$$\begin{matrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} \end{matrix} = \begin{bmatrix} 1+x_2 & x_1 \\ x_2 - 3x_1^2 & -1+2x_2+x_1 \end{bmatrix}$$

for $(0,0)$

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

Saddle

for $(0,1)$

$$\begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}$$

unstable node

for $(1,-1)$

$$\begin{bmatrix} 0 & 1 \\ -4 & -2 \end{bmatrix}$$

Stable focus.

$$-x_2 + x_2^2 + x_1 x_2 - x_1^3 = 0.$$

$$x_1 + x_1 x_2 = 0$$

$$x_1 x_2 = -x_1$$

$$x_2 = -1$$

8. Check for the exist

$$\dot{x}_1 = -x_1 + x_1^3 + x_1 x_2^2$$

$$\dot{x}_2 = -x_2 + x_2^3 + x_1^2 x_2$$

$$\frac{\partial f_1}{\partial x_1} + \frac{\partial f_2}{\partial x_2} = -1 + 3x_1^2 + x_2^2 - 1 + 3x_2^2 + x_1^2$$

$$= (x_1^2 + x_2^2) + 3(x_1^2 + x_2^2) - 2$$

$$= 4x_1^2 + 4x_2^2 - 2$$

for (0, 0) or any other point

$$\neq 0$$

So, it doesn't vanish.

So, limit cycle doesn't exist.

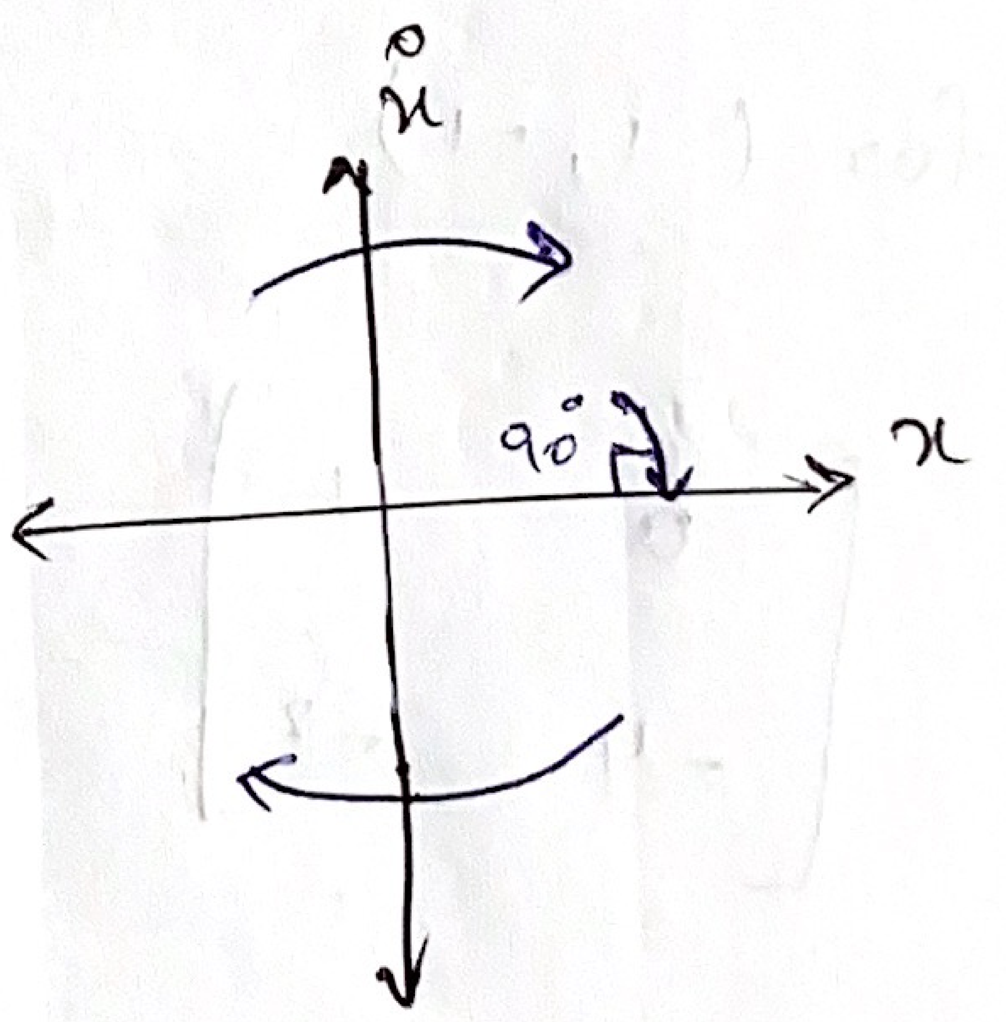
Construction

~~Direction~~ of phase trajectories.

12/10/22

$\dot{x} > 0$ left to right

$\dot{x} < 0$ right to left



Analytical methods.

$$\dot{x}_1 = f_1(x_1, x_2)$$
$$\dot{x}_2 = f_2(x_1, x_2)$$

$$\frac{\dot{x}_1}{\dot{x}_2} = \frac{f_1(x_1, x_2)}{f_2(x_1, x_2)}$$

$$\Rightarrow \frac{dx_2}{dx_1} = \frac{f_2(x_1, x_2)}{f_1(x_1, x_2)}$$

Slope of the phase trajectory at every point on the phase plane.

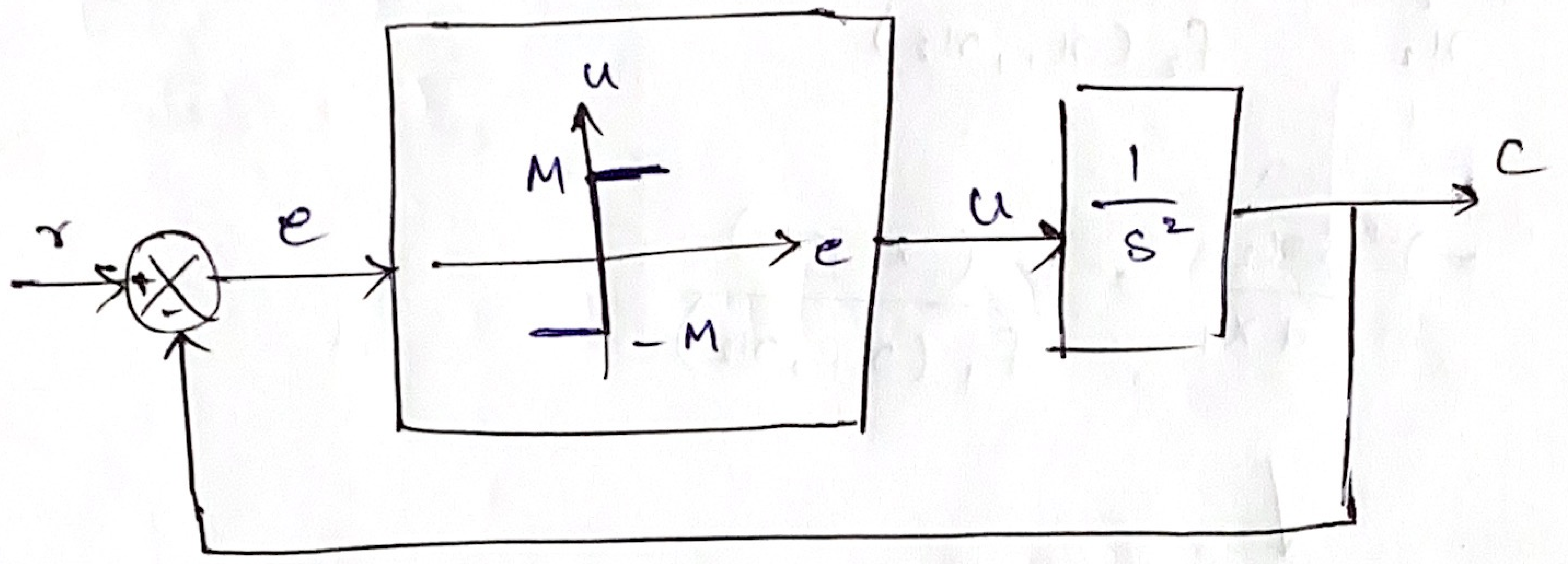
This method is useful onlf if the diff. eqns can be approximated by piecewise linear diff. eqns.

example:-

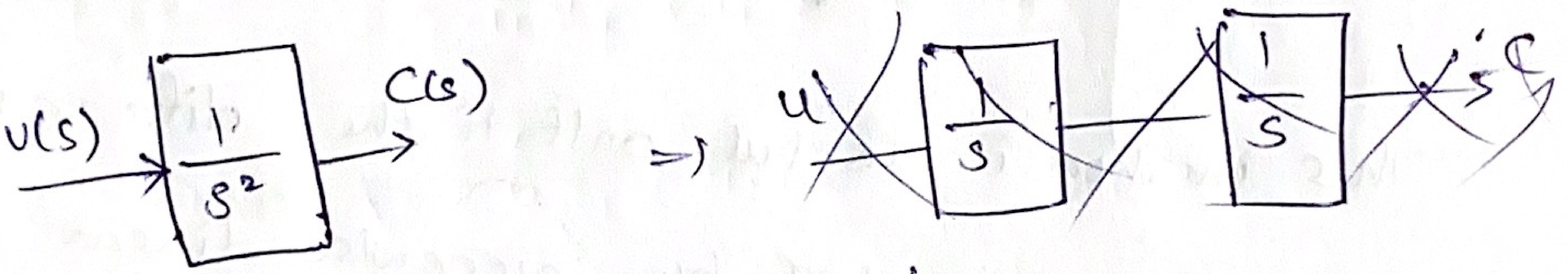
Consider a system with an ideal relay. Determine the singular points. Construct phase trajectories corresponding to initial condt.

