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ICE 2154 NETWORK ANALYSIS AND SIGNALS

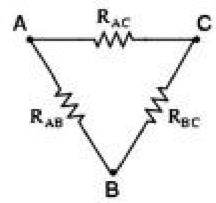
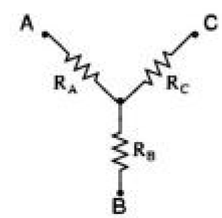
Symbols and Units:

Quantity	Symbols	Unit	Equivalent Unit	Unit Abbreviation
Charge	q	Coulomb	-	C
Current	I , i	Ampere	Coulomb/Second	amp
Flux Linkages	Ψ	Weber-Turn	-	Wb
Energy	W , w	Joul	Newton-Meter	J
Voltage	V ,v	Volt	Joul/Coulomb	V
Power	P , p	Watt	Joul/Second	W
Capacitance	C	Farad	Coulomb/Volt	F
Inductance	M , L	Henry	Weber/Ampere	H
Resistance	R	Ohm	Aolt/Ampere	Ω
Conductance	G	Mho	Ampere/Volt	
Time	t	Second	-	sec
Frequency	f	Hertz	cycles/second	Z H
Frequency	ω	Radian/second	-	-

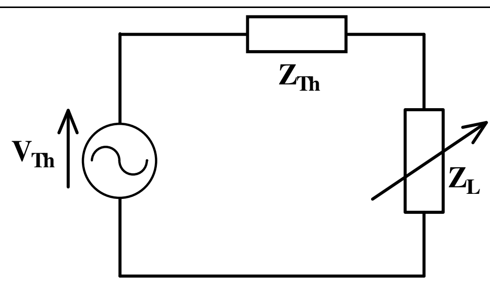
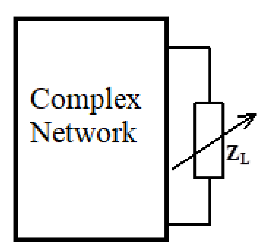
Relationships between the network parameters:

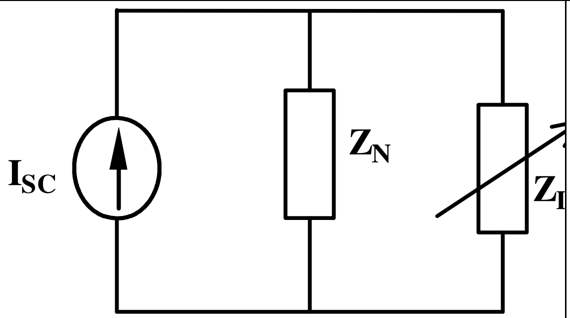
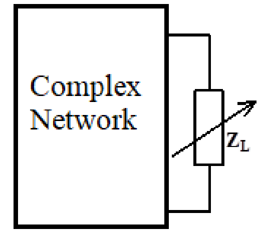
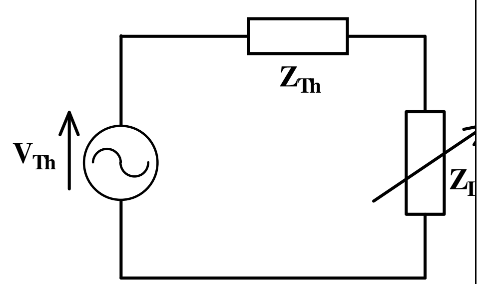
Parameter	Basic Relationship	Voltage-Current Relationship	Energy
R $G = \frac{1}{R}$	$v = Ri$	$v_R = i_R R$ $i_R = G\, v_R$	$w_R = \int_{-\infty}^{t} v_R i_R \; dt$
$L\ (\ or\ M)$	$\Psi = Li$	$v_L = L\frac{di}{dt}$ $i_L = \frac{1}{L}\int_{-\infty}^{t} v_L \; dt$	$w = \frac{1}{2}Li^2$
C $D = \frac{1}{C}$	$Q = Cv$	$v_C = \frac{1}{C}\int_{-\infty}^{t} i_C \; dt$ $i_C = C\frac{dv}{dt}$	$w = \frac{1}{2}Cv^2$

Star Delta Conversion:

1.	Start - delta (wye-delta) conversion	Star to delta conversion: $$R_{AB} = \frac{R_A R_B + R_B R_C + R_A R_C}{R_C}$$ $$R_{BC} = \frac{R_A R_B + R_B R_C + R_A R_C}{R_A}$$ $$R_{AC} = \frac{R_A R_B + R_B R_C + R_A R_C}{R_B}$$ Delta to star conversion: $$R_A = \frac{R_{AB} R_{AC}}{R_{AB} + R_{BC} + R_{AC}}$$ $$R_B = \frac{R_{AB} R_{BC}}{R_{AB} + R_{BC} + R_{AC}}$$ $$R_C = \frac{R_{BC} R_{AC}}{R_{AB} + R_{BC} + R_{AC}}$$	***Delta (Δ) Network***  ***Star (Y) Network*** 

