

CD-Johnson Pg 536 ex 9.

PID

η_P range 0 - 4 V

η_{OP} 0 - 8 V

$$k_p = 2.4\%/\text{V}$$

$$K_I = 9\%/\text{(V/min)} \quad \text{min period} = 8 \text{ sec}$$

$$K_D = 0.7\%/\text{(V/min)}$$

SOL.

$$G_p = \frac{R_2}{R_1}$$

$$\frac{\cancel{2.4\% \text{ of } 4}}{2.4\% \text{ of } 4} \times 8$$

$$\frac{2.4\% \text{ of } 8}{1\% \text{ of } 4}$$

$$G_p = 4.8$$

$$G_I = K_I \cdot \text{V} / (\text{V} - \text{S}) = 9 \times \frac{1}{60} = 0.15\%/\text{(V/s)}$$

$$G_I = \frac{0.15\% \text{ of } 8}{1\% \text{ of } 4} = 0.3 \text{ s}^{-1}$$

$$K_D \cdot \text{V} / (\text{V} - \text{S}) = 0.7 \times \frac{1}{60} = 0.0116\% = 0.7 \times K_D = 0.0116$$

$$G_D = \frac{0.0116 \cdot 1\% \text{ of } 8}{1\% \text{ of } 4} = \frac{0.000928}{0.04} = 0.0232 \text{ s}^{-1}$$

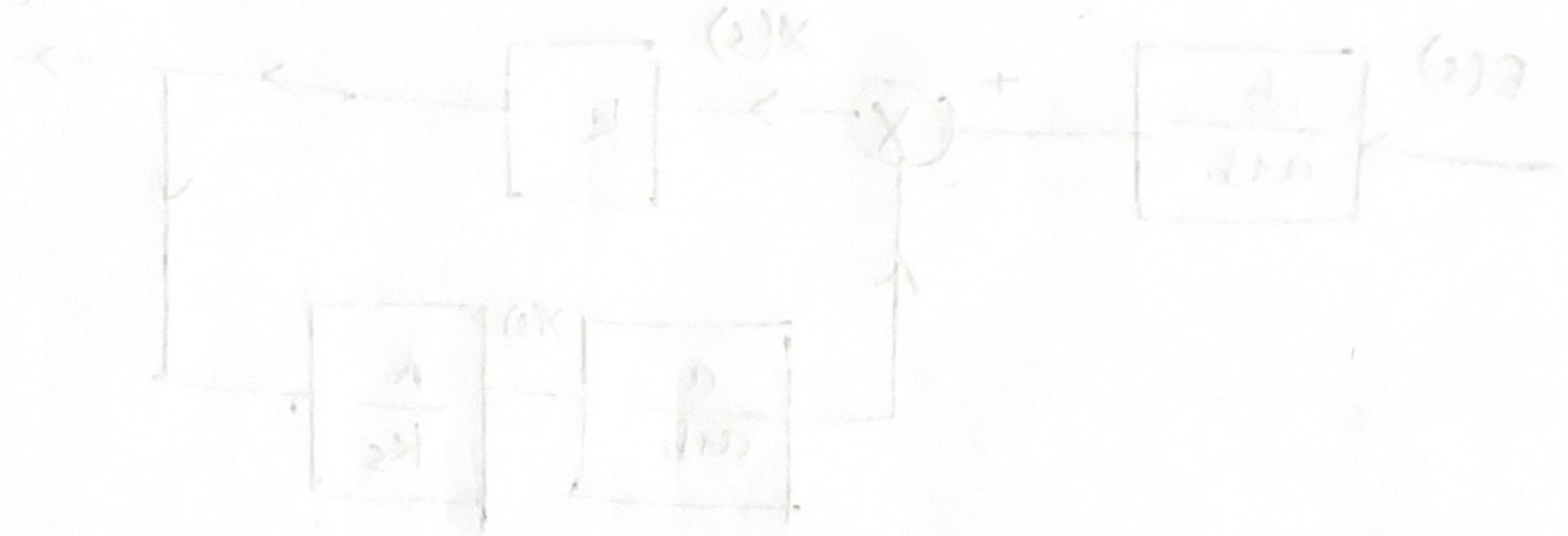
$$T_{op} = \frac{0.0232 \text{ s}^{-1}}{0.04} \times \frac{3.36}{0.04} = 84 \text{ sec}$$

$$2\pi f R_3 C_D = 0.1$$

$$2\pi R_3 C_D = (0.1)(8)$$

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(2)g Pneumatic controllers.

