Activity

1) Let A be the set of all triangles in a Phone and let R be a relation if it is reflexive, Symmetric and transitive, show that R is an equivalence relation in A.

Sol:- The given relation satisfies the following properties:-

i) Reflexivity

Let Δ be the arbitary triangle in A. Then,

$\Delta \cong \Delta \Rightarrow (\Delta, \Delta) \in R$ for all values of Δ in A.

$\therefore$ R is Reflexive

ii) Symmetry

Let $\Delta_1, \Delta_2 \in A$ Such that $(\Delta_1, \Delta_2) \in R$. then

$(\Delta_1, \Delta_2) \in R \Rightarrow \Delta_1 \cong \Delta_2$

$\Rightarrow \Delta_2 \cong \Delta_1$

$\Rightarrow (\Delta_2, \Delta_1) \in R$

$\therefore$ R is Symmetric

iii) Transitivity

Let $\Delta_1, \Delta_2, \Delta_3 \in A$ Such that $(\Delta_1, \Delta_2) \in R$ and $(\Delta_2, \Delta_3) \in R$.

then, $(\Delta_1, \Delta_2) \in R$ and $(\Delta_2, \Delta_3) \in R$

$\Rightarrow \Delta_1 \cong \Delta_2$ and $\Delta_2 \cong \Delta_3$

$\Rightarrow \Delta_1 \cong \Delta_2$

$\Rightarrow (\Delta_1, \Delta_3) \in R$

$\therefore$ R is transitive

Thus, R is reflexive, Symmetric and transitive.

Hence, R is an equivalence Relation

2) Let A be the set of all line in x y Plane and Left R be a relation in A, defined by

$$R = \{(L_1, L_2) : L_1 \parallel L_2\}$$

Show that R is an Equivalence relation in A.
Find the set of all lines related to the line $y = 3x + 5$.

A) The given related Satisfied the following Properties:

i) Reflexitivity

Let L be an arbitrary line in A, then

$L \parallel L \Rightarrow (L, L) \in R \quad \forall L \in A$

Thus, R is reflexive

ii) Symmetry

Let $L_1, L_2 \in A$ Such that $(L_1, L_2) \in R$, then

$(L_1, L_2) \in R \Rightarrow L_1 \parallel L_2$
$\Rightarrow L_2 \parallel L_1$
$\Rightarrow (L_2, L_1) \in R$

$\therefore$ R is Symmetric

iii) Transitivity

Let $L_1, L_2, L_3 \in A$ such that $(L_1, L_2) \in R$ and $(L_2, L_3) \in R$

Then $(L_1, L_2) \in R$ and $(L_2, L_3) \in R$

$\Rightarrow L_1 \parallel L_2$ and $L_2 \parallel L_3$
$\Rightarrow L_1 \parallel L_3$
$\Rightarrow (L_1, L_3) \in R$.

$\therefore$ R is transitive.

Thus R is reflextive, Symmetric and Transitive.

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3) Let S be the set of all real numbers and let R be a relation in S, defined by $R = \{(a,b) : a \le b^2\}$
Show that R satisfies none of reflexivity, symmetry and transitivity.

A) i) Non reflexivity
Clearly, $\frac{1}{2}$ is a real number and $\frac{1}{2} \le \left(\frac{1}{2}\right)^2$ is not true

$\therefore \left(\frac{1}{2}, \frac{1}{2}\right) \notin R$
Hence, R is not reflexive

ii) Non Symmetry
Consider the real number $\frac{1}{2}$ and 1
Clearly, $\frac{1}{2} \le 1^2 \Rightarrow \left(\frac{1}{2}, 1\right) \in R$.

But, $1 \le \left(\frac{1}{2}\right)$ is not true and so $\left(1, \frac{1}{2}\right) \notin R$
Thus, $\left(\frac{1}{2}, 1\right) \in R$ but $\left(1, \frac{1}{2}\right) \notin R$.
Hence, R is not symmetric

iii) Non Transitivity.
Consider the real number 2, −2 and 1
Clearly, $2 \le (-2)^2$ and $-2 \le (1)^2$ but $2 \le 1^2$ is not true
They, $(2,-2) \in R$ and $(-2,1) \in R$, but $(2,1) \notin R$
Hence, R is not transitive

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4) Which of these collection of subset are partitions of:

$S = \{-3, -2, -1, 0, 1, 2, 3\}$

(a) $\{-3, -1, 1, 3\}, \{-2, 0, 2\}$

(b) $\{-3, -2, -1, 0\}, \{0, 1, 2, 3\}$

(c) $\{-3, 3\}, \{-2, 2\}, \{-1, 1\}, \{0\}$

(d) $\{-3, -2, 2, 3\}, \{-1, 1\}$

A) a) $S_1 = \{-3, -1, 1, 3\}$ and $S_2 = \{-2, 0, 2\}$

$S_1 \cap S_2 = \emptyset$ and $S_1 \cup S_2 = \{-3, -1, 1, 3, -2, 0, 2\} = S$

Also $S_1 \neq \emptyset$ and S_2.

