymph
12:00	Norris, M.	SDA	Smythe, C.	Adomen:Laparotomy-Staging
14:00	Barilko, W.	SDA	Smythe, C.	Perineum: Excision Vulvar Lesion
Room: 03		ANAES: Cobb, T.		ANAES RES: Jackson, J.
8:00	Ruth, B.	7B	Young, C.	Ear: Tympanomastoid Revision
11:00	Gerhig, L.	SDA	Young, C.	Nose: Septoplasty
14:15	Robinson, J.	SDA	Young, C.	Ear: Myringotomy & Tubes
15:30	DiMaggio, J.	SDA	Young, C.	Throat: Tonsillectomy
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Table 2: The daily operating room schedule lists cases to be performed, start times, end times, attending physician, and anaesthetist on any given day. Once a master surgical schedule has been completed (Table 1), surgeons are free to book cases into their assigned rooms in the manner and order that they see fit. (The names in this table are fictitious.)

available. (The number of operating room hours available is a function of the budget provided by the hospital for perioperative nursing.) The nurse manager, using this figure as a guide, develops a small number of alternative schedule arrangements that meet the gross number of hours and are feasible in terms of the nurses' collective agreement. At this point, the schedule is simply a template indicating the number of rooms that will be made available each day of the week and the number of hours the rooms will be open.

Once the manager has developed the template for the MSS, he or she must assign the available time to the surgical departments. This is a complex and often contentious task. Managers defining a new MSS are caught between the competing objectives of the hospital to constrain costs by reducing operating room time and the objectives of physicians to maximize income by expanding their use of available time. To reduce the inevitable conflict between surgical departments, any addition to, or reduction from, the master surgical schedule must be equitably distributed among the departments. While it is possible to define equity

in many ways, in this context, equity means that the proportion of the total time assigned to a department before a schedule revision must be the same as the proportion assigned after the schedule has been revised.

In addition to being equitable, a good schedule is assumed to have the following characteristics:

(1) Wherever possible, only one department should be assigned to any operating room on any day. While it is possible to split blocks into morning and afternoon sessions, afternoon sessions are considered undesirable, because the department working in the morning may run late and cut into the time assigned to the afternoon department. If it is necessary to allocate a block of time between two departments, it is preferable to assign entire blocks to different departments on different weeks. (For example, one department has the block on weeks 1 and 2 of the month, and the other department has the block on weeks 3, 4 and 5.)

(2) The schedule should, if possible, be consistent from week to week. While it is acceptable to allocate blocks to different departments on different weeks, the number of such allocations should be minimized.

(3) The schedule must respect bounds on the minimum and maximum number of blocks that can be assigned to a department on any given day or over the course of a week.

Prior to 1995, the master surgical schedule was generated by hand at Mount Sinai. The nurse manager, working in conjunction with the operating room management committee, would ascertain the total number of rooms per week that could be run based on the available budget for perioperative nursing staff and the current staff-to-room ratio. From this, he or she determined how many rooms would be available daily and the length of time each room would be open. After creating this template, the nurse manager assigned the blocks to the various departments. This involved a piece of graph paper and a pencil with a large eraser to fill the various slots. In general, the nurse manager would modify an existing schedule to create the new schedule. However, changes in the number of rooms available, the duration of rooms, and constraints on specialty equipment meant a great deal of trial and error was necessary to create a new feasible schedule.

The draft schedule derived from the manual process was circulated to the surgical departments for comments and suggestions for revision. Many iterations of the process were necessary because each revision to accommodate one service's request invariably affected another. After many weeks of negotiation and maneuvering, an acceptable schedule would eventually emerge. This was never a simple process; intense competition for available operating room time inevitably created friction among surgeons and between departments. The nurse manager was frequently caught in the middle of these disputes. Furthermore, an apprehension existed amongst physicians that the manual process was either biased or flawed.

The Model

A number of unique features allow us to formulate the issue of equitably allocating operating room time to the various surgical departments at Mount Sinai as an integer programming problem. Since the total number of operating rooms to be made available each day and the number of hours that each room will be open is predetermined, the problem can be seen as an allocation of departments to a set of rooms over a fixed planning horizon.