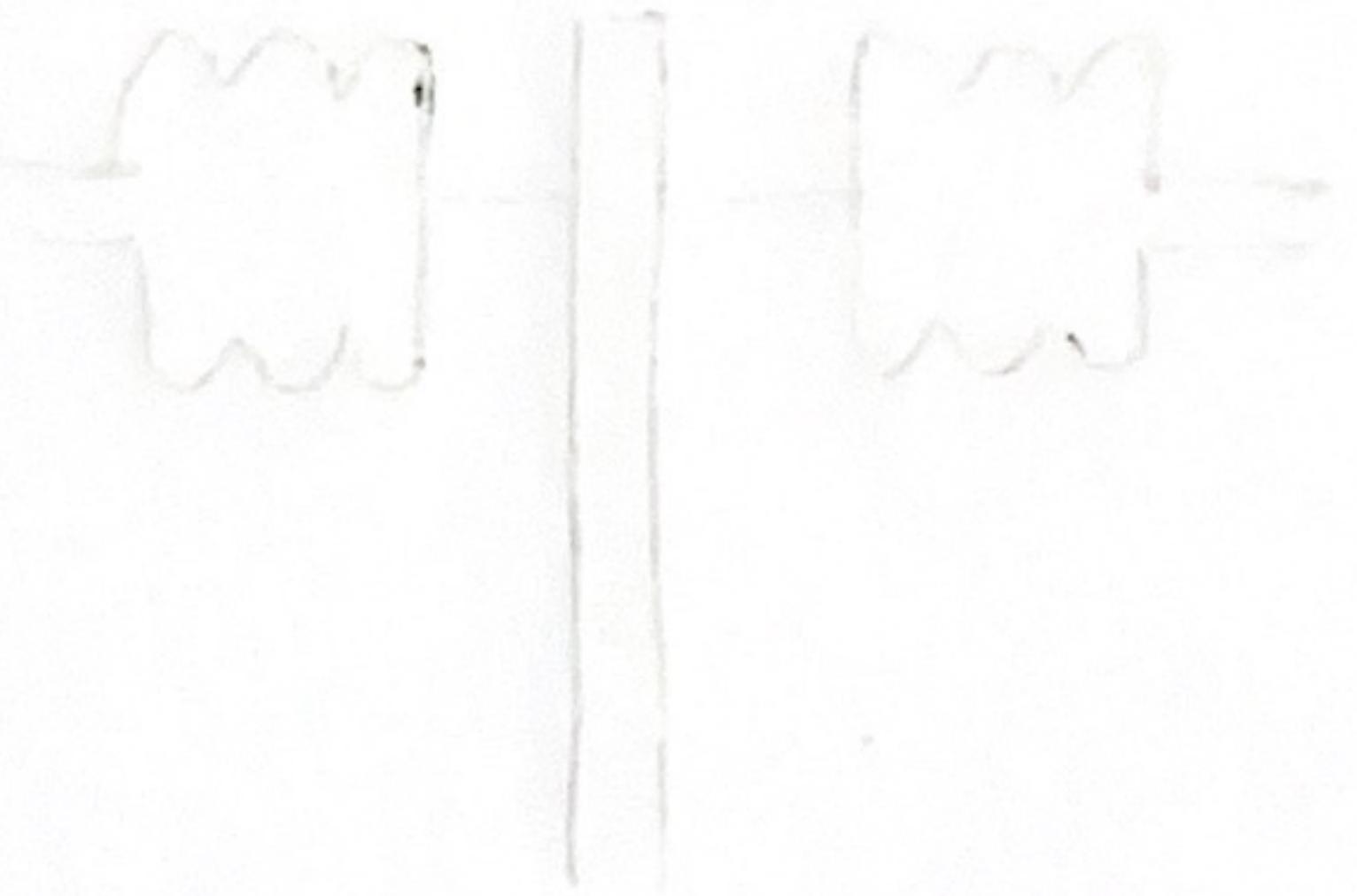


(bottom box) - relation 7

$$M(g_1 + g_2) = m \cdot h (g_1 + g_{out})$$

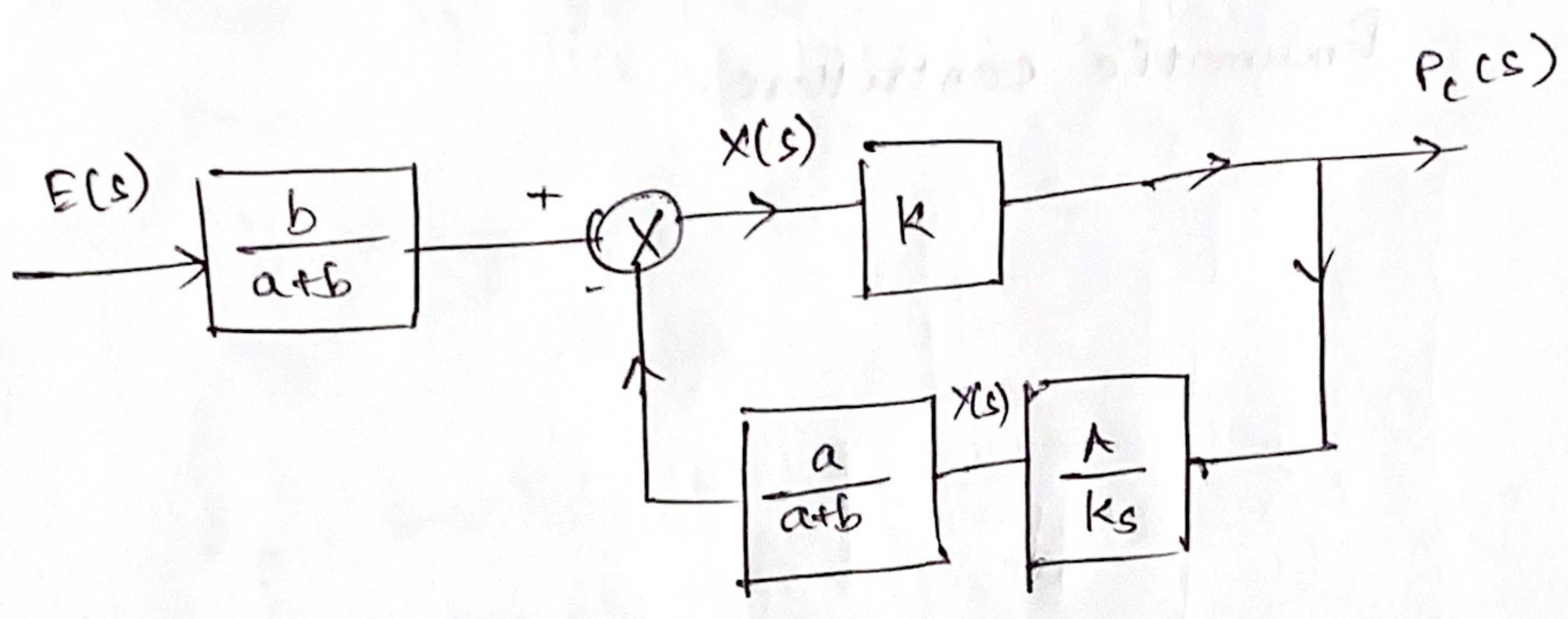
Proportional

$$P + (g_1 + g_2) = \frac{m \cdot h + g_{out}}{A_{out}}$$

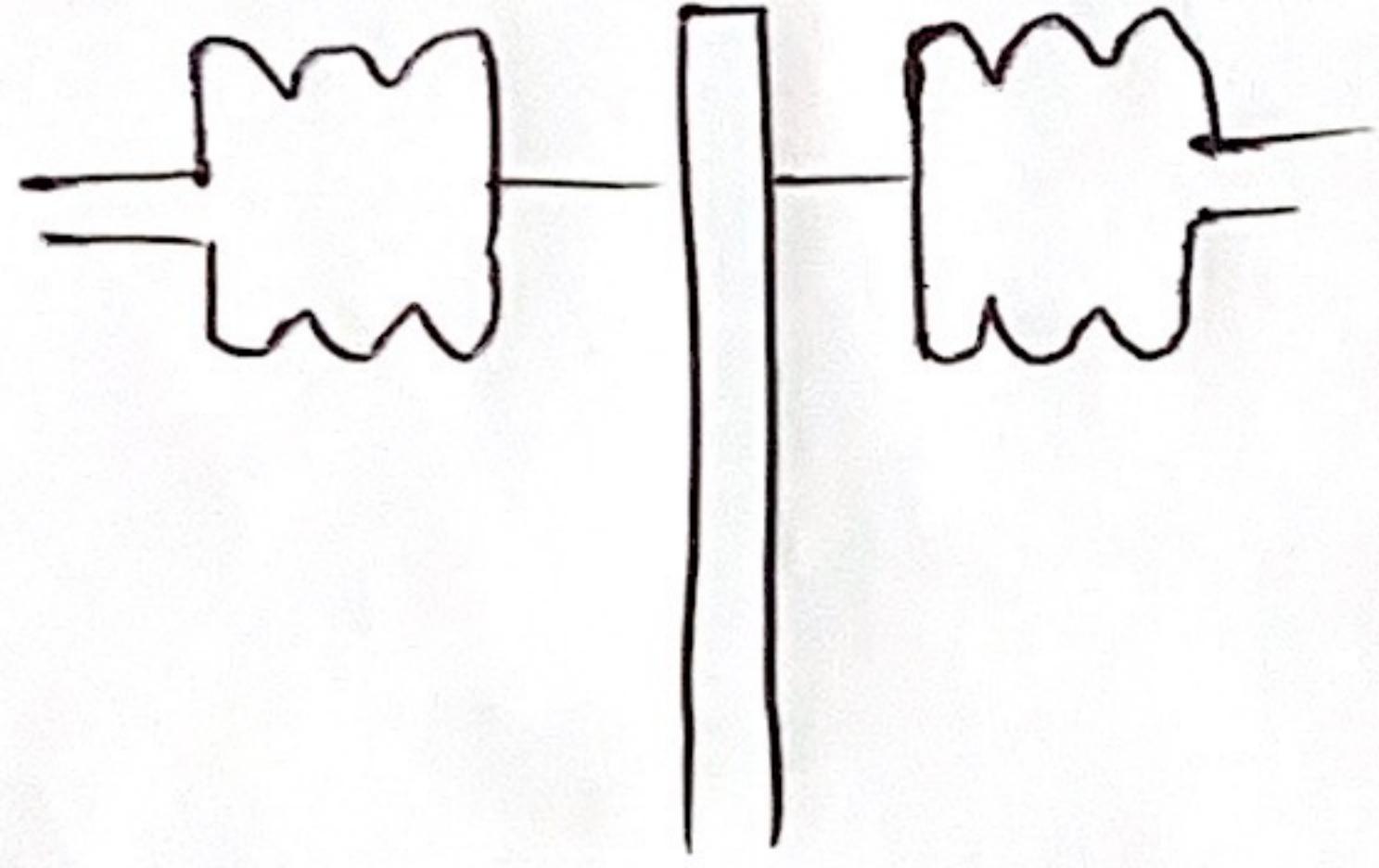


$$x = \frac{b}{a+b} e - \frac{a}{a+b} y$$

$$\frac{P_c(s)}{E(s)} =$$



P controller (2nd method)



