

# Improving the Distribution of Industrial Gases with an On-line Computerized Routing and Scheduling Optimizer

---

WALTER J. BELL

*Air Products and Chemicals, Incorporated  
Box 538, Allentown, Pennsylvania 18105*

LOUIS M. DALBERTO

*Air Products and Chemicals, Incorporated*

MARSHALL L. FISHER

*Wharton School, University of Pennsylvania  
Philadelphia, Pennsylvania 19104*

ARNOLD J. GREENFIELD

*Wharton School, University of Pennsylvania*

R. JAIKUMAR

*Graduate School of Business, Harvard University  
Cambridge, Massachusetts 02138*

PRADEEP KEDIA

*Wharton School*

ROBERT G. MACK

*Air Products and Chemicals, Incorporated*

PAUL J. PRUTZMAN

*Air Products and Chemicals, Incorporated*

---

For Air Products and Chemicals, Inc., inventory management of industrial gases at customer locations is integrated with vehicle scheduling and dispatching. Their advanced decision support system includes on-line data entry functions, customer usage forecasting, a time/distance network with a shortest path algorithm to compute intercustomer travel times and distances, a mathematical optimization module to produce daily delivery schedules, and an interactive schedule change interface. The optimization module uses a sophisticated Lagrangian relaxation algorithm to solve mixed integer programs with up to 800,000 variables and 200,000 constraints to near optimality. The system, first implemented in October, 1981, has been saving between 6% to 10% of operating costs.

The principal products of the industrial gas industry are oxygen, nitrogen, hydrogen, argon, and carbon monoxide. In the industry's formative years, these were sold as highly compressed gases stored in heavy metal cylinders. Air Products and Chemicals, Inc. (Air Products), a leading supplier of

industrial gases, was founded by the late Leonard P. Pool in 1940, on the strength of a simple, but then revolutionary idea: the "on-site" production of industrial gases, primarily oxygen. Air Products built oxygen generating facilities adjacent to large volume users so that oxygen could be piped directly from the generator

## AIR PRODUCTS

to the point of use. This reduced distribution costs and was a technical and economical success. It launched a history of rapid growth at Air Products based upon high technology innovations in distribution.

During the 1950s, Air Products acquired several independent regional gas distributors, and a marketing concept known as "piggybacking" was introduced in which extra gas liquefaction capacity was added to the "on-site" plants. This enabled Air Products to serve its "on-site" tonnage base-load gas customers, such as steelmakers, while serving additional smaller customers in the surrounding area by distributing the liquefied gas in insulated tank trucks.

During the past 20 years, Air Products has been instrumental in developing new applications for industrial gases. These include oxygen in cupolas; oxygen for copper smelting; inert atmospheres for heat treating; liquid nitrogen for the quick freezing of meats and other food products; and liquid nitrogen for grinding various materials and recovering scrap metals from them.

Today, Air Products is one of the world's largest industrial gas producers supplying world-wide a broad range of industrial gases and related production and distribution equipment. Air Products also produces industrial and specialty chemicals and provides engineering, construction, and maintenance services.

Air Products has grown from a company with sales of \$8,300 in 1940 to an international corporation with sales exceeding \$1.5 billion in 1982, 18,900 employees, and facilities in 13 countries.

A spirit of innovative progress has flourished in the distribution function of the Industrial Gas Division. Headed by Vice President of Operations Stan Roman and Manager of Distribution Byron Trammell, the department has been a leader in developing computer applications that improve productivity and enhance management reporting for private fleet transportation. Examples include regular identification of chronically underutilized cylinder delivery trucks and routes, exception reports to identify trips not meeting established goals, a matrix of fuel mileage by driver and tractor, and an integrated product line vehicle activity and accounting system. Programs in driver safety and energy efficiency have been recognized by a presidential award for energy efficiency and the American Trucking Association's president's award for safety programs. The latter award is especially significant because it is the only one ever awarded to a private carrier.

By the 1970s, the plants, trucks and other equipment used by Air Products for the manufacture and distribution of liquid gases were highly engineered and automated. This was in sharp contrast to the completely manual system for scheduling the delivery of these liquid gases. It was clear that a further competitive edge in the industry would depend on automation of delivery scheduling. This goal was first officially recognized in the company's strategic plan written in 1975. Fuller appreciation of the importance of this goal requires a detailed understanding of the economics of industrial gas distribution.

### Economics of Industrial Gas Distribution

Liquid oxygen and nitrogen are manufactured in highly automated plants by repetitive compression and expansion of air to lower its temperature. At specific temperatures, the cooled gases liquefy and are removed via a distillation column. The plants also serve as supply depots where liquefied gases are stored in large tanks at temperatures less than  $-320^{\circ}\text{F}$ . The liquefied gases are distributed in cryogenic bulk tankers to industrial users and hospitals. Storage tanks at customer sites are provided under long-term contracts by the supplier, who monitors the inventory in the tank and delivers product as needed. Air Products currently operates 23 plant/depots throughout the United States supplying about 3,500 customers. The depots operate independently and serve their own designated set of customers. Customers are assigned to depots by a linear programming model which minimizes total system production and distribution costs. Liquid oxygen and nitrogen customer deliveries are made by a fleet of about 340 trucks that travels over 22 million miles a year. The trucks assigned to each depot are controlled by a single scheduler, aided by coordinators.

Because the cost of manufacture is fairly uniform across different suppliers, competition in the industry is based on service, technical marketing, pricing and lower costs obtained through more efficient distribution. The degree of freedom available to distribution management at Air Products is greater than in any other industry. They decide when to supply a customer based on the inventory level in the customer tank, how much to deliver,

how to combine the different loads on a truck and how to route the truck. Thus inventory management at customer locations is integrated with vehicle scheduling and dispatching.

The ability to control inventory makes this fleet scheduling problem different from the traditional and much studied vehicle routing problem. In the traditional

---

### Air Products built oxygen generating facilities adjacent to large volume users . . .

---

problem, customer demands are fixed and must be served by a fleet of trucks within a fixed period of time, frequently a day. Usually, customer orders cannot be split between two or more trucks. The goal in the traditional problem, to minimize the distance traveled in making the required deliveries, is usually accomplished by serving customers that are near to each other with the same truck.

