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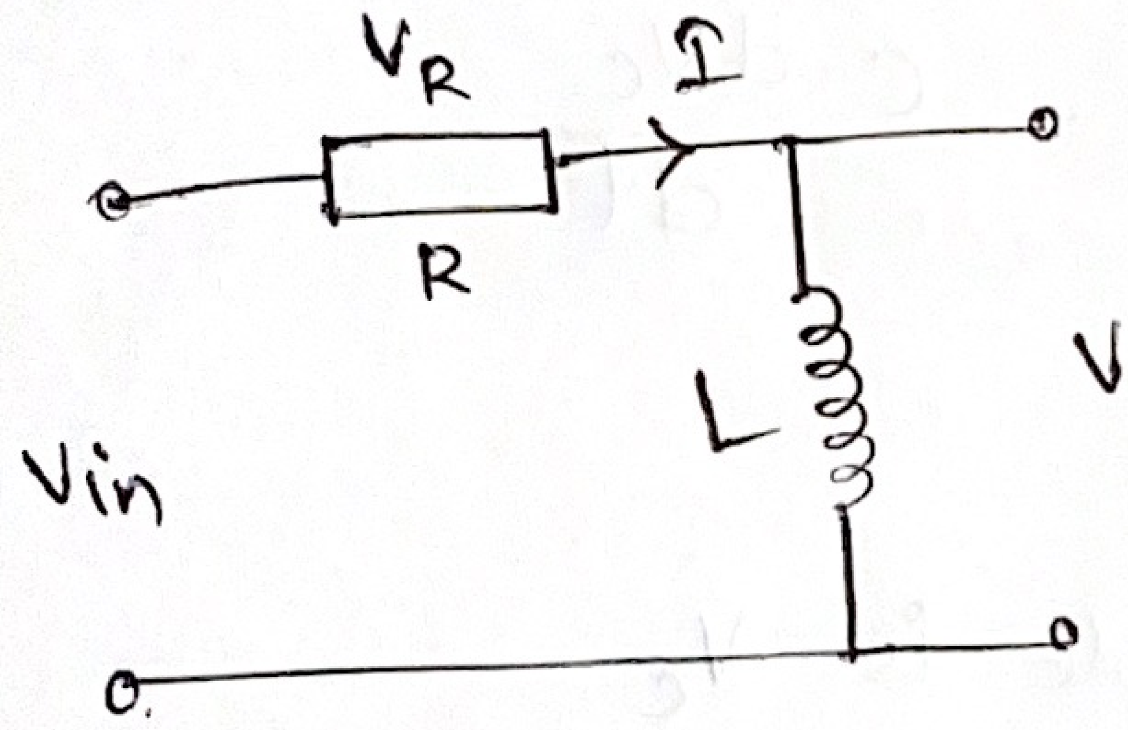
State Space modelling of Electrical systems.

i) RL circuit.



State variables.

only storage elements: { inductor - current ; capacitor - voltage }

Apply KVL

$$V_{in} - IR - V_L = 0$$

$$\Rightarrow V_{in} = V_R + V_L$$

$$x_1 = i_L$$

$$V_{in} = IR + L \cdot \frac{dI}{dt}.$$

$$V_{in} = x_1 R + L \cdot \dot{x}_1$$

$$\Rightarrow \dot{x}_1 = \frac{V_{in}}{L} - x_1 \cdot \frac{R}{L}$$

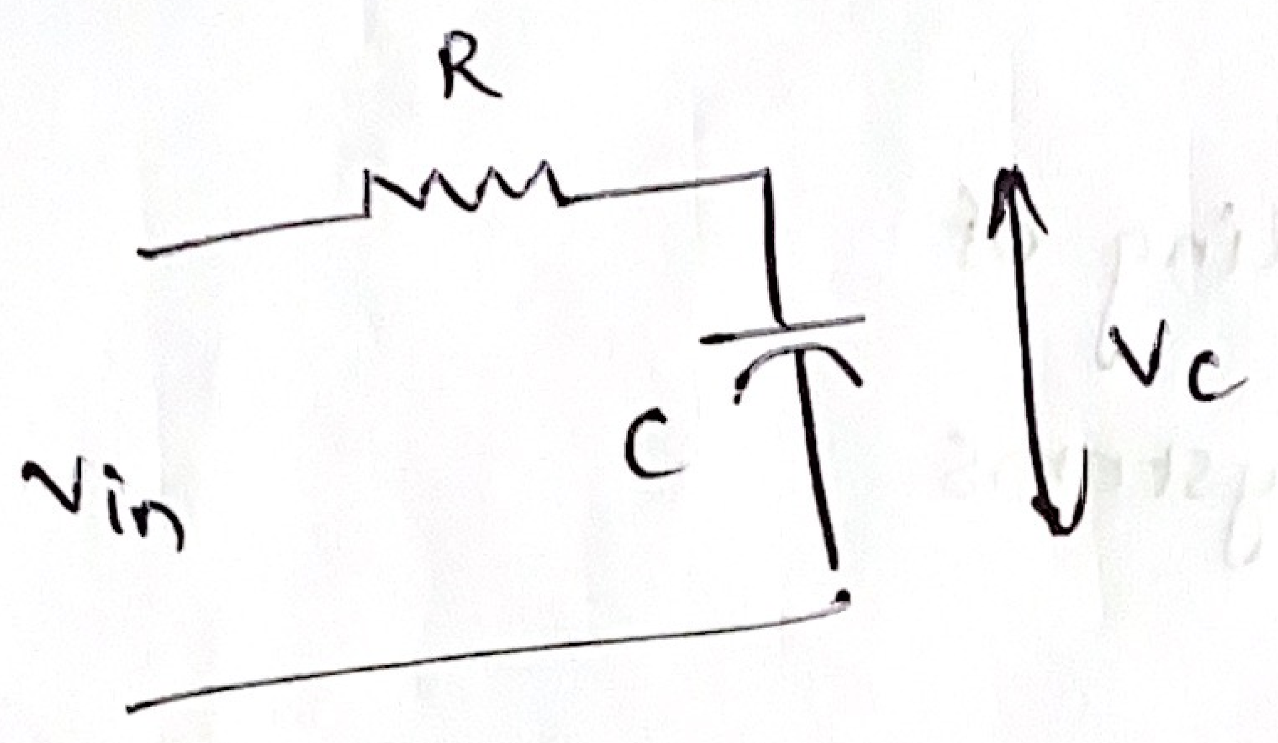
Output equation

$$y = x_1$$

State space model

$$\dot{x}_1 = \left[-\frac{R}{L}\right][x_1] + \left[\frac{1}{L}\right][V_{in}]$$

$$y = [1][x_1]$$



$V_{in} = iR + V_c$

$V_c = \frac{1}{C} \cdot \int i \, dt$

$i = C \cdot \frac{dV_c}{dt}$

$V_{in} = R \cdot C \frac{dV_c}{dt} + V_c$

here state variable is V_c

$\therefore V_c = x_1$

$V_{in} = R.C.\dot{x}_1 + x_1$

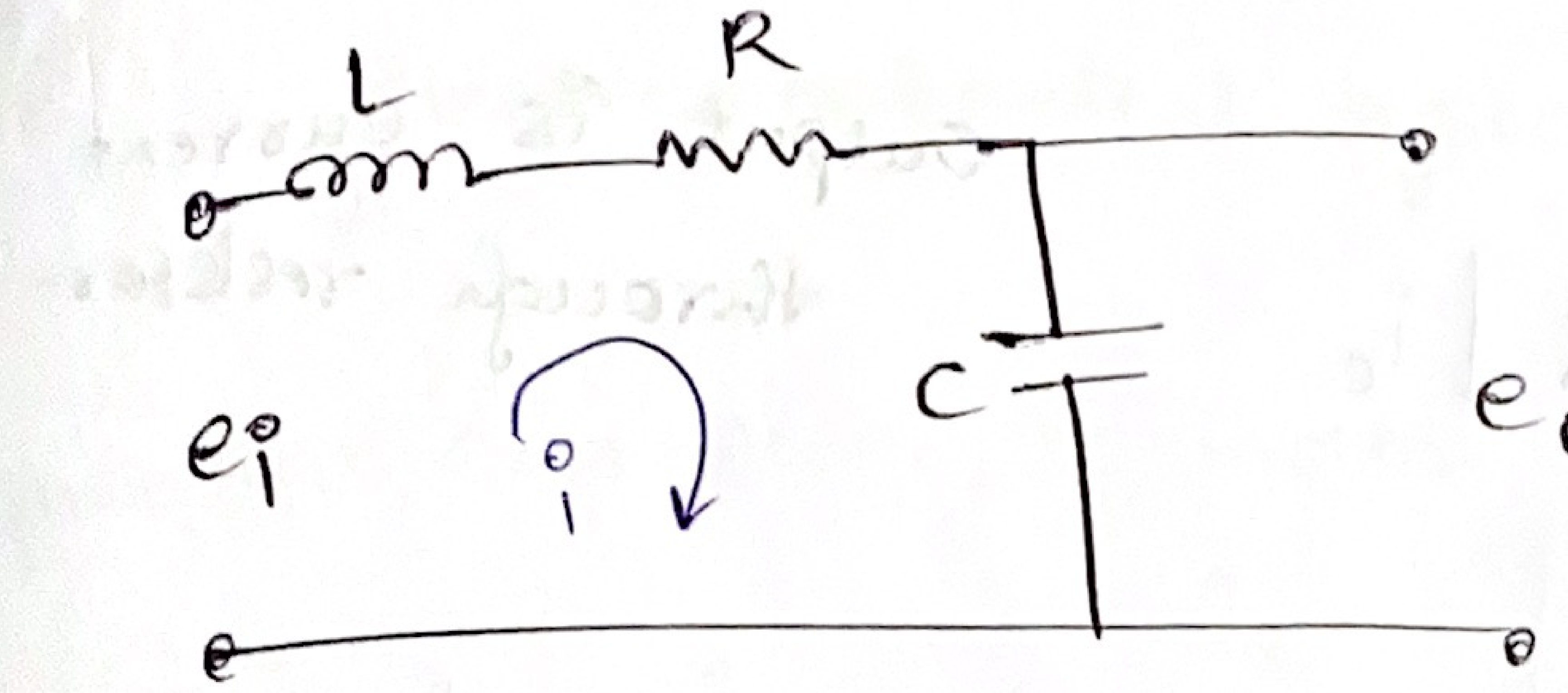
State space model.

$$\dot{x}_1 = \frac{V_{in} - x_1}{RC}$$

$$\Rightarrow \dot{x}_1 = \left[\frac{-1}{RC}\right][x_1] + \left[\frac{1}{RC}\right][V_{in}]$$

$$y = [1][x_1]$$

3.



$$e_i = V_L + V_R + V_C$$

$$V_L = L \cdot \frac{di}{dt}$$

State variables are

$$V_R = iR$$

$$x_1 = i$$

$$V_C = \frac{1}{C} \int i \cdot dt$$

$$x_2 = V_C$$

$$\boxed{i = C \frac{dV_C}{dt}} \longrightarrow (2)$$

$$e_i = iR + L\frac{di}{dt} + \underbrace{\frac{1}{C}\int i \cdot dt}_{V_C} \longrightarrow (1)$$

from (1)

$$e_i = x_1 R + L\dot{x}_1 + x_2$$

$$\Rightarrow \dot{x}_1 = \frac{e_i - x_1 R - x_2}{L}$$

from (2)

$$x_1 = C \cdot \dot{x}_2$$

$$\Rightarrow \dot{x}_2 = \frac{x_1}{C}$$

State space model.

output variables.

