1. Interference (The variation of intensity due to superposition of waves from coherent sources are known as Interference.)

Talk [It is superposition of two waves with same frequency travelling with same velocity in same direction.) When the wavelengths of the waves lie in visible region, the interference is known as interference of light.] ✱

(i) ✱ Let, $\vec{E}$ = electromagnetic field, vector associated with light.

Let it be assumed light travels in (+) X direction.

$$\vec{E} = \vec{E_0} \sin(\omega t - kx)$$

Let, E_1 & E_2 be two waves travelling in X direction with same frequency & velocity but with a phase difference.

$$E_1 = E_{01} \sin(\omega t - kx)$$

$$E_2 = E_{02} \sin(\omega t - kx + \delta)$$

$\delta \rightarrow$ phase diff.

$E_{01}, E_{02} \longrightarrow$ amplitude of E_1 & E_2 resp.

As the waves travels in same direction, the magnitude of the resultant wave is the algebraic sum of mag. of E_1 & E_2.

$$E = E_{01} \sin(\omega t - kx) + E_{02} \sin(\omega t - kx + \delta)$$

$$= E_{01} \sin(\omega t - kx) + E_{02} \sin(\omega t - kx) \cos\delta$$
$$+ E_{02} \sin\delta \cos(\omega t - kx)$$

$$= (E_{01} + E_{02} \cos\delta) \sin(\omega t - kx)$$
$$+ E_{02} \sin\delta \cos(\omega t - kx)$$

Let, $\left. \begin{aligned} R\cos\theta &= E_{01} + E_{02}\cos\delta \\ R\sin\theta &= E_{02}\sin\delta \end{aligned} \right\}$ — (1)

$$\therefore R^2 = (E_{01} + E_{02}\cos\delta)^2 + (E_{02}\sin\delta)^2$$

$$\therefore E = R \sin(\omega t - kx + \theta)$$

$\swarrow$
Resultant Amplitude

$$R = \sqrt{E_{01}^2 + E_{02}^2 + 2E_{01}E_{02}\cos\delta} \quad \text{— (2)}$$

Intensity of wave $\propto$ (Amplitude)^2

$\therefore \quad I_1$ = Intensity of E_1

$= k\, E_{01}^2$ —— (3)

I_2 = Intensity of E_2

$= k\, E_{02}^2$ —— (4)

I = Resultant Intensity

$= k R^2$ . . . . (5)

From eqn (2), (3), (4) & (5),

$$\boxed{I = I_1 + I_2 + 2\sqrt{I_1 I_2}\, \cos\delta} \quad \ldots (6)$$

I depends on δ.

Maximum I occurs ~~at~~ ~~as~~ at $\cos\delta = 1$

$\delta = 0$ or $2n\pi$

$$\therefore \quad I_{max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2 \quad \ldots (7)$$

at $\delta = 2n\pi$, $n = 0, \pm 1, \pm 2, \ldots$

Minimum I occurs at $\cos\delta = -1$

$\delta = (2n+1)\pi$

$$I_{min} = \left(\sqrt{I_1} \sim \sqrt{I_2}\right)^2 \quad (8)$$

Let it be assumed that phase difference arises due to path difference only.

$$E_1 = E_{01} \sin(\omega t - kx_1)$$

$$E_2 = E_{02} \sin(\omega t - kx_2)$$

Phase difference = $\delta = k(x_1 - x_2)$

$\delta = k\Delta x$ $\qquad k = 2\pi/\lambda$

Maxima occurs at $\delta = 2n\pi = k\Delta x$

or, $\dfrac{2\pi\Delta x}{\lambda} = 2n\pi$

$$\boxed{\Delta x = n\lambda} \quad \cdots (9)$$

for minimum, $\delta = (2n+1)\pi = k\Delta x$

$$\frac{2\pi}{\lambda}\Delta x = (2n+1)\pi$$

$$\Delta x = \left(\frac{2n+1}{2}\right)\cdot\lambda$$

$$\boxed{\Delta x = (2n+1)\frac{\lambda}{2}} \quad \cdots (10)$$

At positions satisfied by (9) Max. intensity is obtained & these positions are very bright.

