Engineering Economics

Decision making
1. Define the problem
2. Decision criteron
3. Allocate weight to criterons.
4. Develop alternatives.
5. Analyze (comparing
6. Selection /Implementation
7. Feedback [Review the decision making process]

Classification of economics

1) Micro economics : study of economics in smaller scale (Profit loss, growth, production rate) Ex: industry, business.
2) Macro economics : Larger scale (GDP, unemployment)

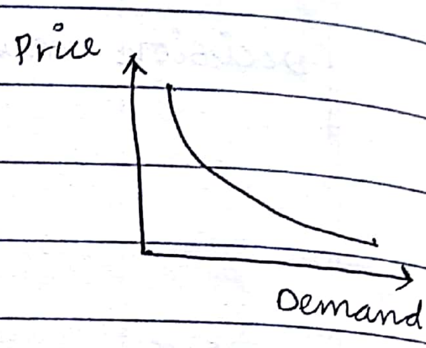
* Macro influences micro economics.

Factors of production

1. Land - industry space and raw material.
2. Labour
3. Capital
   - Real capital (assests (building))
   - Financial capital (money)
4. Enterpreneurship.

Demand : Need + purchasing power

Law of demand :



$$\text{demand} \propto \frac{1}{\text{Price}}$$

Determinants of demand :

1. Cost of the product : cost $\propto$ 1/demand.
2. Income of consumer : Income $\propto$ demand.
3. Prices of related goods :

Related goods — Substitutes (diff^{t} products available)
— Complements (accessories with the products)

4. Taste and preferences
5. Advertisements.

Exceptions to the law of demand :

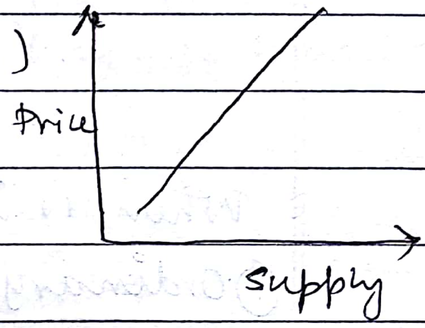
1. Emergency/ Necessary good (Medicine, petrol/diesel, oil)
demand ↑, price ↑.
2. Status symbol (Gold, luxury cars, land)
3. Expectation – Price of product (If consumer expects price to ↑, demand ↑)
└ Income (If consumer expects his income to ↑, demand may ↑ or ↓)
4. Gefens goods / Inferior goods.

When high priced goods demand ↑s, which is the demand of other goods (inferior goods).

Supply: Production & transportation market.

Law of supply:

Price $\propto$ supply (↑ profit)



Determinants of supply:
1. Cost of production
2. Resources available
3. Technology used.

* Dynamics of demand and supply.

Time value of money

Money has time value → it can earn money over time (earning power)
→ its purchasing power changes over time (inflation)

* Time value is measured in terms of interest rate.
* Interest is cost of money - a cost to the borrower & an earning to the lender.
* Relationship between interest & time leads to concept of time value.

Interest: Rental amount charged by financial institutions for use of money.

Simple Interest: Charging an interest rate only to an principal amount.

$$SI = PiN/100$$

$$F = P(1+iN)$$

When N is not a full year, SI is calculated.

(1) Ordinary SI: Year is divided into 12 months having 30 days and year is 360 (12×30) days.

(2) Exact SI: Exactly calendar no. of days N → fraction of days the loan with its effect in that year.

(1) Future sum of money to be paid, if P=₹1000, N = 2 months, i = 10%. (31, 28 days)

Use both methods -

$$SI = PNi/100$$

$$= 1000 \times 10 \times \left(\frac{30+30}{360}\right)/100$$

$$= ₹16.66$$

$$F = P + SI = ₹1016.67$$

Exact SI: $F = P(1+iN)$

$$= 1000\left(1 + 0.1\left(\frac{31+28}{365}\right)\right)$$

$$F = ₹1016.16$$

* For loan → exact
For deposit → ordinary.

Compound Interest

Charging interest rate to initial sum & to any previously accumulated interest that has been withdr-awn.

* Principal changes every year.

$$F = P(1+i)^n$$

Year	Principal (P)	Interest (i%)	Compound Amount
1	P	$P \times i$	$P + P \times i$
2	$P(i+1)$	$P(i+1) \times i$	$P(1+i)^2$
n	$P(1+i)^{n-1}$	$P(1+i)^{n-1} \times i$	$P(1+i)^n$.

① Find compound amount if ₹5000 is charged $i = 18\%$. after 5 yrs.

$$F = P(1+i)^n = 5000(1+0.18)^5 = ₹\,11438.78.$$

Time value equivalence

* Equal deposit, deposit < total money to be withdrawn
* Loan ~~D~~

Cash flow diagram:

Pictorial representation of receipt & payments associated with economic situations.

steps to construct cash flow diagram:

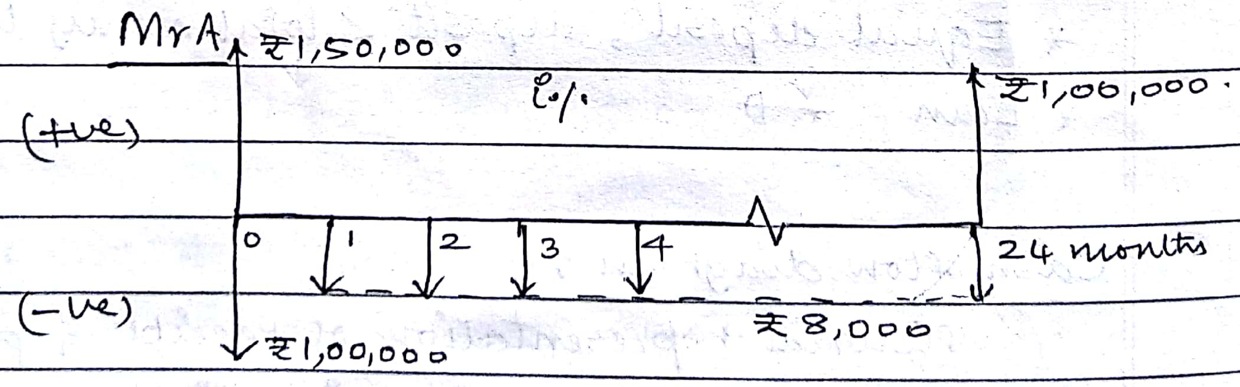
① Define time frame over which cash flows occur.

② Establish a horizontal scale which is divided into periods usually years.

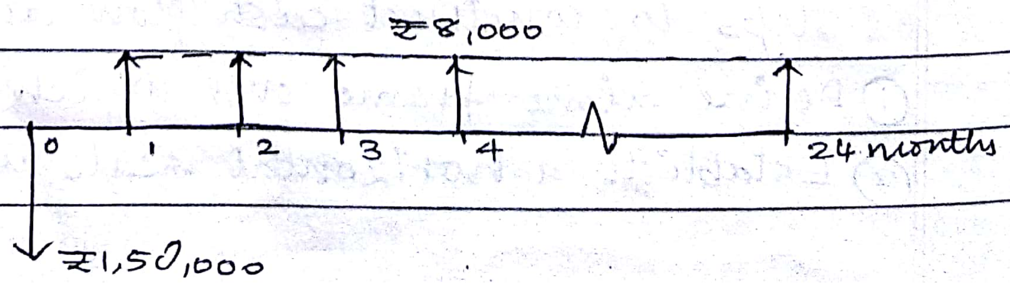
(3) Receipts & payments are then located on the time frame as follows

* All receipts are represented on +ve side.
* All payments are represented on -ve side.
* Magnitude of receipts & payments are represented by varying the heights of lines.

(1) Mr A buys a car by making a down payment of ₹1,00,000 & taking a loan of ₹1,50,000 from a bank. He makes equal monthly repayments of ₹8,000 to the bank to clear the loan in full for a period of 2yrs. After making the last payment he sells the car for ₹1,00,000. Draw 2 cashflow diagram, one for Mr A & another for bank using above cashflows.



Bank:

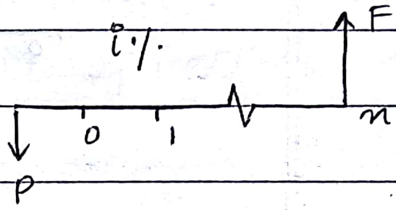


① Single payment Series Compound Amount

Ex: F

Cashflow diagram

CP is known, F is unknown



$$F = P(1+i)^n$$

$$F = P \times \boxed{(F/P, i, n)}$$

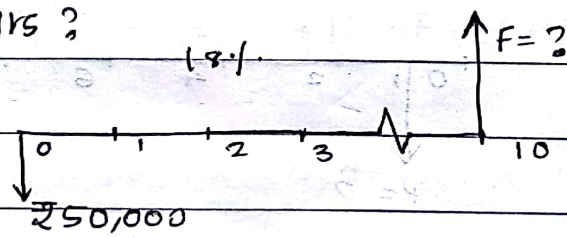
→ Factor; P: Known; F/P: objective; i: Interest rate

If an amount P is invested now and earns at the rate of i% / yr, the total amount accumulated after n yrs,

$$F = P(1+i)^n$$

$$F = P \times (F/P, i, n)$$

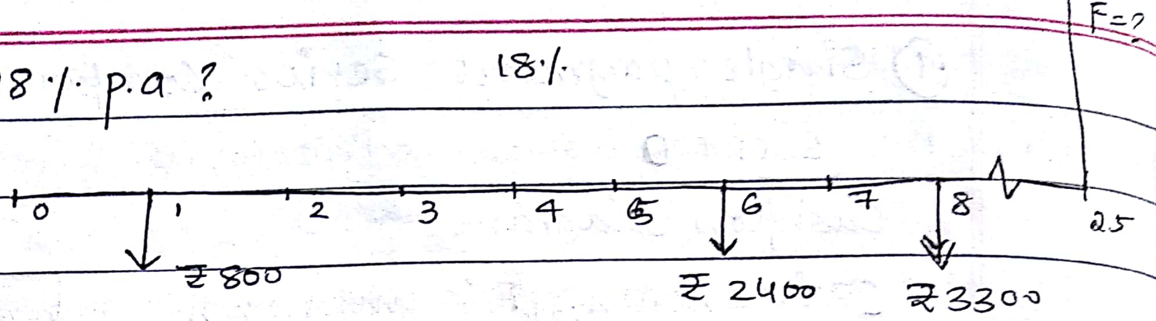
① A person deposits a sum of ₹50,000 at an interest rate of 18% p.a for a period of 10 yrs. Find maturity value after 10 yrs?



$$F = P(F/P, i, n)$$

$$= 50000 \times 5.234 = \underline{₹2,61,700}.$$

② How much money will be accumulated in 25 yrs if ₹800 is deposited one year from now ₹2400 6yrs from now & ₹3300 8yrs from now all at an interest rate of 18% p.a ?

