Standard normal variate:

$X \sim N(0,1)$ i.e mean $=0$

variance $=1$.

Sub $\mu = 0$ $\sigma = 1$ in ①

we get $\phi(x) = \frac{1}{\sqrt{2\pi}} \cdot e^{-\frac{x^2}{2}}$.

The cdf of standard normal variate is given by

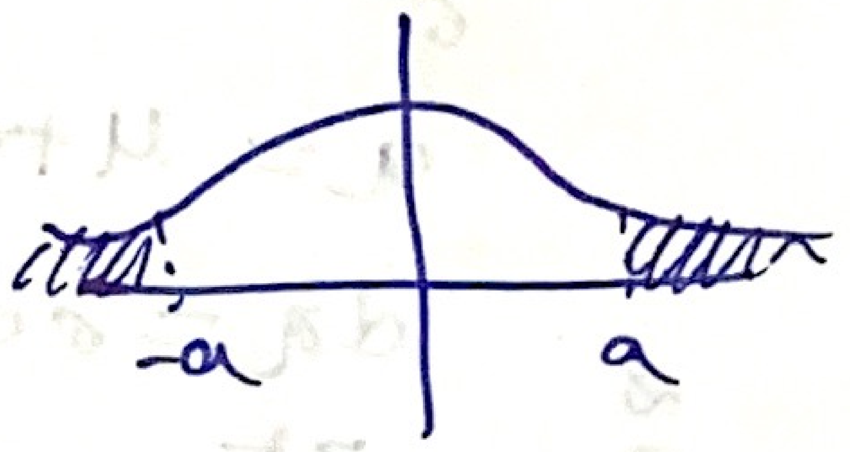
$\phi(a) = P(z \le a)$

$\phi(-a) = P(z \le -a)$

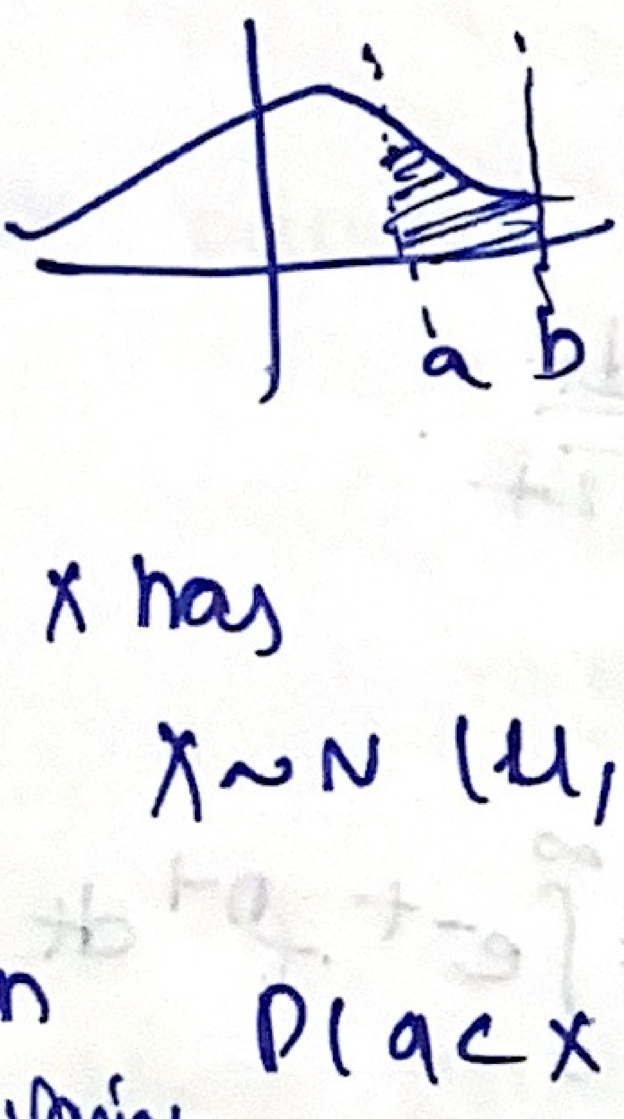
$= P(z > a)$

$= 1 - P(z \le a)$

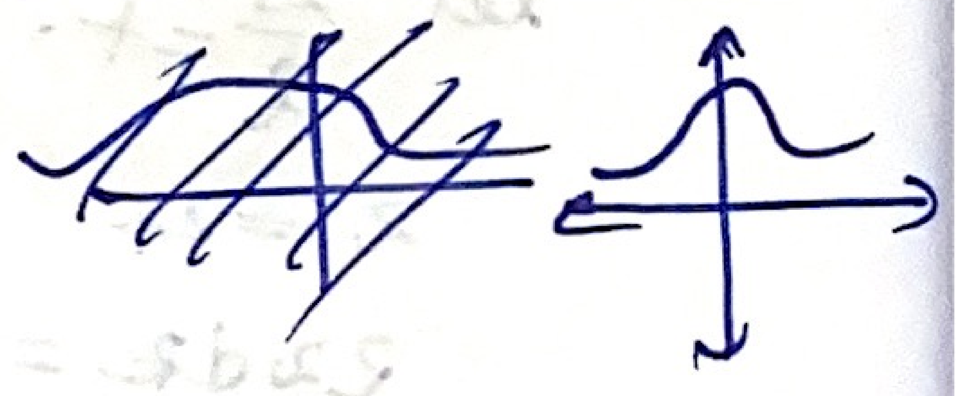
$\phi(-a) = 1 - \phi a$



$P(a < z < b) = \phi(b) - \phi(a)$



Graph of normal distribution is symetrical abt x-axis



If X has

$X \sim N(\mu, \sigma^2)$ then, $z = \frac{X-\mu}{\sigma} \sim N(0,1)$

then $P(a < x < b) = P\left(\frac{a-\mu}{\sigma} < z < \frac{b-\mu}{\sigma}\right)$

$= \phi\left(\frac{b-\mu}{\sigma}\right) - \phi\left(\frac{a-\mu}{\sigma}\right)$

normal variat

Paper 2M

27/5/22

Binomial Poission 3M

Q) Suppose the 'x' has normal distribution with Mean 75 and variance 100 find (i) $P(X < 60)$

(ii) $P(70 < X < 100)$.

Ans)

Let $z = \frac{X-\mu}{\sigma} \sim N(0,1)$ → 0.5 Mars ★

$\mu = 75$, $\sigma = 10$.

$z = \frac{X-75}{10} \sim N(0,1)$

(i) $P(X<60) = P\left(z < \frac{60-75}{10}\right).$

$= P(z < 1.5)$

$= \phi(-1.5)$

$= 1 - \phi(1.5)$

$= 1 - 0.9332.$

$= 0.0668.$

