KATHMANDU UNIVERSITY

DHULIKHEL, KAVRE



Assignment-4

Subject- MATH 104

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1. Define limit, continuity and derivative of the vector function $\vec{r}(t) = f(t)^{\frac{1}{2}} + g(t)^{\frac{1}{2}} + b(t)^{\frac{1}{2}}$. State the component test for continuity of the vector function. Otj - 7k . At what value of t is the vector function T' continuous? Explain reason. Solution: -

Limit of a vector function: Let 7'(t) = f(t) i' + g(t) i' + h(t) R be a vector function with domain D, and I a vector, we Say that I has a limit I as & approaches to to and write (im) 7 (t) = [(a, a) 8

t-> to

if, for every number &>0, there exists a conesponding number 878 such that for all tED 7(t) - L1 < E whenever 041t-to148

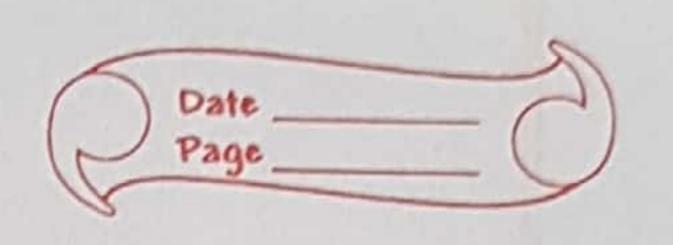
Continuity of a vector function: -

A Vector function of (t) is continuous at a point toto in its domain if lim F (tt) = 7 (to)

The function is continuous if it is continuous at every point in its domain.

Derivative of a vector function:-

The vector function T(+) - f(+) if + g(+) if + h(+) if has a derivative (is differentiable) at & if f, 9 and he have derivatives at t.



The derivative is the vector function

7'(t) = d?
dt = lim 7 (++ A+) - 7 (+) A+->0

At ->0 At Goranos

Continuity + (1) Component test for continuity of vector Function: The vector function of (t) = f(t) it g(t) it h(t) k is continuous at t=to if and only if f, g and h one continuous at to. 五十五年(中国(中国(中国)) 中国(中国) 中国(中国) 中国(中国)

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Here, get) = 3t and hlt) = -7 are continuous over

every values of K.

But,

f(t) = \(\subseteq \frac{12}{18} \) is only defined over the interval

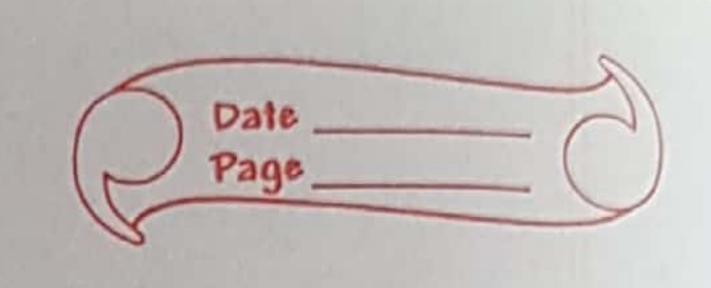
[-1,1] and beyond this interval, there exist no

functional value of given component in vector

... Vector function 7 is continuous in [-1,1]

Solution:

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Solution: 1 (1-t); + 11-t2;

Solution:

· 1 (-1) i + 1 (-2+) i 2 J1-t2 J1-+ 2J1-+

2 (1-t) VI-t2

puoundand ono $\vec{r}(t) = (gin^{-1} 2t)\vec{i} + (tan^{-1} 3t)\vec{j} + 1\vec{k}$ (b)

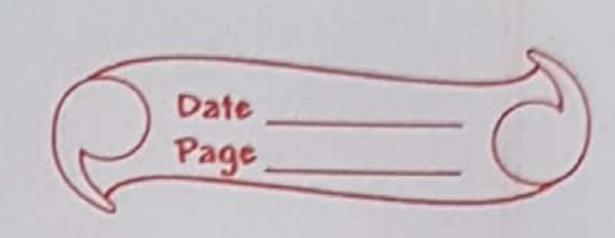
Solution:

