ON NEAR-MARTINGALES AND A CLASS OF ANTICIPATING LINEAR SDES

HUI-HSIUNG KUO, PUJAN SHRESTHA*, SUDIP SINHA, AND PADMANABHAN SUNDAR

ABSTRACT. The primary goal of this paper is to prove a near-martingale optional stopping theorem and establish solvability and large deviations for a class of anticipating linear stochastic differential equations. We prove the existence and uniqueness of solutions using two approaches: (1) Ayed–Kuo differential formula using an ansatz, and (2) a novel braiding technique by interpreting the integral in the Skorokhod sense. We establish a Freidlin–Wentzell type large deviations result for solution of such equations.

1. INTRODUCTION

Anticipating stochastic calculus has been an active and important research area for several years, and lies at the intersection of probability theory and infinite-dimensional analysis. Enlargement of filtration, Malliavin calculus, and white noise theory provide three distinct methodologies to incorporate anticipation (of future) into classical Itô theory of stochastic integration and differential equations.

It is to the credit of Itô who constructed an anticipating stochastic integral in 1976[6], and laid the foundation for the idea of enlargement of the underlying filtration. Ever since, the method was embraced by several researchers that led to many important works (see articles in [7]). The advent of an integral invented by Skorokhod resulted in an impressive edifice built by Malliavin on stochastic calculus of variations in order to prove Hörmander's hypoellipticity result by stochastic analysis. Malliavin calculus provided a natural basis for the development and study of anticipative stochastic analysis and differential equations. Around the same time, a systematic study of Hida distributions gave rise to white noise theory and a general framework for stochastic calculus.

Malliavin calculus and white noise theory have vast applicability to the theory of stochastic differential equations with anticipation. However, the results obtained by these theories are primarily abstract though general. A more tractable theory was envisaged by Kuo based on a concrete stochastic integral known as the Ayed–Kuo integral[1]. Under less generality, the latter allows one to obtain results under easily understood, verifiable hypotheses.

In this article, we prove some results about stopped near-martingales, which are generalizations of martingales. We then study existence, uniqueness and large deviation principle for linear stochastic differential equations with anticipating initial conditions and drifts. While we rely mostly on the Ayed–Kuo formalism, other theories are minimally used either out of necessity, or to compare and contrast the conclusions of certain results.

* Corresponding author.