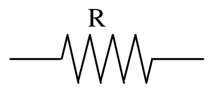
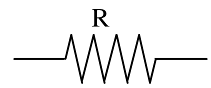
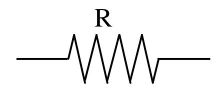
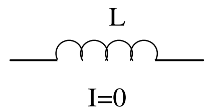
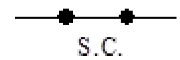
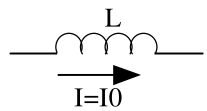
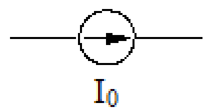
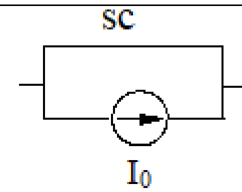
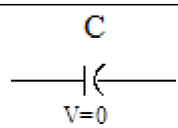
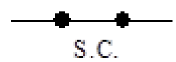
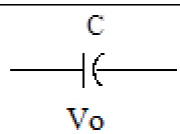
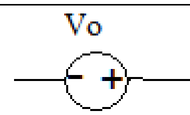
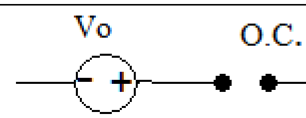
Network Theorems:

1	Thevinin's Theorem	 V_{th}=Open Circuit voltage at the load terminals Z_{th}=. Open circuit equivalent impedance at the load terminals after deactivating all the independent sources	

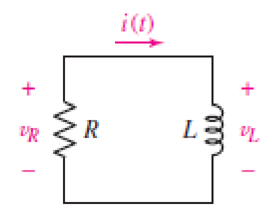
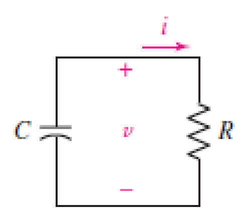
2	**Norton's Theorem (Dual of Thevenin's theorem)**	 I_{sc}=Short circuit current through the load terminals $Z_N=Z_{th}$ = Open circuit equivalent impedance at the load terminals after deactivating all the independent sources	
3	**Maximum Power Transfer theorem**	DC Circuit-- $R_L=R_{TH}$ $$P = \frac{V_{TH}^2}{4R_L}$$ AC Circuit (1) both load resistance and reactance are variable $Z_L=Z_{TH}^*$ (2) load being purely resistive $R_L=\lvert Z_L\rvert= \lvert Z_{th}\rvert$ (3) load being variable resistance and fixed reactance $Z_{th}'=Z_{th} \pm jX_L$ $R_L=\lvert Z_{th}\rvert$	Given circuit is to be replaced by Thevenin's equivalent circuit 

TRANSIENT ANALYSIS:

Equivalent form of elements in terms of the initial and final condition of element:

Element	Equivalent circuit at t=0 $^+$	Equivalent circuit at t = ∞
		
		
		
		
		

First order differential equations:

Source free RL circuit	$\frac{di}{dt}+\frac{R}{L}i=0$ $i(t)=I_0e^{-\frac{Rt}{L}}$ where, I_0 is the initial current at time $t=0$	
Source free RC circuit	$\frac{dv}{dt}+\frac{v}{RC}=0$ $v(t)=v(0)e^{-\frac{t}{RC}}$ v_0 is the initial voltage across capacitor at time $t=0$	
Solution of standard first order differential equation	$\frac{di}{dt}+Pi=Q$ $i=e^{-Pt}\int Q\,e^{Pt}dt+Ae^{-Pt}$	Total response = Forced response + Natural response

Solution of second order differential equation:

Second order differential equation representing **<u>source free</u>** RLC circuit is of the form

$\frac{d^2x(t)}{dt^2}+a_1\frac{dx(t)}{dt}+a_2x(t)=0$ and characteristic equation $s^2+\ a_1s+a_2=0$

Roots of the characteristic equation are

$$s_1 , s_2 = \frac{-a_1}{2} \pm \sqrt{\left(\frac{a_1}{2}\right)^2 - a_2}$$

Response in terms of coefficient conditions:

case	Coefficient Condition	Nature of Roots	Descriptive Name	Form of Solution
1	$a_1^2 > 4a_2$	Negative real and unequal	Overdamped	$x(t) = K_1 e^{s_1 t} + K_2 e^{s_2 t}$
2	$a_1^2 = 4a_2$	Negative real and equal	Critically damped	$x(t) = K_1 e^{s_1 t} + K_2 t e^{s_2 t}$
3	$a_1^2 < 4a_2$	Conjugate Complex	Underdamped	$x(t) = e^{\sigma t} (K_1 cos\omega t + K_2 sin\omega t)$ $s_1 , s_2 = \sigma \pm j\omega$
4	$a_1 = 0 \; and \; a_2 \neq 0$	Conjugate imaginary	Oscillatory	$x(t) = K_1 cos\omega t + K_2 sin\omega t$ $s_1 , s_2 = \pm j\omega$

