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Nu-kote's Spreadsheet Linear-Programming Models for Optimizing Transportation

Larry J. LeBlanc, James A. Hill Jr.
Owen Graduate School of Management, Vanderbilt University, 401 21st Avenue South, Nashville, Tennessee 37203
{larry.leblanc@owen.vanderbilt.edu, james.hill@owen.vanderbilt.edu}

Gregory W. Greenwell
Nu-kote International, Inc., 200 Beasley Drive, Franklin, Tennessee 37064, greenwellg@nukote.com

Alexandre O. Czesnat
Owen Graduate School of Management, Vanderbilt University, 401 21st Avenue South, Nashville, Tennessee 37203,
alexandre.czesnat@owen.vanderbilt.edu

Nu-kote International manufactures ink-jet, laser, and toner cartridges; ribbons; and thermal fax supplies. We developed spreadsheet linear-programming models for planning shipments of finished goods between vendors, manufacturing plants, warehouses, and customers to minimize overall cost subject to maximum-shipping-distance policies. Nu-kote used versions of these linear programs (LPs) to model supply chains with different warehouse configurations. The LPs have between 5,000 and 9,700 variables and 2,452 constraints. Nu-kote has used these LPs for an inherently nonlinear problem to identify improved shipments that will reduce annual costs by approximately $1 million and customer transit time by two days. It has already saved $425,000 from insight gained from the model.

Key words: programming: linear, large-scale systems; transportation: freight-materials handling.
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Nu-kote International is the largest independent manufacturer and distributor of aftermarket imaging supplies for home and office printing devices. Its major products are ink-jet, laser, and toner cartridges; ribbons; and thermal fax supplies. The company manufactures more than 2,000 products for use in over 30,000 types of imaging devices. In addition to remanufacturing for the aftermarket, Nu-kote partners with original equipment manufacturers to develop and manufacture imaging supplies for new equipment. Nu-kote serves over 5,000 customers (commercial dealers, retail stores, and so forth) around the world from a network of five plants, five major vendors, and four warehouses. This privately-held company's headquarters are in Franklin, Tennessee, just south of Nashville (Figure 1).

Distribution Challenges

Nu-kote's strategy is to be the low-cost producer in the imaging supplies industry. In the past, Nu-kote reduced costs by transferring production to China, but distribution and transportation costs remained high. Nu-kote wanted to optimize shipping of finished goods in its transportation network for both inbound transportation (from vendors and plants to warehouses) and outbound transportation (from warehouses to customers).

During 2001 and 2002, an increasing amount of production in China, a drastic change in product mix, and double-digit growth in the retail sector forced the company to analyze the product flows from its vendors and manufacturing plants to its warehouses and customers. Today, Nu-kote fills most customer orders (outbound shipments) from its warehouse in Franklin, Tennessee because 50 percent of the US population is within 500 miles of Franklin and more than 80 percent of its top customers are within 1,000 miles. However, nearly 80 percent of Nu-kote's products are manufactured in Chatsworth, California or imported from China through the port of Long Beach, California. Fewer than 20 percent of its customers are located within 1,000 miles of Chatsworth and Long Beach. Thus, although Nu-kote provides adequate service to its customers by shipping everything from Franklin, managers recognized that this practice might not be cost effective. They decided to investigate the use of a formal optimization-based approach to analyze shipments within their supply chain.

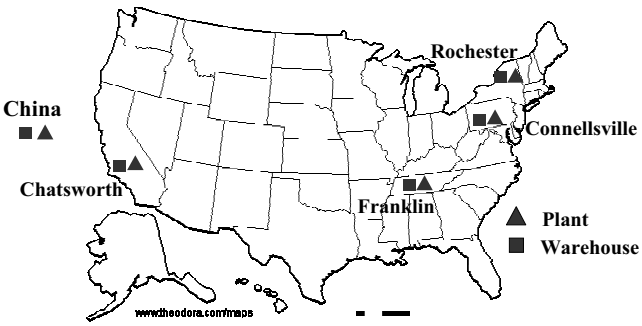

Figure 1: Nu-kote has both plants and warehouses in Rochester, New York; Connellsville, Pennsylvania; Franklin, Tennessee; Chatsworth, California; and Zhu Hai City, China. (Map source: www.theodora.com/maps)

