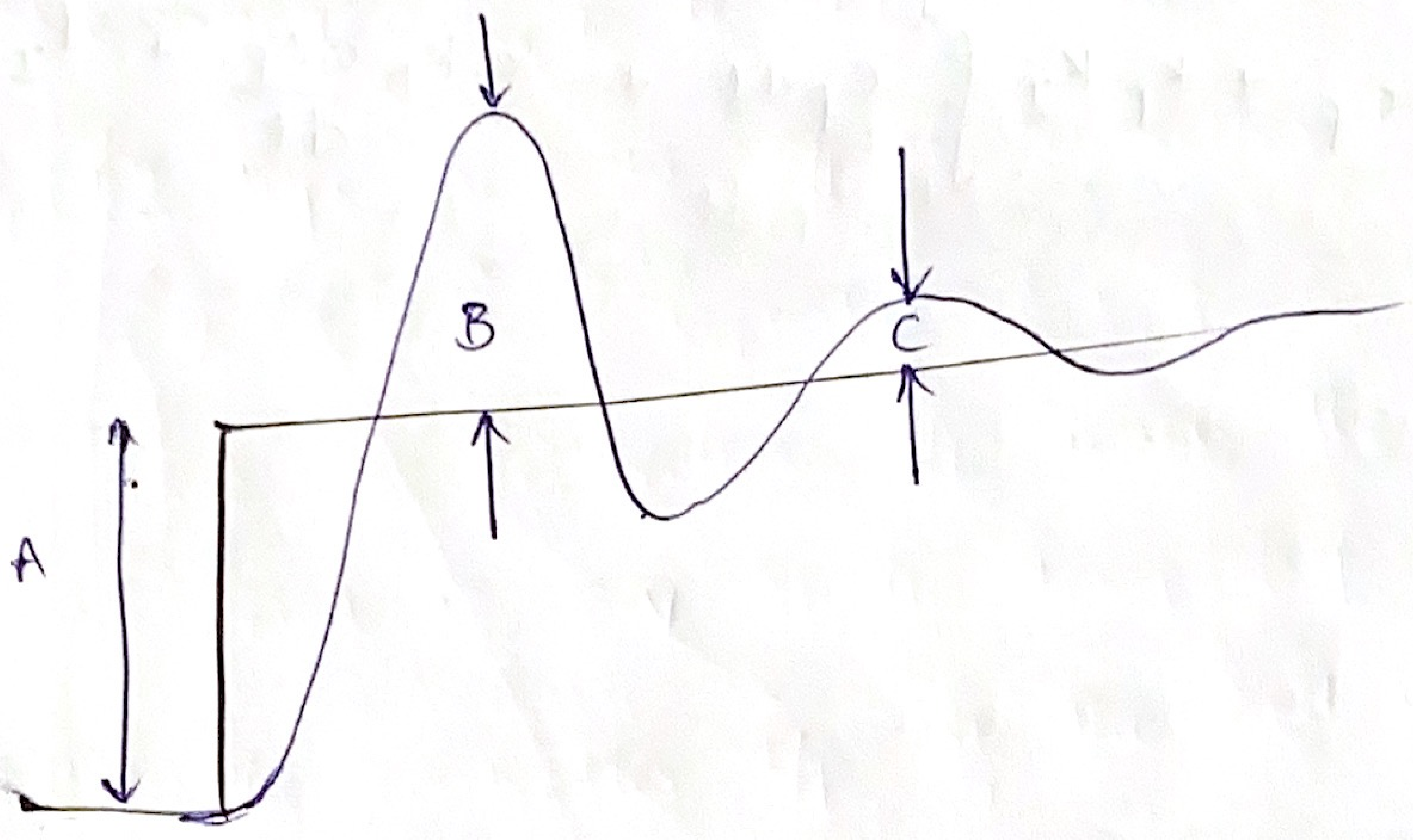
Performance Analysis



Rise time =

Peak time =

Overshoot ratio = B/A

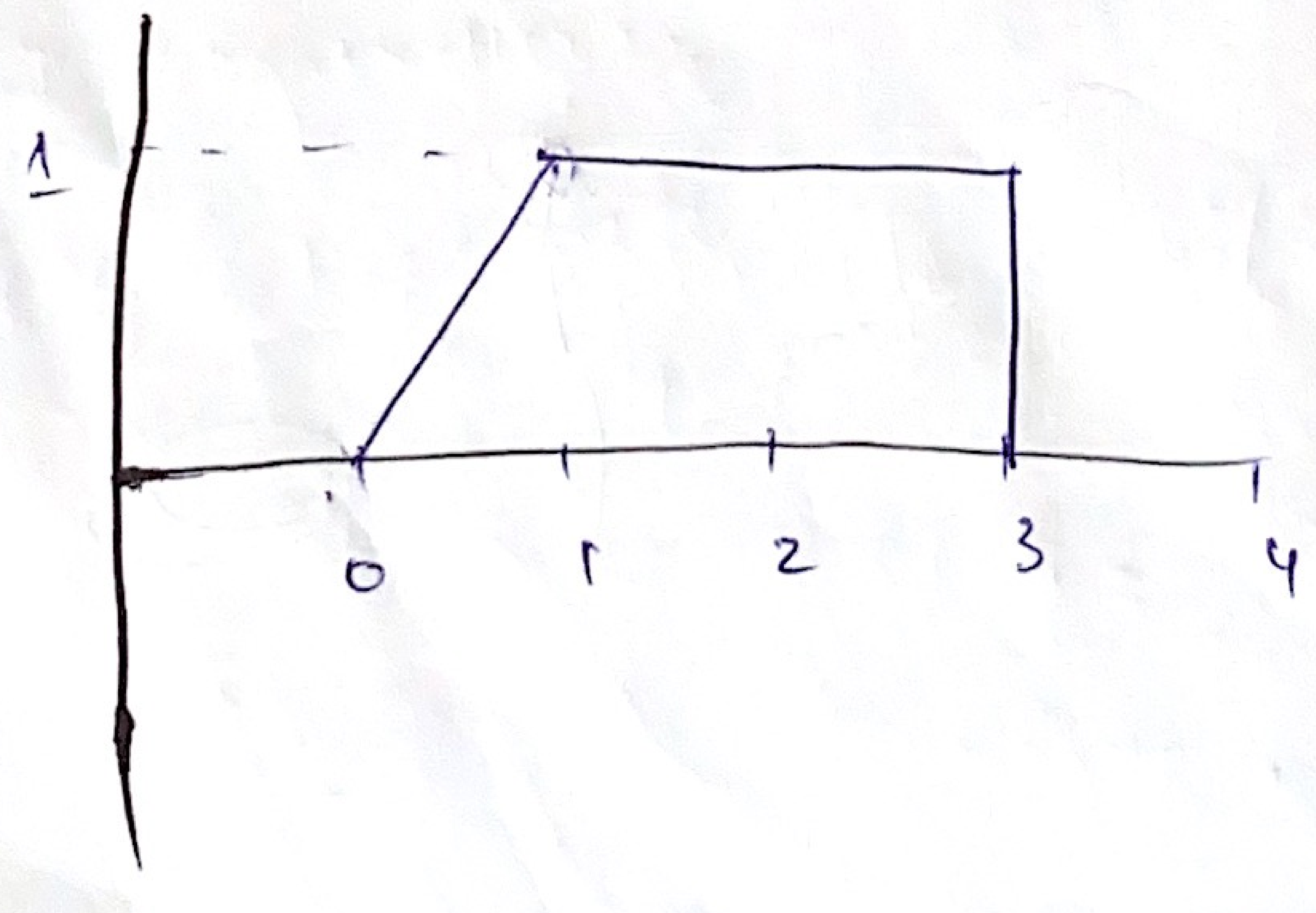
Decay ratio = C/B

Settling time =

Problems

1. Given error, plot a graph of a PI controller o/p as a function of time

$K_p = 5 \quad K_I = 1\ s^{-1} \quad P_i(0) = 20\%$



Sol. $p = k_p e_p + k_p K_I \int_0^t e_p dt + P_I(0)$

(1) $0 \le t \le 1$; $e_p = t$

$P =$

(2) $1 \le t \le 3$; $e_p = 1$

(3) $t \geq 3$; $e_p = 0$

$P_{(1)} = 5(t) + 5(1) \cdot \int_0^1 t \cdot dt + 20$

$= 5t + 5\left[\frac{t^2}{2}\right]_0^1 + 20$

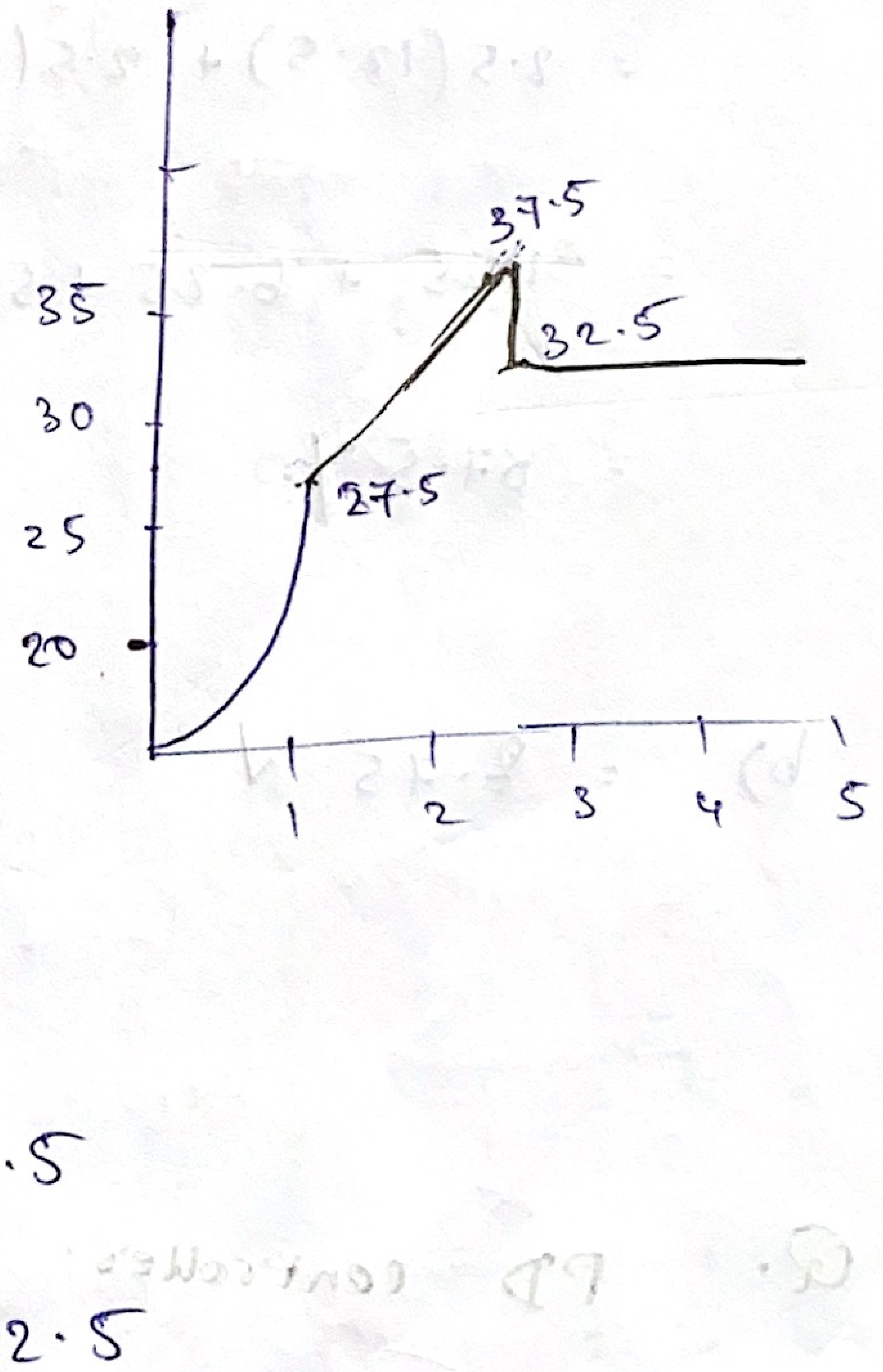
$P_1(t) = 5t + 2.5t^2 + 20$

$P_1(1) = 27.5$

$P_1(0.3) = 21.725$

$P_1(0.5) = 23.125$

$P_1(0.8) = 25.6$



Case (2)

$P_I(0) = k_p k_I \int_0^1 e_p dt + 20$

$= 5\left(\frac{1}{2}\right) + 20 = 22.5$

$P_{(2)} = 5(1) + 5(1) \cdot \int_1^3 1 \cdot dt + 22.5$

$= 5 + \frac{5}{\cancel{8}}(2) + 22.5 = 3\cancel{7}.5$

Case (3)

~~$P_I(0) = k_p k_I \int_1^3 1 \cdot dt + 22.5$~~

~~$= \frac{5 \cdot (9-1)}{2} + 22.5$~~

$P_I(0) = 5(1) \cdot \int_1^3 1 \cdot dt + 22.5$

$= 32.5$

$P(3) = 5(0) + 5(1) \int_3^4 0 \cdot dt + 32.5 = 32.5$

24/9/

For a temp measuring system,

Q. temp range is 0 to 160°C. The controller is of PI type and the o/p of controller is 0 – 10 V

PB = 40% K_I = 1 repeat in 5 min. Desired temp is 80°C. The controller o/p when the temp is at set point is 5 V. If the temp drops to 60°C, at the end of 1 min

Calculate the controller o/p

a) in %

b) in Volts (V).