(ii) $P(70<X<100) = P\left(\frac{70-75}{10} < z < \frac{100-75}{10}\right)$

$= P\left(-\frac{1}{2} < z < 2.5\right). = P(-0.5 < z < 2.5).$

$= \phi(v) - \phi(a)$

$= \phi(2.5) - \phi(-0.5)$

$= \phi(2.5) - (1 - \phi(0.5)).$

$= 0.9938 - 1 + 0.6915$

$= 0.6853.$

Q) It x has Normaldistribution $X \sim N(1,4)$.
Find $P(|x|>4)$.

Ans) $P(|X|>4) = 1 - P(|X| \le 4)$

$= 1 - P(-4 \le X \le 4).$

$z = \frac{X-\mu}{\sigma} = \frac{X-1}{2} \sim N(0,1).$

$P\left(\frac{-4-1}{2} \le z \le \frac{4-1}{2}\right) = P\left(\frac{-5}{2} \le z \le \frac{3}{2}\right)$

$= P(-2.5 \le z \le 1.5)$

$= 1 - (\phi(1.5) - \phi(-2.5))$

$= \cancel{1} - (\phi(1.5) - \cancel{1} + \phi(2.5))$

$= \phi(2.5) - \phi(1.5)$

$= 0.9938 - 0.9332$

$= 0.0606.$

Q) If $X \sim N(75, 25)$
find $P(X > 80 \mid X > 77)$

Ans) $\dfrac{P\{(X>80) \cap (X>77)\}}{P(X>77)} = \dfrac{P(X>80)}{P(X>77)}$.

$= \dfrac{1-P(X \le 80)}{1-P(X \le 77)}$. $= \dfrac{1-P\left(Z \le \frac{80-75}{5}\right)}{1-P\left(Z \le \frac{77-75}{5}\right)}$

$= \dfrac{1-P(Z \le 1)}{1-P(Z \le 0.4)} = \dfrac{1-\phi(1)}{1-\phi(0.4)}$

$= \dfrac{1-0.8413}{1-0.6554} \approx 0.460533.$

Q) In the normal distribution 31% of items are under 45 & 8% are over 64. find the Mean and Standard deviation of distribution.