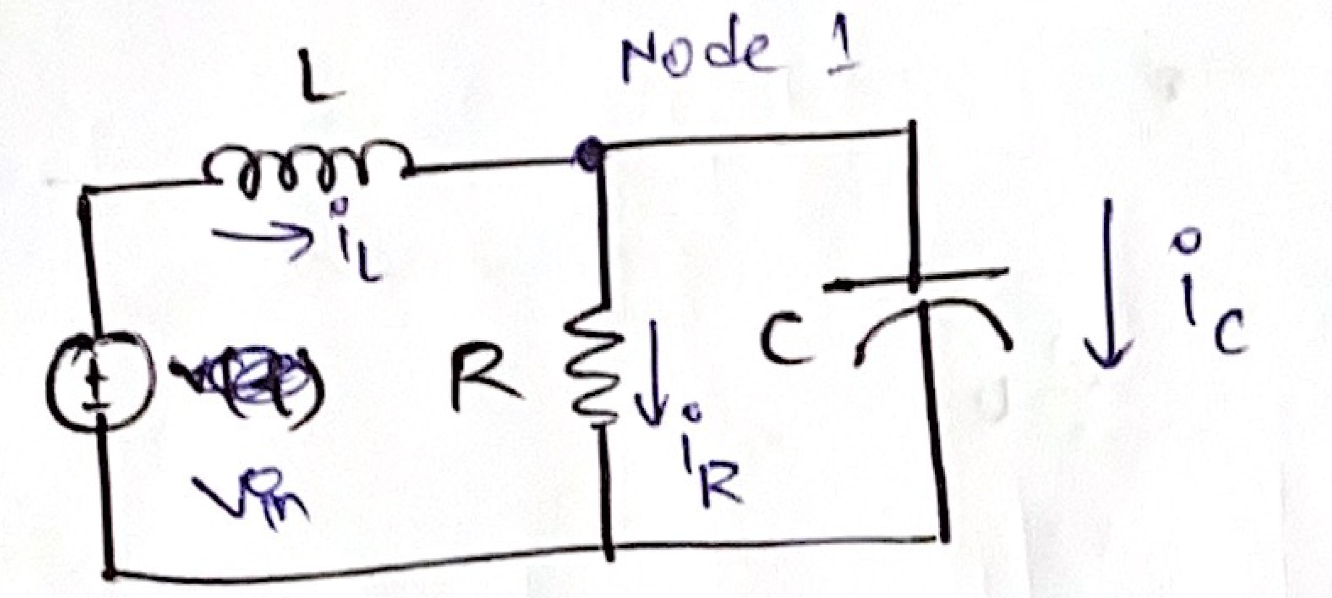
$y_1 = i$ (current through inductor)

$y_2 = V_C$ (~~curr.~~ voltage across capacit)

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -R/L & -1/L \\ 1/C & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1/L \\ 0 \end{bmatrix} [V_i]$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

4.



Output is current through resistor.

At node 1,

$$i_L = i_R + i_c \quad \Rightarrow \quad i_c = i_L - i_R = i_L - \frac{V_c}{R} \rightarrow (1)$$

~~loop 1~~ $\cancel{V(t) = V_L + V_R}$ ~~loop 2~~ $\cancel{V_R}$

here R branch doesnt store energy

So, consider only the outer mesh.

So,

$$V_{in} = V_L + V_c$$

$$V_{in} = L \cdot \frac{di}{dt} + V_c \rightarrow (2)$$

State variables.

$V_c = x_2$

$i_L = x_1$

from (1)

$$i_c = i_L - \frac{V_c}{R}.$$

$$C \cdot \frac{dV_c}{dt} = i_L - \frac{V_c}{R}$$

$$C \cdot \dot{x}_2 = x_1 - \frac{x_2}{R}$$

$$\dot{x}_2 = \frac{x_1}{C} - \frac{x_2}{RC}$$

from (2).

$$L \cdot \frac{di}{dt} = V_{in} - V_c$$

$$L \cdot \dot{x}_1 = V_{in} - x_2$$

$$\dot{x}_1 = \frac{V_{in}}{L} - \frac{x_2}{L}$$

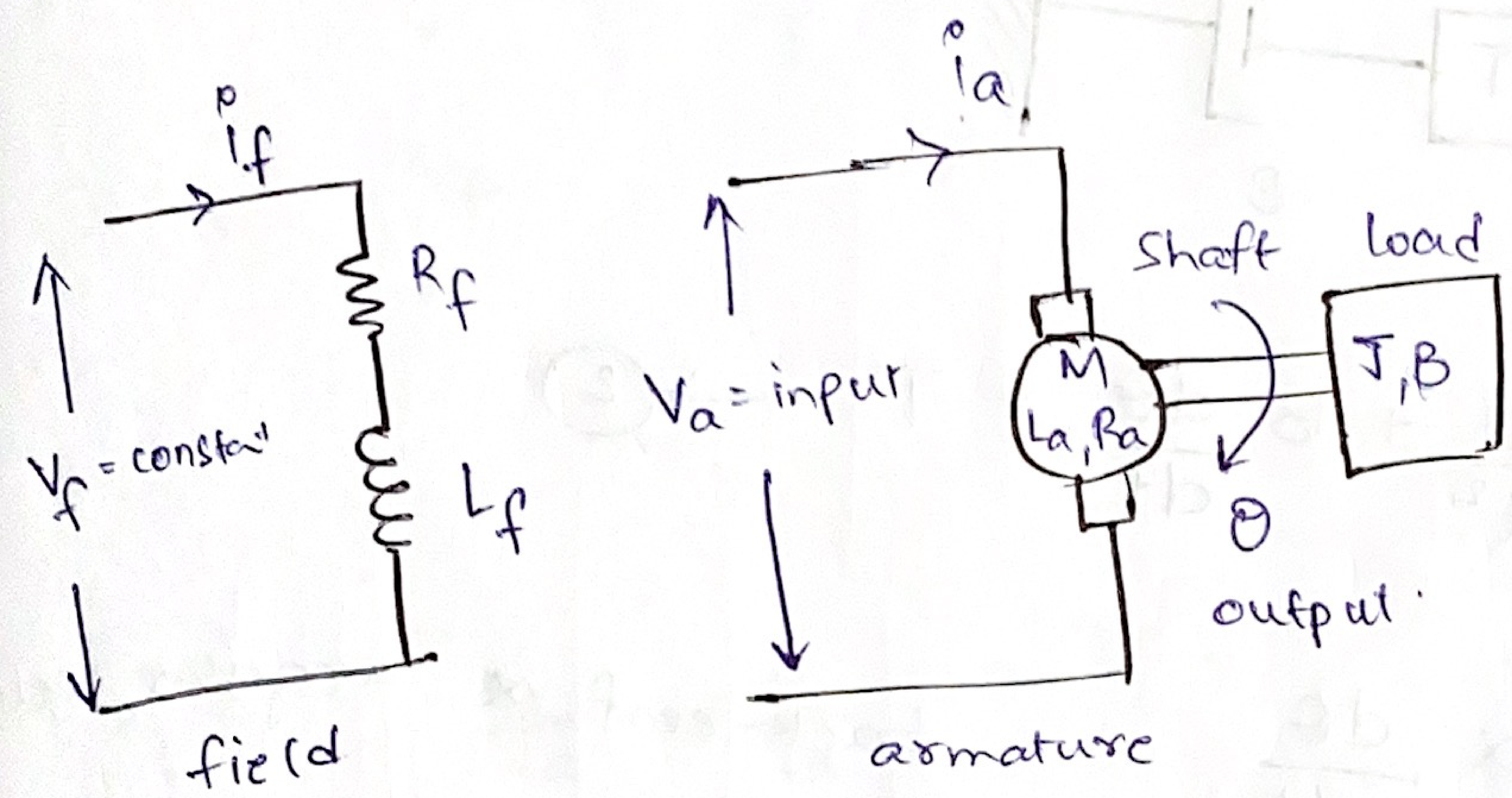
Output eq. $i_R = \frac{V_c}{R}$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -1/RC & 1/C \\ -1/L & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1/L \end{bmatrix} [V_{in}]$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1/R & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Electromechanical systems.

Armature controlled DC motor.