$$(P_{out} - P_o) A_2 n_2 = (P_{in} - P_{sp}) A_1 n_1$$

$$P_{out} = \frac{n_1 A_1}{n_2 A_2} (P_{in} - P_{sp}) + P_o$$

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Tuning PID controllers

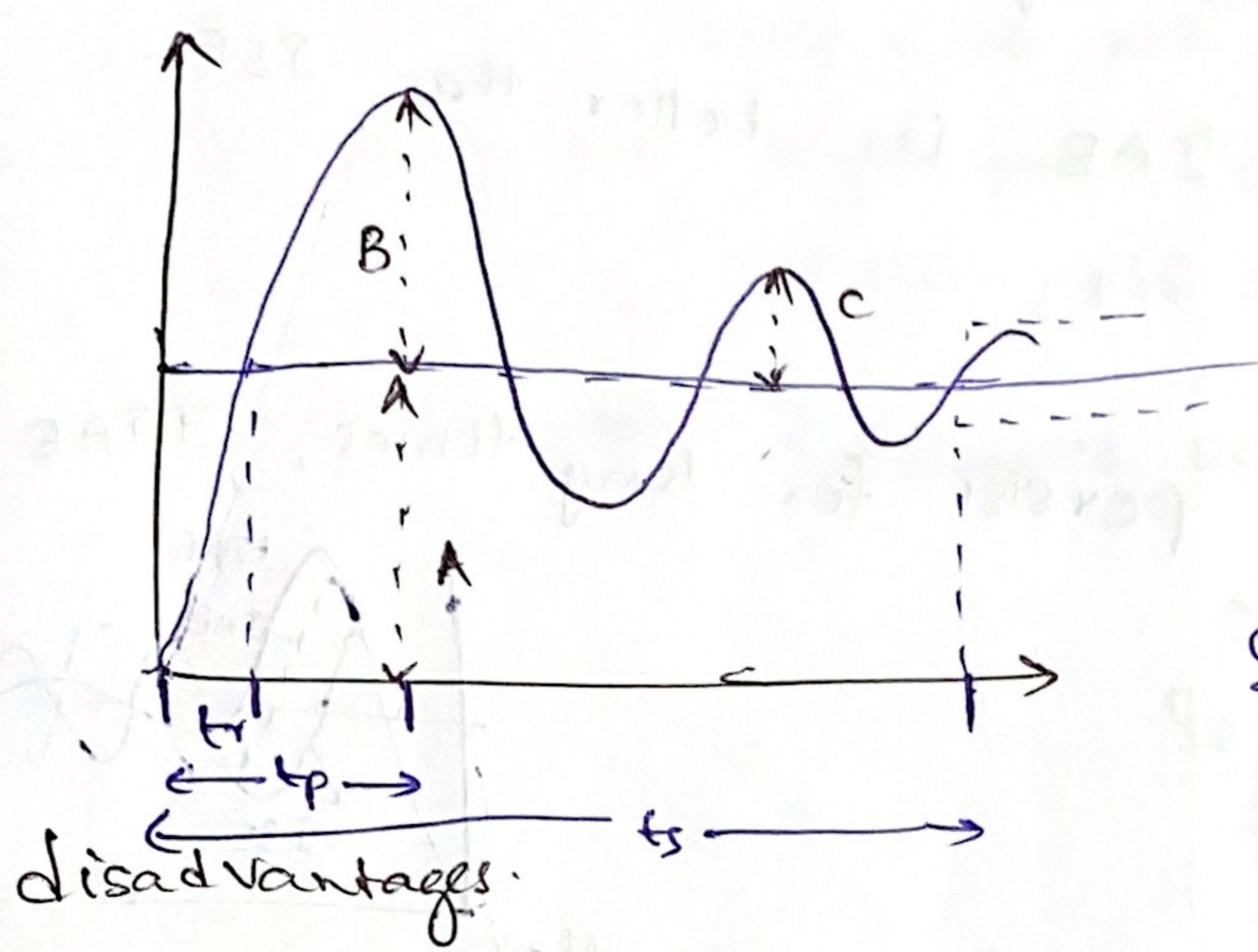
Disturbances

setpoint
load
Noise

The process of experimentation for obtaining the optimum values of the controller parameters with respect to a particular process is known as controller tuning.

Simple Performance criteria.

One Quarter Decay Ratio criterion.



$$\text{decay ratio} = \frac{C}{B} = \frac{1}{4}$$

$$\frac{C}{B} = e^{-\frac{2\pi\xi}{\sqrt{1-\xi^2}} \Delta t} = \frac{1}{4}$$

fast rise time &
reasonable
settling time

→ Responses with $\frac{1}{4}$ decay ratios are often judged to be too oscillatory by plant operators.

→ This criteria uses only two points of the response.

Alternative to this, that uses entire response.

Time Integral Performance criteria.

3 performance indices.

