Computational Physics

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Project 3, due October 20, 2021 at 11:59 p.m.
to be uploaded to https://elearning.jacobs-university.de

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3. Potential within a rectangular region: the Jacobi, Gauß-Seidel and SOR approaches [100 points]

a) Use the Jacobi approach to determine the potential $V(x,y)$ in a rectangular region with linear dimension $L_x = 11$ and $L_x = 21$ without additional charges. The boundary of the rectangle is at a potential $V = 2$. Choose the grid size $\Delta x = \Delta y = 1$. Before you run the program, guess some reasonable initial values. How many iterations are necessary to achieve an absolute precision of 10^{-4}? Decrease the grid size by a factor of two, and determine the number of iterations that are now necessary to achieve an absolute precision of 10^{-4}. As convergence measure use the sum $\sum_{ij} |V_{i,j}^{n+1} - V_{i,j}^{n}|$ over all grid points.
b) Consider the same geometry as in part a), but set the initial charges at the interior sites equal to zero except for the central sites whose charge is set equal to 5. Are the final results independent of your initial guess?
c) Modify the above program so that the potential at each site is updated sequentially. That is, after the average potential of the nearest neighbor sites of site i is computed, update the potential at i immediately. In this way the new potential of the next site is computed using the most recently determined values of its nearest neighbor potentials (Gauß-Seidel method). How many iterations are necessary to achieve an absolute precision of 10^{-4} for the data given in part a) ?
b) Implement the successive overrelaxation approach (SOR) and compare the convergence rate (number of steps needed for same accuracy) to the Jacobi and the Gauß-Seidel methods for the two different system configurations described in a) and b) and initial conditions of your choice.

General remarks for all Projects

You will have to (i) analyze the problem, (ii) select an algorithm (if not specified), (iii) write a Python program, (iv) run the program, (v) visualize the data numerical data, and (vi) extract an answer to the physics question from the data.
Which checks did you perform to validate the code? State the results you got for these tests.
For each project you will submit a short report describing the physics problem, your way of attacking it, and the results you obtained. Provide the documented Python code in such a form that we can run the code. A Jupyter Notebook including the code and report is fine but not necessary.