MANIPAL INSTITUTE OF TECHNOLOGY
MANIPAL
(A constituent unit of MAHE, Manipal)

Modern Control Theory (ICE 3153)

Introduction to State Space Analysis

Bipin Krishna
Assistant Professor (Sr.)
ICE Department
Manipal Institute of Technology
MAHE, Karnataka, India

Syllabus

Hours/ week: 3L+1T **Number of credits: 4**

- State Space Analysis: Introduction, definitions, concept and advantages of state variable analysis, state equation of linear continuous data systems-electrical, mechanical, electromechanical systems. Phase variable and canonical form of state representation. Derivation of state models from transfer function and ordinary differential equation, direct, cascade, parallel realizations. **(10 hrs)**
- Stability analysis, Eigen values, eigen vectors, diagonalization, transformation into canonical forms, Solution of state equations, State transition matrix-Cayley Hamilton theorem. **(08 hrs)**
- Controllability and observability. Pole placement –Ackerman's formula, state feedback and output feedback, Observer design (qualitative). **(06 hrs)**

- Non Linear Systems: Characteristics of non- linear systems, common type of non-linearities, Linearization of Non-linear systems, Phase plane analysis of non-linear systems- Singular points on phase plane. Construction of the phase trajectory by Isocline method, Pell's method and delta method. **(08 hrs)**
- Describing function: definition, determination of describing function for various nonlinearities viz, On-off, dead zone, on-off with dead zone, saturation. Determination of existence and stability of limit cycle. **(08 hrs)**
- Lyapunov's stability analysis: Sign definiteness, quadratic functions. Sylvester's criterion, Definitions and Lyapunov Theorems of stability, instability and asymptotic stability. Stability in the large. Solution by direct method, generation of Lyapunov function for continuous time state equations. **(08 hrs)**

References:

- K. Ogata, Modern Control Engineering, Prentice Hall India, (5e), 2011.
- Nagrath and Gopal, Control System Engineering, New age international Limited, (2e), 1984.
- K. Ogata "State space analysis of control systems" Prentice Hall India 1967.
- H.K. Khalil, *Nonlinear Systems*, (3e), PHI, 2002
- J.J.E. Slotine and W.Li, *Applied Nonlinear control*, Prentice Hall, 1998.

Course Outcomes (COs)

	At the end of this course, the student should be able to:	**No. of Contact Hours**	**Marks**
CO1:	Understand the basic concept of state space analysis and apply to obtain the stae model in different forms.	10	20
CO2:	Analyze the stability, performance and apply state transformation technique.	8	18
CO3:	Design observer and state variable feedback control law.	6	12
CO4:	Understand different types of nonlinearities and analyse non linear system behaviour	16	32
CO5:	Analyse the stability of system using Lyaponuv Theory.	8	18
	Total	48	100

Quick Recap..

- What is the role of a Control Engineer?
    - Develop model
    - Check stability, if its unstable design a controller
    - Change the performance

Modern Vs Conventional control theory

- In case of state space we have to know the, what are the internal parameter of this systems along with input and output.

- Where as in case of transfer function approach we we cannot get any information about the plant only we are are applying input and we are getting the output.

- Modern control theory is applicable to multiple-input-multiple-output systems, which may be linear or nonlinear, time invariant or time varying.

- while the conventional is applicable only to linear time-invariant single-input-single-output systems.

- State space approach can handle the system with non-zero initial conditions.
- Classical approach is base on transfer function approach, and you know that in transfer function we have to neglect the initial conditions
- Modern control theory is essentially a time-domain approach.
- While conventional control theory is a complex frequency-domain approach.
- State equation are always first order irrespective of the system’s order, input and output.

- The conventional indicators of the closed loop performance are the closed loop poles or the location of the closed loop poles.
- For high order system by varying limited number of constants in the control transfer function, one can vary the location of only a few of closed loop poles not all of them.

State of a Dynamic System

- The state of a dynamic system is the smallest set of variables (called *state variables)* such that the knowledge of these variables at *t* = *to,* together with the knowledge of the input for *t* **>=** *to,* completely determines the behaviour of the system for any time *t* **>=** *to.*

- Note that the concept of state is by no means limited to physical systems. It is applicable to biological systems, economic systems, social systems, and others.