1. Integral of the square (ISE)
error.

$$ISE = \int_0^\infty e^2(t) \cdot dt$$

2. Integral of the Absolute value (IAE)

of the error

$$IAE = \int_0^\infty |e(t)| \cdot dt$$

3. Integral of the time-weighted absolute error (ITAE)

$$ITAE = \int_0^\infty t |e(t)| \cdot dt$$

General guidelines

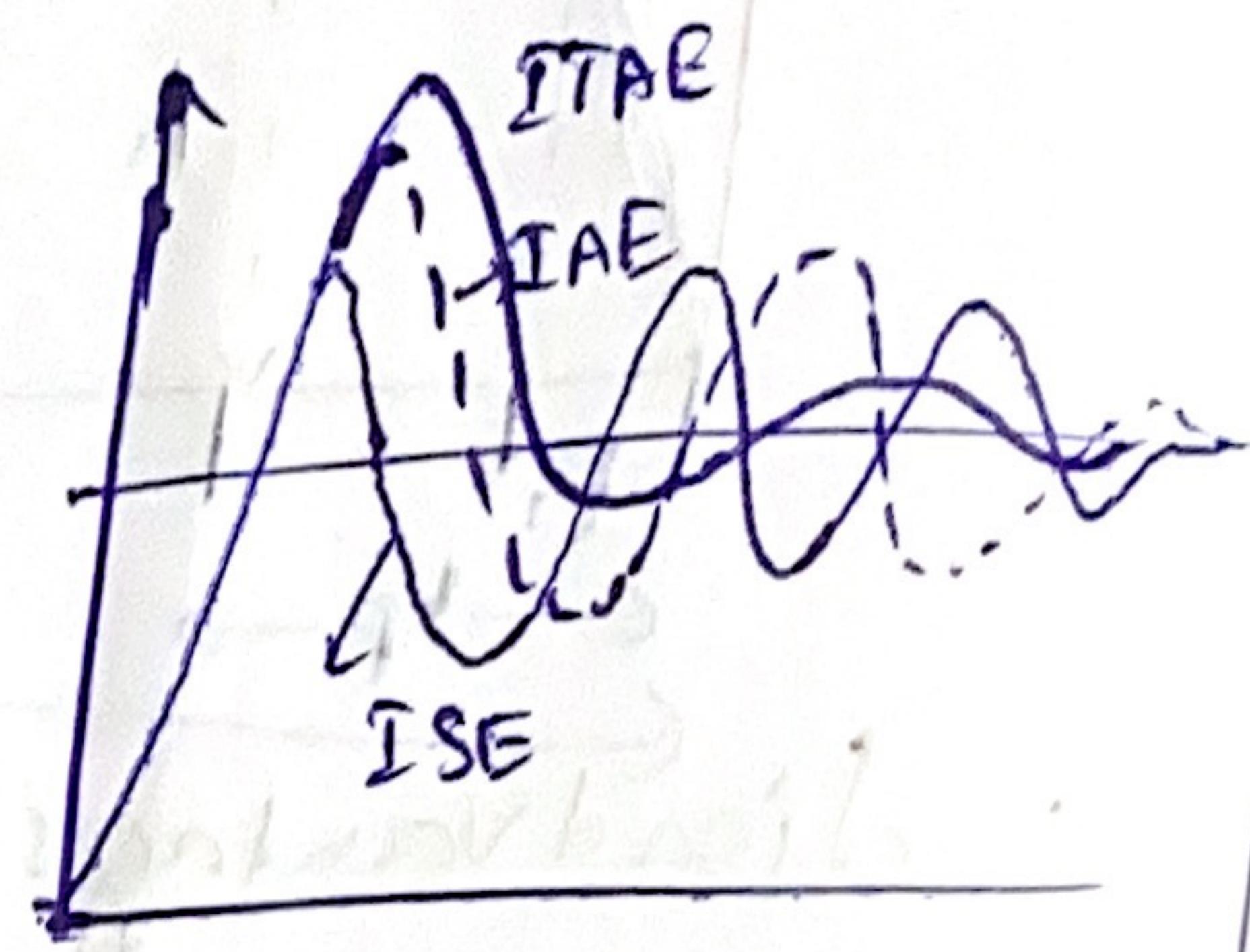
If you want to suppress

1. For large errors, ISE is better than IAE

because errors are squared & contribute large values

2. Small errors, IAE is better than ISE.

3. for errors that persist for long times, ITAE is better



Steps for selecting a feedback controller

1. Define an appropriate performance criteria

2. Compute those values of perf. criteria using a P, PI or PID with best settings

3. Select the controller which gives the best value for the performance criterion.

Drawback of this procedure

1. It is very tedious
2. It relies on the models of the process, sensors, FCE which can't be known exactly.
3. It incorporates certain ambiguities.

	P	I	D
flow level	Always	Usually	Never
temp	"	"	Rare
analytical	"	"	Usually
pressure	"	"	Sometimes