$K_P = \frac{100}{PB} = \frac{100}{40} = 2.5$

$P_0 = 5V = 50\%$

Sol. $P = K_P e_p + K_P K_I \int e_p \, dt + P_0$

$e_p = \frac{r-b}{b_{max} - b_{min}} = \frac{80-60}{160} \times 100$

$= \frac{2}{16} \times 100 = 12.5\%$

$= 2.5(12.5) + 2.5(0.2)\int_0^{1\,min} 12.5 \cdot dt + 50$

$K_I = \frac{1}{5} = 0.2$ repeats/min

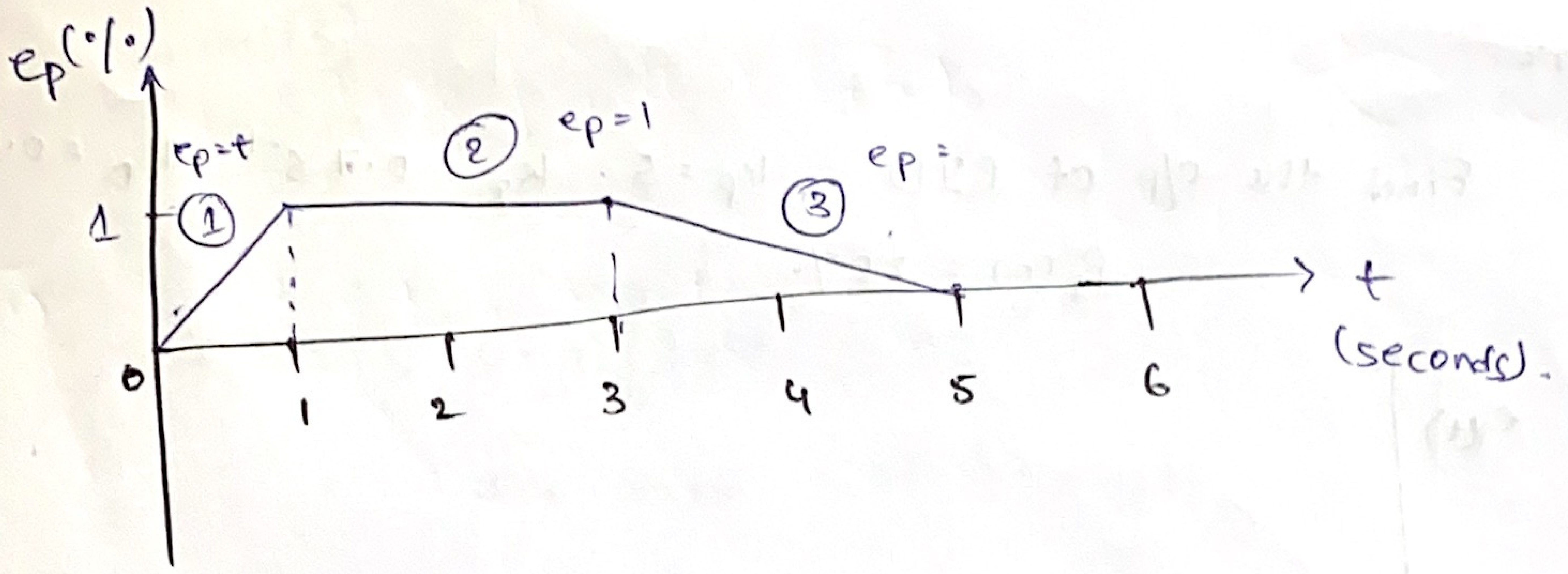
$= 31.25 + 6.25 + 50$

$= 87.5\%$

b) $= 8.75$ V

Q. PD controller:

$K_P = 5$ $K_D = 0.5\,s$ $P_0 = 20\%$



$$p = k_p e_p + k_p k_d \cdot \frac{de_p}{dt} + P_o$$

region 1

$$p = 5t + 5(0.5) \cdot \frac{d}{dt}(t) + 20$$

$$= 5t + 2.5 + 20$$

$$p = 5t + 22.5 \qquad \text{at } t = 1 \quad p = 27.5$$

region 3

$$(y - y_1) = m(x - x_1)$$

$$(y-1) = \frac{0-1}{2}(x-3)$$

$$(y-1) = -\frac{1}{2}(x-3)$$

$$2y - 2 = -x + 3.$$

$$x + 2y - 5 = 0.$$

$$2y = 5 - t$$

$$y = \frac{5-t}{2}$$

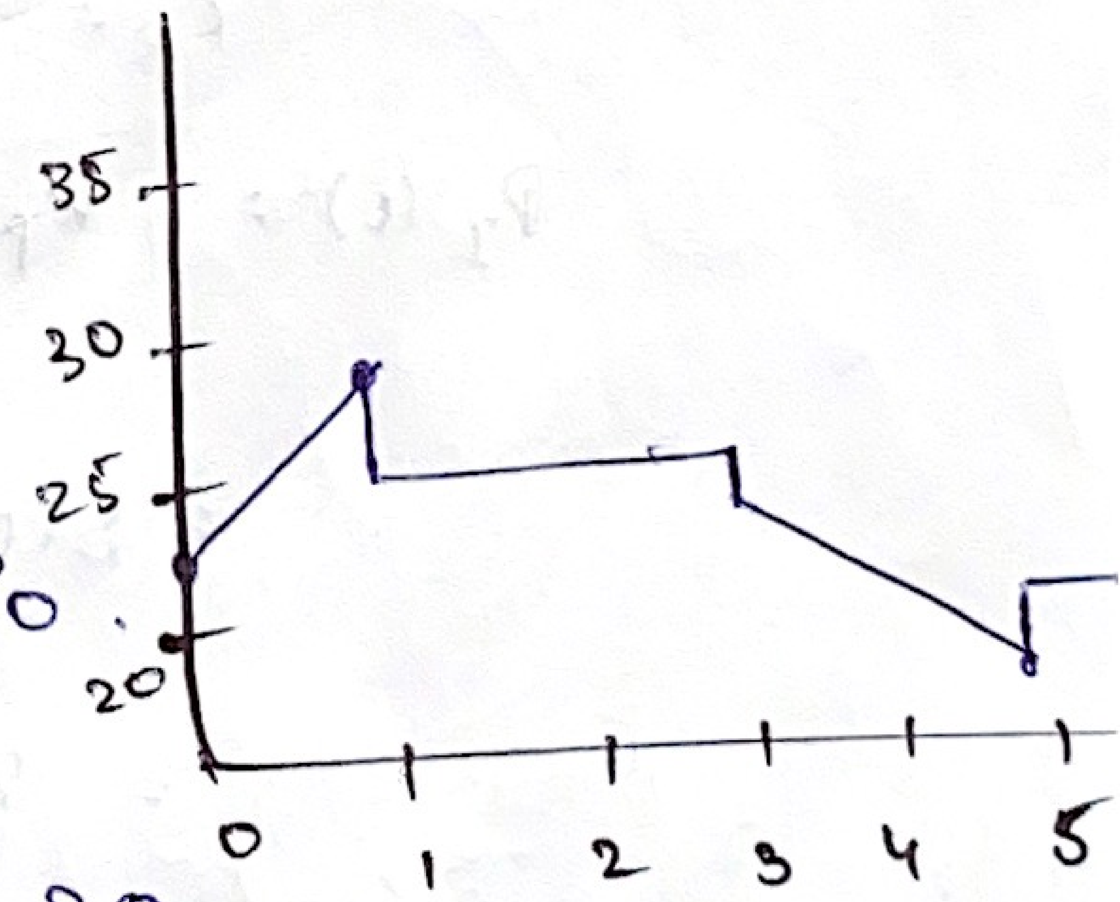
$$e_p = \frac{5-t}{2}$$

%. region 2 $\quad e_p = 1$

$$p = k_p e_p + k_p k_d \cdot \cancel{\frac{de_p}{dt}}^{\,0} + P_o.$$

$$p = k_p e_p + P_o$$

$$p = 5(1) + 20 = 25$$



region 3 $\quad$ at $t = 3$.

$$p = k_p e_p + k_p k_d \cdot \frac{d}{dt}\left(\frac{5-t}{2}\right) + P_o.$$

$$= 5(1) + 5(0.5) \cdot \frac{d}{dt}\left(\frac{5-t}{2}\right) + 20.$$

$$= 5 + 2.5 \cdot \left(\frac{1}{2}(0-1)\right) + 20$$

$$= 5 - 1.25 + 20$$

$$p = 25 - 1.25 = 23.75$$

at $t = 5$.

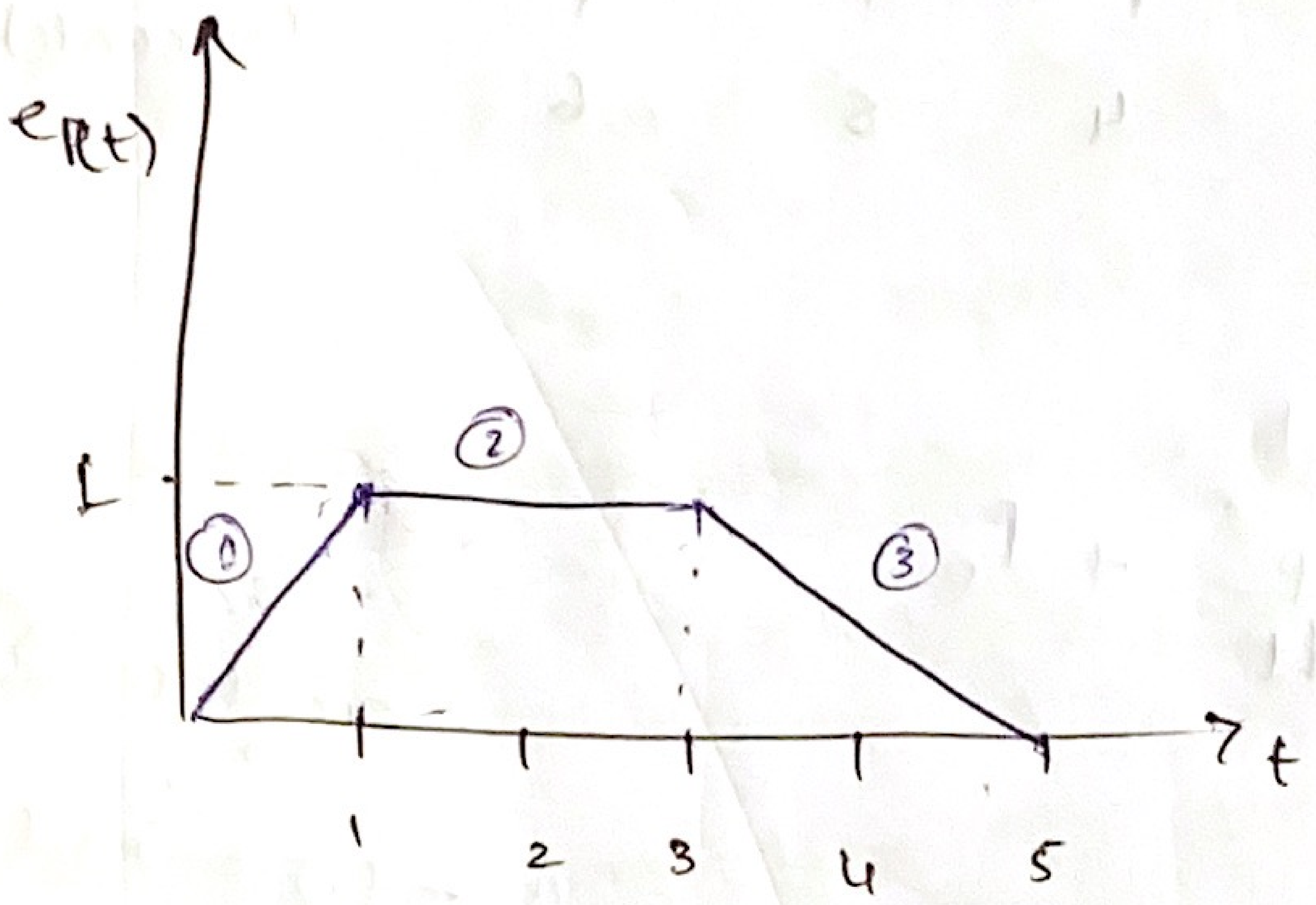
$$p = 5(0) - 1.25 + 20$$

$$p = 18.75.$$

30/9/22

Q. Find the o/p of PID, $k_p = 5$, $k_I = 0.7\ s^{-1}$, $k_D = 0.5\ s$

$P_I(0) = 20\%$.