= dsin-12t x d2t i + dtan 13t x d3t i
d2t dt d3t dt

en (1-4t2); + (sect) K

Solution: -

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 $= \frac{(2t+1)}{dt} \frac{1}{(2t+1)^2} - \frac{(2t+1)}{dt} \frac{1}{(2t+1)} \frac{1}{(2t+1)^2} + \frac{1}{(2t+1)^2} \frac{1}{(2t+1)^2}$ $\frac{(2t+1)^2}{(2t+1)^2} \frac{1}{(2t+1)^2} \frac{1}{(2$

+ Sect tant R

$$(2t-1) \times 2 - 2(2t-1))$$
 $\overrightarrow{1} - 8t$ $\overrightarrow{j} + sect + tant \overrightarrow{k}$
 $(2t+1)^2$ $1-4t^2$

(3)c. The vector T'(t) defines the pasition of a particle moving in the plane I space at time t. Find the particles velocity, acceleration, speed and direction of motion of particle at time specified.

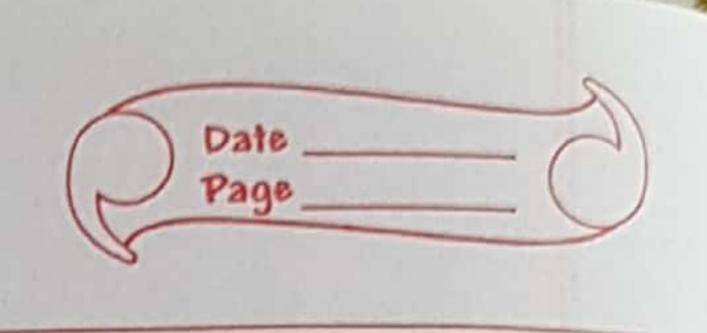
(a)
$$\vec{7}(t) = (t^2+1)\vec{1} + (2t-1)\vec{1}$$
, $t=1$
Solution: $\frac{2}{3}$

Velocity of particle (V(t)) = d7(t)
dt

when
$$t=\frac{1}{2}$$

 $\vec{v} = 2 \times 1 \vec{i} + 2\vec{j}$

$$=\frac{7}{7}+27$$



Direction of motion
$$(\hat{V}) = \hat{V}$$

$$(\hat{V}) = \hat{V} + 2\hat{V}$$

$$= 1\hat{V} + 2\hat{V}$$

$$= 1\hat{V} + 2\hat{V}$$

$$= 1\hat{V} + 2\hat{V}$$

$$= 1\hat{V} + 2\hat{V}$$

30 = 2i

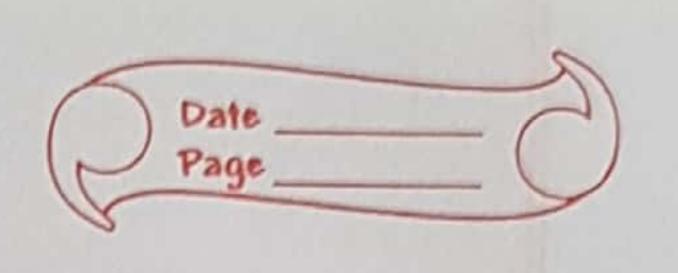
(b)
$$\overrightarrow{r}(t) = (\cos 2t) \vec{i} + (3\sin 2t) \vec{j}$$
, $t = 0$

Solution:

(1) (4) 4- (1) (4) (4) (4) (7) (7) relocity of particle v(t) = d7(t)

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At +=0, V= 6; (1) 815 (1)



with the initial conditions:

Solution: + State 18

Given,
$$\frac{dr^2}{dt^2} = -32R^2$$
 — U)