prediction

PID

$$K_P = 1 + 2 \frac{D}{N}$$

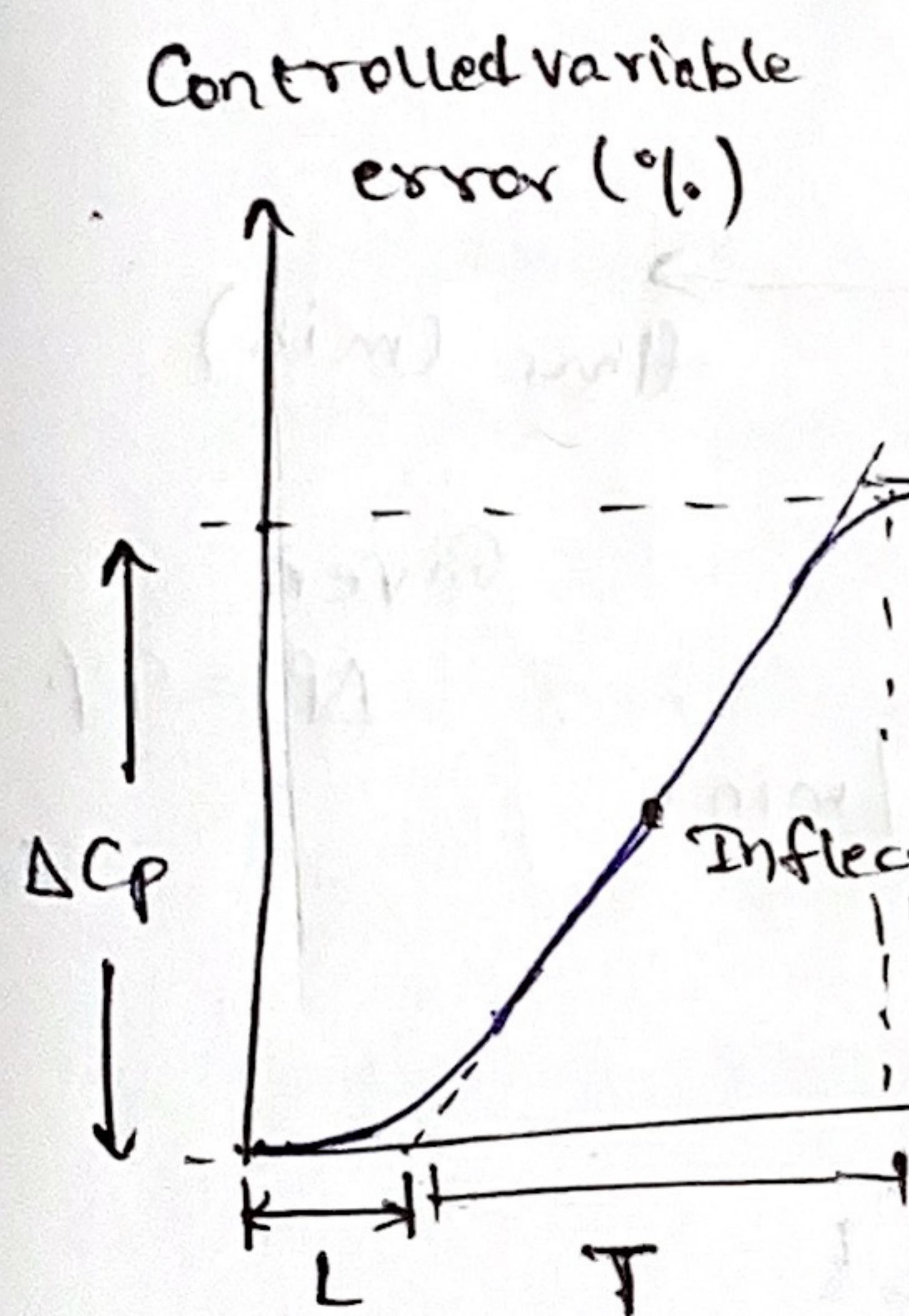
$$T_I = 2L$$

$$T_D = 0.5L$$

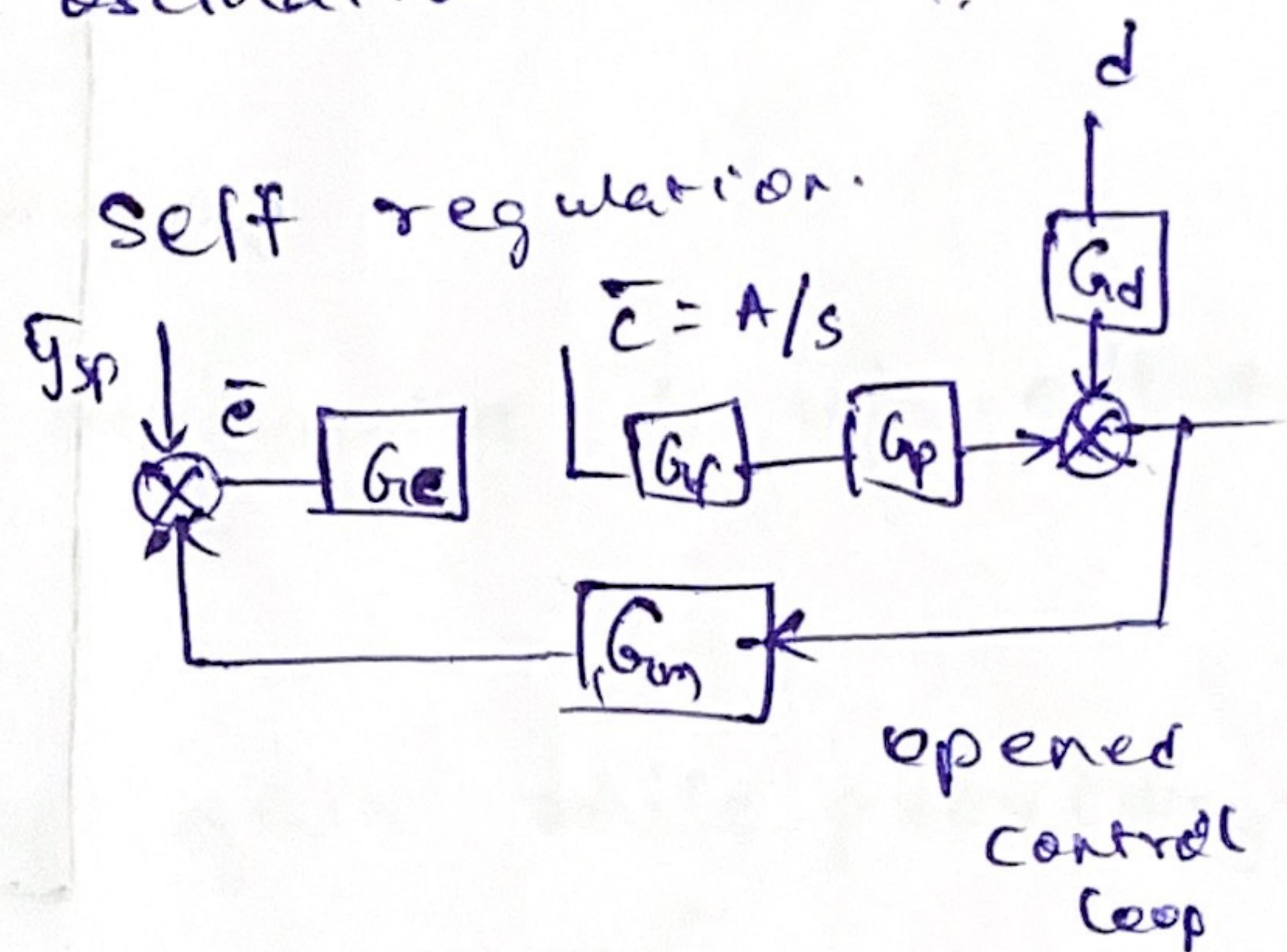
Controller tuning.

- i) Reaction curve Technique (open loop)
(only with self regulation)
- ii) closed loop technique (Continuous cycling method)
(without self regulation).
- iii) closed loop (Damped oscillation method).

ii) Reaction curve.



Sigmoidal shape.



$$N = \frac{\Delta C_p}{T}$$

N = reaction rate in %/min.

the point where slope stops increasing & begins to decrease.

Ziegler - Nicholas

P mode

$$k_p = \frac{\Delta P}{NL}$$

Cohen-coon

cohen coon

$$k_p = \frac{\Delta P}{NL} \left[1 + \frac{NL}{3\Delta C_p} \right]$$

P I

$$k_p = 0.9 \frac{\Delta P}{NL}$$

$$T_I = 3.3L$$

$$k_p = \frac{\Delta P}{NL} \left[0.9 + \frac{1}{12} R \right], \quad R = \frac{NL}{\Delta C_p} \quad (\log \text{ratio})$$

$$T_I = \left[\frac{30 + 3R}{9 + 20R} \right] L$$

P ID

$$k_p = 1.2 \frac{\Delta P}{NL}$$

$$T_I = 2L$$

$$T_D = 0.5L$$

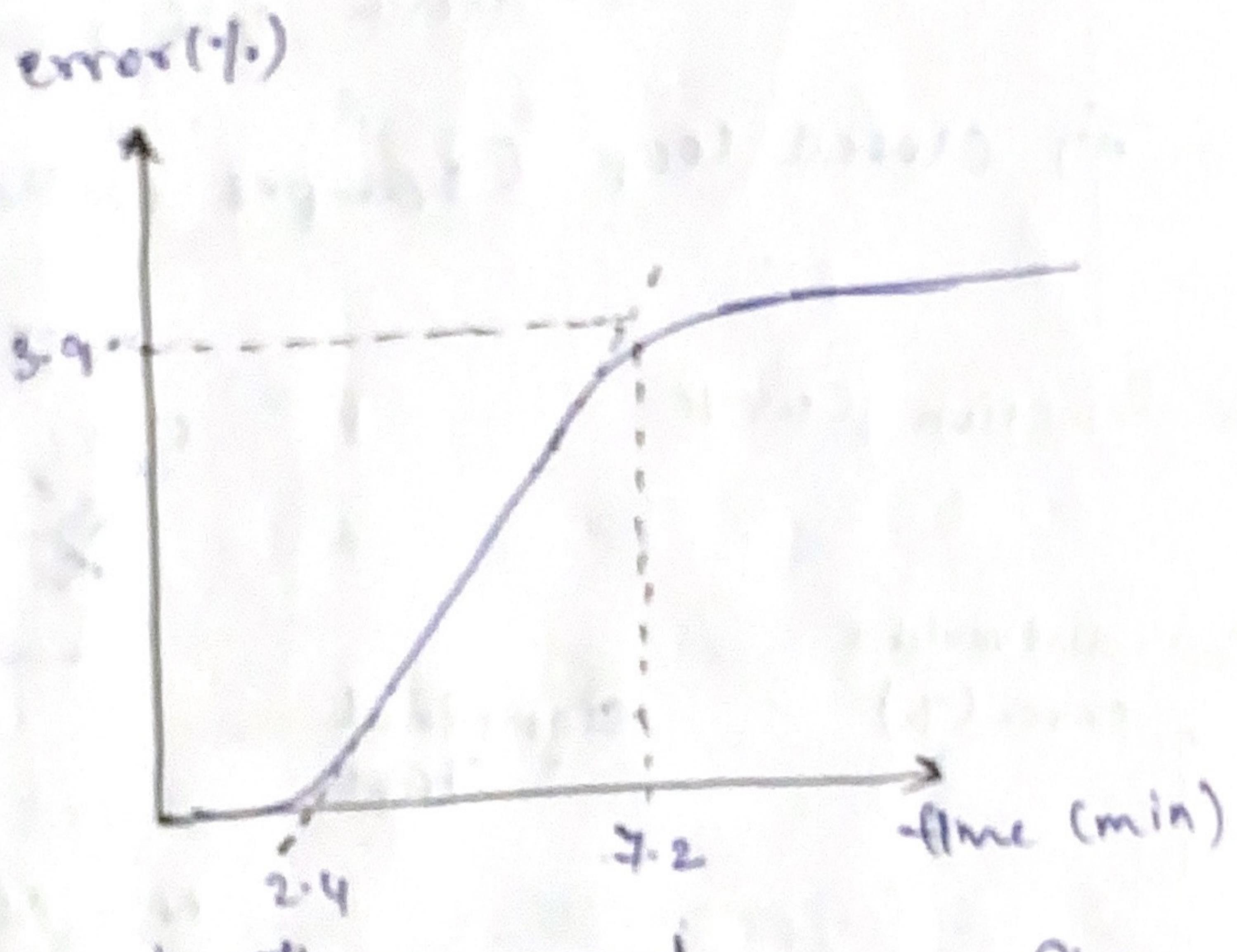
$$k_p = \frac{\Delta P}{NL} \left[1.33 + \frac{1}{4} R \right]$$

$$T_I = \left[\frac{32 + 6R}{13 + 8R} \right] L$$

$$T_D = \left[\frac{4}{11 + 2R} \right] L$$

Q. A transient disturbance test is run on a process loop. The result of a 9% controlling variable change gives a process-reaction graph. Find the settings for a 3-mode action.