Sol. $P = k_p e_p + k_p k_I \int_0^t e_p\,dt + k_p k_d \frac{de_p}{dt} + P_I(0)$

Region 1.

$P = 5(t) + 5(0.7)\int_0^t t\cdot dt + 5(0.5)\frac{d}{dt}(t) + 20$

$= 5t + 3.5\left(\frac{t^2}{2}\right) + 2.5 + 20$

$P_1 = 5t + 1.75t^2 + 22.5$

$p(1) = 29.25$

$p(0.75) = 27.23$

$p(0.5) = 25.43$

$p(0) = 22.5$

Region 2.

$e_p = 1$

$P_I(0) = k_p k_I \int_0^1 t\cdot dt + P_I$

$= 5(0.7)\cdot \frac{1}{2} + 20$

$= 21.75$

$p(t) = 5(1) + 5(0.7)\int_1^3 1\cdot dt + 5(0.5)\cdot\frac{d}{dt}(1) + 21.75$

$= 5 + 3.5(2) + 2.5(0) + 21.75$ $\qquad P_2 = 3.5t + 23.25$

$= 5 + 7 + 21.75 = 33.75$

region (3)

$e_p = \frac{5-t}{2}$

$P_I(0) = 5(0.7)\int_1^3 1 \cdot dt + 21.75$

$= 7 + 21.75 = 28.75$

$P = 5\left(\frac{5-t}{2}\right) + 5(0.7)\int_3^5 \left(\frac{5-t}{2}\right) \cdot dt + 5(0.5) \cdot \frac{d}{dt}\left(\frac{5-t}{2}\right) + 28.75$

$P_3 = 5\left(\frac{5-t}{2}\right) + 3.5 \times \frac{1}{2}\left(5t - \frac{t^2}{2}\right)_3^t + \frac{2.5}{2}(0-1) + 28.75$

$= \frac{25-5t}{2} + 1.75 \left[5t - \frac{t^2}{2} - \left(15 - \frac{9}{2}\right)\right] - 1.25 + 28.75$

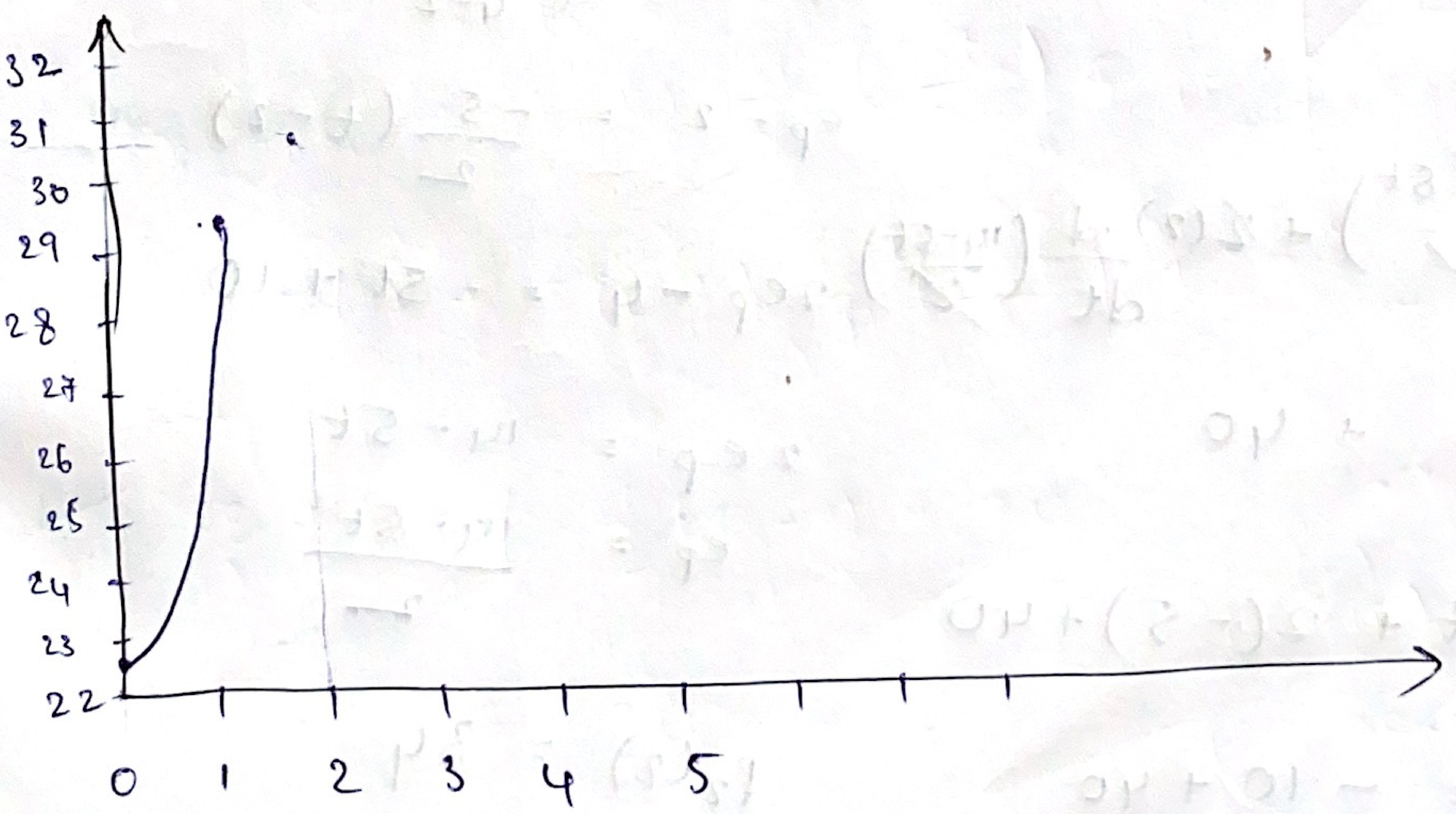
$= 12.5 - 2.5t + 1.75\left(5t - \frac{t^2}{2} - 10.5\right) - 1.25 + 28.75$

$= 12.5 - 2.5t + 8.75t - 0.875t^2 - 18.375 - 1.25 + 28.75$

$P_3 = 6.25t - 0.875t^2 + 21.625$

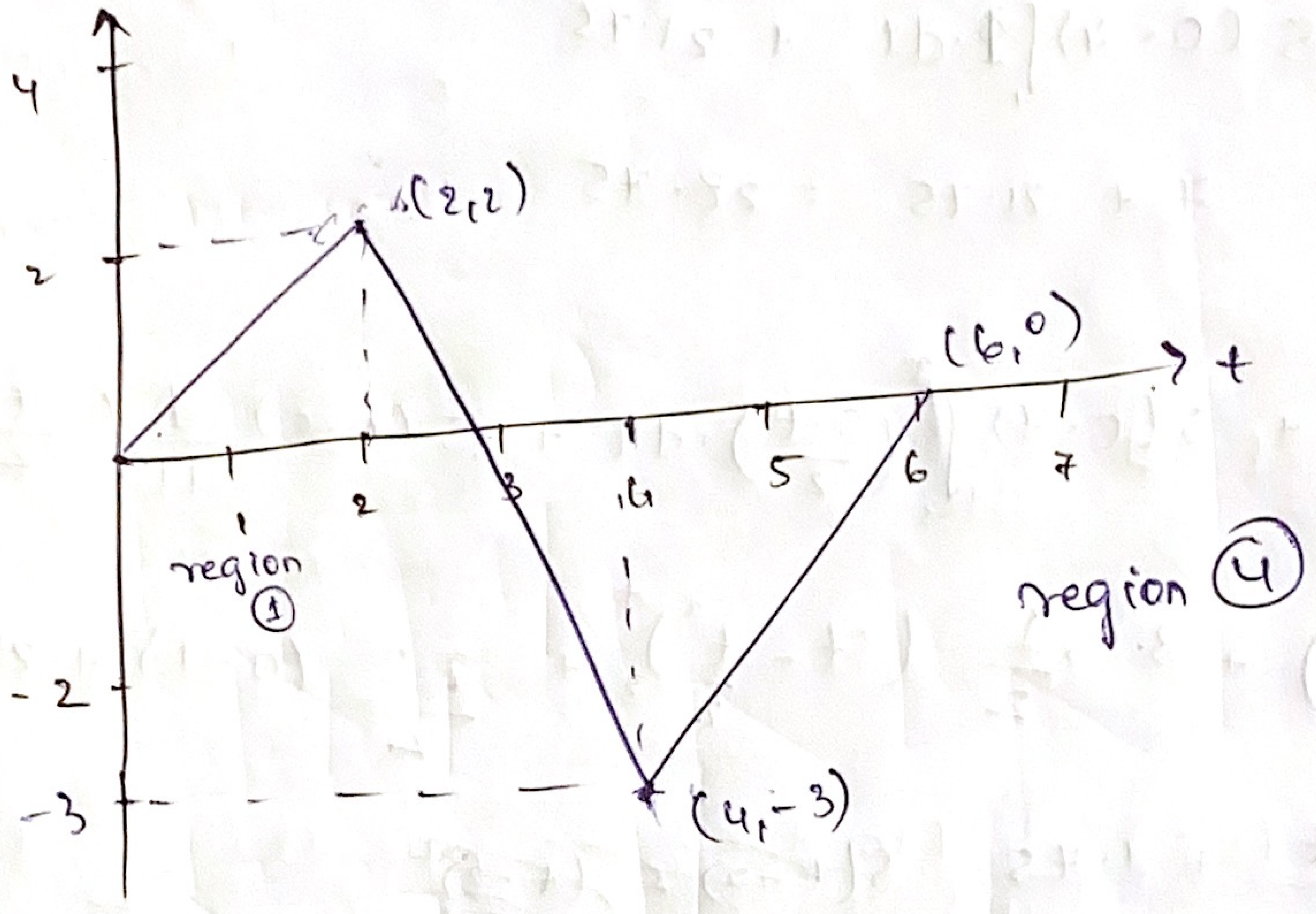
$P_3(3) = 32.5$

$P_3(5) = 31$



1/10/22

Q. PD controller has $K_P = 2$ $K_D = 2$ $P_0 = 40\%$



$P = K_P e_p + K_P K_d \cdot \frac{de_p}{dt} + P_0$

1 0 to 2 $e_p = t$

$P_1 = 2(t) + 2(2) \cdot \frac{d}{dt}(t) + 40$

$= 2t + 4 + 40 \quad = 2t + 44 \Big]_0^2$

$P_1(0) = 44 \quad P_1(2) = 48$

2 2 to 4

$(e_p - 2) = \left(\frac{-3-2}{4-2}\right)(t-2)$

$e_p - 2 = \frac{-5}{2}(t-2)$

$2e_p - 4 = -5t + 10$

$2e_p = 14 - 5t$

$e_p = \frac{14-5t}{2}$

$P_2 = \cancel{2}\left(\frac{14-5t}{\cancel{2}}\right) + \cancel{2}(2) \cdot \frac{d}{dt}\left(\frac{14-5t}{\cancel{2}}\right) + 40$

