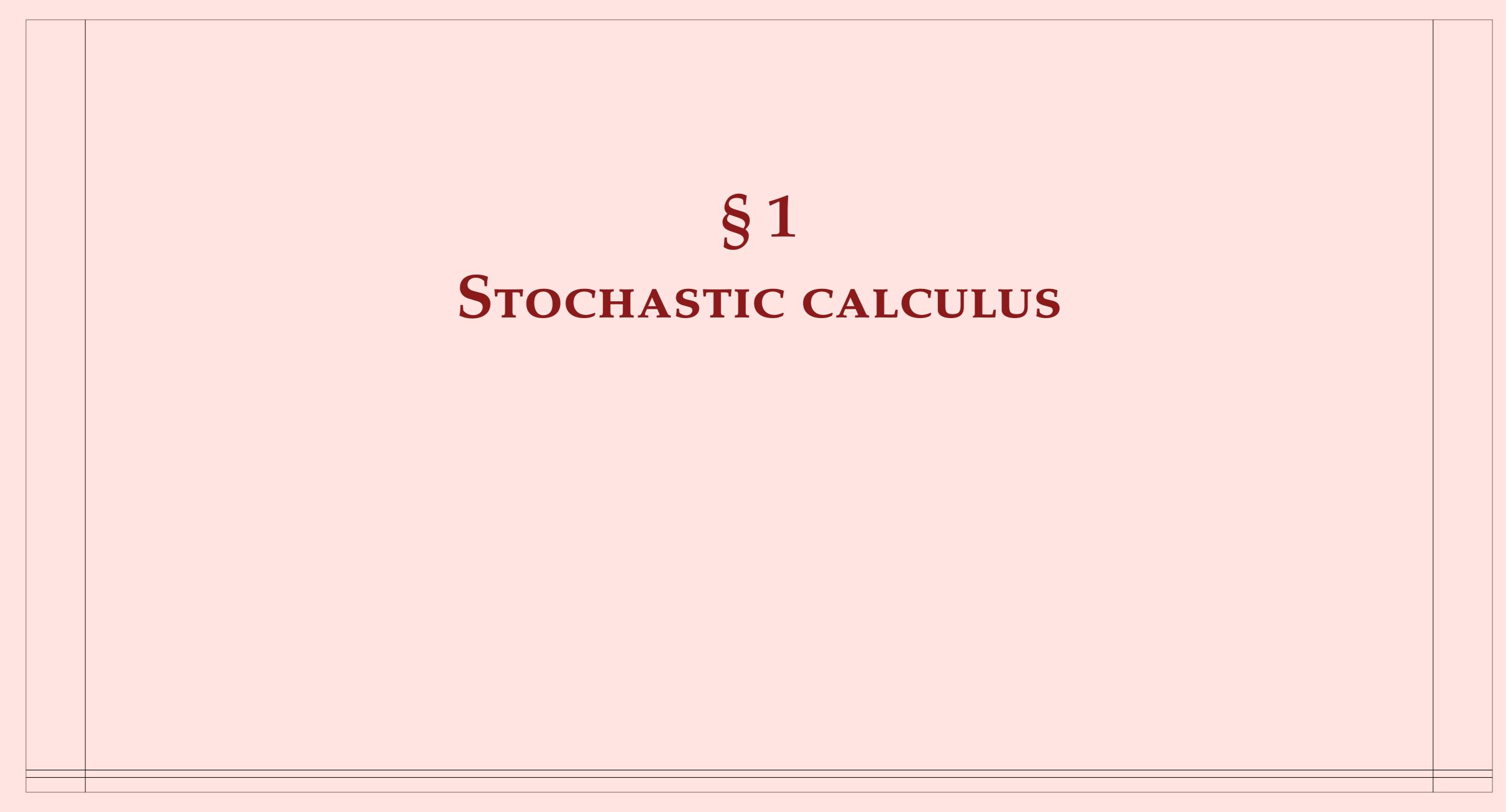
Generalization of stochastic calculus and its applications in large deviations theory

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		Outline	
1 2 3	Stochastic calculus Large deviations theory Conclusion		3 13 22



Quick revision

- 1. Properties of Brownian motion B(t)
 - starts at 0
 - continuous paths
 - independence of increments
 - $B(t) B(s) \sim N(0, t s)$
 - infinite linear variation
 - finite quadratic variation $[B(t), B(s)] = t \land s$.
 - is a martingale
- 2. Naive stochastic integration w.r.t. B(t): not possible