i) $c(0) = 2$, $\dot{c}(0) = 1$ $r = 2V$ $M = 1.2V$

$c(0) = 2$, $\dot{c}(0) = 1.5$



Sol Consider the linear part



$\cancel{x} = \cancel{c}$

$$\frac{C(s)}{U(s)} = \frac{1}{s^2}$$

$s^2 C(s) = U(s)$

$\ddot{c} = u$

$\ddot{c} - u = 0$

Phase variables

$x_1 = c$

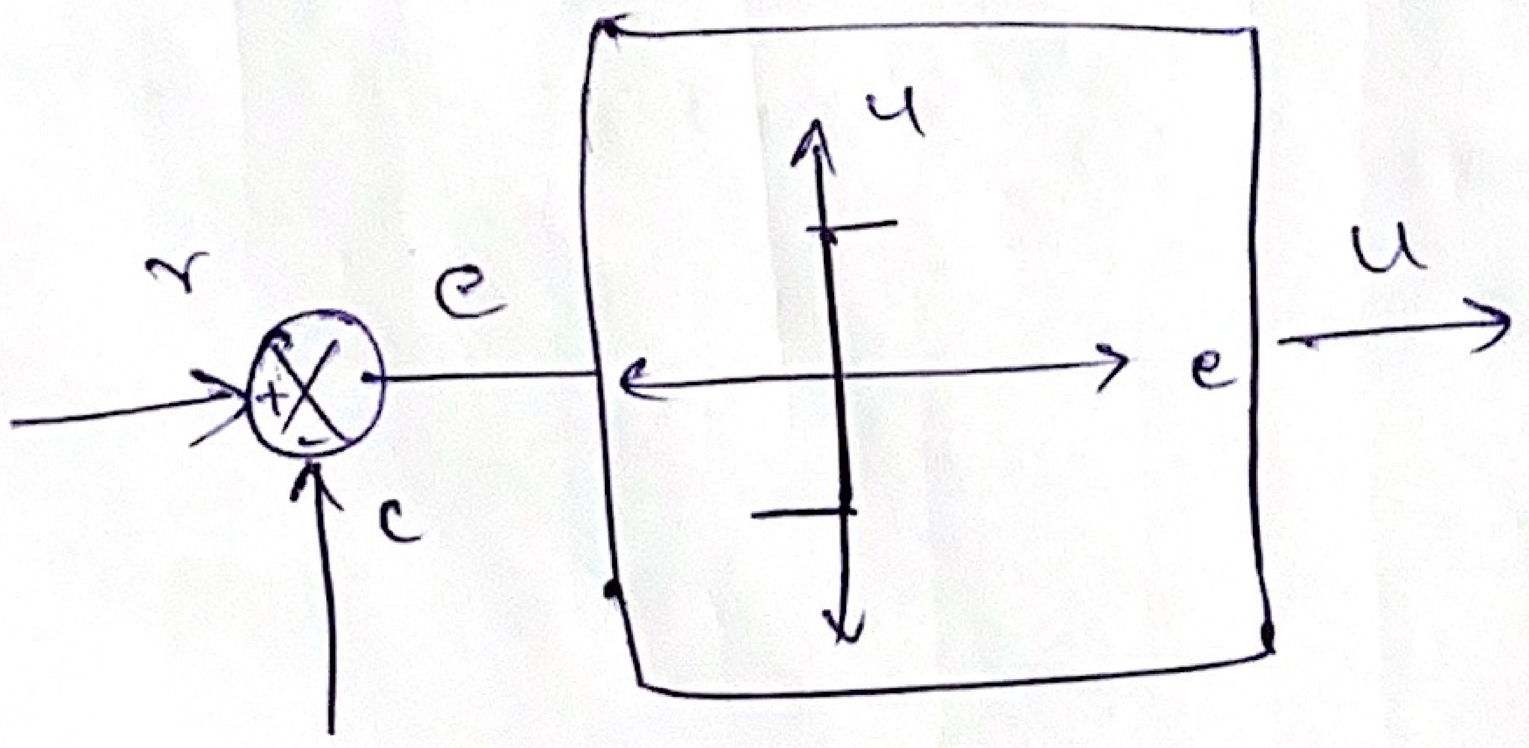
$x_2 = \dot{c}$

State eqn's.

$\dot{x}_1 = x_2$

$\dot{x}_2 = \ddot{c} = u$

Consider the non-linear part.



$e = r - c$

$e = r - x_1$

$x_1 > r$ $e = -ve$, $u = -M$

$x_1 < r$ $e = +ve$, $u = +M$

Sub these values in state eqn's of linear part.

$\dot{x}_1 = x_2$

$\dot{x}_2 = M$: for $x_1 < r$

$-M$ for $x_1 > r$.

for finding eq. points.

$\cancel{\dot{x}_1 = 0}$ $\quad$ $\dot{x}_1 = x_2 = 0$

$\cancel{\dot{x}_2 = 0}$ $\quad$ $\dot{x}_2 = u = 0.$

for $u = 0$, $e = 0$

but $e = r - C$

$\Rightarrow e = r - x_1$

$0 = r - x_1$

$\Rightarrow x_1 = r.$

eq. point = $(r, 0)$

i) $(2, 1)$
$x_1(0)$ $x_2(0)$

$$\frac{dx_2}{dx_1} = \frac{f_2(x_1, x_2)}{f_1(x_1, x_2)} = \frac{\pm M}{x_2}$$

$$x_2 \cdot dx_2 = \pm M dx_1$$

Integrate

$$\int_{x_2(0)}^{x_2} x_2 \cdot dx_2 = \int_{x_1(0)}^{x_1} \pm M dx_1$$

$$\left[\frac{x_2^2}{2}\right]_{x_2(0)}^{x_2} = \pm M \left[x_1\right]_{x_1(0)}^{x_1}$$

$$\frac{x_2^2}{2} - \frac{x_2(0)^2}{2} = \pm M \left[x_1 - x_1(0)\right]$$

$$\frac{x_2^2}{2} = \pm M(x_1 - x_1(0)) + \frac{(x_2(0))^2}{2}$$

slope eqn $\leftarrow$ $x_2^2 = \pm 2M(x_1 - x_1(0)) + (x_2(0))^2$

$$x_2^2 = \pm 2 \cdot 4 (x_1 - 2) + (1)^2$$

$$x_2^2 = \pm(2 \cdot 4 x_1 \mp 4 \cdot 8) + 1$$

for case (i)

$$x_2^2 = 2.4x_1 - 4.8 + 1$$

$$x_2 = \pm\sqrt{2.4x_1 - 3.8} \qquad \text{for } x_1 < r$$

$$x_2^2 = -2.4x_1 + 4.8 + 1$$

$$x_2 = \pm\sqrt{-2.4x_1 + 5.8} \qquad \text{for } x_1 > r.$$

Case (ii)

$(2, 1.5)$

$x_1(0) \quad x_2(0)$

$$x_2^2 = \pm 2.4(x_1 - 2) + (1.5)^2$$

$$x_2^2 = +2.4x_1 - 4.8 + 2.25$$

$$x_2 = \pm\sqrt{2.4x_1 - 2.55} \qquad x_1 < r$$

$$x_2^2 = -2.4x_1 + 4.8 + 2.25$$

$$x_2 = \pm\sqrt{-2.4x_1 + 7.05} \qquad x_1 > r.$$

2.4166
0.012

for initial condition $(2, 1)$

$x_1 > r$			$x_1 < r$		
x_1	x_2		x_1	x_2	
2	1	-1	1.5834	~~0.012~~ 0	0 ~~-0.012~~
2.1	0.87	-0.87	1.6	0.2	-0.2
2.2	0.72	-0.72	1.7	0.53	-0.53
2.3	0.53	-0.53	1.8	0.72	-0.72
2.4	0.2	-0.2	1.9	0.87	-0.87
2.4167	0	0	2	1	-1

||rly for (ii) initial condition.

$x_1 > r$			$x_1 < r$		
x_1	x_2		x_1	x_2	
2	1.5	-1.5	1.0625	0	0
2.1	1.41	-1.41	1.3	0.75	-0.75
2.3	1.23	-1.23	1.5	1.02	-1.02
2.5	1.02	-1.02	1.7	1.23	-1.23
2.7	0.75	-0.75	1.9	1.41	-1.41
2.9357	0	0	2	1.5	-1.5

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Isocline method.

ex:- A linear second order $\ddot{x} + \dot{x} + x^2 = 0$

$$\ddot{e} + 2\xi\omega_n\dot{e} + \omega_n^2 e = 0.$$

where $\xi = 0.15$ $\omega_n = 1$ rad/sec $e(0) = 1.5$

$\dot{e}(0) = 0.$

Determine the singular point. Construct the phase trajectory.