To see how the inventory dimension changes the nature of the problem, consider the simple example illustrated in Figure 1. There are four customers, each with a specified tank capacity and daily usage. The numbers on lines joining customers specify mileages. Suppose we have a single truck that can transport 5,000 gallons per trip and can make up to two trips per day. Currently, all customer tanks are full. How should we schedule our truck to accomplish required deliveries while traveling minimum miles? The most obvious schedule in this example would have two trips a day. One trip would deliver 1,000 gallons to customer 1

# AIR PRODUCTS

and 3,000 gallons to customer 2. The other trip would deliver 2,000 gallons to customer 3 and 1,500 gallons to customer 4. The total distance traveled each day under this schedule is 420 miles. Although this schedule looks reasonable, it is possible to do better. On day one, we drive a single trip to customers 2 and 3 delivering 3,000 and 2,000 gallons respectively. On day two, we drive two trips. The first trip delivers 2,000 gallons to customer 1 and 3,000 gallons to customer 2. The second trip delivers 2,000 gallons to customer 3 and 3,000 gallons to customer 4. On successive days, the schedule repeats, using the day one schedule on odd days and the day two schedule on even days. The total mileage for two days is 760 miles, or a daily average of 380 miles. This is nearly 10% less than the daily average of 420 miles in the more obvious schedule.

... competition in the industry is based on service, technical marketing, pricing, and lower costs obtained through more efficient distribution.

In this simple example, a scheduler would have little difficulty in finding the superior solution by simply enumerating the various possibilities. In reality, things are not so simple. Typical problems involve several hundred customers and about 20 trucks, so that the scheduler must make simplifying assumptions. The most natural assumption is to focus on the spatial dimension of the problem and place customers who are near each other on the same trip. The possibility of driv-

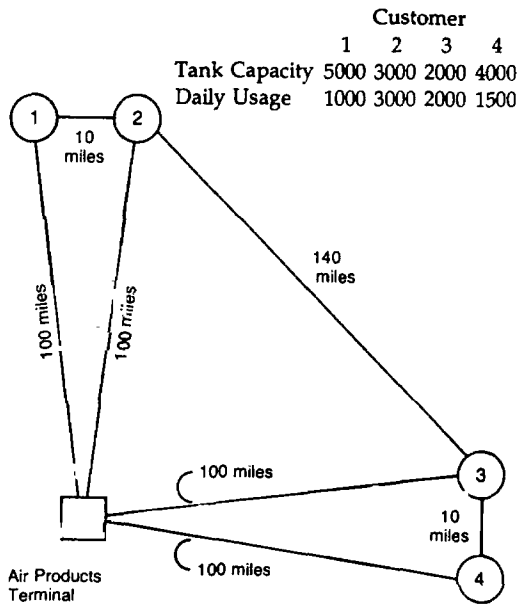


Figure 1. A simple example of the bulk delivery problem.

ing 140 miles from customer 2 to customer 3 when there is a customer 10 miles away is unlikely to be considered by a scheduler who has limited time to make a large number of decisions.

In addition to the large number of customers and trucks, several other factors complicate the real situation. To start with, just getting the data that is presented in Figure 1 is a challenging task. Before the development of a computer system, mileages between customers were known only approximately from a plot of the customer locations on a map. Usage rates are difficult to estimate and vary considerably over time as a function of the customer's operation. Because of the uncertainty in customer demand, it is not possible to let customer inventory drop to 0 as we've assumed in the example. Rather, inventory must be maintained

above a specified safety-stock level. Customers are not open for delivery on every day of the week or during every hour of the day and trucks must make their deliveries within certain prescribed time windows which can vary among customers. The trucks in the fleet differ in characteristics such as capacity and operating costs. Moreover, the capacity of a truck is imposed by state laws specifying the maximum loaded truck weight. These legal weight limits differ from state to state, so the capacity of the truck actually changes when it crosses a state boundary. Finally, some trucks are incapable of serving certain customers because they are too big, require an external power source for an electric pump, and so forth. The availability of trucks, drivers and product is limited. At any particular time, any of these factors can constrain the set of feasible scheduling options.

The costs that must be considered in scheduling include driver pay, tolls, and vehicle-related costs such as depreciation, fuel, and maintenance. In addition to costs that are directly related to mileage, there are costs that do not vary directly with mileage but depend upon the time spent by drivers loading the truck at the terminal, unloading at customer sites and performing various set-up functions. The rules governing driver pay are complicated and vary considerably. For example, depending on the nature of the trip, a driver may be paid by the hour or by the mile.

Prior to the development of the system described here, the complicated scheduling function was performed completely manually by a staff of dedicated and ca-

pable individuals at the depots. The concept of using a computer to assist these schedulers was appealing. At the same time, it was clear that any computerized vehicle scheduling system had to be carefully designed to consider all of the important complexities of the problem, to insure the availability of relevant data, and to complement, rather than replace, the efforts of the existing schedulers.

### **The Vehicle Scheduling System**

The increasing attention that was given by Air Products to the possibility of a computerized vehicle scheduling system culminated in January 1980 with the formation of a team (the authors) charged with developing a computerized vehicle scheduling system. This team has since been involved in the design, development and implementation of the vehicle scheduling system described in this section. The system first went live at the Wharton, New Jersey, depot in October 1981. Subsequently, depots have been added to the system at a steady rate until currently 11 depots are using the system.

The vehicle scheduling system is depicted in Figure 2. The program modules and data of the system reside on an AM-DAHL 470/V8 at Air Products corporate headquarters in Allentown, Pennsylvania. The system is accessed by schedulers at the plants via CRTs linked to the AM-DAHL by dedicated telephone lines. The scheduling module is used daily at each depot to produce a detailed schedule for a two- to five-day horizon, with the first day's schedule being the most important one. Delivery data and customer requirements are updated interactively as they are received. The schedule is also changed

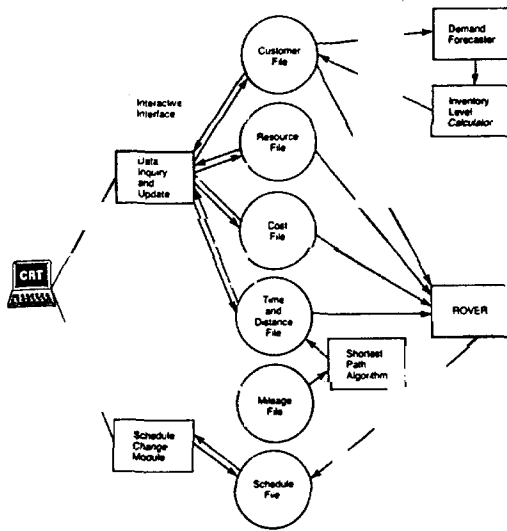


Figure 2. The vehicle scheduling system.

interactively as required.