Itô integral: $f \in L^2$

Definition

Properties of the associated process $X_t = \int_0^t f(t) dB(t)$

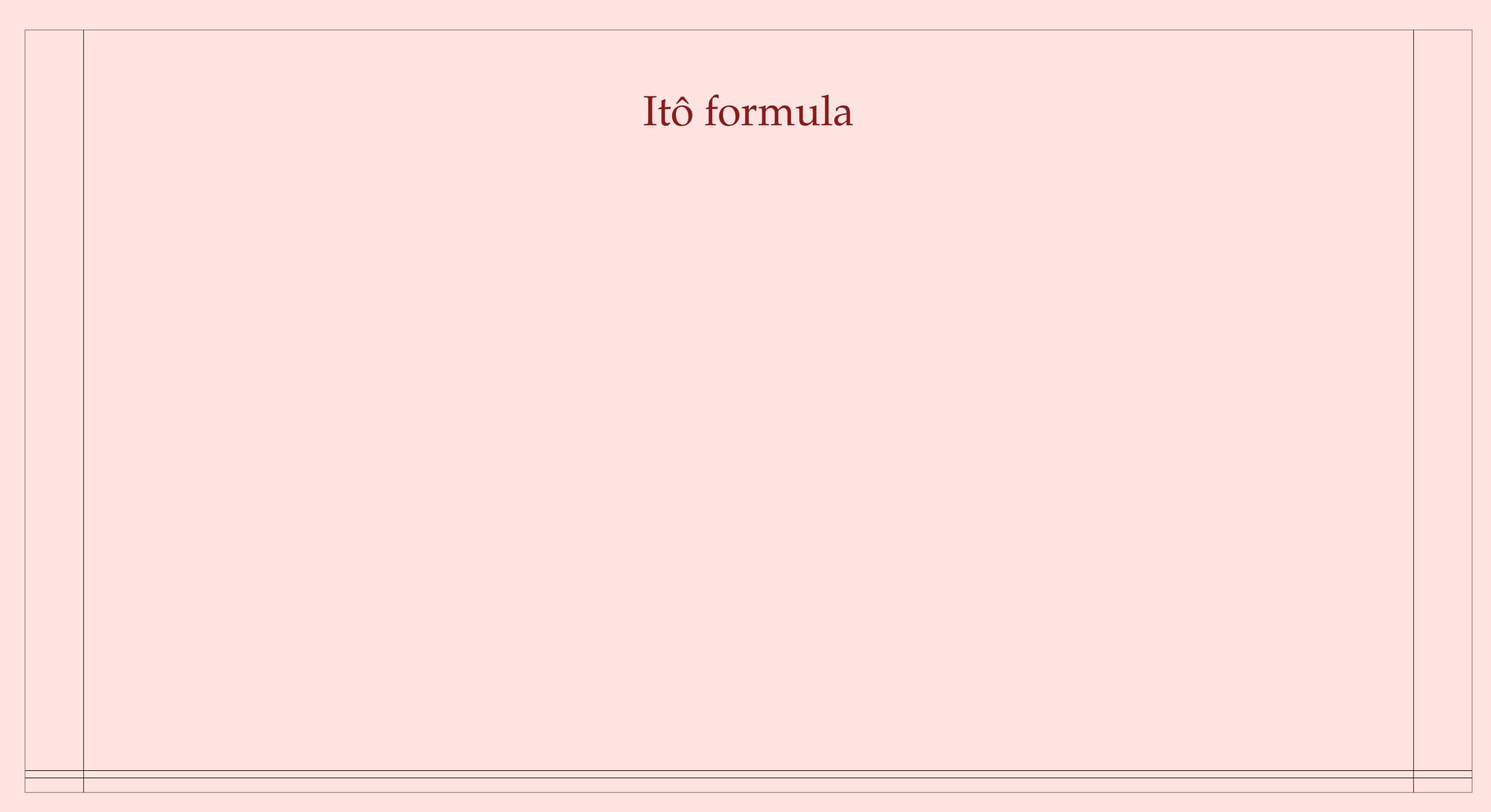
- 1. continuity