Sol.

$x_1 = e$ $\dot{x}_1 = x_2$

$x_2 = \dot{e}$ $\dot{x}_2 = \ddot{e} = -2\xi\omega_n x_2 - \omega_n^2 x_1$

$\dot{x}_2 = -0.3x_2 - x_1$

eq. point = $(0, 0)$ ($\because$ linear system).

$$S = \frac{dx_2}{dx_1} = \frac{f_2}{f_1} = \frac{-0.3x_2 - x_1}{x_2} \Rightarrow S = -0.3 - \frac{x_1}{x_2}$$

$$Sx_2 + 0.3x_2 = -x_1$$

$$S = -\frac{x_1}{x_2} - 0.3$$

$$\Rightarrow x_2 = \frac{x_1}{-0.3 - S}$$

S	-2		-1		-0.5		0		0.5		1		2	
α	-63°		-45°		-27°		0		27°		45°		63°	
Points	x_1	x_2	x_1	x_2	x_1	x_2	x_1	x_2	x_1	x_2	x_1	x_2	x_1	x_2
α	1	0.6	1	1.4	0.25	1.25	0.25	-0.8	1	-1.25	1	-0.77	1	-0.43
α	2	1.2	1.5	2.1	0.5	2.5	0.75	-2.5	2	-2.5	2	-1.54	2	-0.86

26/10/22

Delta method.

Consider a nonlinear 2nd order system

$$\ddot{x} + f(x, \dot{x}, t) = 0.$$

It can be converted to

$$\ddot{x} + \omega_n^2 \left[x + \delta(x, \dot{x}, t)\right] = 0$$

the changes in variable of function δ in short intervals are negligible & can be considered as a constant.

$$\ddot{x} + \omega_n^2 (x + \delta) = 0$$

Choose the state variables.

$x_1 = x \quad \Rightarrow \quad \dot{x}_1 = \dot{x} = x_2 \omega_n$

$x_2 = \dfrac{\dot{x}}{\omega_n} \quad \Rightarrow \quad \dot{x}_2 = \dfrac{\ddot{x}}{\omega_n} = \dfrac{-\omega_n^{\cancel{2}}(x_1 + \delta)}{\cancel{\omega_n}}$

$$\dot{x}_2 = -\omega_n (x_1 + \delta)$$

$$\text{slope} = \frac{\dot{x}_2}{\dot{x}_1} = \frac{-(x_1 + \delta)}{x_2}$$

Ex. 2.6 Advanced Control Theory $\ddot{x} + \dot{x} + x^2 = 0$

2nd edi. Nagoor Kani

Construct a trajectory by delta method.

$\ddot{x} + 4|\dot{x}|\dot{x} + 4x = 0$. Initial conditions are

$x(0) = 1$ $\dot{x}(0) = 0$.

Sol. $\ddot{x} + \omega_n^2(x+\delta) = 0. \rightarrow (1)$

$\ddot{x} + 4|\dot{x}|\dot{x} + 4x = 0$

$\Rightarrow \ddot{x} + 4(x + |\dot{x}|\dot{x}) = 0$ compare with (1)

$\omega_n = 2$ $\delta = |\dot{x}|\dot{x}$

Sub $\dot{x}$ value

$x_1 = x$ $\dot{x}_1 = 2x_2$ $\delta = |2x_2|\,2x_2$

$x_2 = \frac{\dot{x}}{\omega_n} = \frac{\dot{x}}{2}$ $\dot{x}_2 =$ $= 4|x_2|x_2$

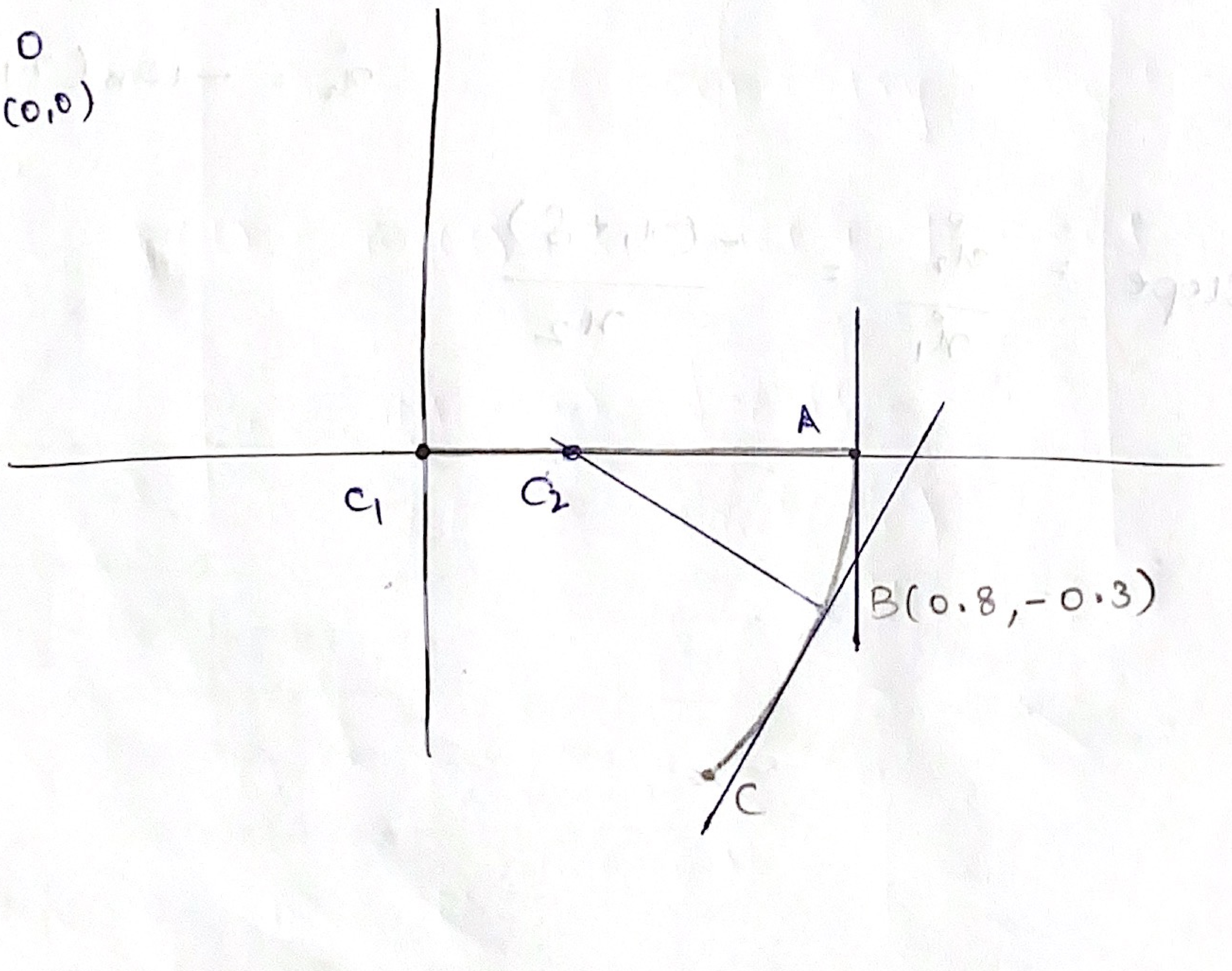
$\Rightarrow \dot{x} = 2x_2$.

initial

Condi $A = (1,0)$.