$= 14 - 5t + 2(-5) + 40$

$= 14 - 5t - 10 + 40$

$P_2 = 44 - 5t$

$P_2(2) = 34$

$P_2(4) = 24$

region ③

$$P_3 = \cancel{2}\left(\frac{3t-18}{\cancel{2}}\right) + \cancel{2}(2)\cdot\frac{d}{dt}\left(\frac{3t-18}{\cancel{2}}\right) + 40$$

$$= 3t - 18 + 2(3) + 40$$

$$P_3 = 3t + 28$$

$$(e_p - 0) = \left(\frac{-3-0}{4-6}\right)(t-6)$$

$$e_p = \frac{+3}{+2}(t-6)$$

$$2e_p = 3t - 18$$

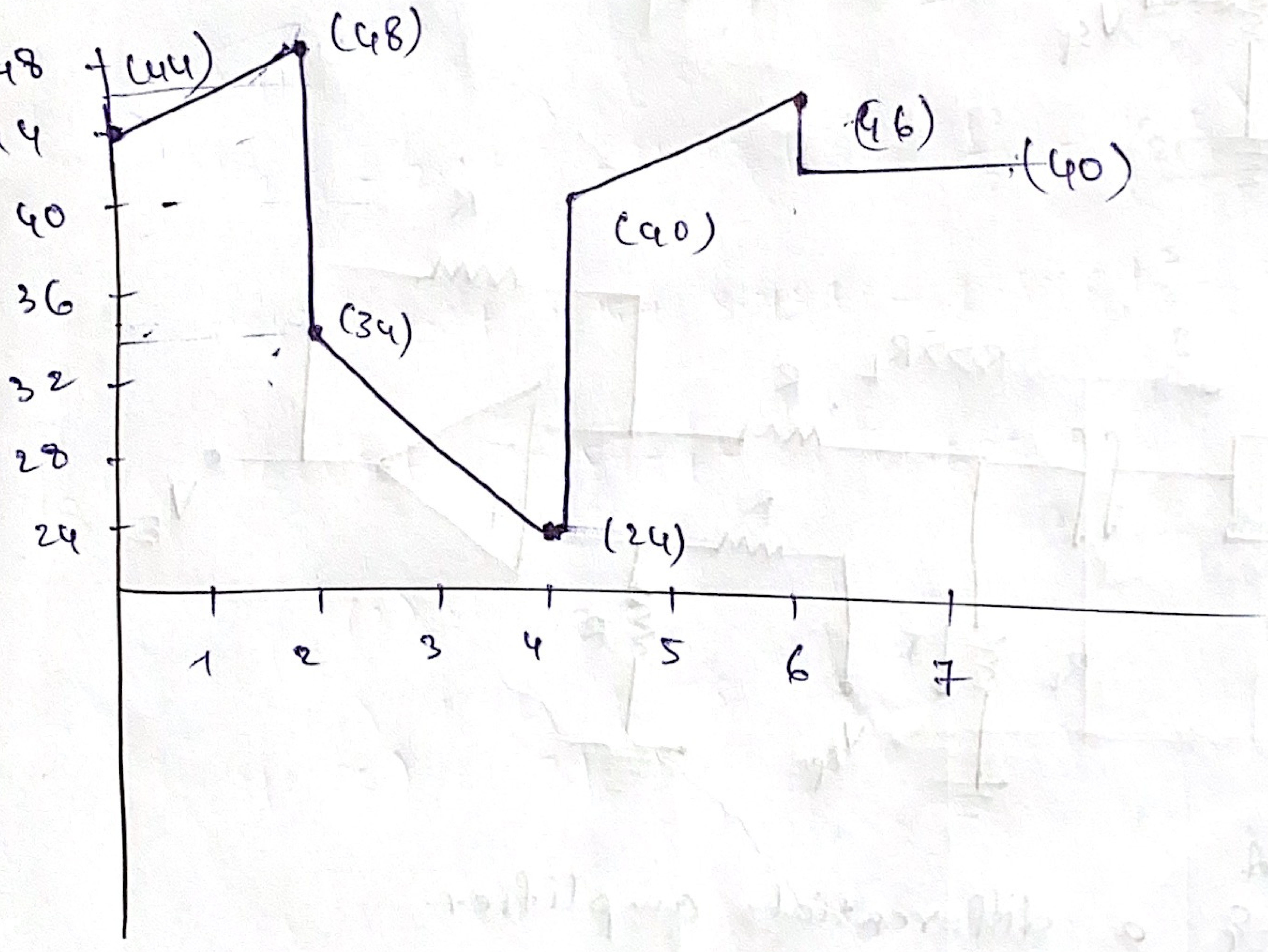
$$e_p = \frac{3t-18}{2}$$

$$P_3(4) = 40$$

$$P_3(6) = 46$$

46
-18

40
-
36



region ④

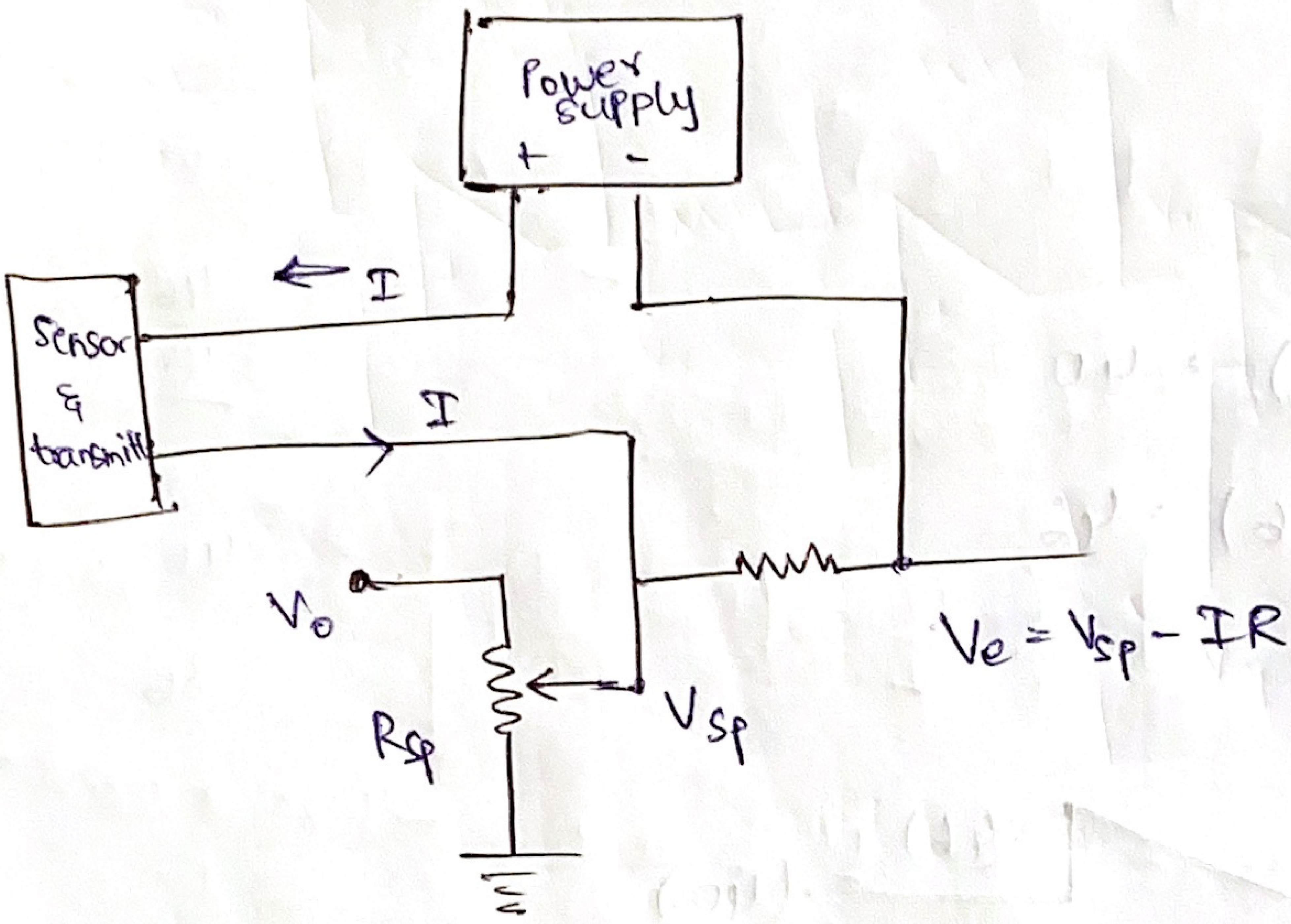
$$P = 2(0) + 2(2)\cdot\frac{d}{dt}(0) + 40$$

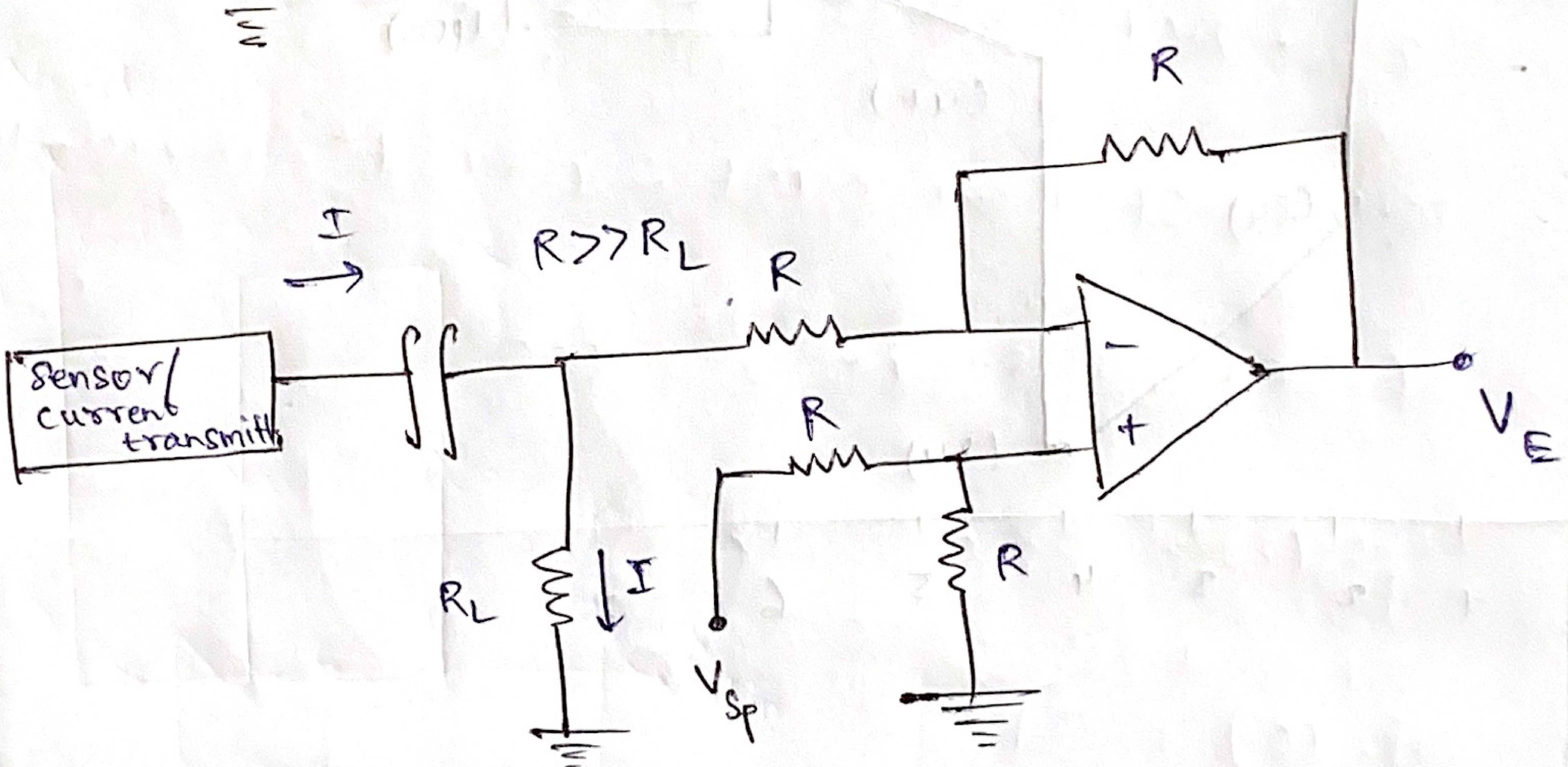
$$P_4 = 40$$

Design of Analog Controllers

Error detector

(floating ground).





Ground based current & a differential amplifier

Q. A sensor converts from 0 to 2 m in to a 4 to 20 mA current. An error detector such as is used with R = 100 $V_0 = 5\,V$ and $R_{sp} = 1\,k\Omega$ pot.

a) If the setpoint is 0.85 m, what is V_{sp}.

b) If $V_{sp} = 1.5\,V$, what is the range of error voltage as position varies from 0 to 2 m.

Sol: 0 to 2 m $\quad (0, 4) \quad (2, 20)$

4 to 20 mA. $\quad x_1 \; y_1 \quad x_2 \; y_2$.

$$(y - y_1) = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)(x - x_1)$$

$$(i - 4) = \left(\frac{20 - 4}{2}\right)(x - 0)$$

$$2(i - 4) = 16x$$

$$2i - 8 = 16x$$

$$i = \frac{16x + 8}{2}$$

$$\Rightarrow i = 8x + 4$$

$$i = 8m\,x + 4m$$

$$\Rightarrow i_{(mA)} = 0.008x + 0.004$$

R = 100

$V = IR$

$V = 100\,I$.