3-mode action



Sol. $N = \frac{\Delta C_p}{T}$, $\frac{3.9\%}{7.2 - 2.4} = 0.8125 \%/\text{min.}$

Given
 $\Delta P = 9\%$

$$k_p = 1.2 \frac{\Delta P}{NL} = 5.54$$

$$T_I = 2L = 4.8 \text{ min.}$$

$$T_D = 0.5L = 1.2 \text{ min}$$

$$R = \frac{NL}{\Delta C_p} = 0.5$$

$$k_p = 6.72$$

$$T_I = 4.94 \text{ min}$$

$$T_D = 0.8 \text{ min.}$$

Closed loop method. (Ultimate cycle method).

(Without self regulation).

Adjusting a closed loop until steady oscillations occur.

Controller settings are based on the conditions that generate cycling.

Steps :-

- Reduce any integral & derivative to min. effect.

$$T_D = 0 \quad T_I = \infty$$

Gradually increase the prop gain while providing periodic small disturbances to the process.

K_c - Critical gain

at which the variable just begins to exhibit steady cycling, that is oscillations about set point.

Note the critical time period, T_c of these oscillations (in min)

PI

$$K_p = 0.45 K_c$$

$$T_I = T_c / 1.2$$

for quarter amplitude

$$\text{make } T_I = T_c$$

& adjust gain to obtain quarter response

PID mode

$$K_p = 0.6 K_c$$

$$T_I = T_c / 2$$

$$T_D = T_c / 8$$

for quarter ampli

$$T_I = T_c / 1.5$$

$$T_D = T_c / 6$$

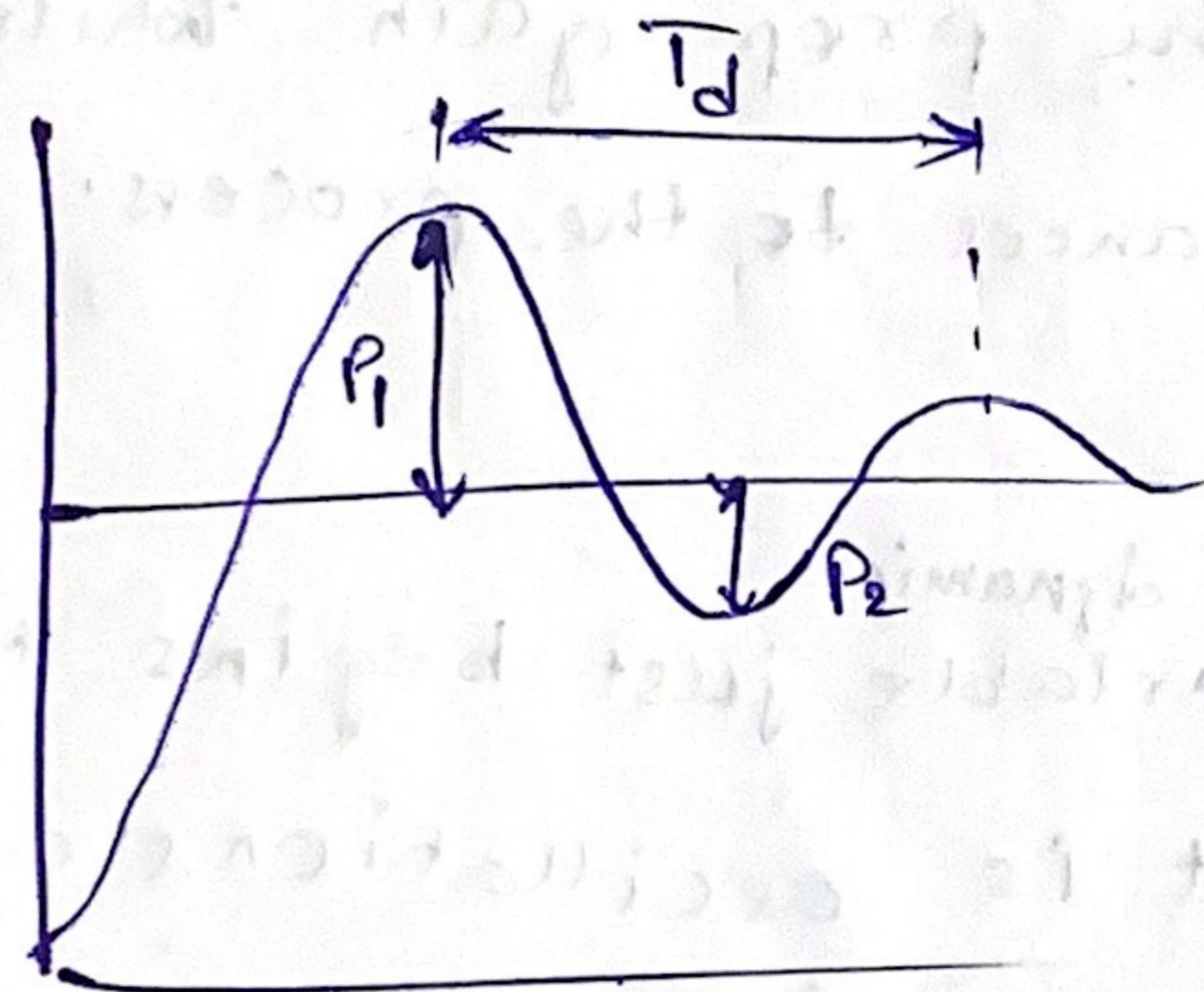
Damped oscillation method.

For plants that do not allow to undergo sustained oscillations.

Initially, closed loop system is operated with low gain P controller with $T_D = 0$ $T_f \approx 0$.

Gain is increased slowly to get a decay ratio of $1/4$.

Under this condition, the period of damped oscillation T_d is also noted & K_d be the prop. gain setting for obtaining $1/4$ decay ratio.



