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ENGINEERING MATHS

1) LINEAR ALGEBRA

(i) Intro to Matrices

→ Linear Algebra: deals with linear equations. [deg = 1] & Matrices are used to analyze them.

eg: $x+y+z=3$

(ii) Types of Matrices

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

→ upper [$i<j$].

→ Principle diag. [$i=j$].

→ lower diag. [$i>j$].

1] Diagonal Matrix: all off diag elements $=0$; Square Matrix.

$\boxed{a_{ij}=0 \; \forall \; i \neq j}$

2] Scalar Martrix $\boxed{a_{ij}=K \; \forall \; i=j}$

Diag Matrix: all diag elements are equal.

3] Identity Matrix/Unit Matrix [I]

- Scalar matrix in which all diag elem $=1$.

$\boxed{AI = IA = A}$; $I \rightarrow$ mul. identity

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4) Null Matrix
- All elements = 0.
- additive identity Matrix ∵
$A+0 = 0+A = A$

5) Upper Triangular Matrix $\boxed{a_{ij}=0, \forall i>j}$
- All elements below Princi Digonal = 0

6) Lower Triangular Matrix
- All elem. above PD = 0.
$\boxed{a_{ij} = 0 \ \forall \ i<j}$

imp

7) Idempotent Matrix ⇒ $A^2 = A$.
8) Involutary Matrix ⇒ $A^2 = I$.

* Multiplication of 2 Matrix

→ if $a \times b * c \times d$ then, $\boxed{b == c}$
(i) may/may not be commutative.
$AB \neq BA$.
(ii) Associative. Vimp
(iii) if $\boxed{AB = 0}$, doesn't mean $A=0$ or $B=0$
(iv) If product of 2 Matrix (Non-Zero) is a Zero Matrix, Both are singular.
(v) $\boxed{A = LU}$; where, $A \to$ Square Matrix
$L \to$ lower Δg
$U \to$ Upper Δg
(vi) $A_{m\times n} * B_{n\times p} = AB_{m\times p}$ imp

$$\boxed{\begin{array}{l} \text{No. of mul} = mp(n) \\ \text{No. of add} = mp(n-1) \end{array}}$$

$\boxed{\text{eg}}$ → Pg 7 (LU decomposition)

$\boxed{eg}$ → Pg 8 [idempotent matrix eg] $\boxed{X^2 = X}$

$\boxed{eg}$ → Pg 9 [Sum of all elements of matrix → imp Q]

(ii) Transpose of Matrix [AT].

- If $A = m \times n$, $A^T = n \times m$.
- element$(A) = a_{ij}$, elem$(A^T) = a_{ji}$

Properties

(i) $(A+B)^T = A^T + B^T$. imp
(ii) $(AB)^T = B^T A^T$.
(iii) $(KA)^T = K \cdot (A^T)$; K = constant.

* Symmetric Matrix ⇒ $\boxed{A^T = A}$

→ $\boxed{a_{ji} = a_{ij}}$ $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}$

* Skew-Symmetric Matrix $\boxed{A^T = -A}$

→ $\boxed{a_{ji} = -a_{ij}}$ $\begin{bmatrix} 0 & 1 & 2 \\ -1 & 0 & 3 \\ -2 & -3 & 0 \end{bmatrix}$

Principle diag elem = 0. (Skew-Symm) Vimp

* Orthogonal $\boxed{A^T = A^{-1}}$

→ Real Matrix
- → Symmetric $\boxed{A^T = A}$
- → Skew-Symmetric $\boxed{A^T = -A}$
- → Orthogonal $\boxed{A^T = A^{-1}}$

Vimp

Properties

- If A = Square Matrix
  (i) $A + A^T$: Symmetric
  (ii) $A - A^T$: Skew Symmetric.
  (iii) $A \cdot A^T$: Symmetric
- If A = Symmetric ; A^n : symmetric
- If A = Skew-Symmetric
  (i) A^n = symmetric ; n = even
  (ii) A^n = Skew-symmet ; n = odd

* Complex Matrix

- Conjugate Matrix

$$A = \begin{bmatrix} 2+3i & 3 \\ 2i & 2-i \end{bmatrix} ; \quad \overline{A} = \begin{bmatrix} 2-3i & 3 \\ -2i & 2+i \end{bmatrix}$$

- Conjugate Transpose Matrix

$$(\overline{A})^T \text{ or } A^\theta = \begin{bmatrix} 2-3i & -2i \\ 3 & 2+i \end{bmatrix}$$

* Complex Matrix →
  - Hermitian $\boxed{A^\theta = A}$
    $\Rightarrow \overline{a_{ji}} = a_{ij}$
  - Skew Hermitian $\boxed{A^\theta = -A}$
    $\Rightarrow \overline{a_{ji}} = -a_{ij}$
  - Unitary $\boxed{A^\theta = A^{-1}}$

imp

Property

* $(KA)^\theta = \overline{K} A^\theta$; K = complex no.

But, $(KA)^T = K(A)^T$; K = real no.

[eg] → Pg 12 (Q 2) [Every diag element of Hermitian is purely real]

[eg] → Pg 13 [A]

(iii) MINOR OF MATRIX

Q How many 2X3 matrices from 4x5 ?

$$\Rightarrow {}^4C_2 \times {}^5C_3 \Rightarrow \frac{4\times 3}{2\times 1} \times \frac{5\times 4}{2} = 60$$

$\boxed{\text{Minor of Matrix} = \text{Submatrix}}$.

∴ 60 minors

* Minor of element

M_{ij} = deleting i^{th} row & j^{th} column & cal. determinant.

$$M_{11} = \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix}.$$

* CoFactor of element

$$\boxed{C_{ij} = (-1)^{i+j} M_{ij}}$$

imp

if even, $C_{ij} = M_{ij}$.

[always of Square matrix]

(iv) DETERMINANTS

calculated in 6 ways. [3!]

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

wrt. R_1 · $\Delta = a_{11}C_{11} + a_{12}C_{12} + a_{13}C_{13}$

* Properties of determinants.

⇒ A must be a Square Matrix for determinant to exist.

(i) $|A| = |A^T|$.

(ii) If 2 ∥ lines interchanged = determinant opp in Sign. (imp)

(iii) $\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 0 & 0 & 0 \end{vmatrix} = 0$. (1 ∥ line = 0).

(iv) $\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 4 & 5 & 6 \end{vmatrix} = 0$ (2 ∥ lines identical to each other)

(v) If 2 ∥ lines proportional to each other = 0.

$$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 2 & 4 & 6 \end{vmatrix} = 2\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 1 & 2 & 3 \end{vmatrix} = 0.$$

(vi) If all consecutive numbers, $|A| = 0$. (Vimp)
(valid for $\geq 3^{rd}$ order determ)

(vii) $\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix} = 27$, $\begin{vmatrix} K & 2K & 3K \\ 4K & 5K & 6K \\ 7K & 8K & 9K \end{vmatrix} = 27K^3$.

(viii) Determinant of Upper Δ / lower Δ / diagonal / Scalar matrix = Product of diagonal elements. (Vimp)

(ix) Det. of Skew Symmetric matrix
if, even order = Perfect Sq.
Odd order = 0.

Vimp

(x) Det. of Orthogonal Matrix $= \pm 1$

(xi) if A, B are 2 Sq. Matrix of same order then $|AB| = |A| \cdot |B|$

(xii) $|A^n| = |A|^n$.
$|A^{-1}| = |A|^{-1} = 1/|A|$.