Previous Supply-Chain Optimization

In many prior studies, researchers have documented the importance of optimization models for helping managers decide how to transport products to destination plants, warehouses, and customers in their supply chains. In an early classic work, Geoffrion and Graves (1974) used mixed-integer programming to design a distribution system for Hunt Wesson Foods, Inc. Blumenfeld et al. (1987) developed a decomposition method to find the minimum cost for transportation and inventory in General Motor's Delco Electronics Division's network. Robinson et al. (1993) developed an optimization-based decision-support system for designing a two-echelon, multiproduct distribution system for DowBrands, Inc. Arntzen et al. (1995) developed a mixed-integer LP that incorporates a global, multiproduct bill of materials for supply chains with arbitrary echelon structures. Epstein et al. (1999) used an LP model to help Chilean forest firms (Bosques Arauco, Forestal Celco, and many others) reduce transportation costs from forests to mills. Koksalan and Sural (1999) developed an optimization model for Efes Beverage Group that considers both the location of new malt plants and the distribution of materials throughout its supply chain. Karabakal et al. (2000) described Volkswagen of America's use of optimization models to evaluate alternative locations for distribution centers to reduce transportation cost. Brown et al. (2001) discussed Kellogg Company's large-scale multiperiod LP for production and distribution of two of its product lines. Sery et al. (2001) used LP models to find the optimal number and location of distribution centers and the corresponding material flows needed to meet anticipated demands at the lowest overall cost for BASF North America's packaged goods group. Chan et al. (2002) proposed two algorithms to find a zero-inventory-ordering policy in a single-warehouse multiretailer scenario in which the warehouse serves as a cross-dock facility. These successful earlier applications of optimization models influenced our decision to use such an approach.

Model Development

Before developing the actual LP model, we decided to use a Microsoft Excel spreadsheet model rather than an algebraic one because Nu-kote's managers, like most managers, tend to think in terms of spreadsheets rather than linearity, functions, and so forth (Powell 1997). Compared to algebraic models, spreadsheet optimization models have the disadvantage of taking longer to solve. However, the speed of modern PCs alleviates this disadvantage for all but the largest of problems.

For our initial conceptualization, we had two distinct objectives:

—To find the least expensive way to transport products from their origins to the customers, taking into consideration inventory holding and handling costs; and

—To provide acceptable responsiveness to customers.

These two objectives may conflict, but that is the case in most supply-chain systems.

To quantify the responsiveness to customers, we used the approach of Robinson et al. (1993), defining acceptable responsiveness as equivalent to each customer being served by a warehouse within a stated maximum distance (the shipping radius). Nu-kote's managers wanted a 1,000-mile radius because this implies a two-day delivery time using the industry standard of 500 miles per day for less than truckload deliveries. (They made exceptions for the few customers located over 1,000 miles from any warehouse.) However, they wanted to know how delivery cost would vary for other radiuses. To implement this service consideration in the model, we did not allow any variable corresponding to a shipment from a warehouse to a customer located farther away than the shipping radius.

In addition, the managers wanted to know whether using an additional warehouse(s) for outbound shipments of finished goods to customers would be less expensive than the current method of using only the Franklin warehouse for all outbound shipments. Currently, Nu-kote uses its other three warehouses for storing materials for manufacturing and for storing finished goods prior to shipping them to Franklin. Thus, both the shipping radius and the warehouse configuration were management decisions, although they were exogenous to the LP models.

We divided our development of Nu-kote's LP models into four steps. The first step was to study Nu-kote's facilities within its current supply-chain system and collect the data for the model. The second

step was to analyze transportation freight costs, gathering data for Nu-kote's less than truckload (LTL) and truckload (TL) carriers and using regression analyses to determine freight-cost relationships. The third step was to design and develop the spreadsheet LP models to minimize costs given a warehouse configuration and a maximum warehouse-to-customer shipping radius. Included in this step was a validation process to test the model's accuracy. Finally, the last step was to solve different versions of the model, evaluate the solutions, and present them to management.