Second order differential equation representing RLC circuit **with excitation** is of the form

$$\frac{d^2x(t)}{dt^2} + a_1 \frac{dx(t)}{dt} + a_2 x(t) = v(t)$$

Has a solution (total)$x(t) = x_c(t) + x_p(t)$: $x_c(t)$ is complementary function and $x_p(t)$ is particular integral

Case	Factor in *v(t)* { excitation}	Necessary choice for the particular integral
1	*V (a Constant)*	A
2	$a_1 t^n$	$B_0 t^n + B_1 t^{n-1} + B_2 t^{n-2} + \cdots\cdots\cdots + B_{n-1} t + B_n$
3	$a_2 e^{rt}$	Ce^{rt}
4	$a_3 \; cos\omega t$ $a_4 \; sin\omega t$	$D \; cos\omega t + E \; sin\omega t$
5	$a_5 t^n e^{rt} \; cos\omega t$ $a_6 t^n e^{rt} \; sos\omega t$	$(F_1 t^n + \cdots + F_{n-1} t + F_n) e^{rt} \; cos\omega t + (G_1 t^n + \cdots + G_{n-1} t + G_n) e^{rt} \; sin\omega t$

Note: The initial condition must always to be applied to the total solution – never to the complementary function alone unless $x_p(t)$= 0.

LAPLACE TRANSFORMS;

Laplace Transform: $L\{f(t\} = F(s) = \int_0^{+\infty} f(t) \, e^{-st} dt$

Inverse Laplace Transform: $f(t) = \frac{1}{2\pi j} \int_{\sigma_0 - j\infty}^{\sigma_0 + j\infty} e^{st} F(s) ds$

Standard Laplace Transforms:

$f(t)$	$F(s)=\int_0^\infty f(t)e^{-st}dt$
$u(t)$	$\frac{1}{s}$
t	$\frac{1}{s^2}$
$\frac{t^{n-1}}{(n-1)!}$ n is a integer	$\frac{1}{s^n}$
e^{at}	$\frac{1}{s-a}$
e^{-at}	$\frac{1}{s+a}$
te^{at}	$\frac{1}{(s-a)^2}$
te^{-at}	$\frac{1}{(s+a)^2}$
$\frac{1}{(n-1)}t^{n-1}e^{at}$	$\frac{1}{(s-a)^n}$
$\frac{1}{(n-1)}t^{n-1}e^{-at}$	$\frac{1}{(s+a)^n}$
$1-e^{at}$	$\frac{-a}{s(s-a)}$
$\frac{1}{\omega}\sin\omega t$	$\frac{1}{s^2+\omega^2}$
$\cos\omega t$	$\frac{s}{s^2+\omega^2}$
$1-\cos\omega t$	$\frac{\omega^2}{s(s^2+\omega^2)}$
$\sin(\omega t+\theta)$	$\frac{s\sin\theta+\omega\cos\theta}{s^2+\omega^2}$
$\cos(\omega t+\theta)$	$\frac{s\cos\theta-\omega\sin\theta}{s^2+\omega^2}$
$e^{-at}\sin\omega t$	$\frac{\omega}{(s+a)^2+\omega^2}$
$e^{at}\sin\omega t$	$\frac{\omega}{(s-a)^2+\omega^2}$
$e^{-at}\cos\omega t$	$\frac{s+a}{(s+a)^2+\omega^2}$

$e^{at}\cos\omega t$	$\frac{s-a}{(s-a)^2+\omega^2}$
$\sinh at$	$\frac{a}{s^2-a^2}$
$\cosh at$	$\frac{s}{s^2-a^2}$

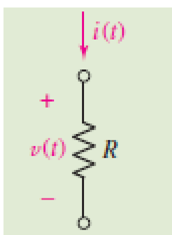
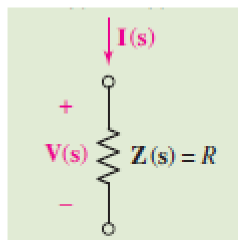
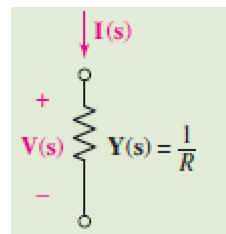
Laplace Transform Operations:

Nature of operation	***f(t)***	***F(s)***
Definition	$f(t)$	$F(s)=\int_0^\infty f(t)e^{-st}dt$
Linear Operation	$af_1(t)\pm bf_2(t)$	$aF_1(s)\pm bF_2(s)$
Differentiation	$\frac{d}{dt}f(t)$	$sF(s)-f(0-)$
	$\frac{d^2}{dt^2}f(t)$	$s^2F(s)-sf(0-)-\frac{d}{dt}f(0-)$
	$\frac{d^3}{dt^3}f(t)$	$s^3F(s)-s^2f(0-)-s\frac{d}{dt}f(0-)-\frac{d^2}{dt^2}f(0-)$
Integration	$\int_0^t f(t)dt$	$\frac{F(s)}{s}$
	$\int_{-\infty}^t f(t)dt$	$\frac{F(s)}{s}+\frac{f(0)}{s}$
Shifting in time domain	$f(t-a)u(t-a)$	$e^{-as}F(s)$
Multiplication by exponential	$e^{-at}f(t)$	$F(s-a)$
Multiplication by t	$t\,f(t)$	$\frac{-d}{ds}F(s)$
	$t^n\,f(t)$	$(-1)^n\frac{d^n}{ds^n}F(s)$
Time scaling	$f(at)$	$\frac{1}{a}F(\frac{s}{a})$
Magnitude scaling	$af(t)$	$aF(s)$
Periodic function	$f(t)=f(t+nT)$ n is an integer	$F(s)=\frac{1}{1-e^{-sT}}F_1(s),$ Where $F_1(s)=\int_0^\infty f(t)e^{-st}dt$: Laplace transform of one cycle of the periodic function

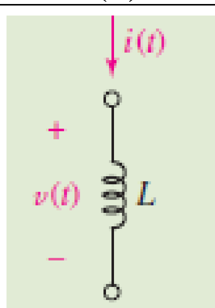
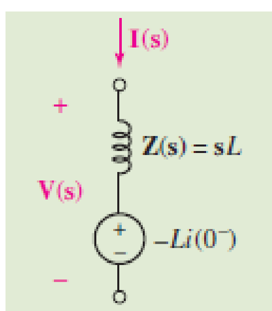
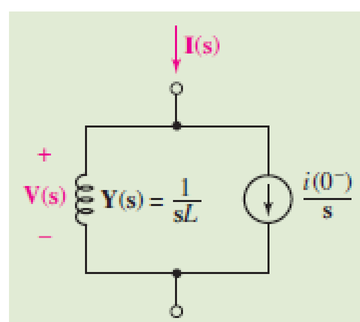
Initial value $f(0^+)$	$\lim_{t\to 0} f(t) =$	$= \lim_{s\to\infty} sF(s)$
Final value $f(\infty)$	$\lim_{t\to\infty} f(t) =$	$= \lim_{s\to 0} sF(s)$

Transformed Network representation for Basic Elements R, L and C:

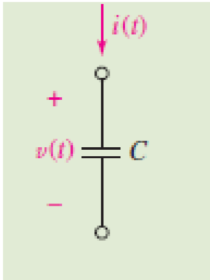
(a) Resistor (R)

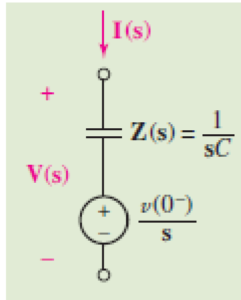
 $v(t) = Ri(t)$	 $V(s) = RI(s)$	 $I(s) = \frac{1}{R}V(s)$

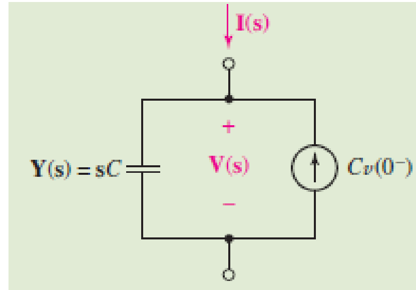
(b) Inductor (L)

 $v(t) = L\frac{di}{dt}$	 $V(s) = sLI(s) - Li(0^-)$	 $I(s) = \frac{V(s)}{sL} + \frac{i(0^-)}{s}$

(c) Capacitor

<table>
<tr>
<td>

$i(t) = C\dfrac{dV}{dt}$</td>
<td>

$V(s) = \dfrac{I(s)}{sC} + \dfrac{v(0^-)}{s}$</td>
<td>

$I(s) = sCV(s) - Cv(0^-)$</td>
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TWO PORT PARAMETERS:

Name	Equations
Z parameters(Open-circuit Impedances)	$V_1 = Z_{11}I_1 + Z_{12}I_2$ $V_2 = Z_{21}I_1 + Z_{22}I_2$
Y parameters(Short-circuit admittances)	$I_1 = Y_{11}V_1 + Y_{12}V_2$ $I_2 = Y_{21}V_1 + Y_{22}V_2$
h(Hybrid) parameters	$V_1 = h_{11}I_1 + h_{12}V_2$ $I_2 = h_{21}I_1 + h_{22}V_2$
T (Transmission) parameters (ABCD)	$V_1 = AV_2 - BI_2$ $I_1 = CV_2 - DI_2$