State Variables

- The state variables of a dynamic system are the variables making up the smallest set of variables that determine the state of the dynamic system.
- If n variables are needed to completely describe the behavior of a dynamic system.
- (so that once the input is given for t **>=**to and the initial state at t = *to* is specified, the future state of the system is completely determined.
- then such n variables are a set of state variables.

State Vector

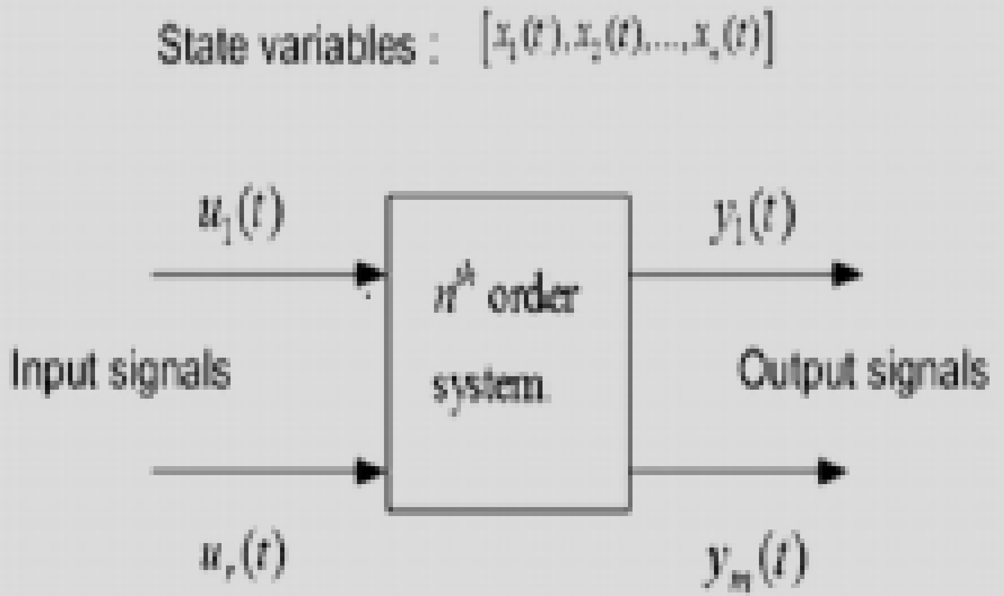
- If n state variables are needed to completely describe the behaviour of a given system, then these n state variables can be considered the n components of a vector x. Such a vector is called a state vector.

- A state vector is thus a vector that determines uniquely the system state x(t) for any time t >=to, once the state at t = to is given and the input u(t) for t >=to is specified.

State Space

- The n-dimensional space whose coordinate axes consist of the x1 axis, x2 axis, . . . , xn axis, where x1 , x2,. . . , xn are state variables; is called a state space.
- Any state can be represented by a point in the state space.

System Block Diagram



System block diagram

State-Space Equations

- In state-space analysis we are concerned with three types of variables that are involved in the modelling of dynamic systems:
- input variables
- Output variables
- and state variables
- The dynamic system must involve elements that memorize the values of the input for t **>=** *t1.*
- Since integrators in a continuous-time control system serve as memory devices, the outputs of such integrators can be considered as the variables that define the internal state of the dynamic system.
- Thus the outputs of integrators serve as state variables.

$$
\begin{aligned}
\dot{x}_1(t) &= f_1(x_1, x_2, \ldots, x_n; u_1, u_2, \ldots, u_r; t) \\
\dot{x}_2(t) &= f_2(x_1, x_2, \ldots, x_n; u_1, u_2, \ldots, u_r; t) \\
&\cdot \\
&\cdot \\
&\cdot \\
\dot{x}_n(t) &= f_n(x_1, x_2, \ldots, x_n; u_1, u_2, \ldots, u_r; t)
\end{aligned}
$$