In the first step in developing the model, we collected information about all supply-chain participants, including vendor capacities and locations, plant production capacities, warehouse storage capacities, and customers' locations and their annual demands for each product. We collected an enormous amount of data and, as a consequence, had to aggregate it. We experimented with aggregating customers into three-digit and five-digit zip code areas, obtaining 253 (using three-digit zip codes) or 394 (using five-digit zip codes) aggregate customer locations. As noted by Simchi-Levi et al. (2000, p. 25), "Various researchers report that aggregating data into about 150 to 200 points usually results in no more than a one percent error in the estimate of total transportation costs." We aggregated using five-digit zip codes because the larger number of customer locations implied greater accuracy than we would obtain using three-digit zip codes. Thus, we represented all customers with the same five-digit zip code as a single customer location and calculated the resulting demand as the total demand of all customers at that location.

Next, because of the very large number of products, we aggregated products into six categories: laser printer cartridges, ink-jet printer cartridges, ribbons, thermal fax supplies, toner cartridges (for copiers and plain paper fax machines), and other (miscellaneous items). It is common practice to use aggregate product categories as an approximation when it is not feasible to forecast demands for individual products (Vollmann et al. 1997). Finally, we calculated the cost of holding one unit of cycle stock as Nu-kote's cost of capital multiplied by the standard product cost. The cost of handling one unit was already available as the standard handling cost at any Nu-kote warehouse.

The transportation costs for TL and LTL shipments exhibit economies of scale and discontinuities, and are thus not linear. Chan et al. (2002) discuss these costs for the case of a single warehouse serving multiple retail stores. However, it is well known that optimal solutions generally cannot be obtained for models with thousands of such nonlinear, nonconvex cost terms. In the initial stages of our study, we ran a nonlinear version of the model to represent costs more accurately. However, solution time for a model with a sample network consisting of only 10 percent of the links in Nu-kote's network was prohibitive (4.5 hours). Therefore, we used a linear approximation for transportation costs (constant cost per unit shipped for any shipment size). We were fortunate that nearly all of Nu-kote's shipments were either small LTL shipments or full TL, which made the linear approximation accurate.

In the next phase of constructing Nu-kote's LP models, we estimated coefficients for these transportation costs to ship products between the various supply-chain locations. We used a simple regression analysis based solely on R^2. We first collected freight-charge data from the first quarter of 2002 from all of Nu-kote's LTL and TL carriers. As a rule, Nu-kote uses TL for shipments from vendors and plants to warehouses and uses LTL for warehouse-to-customer shipments. For TL and LTL shipments, we used separate regression analyses to calculate the relationship between the freight charge and weight and distance for shipping one unit of product. To model the costs of holding and handling inventory at warehouses, we added coefficients for these costs to the target cell (objective function) for legs pointing into any warehouse (Table 1).

Then, to validate the regression analyses, we used a much larger data set covering shipments from June 2001 to May 2002. We put the actual historical shipment amounts into the model and compared the model's calculated costs to the costs that Nu-kote actually incurred during this period. The model's linear cost formulas gave total cost (inbound and outbound transportation costs plus inventory holding and handling costs) within five percent of the actual historical cost, and therefore, we judged the model to be quite satisfactory. Undoubtedly, Nu-kote's shipment characteristics (only small and TL) were a big part of the reason for its accuracy. In spite of this simple approach, the model was very successful in identifying better solutions.

The regression equation we used in the case shown in Table 1 was $Shipping\ Cost = 0.000133 * Distance * Weight$, or equivalently, $0.352636 * Weight$. In the LP

Distance (Miles)	Weight (Pounds Shipped)	Distance ∗ Weight	Actual Shipping Cost	Predicted Shipping Cost
2,651.4	1,055	2,797,227	$352	$372

Table 1: This shows the input data for one of the legs in Nu-kote's supply chain and the regression predicted shipping cost per unit (the numbers are disguised for confidentiality).

V = vendor, W = warehouse, P = plant, C = customer location

Unit shipping cost of product 4 from V1 to W2, including inventory holding and handling cost at W2

Target cell is total cost = SUMPRODUCT(L102:Q...,E102:J...)