Relationship between parameter sets:

	$[z]$	$[y]$	$[T]$	$[h]$
$[z]$	$\begin{matrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{matrix}$	$\begin{matrix} \dfrac{y_{22}}{\Delta_y} & -\dfrac{y_{12}}{\Delta_y} \\ -\dfrac{y_{21}}{\Delta_y} & \dfrac{y_{11}}{\Delta_y} \end{matrix}$	$\begin{matrix} \dfrac{A}{C} & \dfrac{\Delta_T}{C} \\ \dfrac{1}{C} & \dfrac{D}{C} \end{matrix}$	$\begin{matrix} \dfrac{\Delta_h}{h_{22}} & \dfrac{h_{12}}{h_{22}} \\ -\dfrac{h_{21}}{h_{22}} & \dfrac{1}{h_{22}} \end{matrix}$
$[y]$				

	$\begin{matrix} \frac{z_{22}}{\Delta_z} & -\frac{z_{12}}{\Delta_z} \\ -\frac{z_{21}}{\Delta_z} & \frac{z_{11}}{\Delta_z} \end{matrix}$	$\begin{matrix} y_{11} & y_{12} \\ y_{21} & zy_{22} \end{matrix}$	$\begin{matrix} \frac{D}{B} & \frac{\Delta_T}{C} \\ -\frac{1}{B} & \frac{A}{B} \end{matrix}$	$\begin{matrix} \frac{1}{h_{11}} & -\frac{h_{12}}{h_{11}} \\ \frac{h_{21}}{h_{11}} & \frac{\Delta_h}{h_{11}} \end{matrix}$
$[T]$	$\begin{matrix} \frac{z_{11}}{z_{21}} & \frac{\Delta_z}{z_{21}} \\ \frac{\Delta_z}{z_{21}} & \frac{z_{11}}{z_{21}} \end{matrix}$	$\begin{matrix} -\frac{y_{11}}{y_{12}} & -\frac{1}{y_{12}} \\ -\frac{\Delta_y}{y_{12}} & -\frac{y_{11}}{y_{12}} \end{matrix}$	$\begin{matrix} A & B \\ C & D \end{matrix}$	$\begin{matrix} -\frac{\Delta_h}{h_{21}} & -\frac{h_{11}}{h_{21}} \\ -\frac{h_{22}}{h_{21}} & -\frac{1}{h_{21}} \end{matrix}$
$[h]$	$\begin{matrix} \frac{\Delta_z}{z_{22}} & \frac{z_{12}}{z_{22}} \\ -\frac{z_{21}}{z_{22}} & \frac{1}{z_{22}} \end{matrix}$	$\begin{matrix} \frac{1}{y_{11}} & -\frac{y_{12}}{y_{11}} \\ \frac{y_{21}}{y_{11}} & \frac{\Delta_y}{y_{11}} \end{matrix}$	$\begin{matrix} \frac{B}{D} & \frac{\Delta_T}{D} \\ -\frac{1}{D} & \frac{C}{D} \end{matrix}$	$\begin{matrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{matrix}$

Some Parameter Simplification for Passive, Reciprocal Networks:

Parameter	*Condition for Passive networks*	*Condition for Electrical symmetry*
z	$z_{12} = z_{21}$	$z_{11} = z_{22}$
y	$y_{12} = y_{21}$	$y_{11} = y_{22}$
$ABCD$	$AD – BC = 1$	$A = D$
h	$h_{12} = - h_{21}$	$\Delta_h = 1$

SIGNALS AND SYSTEMS

Elementary signals:

Name	Continuous time	Discrete time
Impulse function	$\delta(t) = 0, t \neq 0$	$\delta[n] = \begin{cases} 1, n = 0 \\ 0, n \neq 0 \end{cases}$
Unit step function	$\mathrm{u(t)} = \begin{cases} 1, \mathrm{t} \geq 0 \\ 0, \mathrm{t} < 0 \end{cases}$	$u[n] = \begin{cases} 1, n \geq 0 \\ 0, n < 0 \end{cases}$
Unit Ramp	$r(\mathrm{t}) = \begin{cases} \mathrm{t}, \mathrm{t} \geq 0 \\ 0, \mathrm{t} < 0 \end{cases}$	$r[n] = \begin{cases} n, n \geq 0 \\ 0, n < 0 \end{cases}$
Exponential	$x(t) = e^{at}$	$x[n] = a^n$
Sinusoid	$x(t) = sin(\omega t + \phi)$	$x[n] = sin(\Omega n + \theta)$
Sinc function	$sinc(\omega_0 t) = \dfrac{sin(\pi\omega_0 t)}{\pi\omega_0 t}$	$sinc[\Omega_0 n] = \dfrac{sin(\pi\Omega_0 n)}{\pi\Omega_0 n}$
Rectangular pulse	$x(t) = \begin{cases} 1, \lvert t\rvert \leq T_0 \\ 0, \lvert t\rvert > T_0 \end{cases}$	$x[n] = \begin{cases} 1, \lvert n\rvert \leq M \\ 0, otherwise \end{cases}$
Triangular pulse	$\Lambda\left(\dfrac{t}{\tau}\right) = \begin{cases} 1 - \left\lvert\dfrac{t}{\tau}\right\rvert, t \leq \lvert\tau\rvert \\ 0, t > \lvert\tau\rvert \end{cases}$	$\Lambda\left[\dfrac{n}{N}\right] = \begin{cases} 1 - \dfrac{\lvert n\rvert}{N}, \lvert n\rvert \leq N \\ 0, otherwise \end{cases}$

