JYOTHY INSTITUTE OF TECHNOLOGY

DEPARTMENT OF MATHEMATICS
Semester- III
ASSIGNMENT 21MAT31

MODULE-1
Laplace Tranforms and Inverse Laplace Transforms

1. Find the Laplace transform of $\frac{\cos at-\cos bt}{t}+\sin at$
2. Find Laplace transform of $2^t+\frac{\cos 2t-\cos 3t}{t}+t\,sint$
3. Evaluate i) $L\left\{\frac{cos2t-cos3t}{t}\right\}$ ii) L$\{t^2e^{-3t}sin2t\}$
4. Find L$\{e^{3t}(2\cos 5t-3\sin 5t)\}$
5. A Periodic function $f(t)$ with period 'a' is defined by $f(t)=\begin{cases} E, 0\le t\le \frac{a}{2} \\ -E\,,\frac{a}{2}\le t\le a\end{cases}$ Show that $L\{f(t)\}=\left(\frac{E}{s}\right)\tanh\left(\frac{as}{4}\right)$
6. If $f(t)=\begin{cases} t, & 0\le t\le a \\ 2a-t, & a\le t\le 2a\end{cases}$, $f(t+2a)=f(t)$ then show that $L\{f(t)]=\frac{1}{s^2}\tanh\left(\frac{as}{2}\right)$
7. Express $f(t)=\begin{cases} cost & 0<t<\pi \\ cos2t & \pi<t<2\pi \\ cos3t & t>2\pi\end{cases}$ in terms of unit step function and hence find its Laplace transform
8. Express $f(t)=\begin{cases} sint & 0\le t<\pi \\ sin2t & \pi\le t<2\pi \\ sin3t & t\ge 2\pi\end{cases}$ in terms of unit step function and hence find its Laplace transform
9. Express $f(t)=\begin{cases} 1, & 0<t\le 1 \\ t, & 1<t\le 2 \\ t^2, & t>2\end{cases}$ in terms of unit step function and hence find its Laplace transform
10. Express $f(t)=\begin{cases} sint, & 0\le t\le \frac{\pi}{2} \\ cost, & t>\frac{\pi}{2}\end{cases}$ in terms of unit step function and hence find its Laplace transform
11. Solve the differential equation $\frac{d^2y}{dx^2}+4\frac{dy}{dx}+3y=e^{-x}$ with $y(0)=y^1(0)=1$ using Laplace transform method
12. Solve the differential equation $\frac{d^2y}{dx^2}+4\frac{dy}{dx}+4y=e^{-t}$ with $y(0)=y^1(0)=0$ using Laplace transform method
13. Solve the differential equation $\frac{d^2x}{dt^2}-2\frac{dx}{dt}+x=e^{2t}$ with $x(0)=x^1(0)=-1$ using Laplace transform method
14. Solve the equation by Laplace transform method $y'''+2y''-y'-2y=0$ Given y(0)= $y'(0)=0, y''(0)=6$
15. Solve the differential equation $\frac{d^2y}{dx^2}+6\frac{dy}{dx}+9y=12\,t^2e^{-3t}$ with $y(0)=y^1(0)=0$ using Laplace transform method
16. Solve the differential equation $\frac{d^2y}{dt^2}+2\frac{dy}{dt}+y=te^{-t}$ with $y(0)=y^1(0)=-2$ using Laplace transform method