Speed ∝ armature voltage

Speed ∝ $\frac{1}{\text{flux}}$

e_b = back emf

K_t = torque constant

T = torque

θ = angular displ

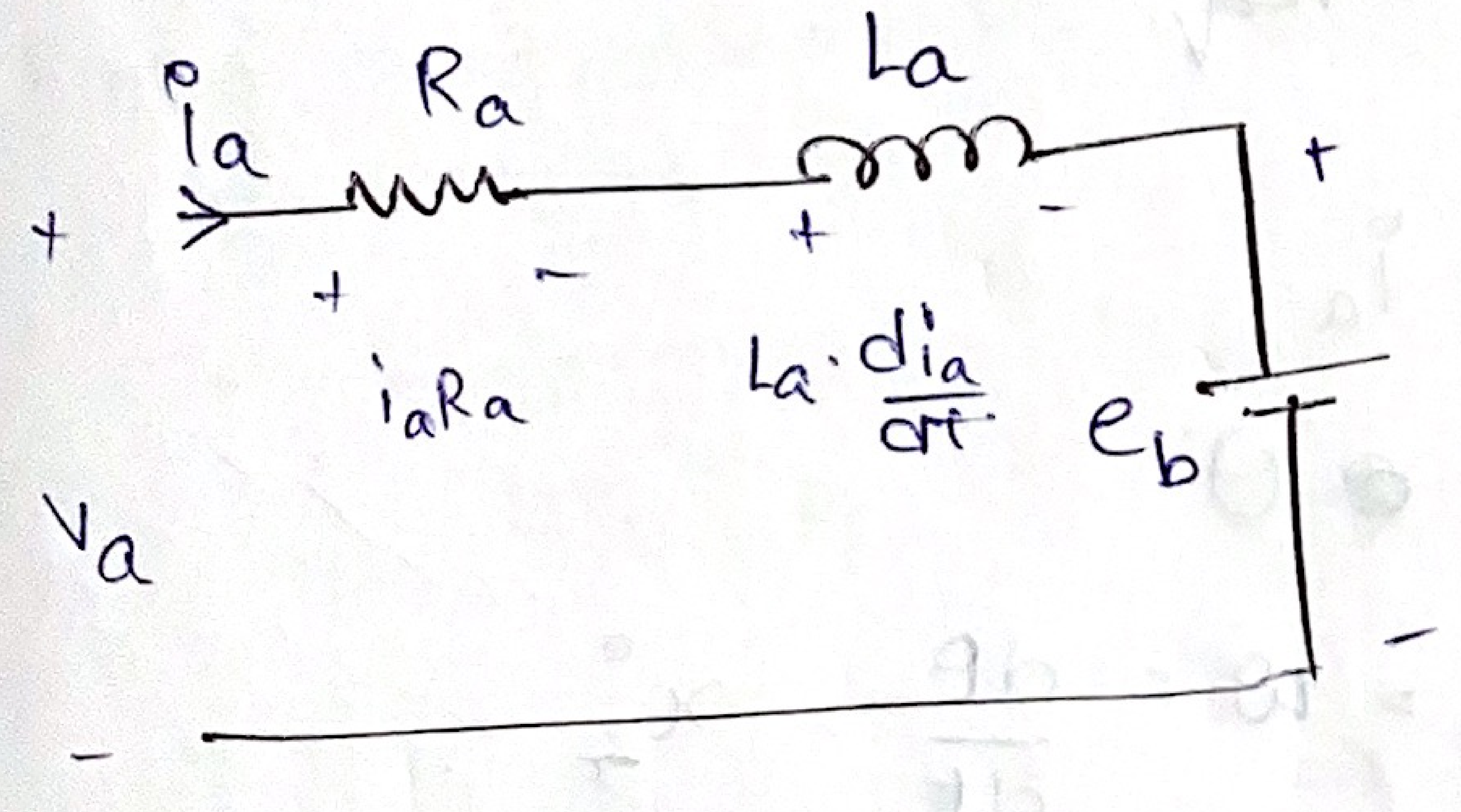
ω = " velocity

J = moment of inertia.

B = frictional coeff.

K_b - Back emf constant.

Armature



Apply KVL

(1) ← $V_a = i_aR_a + L_a\frac{di_a}{dt} + e_b$

$i_a = x_1$ (state variable).

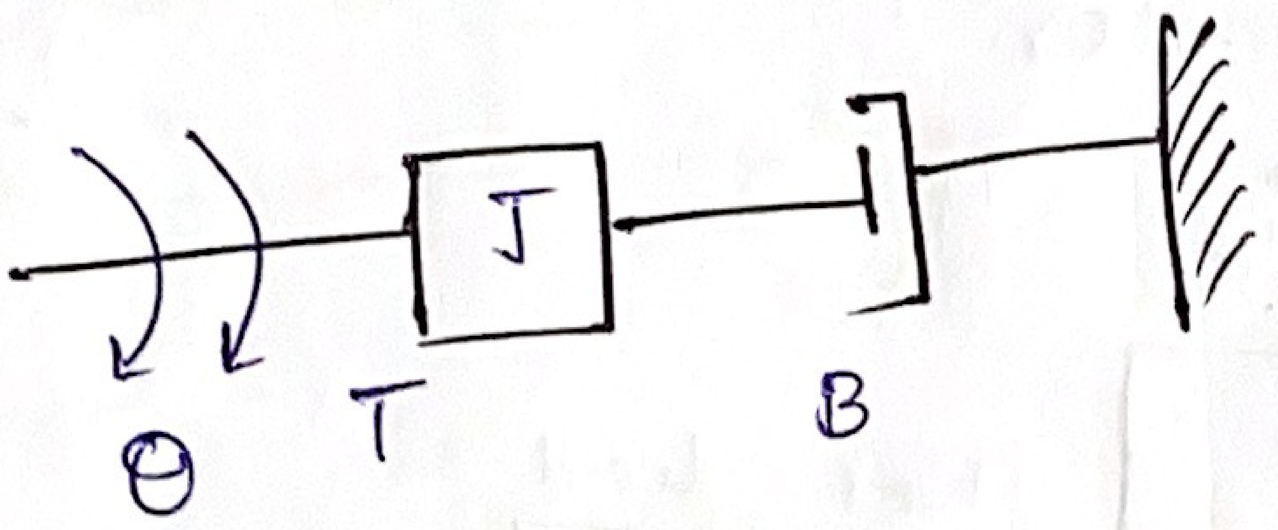
Torque ∝ flux * current

but flux is constant here.

$T \propto i_a$

$$\boxed{T = K_t \cdot i_a}$$

Mechanical system.



$$J \cdot \frac{d^2\theta}{dt^2} + B \cdot \frac{d\theta}{dt} = T \rightarrow (2)$$

Also, $e_b \propto \frac{d\theta}{dt}$ back emf $\propto$ angular velocity

$$e_b = K_b \frac{d\theta}{dt}$$

$$\boxed{e_b = K_b \cdot \dot{\theta}}$$

from (1)

$$V_a = i_a R_a + L_a \cdot \frac{di_a}{dt} + K_b \cdot \frac{d\theta}{dt} \rightarrow (A)$$

from (2)

$$J \cdot \frac{d^2\theta}{dt^2} + B \cdot \frac{d\theta}{dt} = K_t \cdot i_a \rightarrow (B)$$

Here 3 State variables req.

inductor $x_1 = i_a$

mass $x_3 = \theta$

damper. $x_2 = \omega = \frac{d\theta}{dt} = \dot{x}_3$

$\Rightarrow \dot{x}_3 = x_2$

from (B)

$$J \cdot \dot{x}_2 + B \cdot x_2 = K_t x_1$$

from (A) $V_a = u$

$$u = x_1 R_a + L_a \cdot \dot{x}_1 + K_b \cdot \dot{x}_3$$

$$\dot{x}_1 = -\frac{R_a}{L_a} x_1 - \frac{K_b}{L_a} x_2 + \frac{1}{L_a} u$$

$$\dot{x}_2 = \frac{K_t}{J} x_1 - \frac{B}{J} x_2$$

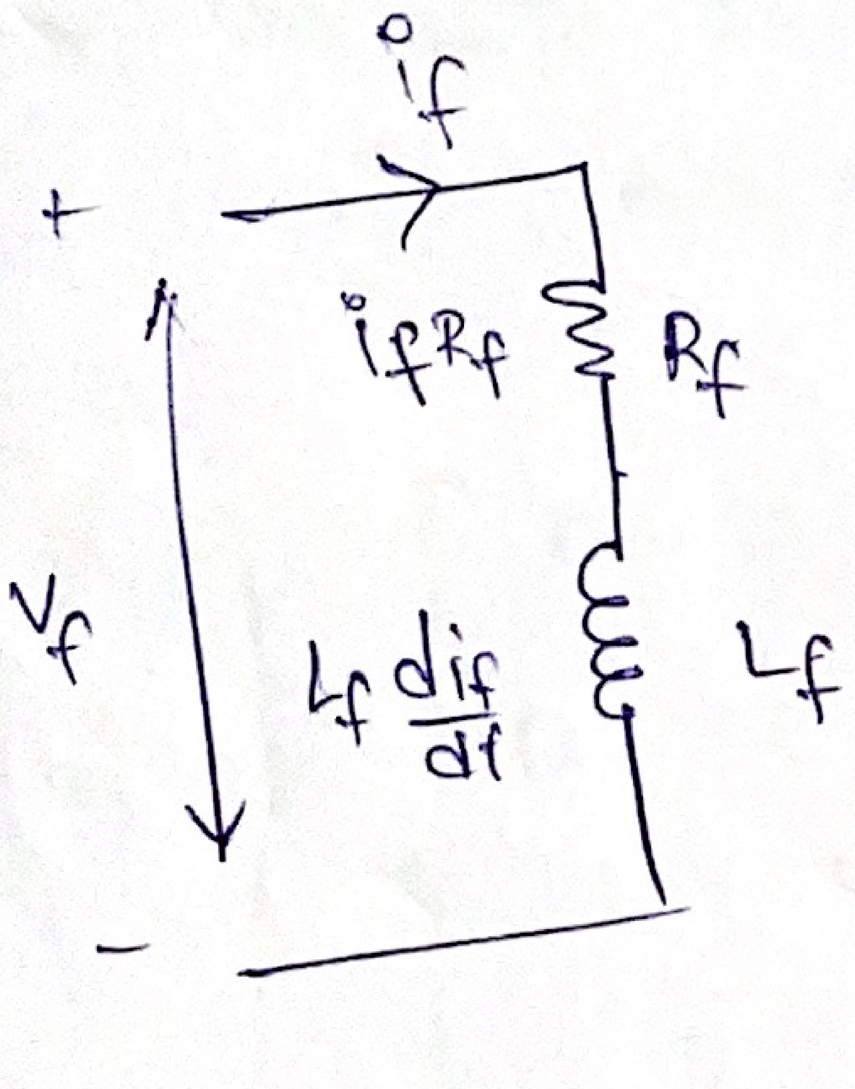
$$\dot{x}_3 = x_2$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -\frac{R_a}{L_a} & -\frac{K_b}{L_a} & 0 \\ \frac{K_t}{J} & -\frac{B}{J} & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} \frac{1}{L_a} \\ 0 \\ 0 \end{bmatrix} [u]$$

$y_1 = x_1 \quad y_2 = x_2 \quad y_3 = x_3$

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Field controlled DC motor



$T \propto i_f$

$\Rightarrow T = K_{tf} \cdot i_f$

$$R_f i_f + L_f \frac{di_f}{dt} = V_f$$

$x_1 = i_f \qquad x_2 = \omega = \frac{d\theta}{dt} \qquad x_3 = \theta$

$$\dot{x}_1 = -\frac{R_f}{L_f} x_1 + \frac{1}{L_f} u$$

$$\dot{x}_2 = \frac{K_{tf}}{J} x_1 - \frac{B}{J} x_2$$

$$\dot{x}_3 = x_2$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -\frac{R_f}{L_f} & 0 & 0 \\ \frac{K_{tf}}{J} & -\frac{B}{J} & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} \frac{1}{L_f} \\ 0 \\ 0 \end{bmatrix} [u]$$

$y_1 = \omega \quad y_2 = \theta$

O/P eq

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

rows = no. of o/p

columns = no. of state variables

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Phase variable & canonical form.