$V = 0.8x + 0.4$

$V_{(0.85\,m)} = 0.8(0.85) + 0.4 = 1.08$

Given $V_{sp} = 1.5\,V$. $\quad$ at 0m = 1.1V

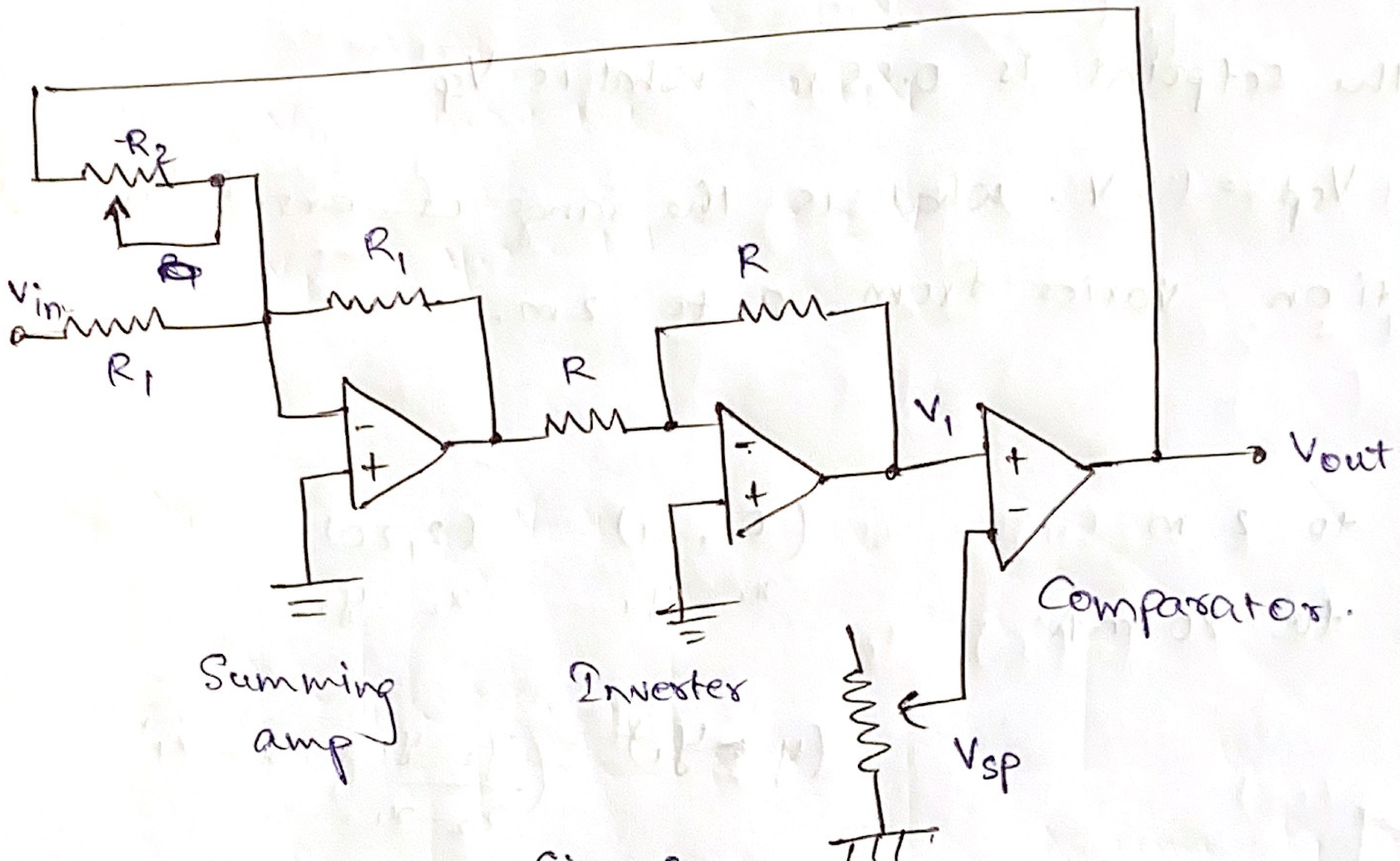
$\quad$ 2m = −0.5

error, $V_e = V_{sp} - IR$

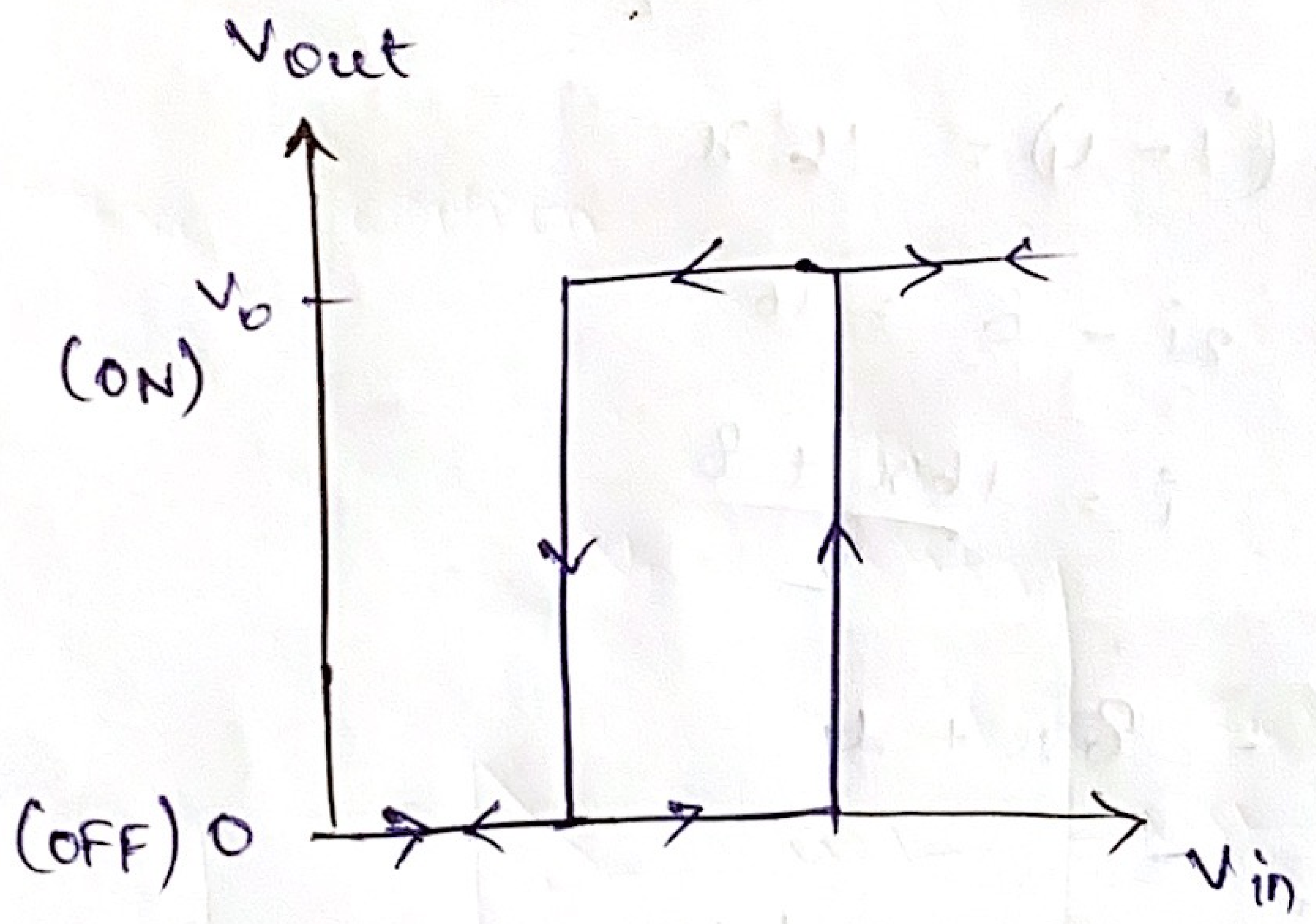
$V_e = 1.5 - (0.8x + 0.4) = 1.1 - 0.8x$

3/10/22

ON/OFF control:



Circuit



Assume that, comp is in OFF state

$V_{out} = 0$

The comp o/p switches when voltage on its input, V_1 is equal to the set point value V_{sp}:

$$V_1 = V_{in} + \frac{R_1}{R_2} V_{out}.$$

Sub $V_{out} = 0$

$V_1 = V_{in}$.

The comp changes to ON state when

$V_1 = V_{in} = V_H$

So, the high (ON) switch voltage is

$V_H = V_{SP}$

$V_{out} = V_o$

With this V_1 changes to

$$V_1 = V_{in} + \frac{R_1}{R_2} V_o$$

If $V_{in} = V_L$ the comp changes to OFF

$$V_L = V_{sp} - \frac{R_1}{R_2} V_o \qquad \left[\frac{R_1}{R_2} V_o + V_i\right] - V_{sp} = V_o$$

Q. Level measurement is provided by a transducer scaled as 0.2 V/m. A pum is to be turned on by application of +5 V when level exceeds 2.0 m. The pump is to be turned back off when level drops to 1.5 m.

Sol. $V_L = 0.2V(1.5m) = 0.3V$

$V_H = 0.2V(2m) = 0.4V$.

$$V_L = V_H - \frac{R_1}{R_2} V_{out}$$

$$0.3 = 0.4 - \frac{R_1}{R_2}(5V)$$

$$\frac{R_1}{R_2} = \frac{0.1}{5} = 0.2$$

Assume $R_1 = 5k$

$R_2 = 25K$

Q. Design a two position control. 0 – 10V input & 0 or 10 V output. The setpoint is 4.3 V & neutral zone is ± 1.1 about this setpoint.