So, $\delta = 0$

$\therefore C_1 = (0,0)$



for $(0.8, -0.3)$

$\delta = 4(0.3)(-0.3)$

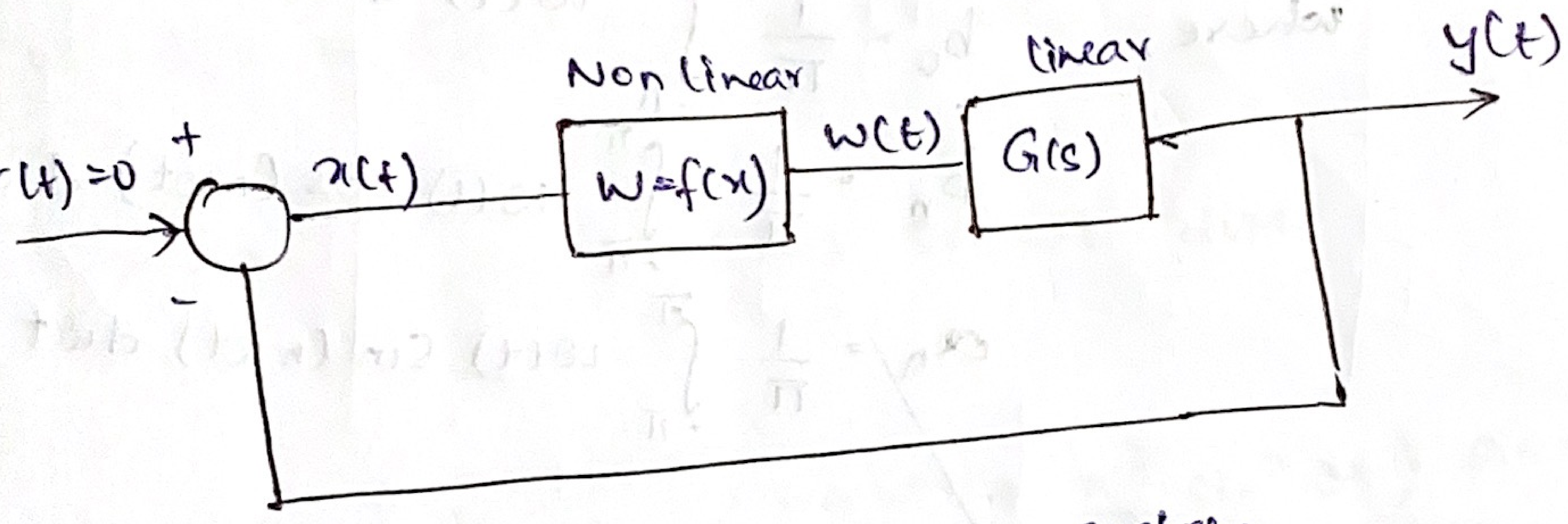
$\delta = -0.36$

$C_2 = (0.36, 0)$

27/10/22

The main use of the describing functions is to find the existence of limit cycles.

Applications of describing functions (DF)



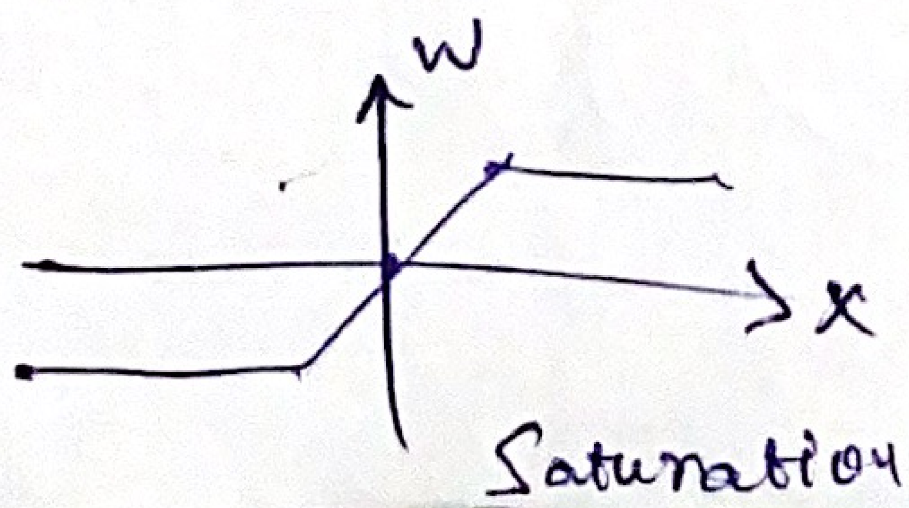
A nonlinear system

Almost ~~non~~linear systems - A system which contain hard nonlinearities. (saturation, dead zone)

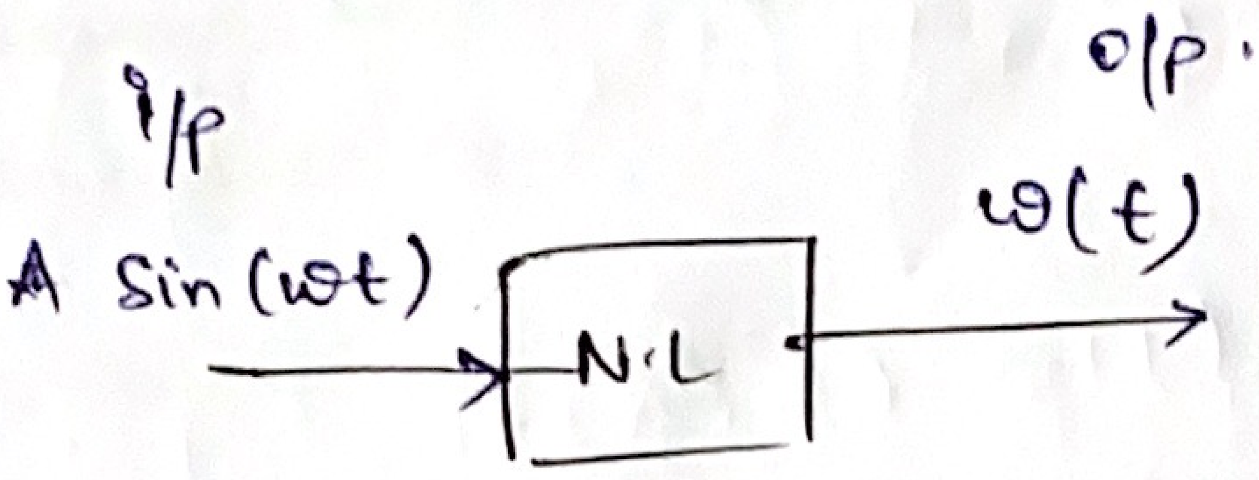
Genuinely linear system - when system dynamics are represented by diff eqn's.

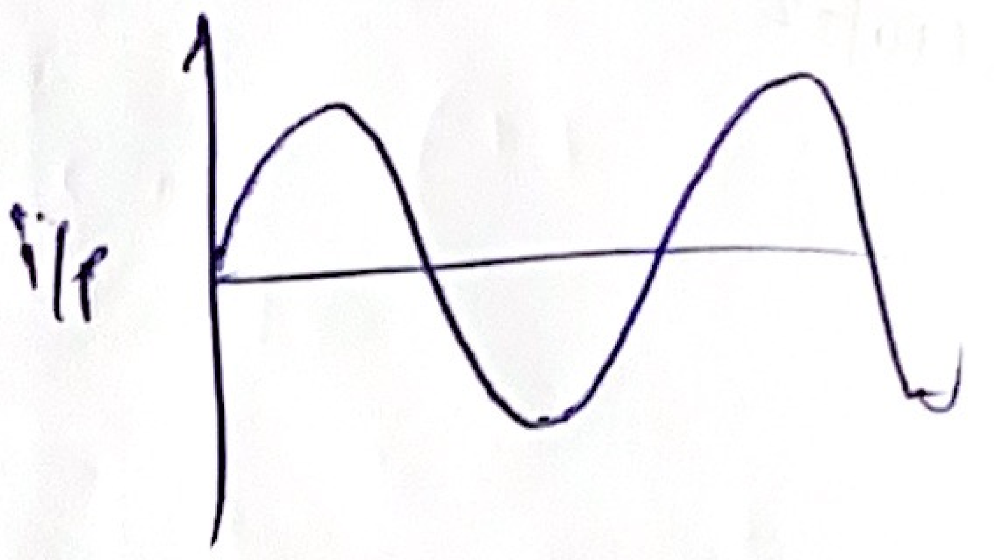
Assumptions

1. Only one non linear component
2. Nonlinear comp. is time invariant (saturation, backlash, couloumb friction)
3. Only the fundamental component in the o/p has to be considered. (Low pass filter)
4. Nonlinearity is odd (symmetry about origin).



Saturation





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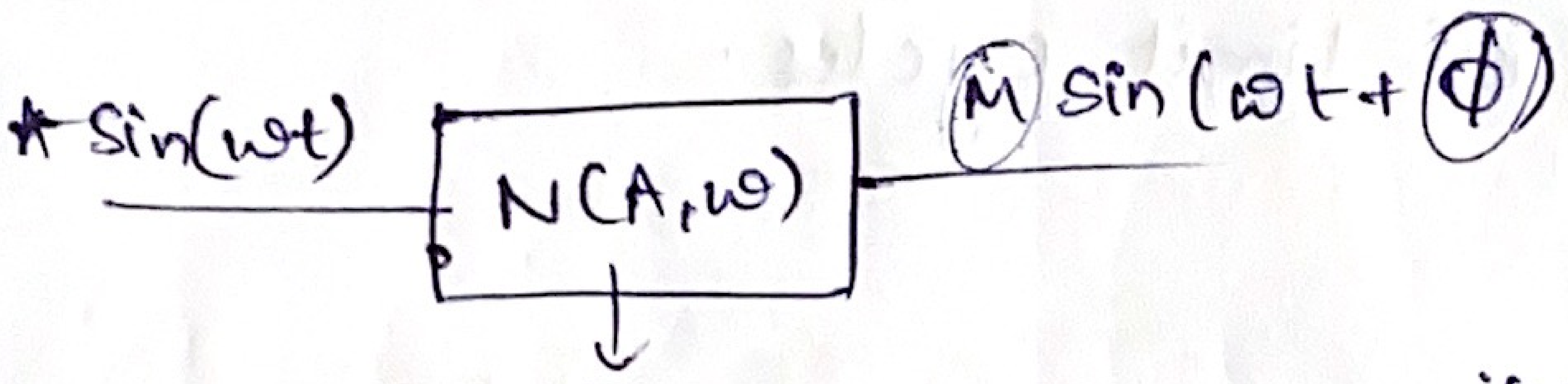


function of non linearity.