At positions satisfied by (10) min. intensity is obtained & these positions are least bright.

Relations (9) & (10) ~~are~~ show that between successive maxima, there will be one minima & vic. versa.

So, if the intensity is obtained on a screen, a pattern will be obtained, showing alternate bright & dark regions. This pattern is called interference fringe.

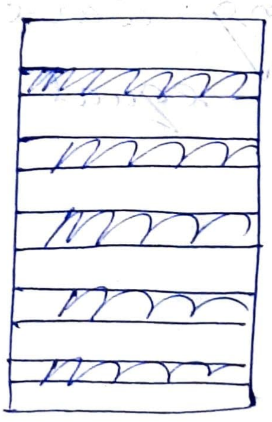
For, $\Delta x = 0 \rightarrow$ ~~1st~~ Zero order max.

$\Delta x = \pm \lambda \rightarrow$ ~~2nd~~ 1st "

$\Delta x = \pm 2\lambda \rightarrow$ ~~3rd~~ 2nd "

$\Delta x = \pm \frac{\lambda}{2} \rightarrow$ 1st order min

$\Delta x = \pm \frac{3\lambda}{2} \rightarrow$ 2nd " "



Case-1

(ii) Let I_1 & I_2 differ by a very large amp. almost such that

$$\sqrt{I_1} >> \sqrt{I_2} \Rightarrow E_{01} >> E_{02}$$

$$E_{01} + E_{02} \simeq E_{01} - E_{02}$$

or, $$\sqrt{I_1} + \sqrt{I_2} \simeq \sqrt{I_1} - \sqrt{I_2}$$

Consequently, $I_{max} \simeq I_{min}$

So the pattern will be too indistinct to differentiate between maxima & minima.

Case-2

if $E_{01} = E_{02}$

$\sqrt{I_1} = \sqrt{I_2} = \sqrt{I_0}$

$I_{max} = 4I_0$

$I_{min} = 0 \longrightarrow$ Complete extinction of light.

So, Bright maxima are easily distinguishable against completely dark minima & interference pattern is distinct.

Thus to get a distinct interference pattern the two interfering waves should have almost equal amplitudes.

$E_{01} = E_{02} = E_0$

$\sqrt{I_1} = \sqrt{I_2} = \sqrt{I_0}$

$I = I_0 + I_0 + 2I_0 \cos\delta$

$= 2I_0(1 + \cos\delta)$

$I = 4I_0 \cos^2\frac{\delta}{2}$

$I_{max} = 4I_0$ for $\cos\delta/2 = \pm 1$

& $I_{min} = 0$ for $\cos\delta/2 = 0$

—x—

2. If δ is time dependent

so, positions of maxima & minima charge continuously & no fixed pattern is obtained on the screen. It is said that sustained interference has not occured. To obtain sustain interference, δ must be constant with time.

i.e. the two interfering waves should be in same phase or bear a phase difference i.e. invariant with time. Two such waves are called Coherent ~~Covarient~~ waves.

Coherence ~~Covarence~~ is a necessary condition for sustained interference

- Coherent Sources – Coherent Sources are the sources which either have no phase difference or have always a constant phase difference between them.

3. (If an electron of the source is excited to a higher energy level it comes back to the ground state by emitting energy in form of electromagnetic waves.

(If wavelength of these waves fall in visible range, light is emitted.)

(Waves are emitted in the form of wave pulses, each pulse arising from a separate transition)

(Phase of emitted wave pulse depends on internal conditions of atom at that time. So, different wave pulses are emitted with different phases varing with time. So, waves from different sources or waves from same source emitted at diff. time may not be coherent & will not produce sustained interference.)