Important properties of signals:

Name	Properties	
	Continuous time	Discrete time
Impulse properties	$\int\limits_{t=-\infty}^{\infty} \delta(t)dt = 1$ $\int\limits_{t=-\infty}^{\infty} x(t)\delta(t-t_0)dt = x(t_0)$ $x(t)\delta(t-t_0) = x(t_0)\delta(t-t_0)$ $\delta(at) = \dfrac{1}{\lvert a\rvert}\delta(t)$	$x[n]\delta[n-n_0] = x[n_0]\delta[n-n_0]$

Even and Odd symmetry	$x_e(t) = x_e(-t)$ $x_o(t) = -x_o(-t)$ $x_e(t) = \frac{x(t) + x(-t)}{2}$ $x_o(t) = \frac{x(t) - x(-t)}{2}$	$x_e[n] = x_e[-n]$ $x_o[n] = -x_o[-n]$ $x_e[n] = \frac{x[n] + x[-n]}{2}$ $x_o[n] = \frac{x[n] - x[-n]}{2}$
Periodicity	For $x(t) = x(t+T),\ fundamental\ period\ T = \frac{2\pi}{\omega}$	$x(n) = x(n+N),$ $fundamental\ period\ N = \frac{2\pi}{\Omega}m,$
Common Periodicity	$T = \frac{T1}{T2} = \frac{n}{m}$ T1 & $T2$ $are\ periods\ of\ functions,$ n & m $are\ the\ integers$	$T = \frac{N1}{N2} = \frac{n}{m}$ N1 & $N2$ $are\ periods\ of\ functions,$ n & m $are\ the\ integers$
Energy of a signal	$E = \lim_{T\to\infty} \int_{-\frac{T}{2}}^{\frac{T}{2}} x^2\,(t)dt$ For non-periodic signal $E = \int_{t=-\infty}^{\infty} \lvert x(t)\rvert^2 dt$	$E = \sum_{n=-\infty}^{\infty} \lvert x[n]\rvert^2$
Power of a signal	$P = \lim_{T\to\infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} x^2\,(t)dt$ For periodic signal $P = \frac{1}{T} \int_{t=-T/2}^{T/2} \lvert x(t)\rvert^2\, dt$	$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{n=N} \lvert x[n]\rvert^2$ For periodic sequence $P = \frac{1}{N} \sum_{n=0}^{N-1} \lvert x[n]\rvert^2$
Linear combination of N signals	$\sum_{i=1}^{N} a_i x_i(t)$	$\sum_{i=1}^{N} a_i x_i[n]$

Convolution between two non-periodic signals	$x_1(t) * x_2(t) = \int_{\tau=-\infty}^{\infty} x_1(\tau)x_2(t-\tau)d\tau$	$x_1[n] * x_2[n] = \sum_{k=-\infty}^{\infty} x_1[k]x_2[n-k]$

LTI system analysis:

	Continuous time system	**Discrete time system**
Name	$x(t) : input$ $y(t) : output$ $h(t) : impulse\ response$	$x[n] : input$ $y[n] : output$ $h[n] : impulse\ response$
Causality and stability in-terms of Impulse response	*Causality:* $h(t) = 0, t < 0$ $stability: \int_{t=-\infty}^{t} \lvert h(t) \rvert d\tau < \infty$ *memoryless:* $h(t) = c\delta(t)$	*Causality:* $h[n] = 0, n < 0$ $stability: \sum_{n=-\infty}^{n} \lvert h[n] \rvert < \infty$ *memoryless:* $h[n] = c\delta[n]$
Response to any input	$y(t) = x(t) * h(t)$	$y[n] = x[n] * h[n]$
Step response	$s(t) = \int_{\tau=-\infty}^{t} h(\tau)d\tau$	$s[n] = \sum_{m=-\infty}^{n} h[n]$
Differential/Difference equation description	$\sum_{k=0}^{N} a_k \frac{d^k}{dt^k} y(t) = \sum_{k=0}^{M} b_k \frac{d^k}{dt^k} x(t)\ ; a_0 \neq 0$	$\sum_{k=0}^{N} a_k y[n-k] = \sum_{k=0}^{M} b_k x[n-k]\ ; a_0 \neq 0$

