$E = {}^{16}C_2$

(a) $P(\text{both articles are good}) = \dfrac{{}^{10}C_2}{{}^{16}C_2}$

(b) $P(\text{both have major defects}) = \dfrac{{}^{2}C_2}{{}^{16}C_2} = \dfrac{1}{{}^{16}C_2}$

(c) $P(\text{atleast one is good}) = \dfrac{{}^{10}C_1\, {}^{6}C_1 + {}^{10}C_2}{{}^{16}C_2}$

(d) $P(\text{atmost one is good}) = \dfrac{{}^{10}C_1 \times {}^{6}C_1 + {}^{6}C_2}{{}^{16}C_2}$

(e) $P(\text{exactly one is good}) = \dfrac{{}^{10}C_1 \times {}^{6}C_1}{{}^{16}C_2}$

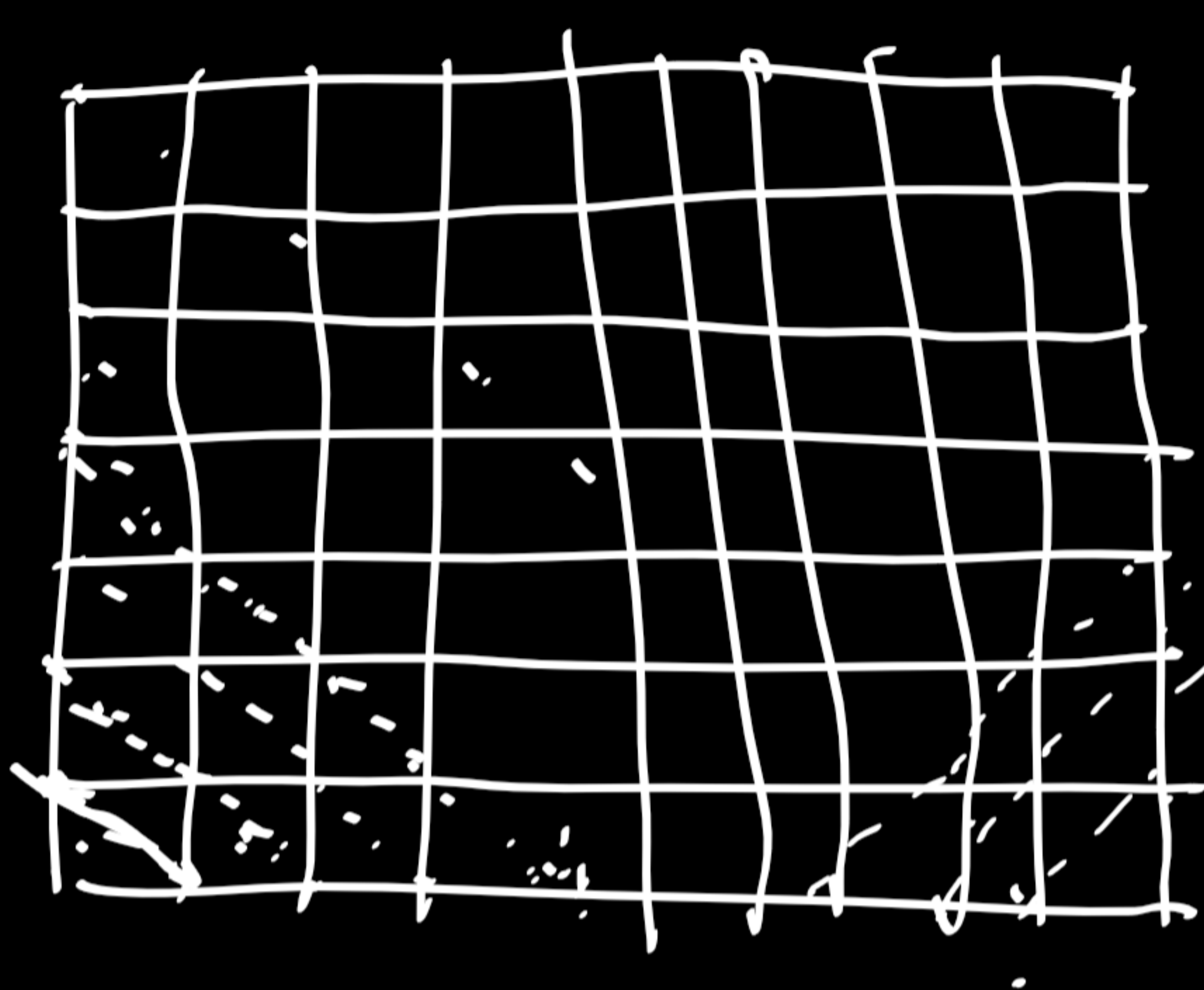
(f) $P(\text{neither has major defects}) = \dfrac{{}^{14}C_2}{{}^{16}C_2}$

(g) $P(\text{neither is good}) = \dfrac{{}^{6}C_2}{{}^{16}C_2}$

5) If four squares are selected at random on a chess board, find the probability that they should be in a diagonal line.

Exhaustive cases = $^{64}C_4$

$A = \{$squares should be in a diagonal line$\}$



$F = 2\left({}^4C_4 + {}^5C_4 + {}^6C_4 + {}^7C_4 + {}^8C_4 + {}^7C_4 + {}^6C_4 + {}^5C_4 + {}^4C_4\right)$

$P(A) = \frac{F}{E} =$

6) A lot consists of 10 good articles, 4 articles with minor defects and 2 articles with major defects. If two articles are selected from the lot then find the probability that

(a) both are good (b) both have major defects

(c) atleast one is good (d) at most one is good

(e) exactly one is good (f) neither has major defects

(g) neither is good.