Integrating both sides wirt. 'till we get,

$$\frac{d}{dt}(t) = -32tR + \frac{2}{3}$$

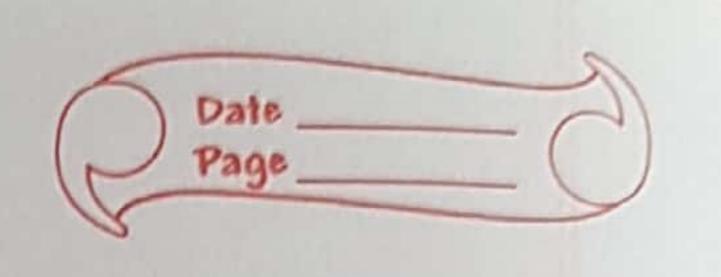
we have,

$$c_{1} - 32 \times 0 + C_{1} = 8\vec{i} + 8\vec{j}$$
 $c_{2} - 32 \times 0 + C_{3} = 8\vec{i} + 8\vec{j}$

Putting value of
$$c_1$$
 ineq (2)
$$d\vec{r} = -32 \pm \vec{R} + 8\vec{j} + 8\vec{j}$$

$$dt$$

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Integrating both sides w.r.t. we get,

7 = 8+7 + 8+7 - 32+2 K + C2

= 8+7+8+7-16+2 R+C2-(4)

(report of the property of the side of th

Again,

We have

 $m, 8 \times 0^{-1} + 8 \times 0^{-1} - 16 \times 0^{-1} \times 100^{-1}$ $m, c_2 = 100^{-1}$

Putring value of C2 in eq (4)

7 = 8+i + 8+j - 16+27 + 100 R

Find the arc length parameter along the curve $\vec{r}(t) = (et cost)\vec{i} + (et sint)\vec{j} + (et)\vec{k}$, from the point where t=0 by evaluating the integrals $S = \int_{-\infty}^{\infty} (U(t)) dt$ and then find

the length of the curve for -ln4 stso.
Solution:-

7'(t) = (et cost) i + (etsint) i + (et) K

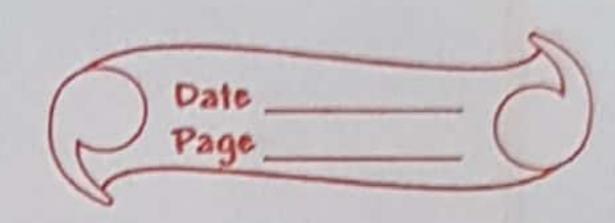
で(t) = dF'(t)

dt

= (etcost-etsint); +(etsint +etcost); + et K

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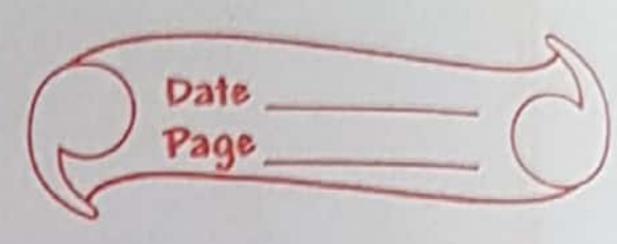




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|
$$\overrightarrow{V}(t)| = | (e^t \cos t - e^t \sin t)^2 + (e^t \sin t + e^t \cot t)^2 + (e^t \sin t + e^t \cot t)^2 + (e^t \cos t + e^t \sin t)^2 + (e^t \cos t + e^t \cos t)^2 + (e^t \cos t + e^t \sin t)^2 + (e^t \cos t + e^t \cos t)^2 + (e^t$$