Geometric Series Formulas:

1	$\sum_{n=0}^{M} a^n = \frac{1-a^{M+1}}{1-a}$
2	$\sum_{n=0}^{\infty} a^n = \frac{1}{1-a}, \qquad \lvert a < 1 \rvert$

3	$$\sum_{n=M_1}^{M_2} a^n = \frac{a^{M_1} - a^{M_2+1}}{1-a}$$
4	$$\sum_{n=1}^{M} n = \frac{M(M+1)}{2}$$

FOURIER ANALYSIS

Fourier representation of continuous time signals:

Time domain representation		Fourier representation		
$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$	• Continuous • Periodic • Period=T, Fundamental frequency ω_0=2π/T rad/sec	$a_k = \frac{1}{T} \int_{t=-T/2}^{T/2} x(t)\, e^{-jk\omega_0 t} dt$	• Discrete • Non periodic	FS
$x(t) = \frac{1}{2\pi} \int_{\omega=-\infty}^{\infty} X(\omega) e^{j\omega t} d\omega$	• Continuous • Non-Periodic	$X(\omega) = \int_{t=-\infty}^{\infty} x(t) e^{-j\omega t} dt$	• Continuous • Non periodic	FT

Properties of Fourier Transform:

Property	*Continuous time signals*	
	Time domain	Frequency domain (FT)
Notation	$x(t)$ $x_i(t)$	$X(\omega)$ $X_i(\omega)$
Linearity	$\sum_{i=1}^{N} a_i x_i(t)$	$\sum_{i=1}^{N} a_i\, X_i(\omega)$
Time shifting	$x(t - t_0)$	$e^{-j\omega t_0} X(\omega)$
Frequency shift	$e^{j\gamma t} x(t)$	$X\big((\omega - \gamma)\big)$

Time reversal	$x(-t)$	$X(-\omega)$
Correlation	$r_{x_1x_2}(\tau) = x_1(\tau) * x_2(-\tau)$	$X_1(\omega)X_2(-\omega)$
Differentiation in time	$\frac{d}{dt}x(t)$	$j\omega X(\omega)$
Differentiation in frequency	$-jtx(t)$	$\frac{d}{d\omega}X(\omega)$
Integration /summation	$\int_{\tau=-\infty}^{t} x(\tau)d\tau$	$\frac{X(j\omega)}{\mathrm{j}\omega} + \pi X(j0)\delta(\omega)$
Convolution	$x_1(t) * x_2(t)$	$X_1(\omega)X_2(\omega)$
Multiplication	$x_1(t)x_2(t)$	$\frac{1}{2\pi}\int_{\vartheta=-\infty}^{\infty} X_1(\vartheta)X_2(\omega - \vartheta)d\vartheta$
Symmetry	$x(t)\ real$	$X^*(\omega) = X(-\omega)$
	$x(t)\ imaginary$	$X^*(\omega) = -X(-\omega)$
	$x(t)\ real\ \&\ even$	$Im\{X(\omega)\} = 0$
	$x(t)\ real\ \&\ odd$	$Re\{X(\omega)\} = 0$
Parseval's Theorem	$\int_{t=-\infty}^{\infty} \lvert x(t)\rvert^2 dt = \frac{1}{2\pi}\int_{\Omega=-\infty}^{\infty} \lvert X(\omega)\rvert^2 d\Omega$	
Duality	$X(t) \xleftrightarrow{FT} 2\pi x(-\omega)$	

Properties of Fourier Series:

Continuous time signal: Periodic, Period=T, Fundamental frequency $\omega_0 = \frac{2\pi}{T}\ radian/sec$

Property	*Continuous time signals*	
	Time domain	Frequency domain (FS)
Notation	$x(t)$ $y(t)$	a_k b_k
Linearity	$Ax(t) + By(t)$	$Aa_k + Bb_k$
Time shifting	$x(t - t_0)$	$e^{-jk\omega_0 t_0}a_k$
Frequency shift	$e^{jk_0\omega_0 t}x(t)$	a_{k-k_0}
Differentiation in time	$\frac{d}{dt}x(t)$	$jk\omega_0 a_k$
Convolution	$x_1(t) * x_2(t)$	$Ta_{k1}a_{k2}$
Multiplication	$x_1(t)x_2(t)$	$\sum_{l=-\infty}^{\infty} a_l b_{k-l}$
Symmetry	$x(t)\ real$	$a_k^* = a_{-k}$
	$x(t)\ imaginary$	$a_k^* = -a_{-k}$
	$x(t)\ real\ \&\ even$	$Im\{a_k\} = 0$
	$x(t)\ real\ \&\ odd$	$Re\{a_k\} = 0$