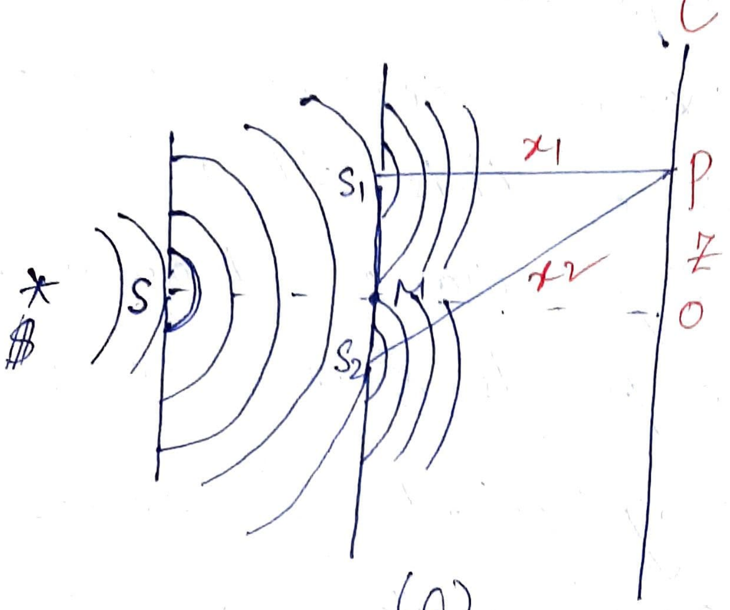
(For sustained interference, the interfering waves should be emitted from same source at same time. To achieve this it is required a single wave should be split into two parts & made to interfere. Being parts of same wave the components are coherent.

There are two ways to split the waves:

1) Division of Wavefront
2) Division of amplitude.)

4. Division of Wavefront:

Young's Double Slit Experiment:



(a)

S → a narrow slit placed in front of source.

S_1 & S_2 → Two other narrow slits in front of S, such that S_1 & S_2 are equidistant.

C → a screen on which interference pattern is obtained.

$d = S_1S_2$

D = distance bet. double slits & screen.

P → a point on C where two waves interfere.

M → mid pt. of S_1S_2

O → ⊥ dropped from M on C.

z = distance PO.

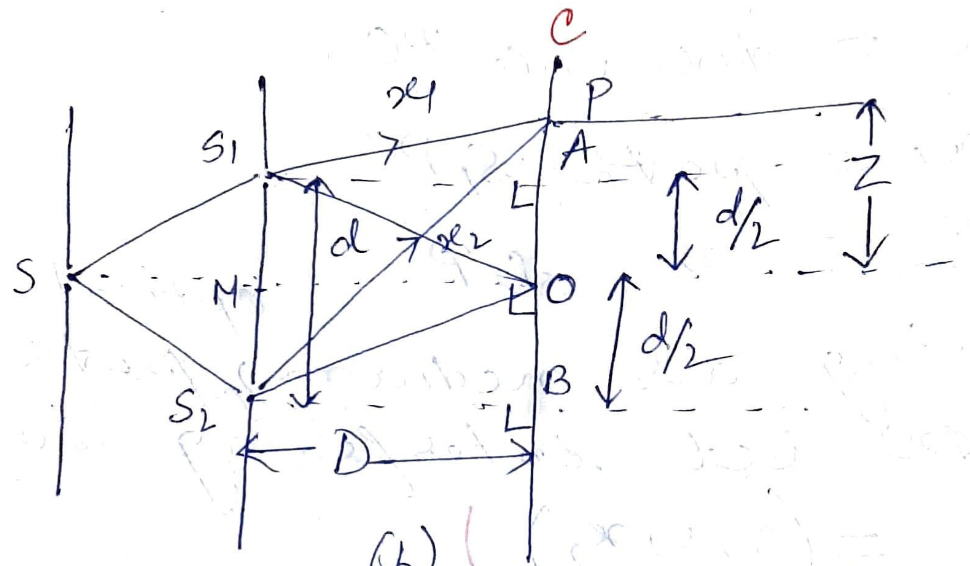
x_1 = path S_1P

x_2 = " S_2P

(Assuming air medium, optical path diff. Δx bet interfering waves
$= (x_1 \sim x_2)$)

(S serves as point source. It sends out spherical wavefront.
The wavefront is obstructed except at S_1 & S_2. Unobstructed part from wavefront at S_1 & S_2 serves as point sources of secondary waves.
These waves are actually primary wavefronts divided in two parts which are coherent.
So, they interfere.)