17. Find the inverse Laplace transform of log $\left(\frac{s(s+5)}{(s^2+25)(s-7)}\right)$

18. Evaluate $L^{-1}\left\{\frac{4s+5}{(s+1)^2(s+2)}\right\}$

19. Find $L^{-1}\left\{\frac{1}{s(s^2+a^2)}\right\}$ by using convolution theorem

20. Find the inverse transform of $\frac{s^2}{(s^2+a^2)^2}$ using convolution theorem

21. Evaluate $L^{-1}\left\{\log\left\{\frac{s^2+1}{s(s+1)}\right\}\right\}$

22. Evaluate $L^{-1}\left\{\frac{s+3}{s^2-4s+13}\right\}$

23. Find the Laplace transform of i) $\frac{\cos at-\cos bt}{t}$ ii) $t\,e^{-t}\sin(4t)$

24. Evaluate $L^{-1}\left\{\frac{s}{(s-1)(s^2+4)}\right\}$

MODULE-2

Fourier series

1. Expand $f(x)=\sqrt{1-\cos x},\ \ 0<x<2\pi$ in the Fourier series. Hence evaluate $\frac{1}{1.3}+\frac{1}{3.5}+\frac{1}{5.7}++$ ....
2. Obtain the Fourier series for the function $f(x)=\begin{cases}1+\frac{2x}{\pi} & in \quad -\pi\le x\le 0\\ 1-\frac{2x}{\pi} & in \quad 0\le x\le \pi\end{cases}$ and hence deduce $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+$ .... $=\frac{\pi^2}{8}$.
3. Find the Fourier series of $f(x)=x-x^2,\quad -\pi\le x\le\pi$. Hence deduce that $\frac{1}{1^2}-\frac{1}{2^2}+\frac{1}{3^2}-$ .... $=\frac{\pi^2}{12}$.
4. Expand $f(x)=x\sin x$ as a Fourier series in the interval $(-\pi,\pi)$, hence deduce the following
   $i)\ \frac{\pi}{2}=1+\frac{2}{1.3}-\frac{2}{3.5}+\frac{2}{5.7}-$ ....
   $ii)\ \frac{\pi-2}{4}=\frac{1}{1.3}-\frac{1}{3.5}+\frac{1}{5.7}-+$ ....
5. Obtain the Fourier expansion of $f(x)=\begin{cases}-\pi & -\pi<x<0\\ x & 0<x<\pi\end{cases}$ and hence $\frac{\pi^2}{8}=\sum_{n=1}^{\infty}\frac{1}{(2n-1)^2}$
6. Obtain the Fourier series of $f(x)=\left(\frac{\pi-x}{2}\right)^2$ in the interval $(0,2\pi)$ and deduce that $\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+$ ...... $=\frac{\pi}{6}$
7. Find the Fourier series for the function $f(x)=|x|$ in $(-\pi,\pi)$, hence deduce that $\frac{\pi^2}{8}=\sum_{n=1}^{\infty}\frac{1}{(2n-1)^2}$
8. Obtain the half range Fourier cosine series for the function $f(x)=x(l-x)$ in $0\le x\le l$. Find the half range Fourier cosine series for the function $f(x)=\begin{cases}kx & in \quad 0\le x\le l/2\\ k(l-x) & in \quad l/2<x\le l\end{cases}$
   Where k is a non-integer positive constant.
9. Find the half range Fourier cosine series for the function $f(x)=(x-l)^2$ in $0<x<l$ and hence show that $\pi^2=8\left\{\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\ ....\right\}$
10. Find the half range Fourier sine series for the function $f(x)=\begin{cases}x & in \quad 0<x<\frac{\pi}{2}\\ \pi-x & in \quad \frac{\pi}{2}<x<\pi\end{cases}$

11. Find the half range Fourier sine series for the function $f(x) = \begin{cases} \frac{1}{4} - x & in \quad 0 < x < \frac{l}{2} \\ x - \frac{3}{4} & in \quad \frac{l}{2} < x < l \end{cases}$

12. Find the half range sine series of $f(x) = e^x$ in $(0,1)$.

13. Find $a_0,\ a_1$ and b_1 in the Fourier expansion of y using the harmonic analysis from the following table

x	0	1	2	3	4	5
y	9	18	24	28	26	20

14. Find the constant term and the first two harmonics in the Fourier series for $f(x)$ given by the following table:

x	0	$\pi/3$	$2\pi/3$	π	$4\pi/3$	$5\pi/3$	2π
$f(x)$	1.0	1.4	1.9	1.7	1.5	1.2	1.0

15. In a machine the displacement y of a given point is given for a certain angle x as follows

x	0	30	60	90	120	150	180	210	240	270	300	330
y	7.9	8	7.2	5.6	3.6	1.7	0.5	0.2	0.9	2.5	4.7	6.8

16. Obtain the constant term and coefficients of first cosine and sine terms in the expansion of y from the following table