Parseval's Theorem	$\frac{1}{T}\int_{t=0}^{T}\lvert x(t)\rvert^2 dt = \sum_{k=-\infty}^{\infty}\lvert a_k\rvert^2$

FT representation for a continuous-time periodic signal*g(t):*

$$g(t) \xleftrightarrow{FT} G(\omega) = 2\pi \sum_{k=-\infty}^{\infty} g_k\,\delta(\omega - k\omega_0)$$

Where g_k are the FS coefficients and ω_0 is the fundamental frequency.

Sampling:

Continuous time signal *x(t)* with FT $X(j\omega)$ is sampled at sampling interval T_s to get $x_\delta(t)$

$$x_\delta(t) = \sum_{n=-\infty}^{\infty} x[nT_s]\,\delta(t - nT_s) \xleftrightarrow{FT} X_\delta(j\omega) = \frac{1}{T_s}\sum_{k=-\infty}^{\infty} X\left(j\left(\omega - k\frac{2\pi}{T_s}\right)\right)$$

Basic Fourier Series pairs:

Time domain	Frequency domain
$x(t) = \sum_{k=-\infty}^{\infty} X(k)e^{jk\omega_0 t}$ $Period = T$	$a_k = \frac{1}{T}\int_{t=-\frac{T}{2}}^{\frac{T}{2}} x(t)\,e^{-jk\omega_0 t}dt$ $\omega_0 = \frac{2\pi}{T}$
$x(t) = \begin{cases} 1, \lvert t\rvert \le T_0 \\ 0, T_0 < \lvert t\rvert \le \frac{T}{2} \end{cases}$	$a_k = \frac{sin(k\omega_0 T_0)}{k\pi}$
$x(t) = e^{jp\omega_0 t}$	$a_k = \delta[k-p]$
$x(t) = cos(p\omega_0 t)$	$a_k = \frac{1}{2}\delta[k-p] + \frac{1}{2}\delta[k+p]$
$x(t) = sin(p\omega_0 t)$	$a_k = \frac{1}{2j}\delta[k-p] - \frac{1}{2j}\delta[k+p]$
$x(t) = \sum_{p=-\infty}^{\infty} \delta(t - pT)$	$a_k = \frac{1}{T}$

Basic Fourier Transform pairs:

Time domain	Frequency domain
$x(t) = \frac{1}{2\pi}\int_{\omega=-\infty}^{\infty} X(j\omega)e^{j\omega t}d\omega$	$X(\omega) = \int_{t=-\infty}^{\infty} x(t)e^{-j\omega t}dt$
$x(t) = \begin{cases} 1, \lvert t\rvert \le T_0 \\ 0, \lvert t\rvert > T_0 \end{cases}$	$X(\omega) = \frac{2sin(\omega T_0)}{\omega}$
$x(t) = \frac{1}{\pi t}sin(Wt)$	$X(\omega) = \begin{cases} 1, \lvert\omega\rvert \le \mathrm{W} \\ 0, \text{otherwise} \end{cases}$
$x(t) = \delta(t)$	$X(\omega) = 1$
$x(t) = 1$	$X(\omega) = 2\pi\delta(\omega)$
$x(t) = u(t)$	$X(\omega) = \frac{1}{j\omega}\pi\delta(\omega)$
$x(t) = e^{-at}u(t) \quad Re\{a\} > 0$	$X(\omega) = \frac{1}{a + j\omega}$
$x(t) = te^{-at}u(t), \quad Re\{a\} > 0$	$X(\omega) = \frac{1}{(a + j\omega)^2}$
$x(t) = e^{-a\lvert t\rvert}, \quad a > 0$	$X(\omega) = \frac{2a}{a^2 + \omega^2}$
$x(t) = \frac{1}{\sqrt{2\pi}}e^{-t^2/2}$	$X(\omega) = e^{-\omega^2/2}$
$x(t) = cos(\omega_0 t)$	$X(\omega) = \pi\delta(\omega - \omega_0) + \pi\delta(\omega + \omega_0)$
$x(t) = sin(\omega_0 t)$	$X(\omega) = \frac{\pi}{j}\delta(\omega - \omega_0) - \frac{\pi}{j}\delta(\omega + \omega_0)$
$x(t) = e^{j\omega_0 t}$	$X(j\omega) = 2\pi\delta(\omega - \omega_0)$
$x(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$	$X(\omega) = \frac{2\pi}{T_s}\sum_{k=-\infty}^{\infty} \delta\left(\omega - k\frac{2\pi}{T_s}\right)$