Six data files are used by the system. The customer file contains a description of each customer served by Air Products, including the capacity of all tanks, safety stock levels, and historical product usage. The demand forecaster uses the historical demand data with a time series model to obtain a forecast of demand for each customer. This forecast is used by the inventory level calculator to project the inventory level in each customer tank at any point in time.

The resource file contains a description of each truck in the system, giving its capacity by state and a list of customers that can be feasibly served by the truck. Also included in the resource file is the availability of liquid nitrogen, liquid oxygen, and delivery drivers by day for at least five days into the future.

The cost file includes all relevant costs, including a per-mile rate for vehicle fuel and maintenance, and extensive data de-

scribing driver pay regulations.

The mileage file is a network representation of the road system of the United States. A shortest route algorithm [Glover and Klingman 1977] is used with this file to obtain the distance, travel time and toll cost between any pair of customers. This data is stored in the time and distance file.

All of this data can be accessed and updated in real-time by the scheduler using the various programs in the data inquiry and update module.

All data is used by the scheduling module which is called ROVER, for Real-time Optimizer for Vehicle Routing. ROVER produces a list of trips to be performed over the next two to five days. The data given for each trip includes the start time, the scheduled vehicle, the amount of product to be delivered to each customer, the time at which delivery to each customer will be completed, the length of the trip and the cost of the trip. The schedule change module allows the scheduler to examine the ROVER output and make changes as necessary due to contingencies. ROVER obtains a schedule by solving a very large (up to 800,000 variables and 200,000 constants) mixed integer program to proven near optimality using a Lagrangian relaxation algorithm developed for this project and reported in Fisher et al. [1982]. The mixed integer programs solved by ROVER, with hundreds of thousands of variables and constraints, are apparently the largest integer programs ever solved to near optimality on a routine basis. The Lagrangian relaxation algorithm developed as part of this project extends the theory of optimization.

The distances between customers are obtained from a detailed network model of the US highway system. This network contains information on 65,000 road segments (links) and 40,000 road intersections (nodes). The length, travel time and

---

Typical problems involve several hundred customers and about 20 trucks . . .

---

any toll charges are stored for each link. A state-of-the-art shortest route algorithm [Glover and Klingman 1977] is used to obtain the shortest path between each pair of customers.

An on-line network is used to link the scheduler at each depot to the vehicle scheduling system at corporate headquarters. This network allows the functions of the system to be performed on-line in real time. This is particularly important since some decisions, such as the appropriate response to an emergency order, are required quickly. A user-friendly interface allows the scheduler to query and update data, execute the ROVER scheduling module, and change the schedule as desired. The decision support system assists, rather than replaces, the scheduler.

**Demand Forecasting and Inventory Level Calculator**

Deciding when to deliver to customers and how much to deliver requires a forecast of the rate at which each customer is consuming product and a calculation of the latest possible time the customer needs a delivery. The forecasting method used is exponential smoothing. Ten values of the smoothing constant are

evaluated each time a new forecast is computed and the value that minimizes the absolute error for the most recent five usage rate forecasts is chosen. The most recent 15 customer inventory levels with corresponding dates and times are stored in the on-line customer file and used to make this point estimate of customer usage.

This process is complicated because usually the only known customer inventory levels are those recorded before and after each delivery. Since the time between deliveries is chosen by the scheduling module on a dynamic basis, the interval between deliveries is generally not uniform (as it is in all other time series forecasting problems of which we are aware). Because of this nonuniformity we must determine the hours of day that a customer is operating and using product and consider this operating pattern in determining usage rates. For example, if a customer does not operate on weekends and a five-day interval between deliveries includes a weekend, the usage per operating day would be one-third of the total usage for the five-day period. In the system, we identify for each eight-hour shift of each day the level of operations at each customer.

Because the forecasted usage rates are subject to forecast error, safety stocks are established at each customer to protect against stockouts. Customers with a high level of variability are often telephoned, as well, to establish their exact inventory level prior to the time at which they have been projected to need the product.

Using forecasted usage rates, the estimated inventory level at 8:00 am is calcu-

lated for all customers daily. We also project when safety and empty levels will be reached. Forecasts and level calculations are then available on-line for inquiry. A daily forecast results report shows on the horizontal time-axis the most recent seven days' history of deliveries and projections of the latest time in the future at which a customer must receive a delivery. The format is similar to that used by the manual system, which helped the schedulers to make the transition from manual to computer operations. On the report, the forecast error for each customer is shown and provides the basis for manual changes.

In addition to receiving the daily batch calculation of the inventory levels, the scheduler can interactively inquire when a customer will reach a specified level or what level is forecasted for a specified time.

### **Time and Distance Data**

The travel time and distance between combinations of customers and between each customer and the depot is an important input to the scheduling module. The common approach of computing straight line distances from coordinates of latitude and longitude and then adjusting by some factor to approximate road miles was judged to be too inaccurate for our application, especially in metropolitan areas, and it provides no information about travel times. At a large depot with 300 customers there are about 45,000 different intercustomer travel times and distances. To achieve the necessary level of detail for distances and travel speeds in a practical manner, a vendor was contracted to develop to our specifications a comprehen-

sive computerized data base of the network of highways in the United States. This data base contains about 65,000 road segments (links) and about 40,000 intersections of road segments (nodes). The roads specified for inclusion in the data base were all the interstate and federal highways, most state highways, and selected county, local, and city roads. For each road segment, the length of the segment and the toll cost is specified. Instead of a travel speed for each segment, five major characteristics are specified

---

---

which are then used to compute travel speed: road type (limited access, divided, undivided, city street), number of lanes, urbanization (as a proxy for traffic density), terrain, and road ownership (interstate, federal primary, state, and so forth).