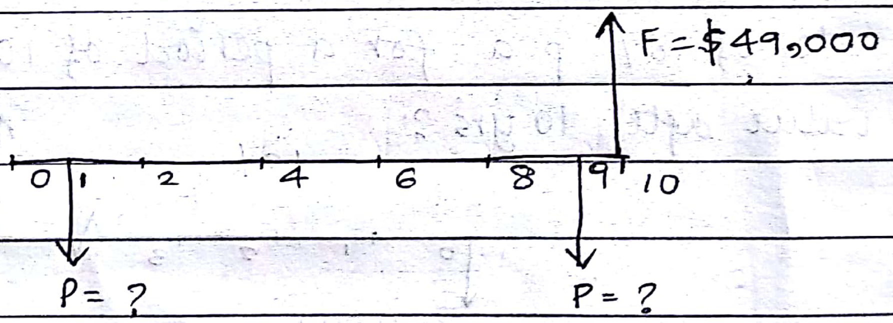


$$F = 800 \times (F/P, i, 24) + 2400\,(F/P, 18, 19) + 3300\,(F/P, 18, 17)$$

$$= 800 \times 53.109 + 2400 \times 23.214 + 3300 \times 16.672$$

$$F = ₹153218.4.$$

③ A company is planning to make 2 equal deposits such that 10yrs from now the company will have \$49,000 to make an investment. If 1st deposit is made at 1 yr from now & 2nd 9 yr from now, how much should be deposited each time at 15% p.a ?



$$F = P\,(F/P, 15, 9) + P\,(F/P, 15, 1)$$

$$49000 = P \times 3.518 + P \times 1.150$$

$$P = \$10497.$$

④ IDBI came out with an issue of deep discount bond in the year 1998. The bonds were offered at a deep discounted price of ₹ 12750. Maturity period for the bond was 30yrs with maturity value of ₹ 5,00,000. Determine the rate of return on the investment.

$F = ₹5,00,000$

$P = ₹12750$

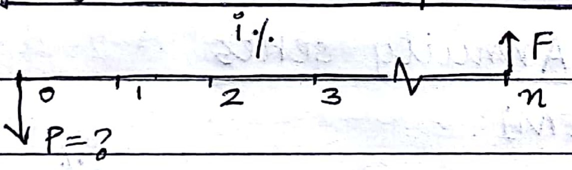
$n = 30 yrs$.

$F = P(1+i)^n$

$50000 = ₹ 12750 (1+i)^{30}$

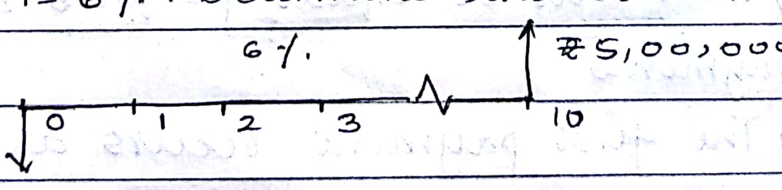
$i = 13\%$

② Single payment series present worth



$P = F/(1+i)^n$ $\qquad$ $P = F \times (P/F, i, n)$

① A person wishes to have a sum of ₹5,00,000 10 yrs from now to make a small Investment. If he plans to deposit a lumpsome amount which will fetch an i=6%. Determine the sum.

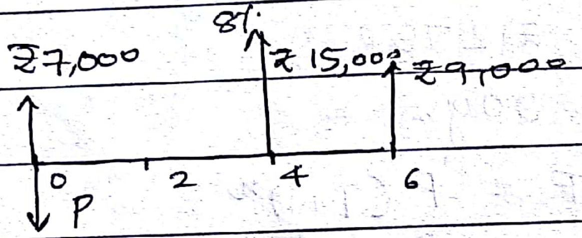


$P = F(P/F, 6\%, 10)$

$= 5,00,000 \times 0.5584 = ₹2,79,200$

* Count the number of spaces to calculate present worth & future worth.

② What is the present worth of ₹7,000 now, ₹15,000 4 yrs from now & ₹9,000 6yrs from now all at 8% p.a?



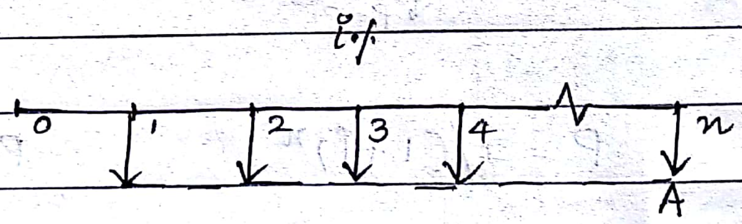
$$P = 7000 + 15000 \times (P/F, 8\%, 4) + 9000 \times (P/F, 8\%, 6)$$

$$= 7000 + 15000 \times 0.7350 + 9000 \times 0.6302$$

$$P = ₹\ 23,698.8$$

Annuity series

Ex: RD, EMI.



Characteristics of an annuity series:

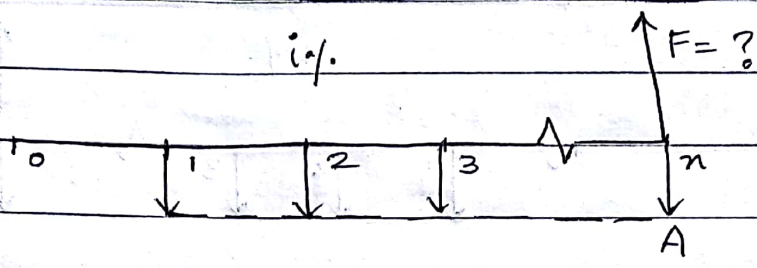
① Magnitude of the payments remain the same & is represented by A.

② There will be equal periods b/w 2 successive payments.

③ The first payment occurs at the end of first period.

(Uniform)

③ Equal payment series compound interest



$$F = A(1+i)^{N-1} + A(1+i)^{N-2} + \dots$$

$$F = A\left[(1+i)^{N-1} + (1+i)^{N-2} + \dots\right] \quad \text{—— ①}$$

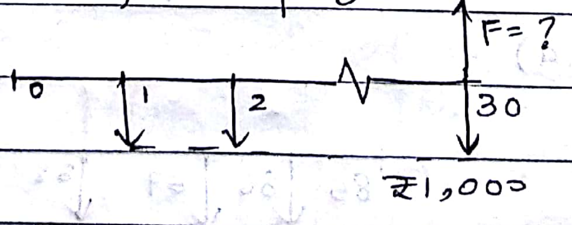
Multiply ① by $(i+i)$ on both sides,

$$F(i+1) = A\left[(1+i)^{N} + (1+i)^{N-1} + \dots\right] \quad \text{—— ②}$$

$$\boxed{F = A\,\frac{\left[(1+i)^N - 1\right]}{i}}$$

$$F = A\,(F/A, i, N)$$

① If annual deposits of ₹1,000 are made into its saving accounts for 30 years begining 1 year from now. How much will be in fund immediately after last payment, if fund pays an interest of 19.25% p.a?

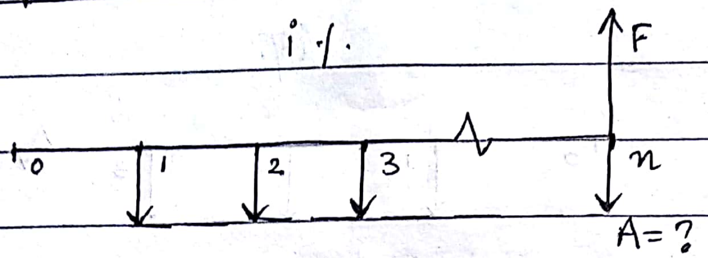


$$F = A\left[\frac{(1+i)^N - 1}{i}\right] = 1000 \times \left[\frac{(0.1925+1)^{30} - 1}{0.1925}\right]$$

$$F = ₹\,10,16,499.32$$

* To arrive at n in annuity, count the no. of transactions.

(4) Equal payment series sinking fund

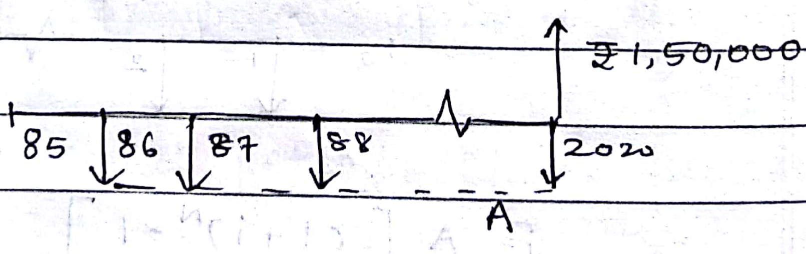


$$A = F\left[\frac{i}{(1+i)^n - 1}\right]$$

$$A = F\,[A/F, i, N]$$

* ~~To arr~~ This factor is used to determine a series of equal payment A occurs at the end of each of n interest periods that are equivalent to known single future amount F.