Phase variables are those particular state variables which are obtained from one of the system variables & its derivatives

Case 1 : Phase variable canonical form.

$$\overset{(n)}{y} + a_1 \overset{(n-1)}{y} + \dots + a_{n-1}\dot{y} + a_n y = u.$$

Choose output $y(t)$ and its $(n-1)$ derivatives as the State variables.

State variables
$$\begin{cases} \boxed{x_1 = y} \text{ o/p eq} \\ x_2 = \dot{y} \\ \vdots \\ x_n = \overset{(n-1)}{y} \end{cases}$$

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = x_3$$
$$\vdots$$
$$\dot{x}_{n-1} = x_n$$
$$\dot{x}_n = y_n$$

State equations

$$\dot{x}_n = y_n = -a_n x_1 - \dots - a_1 x_n + u$$

Phase variable form of the state equations

$$\dot{x} = Ax + Bu$$ $\boxed{\text{COMPANION FORM}}$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \vdots \\ \dot{x}_{n-1} \\ \dot{x}_n \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & & & & \vdots \\ 0 & 0 & 0 & \dots & 1 \\ -a_n & -a_{n-1} & -a_{n-2} & \dots & -a_1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{n-1} \\ x_n \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} u$$

- Output eq :

$$y = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_n \end{bmatrix}$$

Transfer function

$$\frac{Y(s)}{U(s)} = \frac{1}{s^n + a_1 s^{n-1} + \cdots + a_{n-1} s + a_n}$$

Consider a 4th order TF.

$$G(s) = \frac{Y(s)}{U(s)} = \frac{b_0}{s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0}$$

Obtain State space rep. Draw the signal flow graph and block diagram rep.

$$y(s)\left[s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0\right] = b_0 U(s)$$

take inv Lap transfo~

$$s^4 y(s) + a_3 s^3 y(s) + a_2 s^2 y(s) + a_1 s\, y(s) + a_0 y(s) = b_0 U$$

$$y^{(4)} + a_3 \dddot{y} + a_2 \ddot{y} + a_1 \dot{y} + a_0 y = b_0 u$$

phase variable: $x_1 = y$

State variable:

$x_2 = \dot{y}$

$x_3 = \ddot{y}$

$x_4 = \dddot{y}$

$\cancel{x_5 = y^{(4)}}$

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = x_3$$

$$\dot{x}_3 = x_4$$

$$\dot{x}_4 = y^{(4)} = b_0 u - a_3 x_4 - a_2 x_3 - a_1 x_2 - a_0 x_1$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -a_0 & -a_1 & -a_2 & a_3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ b_0 \end{bmatrix} u$$

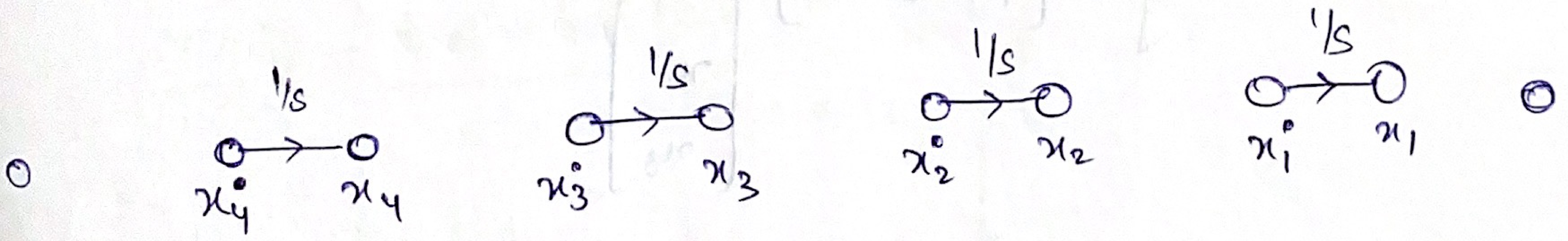
Mason's signal flow-gain formula.

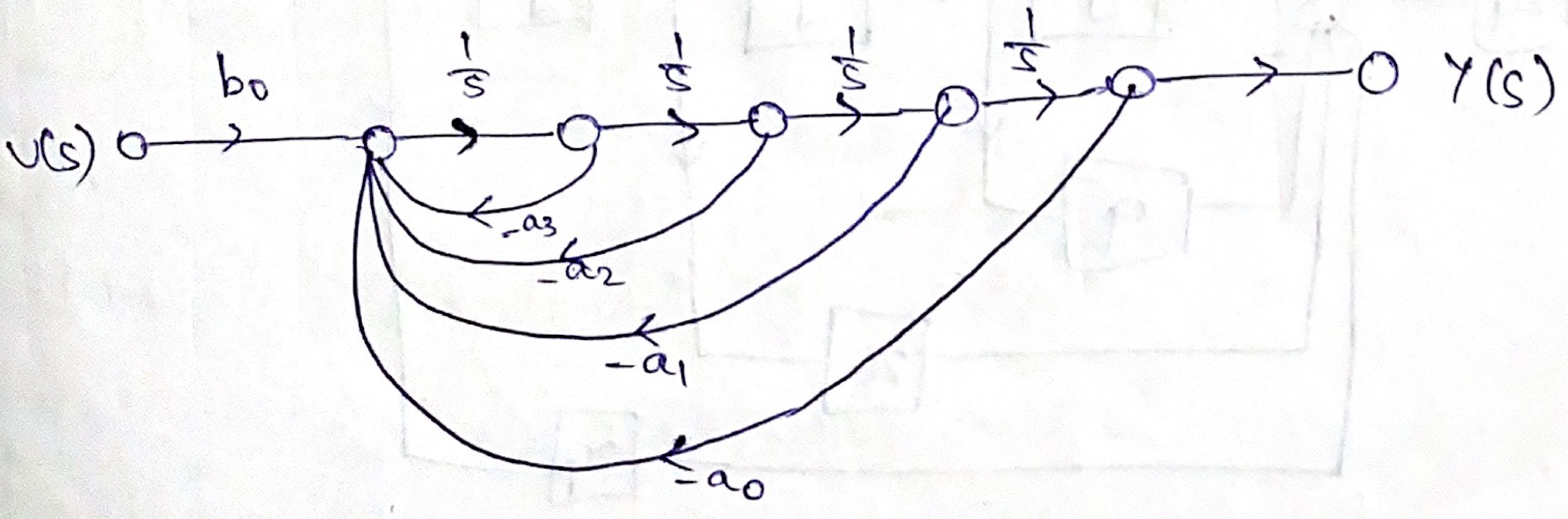
$$= \frac{\text{Sum of forward path gain}}{1 - \text{Sum of feedback path gain.}}$$

Here,

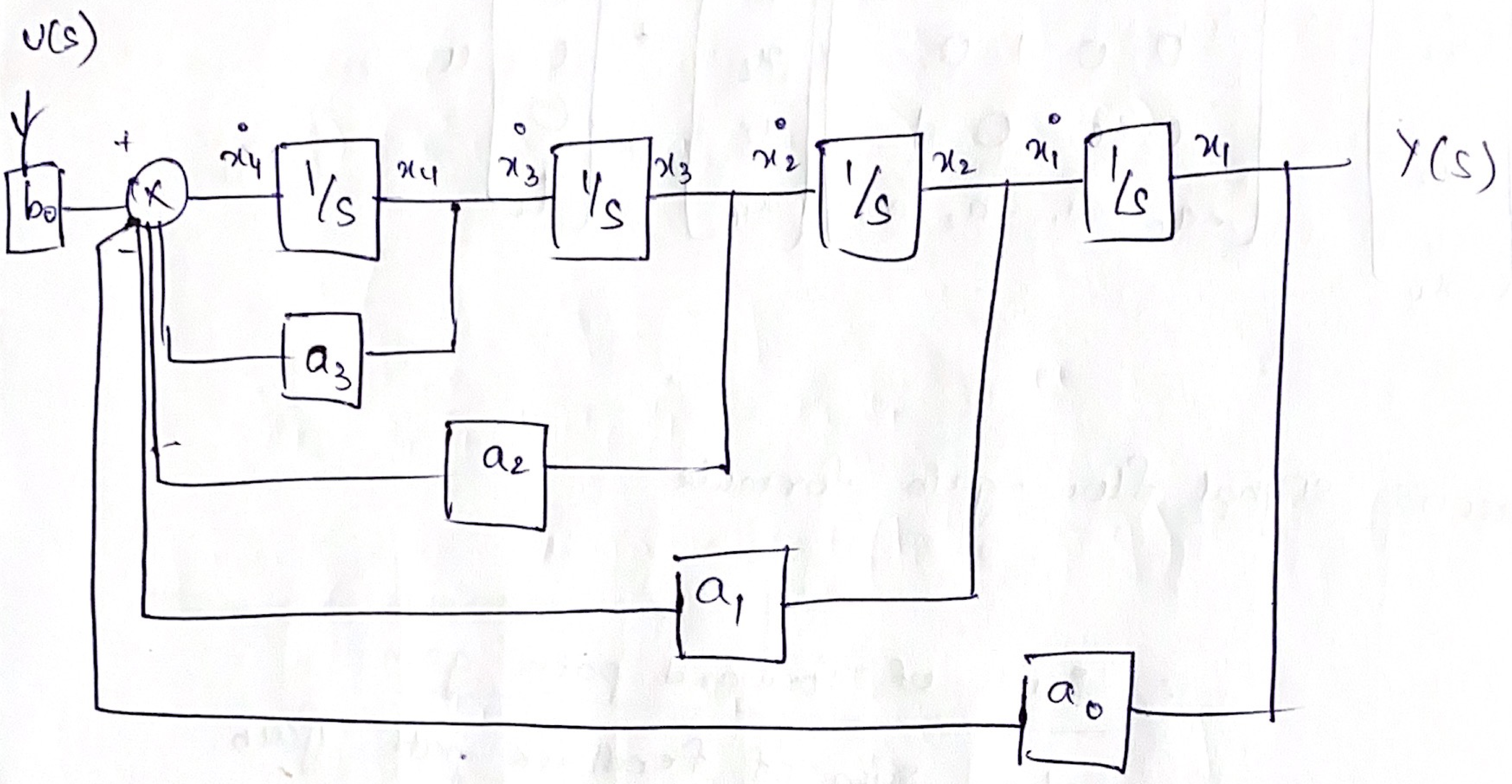
$$= \frac{b_0}{s^4\left(1 + \frac{a_3}{s} + \frac{a_2}{s^2} + \frac{a_1}{s^2} + \frac{a_0}{s^4}\right)}$$

$$= \frac{b_0 s^{-4}}{1 - \left(-\frac{a_0}{s^4} - \frac{a_1}{s^3} - \frac{a_2}{s^2} - \frac{a_3}{s}\right)}$$