Each possible intercustomer distance and travel time is then determined by finding the "shortest" path between pairs of customers through the highway network. The criterion used to determine the "shortest" path is not solely distance, but is a travel-cost value found from a combination of travel time, distance, and tolls. The time-related component of travel cost reflects the case when drivers are paid an hourly rate while the distance component primarily reflects vehicle costs such as fuel, maintenance, and tractor deprecia-



tion. The shortest route algorithm itself is an efficient dynamic programming algorithm developed by Glover and Klingman [1977].

#### **Real-time Optimizer for Vehicle Routing (ROVER)**

ROVER uses all the data described above on customer usage rates, costs, distances, travel times and availability of drivers, product, and trucks to produce a schedule for the next two to five days. ROVER is based on a large mixed-integer-programming formulation of the problem of making deliveries efficiently subject to resource constraints. To formulate the bulk-delivery problem as a mixed-integer problem, we first generate a set of possible vehicle routes, where a route is a set of customers to be delivered on one trip that begins and ends at the depot. The order of customer stops on the route is specified, but not the delivery amounts or the start time of the route. The model is formulated to optimally

---

**That monster problem is solved implicitly, not explicitly.**

---

select from this menu of possible routes a subset to actually be driven, specifying the time each route should start, the vehicle to be used, and the amount to be delivered to each customer on the route.

The major advantage of the route generation approach is that essentially any route constraints and cost function can be accommodated. An obvious limitation is the potentially large number of possible routes. This is not a difficulty here be-

cause the number of customers on a route is small, on average about two and rarely more than four. This means that the number of technically feasible routes is not unreasonably large. Further, many routes can be discarded as uninteresting because they have a high cost or because their combined demand requirements differ greatly from truck capacity.

A program was written to generate the set of possible routes to be included in the mixed-integer-programming model. Not all technically-feasible routes are generated, and the program uses some complicated heuristics to decide whether or not to include a particular route. For example, a route would be excluded if the amount of product that could be delivered to the customers on the route was significantly less or significantly more than a truckload. We would exclude a route to two customers each of whom could take a full truckload of product on the assumption that a single stop trip to each of these customers would be more efficient. Once a route has been generated, the least-cost sequence in which to visit the customers on the route is determined by complete enumeration.

The mixed-integer-programming formulation contains two kinds of variables. A set of 0-1 variables is used to select routes. A particular 0-1 variable equals one if a designated route is selected to be driven by a particular vehicle at a fixed starting time. The two-to five-day horizon of the model is divided into one hour intervals and a separate 0-1 variable is defined for each route and vehicle for each possible starting hour. The other variables in the model are continuous variables that

For each customer a value per unit of product delivered by the end of the planning horizon has been defined. The object of the model is to maximize the value of the product delivered less the costs incurred in making these deliveries. The costs considered by the model include driver pay, vehicle depreciation, fuel, and specify the amount of product to be delivered to each customer on a particular route-vehicle-starting time combination.

The constraints of the model consist of customer-demand constraints and resource constraints imposed by the availability of trucks, drivers, and product. The customer demand constraints impose lower and upper limits on the amount of product that must be delivered by any point in time to each customer, limit the time at which a customer is open for delivery, and define the set of vehicles that can feasibly serve a customer. The resource constraints in the model impose vehicle capacity, the times at which each vehicle is available, the availability of driver time, and the availability of the two products scheduled by the model, liquid oxygen and liquid nitrogen.

tolls.

The size of the problems that are run daily varies from depot to depot. The number of trucks ranges from about 10 to 30, and the number of customers from 150 to 400.

The dimensions of the mixed-integer-programming formulation are enormous. The number of 0-1 variables has ranged from about 100,000 to 200,000; the number of continuous variables from about 300,000 to 600,000; and the number of constraints from about 100,000 to 200,000.

Clearly, problems of this magnitude are far beyond the current state-of-the-art in mixed-integer programming. Fortunately, large problems such as this are usually endowed with a sparse coefficient matrix and special structure that can be exploited in their solution. We were able to apply Lagrangian relaxation to decompose the problem into a set of knapsack problems — one for each vehicle. An algorithm based on this Lagrangian relaxation has obtained feasible solutions that are proven to be within one-half to two percent of optimality in a modest amount of computing time. By working with a relaxed version of the problem one avoids setting up the 800,000 variable mixed-integer program alluded to previously. That monster problem is solved implicitly, not explicitly. A statement of the integer-programming

---

On the day when vehicle scheduling went live at Wharton, New Jersey, in October 1981, the authors of this paper breathed a collective sigh of relief.

---

model algorithm is provided in the Appendix. Further technical details are available in Fisher et al. [1982]. A technical spin-off of this work is reported in Fisher et al. [1983].

Our Lagrangian algorithm appears to have expanded the size problems that are solvable to near optimality on a routine basis. Our computational experience is illustrated by a recent typical run. The problem had 15 vehicles, 54 customers,

and 120 time periods (five days). The route generator produced a menu of 377 possible routes. If all combinations of routes, trucks and starting times were feasible, the number of 0-1 variables would be  $377 \times 120 \times 15 = 678,600$ . However, because constraints on when a customer can receive a delivery and from which trucks exclude many combinations of routes, trucks, and starting times, the number of 0-1 variables was 164,196. The number of continuous variables was 380,481. The model contained 166,166 constraints.

The AMDAHL 470/V8 required 2 minutes 51.83 seconds of CPU time to execute the input of data, the generation of routes, and the Lagrangian relaxation algorithm described [Fisher et al. 1982, Section 2.4]. This produced a feasible solution with a total cost of \$9,064.67. Solution of the Lagrangian relaxation also produced a bound on the optimal solution that exceeded the feasible value by \$46.80

---

## The psychological impact of computerization on individuals was a major consideration in our training approach.

---

or .5 percent of total cost. The daily runs of the system usually require 20 seconds to 3 minutes of computer time and produce feasible solutions that differ from the upper bound by .5 — 2 percent of total costs.