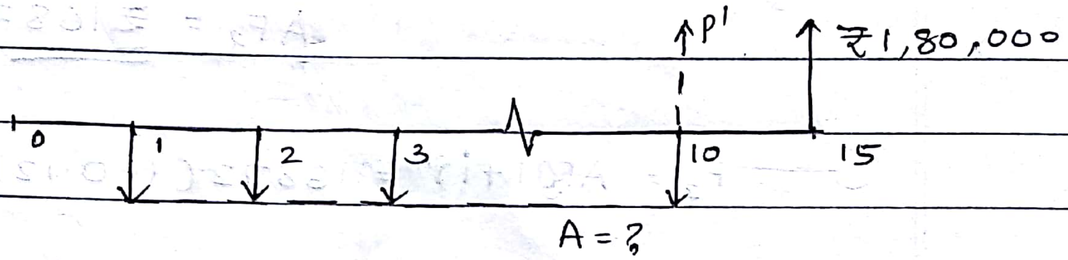
(1) How much money Mr A must deposit into a savings account each year starting from 1986 if he wants to have \$1,50,000 in the year 2020? (16% p.a)



$$A = F \times \left[\frac{(i)}{(1+i)^n - 1}\right] = 150000\left[\frac{0.16}{(1+0.16)^{35} - 1}\right]$$

$$A = \$133.84$$

(2) Determine the equal amount ~~of~~ that would have to be deposited at the end of each year for 10 years starting from 1^{st} yr into a fund in order to have ₹1,80,000 at the end of 15^{th} year at 14% p.a ?



$$P' = F / (i+1)^n$$

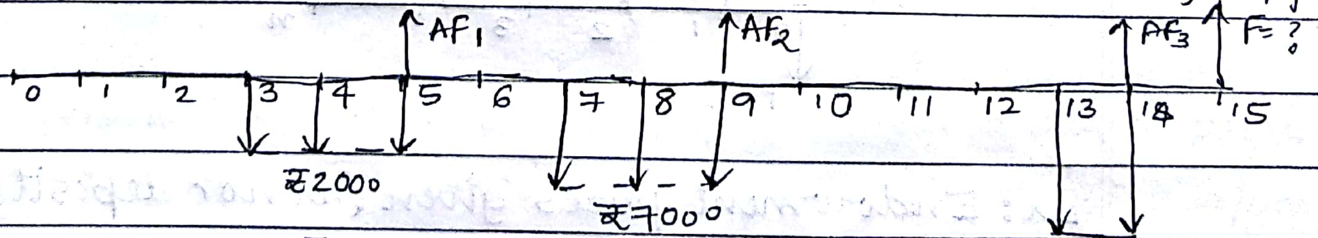
$$= 1,80000 / (1+0.14)^5 = ₹\ 93486.35$$

$$A = F \times \left[\frac{i}{(1+i)^n - 1} \right]$$

$$= 93486.35 \times \left[\frac{0.14}{(0.14+1)^{10} - 1} \right]$$

$$A = ₹\ 4834.5$$

(3) Find total amount accumulated in the year in the account at end of 15 year at an interest rate of 12% p.a?



$$AF_1 = A \left[\frac{(1+i)^n - 1}{i} \right] = 2000 \times \left[\frac{(1+0.12)^3 - 1}{0.12} \right]$$

$$AF_1 = ₹6748.8$$

$$F_1 = AF_1(1+i)^n = 6748.8(1+0.12)^{10} = ₹20960$$

$$AF_2 = A\left[\frac{(1+i)^n - 1}{i}\right] = 5000\left[\frac{(1+0.12)^3 - 1}{0.12}\right]$$

$$AF_2 = ₹16872.$$

$$F_2 = AF_2(1+i)^n = 16872(1+0.12)^6 = ₹33302.33$$

$$AF_3 = A\left[\frac{(1+i)^n - 1}{i}\right] = 10000\left[\frac{(1+0.12)^2 - 1}{0.12}\right]$$

$$AF_3 = ₹21200.$$

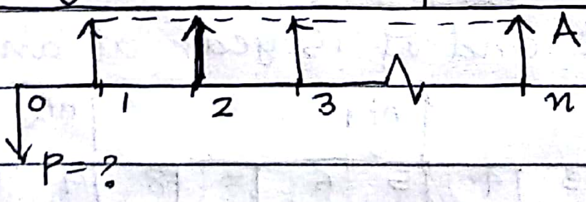
$$F_3 = AF_3(1+i)^n = 21200(1+0.12)^1$$

$$F_3 = ₹23744.$$

$$F = F_1 + F_2 + F_3 = 20960 + 33302.33 + 23744$$

$$F = ₹78007.08$$

(5) Equal payment series present worth.



Ex: Endowment prizes given (Donor deposits amount & equal amount is taken every year).

$$F = A\left[\frac{(i+1)^n - 1}{i}\right]$$

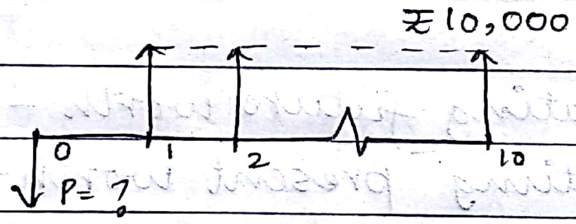
$$P(1+i)^n = A\left[\frac{(1+i)^n - 1}{i}\right]$$

$$P = A = \left[\frac{(1+i)^n - 1}{i(1+i)^n}\right]$$

or

$$P = A\,(P/A, i, N)$$

① A firm is available for sale for ₹65,000 now. The firm yields a cash inflow of ₹10,000 / year for next 10 years. If money is worth 12%, can the firm be bought?

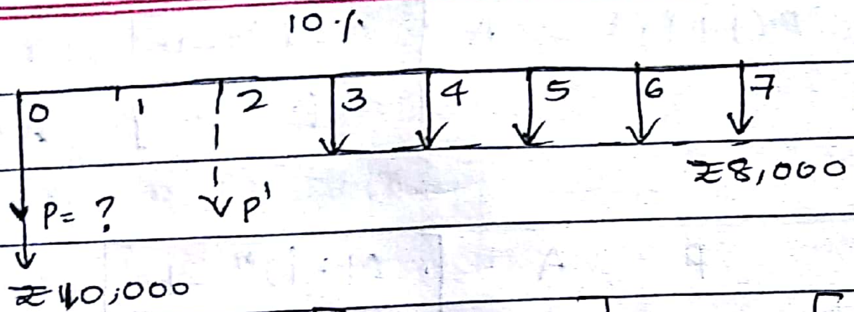


$$P = 10000 \times \left[\frac{(1+i)^n - 1}{i(1+i)^n}\right] = 10000 \times \left[\frac{(1+0.12)^{10} - 1}{0.12\,(0.12+1)^{10}}\right]$$

$$P = ₹56,502$$

The firm cannot be bought.

② A person buys a machine by making a down payment of ₹10,000 & the balance in payments of ₹8,000/year for 5 years starting 3 years from now at 10% p.a. What is cost of the machine?



$$P' = 8000 \times \left[\frac{(1+i)^n - 1}{i(1+i)^n}\right] = 8000\left[\frac{(1+0.1)^5 - 1}{0.1(0.1+1)^5}\right]$$

$$P' = ₹\ 30,326$$

$$P^* = P'/(1+i)^n$$

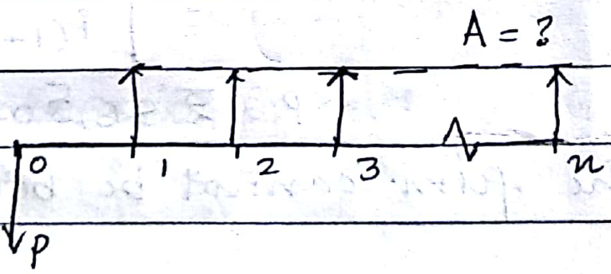
$$= 30,326/(1+0.1)^2 = ₹\ 25,063.$$

$$\therefore\ P'' = 10000 + 25063 = ₹35063$$

* Calculating future worth – compounding
Calculating present worth – discounting

⑥ Equal payment capital recovery factor

Ex: Loan



$$A = P\left[\frac{i(i+1)^n}{(i+1)^n - 1}\right]$$

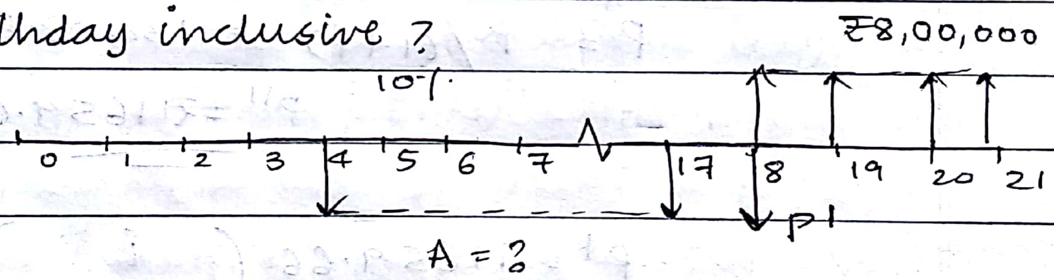
$$A = P\,[A/P,\ i,\ n]$$

① Mr A borrows ₹45,000 with a promise to repay it in 10 equal annual installment, starting 1 year from now, how much would his payments be if $i = 20\%$ p.a?

$$A = P\left[\frac{i(1+i)^n}{(1+i)^n - 1}\right] = 45000\left[\frac{0.2\,(0.2+1)^{10}}{(0.2+1)^{10} - 1}\right]$$

$$A = ₹10733.5$$

② A person decides to make advance plans to finance his 3 year old son's education. Money has to be deposited at 10% p.a. What annual deposit on each birthday from 4 to 17th year inclusive must be made to provide ₹8,00,000 on each birthday from 18th to 21st birthday inclusive ?