(xiii) If $A = n \times n$ Square Matrix imp
(i) $|\text{Adj} A| = |A|^{n-1}$.
(ii) $|\text{Adj}(\text{Adj} A)| = |A|^{(n-1)^2}$.

(xiv) $|KA| = K^n |A|$.

eg → Pg 21 [Q2]

eg → Pg 21 [Q3 (ii)] (Choose the row with most no. of 0s & calculate determ wrt. to that row)

eg → Pg 22 [Q4] (Value of det. by ~~row~~ column transformation)

(V) INVERSE OF A MATRIX

- A must be a non-singular matrix

$$\boxed{A^{-1} = \frac{1}{|A|} \text{adj} A}$$

* Cofactor Matrix → $\begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}$

imp $\boxed{\text{adj} A = [\text{Co} A]^T}$

* Properties of Inverse of Matrix:

(1) $A * A^{-1} = A^{-1} * A = I$ ← Identi matri

(2) $(A^{-1})^T = (A^T)^{-1}$ → imp

(3) $(ABC)^{-1} = C^{-1} B^{-1} A^{-1}$.

(4) $AB = I \Rightarrow A = B^{-1}$ and $B = A^{-1}$

* $A(\text{adj} A) = \text{adj} A(A) = |A| I_n$

* $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$; $\text{adj} A = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

Vimp → (only for 2x2).

(5) $ABCD = I$; then $B^{-1} = CDA$, $A^{-1} = B$

imp → $C^{-1} = DAB$, $D^{-1} = AB$

eg → Pg 27 (Q 4) [Skew Symm]

eg → Pg 27 (Q5) [compute Adj A_{32}]

* Calculate Adj A by Intermediate Matrix Method: → for 3x3 imp

eg → Pg 28 [Q6]

* Calculating |A| from A & Adj A

$A = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & -1 \\ 2 & 3 & 2 \end{bmatrix}$; $\text{Adj} A = \begin{bmatrix} 5 & -3 & 1 \\ -6 & 4 & -1 \\ 4 & -3 & 1 \end{bmatrix}$

$\boxed{A \cdot \text{Adj}(A) = |A| I_n}$

$$|A| = [1\ 0\ -1]\begin{bmatrix}5\\-6\\4\end{bmatrix} \Rightarrow 1$$

(vi) RANK OF A MATRIX

- Order of the highest non-zero Square Submatrix.

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}_{3\times3}$$

if $|A| \neq 0 \rightarrow R(A) = 3$.
if $|A| = 0$; $R(A) < 3$
↓
Check in the 9 2x2 matrix, if atleast one of them is nonzero : $R(A) = 2$.
if all zero, $R(A) = 1$.
∴ only Null Matrix → $R(A) = 0$
Q if $R(A_{7\times7}) = 3$, $|A| = 0$ ← Vimp

Properties

1) $R(O_{N\times N}) = 0$.
2) $R(I_{n\times n}) = n$ ∵ $|I| = 1 \neq 0$.
3) $R(AB) \leq \min(R(A), R(B))$ ~~imp~~
4) Rectangular Matrix : rank defined
$R(A) \leq \min(m, n)$ | $A_{m\times n}$
5) $R(A) = R(A^T)$. $R(A \cdot A^T) = R(A) = R(A^T)$
6) Matrix with rank 'r' contains r linearly independent vectors

imp

Test

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* Linearly dependent Vectors

if $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0 \rightarrow$ linearly dependent

$\boxed{eg} \longrightarrow$ Pg 32 [Q1]

- 1 vector can be expressed by other vectors linearly.

* Calculate rank of Matrix:

$\boxed{eg} \longrightarrow$ Pg 33(b) [Rank of a rectangular Matrix] (imp) (Row echelon form)

Note: In a matrix, if all the rows are proportional to each other, $\boxed{R(A)=1}$ (Vimp)

$\boxed{eg} \longrightarrow$ Pg 34

* $\begin{bmatrix} a & a^2 & a^3 & \dots & a^n \\ a & a^2 & a^3 & \dots & a^n \\ \vdots & \vdots & \vdots & & \vdots \\ a & a^2 & a^3 & \dots & a^n \end{bmatrix}$

$R(A) = 1, \; a \neq 0$
$\quad\quad = 0, \; a = 0$ [proportional]

$\therefore$ R(A) depends on a value

(vii) SYSTEM OF EQUATIONS

- If system of eq^n's has a solution consistent else inconsistent.

Unique   Infinite

1) HOMOGENEOUS System of Eq^ns

→ always have solution ∴ consistent

→ form: $AX=0$

(RHS) is zero.

imp

• Consistent → Unique / trivial sol^n.

$R(A)=n$ (no. of var)

or $|A| \neq 0$.

↓

Infinite / non-trivial.

$R(A)<n$ or $|A|=0$.

2) Non-Homogeneous System of Eq^ns

→ may / may not have solution.

→ form: $AX=B$.

imp

Non-Homogeneous

consistent

$R(A)=R(A/B)$

Unique

$R(A)=R(A/B)$
$=n$

$\Delta \neq 0$.

Infinite

$R(A)=R(A/B)$ → augmented Matrix.
$<n$.

$\Delta=0, \Delta_1=0$

Inconsistent.

$R(A) \neq R(A/B)$

$\Delta=0, \Delta_1 \neq 0$

* Non-Homogeneous System of 2 variables [AX=B]

[2 eq^ns of 2 variables]

$$a_1x + b_1y = c_1$$
$$a_2x + b_2y = c_2$$
} RHS $\neq 0$

[$\Delta \neq 0$] Unique	Infinite [$\Delta = 0, \Delta_1 = 0$]	[$\Delta = 0, \Delta_1 \neq 0$] No soln
$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$	$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$	$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$
[intersecting]	[same line]	[parallel]

$\boxed{\text{eg}}$ → Pg 39 [Q 3] $K = 0, 2$ for non-trivial / ∞ solutions.

* Solve system of eqⁿs by Row transformation method.

$\boxed{\text{eg}}$ → Pg 41 ($x_3 = -2$)

* Identify type of Solution using Δ and Δ_1 method

$\Delta \neq 0$: unique Soln
$\Delta = 0, \Delta_1 = 0$: infinite soln
$\Delta = 0, \Delta_1 \neq 0$: No soln

$\boxed{\text{eg}}$ → Pg 43 [Q3] [AX=B] → for this method:

It must be of Type: [3 Eqⁿs of 3 Var]

$$x_1 - 4x_2 - x_3 = -2$$
$$2x_1 - x_2 + 3x_3 = 1$$
$$3x_1 + 2x_2 + 5x_3 = 2$$

[2 var, 3 eqns]

$\boxed{\text{eg}}$ → Pg 44 (Unique Soln)

imp

$\boxed{\text{eg}}$ → Pg 44 (Q2). The System has a solution — a?

[Solve it by A|B method]

(viii) EIGEN VALUES & EIGEN VECTORS

→ A: n×n Sq. matrix, λ: Scalar M

imp → $|A - \lambda I| \Rightarrow 0$ // Character. Eqⁿ.

. roots of this characteristic eqⁿ is called Eigen Values.

→ For every eigen Value, there exists an Eigen Vector. Vimp

$\boxed{AX = \lambda X}$; X = Eigen Vector
λ = Eigen Value.

* Properties.