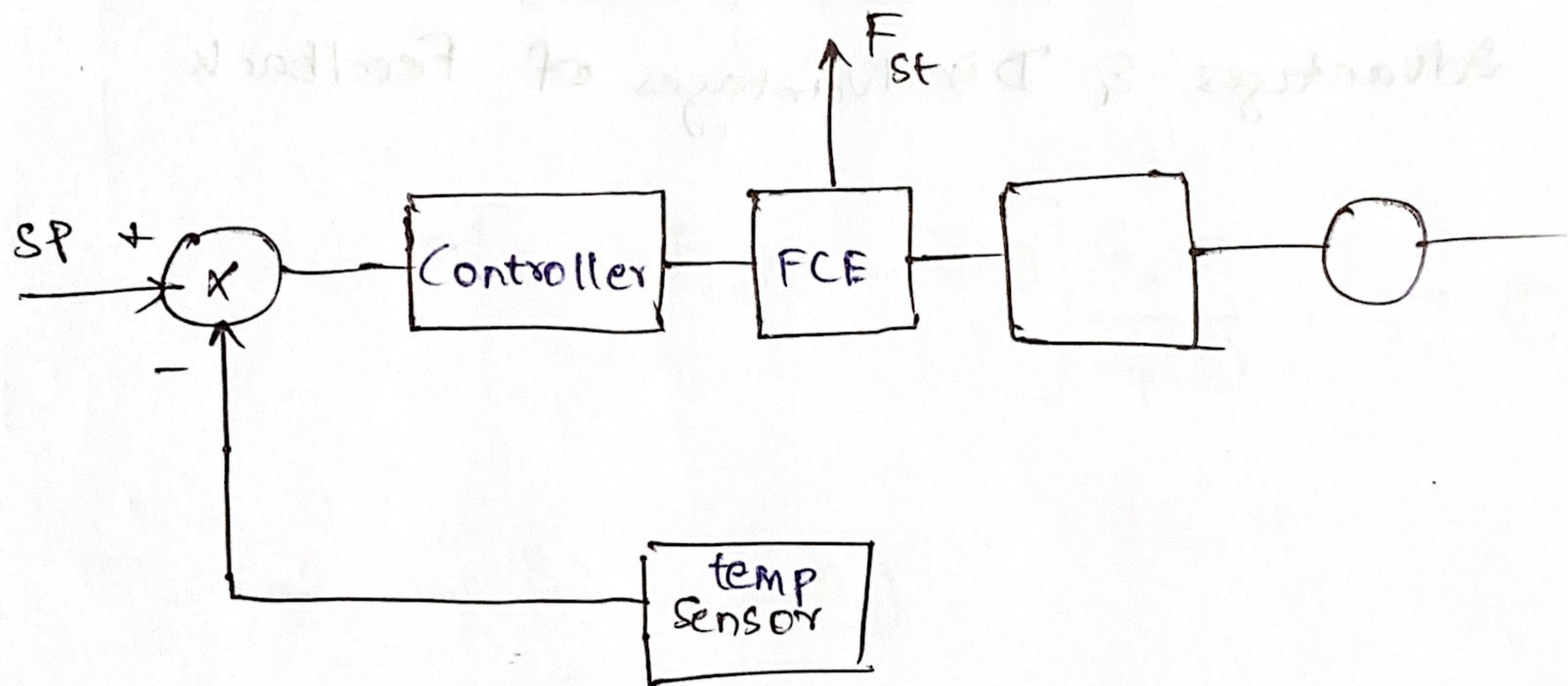
$$T_i = T_d / 1.5$$

$$T_D = T_d / 6$$

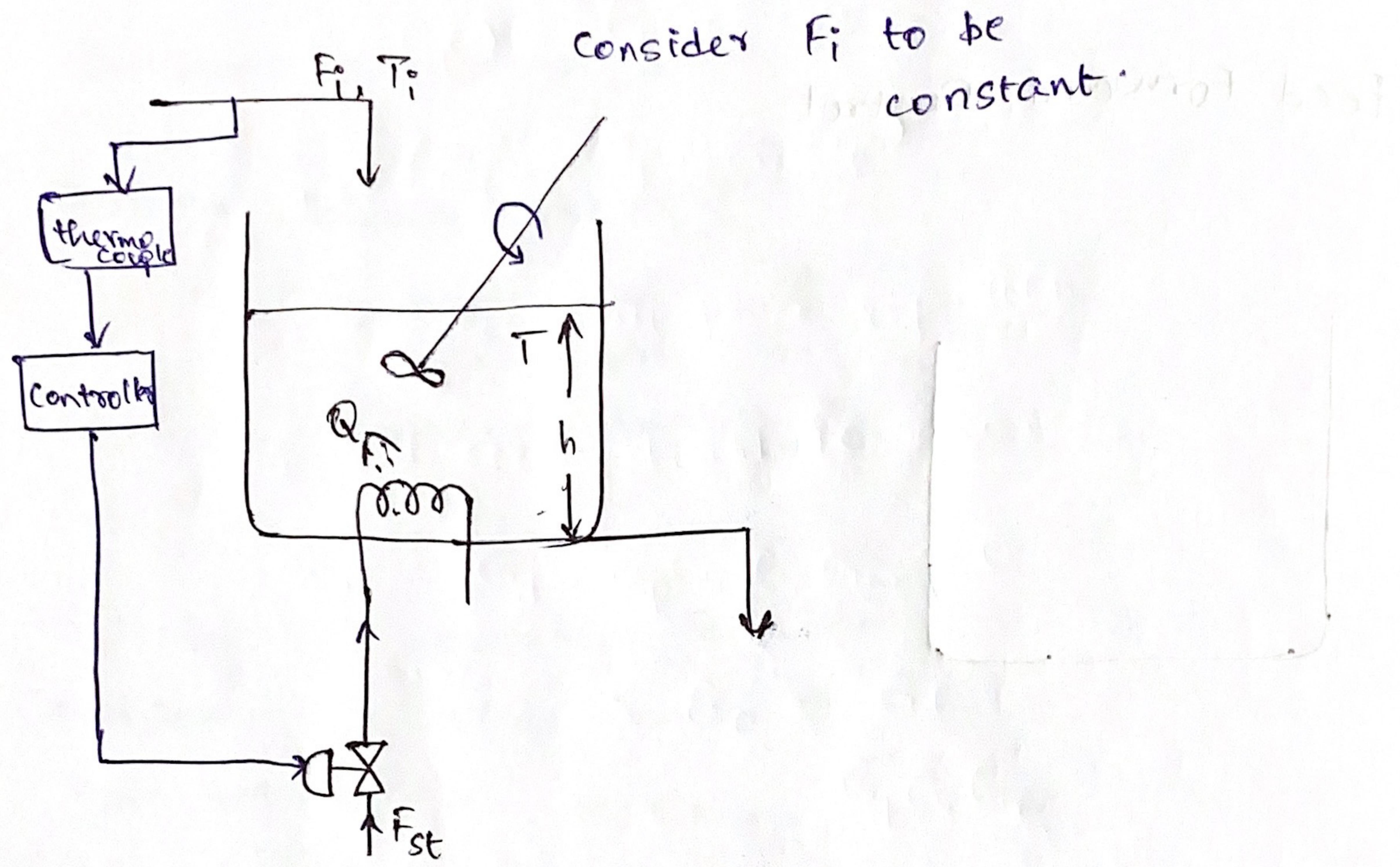
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Advanced Controllers.