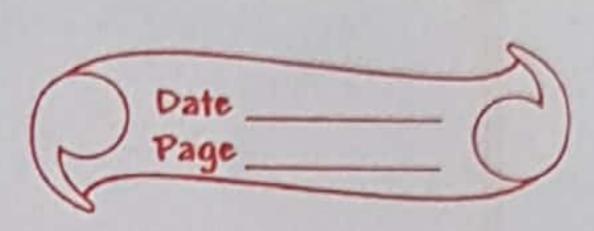
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1- singe +1+ singe Unit Tangential vector et (cost-sint) et Jz = (cost + slot) i + (cost + sint) i 12 Principal unit normal vector N = dT dt NOW dt = d (cost-sint); +d (cost+sint);
dt dt J2 dt J2 = 1 (-sint-cost) +1 (cost-sint) 12 Plinis [1 (-sint-60st)] 2 + [1 (cost-sint)]2 S. Addensia M. K. T. Hari the black of (1+ sin2+) +1 (1-sin2+) 111976 711

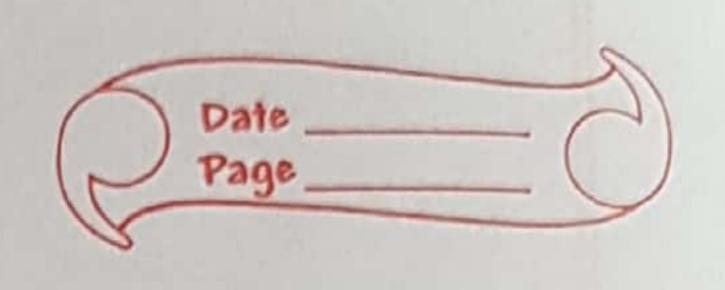
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7'(t) = (cost + tsint) i' + (sint - tcost) j' + 3K solution, (1) Th = (1) (1 VIEF dr'(t) = (-sint + t(cox) + sint) i) + (cost + tsint-cost) = (tcost) i + (tsint); = 1+2 cos2t ++2 sin2t = 1+2 , WO(1 (= () notony tapport tial Unit Tangential vector (T) = v = cost ? + sint; 1 07 Now = - sinti + cost i Stb Jesin 4 cost dt 1 = 1150000 21 Normal vector (N) = d? = - sint 1 + cost ? Curvature K = 1 at I don't have

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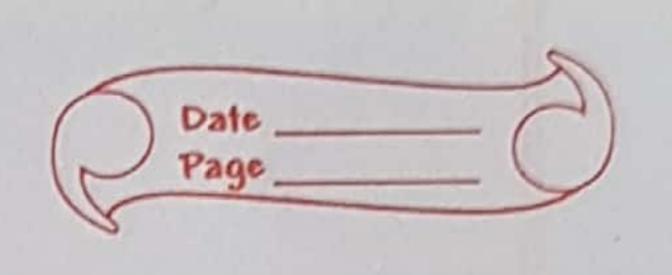
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7 (t) = (cosht) i + (sint) j + tk Solution V(t) = dF(t) = sinht it cosht it + K 1 v(t) 1 = 1 sinh2t + cosh2t +1 = I sinh 2t + cosh 2t + cosh 2t - sinh 2t = cosht \v2 Now, unit Tangent vector (7) = 0 = sinhti + coshti + k

cosht Jz cosht Jz cosht Jz = 1 tanht i t 1 j + 1 secht k 1 500 f Hore - 76 Then, = 1 d (tanht) i' + 1 d (1) j' + 1 d secht k' \[\sqrt{2} \dt \quad \tanht) i \ \dt \quad \tanht \quad \quad \tanht \quad \quad \quad \tanht \quad \tanht \quad \quad \quad \quad \quad \tanht \quad = 1 sech2+ i + 0 + 1 (-secht. +anbt) R = 1 sechet it + 1 (-seche tanhe) K sec. hyt + 1 sechet .tan hyt 1 sech2t (sech2t + tank2t) secht



Now,

12 = 27 #

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= 1 Secht it 1 (-secht tan ht) K

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1 secht

= secht ? - tan ht K

 $K = 1 \left(\frac{d\tilde{T}}{dt} \right)$

= 1, 1 secht cosht $\sqrt{2}$, $\sqrt{2}$

10

 $= 1 \operatorname{sech}^{2}t$

Normal Valle (N.)

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