(1) Sum of Eigen Values = Trace.
(2) Product of Eigen Values = Determinant
(3) Eigen Values of A & A^T = equal.
(4) EV of upper Δ, lower Δ, diagonal, Scalar ⇒ diagonal elements. Vimp
(5) EV of Symmetric matrix ⇒ real
(6) EV of Skew-Symmetric ⇒ purely img. or 0.
(7) EV of orthogonal matrix ⇒ unit mod.
(8) If $a+ib$ → EV of matrix
$a-ib$ → must be another EV.
(9) $\lambda \longrightarrow A$. if, λ is EV of A.
$\lambda^n \longrightarrow A^n$ λ^n is EV of A^n
$1/\lambda \longrightarrow A^{-1}$
$|A|/\lambda \longrightarrow adj A$
$\lambda \pm K \longrightarrow A \pm KI$ VI

$\underbrace{\hspace{6cm}}$
Eigen Values.

10) Eigen Vectors of A and A^n are s

if $\lambda \to A$. $X \to A$.

$\underbrace{\lambda^n \to A^n}_{\text{eigen Value}}$ $\underbrace{X \to A^n}_{\text{eigen Vecto}}$

11) EV corresponding to distinct eigen value of Symmetric matrix ar orthogonal to each other. Vim

* For 2x2 Matrix : characteristic eq

$$\boxed{\lambda^2 - \text{Trace}(A)\lambda + |A| = 0}$$

* For 3x3 Matrix : minors.

imp

$$\boxed{-\lambda^3 + \text{Trace}(A)\lambda^2 - [m_{11} + m_{22} + m_{33}]\lambda + |A| = 0.}$$

* Calculate eigen Values :
[If options are given, calculate Su & product, & match with Trace & determ of matrix]. Tricks

eg → Pg 49 (Q3)

eg → Pg 50 (Q5) [Just check the options see if conjugates of every complex root exits]

If Sum of all the rows = constant ; then that constant must be one of the eigen Value

vimp

$\boxed{eg} \longrightarrow$ Pg 51 [Q 1]

$\boxed{eg} \longrightarrow$ Pg 51 [Q 2] $B = A - A^{-1} + A^2$.

imp

$\lambda \longrightarrow A$ (if λ is EV of A).

$\lambda^2 \longrightarrow A^2$

Note: $\boxed{\text{EV can't be } \infty \because \text{Product} = \infty = |D|}$

$\boxed{eg} \longrightarrow$ Pg 52 [Q 2] Applicⁿ of Property:

$\lambda \longrightarrow X$

$\lambda + K \longrightarrow X + KI$

$\boxed{eg} \longrightarrow$ Pg 53 [Q 1] $|A| = -1$. & options.

Vimp

* Problems on Eigen Vectors

$\boxed{eg} \longrightarrow$ Pg 54 [Q 1] (Given is an upper triangular matrix)

* Eigen Values = $[-1, -2]$

Eigen Vector = $\begin{bmatrix} 1 \\ -1 \end{bmatrix} \begin{bmatrix} 1 \\ -2 \end{bmatrix}$; matrix ?

Trace = -3

determ = 2. $\Big\}$ eliminate some optⁿ.

Now, $AX = \lambda X$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix} = -1 \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$a - b = -1$ —— (1)

$c - d = 1$ —— (2).

Check options

EV are:

$\boxed{eg}$ → Pg 56 [Orthogonal, Symmetric Matrix]

imp

$\boxed{eg}$ → Pg 57 [Q 1] [Find out eigen Vectors].

* Shortcut to find out the Eigen Vector of Matrix :

$\boxed{eg}$ → Pg 58 [Q 1] : (d) $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ (eigen Vector)

$AX = \lambda X$

(ix) CALEY HAMILTON THEOREM

→ Every Square Matrix satisfies its characteristic Equation [satisfied by EV]

$|A - \lambda I| = 0$.

$\boxed{eg}$ → Pg 60 [Q 2]

3 eigen Values ; 3 eigen Vectors

Q A 3x3 Matrix P, such that $P^3 = P$. eigen Values are ?

$\lambda \longrightarrow P$

$\lambda^3 \longrightarrow P^3$

$\therefore \lambda = \lambda^3$

imp

$\Rightarrow \lambda(\lambda^2 - 1) = 0$; $\lambda = 0, \pm 1$

$\boxed{eg}$ → Pg 63 . (a) $511A + 510I$

imp

EV should also satisfy the characteristic equation.

2) CALCULUS

1] LIMITS

$\lim_{x \to a} f(x)$: $f(x)$ approaches to $x=a$, but $x \neq a$.

$\therefore \boxed{\lim_{x \to a} f(x) = f(a)}$

* Left Hand & Right Hand Limit

imp

$\lim_{x \to a^-} f(x) = \lim_{h \to 0} f(a-h) \leftarrow$ LHL

$\lim_{x \to a^+} f(x) = \lim_{h \to 0} f(a+h) \leftarrow$ RHL

$\therefore$ If $\boxed{LHL = RHL}$, limit exists.

eg $\longrightarrow$ Pg 66 [Q 1]

* Standard limits

1) $\lim_{x \to 0} \frac{\sin x}{x} = 1$.

2) $\lim_{x \to 0} \frac{\tan x}{x} = 1$.

3) $\lim_{x \to 0} \frac{\sin ax}{x} = a$; $\lim_{x \to 0} \frac{x}{\sin ax} = \frac{1}{a}$

4) $\lim_{x \to 0} \frac{x^n - a^n}{x - a} = na^{n-1}$.

imp

Vimp

5) $\underset{x \to 0}{Lt} \frac{e^{ax}-1}{x} = a$; $\underset{x \to 0}{Lt} \frac{e^{x}-1}{x} = 1$

6) $\underset{x \to 0}{Lt} \frac{a^{x}-1}{x} = \log_e a$.

7) $\underset{x \to a}{Lt} \log[f(x)] = \log\left[\underset{x \to a}{Lt} f(x)\right]$ vimp

eg → Pg 67 [Q2]

* Intermediate Forms

- $0/0, \infty/\infty, 0 \times \infty, \infty \times 0$: L Hospital
- $\infty - \infty$: LCM
- $0^0, \infty^0$ vimp : 1 (Ans).

- Apply L Hospital, when in $\frac{0}{0}$ or $\frac{\infty}{\infty}$ form.

eg → Pg 71 [Q4] $\underset{x \to 0}{Lt} \frac{\sin x}{x e^x}$

eg → Pg 71 [Q6] → $\frac{a^x - 1}{x}$ form.

Q $\underset{x \to 0}{Lt} x \log(\sin x)$

$\Rightarrow \underset{x \to 0}{Lt} \log(\sin x)^x$ → 0^0 form

$\Rightarrow \log\left[\underset{x \to 0}{Lt} \sin x^x\right] \Rightarrow \log 1 = \underline{\underline{0}}$.

vimp

$$\underset{x\to 0}{Lt} \frac{1-\cos ax}{x^2} = \frac{a^2}{2!}$$ even

$$\underset{x\to 0}{Lt} \frac{1-\cos ax}{x^4} = \frac{a^2}{4!}$$

$$\underset{x\to 0}{Lt} \frac{1-\cos ax}{x^3} = 0.$$ $x^3 \to$ odd

Concept.

Vimp

eg → Pg 73 [Q 1]

* $\infty - \infty$ Model

LCM

Q $\underset{x\to 0}{Lt} \left[\frac{1}{x} - \frac{1}{e^x - 1}\right]$ $\infty - \infty$

imp

eg → Pg 73

just rem formula.

* 0^0 Model / ∞^0 Model = 1.