$$p' = 8,00,000\left[\frac{(1+i)^n\, i}{(1+i)^n - 1}\right]$$

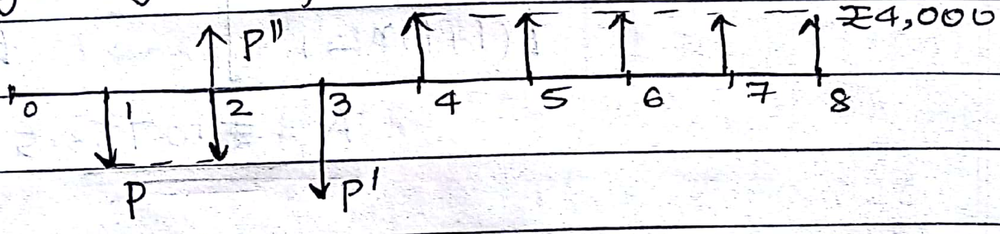
$$= 8,00,000\left[\frac{(1+0.1)^4 \times 0.1}{(1+0.1)^4 - 1}\right]$$

$$p' = ₹2535892$$

$$A = p'\left[\frac{i}{(i+1)^n - 1}\right] = 2535892\left[\frac{0.1}{(1+0.1)^{14} - 1}\right]$$

$$A = ₹90,648.56$$

③ Determine equal amounts P that should be deposited into an account during 1st & 2 years from now in order to withdraw ₹4000/year for 5 years starting 4 years from now at 15% p.a ?



$$P' = 4000\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right] = 4000\left[\frac{(0.15+1)^5 \times 0.15}{0.15(0.15+1)^5}\right]$$

$$\underline{P' = ₹13408.62}$$

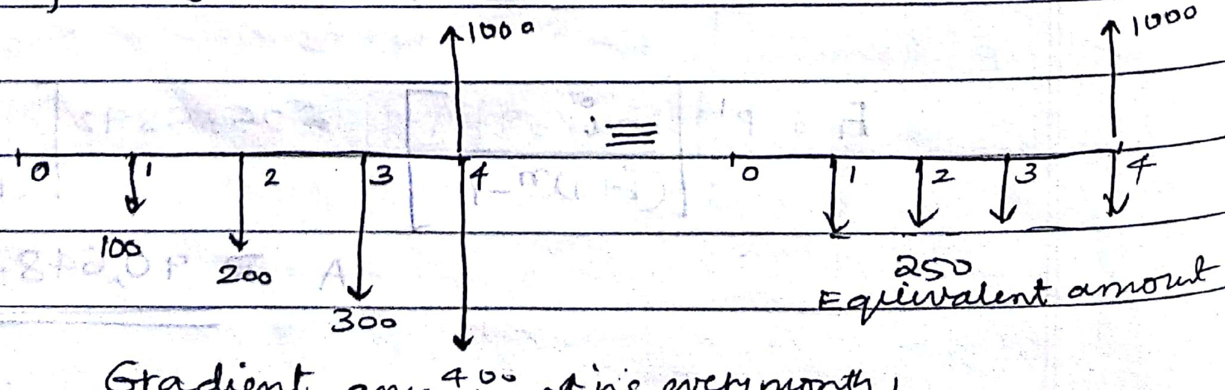
$$P'' = P/(1+i) = 13408.62/(1+0.15)$$

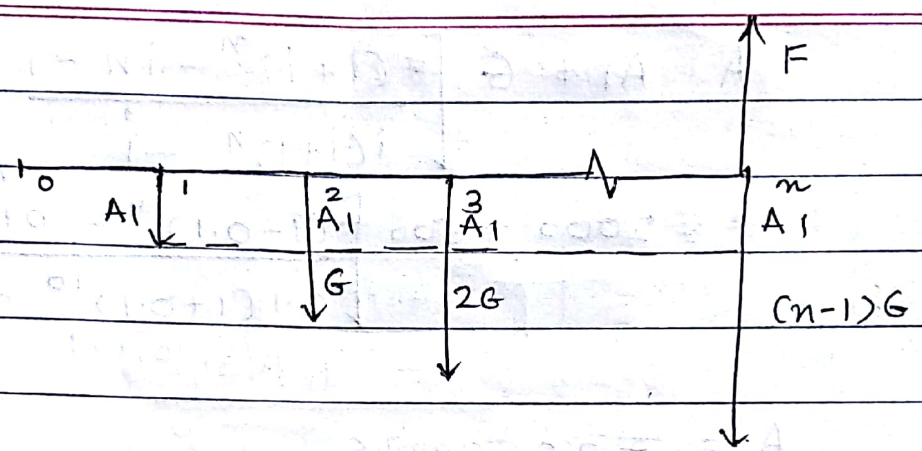
$$\underline{P'' = ₹11659.66}$$

$$P' = 11659.66\left(\frac{i}{(1+i)^n - 1}\right)$$

$$= 11659.66\left(\frac{0.15}{(1+0.15)^2 - 1}\right) = \underline{₹5423}$$

⑦ Uniform gradient series annual equivalent amount.





An uniformly ↑ing / ↓ing series can be evaluated by calculating F or P for each individual payment & then summing the collection.

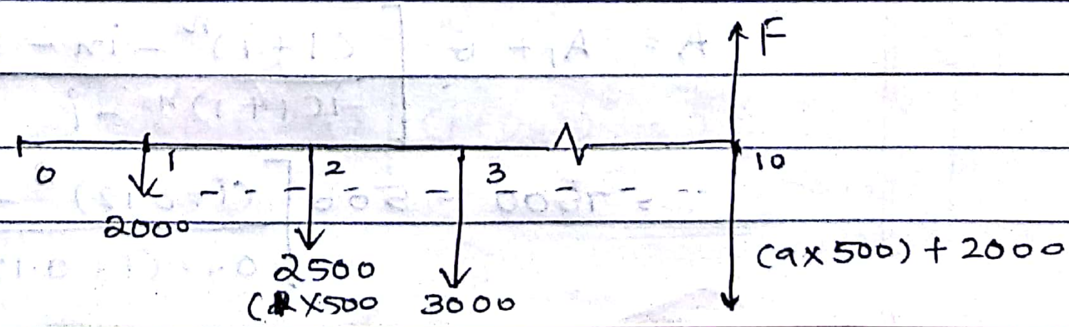
Calculation time can be reduced by converting this gradient series into an equivalent annuity of equal payments A.

The formula for translation is given by

$$A = A_1 + G\left[\frac{(1+i)^n - in - 1}{i(1+i)^n - i}\right]$$

$$A = A_1 + G\,(A/G, i, n)$$

① Mr A has 10 yrs of service before he ~~retaires~~ retires. He now plans to deposit ₹25,000 in the 1st year & ↑s his deposit by ₹500 for the remaining 9 years. If the $i = 10\%$ p.a, find amount accumulated at the end of 10th year?



$$A = A_1 + G\left[\frac{(1+i)^n - in - 1}{i(1+i)^n - i}\right]$$

$$= 2000 + 500\left[\frac{(1+0.1)^{10} - 0.1\times 10 - 1}{0.1(1+0.1)^{10} - 0.1}\right]$$

$(A/G, 10, 10)$

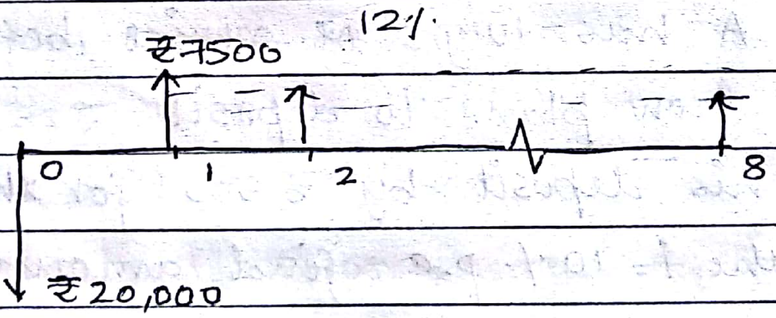
$A = ₹\,26{,}862.73$

$$F = A \times F(F/A, i, n) = A\left[\frac{(1+i)^n - 1}{i}\right]$$

$= 26862.73 \times 15.937$

$F = ₹\,4{,}28{,}111.32$

② A new piece of material handling equipment costs ₹20,000 and is expected to save ₹7,500 in the 1 year of operation. The savings would reduced by ₹500 each following year until the equipment is worn out at the end of 8 years, determine net present worth of machine at interest rate of 12%.



$$A = A_1 + G\left[\frac{(1+i)^n - in - 1}{i(1+i)^n - i}\right]$$

$$= 7500 - 500\left[\frac{(1+0.12)^8 - 0.12\times 8 - 1}{0.12(1+0.12)^8 - 0.12}\right]$$

$A = ₹6043.42$

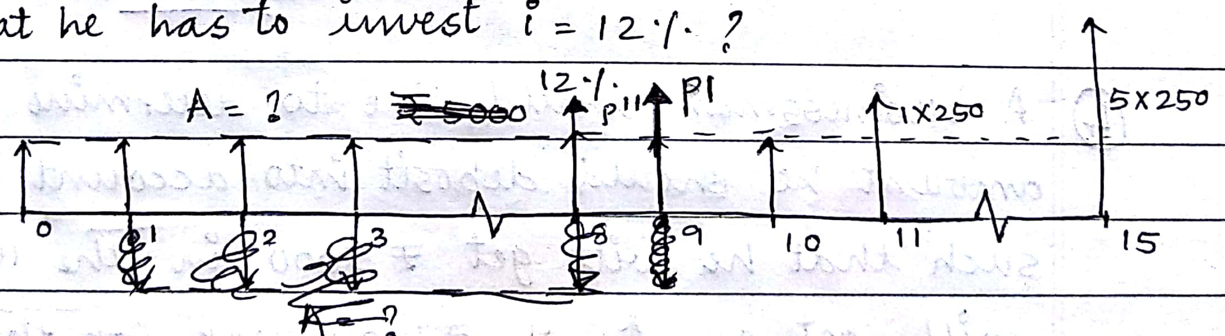
$$P = A \times \left[\frac{(1+i)^n - 1}{i(1+i)^n}\right] = 6043.42 \left[\frac{(1+0.12)^8 - 1}{0.12(0.12+1)^8}\right]$$

$P = ₹30021.63$.