$$
\begin{aligned}
y_1(t) &= g_1(x_1, x_2, \ldots, x_n; u_1, u_2, \ldots, u_r; t) \\
y_2(t) &= g_2(x_1, x_2, \ldots, x_n; u_1, u_2, \ldots, u_r; t) \\
&\cdot \\
&\cdot \\
&\cdot \\
y_m(t) &= g_m(x_1, x_2, \ldots, x_n; u_1, u_2, \ldots, u_r; t)
\end{aligned}
$$

$$\mathbf{x}(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_n(t) \end{bmatrix}, \qquad \mathbf{f}(\mathbf{x}, \mathbf{u}, t) = \begin{bmatrix} f_1(x_1, x_2, \dots, x_n; u_1, u_2, \dots, u_r; t) \\ f_2(x_1, x_2, \dots, x_n; u_1, u_2, \dots, u_r; t) \\ \vdots \\ f_n(x_1, x_2, \dots, x_n; u_1, u_2, \dots, u_r; t) \end{bmatrix},$$

$$\mathbf{y}(t) = \begin{bmatrix} y_1(t) \\ y_2(t) \\ \vdots \\ y_m(t) \end{bmatrix}, \qquad \mathbf{g}(\mathbf{x}, \mathbf{u}, t) = \begin{bmatrix} g_1(x_1, x_2, \dots, x_n; u_1, u_2, \dots, u_r; t) \\ g_2(x_1, x_2, \dots, x_n; u_1, u_2, \dots, u_r; t) \\ \vdots \\ g_m(x_1, x_2, \dots, x_n; u_1, u_2, \dots, u_r; t) \end{bmatrix}, \qquad \mathbf{u}(t) = \begin{bmatrix} u_1(t) \\ u_2(t) \\ \vdots \\ u_r(t) \end{bmatrix}$$

$$\dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}, \mathbf{u}, t)$$

$$\mathbf{y}(t) = \mathbf{g}(\mathbf{x}, \mathbf{u}, t)$$

- If the above state and output Equations are linearized about the operating state, each state variables now becomes a linear combination of the system's state and input, i.e.

$$
\begin{aligned}
\dot{x}_1 &= a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n + b_{11}u_1 + b_{12}u_2 + \ldots + b_{1r}u_r \\
\dot{x}_2 &= a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n + b_{21}u_1 + b_{22}u_2 + \ldots + b_{2r}u_r \\
&\vdots \qquad\qquad\qquad\qquad \vdots \\
\dot{x}_n &= a_{n1}x_1 + a_{n2}x_2 + \ldots + a_{nn}x_n + b_{n1}u_1 + b_{n2}u_2 + \ldots + b_{nr}u_r
\end{aligned}
$$

- In vector matrix form, $\dot{\mathbf{x}}(t) = \mathbf{A}(t)\mathbf{x}(t) + \mathbf{B}(t)\mathbf{u}(t)$

- where *A(t)* is called the state matrix and *B(t)* the input matrix

[A] is (n x n) matrix which is defined as

$$[A]_{(n\times n)} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}$$

[B] is (n x r) input matrix which is defined as

$$[B]_{(n\times r)} = \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1r} \\ b_{21} & b_{22} & \cdots & b_{2r} \\ \vdots & \vdots & & \vdots \\ b_{n1} & b_{n2} & \cdots & b_{nr} \end{bmatrix}$$

$$y_1(t) = c_{11}x_1(t) + \ldots + c_{1n}x_n(t) + d_{11}u_1(t) + \ldots + d_{1r}u_r(t)$$