* 1^∞ Model.

$$\boxed{\lim_{x\to a}[f(x)]^{g(x)} = e^{\lim_{x\to a} g(x)[f(x)-1]}}$$

eg → Pg 75 (Q 1)

2] CONTINUITY

f(x) is continuous at x = a if:

(1) f(x) must have defined value at a.

(2) $$\boxed{\lim_{x\to a} f(x) = f(a)}$$

or, $\boxed{\lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x) = f(a)}$

- LHL = RHL = fun value $|_a$
  [funct^n is continuous]

[eg] → Pg 77 [Q 1]

[eg] → Pg 77 [Q2] [Which one of the following funct is continuous at $x=3$]

3] DIFFERENTIATION

- A funct^n is said to be differentiable at $x=a$; if

$$\boxed{LHD|_{x=a} = RHD|_{x=a}}$$ (imp)

If f(x) is different. at $x=a$, it must be continuous at $x=a$ not converse

* |x| is continuous everywhere, differ. everywhere except $x=0$ [sharp point]

[eg] → Pg 79 [Q 1] f(x) is continuous but not differentiable at $x=0$.

|x| → x ($x>0$), $-x$ ($x<0$)

[eg] → Pg 80 $f(x) = 2x+1$

[eg] → Pg 81 [Q2 (imp)

* Parametric Differentiation

If $x = a(\theta - \sin\theta)$

$y = a(1-\cos\theta)$; $dy/dx = ?$

$\Rightarrow \frac{dx}{d\theta} = a(1-\cos\theta)$

$\frac{dy}{d\theta} = a\sin\theta$.

$\therefore \frac{dy}{dx} = \frac{\sin\theta}{1-\cos\theta} = \cot\frac{\theta}{2}$. Ans.

4 PARTIAL DIFFERENTIATION

$z = f(x, y)$.

- first order PD = $\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}$
- Second order PD = $\frac{\partial^2 z}{\partial x^2}, \frac{\partial^2 z}{\partial y^2}, \frac{\partial^2 z}{\partial x \partial y}$

eg $\rightarrow$ Pg 83 $f = y^x$ [Q1]

eg $\rightarrow$ Pg 84 [$u = f(x+cy) + g(x-cy)$]

* Total Derivative

If $z = f(x, y)$; $x = g(t)$, $y = h(t)$

TD of z wrt t is given by : imp

$$\boxed{\frac{dz}{dt} = \frac{\partial z}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial z}{\partial y} \cdot \frac{dy}{dt}}$$

eg $\rightarrow$ Pg 85

5] MAXIMA – MINIMA

imp

→ $f'(x) = 0$ & calculate roots (a)
if, $f''(a) < 0$: a = maxima
$f''(a) > 0$: a = minima.
$f''(a) = 0$; a = Saddle Point

eg → Pg 87 [Q 1]

eg → Pg 87 [Q 2] ∵ only 1 value, it must be the maxima.

V imp

$\boxed{\text{Max value of } a\sin x + b\cos x = \sqrt{a^2+b^2}}$

eg → Pg 89 $f(x) = e^{\sin x - \cos x}$

* Local maxima / minima in the interval [a, b]

Mistake in Test

- $f'(x) = 0$.
- Consider the roots as $x = c, d$.
- if $c, d \in [a, b]$
  max value = $\max\{f(a), f(b), f(c), f(d)\}$
- if $c, d \notin [a, b]$
  max value = $\max\{f(a), f(b)\}$.

V imp

same for min

eg → Pg 90 [Q 2]

[Test]

6] MEAN VALUE THEOREM

1) Rolle's Mean Value Theorem

If $f(x)$ is:

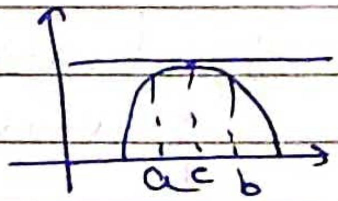
(i) continuous in $[a,b]$

(ii) differentiable in (a,b).

(iii) $f(a)=f(b)$.

there exists $c \in (a,b)$

such that: $\boxed{f'(c)=0}$



2) Lagrange's Mean Value theorem

If $f(x)$ is:

(i) continuous in $[a,b]$.

(ii) differentiable in (a,b).

there exists $c \in (a,b)$.

$$\boxed{f'(c)=\frac{f(b)-f(a)}{b-a}}$$

imp

3) Cauchy's Mean Value Theorem

If $f(x)$ & $g(x)$ are:

(i) continuous in $[a,b]$.

(ii) differentiable in (a,b)

(iii) $g'(x) \neq 0$.

there exists $c \in (a,b)$.

$$\boxed{\frac{f'(c)}{g'(c)}=\frac{f(b)-f(a)}{g(b)-g(a)}}$$

imp

eg $\longrightarrow$ pg 93 $c=\pi$ (Rolle's MVT).

$c \in (\pi/4, 5\pi/4)$

[eg] → Pg 93 [Q2] $c \in (a, b)$; a is not included.

imp

7] SERIES EXPANSION

1) Taylor Series Expansion of $f(x)$ about $x = a$

imp

$$f(x) = f(a) + \frac{x-a}{1!} f'(a) + \frac{(x-a)^2}{2!} f''(a).$$

$$\boxed{\text{coeff of } (x-a)^n = \frac{f^n(a)}{n!}}$$

2) $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} \dots$

3) $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} \dots$

4) $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} \dots$

imp

Q Taylor abt $x = 2$.

$$f(x) = f(2) + (x-2) f'(2) + \frac{(x-2)^2}{2!} f''(2).$$

$\therefore$ coeff of $(x-2)^4 = \frac{f^4(2)}{4!}$

$\because f(x) = e^x$ $\Rightarrow \frac{e^2}{24}$

8] INTEGRAL CALCULUS

1) Idefinite Integration.

1. $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

2. $\int e^{ax} dx = \frac{e^{ax}}{a} + c$.

* 3. $\int a^x dx = \frac{a^x}{\log a} + c$

4. $\int \frac{1}{x} dx = \log x + c$

5. $\int \sin x \, dx = -\cos x + c$

6. $\int \cos x \, dx = \sin x + c$

7. $\int \sec^2 x \, dx = \tan x + c$

8. $\int \operatorname{cosec}^2 x \, dx = -\cot x + c$

9. $\int \sec x \tan x \, dx = \sec x + c$

10. $\int \operatorname{cosec} x \cot x \, dx = -\operatorname{cosec} x + c$.

11. $\int \tan x \, dx = \ln|\sec x| + c$.

12. $\int \frac{1}{\sqrt{a^2 - x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + c$

13. $\int \frac{1}{\sqrt{a^2+x^2}} = \sinh^{-1}\left(\frac{x}{a}\right) + c$ imp

$$\boxed{\int \frac{f'(x)}{\sqrt{f(x)}} = 2\sqrt{f(x)} + C}$$

14. $\int \frac{1}{a^2+x^2}dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$

15. $\int \frac{f'(x)}{f(x)} = \log f(x) + C$ imp

16. $\int e^x[f(x)+f'(x)]dx = e^x f(x) + C$

* Integration by Parts

$$\int f(x)\,g(x)\,dx =$$

$$f(x)\int g(x)\,dx - \int\left[f'(x)\int g(x)\,dx\right]dx$$

$\boxed{eg} \rightarrow$ Pg 99 [Q2] [Q3] $\rightarrow \frac{x^2}{x-1} \Rightarrow \frac{(x^2-1)+1}{x-1}$