(Describing Function) DF

Using fourier series,

$$w(t) = \frac{b_0}{2} + \sum_{n=1}^{\infty} \left[a_n \sin n\omega t + b_n \cos(n\omega t) \right]$$

where,

$$b_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} w(t)\, d\omega t$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} w(t) \cos(n\omega t)\, d\omega t$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} w(t) \sin(n\omega t)\, d\omega t$$

As the assumption of LPF, only the fundamental terms ie. lower oder terms will be present.

Also, because of symmetry about origin.

So, $w(t) = w_1(t) = a_1 \sin\omega t + b_1 \cos\omega t = M \sin(\omega t + \phi)$

Now, $M(A,\omega) = \sqrt{a_1^2 + b_1^2}$

$$\phi(A,\omega) = \tan^{-1}\left(\frac{b_1}{a_1}\right)$$

In complex representation,

$$w_1 = M e^{i(\omega t + \phi)}$$

So, $$N(A,\omega) = \frac{o/p}{i/p} = \frac{M e^{i(\omega t + \phi)}}{A e^{i\omega t}} = \frac{M}{A} e^{i\phi}$$

ex:. Describing function of a hardening spring.

sol: The char eq is

$$w = x + \frac{x^3}{2}$$

x – i/p

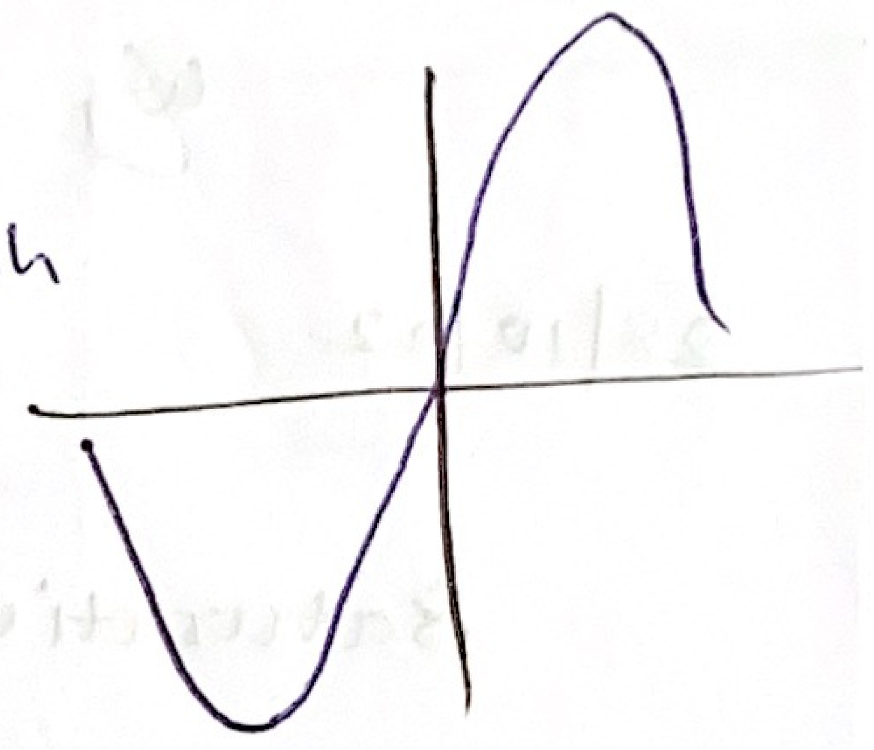
w = o/p.

Assume i/p

$$x(t) = A \sin \omega t$$

$$w(t) = A \sin \omega t + \frac{A^3}{2} \sin^3 \omega t$$

Plot the graph



So, by fourier series

we can get

4th assump is true.

$$w(t) = a_1 \sin \omega t + b_1 \cos \omega t$$

$$a_1 = \frac{1}{\pi} \int_{-\pi}^{\pi} w(t) \sin \omega t \cdot d\omega t$$

$$= \frac{1}{\pi} \int_{-\pi}^{\pi} \left(A \sin \omega t + \frac{A^3}{2} \sin^3 \omega t \right) \sin \omega t \cdot d\omega t$$

$$= \frac{1}{\pi} \int_{-\pi}^{\pi} \left(A \sin^2 \omega t + \frac{A^3}{2} \sin^4 \omega t \right) \cdot d\omega t$$

$$= \frac{1}{\pi} \cancel{A \sin \omega t} \int_{-\pi}^{\pi} \left(A \frac{1 - \cos 2x}{2} + \frac{A^3}{2} \frac{\cos 4x - 4\cos 2x + 3}{8} \right.$$

$$a_1 = A + \frac{3}{8} A^3$$

$w_1 = \left(A + \frac{3}{8}A^3\right)\sin(\omega t) \Rightarrow w_1 = \left(1 + \frac{3}{8}A^2\right)A\sin(\omega t)$