By the definition of partition, the given collection of subset is a partition.

$\{-3, -1, 1, 3\} \cap \{-2, 0, 2\} = \emptyset$ (yes)

b) $S_3 = \{-3, -2, -1, 0\}$ and $S_4 = \{0, 1, 2, 3\}$

$S_3 \cap S_4 \neq \emptyset$

Therefore, the given collection of subset is not a partition

c) $S_5 = \{-3, -2\}, S_6 = \{-2, 2\}, S_7 = \{-1, 1\}, S_8 = \{0\}$

$S_5 \cap S_6 \cap S_7 \cap S_8 = \emptyset$ (yes)

$S_5 \cup S_6 \cup S_7 \cup S_8 = \{-3, 3, -2, 2, -1, 1, 0\} = S$

Also, $S_5 \neq \emptyset$, $S_6 \neq \emptyset$, $S_7 \neq \emptyset$, $S_8 \neq \emptyset$

By the definition of partition the given collection of subset is a partition.

d) $S_9 = \{-3, -2, 2, 3\}$ and $S_{10} = \{-1, 1\}$

$S_9 \cap S_{10} = \emptyset$ and $S_9 \cup S_{10} = \{-3, -2, 2, 3, -1, I\} \neq S$

$\therefore$ The given collection of subset is not a partition.

5) Show that the relation R on the set of all bit strings, such that sRt iff s and t contain the same number of 1s is an equivalence relation. Also, determine the equivalence class of the bit string all for the equivalence relation R.

A) A = set of all bit string

$R = \{(s,t) \mid s \text{ and } t \text{ have the same number of 1s}\}$

i) Reflexivity : $\forall s \in A \ (s,s) \in R$

$(s,s) \in R$ means s and s have same number of 1's. Therefore, R is reflexive

ii) Symmetry : $\forall s, t \in A \ [(s,t) \in R \rightarrow (t,s) \in R]$

$(s,t) \in R$ means s and t have same number of 1's.

$(t,s) \in R$ means t and s have same numbers of 0's.

$\therefore$ R is symmetric

iii) Transitivity : $\forall s, t, u \in A \ [((s,t) \in R \wedge (t,u) \in R) \rightarrow (s,u) \in R]$

if $(s,t) \in R \wedge (t,u) \in R$ then $(s,u) \in R$ because

s and t have the same number of 1's, t and u have the same number of 1's. then it is obvious that s and u have the same number of 1's.

$\therefore$ Therefore, R is transitive.

6) Show that the function $f: N \to N$, defined by

$$f(x) = \begin{cases} x+1, & \text{if } x \text{ is odd} \\ x-1, & \text{if } x \text{ is even} \end{cases}$$

is one-one and onto

A) Suppose $f(x_1) = f(x_2)$.

Case 1: When x_1 is odd and x_2 is even

In this case, $f(x_1) = f(x_2) \Rightarrow x+1 = x_2 - 1$

$\Rightarrow x_2 - x_1 = 2$

This is a contradiction, since the difference between an odd integer and an even integer can never be 2.

Thus, in this case, $f(x_1) \neq f(x_2)$

$\therefore$ Similarly, when x is even and x_2 is odd, then $f(x_1) \neq f(x_2)$

Case-2: When x_1 and x_2 are both odd,

In this case, $f(x_1) = f(x_2) \Rightarrow x_1 + 1 = x_2 - 1$

$\Rightarrow x_1 = x_2$

$\therefore f$ is one-one.

Case-3:- When x_1 and x_2 are both even

In this case, $f(x_1) = f(x_2) \Rightarrow x_1 - 1 = x_2 - 1$

$\Rightarrow x_1 = x_2$

$\therefore f$ is one-one

In order to show that f is auto, let $y \in N$ (the co domain)

Case 1:- When y is odd

In this case, $(y+1)$ is even

$\therefore f(y+1) = (y+1) - 1 = y$

Case 2:- when y is even
In this case, $(y-1)$ is even
$\therefore f(y-1) = y-1+1 = y$

Thus, each $y \in N$ (co domain of f) has its pre-image in dom(f).