- 2. martingale

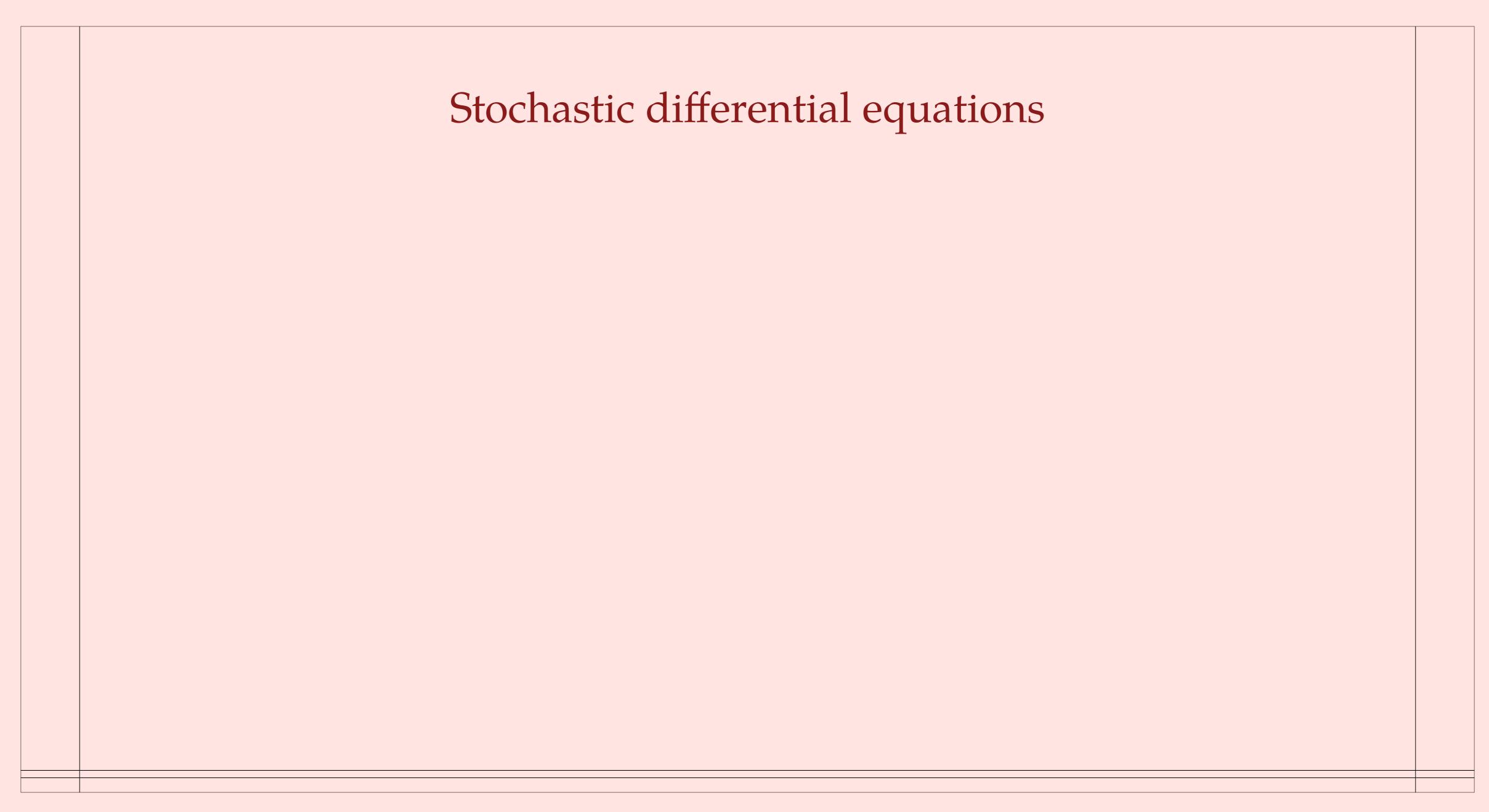
Itô integral: $f \in \mathcal{L}^2$

Definition

Properties of the associated process $X_t = \int_0^t f(t) dB(t)$