$w_1 = N(A, \omega)\, x(t)$

~~w_1~~ $\quad N(A, \omega) = N(A) = 1 + \frac{3}{8}A^2$

$a_1 = M$

$w_1 = 1 + \frac{3}{8}$

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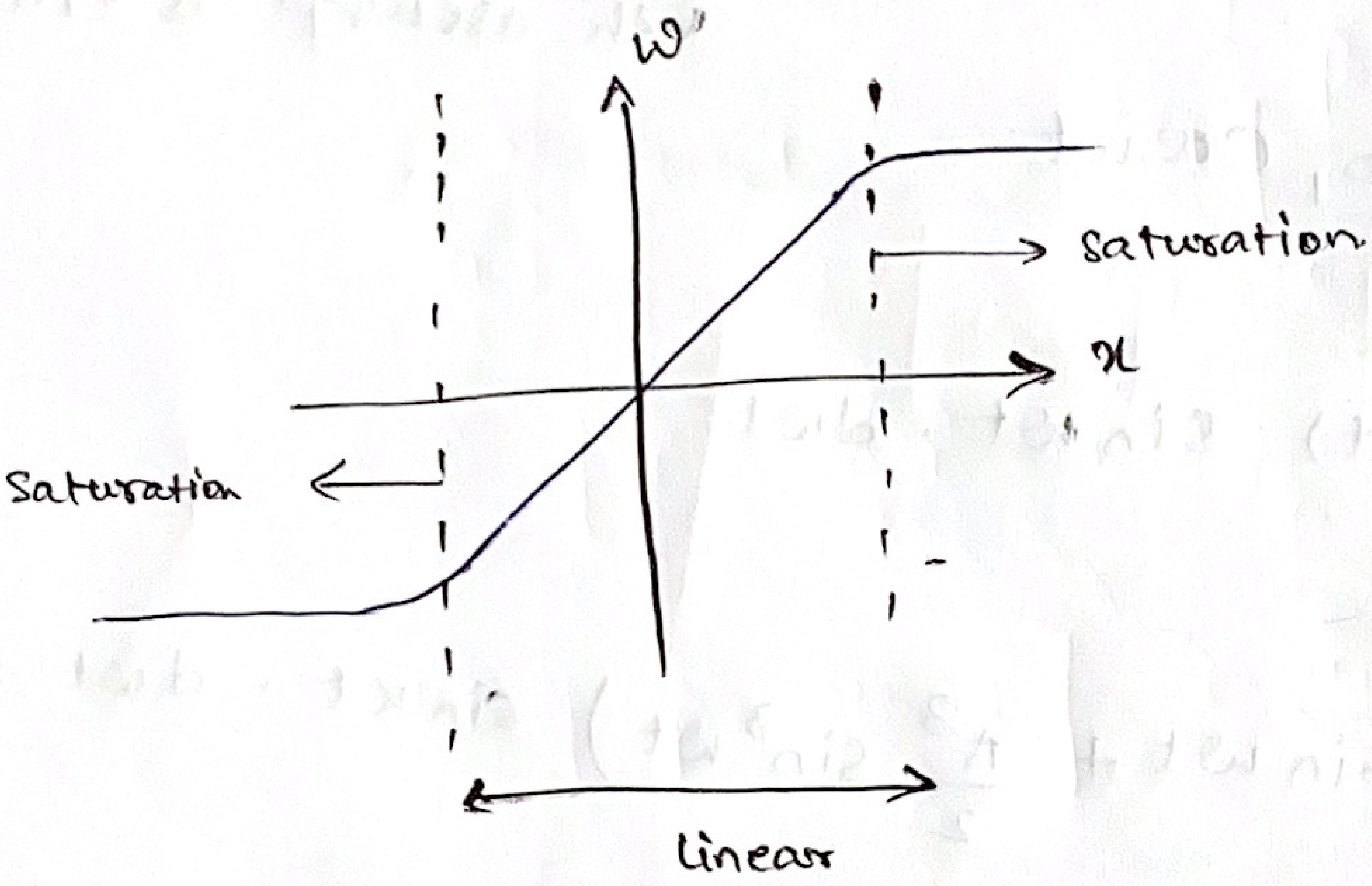
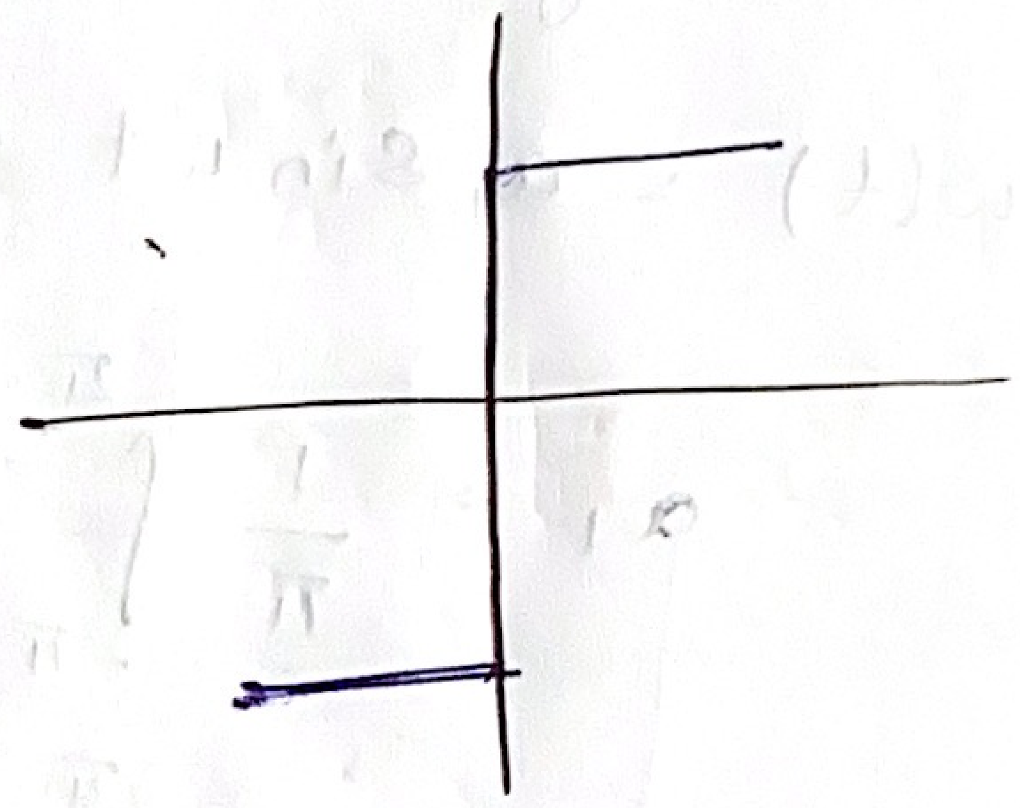
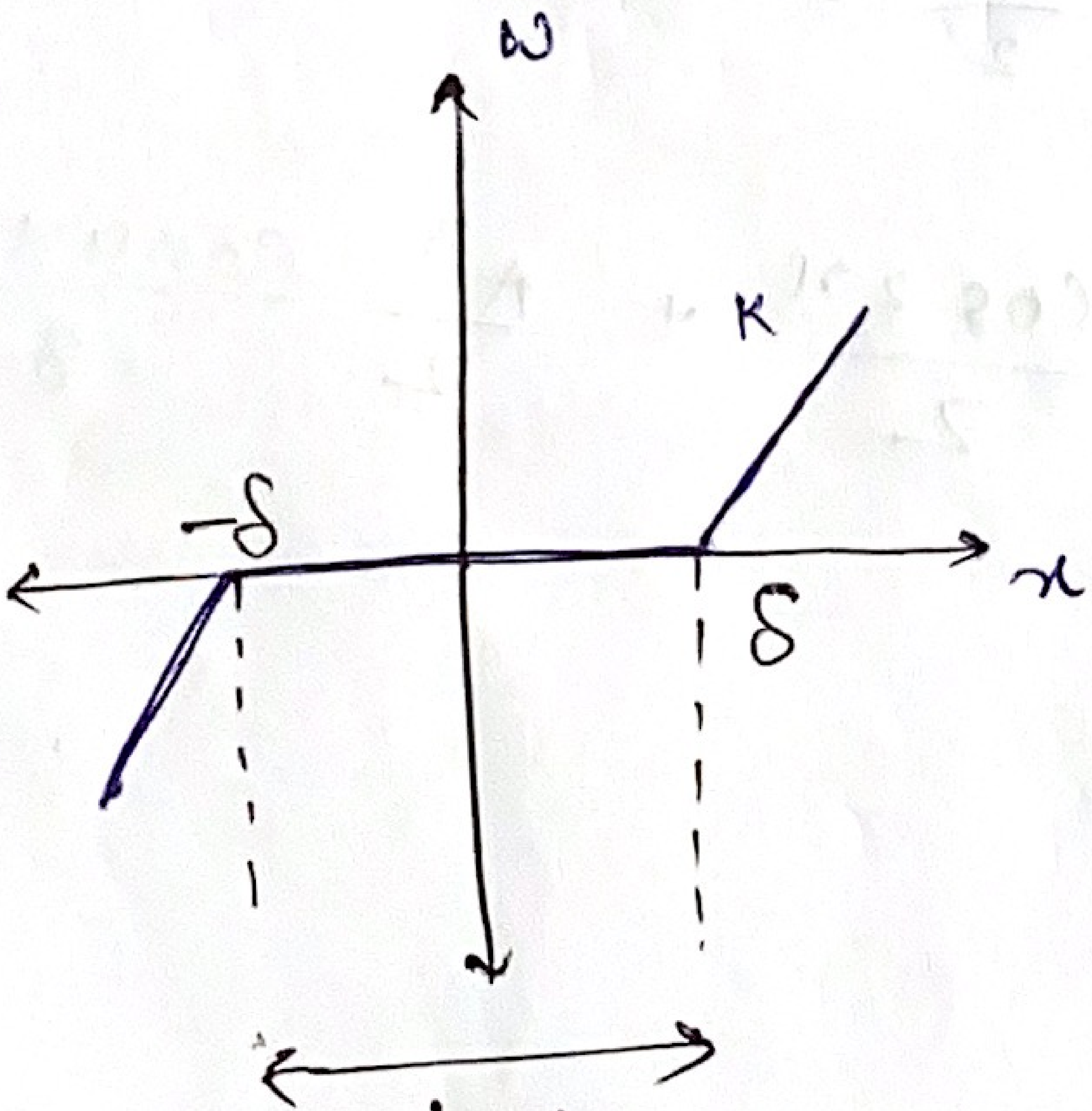
Saturation.