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q
99					$1,234,567												
100																	
101	**From**	**To**	**Miles**		**Product 1**	**Product 2**	**Product 3**	**Product 4**	**Product 5**	**Product 6**		**Unit Cost1**	**Unit Cost2**	**Unit Cost3**	**Unit Cost4**	**Unit Cost5**	**Unit Cost6**
102	V1	W1	1164.5		7,465	0	0	0	0	182		0.04	0.16	0.04	0.25	0.43	0.04
103	V1	W2	961.0		0	0	0	0	0	61		0.03	0.14	0.04	0.23	0.39	0.03
104	V1	W3	1628.0		224	0	0	0	0	2		0.05	0.19	0.05	0.31	0.52	0.05
105	V1	W4	1530.1		0	0	0	0	0	0		0.04	0.18	0.05	0.30	0.50	0.04
Etc.																	
117	V5	W1	2041.2		3,174	0	1,617	0	0	0		0.08	0.28	0.14	0.82	0.67	0.05
118	V5	W2	0.0		3,109	0	401	0	0	0		0.01	0.05	0.01	0.08	0.14	0.01
119	V5	W3	2587.3		0	0	0	0	0	0		0.10	0.34	0.17	1.01	0.81	0.06
120	V5	W4	2473.3		0	0	8	0	0	0		0.09	0.33	0.16	0.97	0.78	0.06
121	P1	W1	2041.2		0	0	0	0	0	0		0.05	0.22	0.06	0.36	0.60	0.05
122	P1	W2	0.0		0	0	0	0	337	0		0.00	0.00	0.00	0.00	0.00	0.00
123	P1	W3	2587.3		0	0	0	0	0	0		0.06	0.26	0.07	0.42	0.71	0.06
123	P1	W4	2473.3		0	0	0	0	0	0		0.06	0.25	0.07	0.41	0.69	0.06
Etc.																	
135	P5	W1	2041.2		0	0	0	0	744	0		0.08	0.28	0.14	0.82	0.67	0.05
136	P5	W2	0.0		0	0	0	0	6	0		0.01	0.05	0.01	0.08	0.14	0.01
137	P5	W3	2587.3		0	0	0	0	10	0		0.10	0.34	0.17	1.01	0.81	0.06
138	P5	W4	2473.3		0	0	0	0	24	0		0.09	0.33	0.16	0.97	0.78	0.06
139	W1	W2	2041.2		0	0	0	0	0	0		0.07	0.23	0.12	0.73	0.52	0.04
140	W1	W3	801.8		0	0	0	0	0	0		0.03	0.09	0.05	0.29	0.21	0.01
141	W1	W4	556.1		0	0	0	0	0	0		0.02	0.06	0.03	0.20	0.14	0.01
142	W1	C1	993.3		25	0	0	0	213	0		0.04	0.15	0.04	0.24	0.41	0.04
143	W1	C2	984.2		28	0	0	0	7	0		0.04	0.15	0.04	0.24	0.41	0.04
144	W1	C4	978.9		3	0	0	0	0	0		0.04	0.15	0.04	0.24	0.41	0.04
145	W1	C7	943.4		0	0	0	0	0	0		0.04	0.15	0.04	0.24	0.40	0.04
Etc.																	

Table 2: The Excel LP model has input data in cells with double borders and changing cells (decision variables) in single heavy borders. The labels in columns A–B of row 101 show that there are changing cells for shipments of each product from vendors to warehouses, plants to warehouses, warehouses to other warehouses, and from warehouses to customer locations. The target cell (objective function) in E99 gives the total cost of shipping all products using the SUMPRODUCT (sum of products) worksheet function. Rows 102–105 show the shipments from vendor V1 to warehouses W1–W4. The mileage and unit cost for shipping are both input data cells and are shown in columns C and L–Q, respectively. Optimal flows of products 1–6 for these vendor-warehouse combinations are shown in columns E–J.

models, if the variable x indicates the units of a product weighing two pounds that are shipped on this leg, then the cost is $2 * 0.352636 * x$, or equivalently, $0.705272 * x$. In this manner, for each separate leg in the supply chain, we found a (separate) constant for shipping one unit of each product category on that leg.

