iii) Shift intersection pt of $\omega = 1$ & 0 dB on 20 log k and draw parallel to -20 dB/dec line. This will continue till first crossover freq i.e $\omega_{c_1} = 2$

iv) At $\omega_{c_1} = 2$, as there is simple pole it will contribute -20 dB/dec. Hence resultant slope after $\omega_{c_1} = 2$ becomes $-20-20 = -40$ dB/dec. This will continue till it intersects next crossover frequency.

i.e $\omega_{c_2} = 20$.

v) At $\omega_{c_2} = 20$, there is a simple pole contributing -20 dB/dec and resulting slope after $\omega_{c_2} = 20$ becomes $-40-20 = -60$ dB/dec. This is resultant of overall G(s)·H(s). The final slope.

vi) Phase angle plot.

Rep. of absence of zeroes

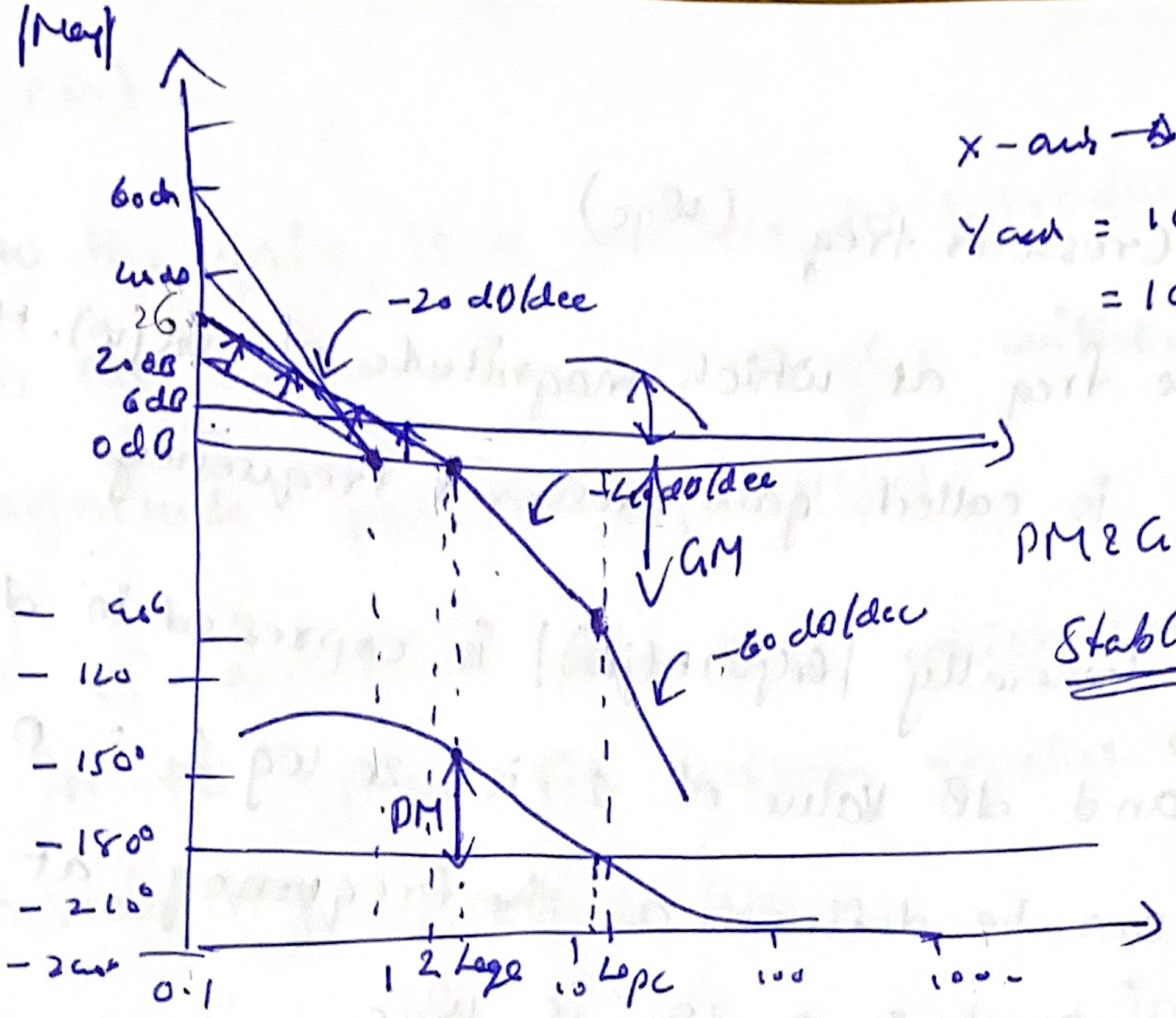
$$G(j\omega)\cdot H(j\omega) = \frac{1}{j\omega\left(1+\frac{j\omega}{2}\right)\left(1+\frac{j\omega}{20}\right)}$$

