Karatsuba Algorithm

The Karatsuba algorithm is a fast multiplication algorithm that was discovered by Anatolii Alexeevitch Karatsuba in 1960. It is a divide-and-conquer algorithm that reduces the multiplication of two n-digit numbers to at most three multiplications of n/2-digit numbers, in addition to some extra additions and subtractions. This leads to a faster multiplication algorithm than the traditional long multiplication algorithm, especially for large numbers.

The Karatsuba algorithm works as follows:

Step 1: Divide the two n-digit numbers, x and y, into two n/2-digit numbers, a, b, c, and d, respectively.

Step 2: Compute three multiplications recursively:

Compute a*c.

Compute b*d.

Compute (a+b)*(c+d).

Step 3: Combine the three multiplication results to get the final product, z:

z = ac + ((a+b)(c+d) - ac - bd) * 10^(n/2) + b*d.

The Karatsuba algorithm reduces the number of multiplication operations required to compute the product of two n-digit numbers. While it may not be faster than the traditional long multiplication algorithm for small n, it becomes faster for larger n. The Karatsuba algorithm is a more efficient algorithm for multiplication than the traditional "long multiplication" algorithm. The time complexity of the Karatsuba algorithm is $O(n^{1.585})$, which is faster than the $O(n^2)$ time complexity of the traditional multiplication algorithm. The space complexity of normal multiplication is $O(n^2)$ whereas the space complexity of Karatsuba algorithm is $O(n)$.

C++ function for Karatsuba Algorithm

```cpp
#include <iostream>

#include <cmath>

using namespace std;

int karatsuba(int x, int y) {

  if (x < 10 || y < 10) {

    return x * y;

  }

  else {

    int n = max((int)log10(x) + 1, (int)log10(y) + 1);

    int half = n / 2;

    int a = x / pow(10, half); // left part of x

    int b = x % (int)pow(10, half); // right part of x

    int c = y / pow(10, half); // left part of y

    int d = y % (int)pow(10, half); // right part of y

    int ac = karatsuba(a, c);

    int bd = karatsuba(b, d);

    int ad_plus_bc = karatsuba(a + b, c + d) - ac - bd;

    return ac * pow(10, 2 * half) + ad_plus_bc * pow(10, half) + bd;

  }
```