For the regression equations we developed, we need distances to calculate the costs of transporting products from specific sources to specific destinations. Because customers were aggregated into zip-code areas, we used the software ZIPFind Deluxe 3.0 (Bridger Systems, Inc. 2003) to calculate the straight-line distances, which are less than actual road distances. To correct for this, we multiplied the straight-line distances by a circuit factor of 1.14, as recommended by Simchi-Levi et al. (2000) for the continental US. This correction introduces an approximation error; exact road distances for each pair of locations or separate circuit factors for each region would be more accurate.

We then developed the Excel spreadsheet linear program (Tables 2, 3, and 4). We show an algebraic formulation in the Appendix. After we deleted all the variables referring to shipments from warehouses to customer locations beyond the allowable shipping radius, Nu-kote's LP models had between 5,000 and 9,700 variables and 2,452 constraints. The exact number of variables depends on the allowable shipment radius and the number of warehouses allowed for outbound shipping.

Solution, Evaluation, and Presentation

Because of the large size of the LP models, we purchased the premium solver platform with the large-scale LP solver (Frontline Systems, Inc. 2003)

=SUMIF(Origins,$A503,Flows1). This sums flows of product 1 whose origin is V1. Named ranges "Origins" and "Flows1" respectively refer to the name of the originating point for flows and the changing cell flows of product 1.

=SUMIF(Origins,$A505,Flows6). This sums all flows of product 6 whose origin is V3.

Capacity of vendor V1 for product 3.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N
501	Vendor Constraints													
502		Outflow, Product 1	Outflow, Product 2	Outflow, Product 3	Outflow, Product 4	Outflow, Product 5	Outflow, Product 6		Capacity1	Capacity2	Capacity3	Capacity4	Capacity5	Capacity6
503	V1	7,688	0	0	0	0	246	<=	7,688	0	127	0	0	246
504	V2	0	244	0	0	0	0	<=	0	410	0	0	0	0
505	V3	559	0	0	0	0	0	<=	559	0	0	0	0	0
506	V4	151	0	0	0	0	0	<=	151	0	0	0	0	0
507	V5	6,283	0	2,027	0	0	0	<=	6,283	0	2,223	0	0	0
508	Plant Constraints													
509		Outflow, Product 1	Outflow, Product 2	Outflow, Product 3	Outflow, Product 4	Outflow, Product 5	Outflow, Product 6		Capacity1	Capacity2	Capacity3	Capacity4	Capacity5	Capacity6
510	P1	0	0	0	0	337	0	<=	0	0	0	0	337	0
511	P2	0	0	588	0	0	0	<=	0	0	588	0	0	0
512	P3	0	701	0	0	0	0	<=	0	8,021	0	0	0	0
513	P4	0	0	0	1,007	0	0	<=	0	0	0	1,511	0	0
514	P5	0	0	0	0	786	190	<=	0	0	0	0	1,348	200

=SUMIF(Origins,$A514,Flows6). This sums all flows of product 6 whose origin is P5.

Table 3: The vendor constraints limit total flow of each product sent from each vendor. Cell B503 has the left side of this constraint for vendor V1, product 1. This constraint uses the SUMIF worksheet function to sum all changing cells involving product 1 whose origin equals V1. The function uses the named ranges, "Origins" and "Flows1," which refer to the names of the originating point for all flows and the changing cell flows of product 1, respectively. The right side of this constraint is the capacity for product 1 given in cell I503. The plant constraints are similar, limiting the total flows of each product sent from each plant. Cell G514 contains the left side of the constraint for plant P5, product 6, giving the sum of all changing cells referring to product 6 (using named range "Flows6") whose origin equals plant P5.

for solving this LP directly in Excel. Solution time on a 900 MHz Pentium 3 PC was approximately 25 minutes. The solution time for reoptimization for a model with a few changes was essentially unchanged, because this software spends almost all of the time on setup for this problem.

We used different versions of the LP model to evaluate Nu-kote using different configurations of its warehouses for outbound shipping of finished goods to customers (with different allowable shipping radiuses (Figure 2)). Currently only the Franklin warehouse is an outbound warehouse. Nu-kote's three other warehouses are factory warehouses used only for storing materials for manufacturing and for storing finished goods prior to shipping them to Franklin. For any configuration, we defined overall costs to be the exogenous annualized fixed costs for expanding each factory warehouse into an outbound warehouse plus the value of the LP target cell corresponding to that warehouse configuration (Figure 2). We did not consider eliminating Franklin, so we calculated the overall costs of all combinations of two, three, and four warehouses that include Franklin. We found that the configuration consisting of the Franklin, Tennessee, and Chatsworth, California warehouses would be best for handling outbound shipments to customers.