4) Suppose that four digits 1, 2, 3 and 4 written down in random order. What is the probability that atleast one digit will occupy its proper place?

$$P\left(\begin{array}{c}\text{atleast one digit occupies} \\ \text{its proper place}\end{array}\right) = 1 - P\left(\begin{array}{c}\text{None of the} \\ \text{digits occupies its} \\ \text{proper place}\end{array}\right)$$

$$\hat{E} = {}^4P_4 = 4! = 24$$

<u>None of the digits</u> occupy its proper place

2	1	4	3	3	1	4	2	4	1	2	3
2	3	4	1	3	4	2	1	4	3	2	1
2	4	1	3	3	4	1	2	4	3	1	2

$$\therefore P\left(\begin{array}{c}\text{atleast one digit occupies its} \\ \text{proper place}\end{array}\right) = 1 - \frac{9}{24} = \underline{\underline{5/8}}$$

(b) Let $B = \{$ greatest badge no: is 5 $\}$

1, 2, 3, 4

5

$F = {}^4C_2$

$$P(B) = \frac{{}^4C_2}{{}^{10}C_3} = 1/20$$

3) What is the probability that in a group of 7 persons

(a) no two were born on the same day of the week.

(b) atleast two were born on the same day of the week.

(c) two were born on sunday and 2 on wednesday.

$F = 7^7$

1 2

7 6

sun

7. 6. 5

(a) $P\left(\text{No two were born on the same day}\right) = \dfrac{7!}{7^7}$

(b) $P\left(\text{atleast two were born on the same day}\right) = 1 - \dfrac{7!}{7^7}$

(C) $P\left(\text{Two were born on Sunday and two on wednesday}\right)$

$$= \frac{{}^7C_2 \times {}^5C_2 \times 5^3}{7^7}$$

1 2 3. 4. 5. 6. 7

Sun Mon Tu Wed Th Fr Sat

5, 6, 7

5 × 5 × 5

2) A class contains 10 boys and 20 girls, of which half the boys and half the girls have brown eyes. Find the probability that the person selected at random is a boy or has brown eyes.

Let $A = \{\text{Selected person is a boy}\}$

$B = \{\text{Selected person has brown eyes}\}$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{10}{30} + \frac{15}{30} - \frac{5}{30} = 2/3$$

3) Ten persons in a room wearing badges are marked 1, 2, ..., 10. Three persons are selected at random and asked to leave the room simultaneously. Their badge no: is noted.

(a) What is the probability that the smallest badge number is 5?

(b) What is the probability that the largest badge no: is 5?

1, 2, 3, 4, 5, $\underbrace{6, 7, 8, 9, 10}$

Exhaustive cases $= {}^{10}C_3$

(a) $A = \{\text{smallest badge no: is } 5\}$

5

$F = {}^5C_2$

$$P(A) = \frac{{}^5C_2}{{}^{10}C_3} = 1/12$$

(b) Let $B = \{\text{sum } 7\}$ and $C = \{\text{sum is } 11\}$

$$P(\text{sum is neither } 7 \text{ nor } 11) = P(\bar{B} \cap \bar{C})$$

$$= P(\overline{B \cup C})$$

$$= 1 - P(B \cup C)$$

$$= 1 - \{P(B) + P(C) - P(B \cap C)\}$$

$$= 1 - \left\{\frac{6}{36} + \frac{2}{36} - 0\right\}$$

$$1 - \frac{8}{36} = \underline{\underline{7/9}}$$

4) Combination (Selection)

Combination is the selection of r objects from n objects.

$$^{n}C_r = \frac{n!}{(n-r)!\, r!}$$

Example

1) If two dice are thrown. What is the probability that

(a) Sum is greater than 8

(b) Sum is neither 7 nor 11

$$S = \left\{ \begin{array}{l} (1,1), (1,2) \ldots (1,6) \\ (2,1), (2,2) \ldots (2,6) \\ \vdots \\ (6,1) \ldots (6,6) \end{array} \right\}$$

Exhaustive cases $E = 36$

(a) $A = \{\text{Sum is greater than } 8\}$

$$= \left\{ \begin{array}{l} (3,6), (4,5), (5,4), (4,6), (6,4) \\ (5,5), (5,6), (6,5), (6,6), (6,3) \end{array} \right\}$$

$f = 10$

$$P(\text{Sum} > 8) = P(A) = \frac{10}{36} = 5/18$$

Methods of Enumeration

1) Multiplication Principle.

Suppose a procedure A can be performed in n_1 ways and a second procedure B can be performed in n_2 ways. Also suppose that each way of doing A is followed by any way of doing B. Then the procedure consisiting of A followed by B can be performed in $n_1 \times n_2$ ways.

2) Addition Principle

Suppose a procedure A can be performed in n_1 ways and a second procedure B can be performed in n_2 ways. Then the no: of ways in which we can perform either A or B is $n_1 + n_2$

3) Permutation (Arrangement)

Permutation is arrangement of n objects taken r at a time.

$$^nP_r = \frac{n!}{(n-r)!}$$