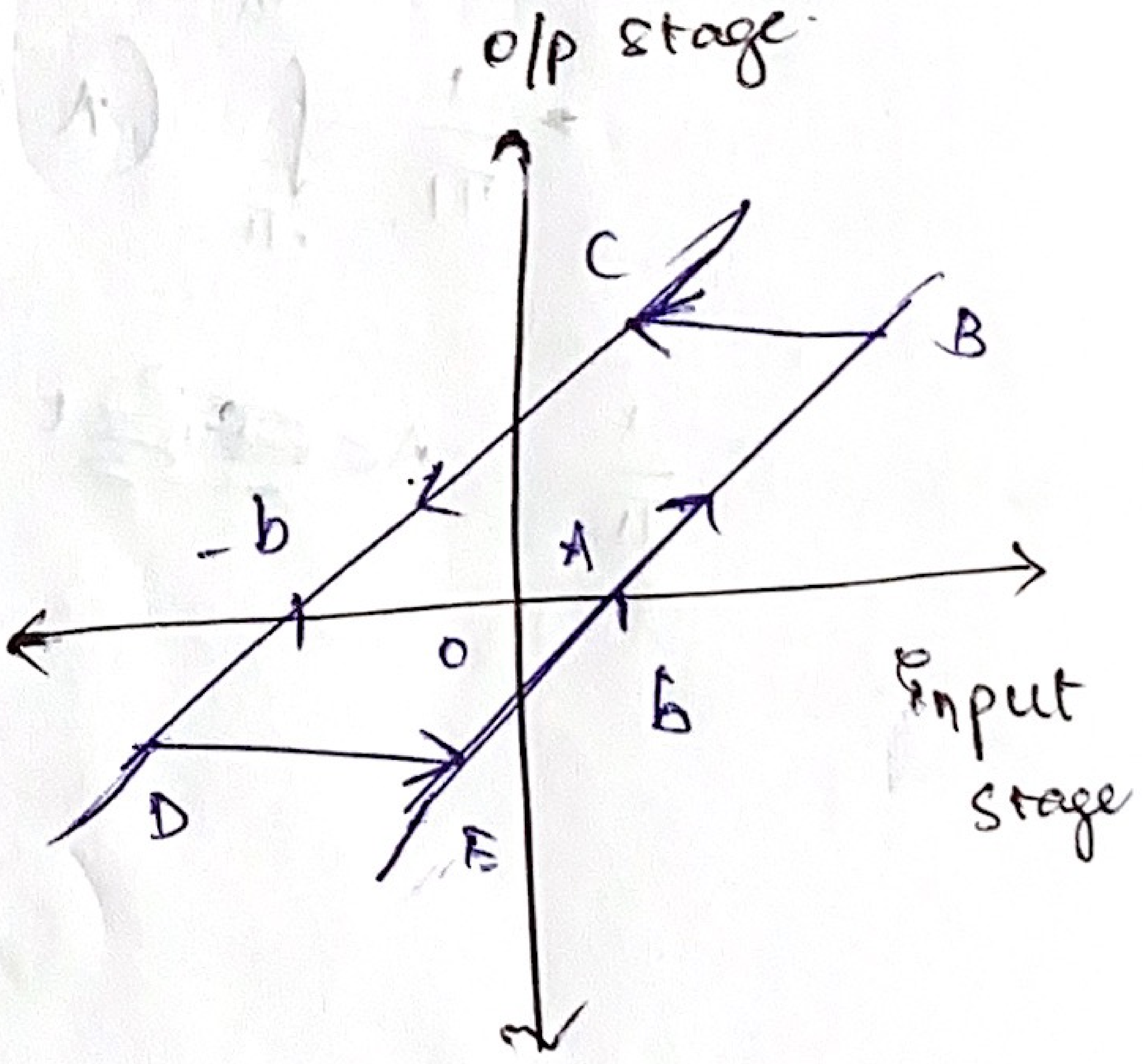


ON-Off





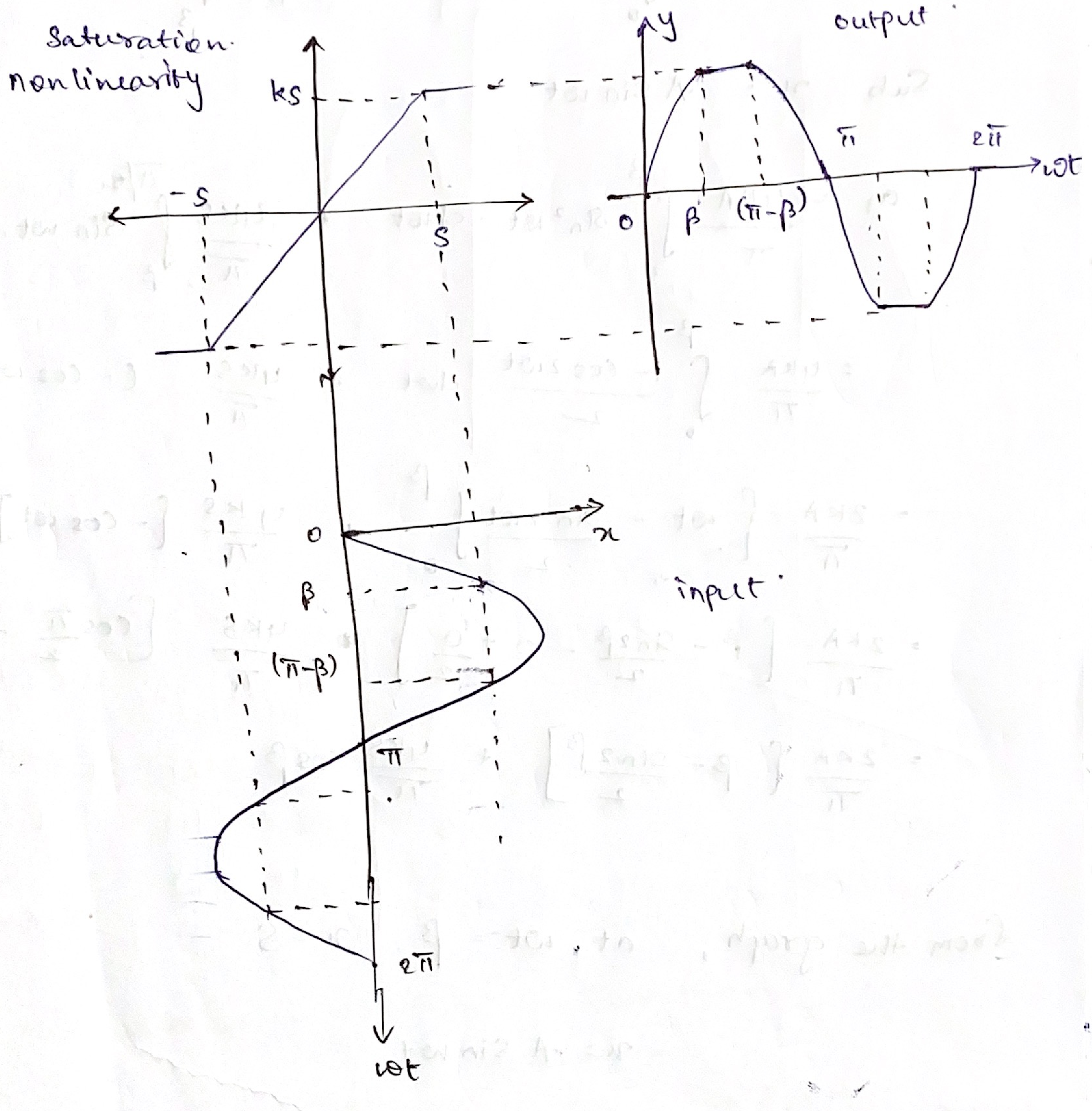
dead Zone



Backlash & hysteresis

2/11/22

Describing functions of Saturation Nonlinearity.



In the region 0 to π

$$y = \begin{cases} kx, & 0 \le \omega t \le \beta \\ ks, & \beta \le (\pi-\beta) \\ kx, & (\pi-\beta) \le \pi \end{cases}$$

where x is i/p.

The o/p has half wave & quarter wave symmetries

$b_1 = 0$ & $a_1 = \frac{2}{\pi/2} \int_0^{\pi/2} y \cdot \sin(\omega t) \cdot d\omega t$.

In the period 0 to $\pi/2$, it has 2 regions.

So, $a_1 = \frac{4}{\pi}\int_0^{\beta} Kx \sin\omega t \cdot d\omega t + \frac{4}{\pi}\int_{\beta}^{\pi/2} KS \sin\omega t \cdot d\omega t$

Sub $x = A \sin\omega t$

$$a_1 = \frac{4KA}{\pi}\int_0^{\beta} \sin^2\omega t \cdot d\omega t + \frac{4KS}{\pi}\int_{\beta}^{\pi/2} \sin\omega t \cdot d\omega t$$

$$= \frac{4KA}{\pi}\int_0^{\beta} \frac{1-\cos 2\omega t}{2} d\omega t + \frac{4KS}{\pi}\left(-\cos\omega t\right)_{\beta}^{\pi/2}$$

$$= \frac{2KA}{\pi}\left[\omega t - \frac{\sin 2\omega t}{2}\right]_0^{\beta} + \frac{4KS}{\pi}\left[-\cos\omega t\right]_{\beta}^{\pi/2}$$

$$= \frac{2KA}{\pi}\left[\beta - \frac{\sin 2\beta}{2} - 0 + \frac{0}{2}\right] - \frac{4KS}{\pi}\left[\cos\frac{\pi}{2} - \cos\beta\right]$$

$$= \frac{2KA}{\pi}\left[\beta - \frac{\sin 2\beta}{2}\right] + \frac{4KS}{\pi}\cos\beta$$

from the graph, at $\omega t = \beta$, $x = S$

$$x = A \sin\omega t$$

$$S = A \sin\beta$$

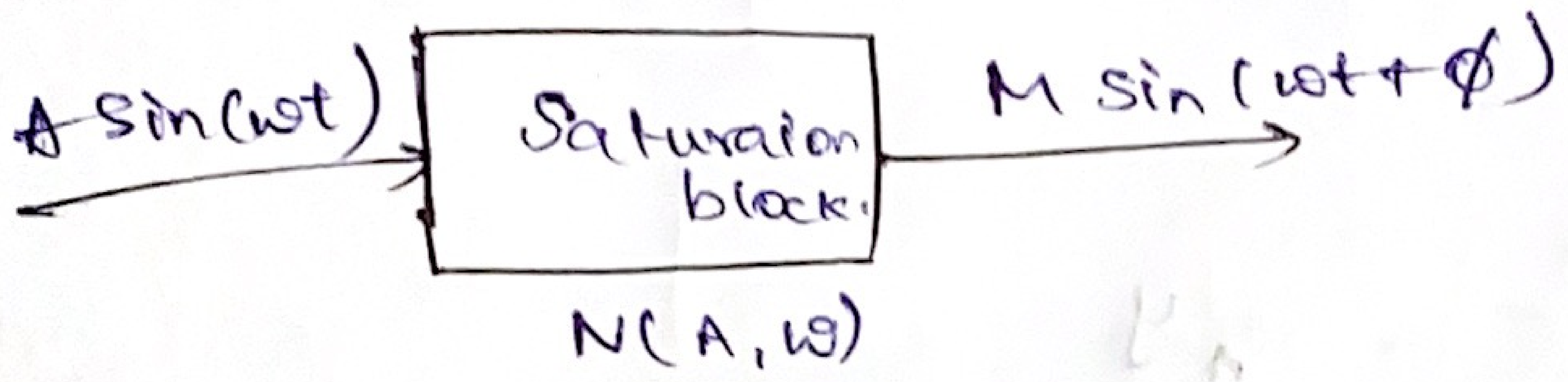
$$\Rightarrow \beta = \sin^{-1}\left(\frac{S}{A}\right)$$

$$a_1 = \frac{2KA}{\pi}\left[\beta - \frac{\sin 2\beta}{2}\right] + \frac{4K}{\pi} A \sin\beta\cos\beta$$

$$a_1 = \frac{2KA}{\pi}\left[\beta - \frac{2\sin\beta\cos\beta}{2}\right] + \frac{4KA}{\pi}\sin\beta\cos\beta$$