We solved the LPs (for all allowable warehouse combinations) with different allowable shipping radiuses to determine their effect on total cost (Figure 3). Although it could have saved several hundred thousand dollars per year by extending the shipping radius, Nu-kote kept the 1,000-mile limit because customer service would have been unacceptable with longer shipping distances.

Based on the solution to the LP models, Nu-kote management decided to expand the Chatsworth warehouse to make it an outbound warehouse as well as a factory warehouse. It expects to complete the expansion by the end of September 2003. In the LP solution, many customers will receive shipments from two warehouses rather than the single warehouse used in the past. Nu-kote has contacted its customers about this, and all major customers have agreed and will be served by the two-warehouse shipments. The new system will reduce annual transportation and inventory costs by approximately $1 million. Furthermore, customer transit times, many of which were four to six days, will decrease by two full days averaged over all customers. This use of optimization

=SUMIF(Origins,$A1001,Flows4) – SUMIF(Dests,$A1001,Flows4)
The first SUMIF sums all flows of product 4 out of W1, and the second SUMIF sums all flows of this product into W1.
The warehouse constraint requires flow out to equal flow in.

=SUMIF(Dests,$A1008,Flows6). This sums all flows of product 6 whose destination is C2

G1017 calculates the number of pallets needed to store product 6 in warehouse W1, using the quantity of product 1 per pallet.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N
	Warehouse Balance Constraints													
1000		**Net Flow, Product 1**	**Net Flow, Product 2**	**Net Flow, Product 3**	**Net Flow, Product 4**	**Net Flow, Product 5**	**Net Flow, Product 6**		**Required**					
1001	W1	0	0	0	0	0	0	=	0					
1002	W2	0	0	0	0	0	0	=	0					
1003	W3	0	0	0	0	0	0	=	0					
1004	W4	0	0	0	0	0	0	=	0					
1005	**Customer Location Constraints**													
1006		**Inflow, Product 1**	**Inflow, Product 2**	**Inflow, Product 3**	**Inflow, Product 4**	**Inflow, Product 5**	**Inflow, Product 6**		**Demand1**	**Demand2**	**Demand3**	**Demand4**	**Demand5**	**Demand6**
1007	C1	19,753	1,463	2,145	571	633	2,168	=	19,753	1,463	2,145	571	633	2,168
1008	C2	4,868	2,359	4,885	0	20,390	0	=	4,868	2,359	4,885	0	20,390	0
1009	C3	1,112	0	0	0	757	0	=	1,112	0	0	0	757	0
1010	C4	5,682	156	0	0	537	0	=	5,682	156	0	0	537	0
1011	C5	0	0	0	443	0	0	=	0	0	0	443	0	0
1012	C6	0	0	0	1,357	0	0	=	0	0	0	1,357	0	0
1013	C7	37,981	461	0	0	1,910	0	=	37,981	461	0	0	1,910	0
1014	C8	5,095	2,425	1,790	0	1,680	0	=	5,095	2,425	1,790	0	1,680	0
1015	**Warehouse Capacity Constraints**													
1016		**Product 1**	**Product 2**	**Product 3**	**Product 4**	**Product 5**	**Product 6**	**Total**		**Capacity**				
1017	W1	975	844	658	440	805	678	4400	<=	4400				
1018	W2	382	692	738	852	958	378	4000	<=	4000				
1019	W3	467	141	0	0	379	643	1630	<=	4500				

Table 4: The warehouse balance constraints require that total flow out equals total flow in for each separate product at each warehouse. Cell E1001 has the left side of this constraint for warehouse W1, product 4. The two SUMIF functions calculate the flow out and the flow in, respectively. Customer location constraints require that flow into the customer location of a given product equals the customer's demand for that product. G1008 and N1008 contain this constraint for customer location C2's demand for product 6. The warehouse capacity constraints link the flows of all products. They prevent the total pallets needed to accommodate the flows from exceeding the capacity of the warehouse.