$\boxed{eg} \rightarrow$ Pg 100 [Q2] $e^x(f(x)+f'(x))$

$\boxed{eg} \rightarrow$ Pg 100 [Q3] $\int \frac{f'(x)}{f(x)} = \ln(f(x)) + C$ imp

$$* \boxed{\int \frac{1}{x^2-a^2} = \frac{1}{2a}\left[\log\frac{(x-a)}{x+a)}\right] + C}$$

$$\boxed{\int \operatorname{cosec} x = \log\left|\tan\frac{x}{2}\right| + C}$$

$$\boxed{\int \sec x = \log|\sec x + \tan x| + C}$$

$$\boxed{\int \frac{1}{a^2-x^2} = \frac{1}{2a}\log\left|\frac{a+x}{a-x}\right| + C}$$

imp

3) PROBABILITY & STATISTICS

① Intro to Probability

- Random exp: An exp whose outcome can't be predicted with certainity
- Sample space: All possible outcomes
- Event: favourable outcome
  $\therefore$ $\boxed{E \subseteq S}$

- Probability of an Event (P(E)):

$$\boxed{P(E) = \frac{n(E)}{n(S)}}$$ (imp)

$\therefore$ $0 \leq P(E) \leq 1$ ← (certain event)
(impos. event)

* Types of Event

1) Complimentary event: if it contains all outcomes present in S, but not E.
E → getting even no.
E^C → getting odd no after rolling dice.
$\boxed{E + E^C = S}$, $\boxed{P(E) + P(E^C) = 1}$

2) Equally likely event: In coin, the probab of getting H/T is equally likel (1/2)
3) Mutually exclusive event: If A occurs, B cannot occur & vic-ver.
$\boxed{P(A \cap B) = \phi}$ (imp)

eg: If H, T can't occur vice-versa.
- correspond to same SS. (imp)

4) Independent events:

$$\boxed{P(A \cap B) = P(A) \cdot P(B)}$$

correspond to different SS.

eg: H → 1st toss, T → 2nd toss

---

* Addition theorem of Prob

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B)$
$- P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$

[either or / neither nor / atleast once]

~~eg~~ → Pg 105 [Q1] (imp)

eg → Pg 106 [Q.2] (Husband & Wife getting Selected).

eg → Pg 107 [Q3] [Similar type]

eg → Pg 108 [Q5] [Application of Addition Theorem & deck of cards].

eg → Pg 109 [Q7] (imp) (Vimp Q)
Prob that req no of tosses is odd.

Bag → 5R Balls, 5B Balls.

eg → Pg 110 [Q10]
(imp) [Total Prob Q]

Q. No is selected randomly from 1 to 100. Prob. that no. picked won't contain 7?

$\therefore$ $\underset{8}{\underline{\ \ }}\ \underset{9}{\underline{\ \ }}\ \underset{9}{\underline{\ \ }} \Rightarrow \dfrac{8\times 9^2}{900} = \dfrac{18}{25}$ ✗

eg → Pg 112 [VK & AS] ← Gate

[Use Cartesian Product & Solve]

$\therefore\ 12/144 = 1/12$

(2) CONDITIONAL PROBABILITY

imp

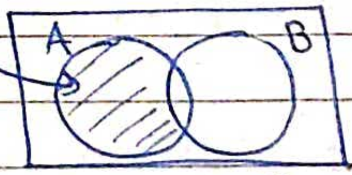
$P(A/B) = \dfrac{P(A\cap B)}{P(B)}$: prob. of event A happ. after event B.

$P(B|A) = \dfrac{P(A\cap B)}{P(A)}$: prob. of event B happ. after A.

* Multiplication Theorem of Probability

imp $P(A\cap B) = P(B)\cdot P(A/B) \Rightarrow P(A)\cdot P(B/A)$

(i) $P(A) = P(A\cap B^c) = P(A) - P(A\cap B)$ → $P(A-B)$.



(ii) $P(B) = P(B\cap A^c) = P(B) - P(A\cap B)$.

(iii) $P(A \text{ and } B) = P(A\cap B)$

(iv) $P(\text{Neither A nor B}) = 1 - P(A\cup B)$

$\Rightarrow P(A\cup B)^c$

$\rightarrow A^c\cap B^c$

imp

imp

eg → Pg 115 [Q2]

Vimp

* 2 dices are rolled. Let r denote sum.

r	2	3	4	5	6	7	8	9	10	11	12
P(X=r)	1/36	2/36	3/36	4/36	5/36	6/36	5/36	4/36	3/36	2/36	1/36

[based on]

eg → Pg 116 [(iii) & (iv)] (conditional Probab → dice Q].

* Reduced Sample Space [Imp Model]

eg → Pg 118 [Q4] ; Given that the first removed ball is white.
[Sample Space reduced]

③ BAYE'S THEOREM

Let A, B, C be 3 mutually exclusive events occurring in a SS.
E → possible in all 3 events. imp

$$P(A/E) = \frac{P(A)\cdot P(E/A)}{P(A)\cdot P(E/A) + P(B)\cdot P(E/B) + P(C)\cdot P(E/C)}$$

eg → Pg 119 [Q1] (Prob that ball came from B1)

eg → Pg 120 [Q2 & Q3] Imp

Tree

(4) DISTRIBUTIONS

imp

Random Variable : Variable which is assigned to all Poss. outcomes in expriment

(i) Discrete RV : RV $\longrightarrow$ integer value.

(ii) Continuous RV $\longrightarrow$ real values.

eg: weight of Student in class.

(1) Discrete Distributions

(i) Basic Discrete Distribution.

a) $\sum P(X) = 1$ $\longrightarrow$ Expectation of X.

b) mean $= E(X) = \sum X P(X)$.

imp

c) Variance $= V(X) = E(X^2) - [E(X)]^2$.

$\Rightarrow \sum X^2 P(X) - [\sum X P(X)]^2$

d) Stand. Deviation $= \sqrt{V(X)}$.

I) $X, Y \rightarrow$ RV.

(i) $E(aX) = aE(X)$.

(ii) $E(aX+b) = aE(X) + b$.

imp

(iii) $V(aX) = a^2 V(X)$.

(iv) $V(aX+b) = a^2 V(X) + 0$.

$\boxed{\text{eg}} \longrightarrow$ Pg 123 [Q2] (Calculate Variance).

$\boxed{\text{eg}} \longrightarrow$ Pg 124 [Q3] (Calculate E(marks)).

imp

Gate

$\frac{X}{P(X)}$

$$\boxed{(1-x)^{-2} = 1+2x+3x^2+4x^3+\ldots\infty}$$

(ii) BINOMIAL DISTRIBUTION

- Prob of obtaining r success out of n Trials.

$$\boxed{P(X=r) = {}^nC_r\, p^r q^{n-r}}$$

p → prob of success, q → prob. of failure.

$\boxed{p+q=1}$

imp

→ Mean = E(X) = np.
→ Variance = V(X) = npq.

∴ $\boxed{\text{Mean} > \text{Variance}}$

* Prob of success must remain constant from trial to trial for BD to be applied. (eg: coin/dice)

eg → Pg 127 (Q2)

atleast one :	$X \geq 1$ → $1-P(X=0)$
atmost one :	$X \leq 1$

eg → Pg 128 [Vimp]

(iii) POISSON DISTRIBUTION

Prob of obtaining r success :

imp

$$\boxed{P(X=R) = \frac{e^{-\lambda}\cdot\lambda^r}{r!}}$$

mean = $E(X) = \lambda$
variance = $V(X) = \lambda$.