### Interactive Interface

The schedule developed by ROVER is used in an operating environment where reaction to contingencies is a daily way of

life. These contingencies include changes in a customer's requirement (either drastic change in usage or a request for a specific delivery) and changes in the delivery resources available (such as trucks, drivers, and product available for delivery). The schedule change module provides the capability to change schedules interactively in response to contingencies. In addition, most schedulers have developed a great deal of expertise at creating opportunities for productivity improvement. For example, depot personnel often anticipate orders and will call customers who otherwise would not permit delivery to arrange early delivery.

The features included in the schedule change module enable the scheduler to add or delete a trip, add or delete a customer on a trip, change the trip start-time, change the vehicle assigned to the trip, and find a customer to add to a trip in progress. With the exception of finding a customer to add to a trip in progress, the scheduler specifies the basic elements of the change and the schedule change module then performs feasibility checks on customer/vehicle compatibility, customer delivery time restrictions, vehicle availability, product availability, and driver availability; determines the sequence of stops; and computes the quantity to be delivered to each customer. Both components of the scheduling module (ROVER) objective function, the cost of the trip and the value of the trip, are calculated and shown to the scheduler together with any feasibility violations. Based on this information, the scheduler can either accept the change or continue with additional changes. The scheduler

also has the option of specifying the sequence of stops and the quantities to be delivered. Finding a customer to add to a trip in progress is used when there is product remaining in the truck after the final scheduled stop has been made.

Because the schedule change process requires extensive interaction between the scheduler and the computer system, we have emphasized ease of use and "friendliness" in the schedule change module to maximize the scheduler's effectiveness. Similarly, in all other on-line functions, especially the frequent entry of delivery data, very special attention has been given to ease of use.

### Implementation

Development work on the vehicle scheduling system began in January 1980 and culminated in October 1981 when the first implementation site, Wharton, New Jersey, began to use the system to produce operational schedules. During the 1980-81 development period the project team created and tested the modules and data bases described in the previous section. Components of the system were tested on historical data from the Wharton, New Jersey and Delaware City, Delaware depots.

The Wharton depot was chosen as the first installation site because it was close to corporate headquarters and had a large, diverse customer population that would thoroughly exercise all features of the system. Since the Wharton installation, eleven additional depots have gone live on the vehicle scheduling system. One of these depots (Delaware City) was closed in May 1983 and all oxygen and nitrogen operations discontinued. The lo-

cation and dates of the implemented depots are listed in Table 1.

Site	Implementation Date	Number of Months Operational as of October 31, 1983
Wharton, NJ	October 1981	25
Glenmont, NY	January 1982	22
Delaware City, DE	May 1982	12
LaSalle, IL	August 1982	15
Granite City, IL	September 1982	14
North Baltimore, OH	October 1982	13
Conyers, GA	November 1982	12
Pryor, OK	February 1983	9
LaPorte, TX	April 1983	6
Dallas, TX	April 1983	6
El Segundo, CA	May 1983	5
Lancaster, PA	July 1983	3
		142

Table 1. The depots at which the computerized vehicle scheduling system has been implemented, their dates of implementation, and the number of months they have been in operation.

It is highly likely that five depots will be added during the fiscal year ending in September 1984. The remaining seven depots are very small and it is not clear at this point if the economic benefits of vehicle scheduling will justify the cost of dedicated phone lines and hardware at the depots. This picture could change as hardware costs decrease and additional applications are added to the on-line network.

The combined use of the vehicle scheduling system by the 12 depots represents a significantly long history of 142 months. Since the system is run at least once every weekday at each implemented depot, the ROVER optimization module and other programs of vehicle scheduling system have now been executed on operational data more than 3,000 times.

On the day when vehicle scheduling went live at Wharton, New Jersey in October 1981, the authors of this paper

breathed a collective sigh of relief. Together with supporting personnel, we had invested 24 man years over a two-year period in developing the vehicle scheduling system. However, our relief was temporary when we realized that the hardest part of the project lay ahead of us — making it work!

The process of “making it work” actually began before the Wharton implementation and continues even today. We can now understand this process; it consists of the following activities:

- Introducing inexperienced computer users to state-of-the-art technology,
- Identifying and solving unforeseen problems, and
- Integrating the system into existing depot activities.

The schedulers who would be using the system were unfamiliar with computers and the discipline and precision required by the management science approach. To facilitate the implementation under such conditions, depot-personnel training was conducted in a phased approach which allowed schedulers the opportunity to become comfortable with the system as each functional area was added. The psychological impact of computerization on individuals was a major consideration in our training approach. We considered the trainees’ feelings and took them into account in our decisions, managing them carefully so that the implementation program would continue at a steady rate. Major emphasis was placed on developing users’ confidence in the computer system, a key to their acceptance of the changes that accompany computerization. It was important to stress that the com-

puter would not “threaten” jobs but rather “enhance” them.

One unforeseen problem resulted from the fact that the integer-programming model of the delivery problem required “hard” constraint limits. By contrast, a human scheduler knows that many of the constraints in the problem are somewhat “soft” and makes judicious adjustments where necessary to improve the quality of the schedule. To illustrate, in one early run of the ROVER model, there were three adjacent customers that each required about one-third of a truckload of product by 12 noon of the first day of the schedule horizon. The obvious decision was to send a single truck to satisfy the three customers’ requirements. However, if such a trip were started at the beginning of the scheduling horizon, delivery would be completed at the first customer by 12 noon, at the second by 12:30 pm and at the third by 1:00 pm. Because this violates the constraints of the model, the trip was not scheduled. Instead, a separate truck was sent to each of the customers, much to the amusement of the scheduler, who pointed out in rather direct language that delaying delivery by one hour to customers receiving deliveries about once a week is no big deal.

What’s at issue here is possibly the most fundamental problem in applying integer-programming models. Because of integrality restrictions, small changes in constraint limits can cause large changes in the optimal value. Because the data in most models can be changed slightly at little cost, these models can give misleading results. To overcome this difficulty, we determined from the nature of our

problem those constraints to which the model was highly sensitive and set their limits to relatively loose values. For example, the problem above was found to occur only for customers that were a considerable distance from the depot and required delivery relatively early in the scheduling horizon. For these few customers, the delivery time-limit was relaxed slightly.