(To achieve same intensity for interfering waves, S_1 & S_2 should be of same length)



(b)

A & B → feet of ⊥ drawn from S_1 & S_2 on C.

<u>ΔS_1AP</u>

$$x_1^2 = S_1A^2 + PA^2$$
$$= D^2 + \left(z - \frac{d}{2}\right)^2$$
$$x_1^2 = D^2\left[1 + \frac{\left(z - \frac{d}{2}\right)^2}{D^2}\right]$$
$$\therefore \quad x_1 = D\left[1 + \frac{(z - d/2)^2}{D^2}\right]^{1/2}$$

Assuming $d, z \ll D$.

$$x_1 = D\left[1 + \frac{(z - d/2)^2}{2D^2}\right]$$
$$= D + \frac{(z - d/2)^2}{2D} \quad \text{--- (1)}$$

Similarly,

$$x_2 = D + \frac{(z + d/2)^2}{2D} \quad \text{--- (2)}$$

$\therefore x_1 \sim x_2 = D + \frac{(z+d/2)^2}{2D} - D - \frac{(z-d/2)^2}{2D}$

$= \frac{1}{2D}\left[z^2 + \frac{d^2}{4} + d\times z - z^2 - \frac{d^2}{4} + d\times z\right]$

$= \frac{d\times z}{D}$

So, $\boxed{\Delta x = \frac{d\times z}{D}}$

Corresponding phase diff.

$\delta = \frac{k\, d.z}{D}$

$\delta = \frac{2\pi}{\lambda} \cdot \frac{d.z}{D}$

For max to accuring,

$\delta = \pm 2n\pi$

$\frac{2\pi}{\lambda} \cdot \frac{dz}{D} = 2n\pi$

$\boxed{\frac{dz}{D} = n\lambda}$ .... (3)

For min to accur,

$\frac{dz}{D} = (2n+1)\frac{\lambda}{2}$ .... (4)

$z \rightarrow$ distance of nth fringe from O.

If $n = 1$ in (3),

$$z = \frac{\lambda D}{d} \rightarrow 1^{st} \text{ order max.}$$

At zero order maxima,

$$n = 0$$

$$\therefore z = 0$$

It is formed at O.

(The screen exhibits alternate maxima or minima. All maxima being of same intensity & all minima being completely dark)

5. Fringe Width

Fringe width is the diff. of z bet. successive maxima or successive minima.

$$\frac{dz}{D} = n\lambda.$$

$$\therefore \frac{d}{D} \cdot \Delta z = \Delta n \lambda$$

For successive maxima,

$$\Delta n = 1, \quad \therefore \Delta z = \beta$$

$\therefore \frac{d}{D} \cdot \beta = \lambda$

$$\boxed{\beta = \frac{\lambda D}{d}}$$

If screen is moved away or S_1 & S_2 are made to come closer, β increases. So the fringes flared out.

If screen is moved closer, or S_1 & S_2 are separated farther, β decreases & it fringes contract.

- If white light is used:

$$\frac{dz}{D} = n\lambda$$

$$z = \frac{n\lambda D}{d}$$

$$z \propto \lambda$$

For non zero n, z will be diff. for diff λ. So colours will split out showing a specturm to each order.

For zero order $n = 0$

$z = 0$ for all λ.

So all colours will ~~meet~~

meet there giving white colour. Zero order max. is called <u>achromatic fringe</u>.

At higher orders max & min of diff colours overlap at random giving at uniform illumination there.

