Contents

1.1 Binomial Theorem

Theorem (**Binomial theorem**)

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k}.$$

Proof (*Inductive*) Homework. □

Proof (*Combinatorial*) Consider the product $(x_1+y_1)(x_2+y_2)\cdots(x_n+y_n)$.

First, note that the expansion consists of 2^n terms, each being a product of n factors.

Secondly, each product contains either x_j xor y_j for each $j \in [n]$.

For example, $(x_1+y_1)(x_2+y_2) = x_1x_2 + x_1y_2 + y_1x_2 + y_1y_2$.

Now, we can we choose k of the x_js and $n-k$ of the y_js in $\binom{n}{k}$ ways, so there are precisely those many terms with mk x_js and $n-k$ y_js in the expansion.

Finally, letting $x_j = x$ and $y_j = y$ for each $j \in [n]$, we get the result. □

Remark *This can be generalized to a finite number of experiments.*

2.1 Discrete Probability Spaces

Notations

Term	Description	Symbol	Coin toss Eg
sample space	set of outcomes	Ω	$\{H, T\}$
outcome	arbitrary outcome	$\omega \in \Omega$	H
event	subset of sample space	E	$\emptyset, \{H\}, \{T\}, \{H, T\}$
prob mass fn	weightage of each outcome	$p : \Omega \to [0, 1]$, with $\sum_{\omega} p(\omega) = 1$	$p(H) = \frac{1}{3}, p(T) = \frac{2}{3}$
probability	(of an event)	$\mathbb{P}(E) = \sum_{\omega \in \Omega} p(\omega)$	$\mathbb{P}(\emptyset) = 0, \mathbb{P}(\{H, T\}) = 1$

Proposition (Basic principle of counting) *Suppose two independent experiments are performed, and there are m possible outcomes of the first experiment and n possible outcomes of the second experiment. Then the total possible outcomes of of the two experiments combined is mn.*

Proof Let (i, j) denote the case when the first experiment gives the ith outcome and the second experiment gives the jth outcome. Enumerating, we get

$$\begin{matrix} (1,1) & (1,2) & \dots & (1,n) \\ (2,1) & (2,2) & \dots & (2,n) \\ \vdots & \vdots & \ddots & \vdots \\ (m,1) & (m,2) & \dots & (m,n) \end{matrix}$$

Since there are m rows and n columns, we have total mn entries. $\square$

Remark *This can be generalized to a finite number of experiments.*

Bibliography