Consider K during taking angles.

$$\angle G(j\omega)\cdot H(j\omega) = \frac{\angle 2+j0}{\angle j\omega \; \angle 1+j\omega/2 \; \angle 1+j\omega/20}$$

$\angle 2+j0 = 0°$ $\quad \frac{1}{\angle j\omega} = -90°$ $\quad \frac{1}{\angle 1+j\omega/2} = -\tan^{-1}\frac{\omega}{2}$

$$\frac{1}{\angle 1+j\omega/20} = -\tan^{-1}\frac{\omega}{20}$$



X-axis → ω
Y axis = 1 CM = 20dB
= 1 CM = 30°

PM & GM
Stable

ω	$1/j\omega$	$-\tan^{-1}\omega/2$	$-\tan^{-1}\omega/20$	Total
0.2	$-90°$	$-5.7°$		
	$-90°$			
∞	$-90°$			

$\phi = -\tan^{-1}\frac{\text{imag}}{\text{real}}$

-20
-20

14/5/22

Gain Crossover freq (ω_{gc})

The freq at which magnitude of $G(j\omega) \cdot H(j\omega)$ is unity i.e 1 is called gain crossover frequency.

Generally $|G(j\omega) H(j\omega)|$ is expressed in dB

and dB value of 1 is $20 \log 1 = 0 \, dB$

It can be defined as the frequency at which $|G(j\omega) \cdot H(j\omega)|$ is 0 dB is ω_{gc}

Phase crossover freq (ω_{pc})

The freq at which phase angle of $G(j\omega) \cdot H(j\omega)$ is -180° is called ω_{pc}.

Gain Margin (GM)

As seen earlier in the root locus, as gain k is increased the system stability reduces and for certain value of k it becomes marginally stable.

So gain margin is defined as the margin in gain allowable by which gain can be increased till system reaches on the verge of instability.

The +ve GM means such increase in k is possible before system become unstable, Hence system is stable

& -ve GM means k is greater than k_{max} and system is unstable. So, K is required to be reduced to make the system stable

Phase Margin (PM)

Similar to the gain, it is possible to introduce phase line in the system i.e -ve angles without affecting magnitude plot of $G(j\omega).H(j\omega)$.

The amount of additional phase line which can be introduced in the system till system reaches on the verge of instability is called PM.

+ve PM means, such -ve angle introduced in the system is possible before system become unstable. Such system is called stable system.

-ve PM means present -ve phase line should be changed by adding +ve angle. Hence PM is said to be -ve and system is unstable.

23/5/22

Factor 4: Quadratic factors

$$G(s).H(s) = \frac{1}{1 + \frac{2\xi}{\omega_n} s + \frac{s^2}{\omega_n^2}}$$

$$G(j\omega).H(j\omega) = \frac{1}{1 + 2\xi j\left(\frac{\omega}{\omega_n}\right) + \left(\frac{j\omega}{\omega_n}\right)^2}$$

where ω is variable & ω_n is constant for that factor.

$$= \frac{1}{1 + 2\xi j\left(\frac{\omega}{\omega_n}\right) - \left(\frac{\omega}{\omega_n}\right)^2} \quad \because j^2 = -1$$

$$= \frac{1}{\left\{1 - \left(\frac{\omega}{\omega_n}\right)^2\right\} + j2\xi\left(\frac{\omega}{\omega_n}\right)}$$