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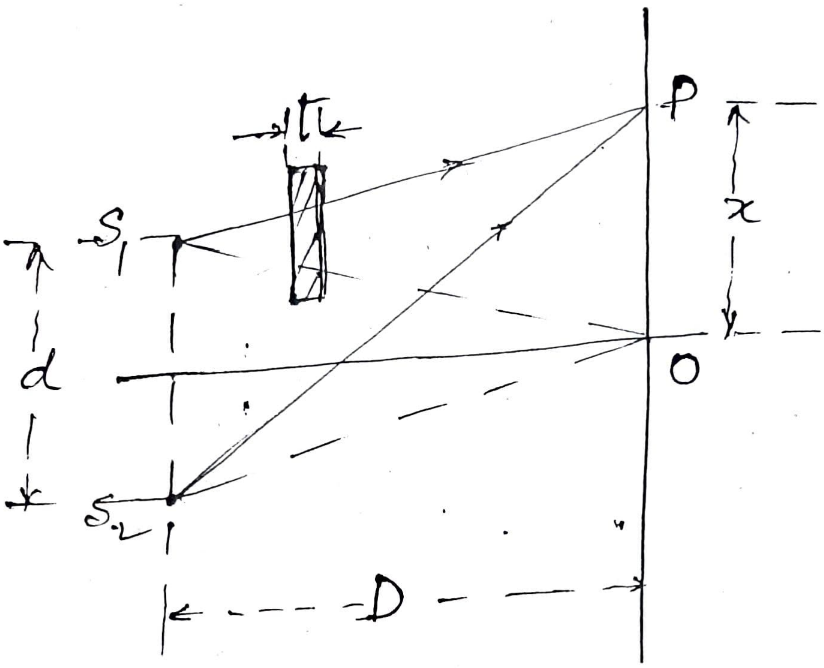
9. Displacement of the fringe system by a thin film



The central (bright) fringe is formed at O in the absence of the film. With the introduction of the film of thickness t and refractive index n in the path S_1O, the central fringe now shifts to a point P where $OP = x$.

It follows that

the optical path S_2P (in air)

= the optical path S_1P through the film.

or, $S_2P = (S_1P - t) + nt$.

or, $S_2P - S_1P = (n-1)t$.

or, $d.\frac{x}{D} = (n-1)t$.

$\therefore \quad x = (n-1)\frac{D}{d}.t$.

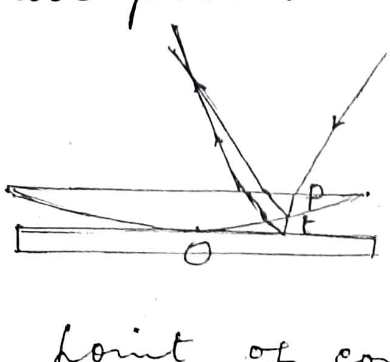
Since $\beta = \frac{D}{d}.\lambda$,

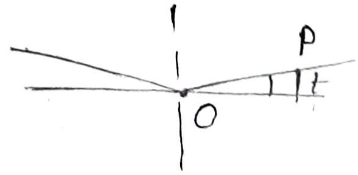
$x = (n-1)\frac{\beta}{\lambda}.t$.

13. ## Newton's rings

(i) Division of amplitude class
(ii) Fringes of equal thickness
(iii) Circular fringes with the central fringe dark (with reflected system)
(iv) Localised fringes.

In Newton's ring experiment, a film (air or liquid) is created by placing a planoconvex lens on a glass plate.





At the point of contact,

$t = 0$,

but the path difference between the two interfering beams originating at the point of contact will be $\pm \lambda/2$, owing to reflection. Hence a dark spot is formed at the point of contact. At a point P corresponding to film thickness t, either constructive or destructive interference would take place depending on the path difference between the interfering beams originating at P. Locus of points like P will be a circle about O. Hence circular fringes, alternately bright and dark with O as centre are formed.