x	0	60^0	120^0	180^0	240^0	300^0	360^0
y	7.9	7.2	3.6	0.5	0.9	6.8	7.9

Module III

Fourier Transform and Z Transform

1. Find the Fourier transform of $f(x) = \begin{cases} 1 - x^2 & for \quad |x| \le 1 \\ 0 & for \quad |x| > 1 \end{cases}$ and evaluate $\int_0^\infty \frac{x\cos x - \sin x}{x^3} \cos\frac{x}{2}\, dx$.
2. Find Fourier transformation of $e^{-a^2x^2}$ $(-\infty < x < \infty)$ hence show that $e^{-\frac{x^2}{2}}$ is self reciprocal.
3. Find Fourier cosine and sine transformation of $f(x) = \begin{cases} x & 0 < x < a \\ 0 & x \ge a \end{cases}$
4. Solve integral equation $\int_0^\infty f(x) \cos sx\ dx = \begin{cases} 1 - s & 0 < s < 1 \\ 0 & s \ge 1 \end{cases}$ hence deduce $\int_0^\infty \frac{1-\cos x}{x^2}\, dx = \frac{\pi}{2}$.
5. Find the Fourier transform of $f(x) = \begin{cases} 1 - |x| & for \quad |x| \le 1 \\ 0 & for \quad |x| > 1 \end{cases}$ hence deduce that $\int_0^\infty \frac{\sin^2 t}{t^2}\, dt = \frac{\pi}{2}$.
6. Find $f(x)$, if $\tau_s\{f(x)\} = \frac{s}{s^2+1}$
7. Find the Fourier cosine transform of e^{-ax} and $x\, e^{-ax}$ where $a > 0$ deduce that, $\int_0^\infty \frac{\cos mx}{x^2+a^2}\, dx = \frac{\pi}{2a} e^{-am}$
8. Find the Fourier sine transform of $\frac{e^{-ax}}{x}$
9. Find the Fourier cosine transform of $f(x) = \begin{cases} 4x & for \quad 0 < x < 1 \\ 4 - x & for \quad 1 < x < 4 \\ 0 & for \quad x > 4 \end{cases}$
10. Find the Fourier transform of the function $f(x) = \begin{cases} 1 & for \quad |x| \le a \\ 0 & for \quad |x| > a \end{cases}$ and hence evaluate $\int_0^\infty \frac{sinx}{x} dx$.
11. Find the Fourier sine transform of $f(x) = e^{-|x|}$ and hence evaluate $\int_0^\infty \frac{x \sin mx}{x^2+1}\, dx, \quad m > 0$.
12. Find the Fourier transform of the function $f(x) = xe^{-a|x|}$.
13. Find the inverse Fourier sine transform of e^{-s^2}
14. Find the z-transform of: $i)\ \sin h\, n\theta;\ \ ii)\ \cos h\, n\theta\,;\ \ iii)\ n \cos n\,\theta;\ \ iv)\ n \sin n\theta$.

Module V

Numerical solution of second order ODE and Calculus of variation

1. Given $y'' - xy' - y = 0$ with initial condition y(0) =1,y'(0) = 0 compute y(0.2) by taking h=0.2 using fourth order Runge Kutta Method.
2. Applying Milne's method compute y (0.8).Given that y satisfies the equation $y'' = 2yy'$

   y(0) = 0, y(0.2) = 0.2027, y(0.4) = 0.4228, y(0.6) = 0.6841

   y'(0) =1, y'(0.2) =1.041, y'(0.4) =1.179, y'(0.6)=1.468 (Apply corrector only once)

3. Obtain the solution of the equation $2\frac{d^2y}{dx} = 4x + \frac{dy}{dx}$ by computing the value of the dependent variable corresponding to the value 1.4 of the independent variable by applying Milne's method using the following the data:

x	1	1.1	1.2	1.3
y	2	2.2156	2.4649	2.7514
y'	2	2.3178	2.6725	3.0657

4. By Runge Kutta method solve $\frac{d^2y}{dx^2} = x(\frac{dy}{dx})^2 - y^2$ for x = 0.2. Correct to four decimal places using the initial conditions y =1 and y'= 0 at x = 0,h = 0.2

5. Using R-k method, find the solution at x = 0.1 of an equation $y'' - x^2y' - 2xy - 1 = 0$ with the conditions y(0) =1,y'(0) = 0 and step size h = 0.1
6. Given that $y'' + xy = 0$, y(0) =1, y(0.1) =1.0998, y(0.2) =1.1987, y(0.3) =1.2955 y'(0) =1, y'(0.1) = 0.9946, y'(0.2) = 0.9773, y'(0.3) = 0.946 Find y(0.4) Using Milne's method

7. Derive Euler's equation in the standard form $(\partial f/ \partial y) - d/dx\,(\partial f/\partial y) = 0$.

8. Find the extremal of the functional $\int_0^{\frac{\Pi}{2}} (y^2 - y'^2 + 2y\sin x)\,dx$

9. Find the geodesics on a surface given that the arc length on the surface is $S = \int_{x1}^{x2} \sqrt{x(1 + y_1{}^2)\,dx}$.

10. Prove that the shortest distance between two points in a plane is along the straight line joining them or prove that the geodesics on a plane are straight lines.

11. Prove that catenary is a curve which when rotated about a line generates a surface of minimum area.

12. Find the path in which a particle, in the absence of friction, will slide from one point to another in the shortest time under the action of gravity.