Net Present Value (NPV) = Present worth of revenue − Present worth cost

$= 30022 - 20000 = ₹10022$

③ A person is planning to withdraw ₹5000 in the 10 years from now & then onwards he ↑s his withdrawal amount by ₹250 upto 15 years. To make these withdrawals he is planning to invest an equal amount for 8 years starting from 1st year. Find this equal amount what he has to invest $i = 12\%$?



$$A = A_1 + G\left[\frac{(1+i)^n - in - 1}{i(1+i)^n - i}\right]$$

$$= 5000 + 250 \left[\frac{(1+0.12)^6 - 0.12 \times 6 - 1}{0.12(1+0.12)^6 - 0.12}\right]$$

$A = ₹5543.68$

p' is present worth of uniform gradient series,

$$p' = A \times \left[\frac{(1+i)^n - 1}{i(1+i)^n}\right]$$

$$= 5543 \left[\frac{(1+0.12)^6 - 1}{0.12\,(1+0.12)^6}\right]$$

$$p' = ₹\ 22789.$$

p'' is future worth of annuity series,

$$p'' = p'/(1+i)^n$$

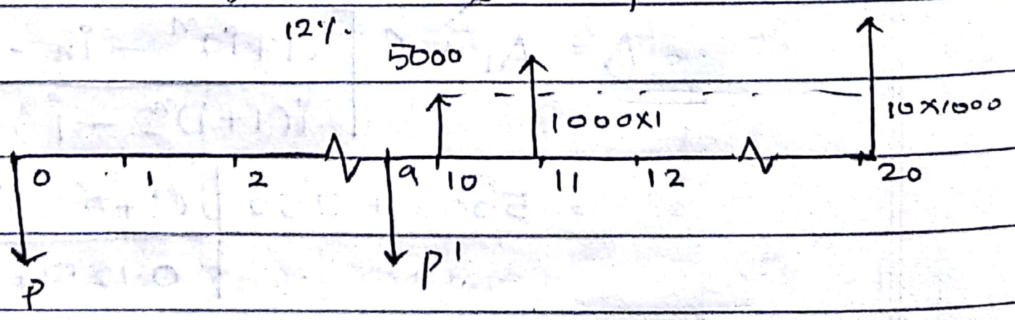
$$p'' = 22789/(1+0.12)^1 = ₹20347$$

To find equal amount A,

$$A = P\left(\frac{i}{(1+i)^n - 1}\right) = 20347 \times \left(\frac{0.12}{(1+0.12)^8 - 1}\right)$$

$$A = ₹1663.21$$

(4) A businessman would like to determine what amount he should deposit into account 12% p.a such that he will get ₹5000 in the 10th year & will get an ↑ of ₹1000/year for the next 10 yrs?



$$A = A_1 + G\left[\frac{(1+i)^n - in - 1}{i(1+i)^n - i}\right]$$

$$= 5000 + 1000\left[\frac{(1+0.12)^{11} - 11\times 0.12 - 1}{0.12(1+0.12)^{11} - 0.12}\right]$$

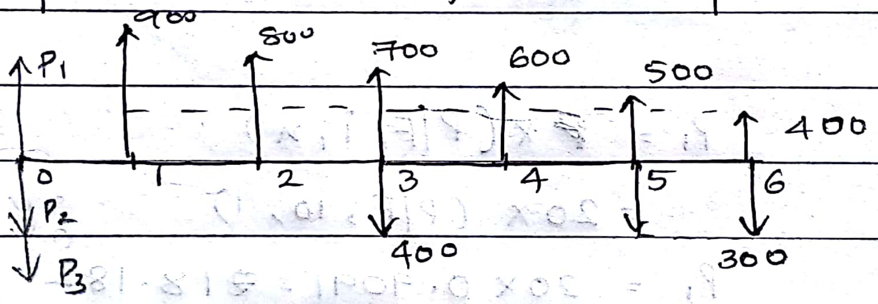
$A = ₹\,8895.24$

$P' = A \times (P/A, i, \overset{11}{n})$

$= 8895 \times 5.938 = ₹\,52820$

$P = P'/(1+i)^n = 52820/(1+0.12)^9 = ₹\,19047$

(4) Calculate present worth if $i = 12\%$ p.a.



$A = A + G(A/G, i, N)$

$= 900 - 100 \times (2.172)$

$A = ₹\,682.8$

$P_1 = A \times (P/A, i, N)$

$= 682.8 \times 4.111 = ₹\,2806.99.$

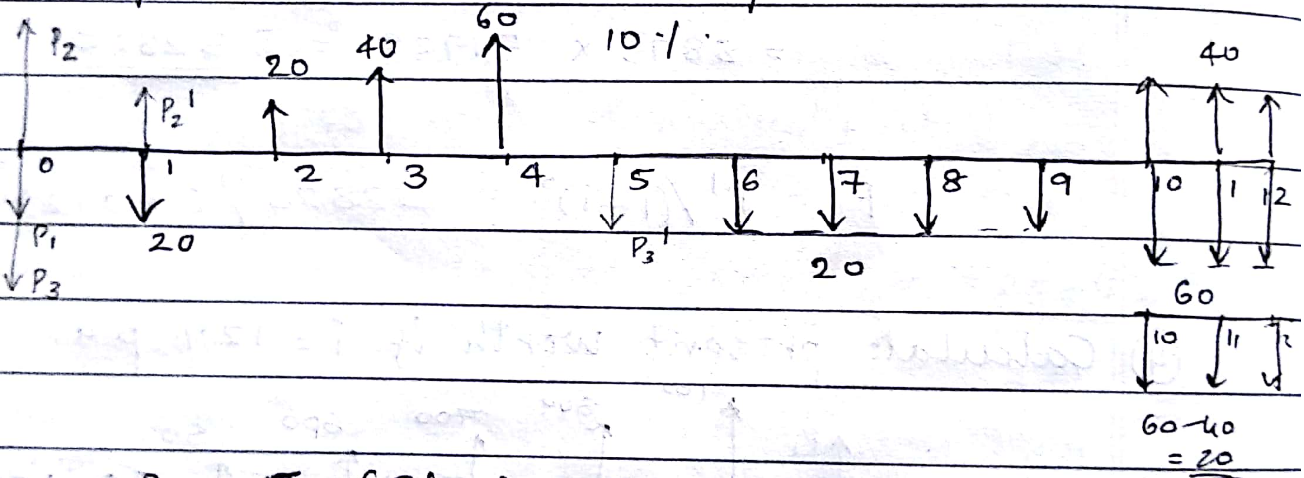
$P_2 = 400 \times (P/F, i, \overset{3}{n})$

$= 400 \times 0.718 = ₹\,284.72.$

$$P_3 = 300 \times (F/A, 12, 2) \times (P/F, 12, 6)$$
$$= 300 \times 2.120 \times 0.5066$$
$$= ₹\,322$$

$$P = P_1 - P_2 - P_3 = 2200$$

(5) Find present worth at 10% p.a.



$$P_1 = F \times (P/F, i, n)$$
$$= 20 \times (P/F, 10, 1)$$
$$P_1 = 20 \times 0.9091 = ₹\,18.182$$

$$A = A_1 + G\,(A/G, i, n)$$ 3
$$A = 20 + 20(0.937) = ₹\,38.74.$$

$$P_2' = A \times (P/A, i, 3)$$
$$= 38.74 \times 2.487 = ₹\,96.34$$

$$P_2 = P_2' \times (F/P, i, 1)$$
$$= 96.34 \times 0.9091 = ₹\,87.58$$

$P_3' = A \times (P/A, i, 7)$

$= 20 \times 4.868 = ₹\,97.36.$

$P_3 = P_3' \times (P/F, i, 5) = 97.36 \times 0.6209$

$= ₹\,60.45.$

$P = P_2 - P_1 - P_3 = 87.58 - 18.482 - 60.45$

$= ₹\,8.948.$

$20\,\overset{0.9091}{(P/F, 10, 1)} + 20\,\overset{3.170}{(P/A, 10, 4)}\,\overset{0.6209}{(P/F, 10, 5)} + 20\,\overset{2.487}{(P/A, 10, 3)}$

$\times \underset{0.4241}{(P/F, 10, 9)} - [20 + 20\,\underset{\cancel{1.38}\;0.937}{(A/G, 10, 3)}] \times \underset{2.487}{(P/A, 10, 3)}\,\underset{0.9091}{(P/F, 10, 1)}$

$P = -8.946$ ( )

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Date
Page

Chapter - 2

Nominal and effective interest rate

Interest rate are usually calculated on an annually basis. However it may be compounded several times in a yr. i.e quarterly, monthly, half yearly etc. But in all the cases, interest charges is represented annually.

For example, if one year is divided into quarters & each quarter is charged with an interest rate of 2% then this is stated as, 8% p.a compounded quarterly. Stated in this fashion the interest is known as nominal interest rate and is given,

$$\text{Nominal interest} = \frac{r}{n}$$

r - rate of interest / yr.

n - No. of times in a yr interest is compounded

The future worth at the end of one year of ₹1000 earning an interest rate of 8% compounded quarterly is,

$$F_{Q_1} = 1000(1+0.02)^1 \qquad i = 8/4 = 0.02$$

$$= 1020.$$

$$F_{Q_2} = 1020\,(1+0.02) = 1040.4$$

IIIly $F_{Q_4} = ₹1082.43$ — (1)

If the interest rate is 8% compounded annually,

$$F = 1000(1+0.08) = ₹1080 \text{ ——— } (2).$$

* Comparing (1) & (2) it is evident that nominal interest gives higher returns.

