Journal of Stochastic Analysis
Vol. 2, No. 2 (2021) Article 5 (18 pages)
DOI: 10.31390/josa.2.2.05

ANTICIPATING LINEAR STOCHASTIC DIFFERENTIAL EQUATIONS WITH ADAPTED COEFFICIENTS

HUI-HSIUNG KUO, PUJAN SHRESTHA*, AND SUDIP SINHA

Abstract. Stochastic differential equations with adapted integrands and initial conditions are well studied within Itô's theory. However, such a general theory is not known for corresponding equations with anticipation. We use examples to illustrate essential ideas of the Ayed–Kuo integral and techniques for dealing with anticipating stochastic differential equations. We prove the general form of the solution for a class of linear stochastic differential equations with adapted coefficients and anticipating initial condition, which in this case is an analytic function of a Wiener integral. We show that for such equations, the conditional expectation of the solution is not the same as the solution of the corresponding stochastic differential equation with the initial condition as the expectation of the original initial condition. In particular, we show that there is an extra term in the stochastic differential equation, and give the exact form of this term.

1. Introduction

Let $B(t)$, where $t \in [a,b]$, be a Brownian motion starting at 0 and let $\{\mathcal{F}_t\}$ be the filtration generated by $B(t)$, that is, $\mathcal{F}_t = \sigma\{B(s); a \le s \le t\}$. In the framework of Itô's calculus, a stochastic differential equation

$$\begin{cases} dX(t) = \alpha(t, X(t))\, dB(t) + \beta(t, X(t))\, dt, & t \in [a,b], \\ X(a) = \xi, \end{cases}$$

with the initial condition ξ being $\mathcal{F}_a$-measurable, is a symbolical representation of the stochastic integral equation

$$X(t) = \xi + \int_a^t b(s, X(s))\, ds + \int_a^t \sigma(s, X(s))\, dB(s), \quad t \in [a,b],$$

where $\int_a^t \sigma(s, X(s))\, dB(s)$ is defined as an Itô integral. In Itô's framework, we require both the coefficients $b(t,x,\omega)$ and $\sigma(t,x,\omega)$ to be adapted apart from usual integrability constraints, and and the initial condition ξ to be measurable with respect to the initial σ-algebra $\mathcal{F}_a$. The question of how the stochastic integral can be defined when any of these quantities are not adapted (called *anticipating*) has been an open question in the field of stochastic analysis for past decades.

Received 2021-4-17; Accepted 2021-5-6; Communicated by the editors.
2010 *Mathematics Subject Classification.* Primary 60H05; Secondary 60H10, 60H20.
Key words and phrases. anticipation, stochastic integral, stochastic integral, Itô formula, stochastic differential equations, conditional expectation, adapted coefficients.
* Corresponding author.