With reference to the figure showing experimental arrangement,

G — optically flat glass plate

L — plano-convex lens of large radius of curvature

G' — a glass plate inclined at 45° with the vertical

M — travelling microscope

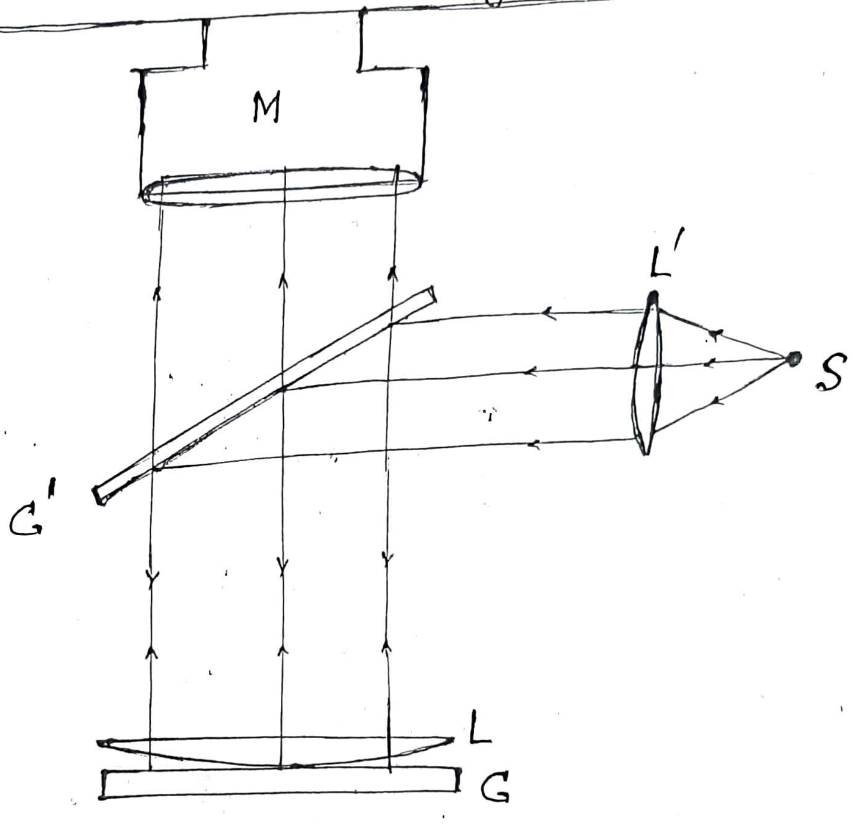
Ex

S

Th

f

Experimental arrangement



S — a wide monochromatic source placed at the focal plane of the lens L'.

Theory

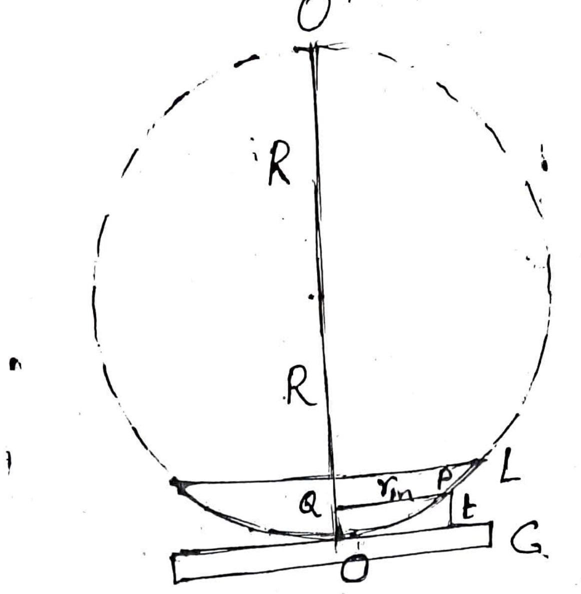
For normal incidence,

$$2nt = \left(m+\tfrac{1}{2}\right)\lambda \qquad m = 0, 1, 2, \ldots$$

for a bright ring.