The project team carefully designed the man-machine interfaces so that the vehicle scheduling system would fit into the daily work flow of existing depot activities. Changes were made to screen-formats and on-line dialogues as we observed bottlenecks, problems, and opportunities for enhancements. One key improvement that became the cornerstone of the implementation process was revamping the daily forecast results report. As initially designed, the report was oriented towards a strict inventory control perspective and was not compatible with the way the scheduler evaluated customer demand patterns. This initial report did not lend itself to replacing manual customer records, and as a result both a manual and automated forecasting system were maintained. A new report was designed which replicated the manual customer records and added other enhancements.

During the implementation phase, the model's solution was critically evaluated in parallel with the manual scheduling process. Schedulers had to accept or revise the model's daily solutions. Each scheduler had certain routes that he was accustomed to using out of habit, even though other more efficient routes existed. It was often troubling to schedul-

ers when ROVER produced routes that differed greatly from those they were used to. At this point we had the choice of "winning the battle" by forcing the scheduler to use the routes produced by the system or "winning the war" by allowing the scheduler to exercise control over the system and thereby gain confidence in it.

We opted for the latter course and were able to employ a technical feature of the system that had not been intended for this purpose. Each customer had a specified "neighborhood" of other customers that could be combined with that customer on the same route. Neighborhoods were originally introduced to reduce the amount of data that needed to be stored and were set up to contain about 50 customers. We now found that by carefully reducing the neighborhoods, we could limit the system to producing routes similar to those the scheduler was used to seeing. This proved invaluable in making schedulers feel comfortable with the system. Once they felt "in control" of the system, most schedulers began to ask that the neighborhoods be expanded in size to allow more options.

## Impact

In 1979 when this project was undertaken, the economic environment was severe. The nation was entering a recession and double digit inflation was having a serious effect on Air Products' distribution expenses. The industrial gas industry was in an overcapacity position that was further squeezing already tight profit margins. Although there was a good manual scheduling system in place, Air Products' management saw room for im-

provement, and people throughout the organization were capable and willing to accept change. Within this background, the management committee of Air Products approved this high technology, high risk project. Senior management decided to undertake this project as a demonstration and a use of high technology computer methods "to enter the 21st century on the run." The impact of the project has completely confirmed the wisdom of their decision.

The most direct tangible benefit has been a reduction in delivery expenses resulting from the increased productivity of the drivers and vehicles. The methodology we employed to measure this reduction is reasonably straightforward. Since delivery costs correlate strongly with miles driven, the miles driven per gallon of product delivered were monitored continuously before and after implementation of the vehicle scheduling system. In interpreting changes in the miles driven per gallon of product delivered, care was taken to control for the fact that customer usage rates change over time. For example, if the usage rate decreased significantly for several customers that were relatively far from a depot, this would cause a reduction in the miles traveled per gallon delivered, even if there were no improvement in scheduling efficiency. To control for this effect, we also computed on a continual basis a "weighted delivery radius." The weighted delivery radius for a period equalled the amount delivered to each tank times the distance of that tank from the depot summed over all tanks and divided by the amount delivered.

The effect of the vehicle scheduling sys-

tem can be evaluated by measuring over time the miles per gallon delivered relative to the weighted delivery radius. The weighted delivery radius for a terminal is computed by multiplying each customer's usage rate times his distance from the terminal, summing over all customers and dividing by total customer usage. Generally, the savings increased over time as the schedulers became familiar with using the system. In the case of liquid nitrogen deliveries at Wharton, the average miles per gallon delivered has decreased by 9.1 percent since introduction of the vehicle scheduling system. To this must be added the impact of the weighted delivery radius which has increased by 1.3 percent, yielding a total mileage savings of 10.4 percent. The average savings for a system of 16 depots is estimated at 6 percent to 7½ percent or \$1.54 million to \$1.72 million annually. Obviously, the dollar value of annual savings will increase as Air Products business continues to grow or if the inflation in the cost of transportation persists.

Another benefit derived from increased tanker productivity is a reduction in future requirements to purchase additional tankers to meet growing business needs. In the future, savings from avoiding capital expenditure should reach \$445,000 annually, representing a reduction in capital spending of \$3.1 million.

A third area for savings lies in greater utilization of the existing telephone line and computer network. Once the network exists, new applications can be added at a small incremental cost. One such application under development would replace the present practice of mailing completed

shipment receipts to regional teleprocessing centers who subsequently transmit them electronically to the billing system at corporate headquarters. Instead, completed shipment receipts would be directly transmitted and result in a net present value cash flow exceeding \$100,000 over several years.

The major intangible benefit is that the complex and powerful decision support system is used continuously each day by depot personnel. The interactive schedule change module and the extensive data base at the customer-tank-level of detail offer schedulers the opportunity to be more comprehensive in evaluating and selecting trips. The ready availability of the cost of alternative pairings is particularly valuable and is a crucial element of the system.

Using the vehicle scheduling system has reduced the clerical effort in scheduling, and thereby increased the time available for planning. The scheduling module (ROVER) also serves as a catalyst in training others to do scheduling, thereby reducing the reliance upon a single individual.

The availability of the time/distance data base and associated road network offers the potential to more effectively determine what roads vehicles will take. The continually shifting schedule means there are few "standard" routes. Initial efforts in this area showed that there are tradeoffs; the shortest route is not necessarily the best route. Work is underway at two depots to better define the criteria for selecting routes, with the ultimate objective of creating a comprehensive cross-reference of distances and routes between

delivery locations.

A number of further benefits are expected in the future. We are continuing to refine and enhance the system, and the schedulers are becoming more familiar with its use. Both tend to further reduce costs. In the future, consideration will be given to extending the system to other industrial gas product lines. We also believe this methodology is applicable outside the industrial gas industry for scheduling delivery of such products as petroleum, liquid propane, and home heating oil, or for scheduling of raw materials and products internally within a company.

Currently the safety-stock inventory level in each customer tank is determined manually by the scheduler. This process could be improved by considering in a more rigorous fashion variability in usage and distance from the depot. Using the vehicle scheduling system to determine the optimal tank size for new and existing customers is also being considered. The vehicle scheduling system would provide very accurate distribution-cost information which could then be combined with other financial considerations, including tank investment and installation cost, to determine the optimal tank size.