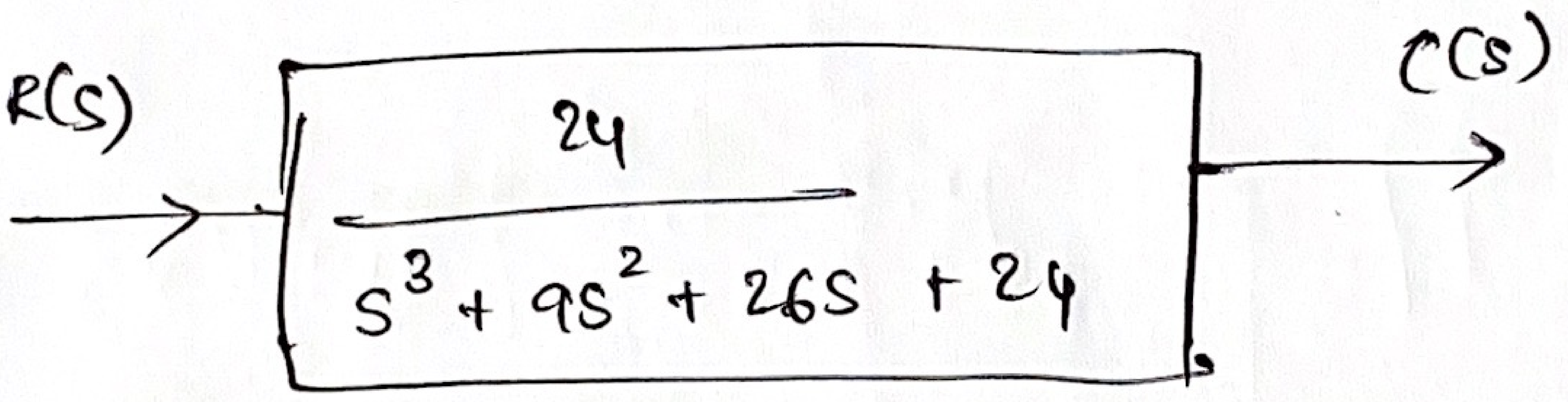




Block diagram.

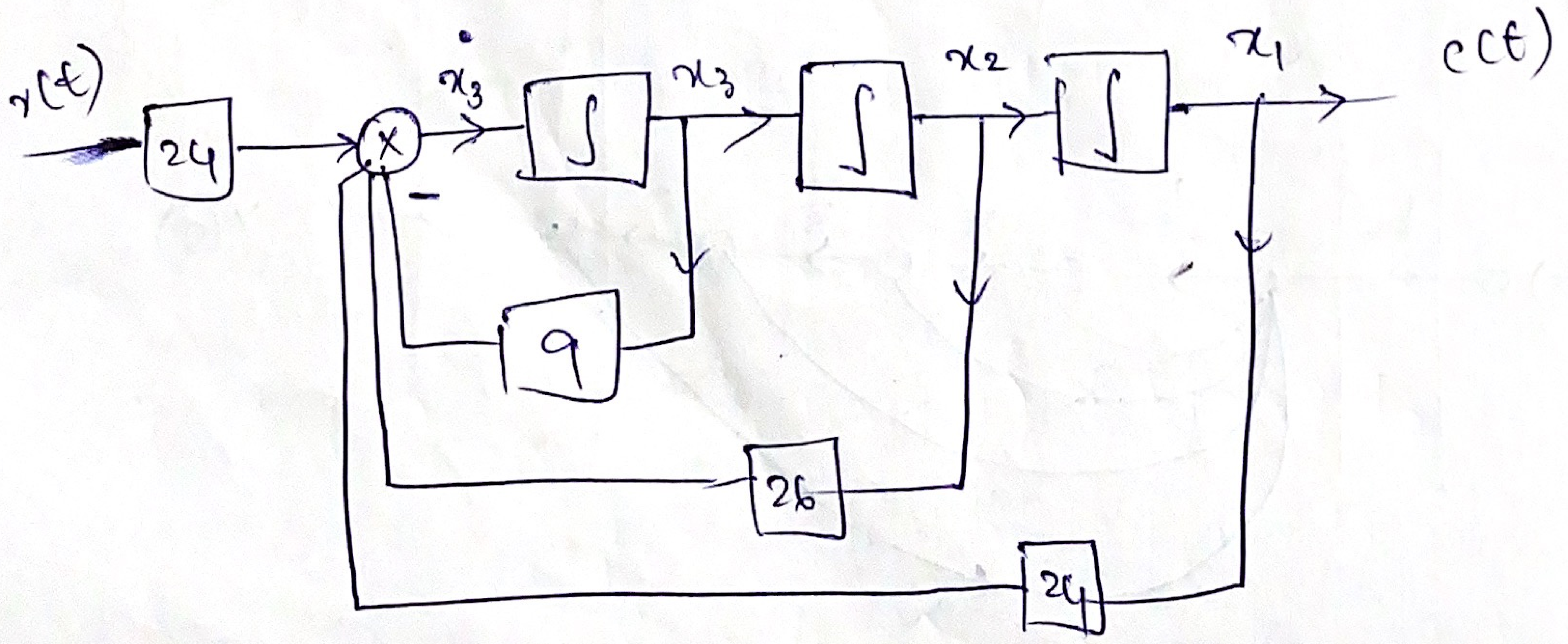


Q.



$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -24 & -26 & -9 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 24 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$



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Controllable canonical form

Consider

$$\overset{(n)}{y} + a_1 \overset{(n-1)}{y} + \cdots + a_{n-1}\dot{y} + a_n y = b_0 \overset{(n)}{u} + b_1 \overset{(n-1)}{u} + \cdots + b_{n-1}\dot{u} + b_n u$$

$$\text{Transfer function} = \frac{b_0 s^n + b_1 s^{n-1} + \cdots + b_{n-1} s + b_n}{s^n + a_1 s^{n-1} + \cdots + a_{n-1} s + a_n}$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \vdots \\ \dot{x}_{n-1} \\ \dot{x}_n \end{bmatrix} = \overset{A}{\begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & & & & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ -a_n & -a_{n-1} & -a_{n-2} & \cdots & -a_1 \end{bmatrix}} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{n-1} \\ x_n \end{bmatrix} + \overset{B}{\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}} u$$

O/P eq:

$$= \overset{C}{\begin{bmatrix} (b_n - a_n b_0) & (b_{n-1} - a_{n-1} b_0) & \cdots & (b_1 - a_1 b_0) \end{bmatrix}} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} + \overset{D}{b_0} u$$

taking ex.

$$\frac{Y(s)}{U(s)} = \frac{b_0 s^3 + b_1 s^2 + b_2 s + b_3}{s^3 + a_1 s^2 + a_2 s + a_3}$$

$$U(s) \longrightarrow \boxed{\frac{1}{s^3 + a_1 s^2 + a_2 s + a_3}} \xrightarrow{X_1(s)} \boxed{b_0 s^3 + b_1 s^2 + b_2 s + b_3} \longrightarrow Y(s)$$

$$\frac{X_1(s)}{U(s)} = \frac{1}{s^3 + a_1 s^2 + a_2 s + a_3} \longrightarrow (1)$$

$$\frac{Y(s)}{X_1(s)} = b_0 s^3 + b_1 s^2 + b_2 s + b_3 \longrightarrow (2)$$

Consider (1) T.F

$$x_1(s)\,(s^3 + a_1 s^2 + a_2 s + a_3) = U(s)$$

$$\dddot{x}_1 + a_1\ddot{x}_1 + a_2\dot{x}_1 + a_3 x_1 = u \rightarrow (A)$$

Using phase variable method.

Phase vari ← $x_1 = x_1$ $\qquad \dot{x}_1 = x_2$

$x_2 = \dot{x}_1$ $\qquad \dot{x}_2 = x_3$

$x_3 = \ddot{x}_1$ $\qquad \dot{x}_3 = -a_3x_1 - a_2x_2 - a_1x_3 + u$

(from (A))

$$\dot{X} = AX + Bu$$

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix} = \begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ -a_3 & -a_2 & -a_1\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix} + \begin{bmatrix}0\\ 0\\ 1\end{bmatrix}u$$

Consider (2) T.F

$$Y(s) = x_1(s)\left[b_0 s^3 + b_1 s^2 + b_2 s + b_3\right]$$

Inverse laplace.

$$y = b_0\dddot{x}_1 + b_1\ddot{x}_1 + b_2\dot{x}_1 + b_3 x_1$$

Replace the differential terms with first order terms from above state eq.

$$y = b_0(-a_3x_1 - a_2x_2 - a_1x_3 + u) + b_1x_3 + b_2x_2 + b_3x_1$$

$$= -a_3b_0x_1 - a_2b_0x_2 - a_1b_0x_3 + b_0u + b_1x_3 + b_2x_2 + b_3x_1$$

$$y = x_1(b_3 - a_3b_0) + x_2(b_2 - a_2b_0) + x_3(b_1 - a_1b_0) + b_0u$$

$$y = Cx + Du$$

$$= \begin{bmatrix} b_3 - a_3 b_0 & b_2 - a_2 b_0 & b_1 - a_1 b_0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + b_0 u.$$