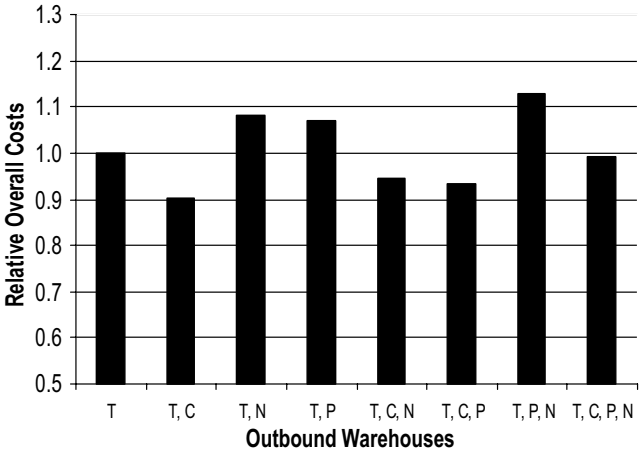

Figure 2: These are normalized overall optimal costs of the current supply-chain network (Franklin, Tennessee is the only outbound warehouse) and other combinations of outbound warehouses for a 1,000 mile shipping radius. The notation is T = Tennessee (Franklin) = 1.0, C = California (Chatsworth), N = New York (Rochester), P = Pennsylvania (Connellsville).

modeling has been the catalyst for a new way of thinking by Nu-kote managers.

Practitioners have often suggested that the real value of modeling does not come from the specific numerical solution but from the insight the solution provides, thus enabling users to ask the right questions (Savage 2002). Based on the new warehouse configuration, Nu-kote and its main LTL carrier, Yellow Freight, discussed the costs of returning empty cartridges to two locations (Chatsworth for some customers and Franklin for others) instead of one. We did not include the flows of these empties in the LP models, yet Nu-kote is saving $425,000 a year from Yellow Freight because of the insight it gained from the LP solution.

Conclusions

Our LP models have helped Nu-kote managers to choose a configuration of outbound warehouses and shipping distances that improves customer service. It has made a major capital investment as a result of the models, has realized significant savings, and anticipates additional savings. In the future, as customer

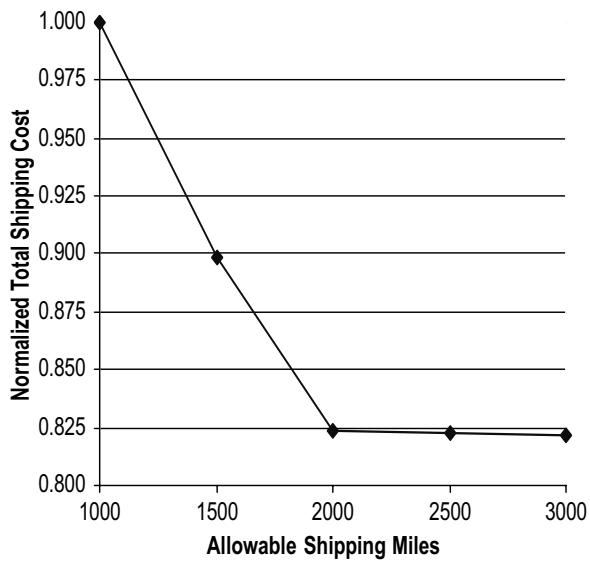

Figure 3: This summarizes the values of the LPs with the Franklin-Chatsworth warehouse configuration. We did not consider 500 miles because many customers are farther than that from any warehouse. Annual shipping costs would drop by nearly 18 percent if Nu-kote allowed shipping distances longer than 1,000 miles. This is equivalent to several hundred thousand dollars per year.

demands change, we will re-solve the LP models several times per year. This use of optimization modeling has been the catalyst for a new way of thinking by Nu-kote managers.

Appendix

We give an algebraic formulation of Nu-kote's LPs for a 1,000-mile allowable shipping distance. Our notation is the following.

Indices and Sets

i = major vendor, $i = 1, 2, 3, 4, 5$.

j = plant, $j = 1, 2, 3, 4, 5$.

k = warehouse, $k = 1, 2, 3, 4$.

l = aggregate customer, $l = 1, 2, \ldots, 394$.

p = product, $p = 1, 2, 3, 4, 5, 6$.