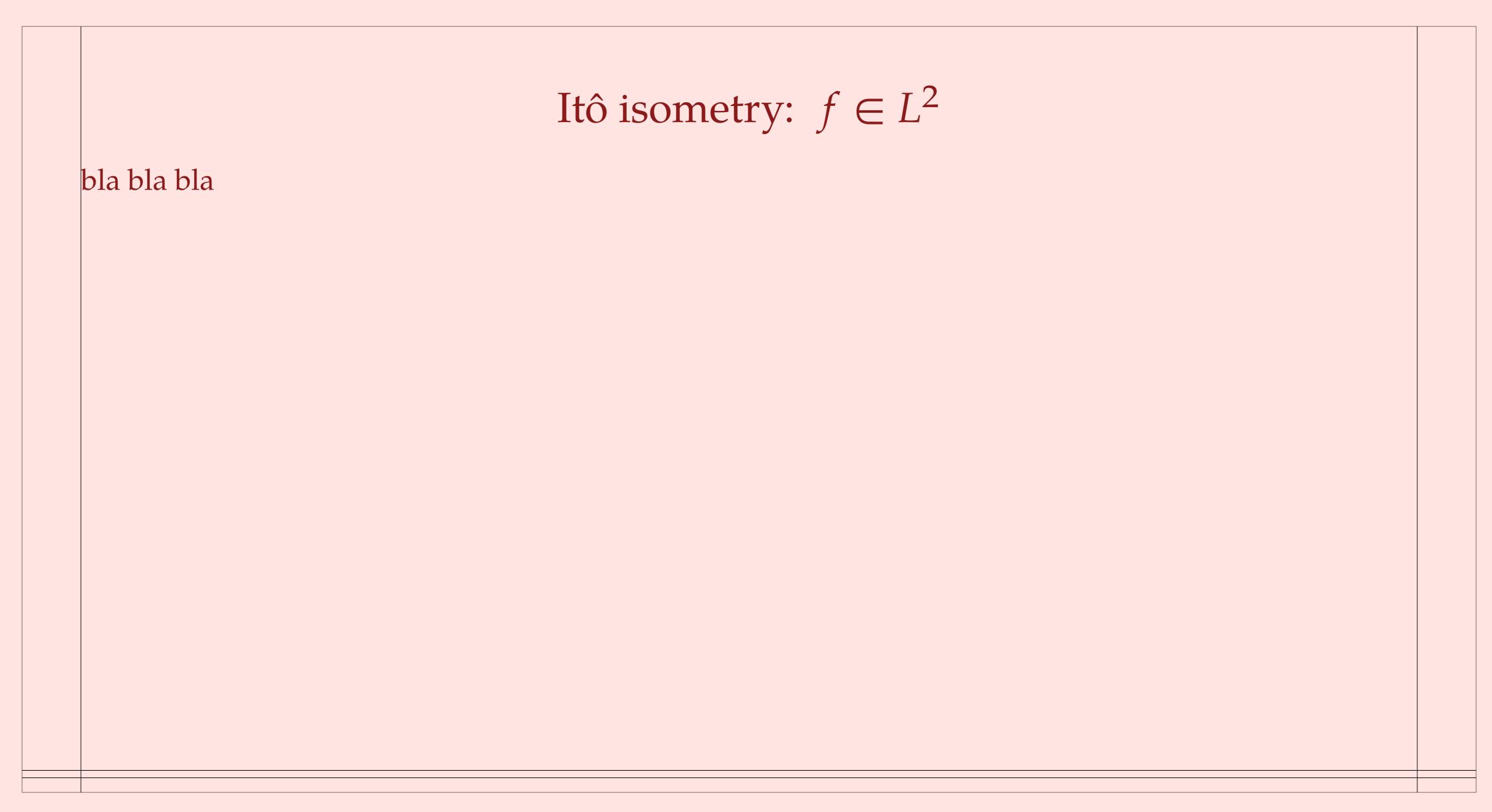
- 1. continuity
- 2. local martingale

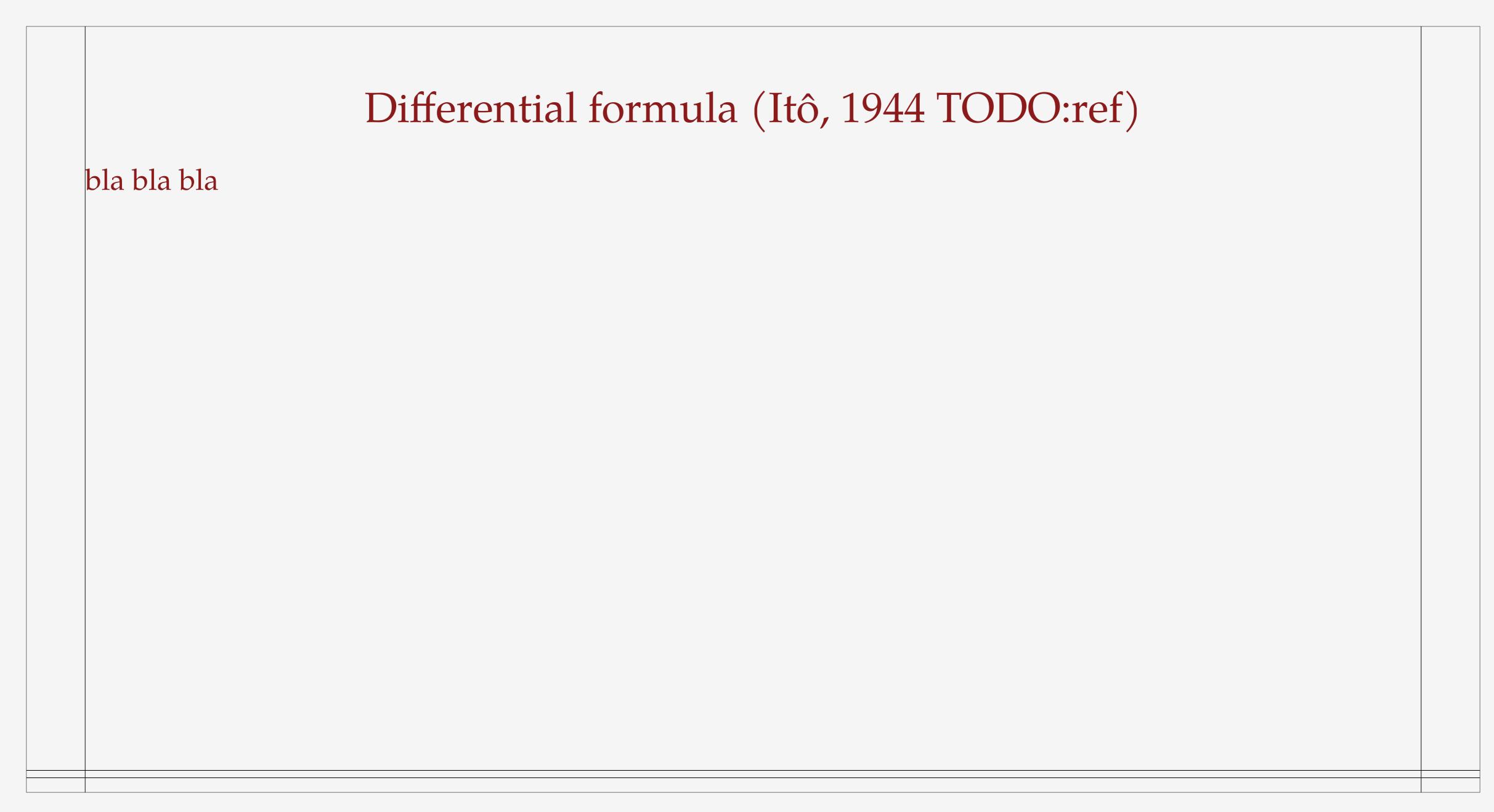