$\therefore$ $\boxed{\text{mean} = \text{Variance}}$

* Prob of success very small, PD is used.

eg → Pg 130 (defective resistor) Test

(2) CONTINUOUS DISTRIBUTION

(i) BASIC CONTINUOUS DISTRIBUTION

if X = RV continuous, then continuous dist. of X =

$$\boxed{P(-\infty < x < \infty) = \int_{-\infty}^{\infty} f(x)\,dx = 1}$$

a) Mean $= E(x) = \int_{-\infty}^{\infty} x f(x)\,dx$. imp

b) Variance $= V(x) = E(x^2) - [E(x)]^2$.

$$\Rightarrow \int_{-\infty}^{\infty} x^2 f(x)\,dx - \left[\int x f(x)\,dx\right]^2$$

c) $P(x > a) = \int_{a}^{\infty} f(x)\,dx$

d) $P(a < x < b) = \int_{a}^{b} f(x)\,dx$

e) $P(x < a) = \int_{-\infty}^{a} f(x)\,dx$.

Vimp

$$\boxed{F(x) = \int_{-\infty}^{\infty} f(x)\,dx}$$

$f(x) \rightarrow$ Prob density function

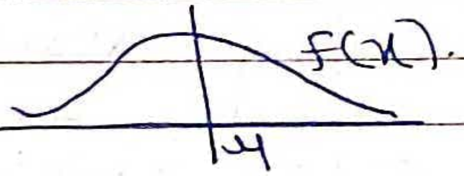
$F(x) \rightarrow$ prob. dist. function.

(iv) NORMAL DIST. / GAUSSIAN DIST

A continuous RV is said to follow ND with parameters $[\mu, \sigma^2]$

$$\boxed{f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}}$$

mean $= \mu$
Variance $= \sigma^2$



* If, $z = \frac{x-\mu}{\sigma}$; $f(z)$ = Stand. ND.

imp

Standard ND ; mean = 0
Variance = 1

$$\boxed{\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-x^2/2}\, dx = 1}$$

eg → Pg 3

(5) STATISTICS

Data → Grouped data : class Int along with f
Data → Ungrouped [No CI, only observations]

1) Mean:

$$\boxed{\text{mean}(\bar{X}) = \frac{\sum x_i f_i}{\sum f_i}}$$

$x_i \rightarrow$ midpoint of class

* Grouped Data:

imp

→ find K?

eg → Pg 133 ($f(x) = K(5x - 2x^2)$)

eg → Pg 134 [$P(0.5 < x < 5)$] ?

(ii) UNIFORM DISTRIBUTION

A continuous RV is said to follow UD in the [a, b]; if its PDF:

$$\boxed{f(x) = \frac{1}{b-a}}$$ (Prob density function).

- Mean → $E(x) = (b+a)/2$
- Variance → $V(x) = (b-a)^2/12$.

eg → Pg 135 [Q 2]

$$E(x^3) = \int_{-\infty}^{\infty} x^3 \cdot f(x) \cdot dx$$

→ $1/_{b-a}$.

imp

(iii) EXPONENTIAL DISTRIBUTION

A continuous RV, X is said to follow ED, with parameter λ, if its pdf:

$$\boxed{\begin{aligned} f(x) &= \lambda e^{-\lambda x} &; \text{ for } x \geq 0 \\ &= 0 &; \text{ otherwise} \end{aligned}}$$

imp

Mean $= 1/\lambda$.
Variance $= 1/\lambda^2$

eg → Pg 1

R ↓ consistency ↑.

eg → pg 9 [coeff. of var. $\propto \frac{1}{\text{consistency}}$]

6) REGRESSION ANALYSIS

Line of regression

1. y on x.

$$y-\bar{y} = r \cdot \underbrace{\frac{\sigma y}{\sigma x}} (x-\bar{x}).$$ (i)

$$\boxed{y-\bar{y} = b_{yx}(x-\bar{x})}$$ $$\boxed{b_{yx} = r \cdot \frac{\sigma y}{\sigma x}}$$

2. x on y

$$\boxed{x-\bar{x} = b_{xy}(y-\bar{y})}$$ $$\boxed{b_{xy} = \frac{r \cdot \sigma_x}{\sigma_y}}$$

r → coefficient of correlation.

$r = -1$: both var. are opp
$= 1$: both var are same
$= 0$: no relation.

(ii) $r = \sqrt{b_{xy} \cdot b_{yx}}$ or GM of 2 reg coeff.

(iii) Angle b/w 2 regression lines

$$\boxed{\tan\theta = \frac{1-r^2}{|r|} \frac{\sigma x \sigma y}{\sigma x^2 + \sigma y^2}}$$

$r = \pm 1$, $\theta = \pi/2$.

$\boxed{r = \pm 1}$

imp

When 2 variables are same/app to each other, regression lines are Perpendicular.

—— X —— X —— X ——

→[Mean of Grouped data]

eg → Pg 5

2) Median :

After sorting.

If n is odd ; $\boxed{\text{median} = \left(\frac{n+1}{2}\right)^{th} \text{val.}}$

If n is even,

$$\boxed{\text{median} = \frac{\left(\frac{n}{2}\right)\text{val} + \left(\frac{n}{2}+1\right)\text{val}}{2}}$$

eg → Pg 6 [Q 1]

3) MODE

Observation with highest freq / repeat max no. of times.

eg → Pg 7 [Vimp]

4) STANDARD DEVIATION.

imp

$$\boxed{\sigma = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2}}$$

- SD indicates spread of data around mean.

eg → Pg 8 [Imp].

5) COEFFICIENT OF VARIATION (R)

imp

$$\boxed{\text{Coeff of Variation} = \frac{\sigma}{\mu}}$$

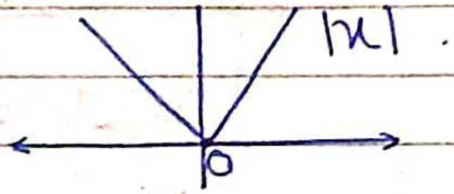
σ → SD

μ → mean.

* $\boxed{|A| = |A^T|}$

---

* Graph of $|x|$



continuous but not differentiable (at $x=0$)

---

* $P+Q = \begin{bmatrix} 0 & -1 & -2 \\ 8 & 9 & 10 \\ 8 & 8 & 8 \end{bmatrix}$ rank = ? imp

$\because$ det $= 0$; Rank < 3

consider any 2×2 minor $\begin{vmatrix} 0 & -1 \\ 8 & 9 \end{vmatrix} \neq 0$ — 9 of them

$\therefore$ $\boxed{\text{Rank} = 2}$

---

eqn: $\lambda^3 - 4\lambda^2 - 11\lambda + 30 = 0$ → 3x3

1 root → 2. [1 eigen Value = 2]

find other roots?

⇒ [By Horner's Synthetic division:]

2	1	−4	−11	30
	0	2	−4	−30
	1	−2	−15	0

$\therefore x^2 - 2x - 15 = 0$

5, −3

$\therefore$ Roots are $(2, -3, 5)$

$\boxed{\text{eg}}$ → Q 6.49 imp

if $a^3 + b^3 + c^3 = 3abc$

⇒ $\boxed{a+b+c=0}$; $\boxed{a=b=c}$

PREVIOUS YEARS

* If the rows/col. of square matrix are linearly dependent, |det| = 0.

[eg] → Q6.11 [Try Substitution to solve such Probs]
[a = 3, b = 2, c = 1] (determinant).