Observable canonical form

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \vdots \\ \dot{x}_{n-1} \\ \dot{x}_n \end{bmatrix} = \begin{bmatrix} 0 & 0 & \cdots & 0 & -a_n \\ 1 & 0 & \cdots & 0 & -a_{n-1} \\ \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & & 1 & -a_1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} + \begin{bmatrix} b_n - a_n b_0 \\ b_{n-1} - a_{n-1} b_0 \\ \vdots \\ b_1 - a_1 b_0 \end{bmatrix} u$$

$$y = \begin{bmatrix} 0 & 0 & \cdots & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{n-1} \\ x_n \end{bmatrix} + b_0 u.$$

$$A_{obs} = A^T_{cont}$$

$$B_{obs} = C^T_{cont}$$

$$C_{obs} = B^T_{cont}$$

$$D_{obs} = D_{cont}$$

Consider the T.F

$$\frac{Y(s)}{U(s)} = \frac{b_0 s^3 + b_1 s^2 + b_2 s + b_3}{s^3 + a_1 s^2 + a_2 s + a_3}$$

Inv. Laplace transform

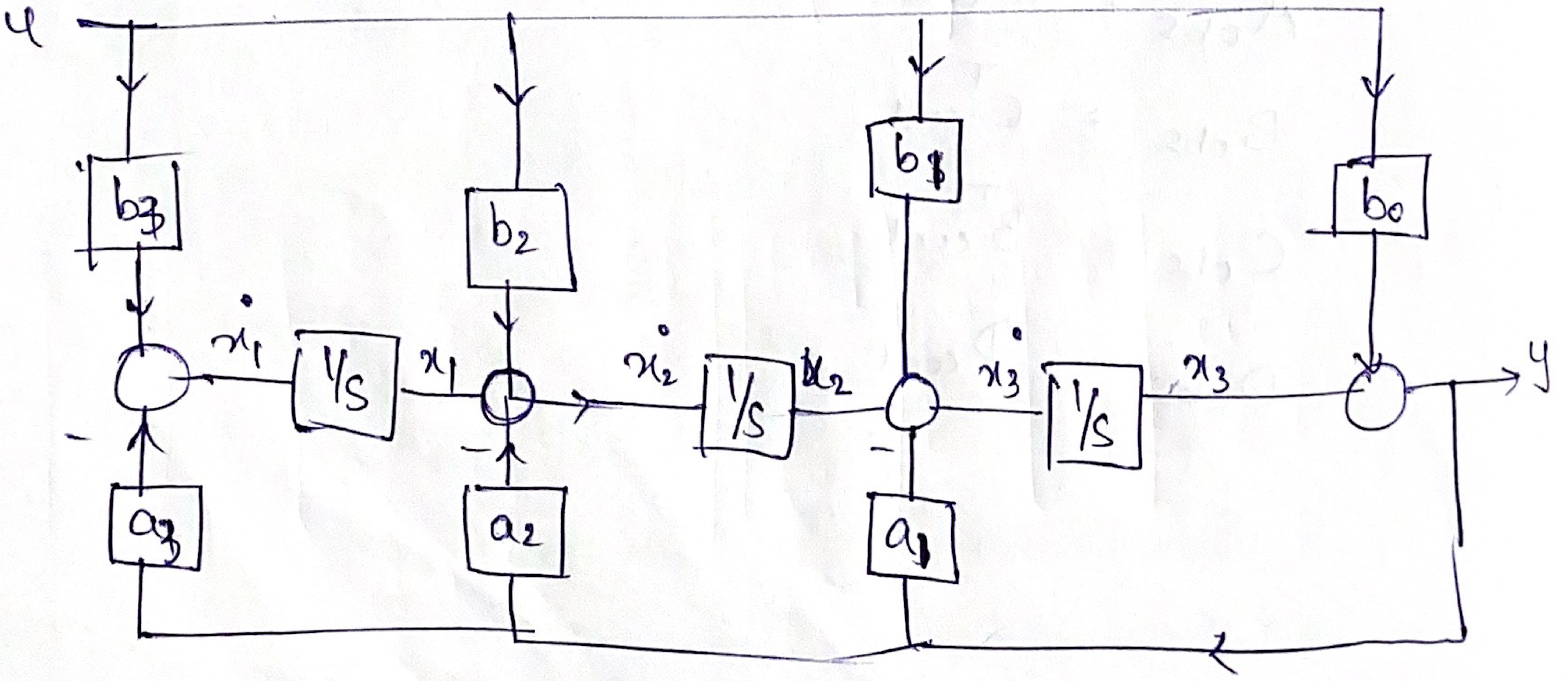
$$\dddot{y} + a_1\ddot{y} + a_2\dot{y} + a_3 y = b_0\dddot{u} + b_1\ddot{u} + b_2\dot{u} + b_3 u$$

$$\frac{d^3y}{dt^3} = b_0\frac{d^3u}{dt^3} + \cancel{b_1\frac{d^2u}{dt^2}}\ \frac{d^2}{dt^2}(b_1u - a_1y) + \frac{d}{dt}(b_2u - a_2y) + (b_3u - a_3y)$$

Integrate 3 times

$$y = b_0u + \int(b_1u - a_1y)\,dt + \iint(b_2u - a_2y)\,dt + \iiint(b_3u - a_3y)\,dt$$

Block diagram rep.



from block diagram.

O/p eq. $\Rightarrow y = x_3 + b_0 u$

$$\dot{x}_1 = b_3u - a_3y$$
$$= b_3u - a_3x_3 - a_3b_0u$$
$$\Rightarrow \dot{x}_1 = -a_3x_3 + u(b_3 - a_3b_0)$$

$$\dot{x}_2 = b_2 u - a_2 y + x_1$$

$$= b_2 u - a_2 (x_3 + b_0 u) + x_1$$

$$\Rightarrow \dot{x}_2 = -a_2 x_3 + (b_2 - a_2 b_0) u + x_1$$

$$\dot{x}_3 = x_2 - a_1 y + b_1 u$$

$$\Rightarrow \dot{x}_3 = x_2 - a_1 x_3 + (b_1 - a_1 b_0) u$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -a_3 \\ 1 & 0 & -a_2 \\ 0 & 1 & -a_1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} b_3 - a_3 b_0 \\ b_2 - a_2 b_0 \\ b_1 - a_1 b_0 \end{bmatrix} u$$

$y =$

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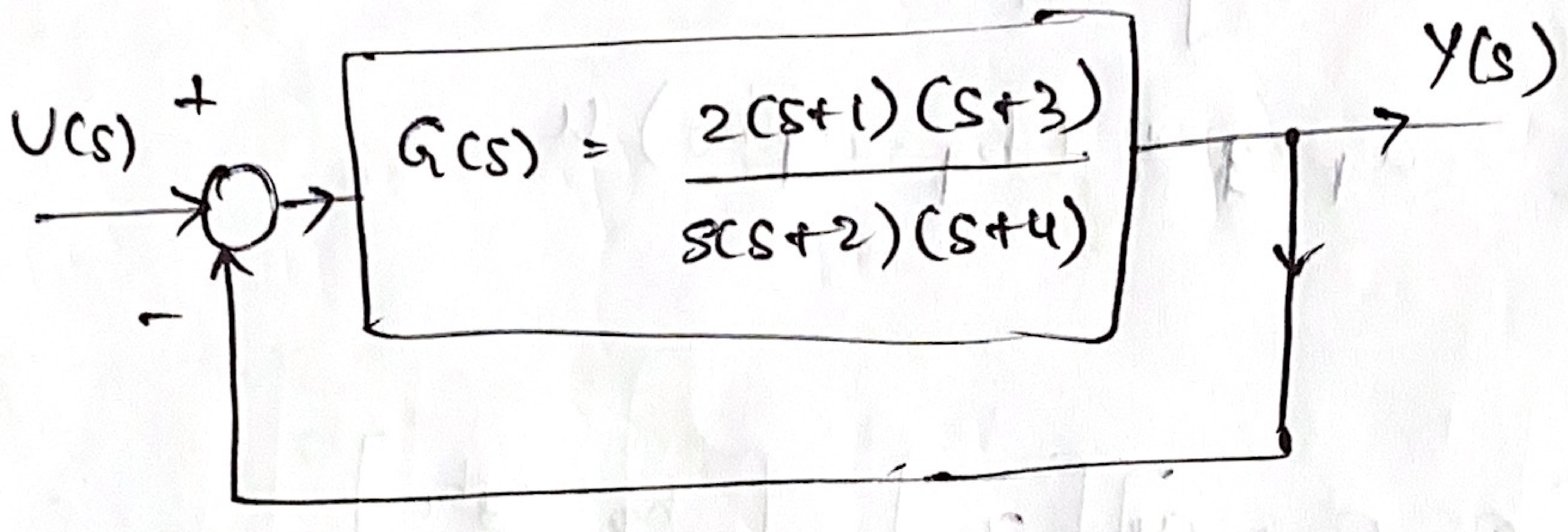
Tutorial - 2

ex. 3.2 MCS - 12th edition, Dorf and Bishop.

Derive the controllable & observable canonical form.



Sol. $\frac{Y(s)}{U(s)} = \frac{2s^2 + 8s + 6}{s^3 + 8s^2 + 16s + 6}$

Controllable

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -16 & -8 \end{bmatrix}}_{A} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}}_{B} u$$

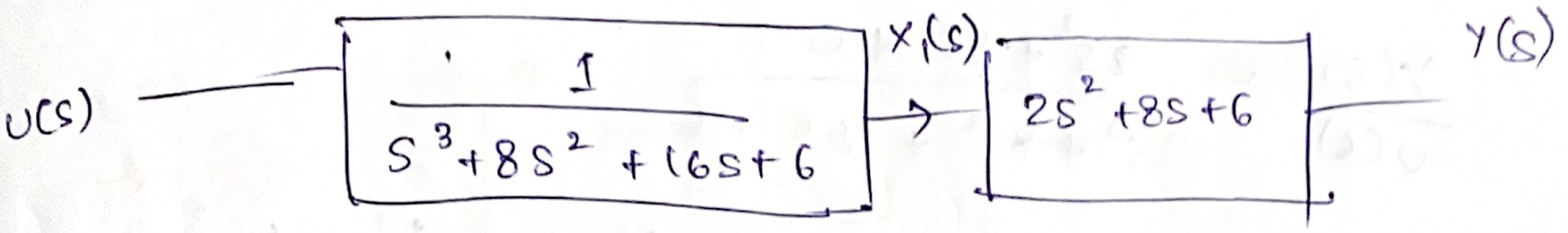
$$y = \underbrace{\begin{bmatrix} 6 & 8 & 2 \end{bmatrix}}_{C} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \underbrace{[0]}_{D}$$