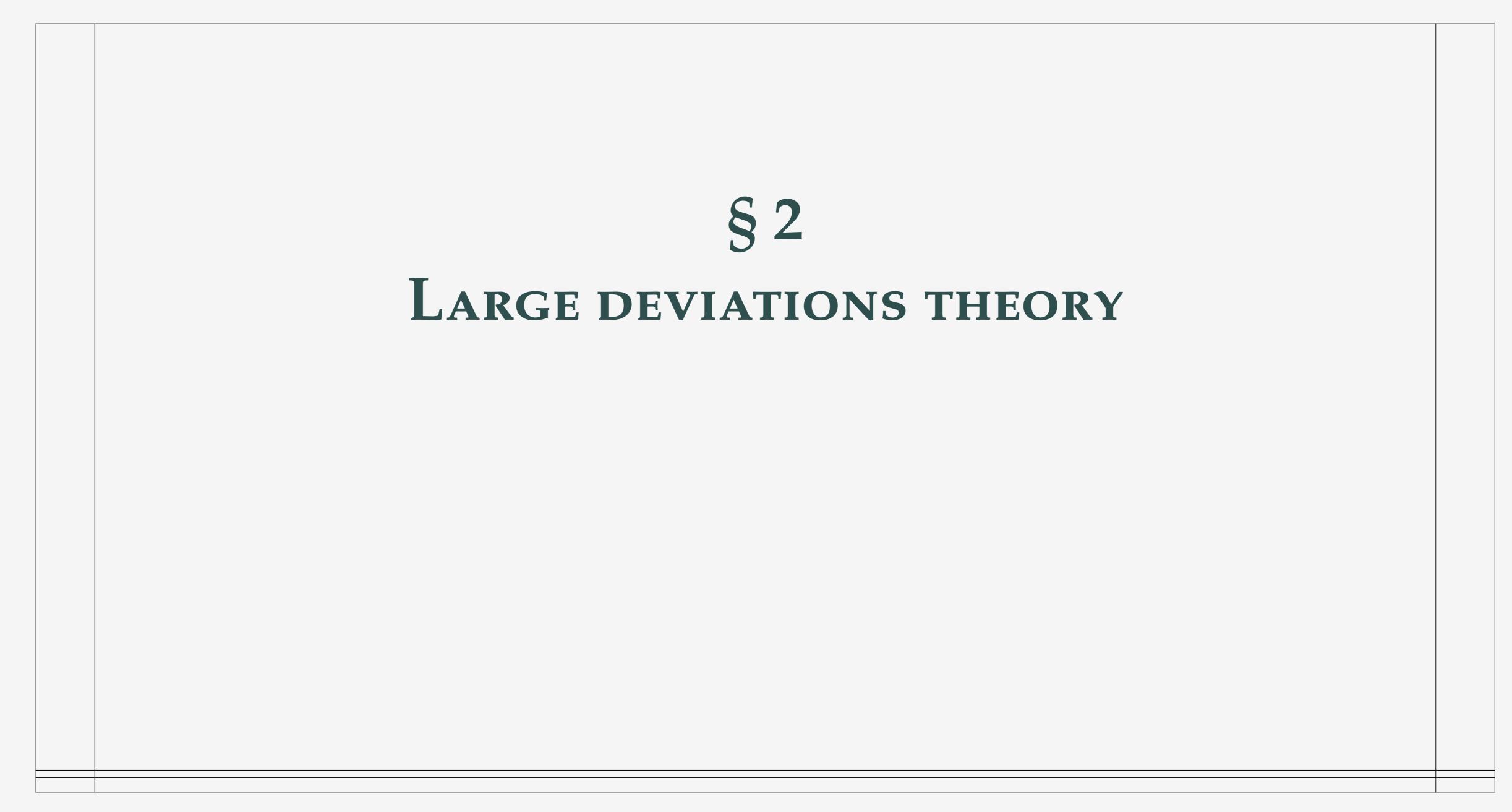




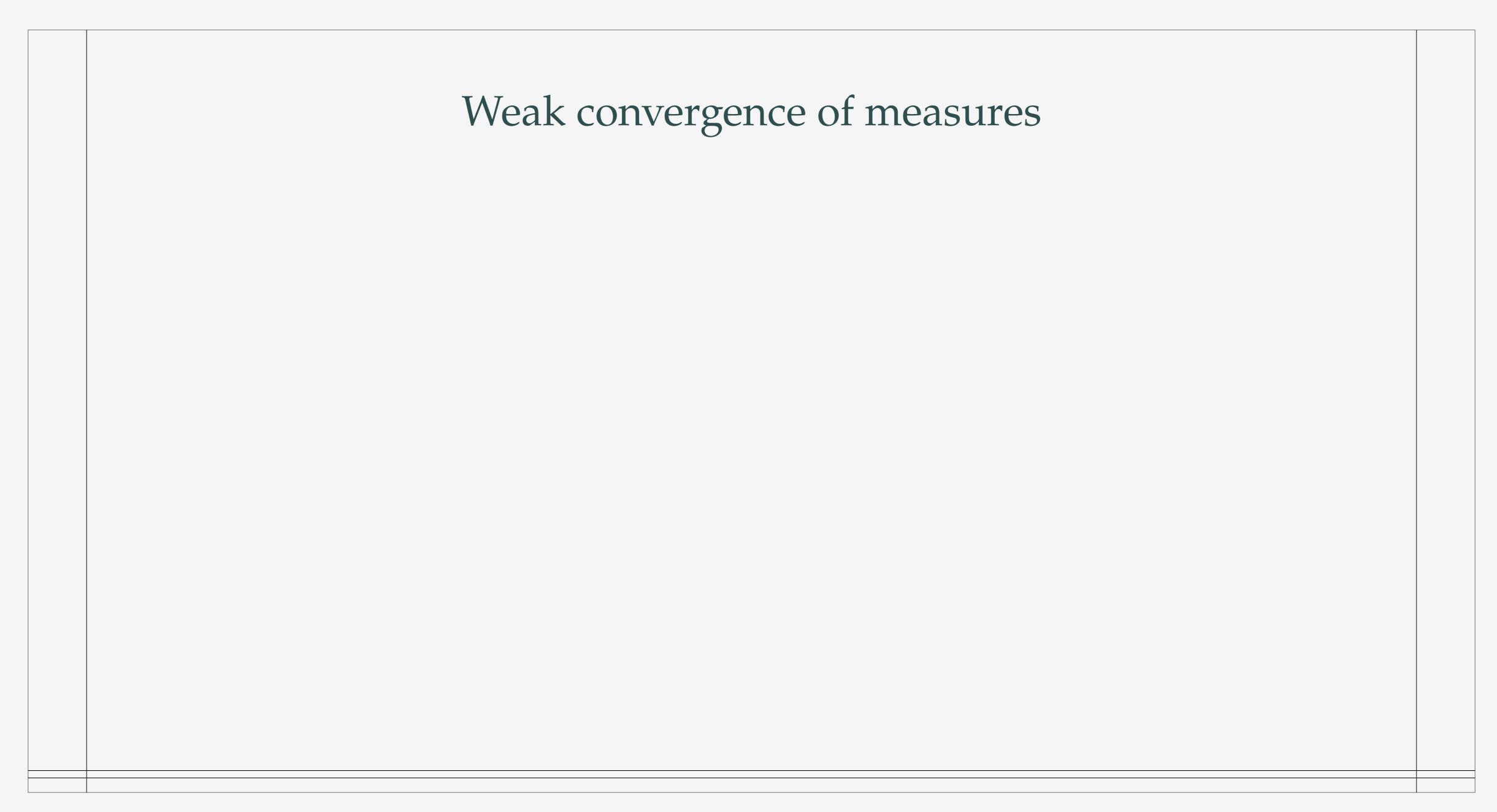
	Itô integral: $f \in \mathcal{L}^2$	
This is a citation [1].		
□ One	□ Three	
□ Two	□ Four	