* Sum of 2 Non-singular matrix may be singular.
Sum of 2 singular matrix may be non-singular. (imp)

* Sum of elements of skew-symmet. matrix = 0

* By Caley-Hamilton theorem, the matrix will satisfy the charact. equation:

$$A^{-1} = \frac{1}{|A|} \text{adj}.A$$

$$\text{adj}.A = \begin{bmatrix} 1-a & -1 \\ a^2-a+1 & a \end{bmatrix} ; |A| = 1$$

[eg] → Q6.21 [inverse using CH theorm].

[rank < n]

* If M is a Square matrix with a Zero determinant. [singular matrix] each row/column of M, can be represented as a linear combination of other rows. [imp]

PROBABILITY

* 2 dice thrown.
Atleast one of them, 6 facing up.

$$\Rightarrow 1-\left(\frac{5}{6}\right)^2 \Rightarrow \frac{11}{36}$$

[None of them]

* Binomial distribution. imp

eg → 5.6, 5.14

ex. 2H in 4 tosses ⇒ 2H, 2T in 4 Tosses

eg → 6.5 [System of linear equations]

A: m×n → [m eqns, n variables]

* Saddle Point: $\frac{d^2x}{dx^2} = 0$.
→ Not an extremum [minima/maxima]

* $\lim_{x\to\infty} \frac{\sin x}{x} = 0$; $\lim_{x\to\infty} \frac{\cos x}{x} = 0$.

* If tangent = $-2x/y$.

$\frac{dy}{dx} = \frac{-2x}{y}$ (integrate ba.

$\int y\,dy = \int -2x\,dx$ imp

$\Rightarrow \frac{y^2}{2} = \frac{-2x^2}{2} + C$

$\frac{y^2}{2} + x^2 = C$ → eqn of ellipse

* imp

$$\boxed{f'(x_0) = \frac{f(x_0+h) - f(x_0)}{h} \Rightarrow \frac{f(x_0) - f(x_0-h)}{h}}$$

$x_0 = x_0 - h$

No of linearly independent rows = rank

Page No.:
Date:

2 eigen values

* If A is a real value square Symmetric matrix (rank = 2).

$$\boxed{\lambda_1^2 + \lambda_2^2 \le \sum_{i=1}^{n} \sum_{j=1}^{n} A_{ij}^2}$$

(V imp)

eg → Q6.54

* No. of non-zero eigen values <= rank of the matrix.

(imp) ∵ rank = 2, matrix can have max 2 eigen values [non-zero EV].

eg → Q6.40 [identify rank using linear independent rows]

$$\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} \xrightarrow[\text{Col Trans.}]{} \begin{vmatrix} 1 & x+1 & x(x+1) \\ 1 & y+1 & y(y+1) \\ 1 & z+1 & z(z+1) \end{vmatrix}$$

$$C_2 \to C_2 + C_1$$
$$C_3 \to C_3 + C_2$$

* A matrix with repeated eigen values may / may not be diagonaliz but if; matrix's eigen values distinct → surely diagonalized.

if dependent eigen vectors, repeated eigen values. (imp)

eg → Q6.58

//

$$\boxed{\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-x^2/2}\,dx = 1}$$

$Z = \frac{x - \mu}{\sigma}$

$\boxed{\mu = 0, \sigma = 1}$

$f(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$ (Eqn of Standard Normal Pdf).

INTEGRATION.

imp

$$\boxed{\int \cot x\,dx = \log|\sin x| + C}$$

$\int \sec x\,dx = \log|\sec x + \tan x| + C$

$\int \operatorname{cosec} x\,dx = \log|\operatorname{cosec} x - \cot x| + C$.

1) Methods of integration.

eg → Pg3 [Q1] $\int \sin^3 x \cot^2 x\,dx$

2) Integration by Parts.

$$\int \underbrace{f(x)\,g(x)}\,dx = f(x)\int g(x) - \int f'(x)\int g(x)$$

→ which functn to choose as $f(x)$ or $g(x)$.

[ILATE]

$\xleftarrow[f(x)]{}$ priority.

eg → Pg4 [Q1]

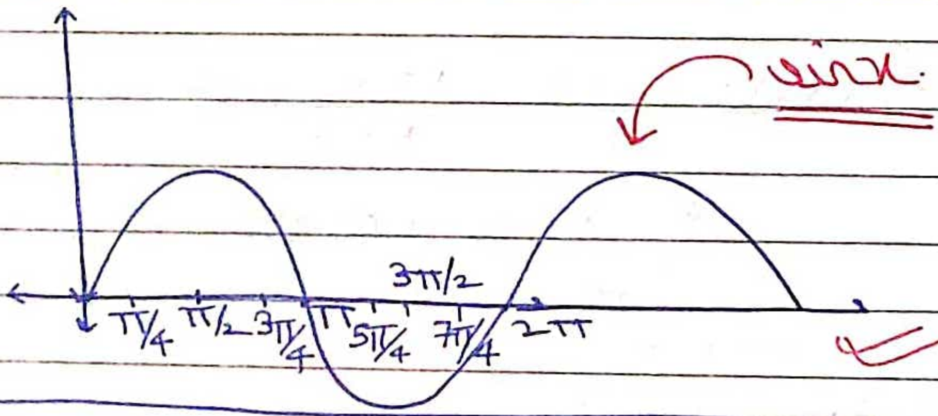
imp $\boxed{\tan^{-1}\left[\frac{2x}{1-x^2}\right] = 2\tan^{-1}x}$

* $\sin(90+\theta) = \cos\theta$
$\cos(90+\theta) = -\sin\theta$
$\tan(90+\theta) = -\cot\theta$
$\operatorname{cosec}(90+\theta) = \sec\theta$

S	A
T	C

$\sin(180-\theta) = \sin\theta$
$\cos(180-\theta) = -\cos\theta$
$\tan(180-\theta) = -\tan\theta$

$\sin(180+\theta) = -\sin\theta$
$\cos(180+\theta) = -\cos\theta$
$\tan(180+\theta) = \tan\theta$

* 

* If a cubic polynomial has 1, 2, 3 as roots:
$f(x) = \alpha(x-1)(x-2)(x-3)$

If, $x^2-13x+36=0$. $\boxed{\text{Deal}}$
coefficients in base b.
$(13)_b = b+3$ (decimal)

$\Rightarrow x^2-(b+3)x+(3b+6)=0$ (decimal)
$\because$ 5 is a root.
$\Rightarrow 25-(b+3)5+(3b+6)=0$.
$\therefore b=8$

Q. $\int_0^{\pi/2} \log(\tan x)\,dx \longrightarrow \boxed{I=0}$

eg $\longrightarrow$ 8 Pic in Cell

Q. $\int_0^1 \frac{\log(1+x)}{1+x^2}\,dx$ imp

$\hookrightarrow$ if $(1+x^2)$ term present in Q, substitute $x=\tan t$; $dx=\sec^2 t\,dt$

eg $\longrightarrow$ 9th Pic in Cell

3) $$\boxed{\int_0^{na} f(x)\,dx = n\int_0^{a} f(x)\,dx \quad \text{if: } f(a+x)=f(x)}$$ imp

4) $$\int_{-a}^{a} f(x)\,dx = \begin{cases} 2\int_0^a f(x)\,dx \;;\; \text{if } f(x)=f(-x) & [\text{even}] \\ 0 \;;\; \text{if } f(x)=-f(-x) & [\text{odd}] \end{cases}$$

eg $\longrightarrow$ Pg 11 [Q 1]

5) $$\boxed{\int_0^{a} x\cdot f(x)\,dx = \frac{a}{2}\int_0^{a} f(x)\,dx \;;\; \text{if: } f(a-x)=f(x)}$$ imp

eg $\longrightarrow$ Pg 11 [Q2]

6) $$\int_0^{\pi/2} \sin^n x\,dx = \int_0^{\pi/2} \cos^n x\,dx$$

3) Integration by Partial Fractions

[Ratio of 2 polynomials].