Effective interest rate: is defined as ratio of interest charged for one year to the principal:

$$i_{eff} = \frac{F-P}{P} \times 100.$$

For example, for a 1 year loan of ₹1000 at a nominal interest rate of 8% compounded quarterly, we have effective interest rate,

$$i_{eff} = \frac{1082.43-1000}{1000} \times 100 = 8.234\%.$$

* Therefore effective interest rate is the equivalent interest rate which gives the same final amount when compounded annually for the interest which is being compounded more than once in a year:

* 8% compounded quarterly ≡ 8.243% comp-ounded annually.

$$i_{eff} = \frac{F-P}{P} \times 100$$

$$= \frac{P(i+1)-P}{P} \times 100 = (1+i)^n - 1$$

$$\boxed{i_{eff} = (1 + r/n)^{n} - 1}$$

(r/n) → represent rate of interest applicable for each compounding period.

(power) n → no, of time in a year / payment period interest is calculated.

① A finance company charges interest at the rate of 18% p.a compounded monthly. Calculate effective interest rate.

$$r/n = 18/12 = 1.5\%$$

$$i_{eff} = (1 + 18/12)^{12} - 1$$

$$i_{eff} = 19.56\%$$

② Calculate effective interest rate if rate of interest 6% is compounded ① yearly ② semi-annually, ③ quarterly ④ Monthly ⑤ Daily.

① $i_{eff} = (1 + r/n)^{1} - 1 = 6\%$.

② $i_{eff} = (1 + 0.06/2)^{2} - 1 \approx 6.09\%$.

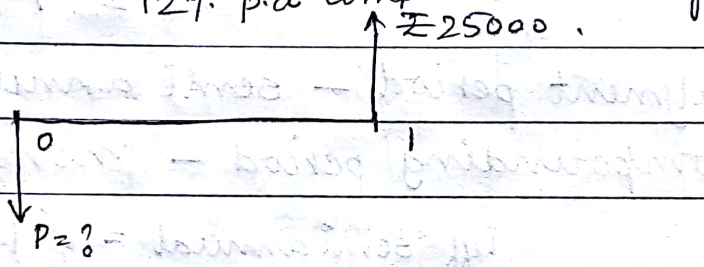
③ $i_{eff} = (1 + 0.06/4)^{4} - 1 = 6.13\%$.

(4) $i_{eff} = \left(1 + \frac{0.06}{12}\right)^{12} - 1 = 6.16\%$

(5) $i_{eff} = \left(1 + \frac{0.06}{365}\right)^{365} - 1 = 6.18\%$

(3) ~~A~~ If person wants to have ₹25000 today, How much he had to deposit 1yr ago, when the investment earned an interest at the rate of 12% compounded monthly?

12% p.a compounded mothly.



$r/n = 12/12 = 1\%$

$~~F~~ = i_{eff} = \left[1 + .12/12\right]^{12} - 1 = 12.68\%$

$P = \frac{F}{(1+i)^n} = \frac{25000}{(1 + 0.1268)^1} = ₹\,22186.7$

$= \frac{25000}{\left(1 + .12/12\right)^{12}} =$

(4) A credit card company charges interest at a rate of 2% per month on the unpaid balance. Calculate effective interest rate if the person is

* Calculation is easier when payment period & compounding period is same.

clearing the dues every semi annual period ?

$$i_{eff} = \left(1 + \frac{r}{n}\right)^n - 1$$

* When payment period & compounding period, i_{eff} is calculated.

Solution:

$$\cancel{i_{eff}} = \left(\cancel{1 + 0.02} \atop 2\right)$$

Payment period – semi annual period
Compounding period – monthly.

$$i_{eff} \text{ semi annual} = \left(1 + \frac{r}{n}\right)^n - 1$$

$$= \left(1 + \frac{0.24}{12}\right)^6 - 1 =$$

2% per month ≡ 24% p.a compounded monthly

$$i_{eff} \text{ semi annual} = 0.126 = \underline{\underline{12.6\%}}$$

(24% p.a compounded monthly & 12.6% compounded annually will give same amount at the end of half ~~an~~ year).

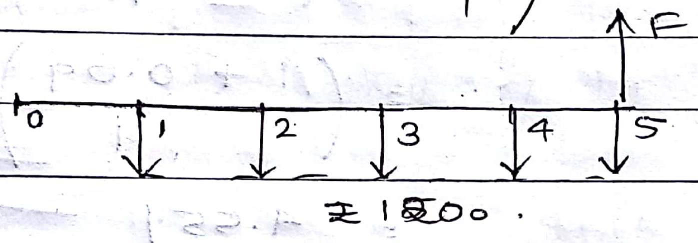
(2) An amount of ₹1200/year is to be paid ~~to an~~ into an account starting one year from now for the next 5 years. Determine total amount accumulated in the account at the end of 5th yr

If interest rate is 12% p.a compounded quarterly?

Payment period = ~~4~~ yearly.
compounding period = quarterly.
No, of times in 1yr ~~payment~~ interest is calculated – 4.

$$i_{eff} = \left(1 + \frac{r}{n}\right)^n - 1$$

$$= \left(1 + \frac{0.12}{4}\right)^{4} - 1 \approx \cancel{15.9\%}\ 12.55\%.$$



$$\cancel{A=₹1} \quad F = A\left[\frac{(1+i)^n - 1}{i}\right]$$

$$= 1200 \times \left[\frac{(1+0.1255)^5 - 1}{0.1255}\right]$$

$$F = ₹7707.$$

(3) Alpha is a vendor for Ford Motor company. An engineer is on Alpha's committee to evaluate bids for a new generatn machine. The vendor's bid include the interest rate as stated below. Alpha will make payments only on semi annual payments, help the engineers to (1) determine

the best bid.

BID 1 : 9% p.a compounded quarterly

BID 2 : 3% p. quarter " " = 12% p.a compounded quarterly

BID 3 : 8.8 p.a " monthly.

(2) Calculate effective interest rate for annual payments.

→ (1) Bid (1) $i_{eff} = \left(1 + \frac{r}{n}\right)^n - 1$

$= \left(1 + \frac{0.09}{4}\right)^2 - 1$

$= 4.55\%$

Bid (2) $i_{eff} = \left(1 + \frac{r}{n}\right)^n - 1$

$= \left(1 + \frac{0.12}{4}\right)^2 - 1$

$= \cancel{1.5\%}\ \cancel{0.0}\ 6.09\%$

Bid (3) $i_{eff} = \left(1 + \frac{r}{n}\right)^n - 1$

$= \left(1 + \frac{0.088}{12}\right)^6 - 1$

$= 4.48\%$

Best bid : BID (3) (Least from buyers POV)

(2) Bid (1) $i_{eff} = \left(1 + \frac{r}{n}\right)^n - 1$

$= \left(1 + \frac{0.09}{4}\right)^4 - 1$

$= 9.3\%$

Bid (2) $i_{eff} = \left(1 + \frac{0.02}{4}\right)^4 - 1$

$= \cancel{3.03\%}$ 2.55%

Bid (3) $i_{eff} = \left(1 + \frac{0.088}{12}\right)^{12} - 1$

$= 9.16\%$

(4) A company is planning to invest ₹6000 every 6 months & savings will be cont, for 5 years. Determine amount accumulated in the account at the end of 5^{th} year in the following case

(1) When i = 12% compounded semi annually

(2) " i = 12% " annually

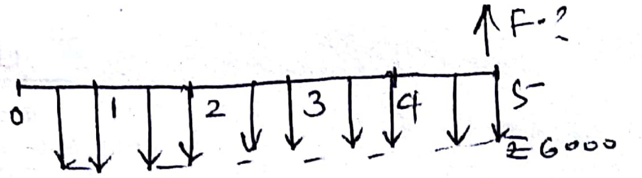
(3) " i = 12% " quarterly

(1) $i_{eff} = \left(1 + \frac{r}{n}\right)^n - 1 = \left(1 + \frac{.12}{2}\right)^1 - 1$

$\left[\frac{(1+i)^n - 1}{i}\right]$ $= 6\%$

$F = A \times \cancel{(1+i)^n} = \cancel{6000 \times (1+0.06)^5}$

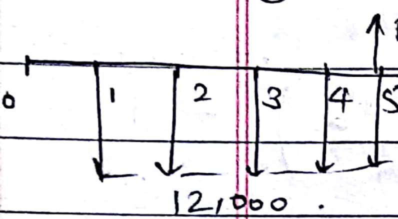
$= \cancel{₹8029.35}$



$$6000 \times (F/A,\ 6\%,\ 10)$$

$$F = 6000\left[\frac{(1+0.06)^{10} - 1}{0.06}\right] = ₹\ 79084.76$$

(2) $i = 12\%$ compounded annually $= i_{eff}$. [₹6000 for semi year will be ₹12000]



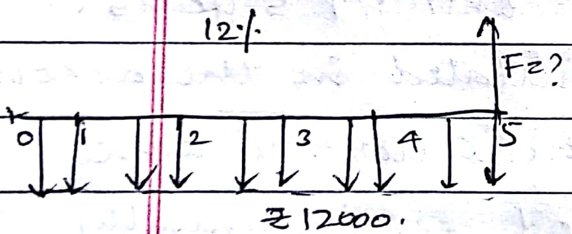
$$F = 1\cancel{2}000 \times \left[\frac{(1+0.\cancel{12})^{5} - 1}{0.12}\right] = \cancel{₹\ 105292.41}\ ₹\ 76234.16$$

(3) $$i_{eff} = \left(1+\frac{r}{n}\right)^{n} - 1 = \left(1+\frac{12}{4}\right)^{2} - 1$$

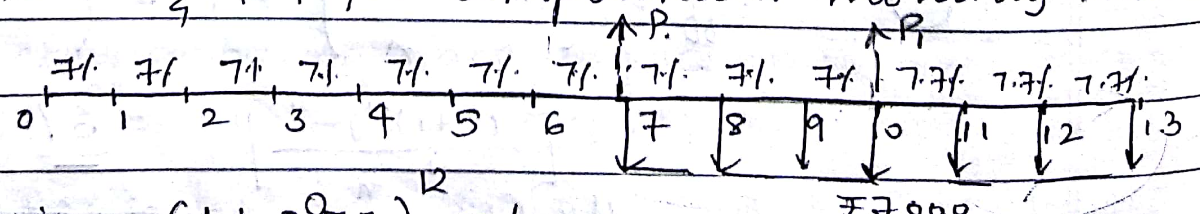
$$= \cancel{15\%}\ 6.09\%$$

$$F = 6000 \times \left[\frac{(1+\cancel{0.15}\ 0.0609)^{10} - 1}{\cancel{0.15}\ 0.0609}\right] = \cancel{₹\ 121822.309}$$

$$= ₹\ 79419.82$$



(5) A seven payment annuity of ₹7000 begins 7 years from now. Determine the worth of this annuity in the 7th yr if interest rate is 7% p.a for 1st 10yrs & 7.7% compounded monthly thereafter



$$i_{eff} = \left(1+\frac{0.077}{12}\right)^{12} - 1$$

$$= 7.97\%$$

Present worth for $11^{th}, 12^{th}, 13^{th}$ year, (at 10^{th} yr)

$P_1 = A\left(\left((1+i)^n - 1\right) / i(1+i)^n\right) = 7000\left(\dfrac{(1+0.0797)^3 - 1}{0.0797(0.0797+1)^3}\right)$

$= ₹\,18049.44$.