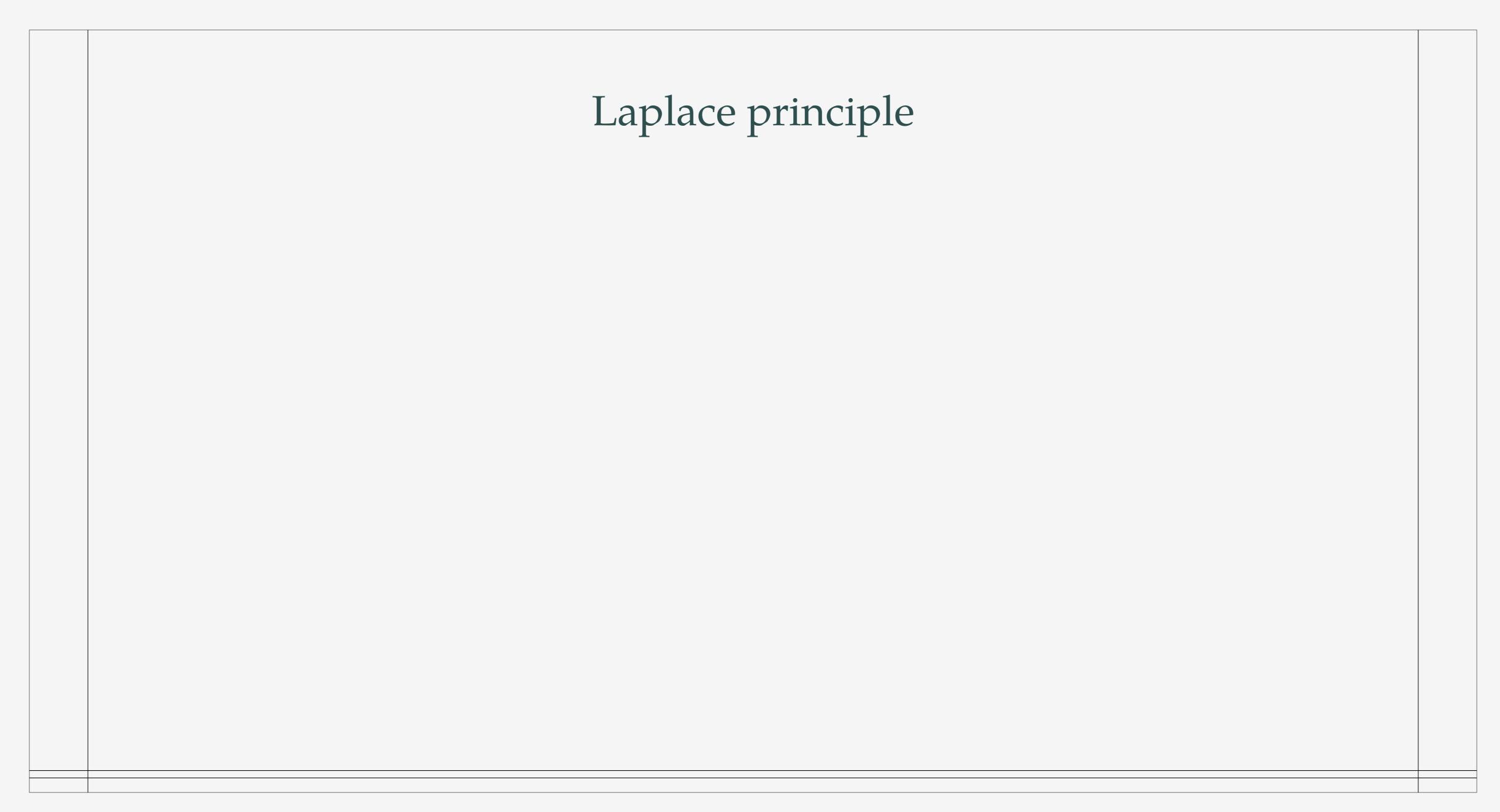












Cramér theorem

This is the first one:

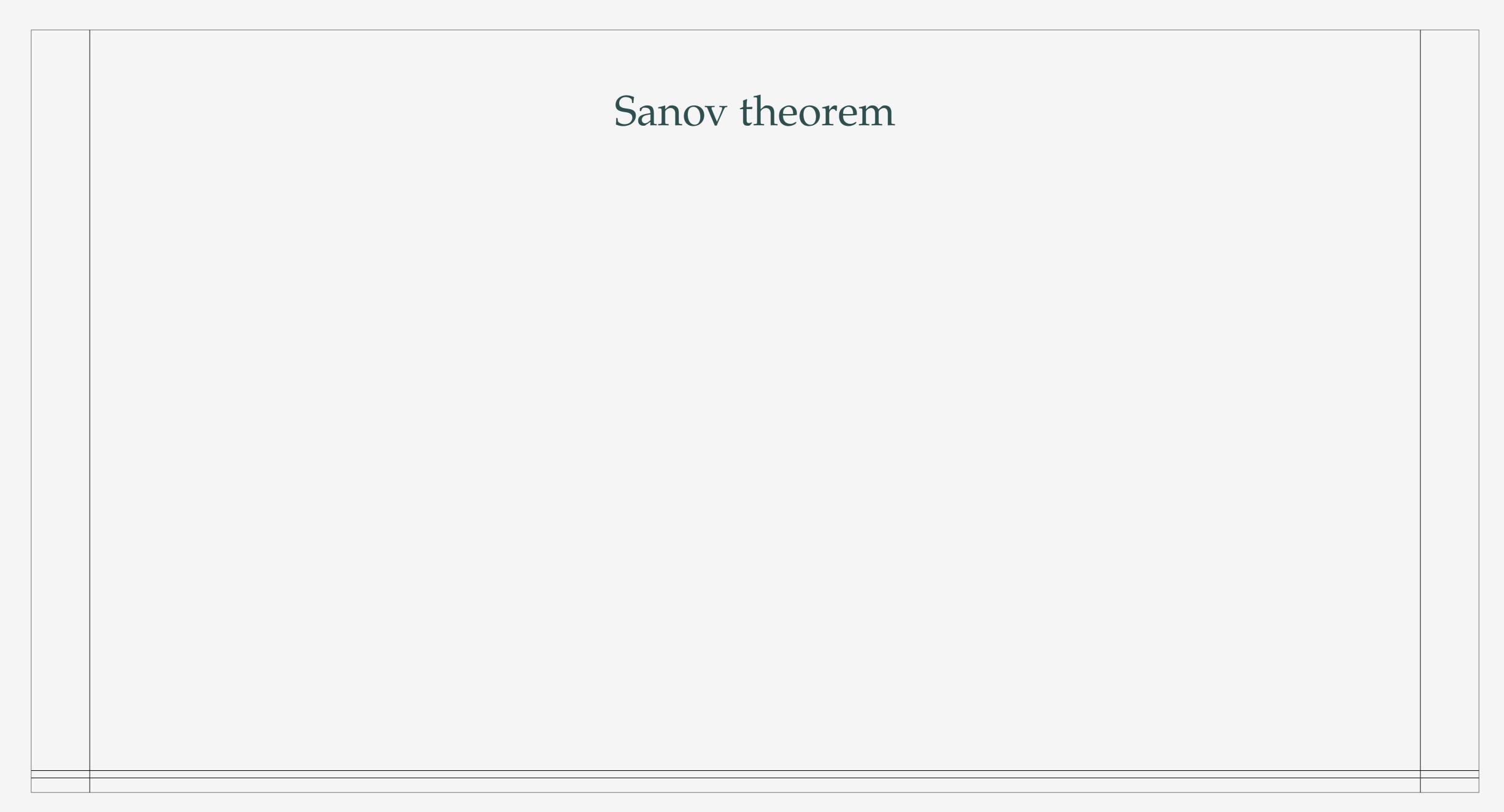
Theorem All conjectures are interesting.

This is the second one:

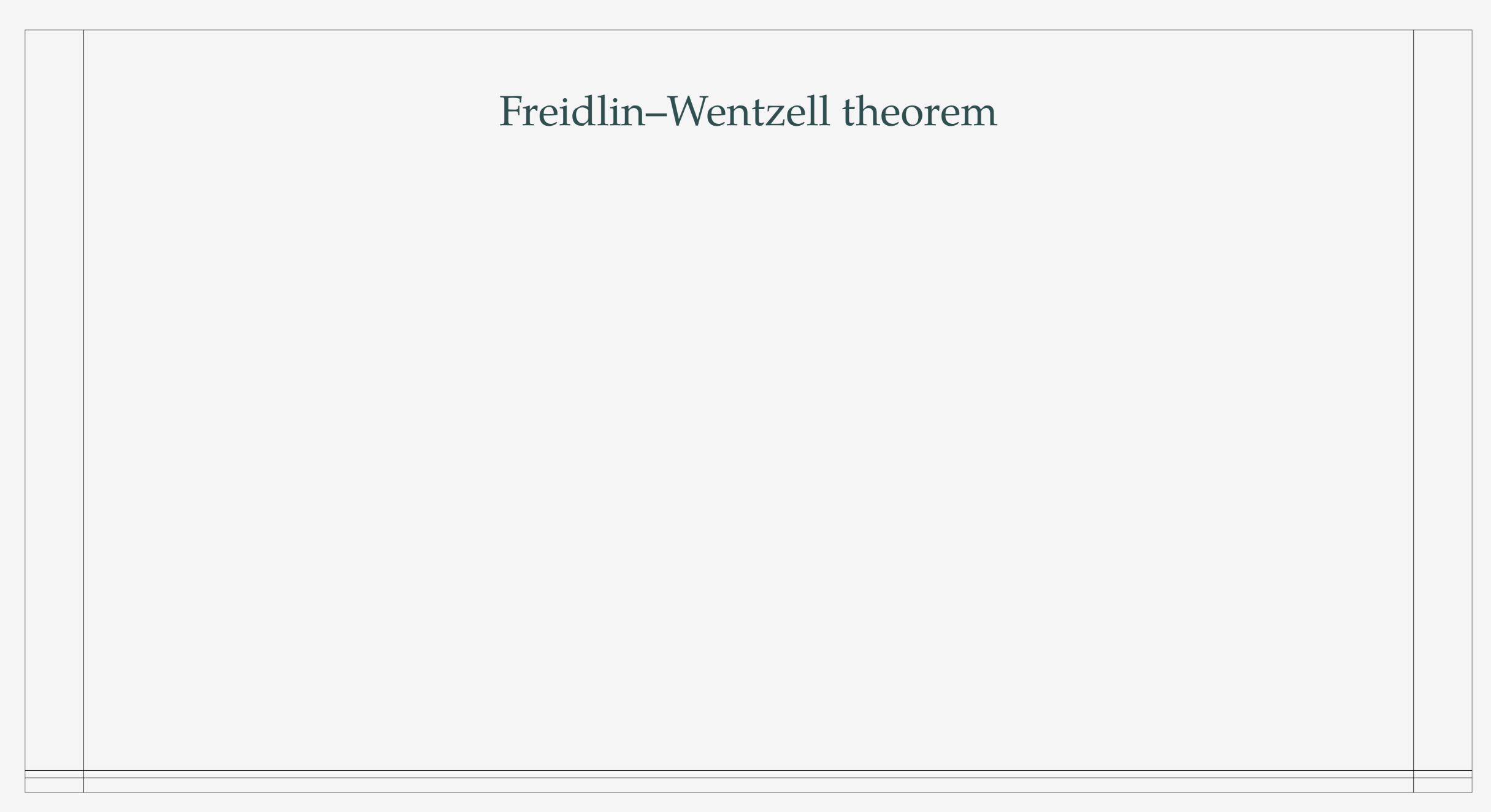
Theorem Let $X_1, X_2, ...$ be a series of i.i.d. real random variables with finite logarithmic moment generating function, e.g. $\Lambda(t) < \infty \ \forall t \in \mathbb{R}$.