A final benefit of this project is the experience we gained in developing this large-scale operational decision support system in an inter-disciplinary environment. The internal project development team included individuals from the management science, business systems, and user communities, acting in consort with outside consultants. The development process was a powerful catalyst cementing the relationship between the various



management information and end-user functional areas. This relationship makes it markedly easier to develop other similar projects. The lessons we all learned from this experience offer valuable insights into ways to better manage a developmental project of this magnitude and scope.

The project has received widespread support and interest among top management of the Company. At a recent financial conference with securities analysts sponsored by Air Products, A. P. Dyer, Air Products Group Vice President — Gases Groups, briefly described the vehicle scheduling system: "I've simplified an extremely complex process which is designed to significantly reduce distribution costs. Thus far we have implemented the program in eight of our terminals. The savings in the first year of implementation including the usual start-up problems have been about 7 percent per terminal. We are highly optimistic for the future of this program. I might mention that other companies involved in similar bulk distribution have exhibited keen interest in the program." [1982]

#### Acknowledgements

We would like to give special acknowledgement to Stan Roman, Vice President of Air Products Industrial Gas Division Operations, and Byron Trammell, Manager of Air Products Industrial Gas Division Distribution, for their continuing support of this project. Significant contributions to the design, development, and implementation of the system were made by the other members of the Air Products project team: Ken Bailey, Paul Fehr, Dennis Houser, Charlie Lewis, Pete Peluso, Al Russell, and Tracey Smith. We

would also like to acknowledge Roger Bast, Manager of Management Sciences at Air Products, for his contributions to this project. Special thanks are due the personnel at the initial system implementation sites in the Eastern Region for their help in making the system work. We would also like to thank Peter Kolesar and Janet Showers for their review and comments on this manuscript.

Marshall Fisher's work was supported in part by ONR contract N00014-78-C-0307 P0007.

#### References

- Fisher, M. L.; Greenfield, A. J.; Jaikumar, R.; and Kedia, P. 1982, "Real-time scheduling of a bulk delivery fleet: practical application of Lagrangian relaxation," University of Pennsylvania, Decision Sciences Working Paper, October 1982.
- Fisher, M. L.; Jaikumar, R.; Kedia, P.; and Solomon, M. 1983, "A Lagrangian relaxation approach to solving large set partitioning-packing problems with side constraints," talk at the Chicago ORSA/TIMS Meeting, April 1983.
- Fisher, M. L. forthcoming, "A practitioner's guide to Lagrangian relaxation," *Interfaces*.
- Glover, F., and Klingman, D. 1977, "Network applications in industry and government," *AIIE Transactions*, Vol. 9, No. 4 (December), pp. 363-376.
- "Managing Growth — Planning Profitable Change" 1982, A compilation of the presentations to representatives of the financial community at Air Products Corporate Headquarters during the week of December 6, 1982, Air Products & Chemicals, Allentown, Pennsylvania.

#### APPENDIX: Mixed-Integer-Programming Formulation and Algorithm Summary

This appendix is extracted from Fisher et al. [1982], which contains a more extensive discussion of the technical aspects of our algorithm. The following definitions are used in our formulation.

#### Indices

$k$  = possible route index

$v$  = vehicle index

$i$  = customer index

$t$  = time index

$T$  = length of the planning horizon.

The index  $t$  ranges from 0 to  $T$ .

## Customer Data

$D_{it}$  = upper limit on the amount that can be delivered to customer  $i$  during the interval from 0 to  $t$

$d_{it}$  = lower limit on the amount that must be delivered to customer  $i$  during the interval from 0 to  $t$

$vd_i$  = per unit value of deliveries to customer  $i$

$VEH_i$  = set of vehicles that can feasibly service customer  $i$ . A vehicle could be infeasible for a customer because, for example, it was too large to fit at the unloading dock or because it required an external power source for unloading that was unavailable at the customer site.

$A_1$  = set of customers that can receive any number of deliveries

$A_2$  = set of customers that must receive exactly one delivery

$A_3$  = set of customers that can receive at most one delivery

( $A_1 \cup A_2 \cup A_3$  is the set of all customers)

$TIME_i$  = set of times at which a delivery at customer  $i$  can be completed.

This set is defined because many customers are not open 24 hours a day and are constrained as to when they can receive deliveries. Also, by definition of  $d_{it}$  and  $A_2$ , delivery to a customer  $i \in A_2$  is infeasible at any time greater than  $t$  if  $d_{it} > 0$ .

## Vehicle Data

$CAP_v$  = capacity of vehicle  $v$

$c_v$  = per unit unloading cost for vehicle  $v$ .

## Possible Routes Data

$S_k$  = set of customers on route  $k$  in delivery order

$R_i$  = set of routes on which customer  $i$

appears

$= \{k \mid i \in S_k\}$

$TL_k$  = time that a vehicle is occupied in driving route  $k$  (assumed integral)

$tc_{ik}$  = time after the start of route  $k$  until delivery to customer  $i$  is completed (assumed integral)

$T_k$  = times at which route  $k$  can start  
 $= [t \in \{0, 1, \dots, T\} \mid t + tc_{ik} \in TIME_i \text{ for all } i \in S_k]$

$V_k$  = vehicles that can drive route  $k$   
 $= \bigcup_{i \in S_k} VEH_i$

$K_v$  = routes that vehicle  $v$  can drive  
 $= \{k \mid v \in V_k\}$

$FC_{kv}$  = fixed cost for vehicle  $v$  driving route  $k$ .  $FC_{kv}$  includes all costs that are independent of the amount delivered to each customer on route  $k$ . In the application considered here,  $FC_{kv}$  is determined from the mileage of route  $k$ , driver pay regulations, fuel costs and vehicle depreciation per mile.

## Variables

$y_{ktv} = 1$ , if route  $k$  starts at time  $t$  on vehicle  $v$ , 0, otherwise

$x_{iktv}$  = amount delivered to customer  $i$  on route  $k$  by vehicle  $v$  starting at time  $t$ .