Feed Forward Control

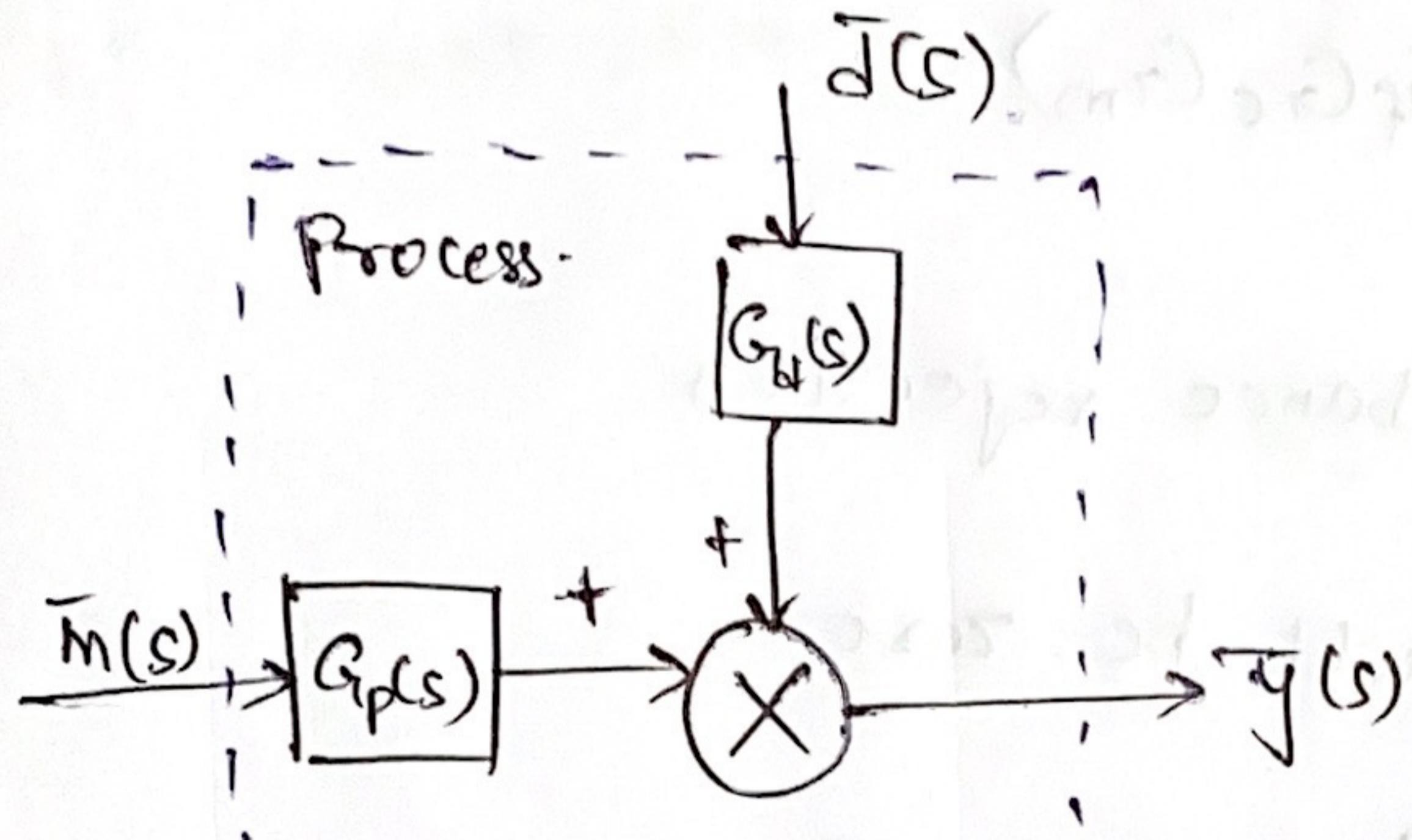


feed forward loop



Advantages & Disadvantages of Feedback

Design Procedures.



$$\bar{y}(s) = \bar{m}(s) G_p(s) + \bar{d}(s) G_d(s)$$

Consider the O/P has reached set point $\bar{y}_{sp}(s)$

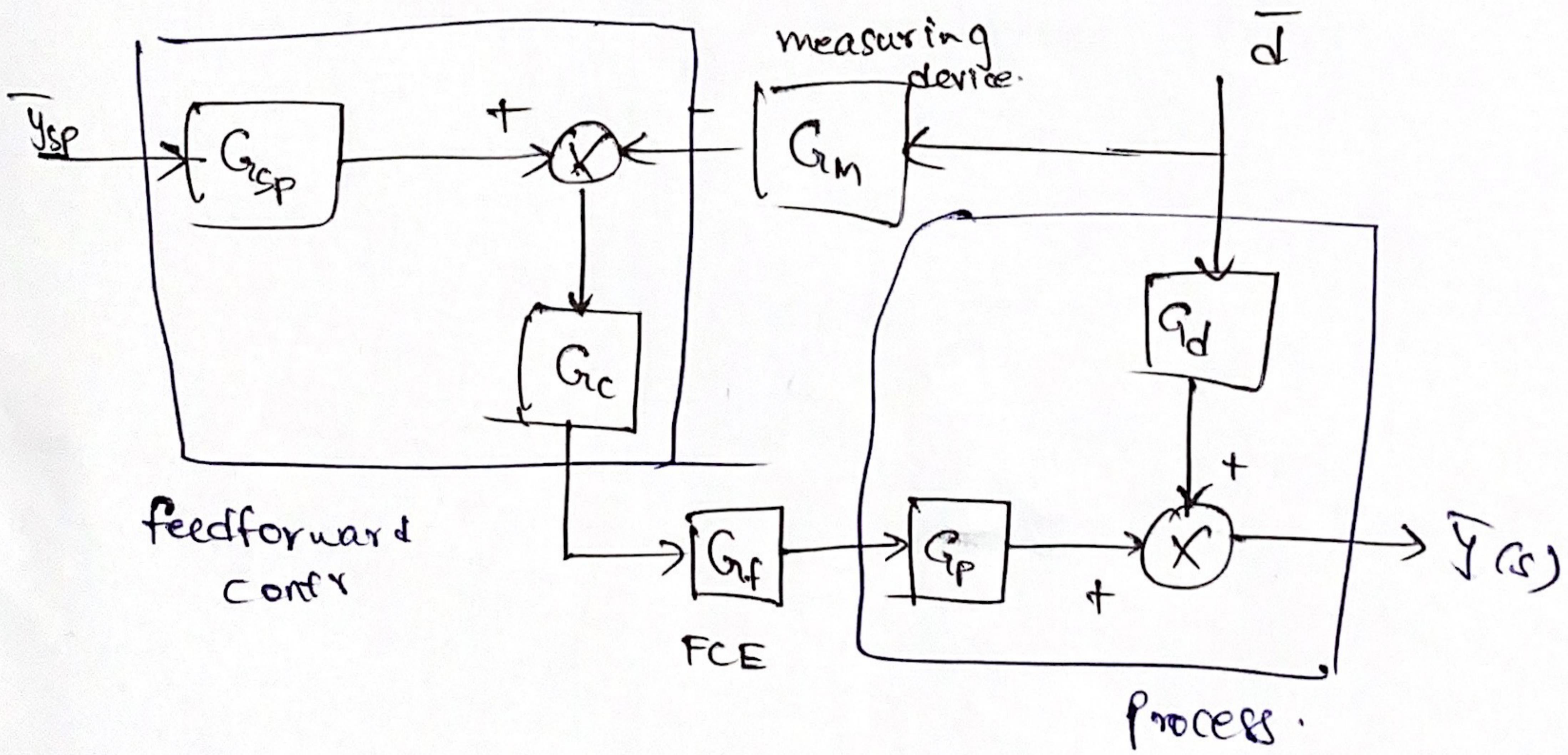
$$\text{So, } \bar{y}_{sp}(s) = \bar{m}(s) G_p(s) + \bar{d}(s) \cdot G_d(s) \rightarrow ②$$

$$\Rightarrow \bar{m}(s) = \frac{\bar{y}_{sp}(s) - G_d(s) \cdot \bar{d}(s)}{G_p(s)}$$

$$\bar{m}(s) = \left[\frac{1}{G_d(s)} \bar{y}_{sp}(s) - \bar{d}(s) \right] \frac{G_d(s)}{G_p(s)} \rightarrow ③$$

$$G_c = \frac{G_d(s)}{G_p(s)} \rightarrow ④$$

$$G_{sp} = \frac{1}{G_d(s)}$$



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$$\bar{y}(s) = \left(G_p G_f G_c G_{sp} \right) \bar{y}_{sp} + \bar{d} G_d(s) - G_p G_f G_c G_m$$

Design

$$\bar{y}(s) = \frac{\left(G_p G_f G_c G_{sp} \right) \bar{y}_{sp}}{1 + d} (G_d - G_p G_f G_c G_m)$$

→ To make disturbance rejection

i.e., d coeff. should be zero.

$$\text{so, } G_d - G_p G_f G_c G_m = 0$$

$$\Rightarrow G_c = \frac{G_d}{G_p G_f G_m}$$

→ for setpoint tracking

Coeff of $\bar{y}_{sp} = 1$

$$\text{so, } G_p G_f G_c G_{sp} = 1$$

$$G_{sp} = \frac{1}{G_p G_f G_c} \cdot \frac{1}{G_p G_f} \left(\frac{G_p G_f G_m}{G_d} \right)$$

$$\Rightarrow G_{sp} = \frac{G_m}{G_d} //$$

example.

Objective: To maintain temp T at T_{sp} . Assume
 F_i doesn't change $F_i = F$

$$V\rho C_p \frac{dT}{dt} = FL C_p (T_i - T) + Q \quad \xrightarrow{\text{Transient energy balance eq.}}$$

$$A \cdot \frac{dh}{dt} = F_i - F$$

$$Ah \frac{dT}{dt} = F_i (T_i - T) + \frac{Q}{\rho C_p}$$

where A - area of tank

h - height of liquid

ρ - density of liquid

C_p - liquid's heat capacitance

Q - Heat input

i) Steady state feed forward
Controller:

$$O = F_i(T_i - T) + \frac{Q}{\rho C_p}$$

$$T = T_i + \frac{Q}{F_i \rho C_p}$$

Design eq, is the eq that relates controlled variable to
the manipulated variable.

$$\Rightarrow Q = F_i \rho C_p (T_{sp} - T_i)$$