Sol. $V_H = 4.3 + 1.1 = 5.4V$

$V_L = 4.3 - 1.1 = 3.2 V$

$$V_L = V_H - \frac{R_1}{R_2} V_{out}$$

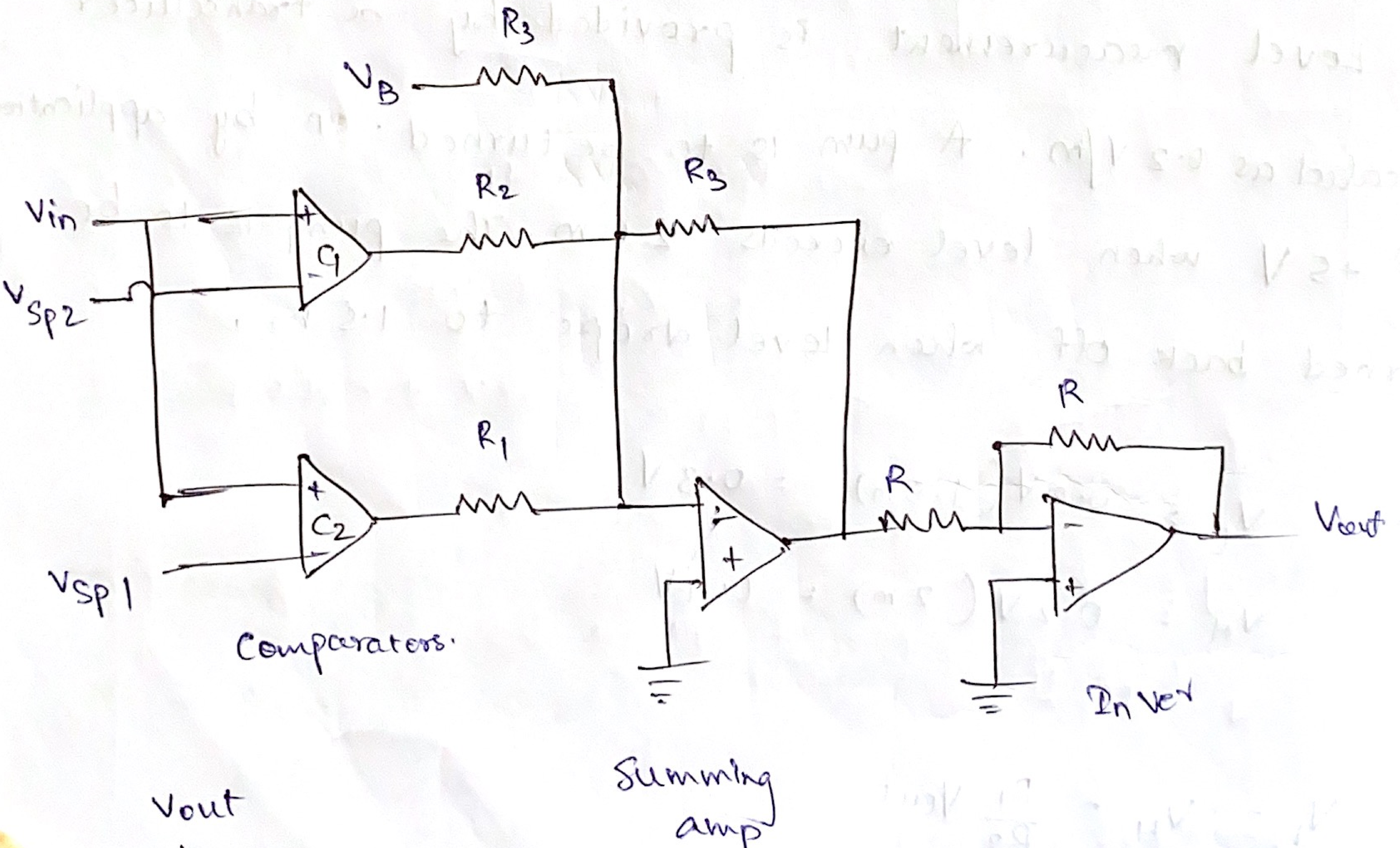
$$3.2 = 5.4 - \frac{R_1}{R_2} 10$$

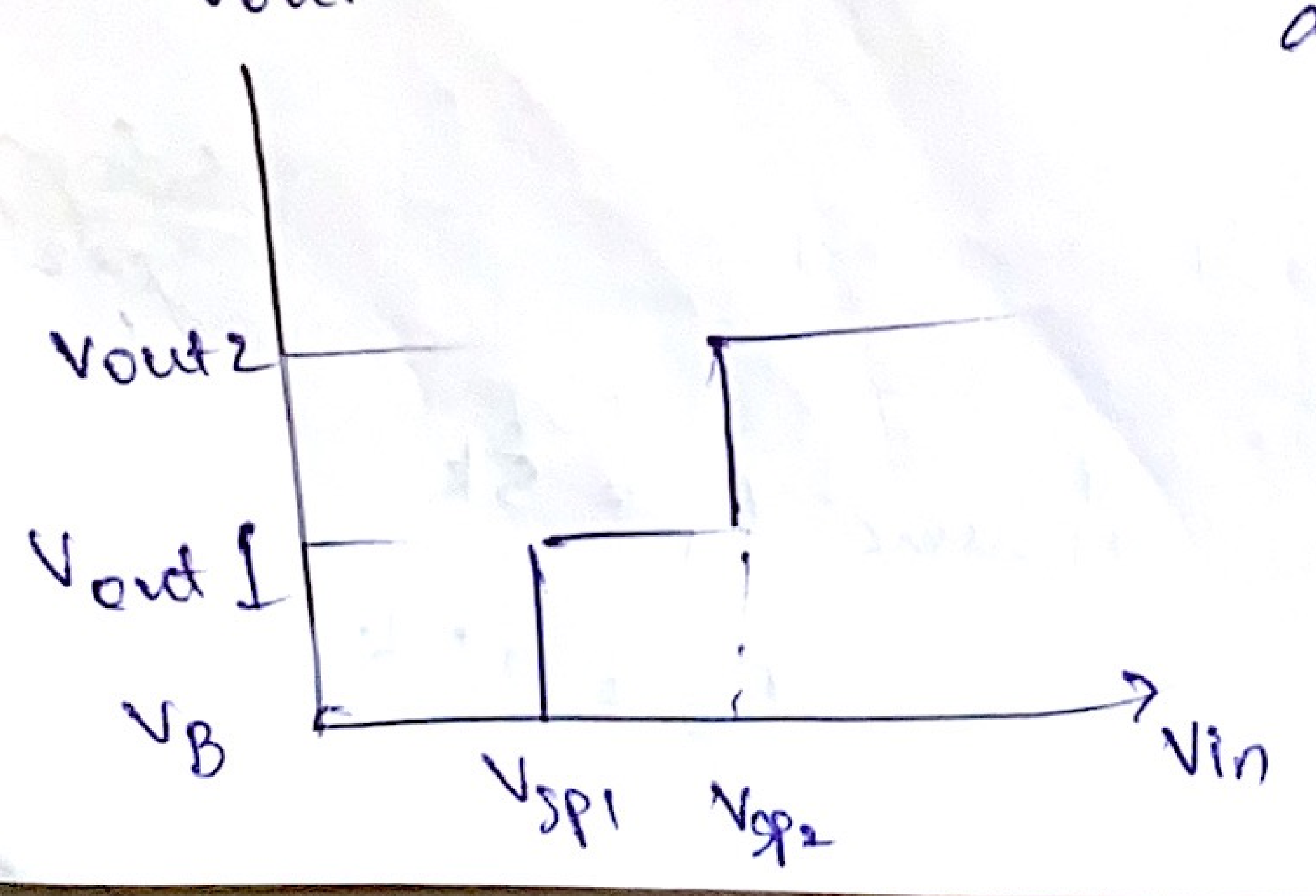
$$\frac{R_1}{R_2} = \frac{2.2}{10} = 0.22$$

Assume $R_1 =$

then $R_2 =$

Three position.





When $V_{in} < V_{sp1}$

C_1 - OFF   o/p = 0.
C_2 - OFF

So, $V_{out} = V_B$.

when. $V_{sp1} < V_{in} < V_{sp2}$.

C_1 - OFF   o/p $C_1 = 0$
C_2 = ON.   o/p $C_2 = \frac{R_3}{R_1} V_o$

$$V_{out} = V_B + \frac{R_3}{R_1} V_o.$$

when $V_{sp2} < V_{in}$

C_1 = ON — $\frac{R_3}{R_2} V_o$
C_2 = ON — $\frac{R_3}{R_1} V_o$

So, $V_{out} = V_B + \frac{R_3}{R_2} V_o + \frac{R_3}{R_1} V_o$

P-mode.

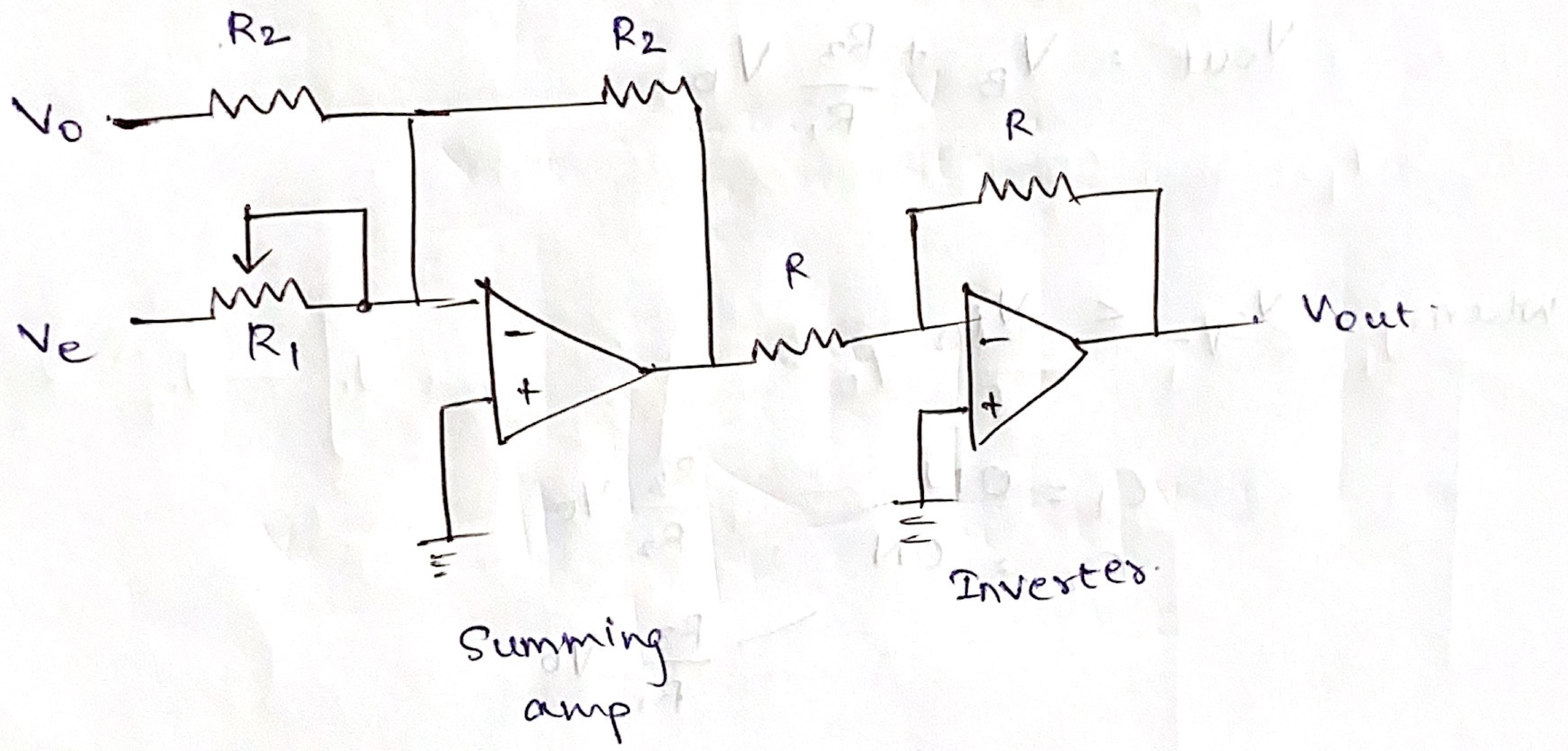
$p = k_p e_p + P_o$

$V_{out} = G_p V_e + V_o$ → Voltag with zero error.

V_{out} ↓ o/p voltge

G_p ↓ Gain $= \frac{R_2}{R_1}$

V_e ↓ error Voltge



$G_p = k_p \frac{\Delta V_{out}}{\Delta V_m}$

10/10/22

Integral mode

$$p(t) = K_I \int_0^t e_p \cdot dt + p(0)$$

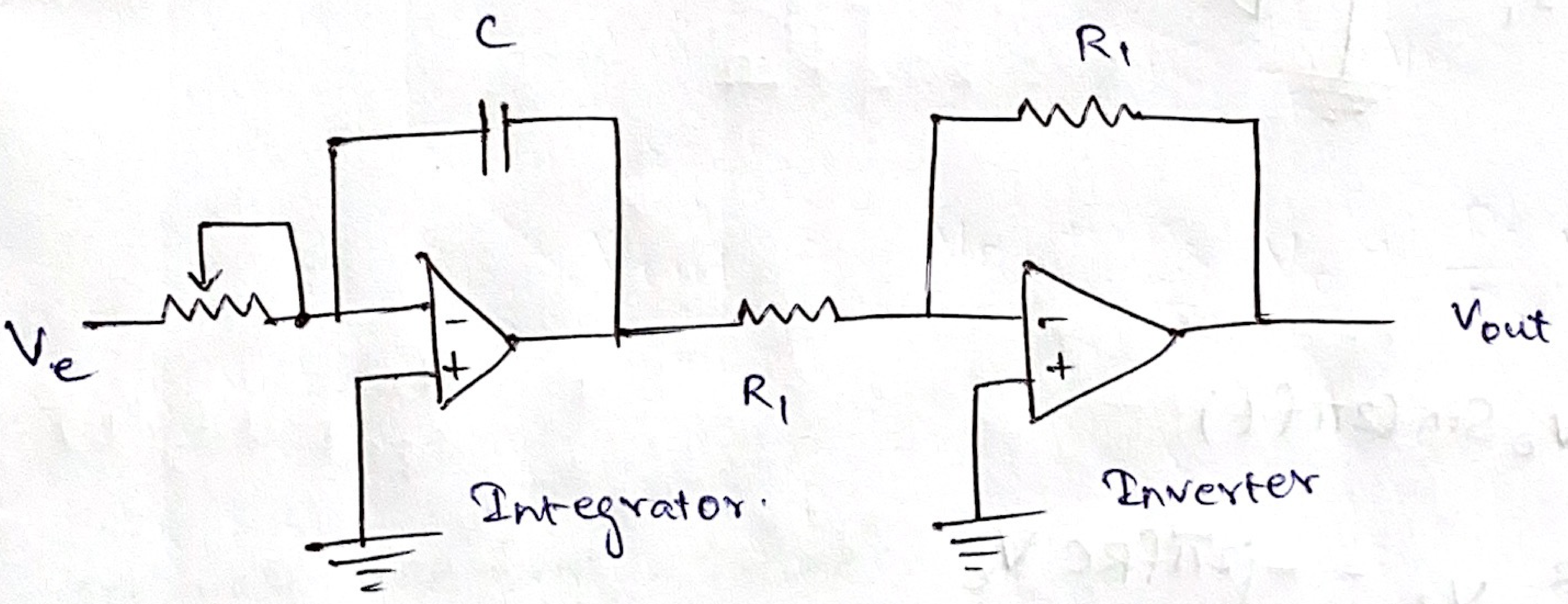
$$V_{out} = G_I \int_0^t V_e dt + V_{out}(0).$$