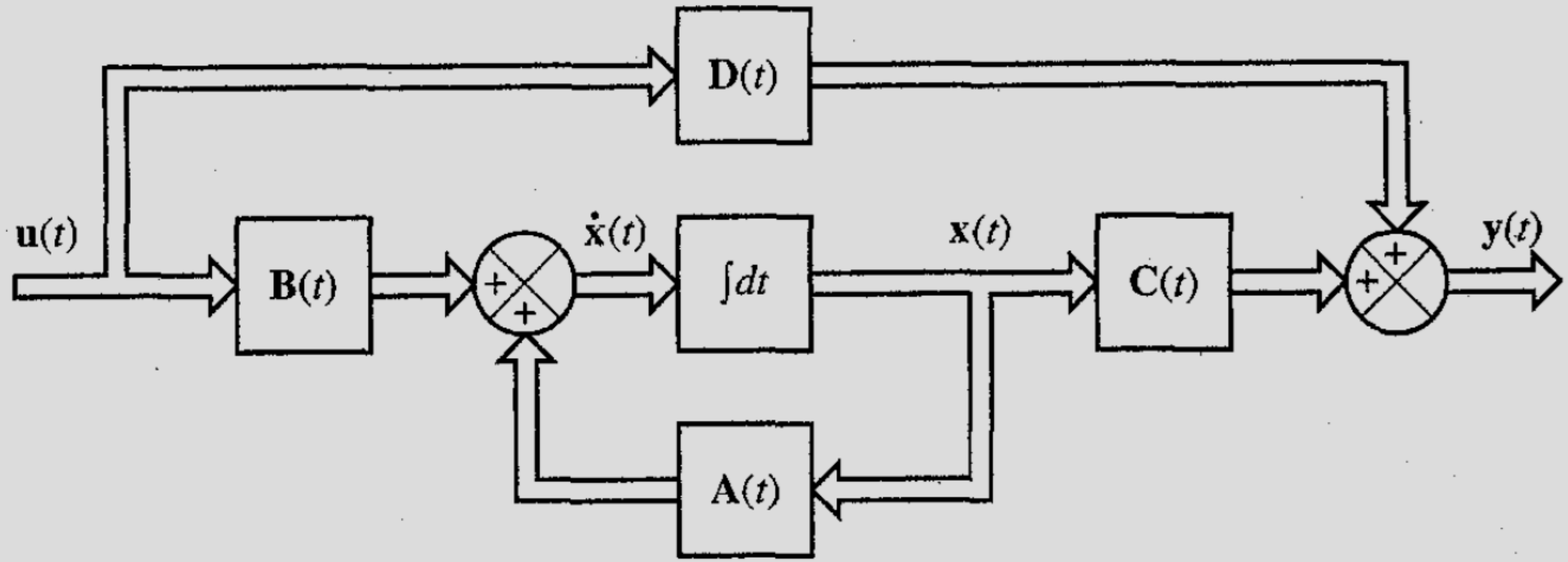
$$\vdots$$

$$y_m(t) = c_{m1}x_1(t) + \ldots + c_{mn}x_n(t) + d_{m1}u_1(t) + \ldots + d_{mr}u_r(t)$$

$$y(t) = Cx(t) + Du(t)$$

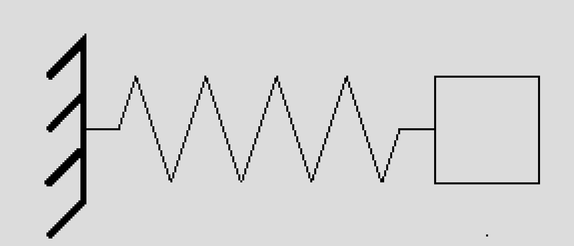
$$[C]_{(m\times n)} = \begin{bmatrix} c_{11} & c_{12} & \ldots & c_{1n} \\ c_{21} & c_{22} & \ldots & c_{2n} \\ \vdots & \vdots & & \vdots \\ c_{m1} & c_{m2} & \ldots & c_{mn} \end{bmatrix}, [D]_{(m\times r)} = \begin{bmatrix} d_{11} & d_{12} & \ldots & d_{1n} \\ d_{21} & d_{22} & \ldots & d_{2n} \\ \vdots & \vdots & & \vdots \\ d_{m1} & d_{m2} & \ldots & d_{mr} \end{bmatrix}$$

- *C(t)* the output matrix
- And *D(t)* the direct transmission matrix.



D(t)
u(t)
B(t)
+
+
ẋ(t)
∫dt
x(t)
C(t)
+
+
y(t)
A(t)

Sate variable selection



- Where the system will be after 1 seconds?
- Initial Conditions:
- 1. Is the mass currently moving?

- 2. How hard is the spring pulling on the mass (or spring force)
- Why not more variables?

- Number of state variables should be minimum.
- Number of state variable must be linearly independent.
- Number of state variables is equal to number independent energy storage element.
- Number of state variables equals to the order of the differential equations.
- Number of state variables are equal to number of integrators.