Analysis

With an emphasis on probability theory

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Our aim is to formulate a large deviations principle for an SDE with anticipating initial conditions. We start of with a very simple case

$$X_t^\varepsilon = B_T + \sqrt{\varepsilon}\int_0^t \sigma(X_t^\varepsilon)\,\mathrm{d}B_t,$$

where $t \in [0,T]$ for some $T < \infty$, and conditions on σ shall be imposed as necessary. We shall look at the method of enlargement of filtration by [<Itô1978>]. We denote the enlarged filtration by $\tilde{\mathscr{F}}_t = \mathscr{F}_t \vee \sigma(B_T)$. Then

$$B_t = \tilde{B}_t + \int_0^t \frac{B_T - B_s}{T-s}\,\mathrm{d}s,$$

where $\tilde{B}_\cdot$ is a Brownian motion w.r.t. $\tilde{\mathscr{F}}_\cdot$. Using this, we write our original SDE as

$$X_t^\varepsilon = B_T + \sqrt{\varepsilon}\int_0^t \sigma(X_t^\varepsilon)\,\mathrm{d}\tilde{B}_t + \sqrt{\varepsilon}\int_0^t \sigma(X_t^\varepsilon)\frac{B_T - B_s}{T-s}\,\mathrm{d}s.$$

Now, let $Y_t^\varepsilon = \sqrt{\varepsilon}(B_T - B_t)$. Then X_t^ε is given by

$$X_t^\varepsilon = B_T + \sqrt{\varepsilon}\int_0^t \sigma(X_t^\varepsilon)\,\mathrm{d}\tilde{B}_t + \int_0^t \sigma(X_t^\varepsilon)\frac{Y_s^\varepsilon}{T-s}\,\mathrm{d}s.$$

Moreover,

$$\begin{aligned} Y_t^\varepsilon &= \sqrt{\varepsilon}B_T - \sqrt{\varepsilon}B_t \\ &= \sqrt{\varepsilon}B_T - \sqrt{\varepsilon}\left(\tilde{B}_t + \int_0^t \frac{B_T - B_s}{T-s}\,\mathrm{d}s\right) \\ &= \sqrt{\varepsilon}B_T - \sqrt{\varepsilon}\tilde{B}_t - \int_0^t \frac{Y_s^\varepsilon}{T-s}\,\mathrm{d}s \end{aligned}$$

Bibliography