$W_l(1{,}000)$ = set of outbound warehouses within 1,000 miles of customer l.

$C_k(1{,}000)$ = set of customers within 1,000 miles of outbound warehouse k.

Decision Variables

VW_{ik}^p = units of product p sent from vendor i to warehouse k.

PW_{jk}^p = units of product p sent from plant j to warehouse k.

WW_{kK}^p = units of product p sent from warehouse k to warehouse K.

WC_{kl}^p = units of product p sent from warehouse k to customer l.

Parameters

$VWCost_{ik}^p$ = cost per unit shipped of product p from vendor i to warehouse k.

$PWCost_{jk}^p$ = cost per unit shipped of product p from plant j to warehouse k.

$WWCost_{kK}^p$ = cost per unit shipped of product p between warehouses k and K.

$WCCost_{kl}^p$ = cost per unit shipped of product p from warehouse k to customer l.

The objective function is to minimize total shipping, holding, and handling costs. The latter two unit costs are included in the parameters $VWCost_{ik}^p$, $PWCost_{jk}^p$, and $WWCost_{kK}^p$.

$$\text{Min} \sum_{i=1}^{5}\sum_{k=1}^{4}\sum_{p=1}^{6} VWCost_{ik}^p VW_{ik}^p \quad \text{(vendors to warehouses)}$$

$$+ \sum_{j=1}^{5}\sum_{k=1}^{4}\sum_{p=1}^{6} PWCost_{jk}^p PW_{jk}^p \quad \text{(plants to warehouses)}$$

$$+ \sum_{l=1}^{394}\sum_{k\in W_l(1{,}000)}\sum_{p=1}^{6} WCCost_{kl}^p WC_{kl}^p$$

(warehouses to customers)

$$+ \sum_{k=1}^{4}\sum_{K=1}^{4}\sum_{p=1}^{6} WWCost_{kK}^p WW_{kK}^p$$

(warehouses to warehouses where $K \neq k$).

Additional notation for the constraints is

Parameters

$VCapacity_i^p$ = capacity for product p at vendor i.

$PCapacity_j^p$ = capacity for product p at plant j.

$WCapacity_k$ = overall capacity (total pallets) of warehouse k.

$Demand_l^p$ = demand for product p from customer l.

$Pallets^p$ = parameter to convert flow of product p to pallets needed to accommodate the resulting inventory.

The following constraints complete the optimization model:

$$\sum_{k=1}^{4} VW_{ik}^p \leq VCapacity_i^p$$

(vendor capacities for each product p for each vendor i),

$$\sum_{k=1}^{4} PW_{jk}^p \leq PCapacity_j^p$$

(plant capacities for each product p at each plant j),

$$\sum_{i=1}^{5} VW_{ik}^p + \sum_{j=1}^{5} PW_{jk}^p + \sum_{K=1}^{4} WW_{Kk}^p$$

$$= \sum_{K=1}^{4} WW_{kK}^p + \sum_{l\in C_k(1{,}000)} WC_{kl}^p$$

(flow in = flow out at each warehouse k for each product p),

$$\sum_{k \in W_l(1,000)} WC_{kl}^p = Demand_l^p$$

(demands for each product p for each customer l),

$$\sum_{i=1}^{5}\sum_{p=1}^{6} Pallets^p VW_{ik}^p + \sum_{j=1}^{5}\sum_{p=1}^{6} Pallets^p PW_{jk}^p + \sum_{K=1}^{4}\sum_{p=1}^{6} Pallets^p WW_{Kk}^p \leq WCAPACITY_k$$

(pallets of all products in each warehouse k cannot exceed warehouse k's capacity).

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Phillip L. Theodore, Senior Vice President Operations, Nu-kote International, 200 Beasley Drive, PO Box 3000, Franklin, Tennessee 37068, writes: "We are confident that the use of this optimization modeling will save the company approximately $1 million from the improved shipments that it has identified. We are quite pleased with the overall results and expect to use this model frequently as our customer demands and supply chain infrastructure change. We are confident that additional savings will accrue in the future as a result of continued use of the model."