In practice, the nurse manager at Mount Sinai creates templates with four room types: a set of "long" and "short" rooms in both the main surgical suite and the EOPS suite. Short rooms are designed to run for the duration of a single nursing shift and are thus simple to staff. Long rooms, which permit surgeons to perform several complex surgeries back to back, generally last longer than a single nursing shift. The nursing staff in the main operating room at Mount Sinai comprises 25 full-time and seven part-time employees. EOPS is staffed separately with seven full-time and two part-time nurses. In both the main operating room and EOPS, 2.5 persons are assigned to each room (a scrub nurse, a circulating nurse, and a relief person who is shared between two rooms) at all times. Shifts are eight hours long, and start times are staggered to allow for long rooms and emergency coverage.

Regularly scheduled surgery takes place on weekdays. In the main surgical suite, long rooms run from 08:00 to 17:00 daily, Monday through Thursday, and short rooms run from 08:00 to 15:30, Monday through Thursday. On Fridays, the nursing staff has an hour-long in-service education meeting. Thus on Fridays in the main surgical suite, long rooms run from 09:00 to 17:00 and short rooms run from 09:00 to 15:30. In the EOPS suite, long rooms run from 08:00 to 16:00, while short rooms run from 08:00 to 15:30, Monday through Thursday. As is the practice in the main surgical suite, a one-hour in-service session takes place on Fridays, and thus start times are one hour later. In keeping with the idea of schedule simplicity, the nurse manager tends to build templates in which the number of long and short rooms in both the main surgical suite and the EOPS suite are constant from Monday through Friday (Table 1).

If we define i to be operating room type, j to be department, and k to be the day of the week, then x_{ijk} can be defined as an integer variable representing the number of blocks of type i to be assigned to department j on day k. If d_{ik} represents the duration of block type i on day k, then the problem of equitably allocating operating room time to surgical departments can be seen as selecting appropriate values of x_{ijk} such that the sum

of all time allocated to a particular department over all days ($\Sigma_i \ \Sigma_k \ d_{ik}x_{ijk}$ for each department) is equal to a target number of hours. Since equity is defined as a consistent share of total operating room time before and after a schedule change, the target time for each department on a new schedule is determined by multiplying the department's existing share of operating room time by the total amount of time available on the new schedule. If t_d is the number of hours assigned to a department on the existing schedule, T_E is the total amount of time available to all departments on the existing schedule, and T_N is the total amount of time available to all departments on the new schedule, then t_j, the target hours for department j on the new schedule, can be calculated as follows:

$$t_j \ = \ T_N \left[\frac{t_d}{T_E} \right].$$

It is unlikely that an integer combination of blocks can be found that exactly equals the target time for each and every department. Thus, the problem is actually to minimize the difference between the assigned operating room time and the target time. We therefore define p_j to represent the penalty associated with department j's allocation of time. Since smaller departments lose proportionately more income per unit of time than larger ones, the value of p_j is calculated as follows:

$$p_j \ = \ \max\left(0, \ 1 \ - \ \frac{1}{t_j} \sum_i \sum_k d_{ik}x_{ijk}\right).$$

Since Mount Sinai considers shortfalls from target time to be extremely undesirable relative to the allocation of surplus time to a department, the penalty p_j applies only in instances when a department is assigned less time than its target. By setting a zero penalty for overallocation of time, the model exhibits no bias in the allocation of any surplus time to any department.

Constraining the solution are bounds on the number of rooms that may be assigned to a department. These limits arise from equipment restrictions, surgeon availability, or assumed patient volumes. Daily bounds on the number of rooms that can be assigned to any department may be global (for example, the Department of Surgery may run no more than eight rooms each

weekday) or type specific (for example, the Department of Gynecology must have at least one EOPS room each weekday). Besides daily limits on the number of rooms that can be assigned to any department, weekly bounds constrain the number of rooms that can be allocated to a department over the entire week. Weekly bounds are all specific to room type (for example, the Department of Ophthalmology must have at least two EOPS blocks over the course of the week). In addition to constraints on the number of rooms that may be assigned to a service, a set of constraints is included in the model to limit the over or under allocation of time to any given service (Appendix).