## Route Selection Model

$$\max \sum_k \sum_{v \in V_k} \sum_{t \in T_k} \left[ \sum_{i \in S_k} (vd_i - c_v) x_{iktv} - FC_{kv} y_{ktv} \right] \quad (1)$$

subject to

$$d_{it} \leq \sum_{k \in R_i} \sum_{v \in V_k} \sum_{\substack{\tau \in T_k \\ \tau \leq t - tc_{ik}}} x_{ik\tau v} \leq D_{it}, \quad \text{for all } t \text{ and } i \in A_1 \quad (2.1)$$

$$\sum_{k \in R_i} \sum_{v \in V_k} \sum_{t \in T_k} y_{ktv} = 1, \quad i \in A_2 \quad (2.2)$$

$$\sum_{k \in R_i} \sum_{v \in V_k} \sum_{t \in T_k} y_{ktv} \leq 1, \quad i \in A_3 \quad (2.3)$$

$$\sum_{i \in S_k} x_{iktv} \leq CAP_v y_{ktv}, \text{ for all } ktv \quad (3)$$

$$\sum_{k \in K_v} \sum_{\tau=t-TL_k}^{\tau \in T_k} y_{ktv} \leq 1, \text{ for all } tv \quad (4)$$

$$y_{ktv} = 0 \text{ or } 1, \text{ for all } ktv \quad (5.1)$$

$$D_{i,t} + tc_{ik} \geq x_{iktv} \geq 0, \text{ for all } iktv \quad (5.2)$$

$$x_{iktv} \geq d_{it} y_{ktv}, \text{ for all } ktv \quad (5.3)$$

and  $i \in S_k \cap A_2$ .

The objective of the route selection model is to maximize the value of all deliveries less the fixed and variable costs of making those deliveries. Constraints (2.1) through (2.3) represent customer demand for the three types of customers. In constraint (2.1) note that  $x_{ik\tau v}$  is not the amount delivered to customer  $i$  at time  $\tau$ , but at time  $\tau + tc_{ik}$ . This explains the requirement for  $\tau \leq t - tc_{ik}$  in the summation over  $t$ . The summation over  $x_{ik\tau v}$  is the amount delivered to customer  $i$  during the time interval from 0 to  $t$ . Constraints (2.2) and (2.3) express the requirement that 1, or at most 1, delivery should be made to customers in  $A_2$  or  $A_3$ . Constraints on the volume of these deliveries are imposed by (5.2) and (5.3). Constraint (5.3) expresses the requirement that the single delivery received by a type 2 customer must be sufficient to supply his needs for the entire planning horizon. The sets  $A_1$ ,  $A_2$  and  $A_3$  would normally be defined by management policy. Generally,  $A_2$  and  $A_3$  will contain smaller customers for whom multiple visits in a single planning horizon would be uneconomical. The customers in  $A_3$  have the further property that  $d_{it} = 0$ .

Constraint (3) is similar to constraints that arise in warehouse location models if we think of  $y_{ktv}$  as the variable that determines whether a warehouse is open and the  $x_{iktv}$  as variables representing

flows from that warehouse. If a particular route-time-vehicle combination  $ktv$  is not selected, then  $y_{ktv} = 0$  and constraint (3) requires that  $x_{iktv} = 0$  for all  $i$  contained in  $S_k$ . On the other hand, if a  $ktv$  combination is selected, then  $y_{ktv} = 1$  and this constraint requires that deliveries on this route not exceed vehicle capacity. It is understood in constraints (3) and (5) that the indices  $ktv$  will run only over values that are allowed by the sets  $T_k$  and  $V_k$ .

Constraint (4) imposes the requirement that vehicle  $v$  cannot be driving more than one route during the time interval from  $t-1$  to  $t$ .

Constraints (5.1) to (5.3) impose integrality requirements and natural lower and upper bounds on the continuous variables. The parameters  $d_{it}$  and  $D_{it}$  are determined from the forecast of the demand for customer  $i$ . As described in the section on demand forecasting and inventory level calculation, the forecast of future demand can be used to predict what the customer inventory level would be at any point in time if no further deliveries were made. Then,  $D_{it}$  is set to the capacity of the storage tank minus the predicted inventory level at time  $t$ . If forecast inventory is above the safety stock level at time  $t$ , then  $d_{it}$  is set to 0. Otherwise,  $d_{it}$  is set to the safety stock level minus predicted inventory at time  $t$ .

The parameters  $vd_i$  are used to represent the effect of model decisions on events that occur beyond the horizon of the model. In the short run horizon considered by the model, there is considerable discretion in the amount of product that can be delivered to a particular customer. However, the amount delivered in the long run is determined by customer demand. Hence, each gallon scheduled for delivery to customer  $i$  within the model horizon reduces the amount that must be delivered in the future. This effect is accounted for by setting  $vd_i$  to an estimate of the cost per unit of delivering

to customer  $i$  at a point in time outside the planning horizon of the model.

Lagrangian relaxation was the central concept used in analyzing the route selection model. As described in Fisher [1984], the goal in formulating a Lagrangian relaxation is to dualize a set of constraints whose removal greatly simplifies the problem, but still admits a tight bound on the optimal objective value. This purpose is accomplished here by dualizing constraints (2.1) — (2.3).

The key observation in solving the Lagrangian problem is that removing constraints (2.1) — (2.3) greatly simplifies the dependence of  $x_{iktv}$  on  $y_{ktv}$ . As a result, we are able to solve for values of  $x_{iktv}$ ,  $i \in S_k$  that are optimal in the Lagrangian problem if  $y_{ktv} = 1$ . Once the  $x_{iktv}$  have been determined, the Lagrangian problem reduces to a problem for each vehicle that resembles the knapsack problem and can be solved by dynamic programming.

In our computational algorithm we first use a multiplier adjustment method to determine values for the Lagrange multipliers that provide a tight upper bound on the optimal value. We then execute a primal heuristic that uses the Lagrange multipliers to obtain a feasible solution to the route selection model.

Finally we have built a branch and bound algorithm around this Lagrangian procedure. However, branch and bound has not played a major role in our computational work because the upper bound and feasible solution that come from the procedures just described have always been sufficiently close for practical purposes.

Ed Note: As we go to press, Marshall Fisher is preparing "A practitioner's guide to Lagrangian relaxation." Look for it early in 1984.