Present worth of ₹ 18049 at 7^{th} year (single payment 7% p.a.)

$P_2 = 18049/(1+0.07)^3 = ₹\,14733.36$

Present worth for $7^{th}, 8^{th}, 9^{th}, 10^{th}$, (at 6^{th} yr)

$P_3 = 7000\left((1+0.07)^4 - 1\right) / 0.07(0.07+1)^4 =$ ₹ 23710

Future worth of ₹ 23710 at 7^{th} yr

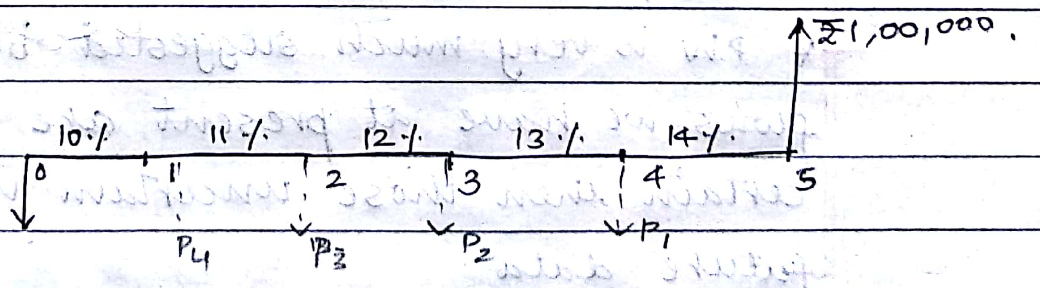
$= 23710 \times (1+0.07) =$ ₹ 25369.7

Total present worth = $14733.36 + 25369.7$

= ₹ 40103

Varying Interest Rate: (due to inflation)

(1) A company wants to have ₹1,00,000 available in the 5^{th} year to make an investment. Determine the lumpsome amount the company should deposit now if the interest rate in the yrs 1, 2, 3, 4, 5 are 10%, 11%, 12%, 13% & 14%.



$P = F/(1+i)^n = 100000 / (1+0.14)(1+0.13)(1+0.12)(1+0.11)(1+0.1)$

$P = F \times (F/P, 14, 1) \times (F/P, 13, 1) \times (F/P, 12, 1) \times (F/P, 11, 1) \times (F/P, 10, 1)$

$P =$ ₹ 56765

Chapter - 3

EVALUATION OF ALTERNATIVES

Objectives: * Select best alternative economically.
* Understand various bases for comparison of alternatives.

Assumptions:

1) Cash flows are known (investment & forecasted revenue of project are known)
2) Cash flows are in constant-value dollars. (Neglecting inflation)
3) The interest rate is known [rate of return / rate]
4) Comparisons are made with before tax cash flows.

COMPARISON OF ALTERNATIVE WITH EQUAL LIFE:

I PRESENT WORTH METHOD

* Equivalent value at present of the asset based on the time value of money.
* Discounted value of future sums i.e all future cash flows are discounted to present & compared.
* PW is very much suggested because any cash flows we have at present are known & are certain then those uncertain values at some future data.
* PW comparison are made only b/w co terminated (ending at same time) proposals to assure equivalent outcomes.

Two methods in calculating PW :

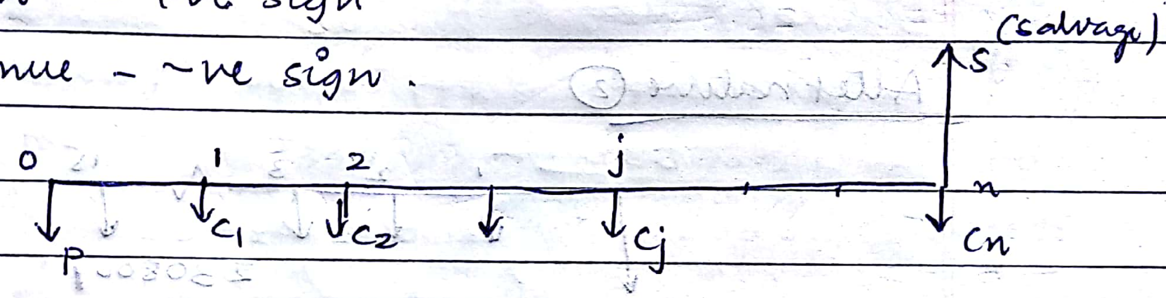
* Cost dominated cash flow diagram.
* Revenue dominated " " "

Cost dominated cash flow diagram :

Salvage value → Return from deposit of assests.

* cost — +ve sign

  Revenue — -ve sign.

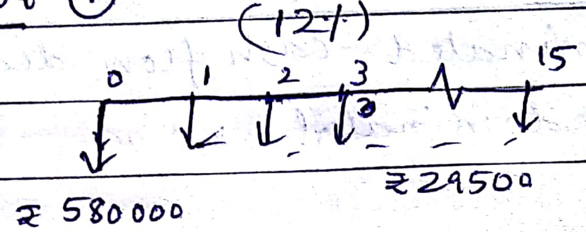


* Alternative with min PW should be selected.

① A construction company receives two bids for an elevator to be installed in their newly constructed apartment, the details of which are given in the table. Determine best bid based on present worth method @ 12% interest rate :

Alternative	Initial cost (Rs)	Service life 15	Annual operat^g cost.
1	580000	15	29500
2	640000	15	30800.

Alternative ①

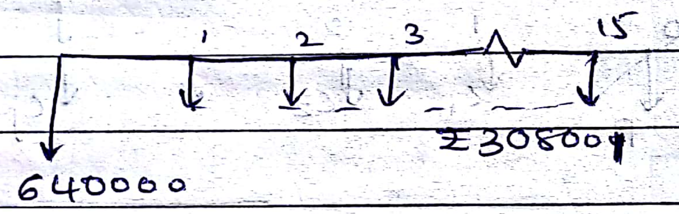


$PW_A = 580000 + 29500 \times (P/A, 12, 15)$

$= 580000 + 29500 \times 6.811$

$= 7,80,924.5$

Alternative ②



$PW_2 = 640000 + 30800 \times 6.811$

$= 8,49,778.8$

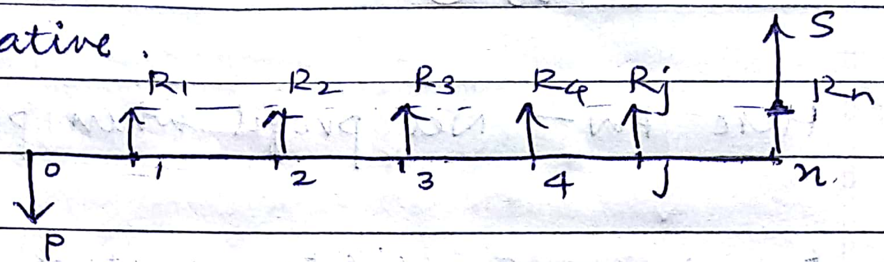
∴ Alternative ① is best bid.

* PW – represent total cost reqd to implement the project & run it for 15 yrs.

Revenue dominated cash flow diagram

* Revenue – +ve
* Cost – ve sign.

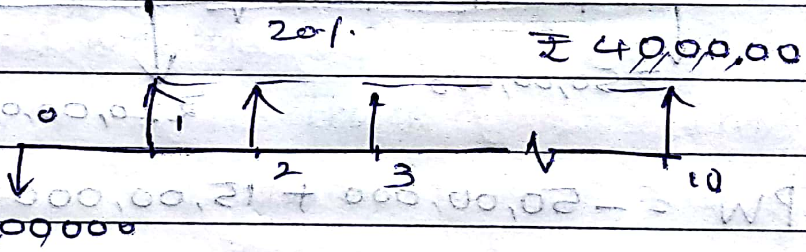
* Alternative with maximum PW of revenue is best alternative.



① Initial outlay & annual revenues of product firm. Find best alternative if i = 20% compounded annually.

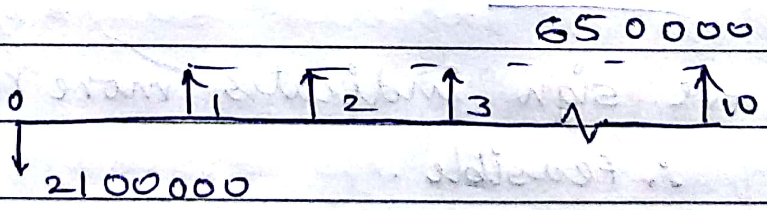
Technology	Initial cost (₹)	Annual Revenue (₹)	Life (yrs)
1	1300000	400000	10
2	2100000	650000	10
3	2300000	860000	10.