When run, this model provides the allocation of whole blocks to departments that minimizes the shortage of time to each. The resulting schedule has a number of nice features. Since the model selects only whole blocks, the resulting schedule has no split blocks and is entirely consistent from week to week. The model's bounds ensure that the resulting schedule is feasible. The schedule, however, is only near equitable in the sense that splitting a whole block assigned to a service

The model has greatly reduced conflict.

with too much time and allocating a portion of that time to a service with too little time may improve the quality of the allocation. With some minor postprocessing, however, the schedule produced by the IP model can be improved.

According to rules specified by the hospital, it is permissible to allocate the same block to different departments on different weeks. Thus, to improve the schedule returned by the IP model, we simply allocate some rooms to different departments on different weeks. However, the number of such allocations should be minimized. In addition, managers and surgeons have stated that they like the schedule to be simple. Through trial and error, we have interpreted a simple schedule to mean that no block is allocated to more than two departments in any calendar month. Using these two rules, we developed a heuristic to improve the schedule produced by the IP model.

The improvement algorithm is essentially enumerative in nature. Taking the assignment produced by

the IP model, the algorithm selects, in list order, a department labeled p, as a potential contributor of time to another department. The algorithm then selects another department, labeled q where $q \neq p$, and scans through its scheduled allotment of time. On each day for which department p has at least one complete block of time available, the algorithm creates a trial schedule in which some portion of department p's block time is allocated to department q. Since the rules for splitting blocks at Mount Sinai are based on the number of weeks in a calendar month, the algorithm tests 10 different allocations between departments p and q. (Department q may be allocated one, two, three, four, or five weeks out of every calendar month.) The algorithm then determines the penalty score for the trial schedule. If making the proposed change decreases the penalty score, the trial schedule is retained as a possible replacement for the current schedule. If not, the trial schedule is discarded and the algorithm continues. If the allocation of time from department p to department q violates any room constraint on any day, the algorithm simply moves on to the next day of the week. This process of developing and evaluating trial schedules continues until schedules have been generated for each service q and each day. After all combinations of donor and receiver departments have been tested, the best trial schedule is accepted as the new schedule. The algorithm is repeated until no improvement can be found. Although the improvement algorithm is enumerative, the restricted problem size makes this approach feasible. Once the IP model has generated an initial solution, the heuristic is able to obtain an improved solution with very little extra effort.

The front end for this system is MS-Excel. Historically, the nurse manager at Mount Sinai has used Excel for generating and evaluating schedules. We retained Excel, because the manager is familiar with it and it provides good graphical capabilities for displaying output. Typically, solutions require about 1.12 minutes. Of this time, 0.90 minutes are required to generate an initial integer solution through a call to a CPLEX solver. The improvement algorithm, running as a VB macro from Excel, terminates in approximately 0.21 minutes. (The model has recently been ported to run entirely under Excel using Frontline System's Extended Solver. We invite interested readers to contact us for more information and an example copy.)

We note that the definition of equity used to define the objective function could create a problem if the model is sequentially applied over several iterations. Equity exists in this problem when a department's share of total operating room time is the same before and after reallocation of time. Each time the model is solved, however, some error is introduced, since an integer solution that precisely achieves each department's target can seldom be found. In the example in Table 1, the Department of Otolaryngology's share of total operating room time decreases from 6.6 percent to 6.4 percent; a shortfall of 42 minutes. If this output were used to set the targets for subsequent runs, department allocations could drift over time because of accumulated error.

In practice, however, accumulated error tends not to be a great concern. Mount Sinai and its medical staff review the distribution of operating room time to surgical departments on a regular basis when surgeons join or leave the medical staff or when programs merge through regionalization. The target share of time given to each service is a management decision that is reviewed frequently and adjusted as necessary. These adjustments tend to reset the target share and thus eliminate accumulated error. The model can be used when the target allocation of time to department j, t_j, is set by management directive rather than calculated from an existing schedule.

Results

The model has enjoyed some success. Initially implemented in 1996, the program has been in full production since 1997. All of Mount Sinai's schedules have been produced using a version of this algorithm since that time. The model provides a number of qualitative and quantitative benefits. The clerical time required to produce a schedule has been reduced from days to one or two hours. In addition, the operating room manager's time for schedule development has been greatly reduced. This has resulted in annual savings of approximately $20,000.