Observable

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -6 \\ 1 & 0 & -16 \\ 0 & 1 & -8 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 6 \\ 8 \\ 2 \end{bmatrix} u$$

$$y = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

controllable canonical form.



$$\frac{X_1(s)}{U(s)} = \frac{1}{s^3+8s^2+16s+6} \rightarrow (1) \qquad \frac{Y(s)}{X_1(s)} = 2s^2+8s+6. \rightarrow (2)$$

Inverse Laplace.

$$\dddot{x}_1 + 8\ddot{x}_1 + 16\dot{x}_1 + 6x_1 = u$$

phase variable

$\rightarrow x_1 = x_1$ $\qquad \dot{x}_1 = x_2$

$x_2 = \dot{x}_1$ $\qquad \dot{x}_2 = x_3$

state variables $x_3 = \dot{x}_2 = \ddot{x}_1$ $\qquad \dot{x}_3 = u - 8x_3 - 16x_2 - 6x_1$

$$\dot{x} = Ax + Bu$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -16 & -8 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

Consider (2)

$$Y(s) = X_1(s)(2s^2+8s+6)$$

Inv. Laplace.

$$y = 2\ddot{x}_1 + 8\dot{x}_1 + 6x_1 \qquad \text{from state variables.}$$

$$= 2x_3 + 8x_2 + 6x_1$$

$$y = \begin{bmatrix} 6 & 8 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Observable canonical form

$$\frac{Y(s)}{U(s)} = \frac{2s^2 + 8s + 6}{s^3 + 8s^2 + 16s + 6}.$$

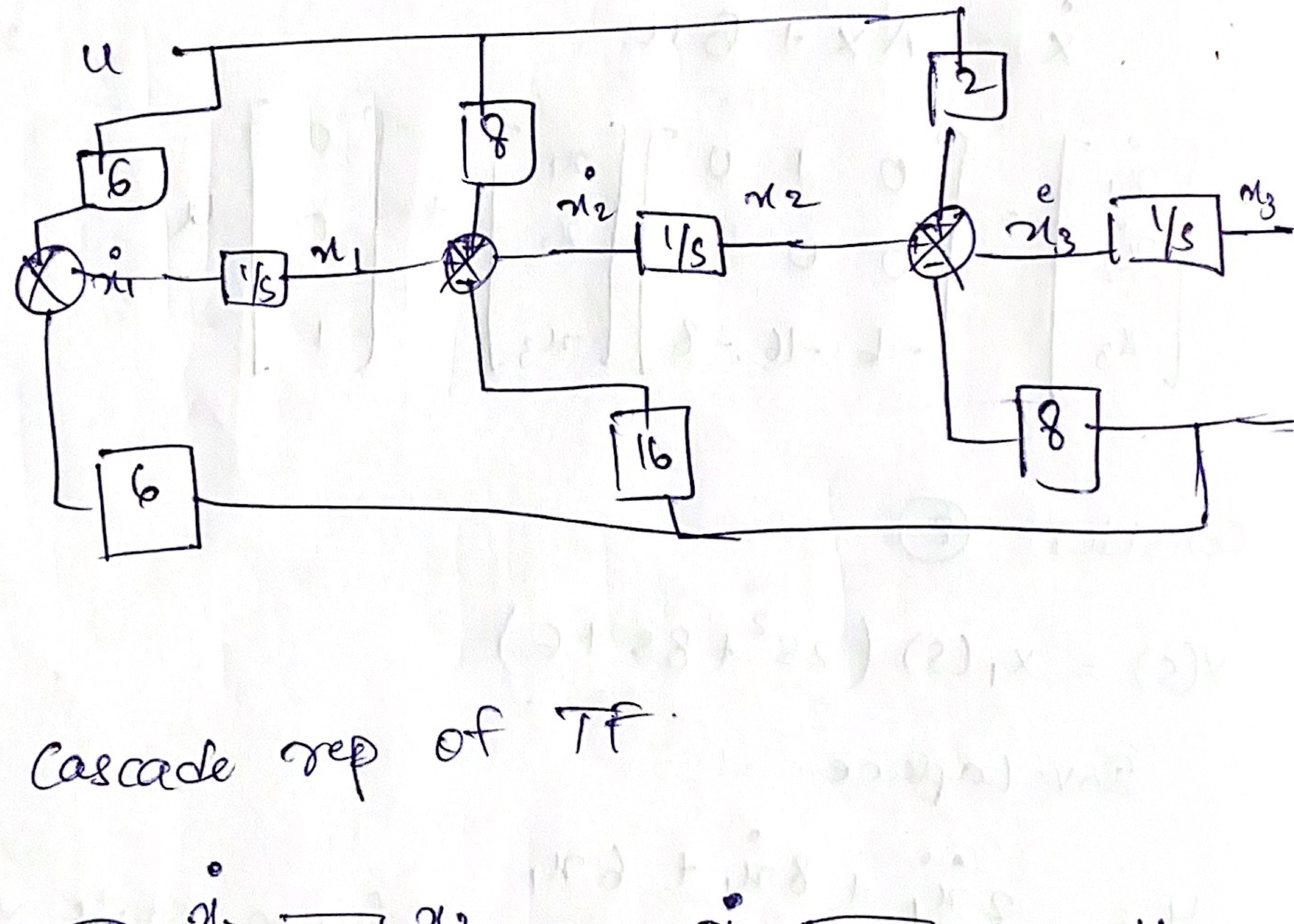
$$\dddot{y} + 8\ddot{y} + 16\dot{y} + 6y = 2\ddot{u} + 8\dot{u} + 6u$$

$$\frac{d^3y}{dt^3} = \frac{d^2(2u - 8y)}{dt^2} + \frac{d(8u - 16y)}{dt} + (6u - 6y)$$