With air film, $n = 1$,

$$2t = \left(m+\tfrac{1}{2}\right)\lambda.$$



From the figure :

$$r_m^2 = OQ \cdot QO'$$

where r_m is the radius of the bright ring corresponding to the film thickness t at P.

or, $$r_m^2 = t(2R-t)$$
$$\simeq 2Rt \quad \because 2R >> t.$$

or, $$r_m^2 = R.\left(m+\tfrac{1}{2}\right)\lambda \quad , m = 0,1,2,\cdots$$

whence we get

$$r_1 = \left(\frac{\lambda R}{2}\right)^{\frac{1}{2}}, \quad r_2 = \left(\frac{3\lambda R}{2}\right)^{\frac{1}{2}}, \cdots$$

$$r_1 : r_2 : \cdots = \sqrt{1} : \sqrt{3} : \cdots$$

For dark rings,

$$r_m'^2 = R \cdot m\lambda \qquad m = 1, 2, \cdots$$

$$r_1' = (\lambda R)^{\frac{1}{2}}, \quad r_2' = (2\lambda R)^{\frac{1}{2}}, \cdots$$

$$r_1' : r_2' : \cdots = \sqrt{1} : \sqrt{2} : \cdots$$

** [Thus, the radii of the bright rings are proportional to the square roots of the odd natural numbers and the radii of the dark rings are proportional to the square roots of the natural numbers.]

Measurement of wavelength of light

Considering bright rings, the diameters of the rings of orders m and $(m+p)$ are given by

$$D_m^2 = \left(m + \tfrac{1}{2}\right) \cdot 4\lambda R$$

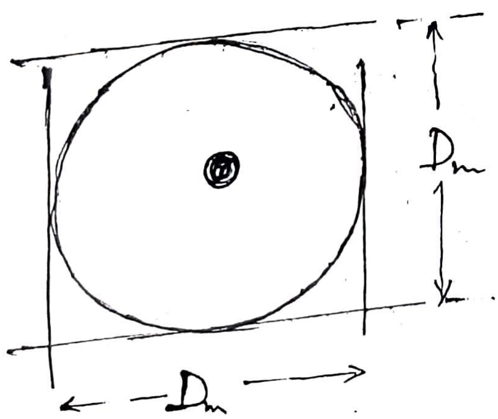
and $$D_{m+p}^2 = \left(m + p + \tfrac{1}{2}\right) \cdot 4\lambda R$$

whence we get

$$D_{m+p}^2 - D_m^2 = 4p\lambda R$$

$$\text{or,} \quad \lambda = \frac{D_{m+p}^2 - D_m^2}{4pR} \quad \cdots\cdots (1)$$

Eq. (1) is used to determine the wavelength of a given monochromatic light by measuring the diameters of two bright rings of different orders using a travelling microscope.



Determination of the refractive index of a liquid

With the liquid film between the glass plate G and the plano-convex lens L, for a bright ring of order m,

$$2nt = (m+\tfrac{1}{2})\lambda$$

or, $$\frac{r_m'^2}{R} \cdot n = (m+\tfrac{1}{2})\lambda$$

Since $r_m'^2 \simeq 2tR$, r_m' being the radius of the ring.

or, $$n \cdot d_m^2 = (m+\tfrac{1}{2}) 4\lambda R.$$

where d_m is the diameter of the ring corresponding to the order m.

Similarly, for the ring of order $(m+p)$ we have

$$n \cdot d_{m+p}^2 = (m+p+\tfrac{1}{2}) \cdot 4\lambda R.$$

Hence we get

$$n \cdot (d_{m+p}^2 - d_m^2) = 4p\lambda R.$$

With air film, we have

$$D_{m+p}^2 - D_m^2 = 4p\lambda R.$$

whence we obtain

$$\frac{D_{m+p}^2 - D_m^2}{n(d_{m+p}^2 - d_m^2)} = 1$$

or,

$$n = \frac{D_{m+p}^2 - D_m^2}{d_{m+p}^2 - d_m^2} \quad \cdots\cdots (2)$$