The faster turnaround means that each time a schedule revision is necessary many different scenarios can be evaluated in the same time it took to generate a single schedule by hand. This gives hospital management greater flexibility and an increased ability to explore creative scheduling options. Thus, the quality of schedules generated has been improved. Each time the manager generates a new schedule, he or she need only glance at the model output that demonstrates the percentage of time allotted to each department to recognize what the implications are for each service. Once the schedule appears equitable, the nursing manager simply checks the schedule to ensure that services are distributed in a manageable way throughout the days of the week and verifies that equipment and supplies criteria have been met. The schedule then goes to each of the department chiefs who distribute their department's time to individual surgeons.

The most obvious benefit of the model is its ability to produce an unbiased, equitable schedule through a consistent, identifiable process. Each time it is run, the model produces a numerical display of the total available hours along with each department's allocated percentage of time and the percentage of time lost or gained by each department. Thus, departments cannot argue about equity because the numbers are always available and can be validated on a schedule-by-schedule basis. The model benefits the nurse manager because it provides a transparent, consistent process, which is credited with being fair. Furthermore, because the model's objectives are straightforward and the assignments it generates are generally very good, there is no apprehension of bias associated with the new process. Thus, the model has greatly reduced conflict amongst surgeons and between surgeons and the nurse manager and has helped to improve the overall organization of perioperative services at Mount Sinai Hospital.

Conclusion

The operating room is the engine that drives the hospital. A proper master surgical schedule is essential to ensure that the surgical operations of a hospital proceed in an efficient and regulated manner. The Association of Operating Room Nurses (1999) notes that accurate scheduling of procedures and collaboration among health-care providers is an essential component of proper planning. With the introduction of this model, the scheduling process at Mount Sinai has been greatly improved. Political maneuvering has been reduced, since surgeons no longer question the fairness of the calculations or the method used to allocate time. Blocks are now assigned to each surgical department, and negotiations for changes take place between the chiefs of the individual departments. The practical benefits are substantial. The nurse manager is no longer tied up in a clerical exercise requiring several days as surgeons nit-pick each entry in the proposed schedule. Conflict among surgeons and between surgeons and the nurse manager has also been greatly reduced.

The drive toward improved efficiency in health care is never ending. Automating time-consuming clerical tasks, such as creating the master surgical schedule, frees managers to concentrate on more important issues. This is of vital importance in the Canadian health-care system as regionalization and rationalization make mergers and amalgamations of programs commonplace. By using our scheduling algorithm, Mount Sinai Hospital has made the work of the operating-room nurse manager less cumbersome, more methodical, and more efficient. The algorithm is a useful tool that the nurse manager finds beneficial for mitigating the sensitive issues surrounding scheduling and that the hospital uses to achieve greater operational efficiency.

Appendix

Let i be operating room type ($i = 0, 1, \ldots, 3$), j be department ($j = 0, 1, 2, 3 \ldots 5$), and k be the day of the week ($k = 0, 1, 2, \ldots, 6$). Then x_{ijk} can be defined as an integer variable representing the number of blocks of type i to be assigned to department j on day k. Assume d_{ik} represents the duration of block type i on day k and t_j represents the target allocation of operating room time to department j and a_{ik} represents the total number of rooms of type i available on day k.

The total amount of operating room time allocated to department j is

$$\sum_{i=0}^{3} \sum_{k=0}^{6} d_{ik} x_{ijk}.$$

Define s_j^+ to be a variable representing the amount of oversupply of operating room time to department j and s_j^- to be a variable representing undersupply of operating room time to department j relative to t_j. Thus,

$$\sum_{i=0}^{3} \sum_{k=0}^{6} d_{ik} x_{ijk} + s_j^- - s_j^+ = t_j.$$

Define p_j to be the penalty associated with the undersupply of operating room time to department j. This penalty can be calculated as follows:

$$p_j = \max\left(0,\ 1 - \frac{\sum_{i=0}^{3} \sum_{k=0}^{6} d_{ik} x_{ijk}}{t_j}\right)$$

$$= \max\left(0,\ \frac{1}{t_j}\left[t_j - \sum_{i=0}^{3} \sum_{k=0}^{6} d_{ik} x_{ijk}\right]\right) = \frac{1}{t_j} s_j^-.$$