V_{out} = o/p voltage

$G_I = \frac{1}{RC}$ = integration gain

V_e = error voltage

$V_{out}(0)$ = initial o/p voltage



Steps to find G_I value,

Derivative mode

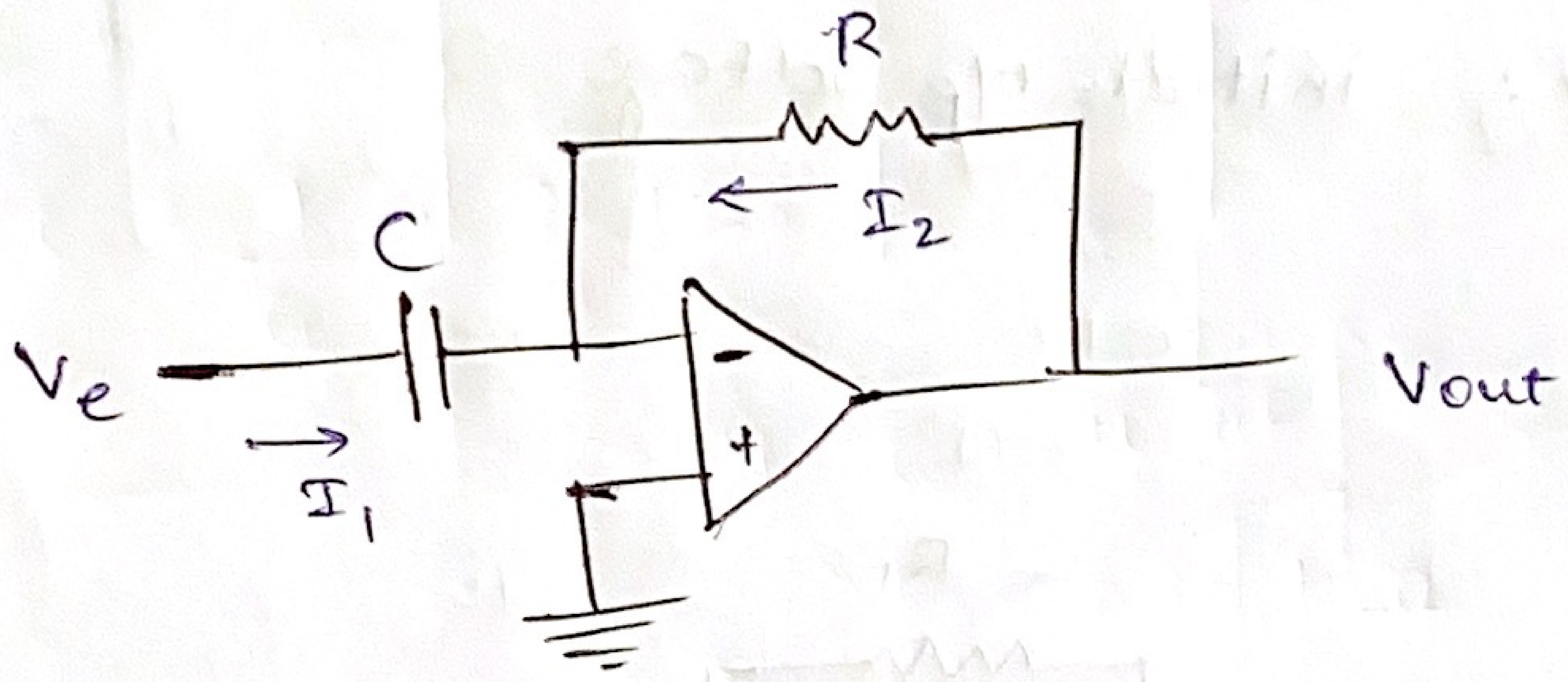
$$p(t) = K_D \cdot \frac{de_p}{dt}$$

p : Controller o/p in % of full o/p.

K_D : Derivative time constant

e_p - error in % of full scale range.

$$V_{out} = -RC\frac{dV_e}{dt}$$



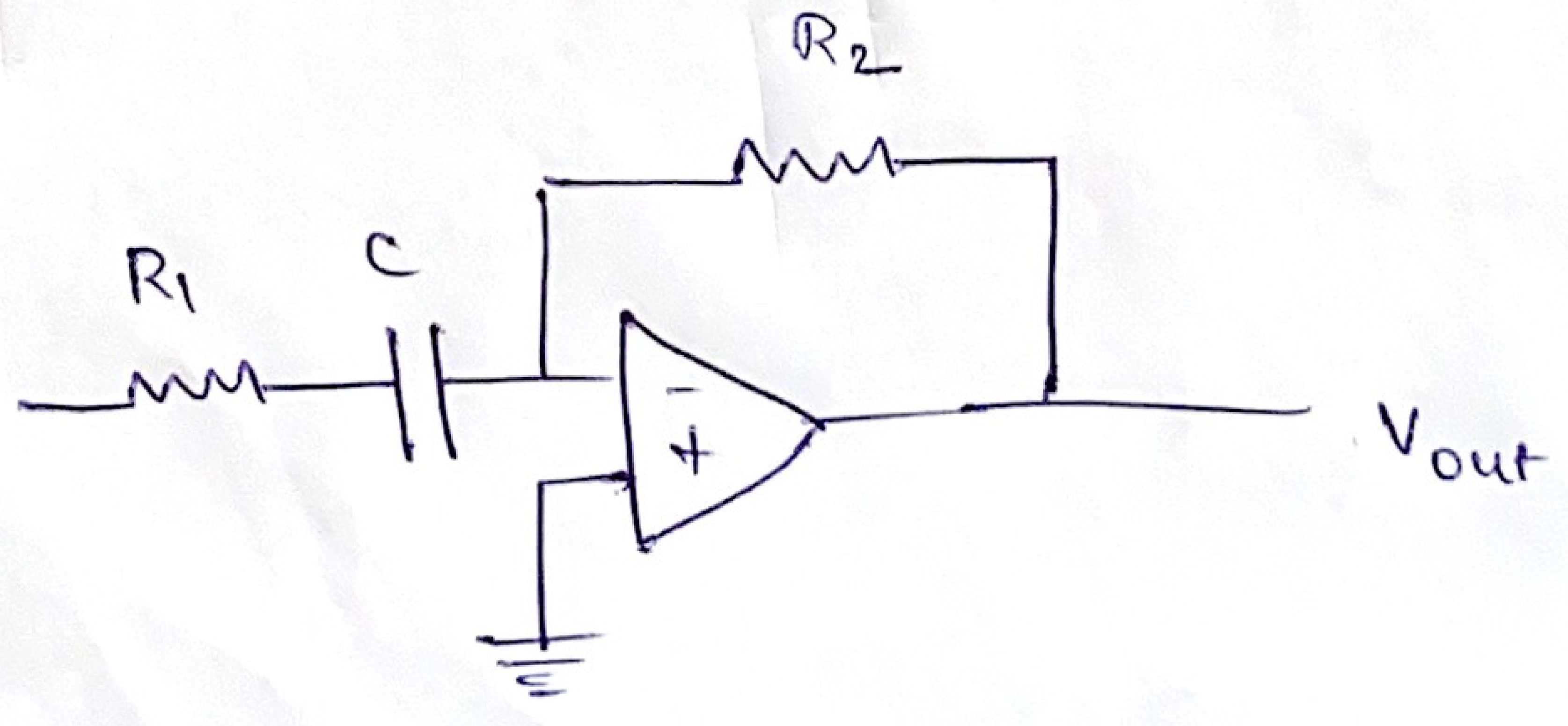
Consider

$V_e = V_0 \sin(2\pi f t)$

$$V_{out} = \frac{R}{-jX_C} V_e = -j2\pi fRC\, V_e$$

mag. of O/p is $|V_{out}| = 2\pi fRC|V_e|$

Consider



$$X_{in} = R_1 + \frac{1}{j2\pi fC} \times \frac{j}{j}$$

$$X_{in} = R_1 - \frac{j}{2\pi fC}$$

Guidelines to design derivative mode controller.

$G_D = R_2 C$

estimate max freq, f_{max}.

Set $2\pi f_{max} R_1 C = 0.1$ and solve R_1.

$f = 0.1 f_{max}$ $V_{out} = 0.995\,(2\pi f R_2 C)\,|V_e|$ Derivative action

$f = f_{max}$ $V_{out} = 0.707\,(2\pi f R_2 C)\,|V_e|$ Transition action

$f = 10 f_{max}$ $V_{out} = 0.0995\,(2\pi f R_2 C)\,|V_e|$ No derivative action.

Q. An integral controller has an i/p range of 1 to 8 V and an o/p range of 0 to 12 V.. If $K_I = 12\% / (\%\text{-min})$ find G_I, R and C

Sol.

$$K_I = \frac{12}{60} = 0.2\% / (\%\text{-sec})$$

$$1\% \text{ of i/p} = \frac{1}{100} \times (8-1) = 0.07\ V\ V\text{-s.}$$

$$0.2\% \text{ of o/p} = (0.002)(12) = 0.024\ V$$

$$G_I = \frac{0.024}{0.07} = 0.3428\ s^{-1}$$

$$RC = \frac{1}{G_I} = 2.92 s$$

If $C = 10\ \mu F$

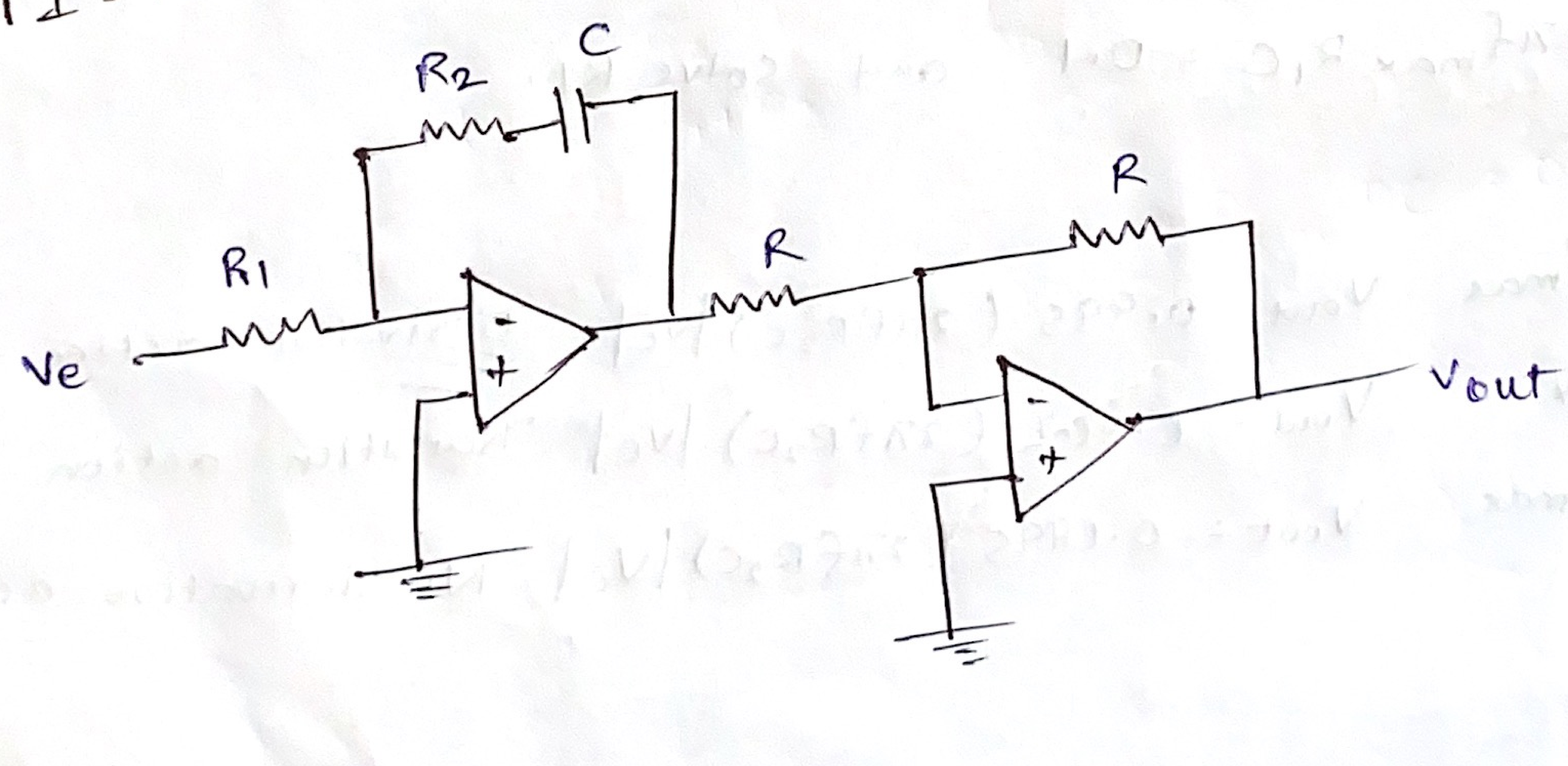
$R = 292\ k\Omega$

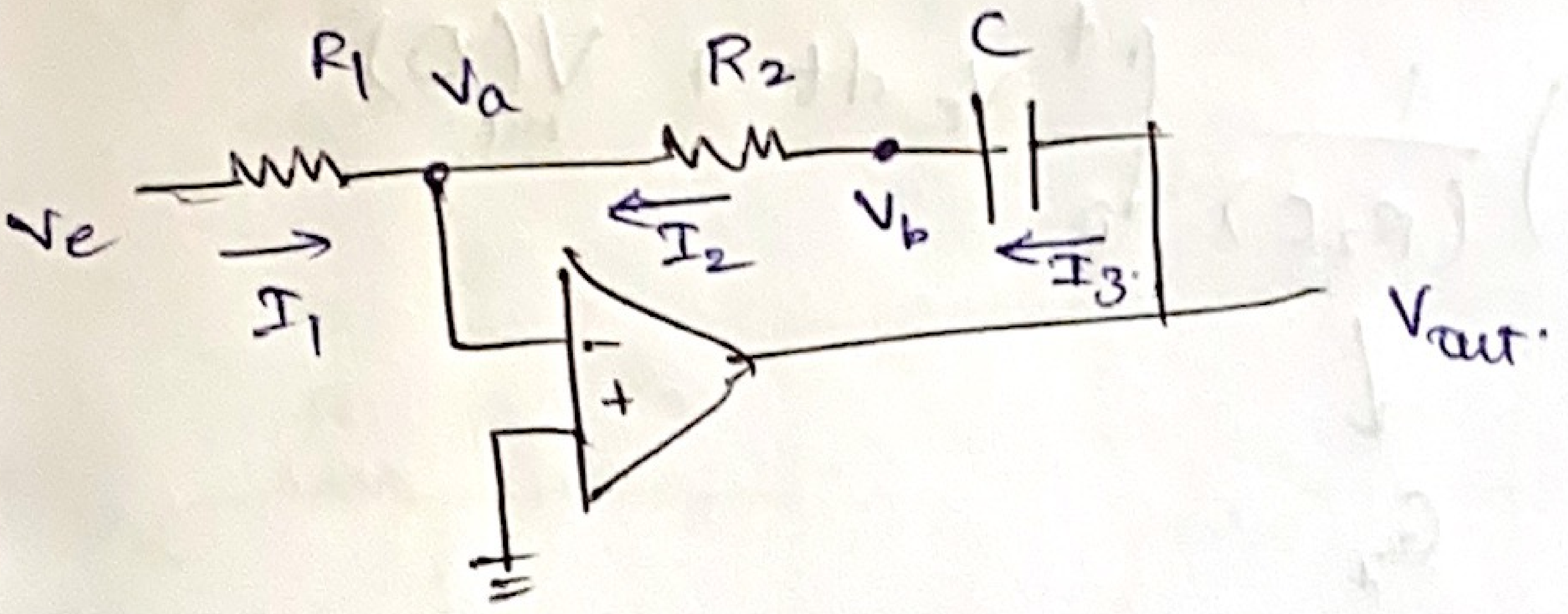
12/10/22

Composite Controller modes

$$P = K_p e_p + K_p K_I \int_0^t e_p dt + P_I(0).$$

PI - mode.