→

① 

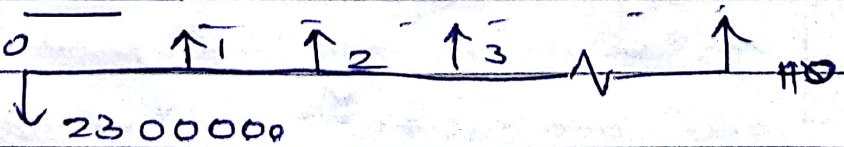
$PW_1 = -1300000 + 400000 \times (P/A, 20, 10)$

$= ₹ 376800.$

② 

$PW_2 = -2100000 + 650000 \times 4.192$

$= ₹ 6,24,800$

③ 

$PW_3 = -2300000 + 860000 \times 4.192 = ₹ 13,05,120$

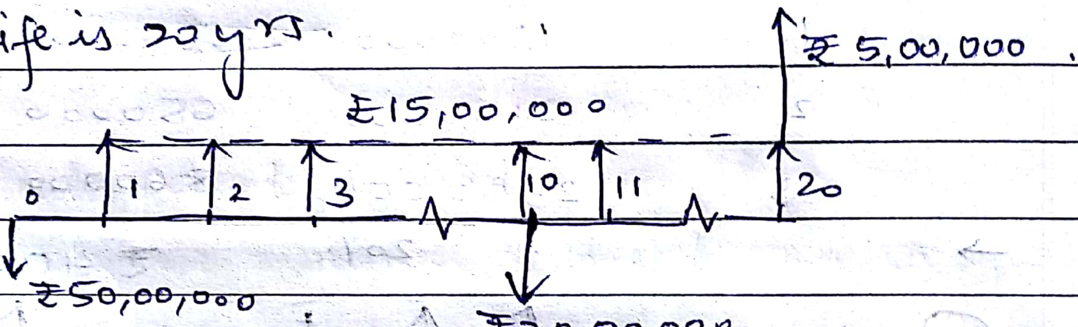
Technology ③ is best bid.

Here PW – Net profit when project runs for 10yrs.

③ Details of feasibility report of a project are as shown below. Check feasibility of project based on present worth method if i = 20%.

Initial outlay	Annual revenue	Modernizing cost at end of 10yrs	Salvage value
₹ 50,00,000.	₹ 15,00,000	₹ 20,00,000	₹ 5,00,000

* Life is 20yrs.



$$PW = -50,00,000 + 15,00,000 \times 4.870 \; (P/A, 20, 20)$$
$$- 20,00,000 \times \cancel{0.026} \; 0.1615 \; (P/F, 20, 10)$$
$$+ 5,00,000 \times 0.0261 \; (P/F, 20, 20)$$
$$= + ₹ 1995050$$

(+ve sign indicates more revenue than investment)

∴ Feasible.

COMPARISON OF ALTERNATIVES ~~W/O~~ WITH UNEQUAL LIFE:

① LCM Method:

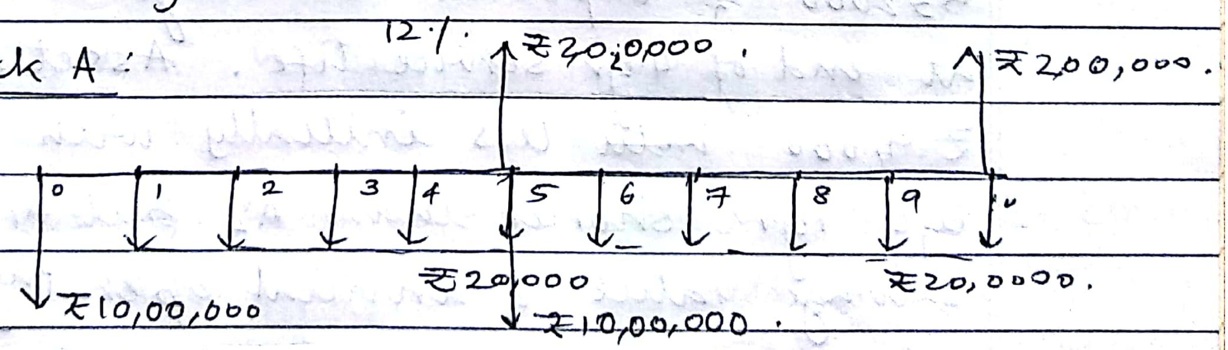
In this method, the alternatives are co-terminated by selecting an analysis period which is the least common multiples of the life's of involved assests. The assumptn here is that the assests will be repeatedly replaced by successor having idintical cost characteristics.

① Two types of trucks are available for mining applicatn. Assume i=12% & determine best truck based on present worth method.

Particulars	Truck A	Truck B
Initial cost	₹ 10,00,000	₹ 15,00,000
Annual operating cost	₹ 20,000	₹ 15,000.
Salvage value	₹ 2,00,000	₹ 5,00,000.
Service life	5 years	10 years.

→ LCM = 10 yrs.

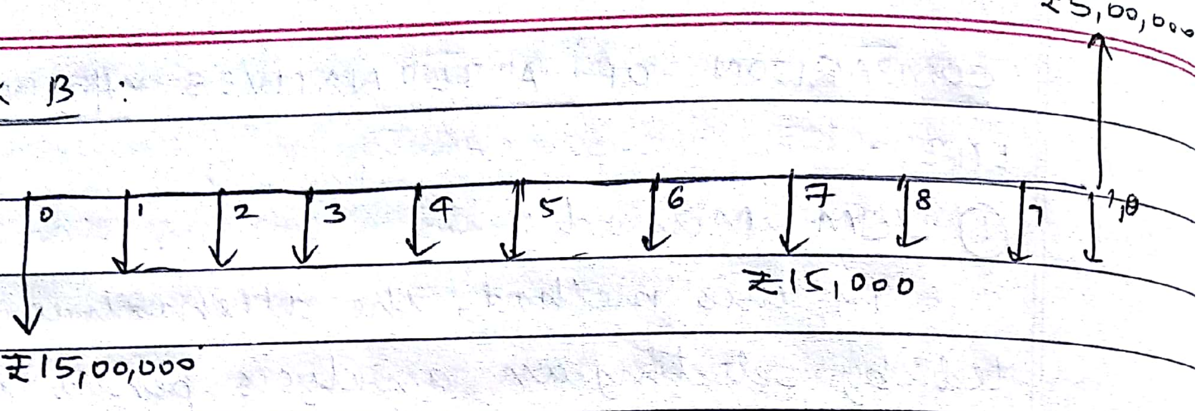
Truck A:



PW = + 10,00,000 + 10,00,000 × (P/F, 12, 5) + 20,000 × (P/A, 12, 10) − 2,00,000 × (P/F, 12, 5) − 2,00,000 × (P/F, 12, 10)

= + 10,00,000 + 10,00,000 × 0.5674 + 20,000 × 5.650 − 2,00,000 × 0.5674 − 2,00,000 × 0.3220 = ₹ + 1400,820

Truck B :



$PW = +1500000 + 15000 \times (P/A, 12, 10) - 5,00,000 \times (P/F, 12, 10)$

$= +15,00,000 + 15000 \times 5.650 - 5,00,000 \times 0.3220$

$= +1423750$

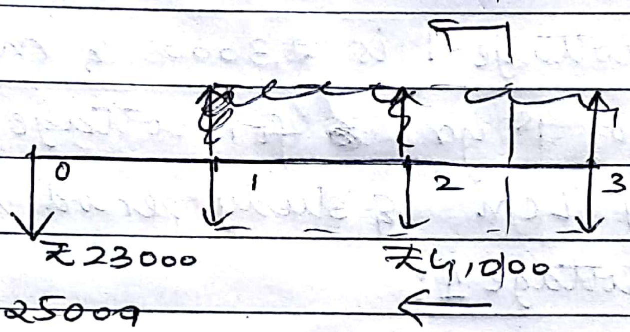
② Study period method

Here the analysis is based on a specified durat^n corresponding to the length of the project / time period during which the assests are in service.

① Assets A_1 & A_2 have the capability of satisfactorily performing a reqd func^n. Initial cost of asset A_2 ₹32000 & expected salvage value ₹ 4,000 at end of 4 yr service life. Asset A_1 costs ₹ 9,000 ~~initi~~ less initially with economic life 1 yr shorter than A_2 but A_1 has no salvage value & annual operating cost exceed those of A_2 by ₹ 2,500. If $i = 15\%$, select the best alternative when comparison is by 2 year study period method.

	A_1	A_2	
~~Asset A_1~~:	23000	32000	Initial cost
	—	4000	Salvage
	2500	—	Operating cost
	3	4	Life

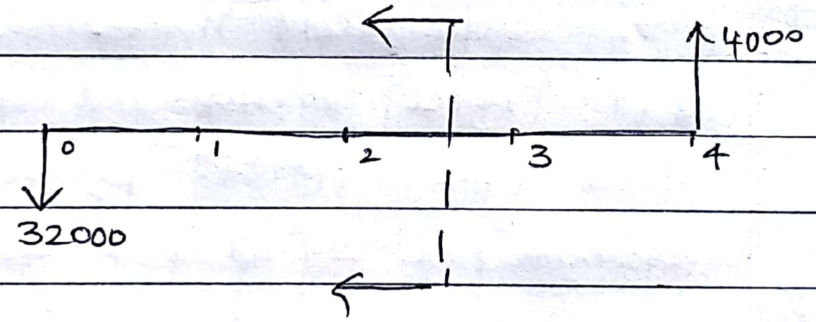
Asset A_1:



$PW = 23000 + \cancel{4{,}000}\ 2500\,(P/A, 15, 2)$

$= 23000 + \cancel{4000} \times 1.626 = \cancel{29504}\ 27065$

Asset A_2:



$PW = 32000$

II FUTURE WORTH METHOD

(1) Two holiday cottages are under considerat^n^. Compare them based on future worth method when i = 12%. when neither of a cottage has a rett realisable salvage value.