A) $31\% \rightarrow \downarrow 45$

$8\% \rightarrow \uparrow 64$

$P(X < 45) = 0.31$

$P(X > 64) = 0.8$

$X \sim N(\mu, \sigma^2)$

Let $Z = \dfrac{X-\mu}{\sigma} \sim N(0,1)$

$P(X < 45) = P\left(Z < \dfrac{45-\mu}{\sigma}\right) = 0.31$

$\phi\left(\dfrac{45-\mu}{\sigma}\right) = 0.31$

$\phi\left(\dfrac{\mu-45}{\sigma}\right) = 1-0.31$

$\phi\left(\dfrac{\mu-45}{\sigma}\right) = 0.69$

$\dfrac{\mu-45}{\sigma} = 0.5$

$\mu - 0.5\sigma = 45 \rightarrow ①$

$\phi\left(\dfrac{64-\mu}{\sigma}\right) = 0.92$

$\dfrac{64-\mu}{\sigma} = 1.41$

$64 - \mu - 1.41\sigma = 0 \rightarrow ②$

Q) The height of 500 soliders are found to have normal distribution of them 258 are found to be within 2 cm of mean height is 170 cm. Find the standard deviation of height.

A) Let,

$X = $ Height

$X \sim N(170, \sigma^2)$.

$$P(170-2 \le X \le 170+2) = \frac{258}{500}$$

$$P(168 \le X \le 172) = 0.516.$$

$$P\left(\frac{168-170}{\sigma} \le Z \le \frac{172-170}{\sigma}\right) = 0.516$$

$$P\left(-\frac{2}{\sigma} \le Z \le \frac{2}{\sigma}\right) = 0.516$$

$$\phi\left(\frac{2}{\sigma}\right) - \phi\left(-\frac{2}{\sigma}\right) = 0.516$$

$$\phi\left(\frac{2}{\sigma}\right) - 1 + \phi\left(\frac{2}{\sigma}\right) = 0.516$$

$$2\phi\left(\frac{2}{\sigma}\right) = 1.516$$

$$2\phi\left(\frac{2}{\sigma}\right) = \phi(0.7)$$

$$\sigma = \frac{2}{0.7} = 2.85$$

HW

Q1) If $X \sim N(0,1)$, find $E(X^2)$ and $V(X^2)$.

Q2) Suppose that temp is normally distributed with mean 50 and variance 4. What is probality that temp lies b/w 48°c to 53°c

Q3) $X \sim N(\mu, \sigma^2)$. Find c as a function of μ and σ. Such that $P(X<C) = 2P(X>C)$.

Q4) Suppose that life lengths of 2 electronic devices. say d_1 and d_2 have distributions $N(40,36)$ & $N(45,9)$ respectively. If the device is to be used for atleast 45 hrs. which device use to be prefer

Exponential Distribution:

A CRV 'X' is said to have exponential distribution with parameter $\alpha > 0$ if its pdf is given by $\alpha e^{-\alpha x}$.

$$f(x) = \begin{cases} \alpha e^{-\alpha x}; & x > 0, \alpha > 0. \\ 0; & \text{elsewhere} \end{cases}$$

Mean and variance:

Mean $E(X) = \frac{1}{\alpha}$ variance $V(X) = \frac{1}{\alpha^2}$

Gamma distribution:

$$f(x) = \begin{cases} \frac{\alpha}{\Gamma(\gamma)} e^{-\alpha x} (\alpha x)^{\gamma - 1} & x > 0, \gamma > 0, \alpha > 0. \\ 0, & \text{elsewhere} \end{cases}$$

Mean and variance:

$E(X) = \frac{\gamma}{\alpha}$ $V(X) = \frac{\gamma}{\alpha^2}$

Chi-square distribution:

$$f(x) = \frac{1}{2^{n/2} \Gamma(n/2)} e^{-x/2} x^{\frac{n}{2} - 1}, \quad x > 0, n > 0$$

$$X \sim \chi^2(n).$$

Mean and variance:

$E(X) = n, \; V(X) = 2n.$