$I_1 + I_2 = 0 \rightarrow (1)$

$I_C = C\frac{dV_c}{dt}$

$I_3 - I_2 = 0 \rightarrow (2)$

So, $I_3 = C\cdot\frac{d}{dt}(V_{out} - V_b)$

from (1).

$\frac{V_e}{R_1} + \frac{V_b}{R_2} = 0. \Rightarrow V_b = \frac{-R_2}{R_1}\cdot V_e$

from (2)

$C\frac{d}{dt}(V_{out} - V_b) - \frac{V_b}{R_2} = 0 \rightarrow (3)$

~~solve for Vb~~

sub V_b in (3)

$C\frac{d}{dt}\left(V_{out} + \frac{R_2}{R_1}V_e\right) + \left(\frac{\cancel{R_2}}{R_1}V_e\right)\left(\frac{1}{\cancel{R_2}}\right) = 0$

$C\cdot\frac{d}{dt}(V_{out}) + \frac{CR_2}{R_1}\frac{d}{dt}V_e + \frac{1}{R_1}V_e = 0.$

$\frac{d}{dt}(V_{out}) + \frac{R_2}{R_1}\frac{d}{dt}V_e + \frac{1}{R_1C}V_e = 0.$

for Vout, Integrate

$V_{out} + \frac{R_2}{R_1}V_e + \frac{1}{R_1C}\int_0^t V_e dt + V(0) = 0$

$\Rightarrow V_{out} = -\frac{R_2}{R_1}V_e - \frac{1}{R_1C}\int_0^t V_e\cdot dt + V(0)$

$= -\frac{R_2}{R_1}V_e - \frac{R_2}{R_1}\cdot\frac{1}{R_2C}\int_0^t V_e\cdot dt + V(0).$

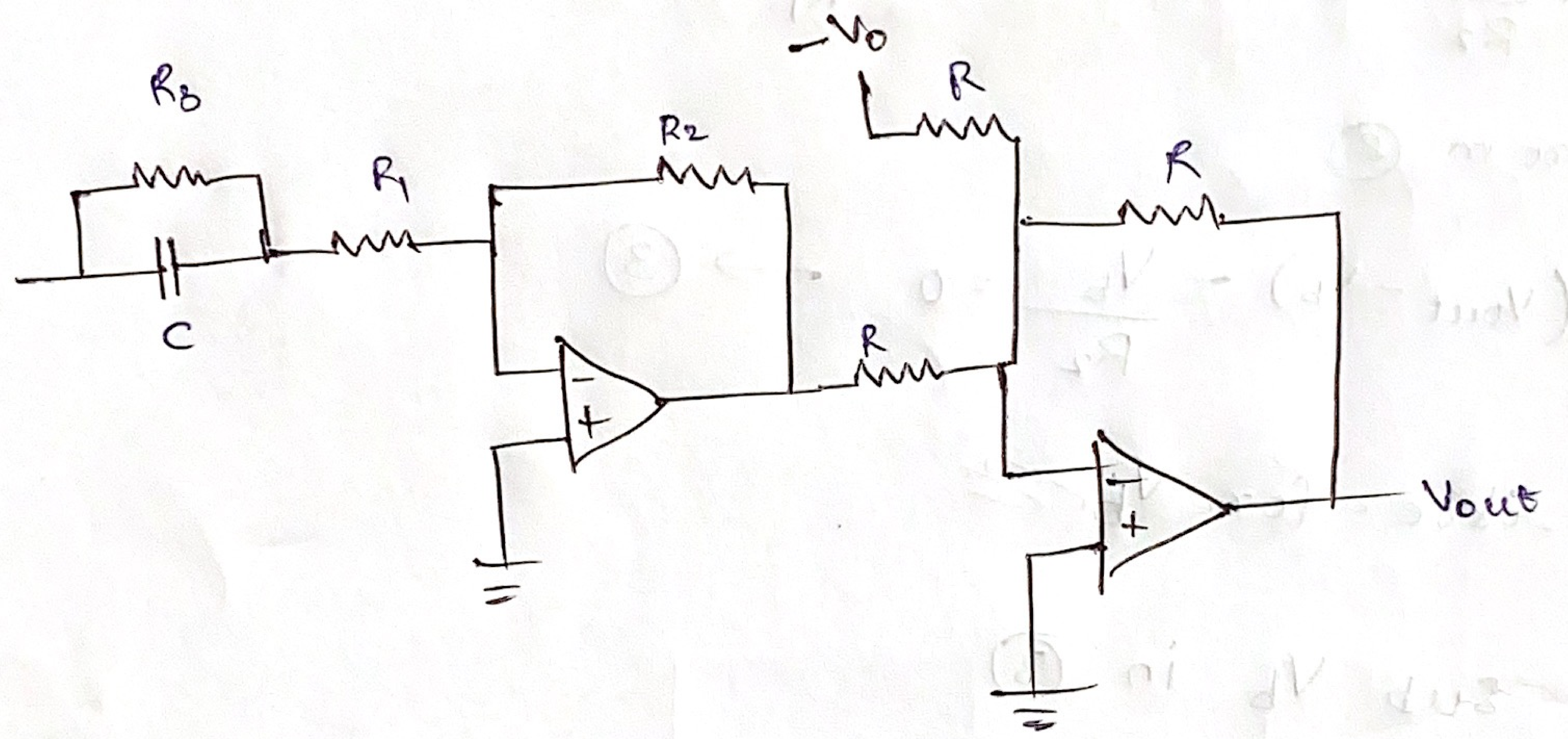
After inverting

$$V_{out} = \left(\frac{R_2}{R_1}\right) V_e + \left(\frac{R_2}{R_1}\right) \frac{1}{(R_2 C)} \int_0^t V_e \, dt + V_{out}(0)$$

$\left(\frac{R_2}{R_1}\right) \downarrow G_P$ $\qquad$ $\frac{1}{(R_2C)} \downarrow G_I$

P-D mode

$$p = \cancel{K_p}\ K_p e_p + K_p \frac{d}{dt} e_p + P(0)$$



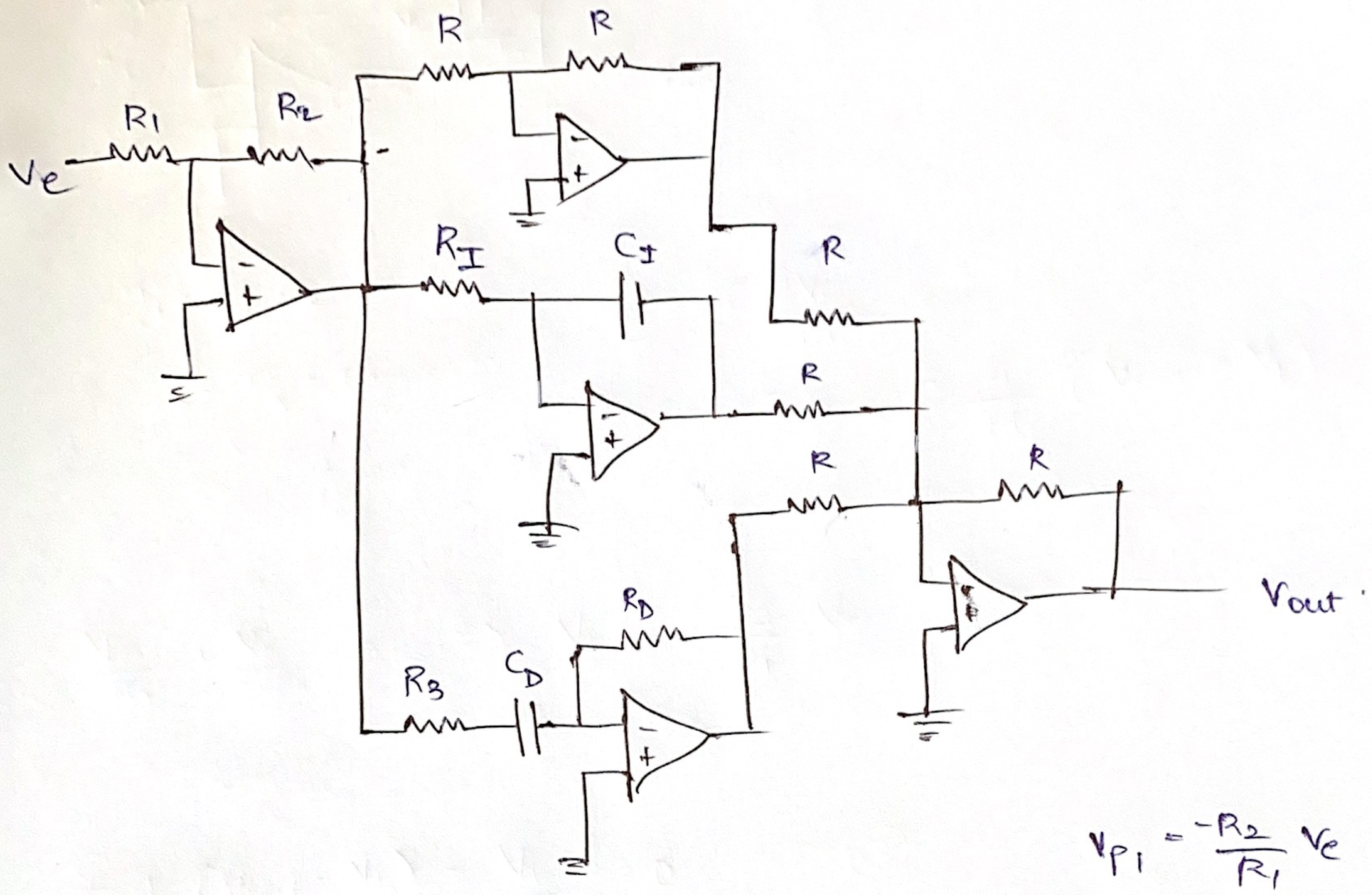
$$V_{out} = \left(\frac{R_2}{R_1+R_3}\right) V_e + \left(\frac{R_2}{R_1+R_3}\right) R_3 C \cdot \frac{d}{dt} V_e + V_o$$

$$G_P = \frac{R_2}{R_1+R_3} \qquad G_D = R_3 C$$

$$2\pi f_{max} RC = 0.1$$

PID mode

$$P = k_p e_p + k_p k_I \int_0^t e_p\,dt + k_p k_D \frac{de_p}{dt} + P_I(0)$$



$$V_{P1} = \frac{-R_2}{R_1} V_e$$

$$V_P = \frac{R_2}{R_1} V_e$$

$$V_I = -\frac{1}{R_1 C_1} \int_0^t V_{P1}\,dt \qquad V_I = \frac{R_2}{R_1} \frac{1}{R_2 C_I} \int_0^t V_e\,dt$$

$$V_D = -R_3 C_D \frac{d}{dt} V_{P1} \qquad V_D = \frac{R_2}{R_1} R_D C_D \frac{d}{dt} V_e.$$

$$-V_{out} = \frac{R_2}{R_1} V_e + \frac{R_2}{R_1}\left(\frac{1}{R_2 C_I} \int V_e\,dt\right) + \frac{R_2}{R_1} R_D C_D \frac{d}{dt} V_e + V_{out}(0).$$

$$G_P = \frac{R_2}{R_1} \qquad G_I = \frac{1}{R_I C_I} \qquad G_D = R_D C_D.$$

After inverting

$$V_{out} = G_P V_e + G_P G_I \int V_e\,dt + G_P G_D \frac{dV_e}{dt} + V_{out}(0).$$