
The Singular Points method for Asian American options for local volatility models

Sudip Sinha

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1 Introduction

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2 Notations

$$[n] = \{0, 1, 2, \dots, n\}$$

3 Basic formulae

Arithmetic average

$$A_n = \frac{\sum_{i=0}^n S_i}{n+1} \tag{3.1a}$$

$$\implies (n+1)A_n = \sum_{i=0}^n S_i \tag{3.1b}$$

4 Results

Definition 1 (Path). *A path is a sequence $(j_i)_{i \in [n]}$ such that $j_{i+1} \in \{j_i, j_i + 1\}$.*

Lemma 1. *Let there be two paths α and β , such that $S_{i,j_i^\alpha} \geq S_{i,j_i^\beta} \forall i$. Then $A^\alpha \geq A^\beta$.*

Proof. Clearly if $S_{i,j_i^\alpha} = S_{i,j_i^\beta} \forall i$, then $A^\alpha = A^\beta$.

We only need to show the case of inequality. Let $S_{i,j_i^\alpha} > S_{i,j_i^\beta} \forall i \in [n] \setminus \{l\}$. That is, $S_{l,j_l^\alpha} > S_{l,j_l^\beta}$.

Now, from equation 3.1, we have:

$$\begin{aligned} (n+1)A_{n,j}^\alpha &= \sum_{i=0}^{l-1} S_{i,j_i} + S_{l,j_l^\alpha} + \sum_{i=l+1}^n S_{i,j_i} \\ (n+1)A_{n,j}^\beta &= \sum_{i=0}^{l-1} S_{i,j_i} + S_{l,j_l^\beta} + \sum_{i=l+1}^n S_{i,j_i} \\ \implies (n+1)(A_{n,j}^\alpha - A_{n,j}^\beta) &= S_{l,j_l^\alpha} - S_{l,j_l^\beta} \\ &= S_{l-1,j_{l-1}} u_l - S_{l-1,j_{l-1}} d_l \\ &= S_{l-1,j_{l-1}} (u_l - d_l) > 0 \\ \implies A_{n,j}^\alpha &> A_{n,j}^\beta \end{aligned}$$

□

Remark. The path α signifies the path above and β signifies the path below. Thus, the path above always has a higher arithmetic mean.

Corollary 1. At each node $N(n, j)$, the average values vary between a minimum average $A_{n,j}^{\min}$ (corresponding to the path with $(n-j)$ down movements followed by j up movements) and a maximum average $A_{n,j}^{\max}$ (corresponding to the path with j up movements followed by $(n-j)$ down movements).

Proof. The path 'min' is the bottom-most one and 'max' is the topmost one. □

Lemma 2 (Lemma 3). The price function at maturity $v_{n,j}$ is convex and piecewise linear.

Proof. By construction. See the paper. □

Lemma 3 (Lemma 4). The price function $v_{i,j}$ is concave and non-linear.

Proof. 3.1 □

5 Conclusion

The singular points method may not be used to price Geometric Asian options.