$\therefore f$ is onto.
Hence, f is one-one onto.

7) Show that $f: N \to N$, defined by

$$f(x) = \begin{cases} \frac{n+1}{2}, & \text{if } n \text{ is odd} \\ \frac{n}{2}, & \text{if } n \text{ is even} \end{cases}$$

is a many-one onto function

A) we have
$f(1) = \frac{(1+1)}{2} = \frac{2}{2} = 1$ and $f(2) = \frac{2}{2} = 1$
Thus, $f(1) = f(2)$ while $1 \neq 2$.

$\therefore f$ is many-one.
In order to show that f is onto, consider an arbitary element $n \in N$
if n is odd then $2n$ is even and $f(2n) = \frac{2n}{2} = n$

Thus, for each $n \in N$ (whether even or odd) there exists its pre-image in N.

$\therefore f$ is onto
Hence, f is many-one onto.

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8) Let $A = R - \{3\}$ and $B = R - \{1\}$

Let $f : A \to B : f(x) = \frac{x-2}{x-3}$ for all values of $x \in A$

Show that f is one-one and onto.

A) f is one-one, since.

$f(x_1) = f(x_2) \Rightarrow \frac{x_1 - 2}{x_1 - 3} = \frac{x_2 - 2}{x_2 - 3}$

$\Rightarrow (x_1 - 2)(x_2 - 3) = (x_1 - 3)(x_2 - 2)$

$\Rightarrow x_1 x_2 - 3x_1 - 2x_2 + 6 = x_1 x_2 - 2x_1 - 3x_2 + 6$

$\Rightarrow x_1 = x_2$

Let $y \in B$ such that $y = \frac{x-2}{x-3}$

then $(x-3)y = (x-2) \Rightarrow x = \frac{(3y-2)}{(y-1)}$

Clearly, x is defined when $y \neq 1$

Also, $x = 3$ will give us $1 = 0$, which is false

$\therefore x \neq 3$

And, $f(x) = \frac{\left(\left(\frac{3y-2}{y-1}\right) - 2\right)}{\left(\left(\frac{3y-2}{y-1}\right) - 3\right)} = y$

Thus, for each $y \in B$, there exists $x \in A$ such that $f(a) = y$

$\therefore$ f is onto

Hence, f is onto one-one

9)

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9) Let A and B be two non empty sets, show that the function $f:(A\times B)\to(B\times A)$ · $f(a,b)=(b,a)$ is a bijective function.

A) f is one-one, since
$f(a_1,b_1)=f(a_2,b_2)\Rightarrow(b_1,a_1)$
$\Rightarrow a_1=a_2$ and $b_1=b_2$
$\Rightarrow(a_1,b_1)=(a_2,b_2)$

In order to show that f is onto, let (b,a) be an arbitary element of $(B\times A)$.

Then $(b,a)\in(B\times A)$
$\Rightarrow b\in B$ and $a\in A$
$\Rightarrow(a,b)\in(A\times B)$

Thus, for each $(b,a)\in(B\times A)$, there exists $(a,b)\in A\times B$ such that
$f(a,b)=(b,a)$
$\therefore$ f is onto
Thus, f is one-one onto and hence bijective.

10) Consider a function $f:X\to Y$ and define a relation R in X by $R=\{(a,b): f(a)=f(b)\}$, show that R is an equivalence relation.

A) Here, R satisfies the following properties.

i) Reflexivity
Let $a\in X$. then,
$f(a)=f(a)\Rightarrow(a,a)\in R$.
$\therefore$ R is reflexive.

ii) Symmetric

Let $(a,b) \in R$, then

$(a,b) \in R \Rightarrow f(a) = f(b) \Rightarrow f(b) = f(a)$

$\Rightarrow (b,a) \in R$.

iii) Transitivity :-

Let $(a,b) \in R$ and $(b,c) \in R$, Then

$(a,b) \in R, (b,c) \in R$.

$\Rightarrow f(a) = f(b)$ and $f(b) = f(c)$

$\Rightarrow f(a) = f(c)$

$\Rightarrow (a,c) \in R$

$\therefore$ R is transitive.

Hence, R is an Equivalence Relation.