Foundations of Mathematics
Exercises from PHIL 4010

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Contents

Part 1
Study: Set theory notes

1.1 Exercises for 2019-09-10

Proposition (Exercise 1.8) *For any sets A and B, we have $A \cap B \subseteq A$.*

Proof. Let $x \in A \cap B$ be arbitrary. This means $x \in A$ and $x \in B$. Therefore $x \in A$. Since every element in $A \cap B$ is also an element of A, we have $A \cap B \subseteq A$.

Proposition (Exercise 1.10) *For any set A, we have $A \cap \emptyset = \emptyset$.*

Proof. **($\subseteq$)** Let $x \in A \cap \emptyset$ be arbitrary. This means $x \in A$ and $x \in \emptyset$. But there does not exist $x \in \emptyset$. Therefore, the statement is vacuously true.
($\supseteq$) Now, let $x \in \emptyset$ be arbitrary. Again, there does not exist $x \in \emptyset$, rendering the statement vacuously true.

Proposition (Exercise 1.13) *For any sets A and B, if $A \subseteq B$, then $A \cup B = B$.*

Proof. **($\subseteq$)** Let $x \in A \cup B$ be arbitrary. This means $x \in A$ or $x \in B$. If $x \in A$, then by the condition $A \subseteq B$, we obtain $x \in B$. Therefore, in either case, $x \in B$.
($\supseteq$) Let $x \in B$ be arbitrary. Then $x \in A$ or $x \in B$, and so $x \in A \cup B$.

Part 2
Book Study: Enderton (Logic)

Bibliography