Then the Legendre transform of Λ , $\Lambda^* = \sup_{t \in \mathbb{R}} (tx - \Lambda(t))$ satisfies

$$\lim_{n \to \infty} \frac{1}{n} \log \mathbb{P}\left(\sum_{i=1}^{n} X_i \ge nx\right) = -\Lambda^*(x) \quad \forall x > \mathbb{E}(X_1)$$







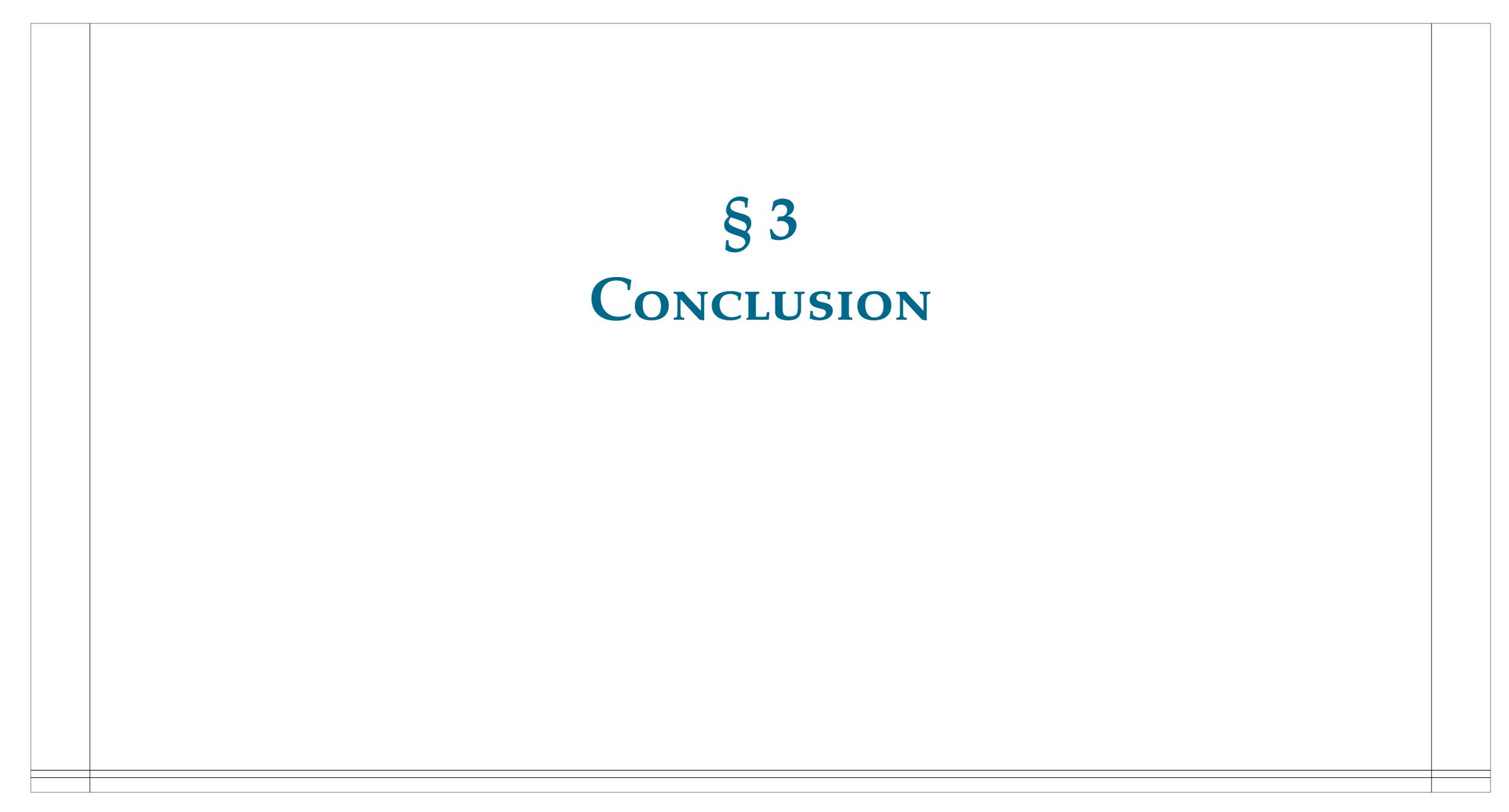
Freidlin-Wentzell theorem

Column 1

The Earth, as a habitat for animal life, is in old age and has a fatal illness. Several, in fact. It would be happening whether humans had ever evolved or not. But our presence is like the effect of an old-age patient who smokes many packs of cigarettes per day—and we humans are the cigarettes.

Column 2

Since the mid-1990s, humans have taken an unprecedented step in Earthly annals by introducing not just exotic flora or fauna from one ecosystem into another, but actually inserting exotic genes into the operating systems of individual plants and animals, where they're intended to do exactly the same thing: copy themselves, over and over.



Possible areas of interest

- * Extension to SDEs with anticipating coefficients
- Near-Markov property
- * Girsanov theorem for generalized integration
- * Freidlin-Wintzell type result for SDEs with anticipating initial conditions

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1 C.R. Hwang, H.H. Kuo, K. Saitô et al., "A general Itô formula for adapted and instantly independent stochastic processes", Communications on Stochastic Analysis 10(3), 2016.