Integrate 3 tim

$$y = \int(2u - 8y) + \iint(8u - 16y) + \iiint(6u - 6y)$$

y =



$y = x_3$

$\dot{x}_1 = 6u - 6y$

Cascade rep of TF

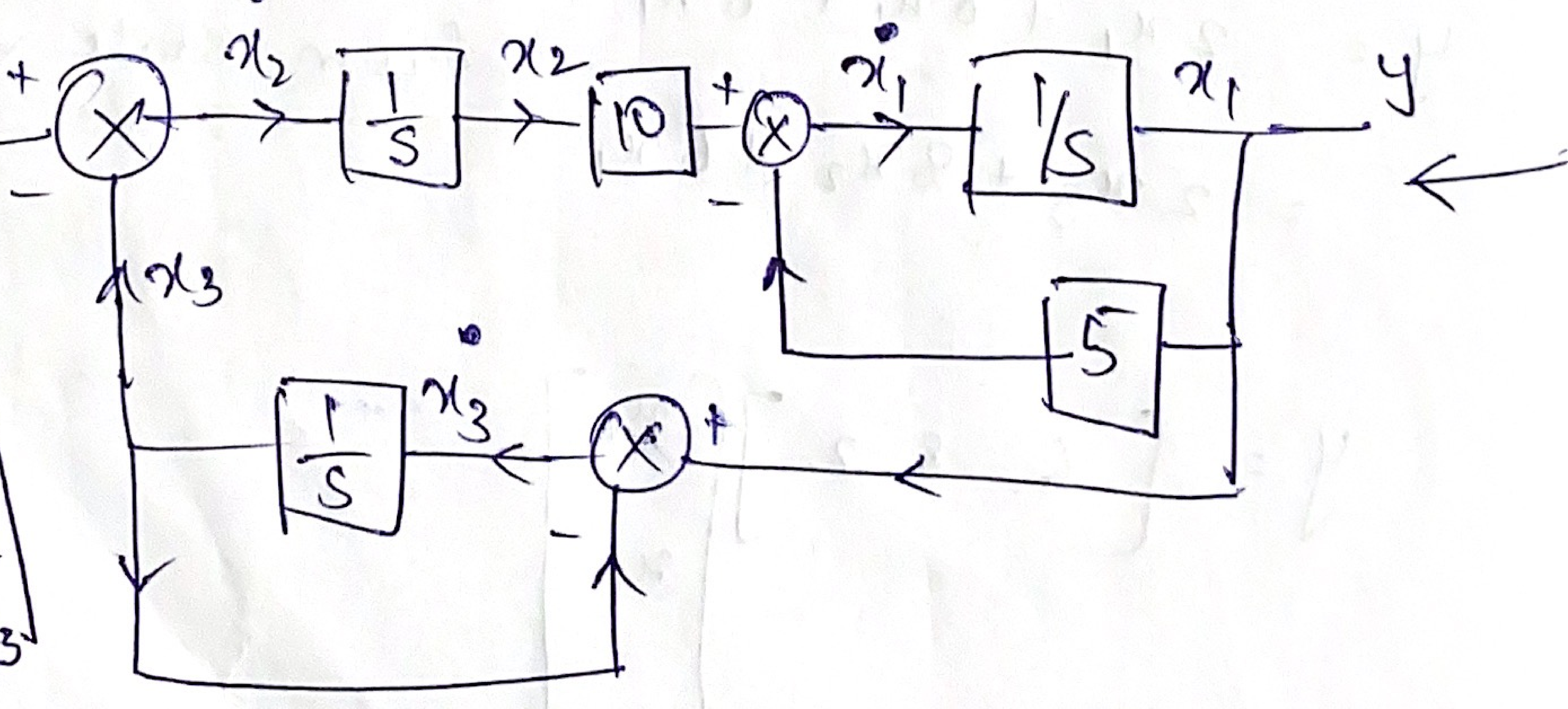
$\dot{x}_1 = -6x_3 + 6u$

$\dot{x}_2 = 8u - 16y + x_1$

$\dot{x}_2 = x_1 - 16x_3 + 8u$

$\dot{x}_3 = 2u - 8y + x_2$

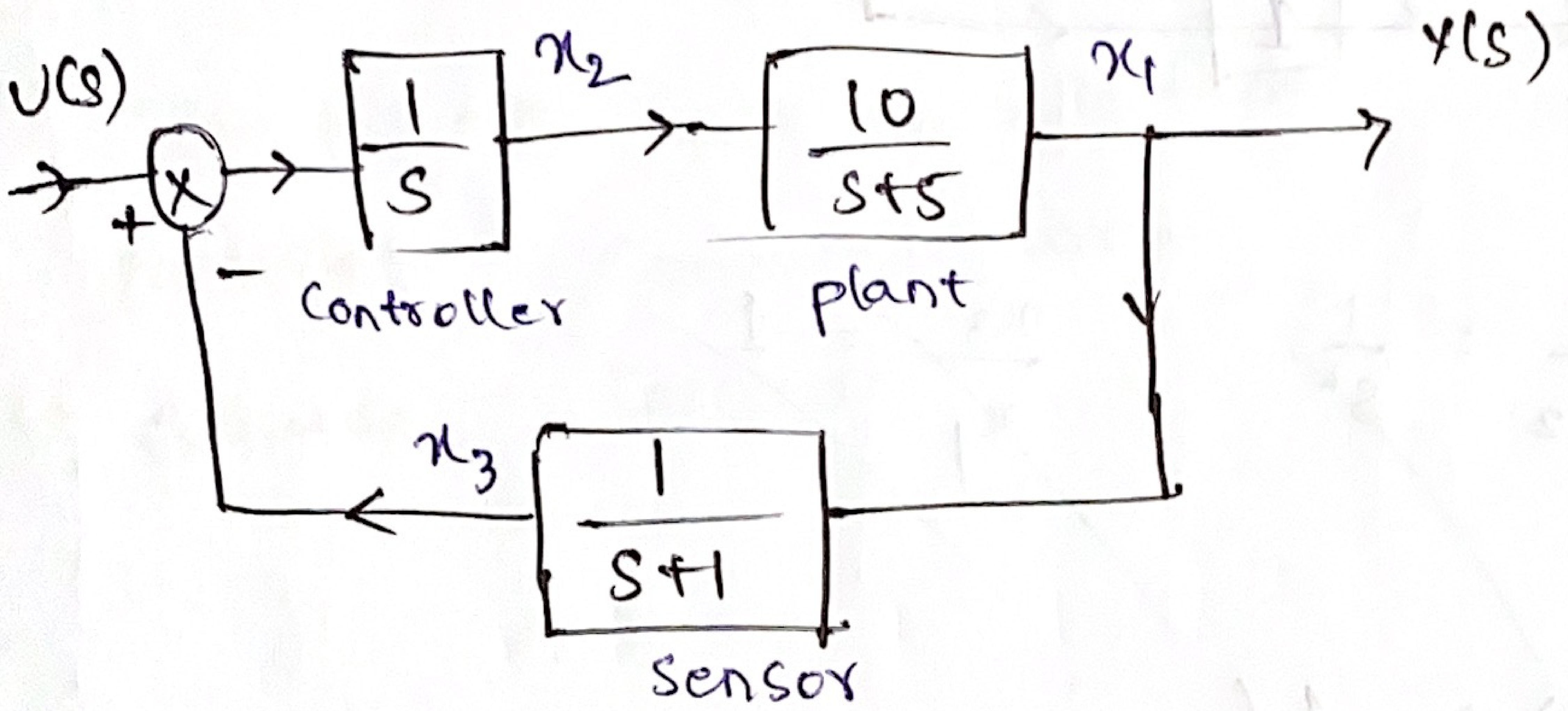
$= x_2 - 8x_3 + 2u$



$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -6 \\ 1 & 0 & -16 \\ 0 & 1 & -8 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 6 \\ 8 \\ 2 \end{bmatrix} u$$

$$y = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + 0u$$

Q. Obtain a state space model.



Sol.

$$\frac{\frac{10}{s^2+5s}}{1+\frac{10}{(s^2+5s)}\frac{1}{(s+1)}} = \frac{\frac{10}{\cancel{(s^2+5s)}}}{\frac{s^3+s^2+5s^2+5s+10}{\cancel{(s^2+5s)}(s+1)}}$$

$$\frac{Y(s)}{U(s)} = \frac{10s+10}{s^3+6s^2+5s+10}$$

(OR)

$$\frac{x_3}{x_1} = \frac{1}{s+1} \rightarrow ① \qquad \frac{x_1}{x_2} = \frac{10}{s+5} \; ② \qquad \frac{x_2}{U(s)-x_3} = \frac{1}{s} \; ③$$

$Y(s) = X_1(s)$ ($\because$ o/p eq)

Inv. Laplace

$\dot{x}_3 + x_3 = x_1 \Rightarrow \dot{x}_3 = x_1 - x_3$

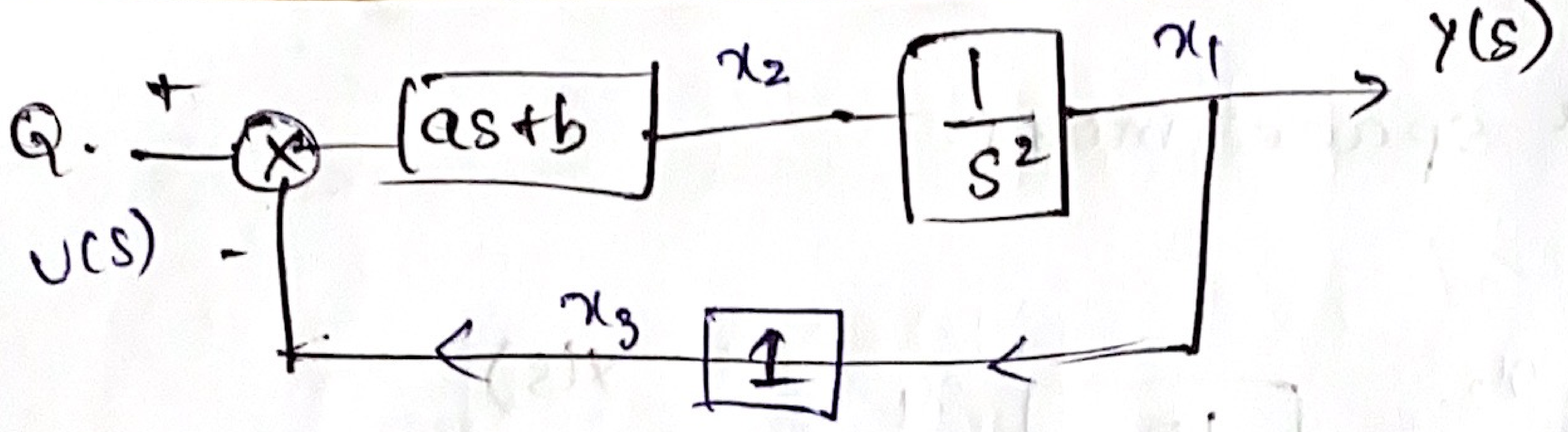
$\dot{x}_1 + 5x_1 = 10x_2 \Rightarrow \dot{x}_1 = -5x_1 + 10x_2$

$\dot{x}_2 = u - x_3 \Rightarrow \dot{x}_2 = -x_3 + u$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -5 & 10 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

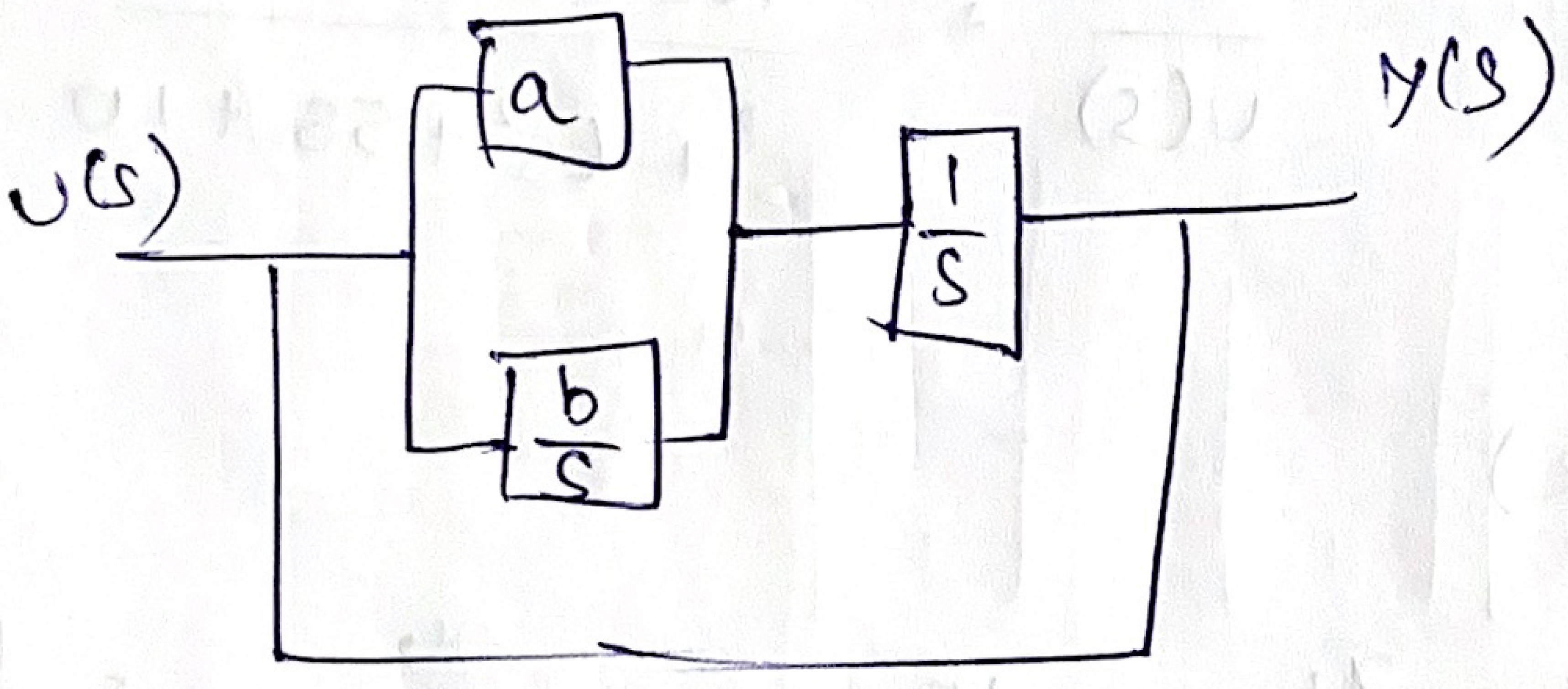
Q.



$$\frac{x_1}{x_2} = \frac{1}{s^2} \qquad \frac{x_3}{x_1} = 1$$

$$(as+b) = \frac{1}{s}\Big($$

$$\frac{as+b}{s^2} = \frac{1}{s}\left(\frac{a}{s} + \frac{b}{s^2}\right) = \frac{1}{s}\left(a + \frac{b}{s}\right)$$



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Decomposition of TF

Direct → Phase variable form

parallel - Canonical form.

Cascade - Cascade.