Exercise 3

Probabilities of characters in normal text

$\Pr(Y = j) = 0.015$

$\Pr(Y = k) = 0.045$

$\implies \Pr((Y = j) \vee (Y = k)) = \Pr(Y = j) + \Pr(Y = k) = 0.06$

Part (a)

The most basic classifier we can choose is the classifier on based only on prior probabilities. Since $\Pr(Y = j) < \Pr(Y = k)$, let this classifier be $f(x) = k \ \forall x \in X$.

Calculating risk: $\mathcal{R}(f) = \Pr(Y = j \mid ((Y = j) \wedge (Y = k))) = \frac{\Pr(Y=j)}{\Pr((Y=j)\vee(Y=k))} = \frac{1}{4} < 0.3$.

Thus, the most basic classifier has a risk lower than the classifier being sold. So we would not buy the classifier being sold.

The lowest probability of error if we do not look at the input data is 0.25.

Part (b)

Again let us choose the most basic classifier, which just predicts $\sim j$ without regard to the input data.

$\mathcal{R}(f) = \Pr(Y = j) = 0.015 < 0.2$

By the same logic of Part (a), we would not buy the classifier being sold.

Exercise 5

Notations

S = message is actually spam

L = message is actually legitimate

$\hat{S}$ = message is classified as spam

$\hat{L}$ = message is classified as legitimate

Part (a)

From the given data, we fill up the confusion matrix.

	$\hat{S}$	$\hat{L}$	
S	$0.51 (= 0.85 \times 0.6)$	$0.09 (= 0.6 - 0.51)$	0.6
L	$0.02 (= 0.4 \times 0.05)$	$0.38 (= 0.4 - 0.02)$	$0.4 (= 1 - 0.6)$
	$0.53 (= 0.51 + 0.02)$	$0.47 (= 1 - 0.53)$	1

From the above table, we can conclude the following[1]:

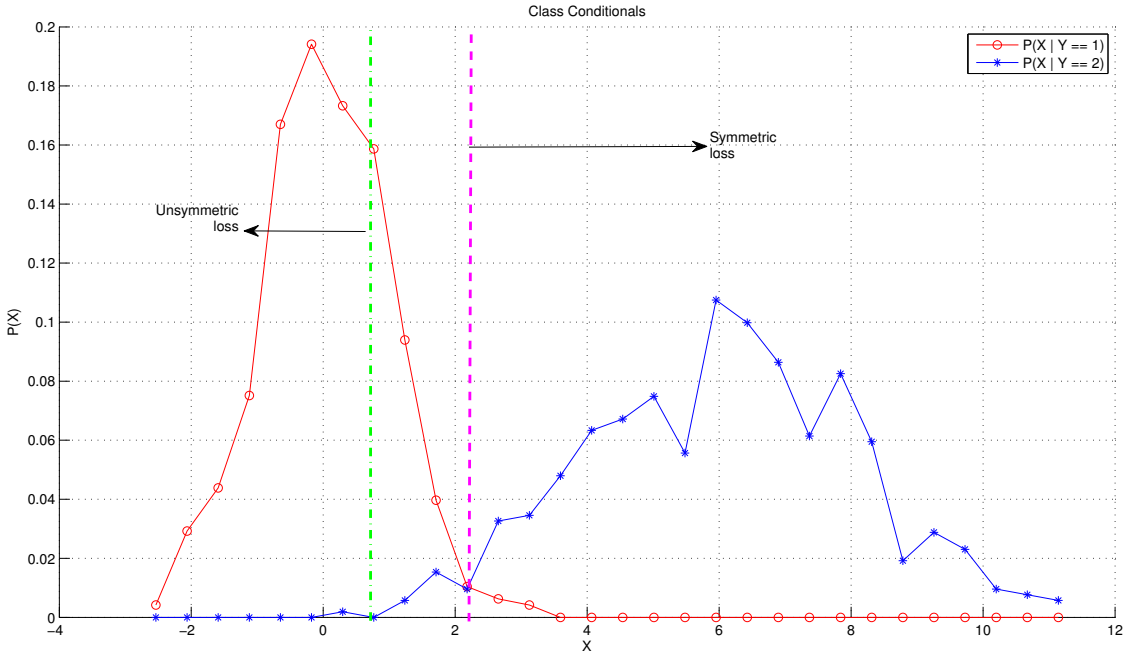
- False positive $= \Pr\left(\hat{S} \cap L\right) = 0.02$
- False negative $= \Pr\left(\hat{L} \cap S\right) = 0.09$
- Average error $= \Pr\left(\hat{S} \cap L\right) + \Pr\left(\hat{L} \cap S\right) = 0.02 + 0.09 = 0.11$

Part (b)

- A classication algorithm with zero false positive rate would be a classifier which simply classifies everything as legitimate.
- A classication algorithm with zero false negative rate would be a classifier which simply classifies everything as spam.

[1] Definition from "Counting performance mea ure (2)" page of the lide

Part (c)


Class Conditionals
P(X | Y == 1)
P(X | Y == 2)
Symmetric loss
Unsymmetric loss
P(X)
X
0.2
0.18
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
−4
−2
0
2
4
6
8
10
12