Given this penalty, we may define the integer programming portion of the problem of equitably allocating operating room time to surgical departments as follows:

$$\text{Minimize } \sum_{j=0}^{5} p_j$$

subject to

$$\sum_{i=0}^{3} \sum_{k=0}^{6} d_{ik} x_{ijk} + s_j^- - s_j^+ = t_j \quad \forall j, \tag{1}$$

$$\sum_{j=0}^{5} x_{ijk} = a_{ik} \quad \forall i,\ \forall k, \tag{2}$$

$$LDGlob_{jk} \le \sum_{i=0}^{3} x_{ijk} \le UDGlob_{jk} \quad \forall j,\ \forall k, \tag{3}$$

$$LDSpec_{ijk} \le x_{ijk} \le UDSpec_{ijk} \quad \forall i,\ \forall j,\ \forall k, \tag{4}$$

$$LWeek_{ij} \le \sum_{k=0}^{6} x_{ijk} \le UWeek_{ij} \quad \forall i,\ \forall j, \tag{5}$$

$$0 \le s_j^- \le 10 \quad \forall j, \tag{6}$$

$$x_{ijk},\ s_j^-,\ s_j^+ \ge 0 \quad \forall i,\ \forall j,\ \forall k, \tag{7}$$

$$x_{ijk} \in I \quad \forall i,\ \forall j,\ \forall k. \tag{8}$$

In the above model, constraint (1) calculates the penalty s_j^- associated with the allocation of operating room time to service j. If the total sum of the time allocated to service j ($\Sigma_i \Sigma_k d_{ik} x_{ijk}$) is equal to the target time (t_j), then s_j^- and s_j^+ are both zero. If the service j is assigned less than its target time, then s_j^- takes on a positive value; if the total time assigned is greater than the target time, s_j^+ takes on a positive value. Since s_j^- is minimized in the objective function, the model's objective is to minimize underallocation of operating room time.

Constraint (2) is designed to ensure that the number of rooms of type i assigned to all departments on day k is equal to the total number of rooms of that type that are available (a_{ik}).

Constraints (3), (4), and (5) are bounds on the number of rooms that can be assigned to a department. Constraint (3) sets daily global bounds on the number of rooms of all types that can be assigned to department j on day k. $LDGlob_{jk}$ represents the lower daily global

bound on rooms of all types that can be assigned to service j on day k; $UDGlob_{jk}$ represents the upper daily global bounds. Constraint (4) is similar to constraint (3), but it sets daily bounds on the number of rooms of a specific type i that can be assigned to department j on day k. $LDSpec_{ijk}$ represents the lower bound, while $LDSpec_{ijk}$ represents the upper bound. Constraint (5) sets bounds on the total number of rooms of type i that can be assigned to department j in a given week.

Constraint (6) is an arbitrary bound on the maximum underallocation of time to service j. In our model, we limit the total underallocation to 10 hours.

Constraint (7) is the usual nonnegativity constraint on model variables. Constraint (8) defines each x_{ijk} variable to be an integer.

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Sharon Ball, RN, BScN, CPN©, Manager, Perioperative Services, Mount Sinai Hospital, 600 University Avenue, Toronto, Ontario, Canada M5G 1X5, writes: "At Mount Sinai Hospital we are required, several times per year, to produce a new operating room schedule. Scheduling the operating rooms by hand was always a difficult and time-consuming task. We had to keep in mind the overall allocation of time to each of the surgical departments as well as equipment restrictions, personal preferences, teaching schedules, etc. The time required to make up a new schedule was enormous. Furthermore, once a schedule was created, it was often difficult to get physician buy-in. Balancing the competing requests for times and dates was very difficult.

"The system produced by Dr. Blake and Dr. Donald has eliminated most of the difficulties associated with the manual system. The computer takes the drudgery out of generating a schedule. The speed with which the system produces schedules furthermore allows us to create and evaluate a number of different alternatives in the time formerly required to create one by hand. This has allowed us to improve the quality of our schedules. However, by far, the greatest benefit of this system is that it is perceived to be unbiased. Since the computer has no alliances, it cannot be considered to favour any one department over any other. Thus, it has been much easier to obtain surgeon buy in with the new scheduling system."