$\int \frac{x^2 \, dx}{(x^2+1)(x^2+4)}$ → Put $x^2 = y$ (for ease)

eg → Call [4th pic]

* Intro to Definite Integral

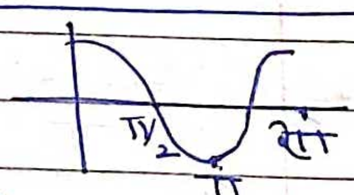
$\int_0^{\pi/2} \cos x \, dx = [\sin x]_0^{\pi/2} \Rightarrow 1$.

[Area of ~~curve~~ $\cos x$ from 0 to $\pi/2$]

1) $$\boxed{\int_a^b \frac{f(x)}{f(x)+f(a+b-x)} = \frac{b-a}{2}}$$

eg → Pg 7 (Pic 7 → Call)

* $\int_0^{\pi} |\cos x| \, dx$ →



$\Rightarrow \int_0^{\pi/2} \cos x + \int_{\pi/2}^{\pi} (-\cos x) \, dx$ ← imp

eg → Pg 8 $\int_0^n [x] \, dx$. (Step function).

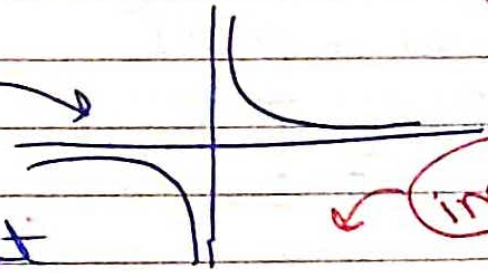
2) $$\boxed{\int_0^a f(x) \, dx = \int_0^a f(a-x) \, dx}$$ imp

FU=b, FV=b. imp |F|=0.

eg → 6.26 [See the approach of Q].

eg → 7.24

$f(x) = x^{-1/3}$ →



∴ This functⁿ is not bounded b/w [-1, 1] ∵ at 0 = ∞. imp

Now, $A = \int f(x)\,dx$.

$$\Rightarrow \left|\int_{-1}^{0} x^{-1/3}\,dx\right| + \left|\int_{0}^{1} x^{-1/3}\,dx\right|.$$

Area → always positive

eg → Q7.16 [Imp Application of Rolle's Theorem].

Given: $af(x) + bf(1/x) = 1/x - 25$.

$af(1/x) + bf(x) = x - 25$.

imp ∴ Eliminate $f(1/x)$ from these 2 equations ∴ to get $f(x) = \ldots$

* PROBABILITY.

gate

Total pdf = convolution of 2 pdfs. $f_1(t)$, $f_2(t)$.

If $f_1 = x$, $f_2 = t - x$ ∵ sequentially

$$\therefore \int_{0}^{t} f_1(x) f_2(t-x)\,dx.$$

When 2 modules are executed sequent

imp

$$\Rightarrow \left[\frac{n-1}{n}\cdot\frac{n-3}{n-2}\cdot\frac{n-5}{n-4}\cdots \tfrac{2}{3} \text{ or } \tfrac{1}{2}\right] * K.$$

$K = \pi/2$, n = even.

$K = 1$, n = odd.

eg → Pg 12 [Q 4]

7) $$\int_0^{\pi/2} \sin^m x \cos^n x = \frac{[(m-1)(m-3)\cdots 2/1] \times [(n-1)(n-3)\cdots 2/1]}{(m+n)(m+n-2)\cdots 2/1} \times K$$

$K = \pi/2$; m & n are even

$K = 1$; otherwise.

8) $$\int_0^{2a} f(x)\,dx = \begin{cases} 2\int_0^a f(x)\,dx & ; \text{ if } f(2a-x) = f(x) \\ 0 & ; \text{ if } f(2a-x) = -f(x) \end{cases}$$

imp

eg → Pg 13 [Q 3]

eg → Pg 15 [Gate 2014]

$\int_0^{\pi} x^2 \cos x\,dx$

Appl^n of:

$\int_0^a x\cdot f(x)\,dx = \frac{a}{2}\int_0^a f(x)\,dx$, if: $f(a-x) = f$

* unit eigen Vector

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \Rightarrow \sqrt{x_1^2 + x_2^2} = 1$$

$\therefore \boxed{x_1^2 + x_2^2 = 1}$

eg → 6.28 [imp] ← GO

imp

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Date

* Expected length of Shorter Stick

$l \in [0, 1/2]$ $\because$ Scale = 1m

$E(l) = \frac{0 + 1/2}{2} = 0.25m$

→ 5.31 [Gaussian dist. (Imp)] Imp

i) pdf of rv:

x	-1	$+1$
$P(X)$	0.5	0.5

cumulative df

x	-1	$+1$
$F(x)$	0.5	1

→ 5.23 [Q on Total Probability]

Imp

$$\text{}\begin{cases} \frac{1}{2}\ T \leq 25 \xrightarrow{0.4} RA \\ \frac{1}{2}\ T \geq 25 \xrightarrow{x} RA \end{cases}$$

$\frac{1}{2} \times 0.4 + \frac{1}{2} \times x = 0.6$

→ Q 5.22 (Unbiased coin tossed repeatedly until successive toss is same)

$E(X) = ?$

in AGP bring into: $2 \cdot \frac{2}{4} + 3 \cdot \frac{2}{8} + 4 \cdot \frac{2}{16} + \cdots$

imp

$\therefore \frac{2}{4}\left[2 + 3 \cdot \frac{1}{2} + 4 \cdot \frac{1}{4} + \cdots \infty\right]$

Q eg: no of LED bulbs of 2 diff types.

Prob of LED bulb > 100 hr given Type 1: 0.7.

Prob of LED > 100 given T-2 : 0.4

Prob of LED > 100 ? [like rain]

$$\text{Bulb}\begin{cases} \frac{1}{2}\ T\text{-}1 \text{ —— } 0.7 \\ \frac{1}{2}\ T\text{-}2 \text{ —— } 0.4 \end{cases}$$

2+3+   2+3+4

$\rightarrow \frac{e^{-\lambda}\cdot\lambda^{r}}{r!}$

* In Poisson's.
if 20 req ⟶ 1hr (avg).
Probability asked is in 45 min;
convert λ ∴ 15 req

[eg] ⟶ Q5.15 Prob = |favourable children| / |total no. of children|
[imp]

[eg] ⟶ Q5.40 [If 1,2,3 → rolled again, else not] (Prob ≥ 6) → Sum.
(imp) $\frac{1}{2} \times \frac{9}{18} + \frac{1}{2} \times \frac{1}{3} \rightarrow \{4,5,6\}$

In Poisson's distribution:
$E(X) = V(X) = \lambda$.
$E(X^2 + 4 + 4X) = \underbrace{E(X^2)}_{\text{(find through } V(X))} + 4E(X) + 4$.

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* Four dice rolled. (imp)
Sum of results = 22, Prob = ?
6 6 6 4   6 6 5 5 → $\frac{4!}{2!2!}$ } Total ways.

[eg] ⟶ 5.43 (ME soln)

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* If E(X) = 5, there is a sample point at which X ≥ 5. (imp)
∵ if all values < 5, E(X) < 5

[eg] ⟶ 5.44
(imp)

10
4W   6NW
4
3W   1NW
4W   0NW
} tree.