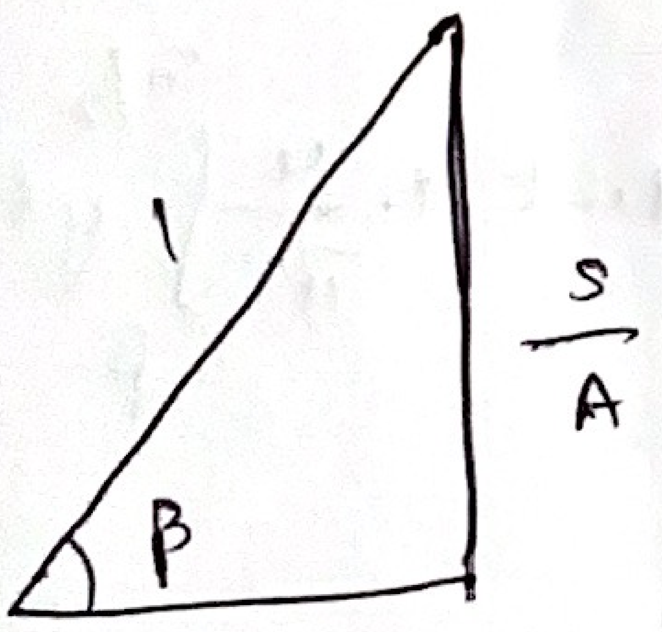
$$= \frac{2KA}{\pi}\left[\beta - \sin\beta\cos\beta + 2\sin\beta\cos\beta\right]$$

$$= \frac{2KA}{\pi}\left[\beta + \sin\beta\cos\beta\right]$$



DF can also be written in another form.

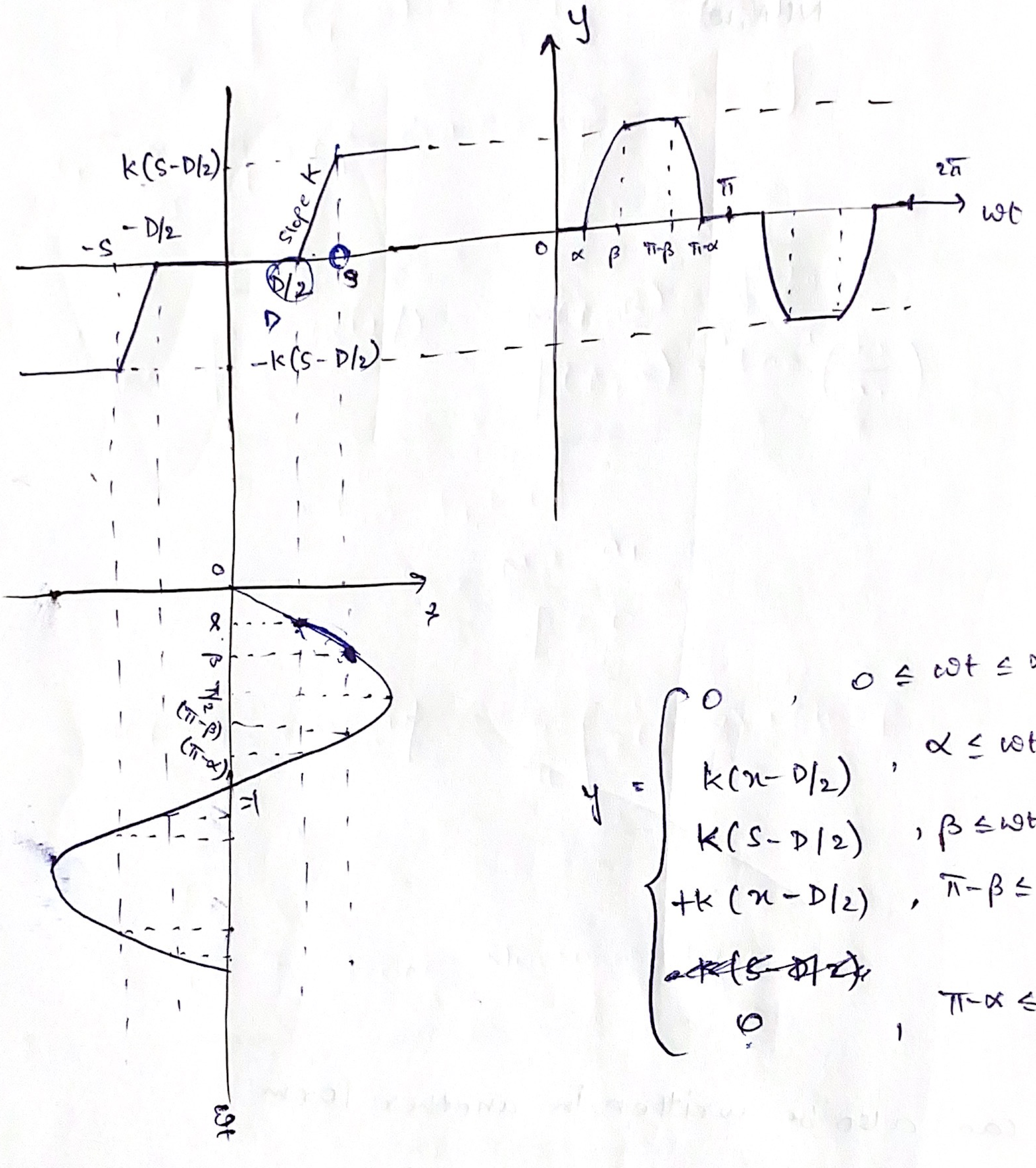
$S = A \sin\beta \qquad \frac{S}{A} = \sin\beta \qquad \beta = \sin^{-1}\left(\frac{S}{A}\right)$



$\cos\beta = \sqrt{1-\left(\frac{S}{A}\right)^2}$

$$N(A,w) = \frac{2K}{\pi}\left[\underbrace{\sin^{-1}(S/A)}_{(\beta)} + \underbrace{\frac{S}{A}}_{(\sin\beta)}\underbrace{\sqrt{1-\left(\frac{S}{A}\right)^2}}_{(\cos\beta)}\right]$$

DF of Dead zone and Saturation nonlinearity.



$$y = \begin{cases} 0 , & 0 \le \omega t \le \alpha \\ k(x - D/2) , & \alpha \le \omega t \le \beta \\ k(S - D/2) , & \beta \le \omega t \le \pi - \beta \\ +k(x - D/2) , & \pi - \beta \le \omega t \le \pi - \alpha \\ \cancel{-k(S-D/2)} \\ 0 , & \pi - \alpha \le \omega t \le \pi \end{cases}$$

3/11/22 The o/p has half & quarter wave symmetries. So, $b_1 = 0$. Also, consider the region 0 to $\pi/2$

$$a_1 = \frac{4}{\pi}\int_0^{\alpha} y \sin\omega t \cdot d\omega t + \frac{4}{\pi}\int_{\alpha}^{\beta} y \sin\omega t \cdot d\omega t + \frac{4}{\pi}\int_{\beta}^{\pi/2} y \sin\omega t \cdot d\omega t$$

$$a_1 = 0 + \frac{4}{\pi}\int_{\alpha}^{\beta} k\left(\sin\omega t - \frac{D}{2}\right)\sin\omega t \cdot d\omega t$$

$$+ \frac{4}{\pi}\int_{\beta}^{\pi/2} k\left(S - \frac{D}{2}\right)\sin\omega t \cdot d\omega t .$$

$$a_1 = \frac{4K}{\pi}\left[\int_{\alpha}^{\beta} A\sin^2 \omega t \cdot d\omega t - \int_{\alpha}^{\beta} \frac{D}{2} \sin \omega t \cdot d\omega t + \int_{\beta}^{\pi/2} \left(S - \frac{D}{2}\right) \sin \omega t \, d\omega t\right]$$

$$= \frac{4K}{\pi}\left[A \int_{\alpha}^{\beta} \frac{(1 - \cos 2\omega t)}{2} \cdot d\omega t - \frac{D}{2}\left(-\cos \omega t\right)_{\alpha}^{\beta} + \left(S - \frac{D}{2}\right)\left(-\cos \omega t\right)_{\beta}^{\pi/2}\right]$$

$$= \frac{4K}{\pi}\left[\frac{A}{2}\left[\omega t - \frac{\sin 2\omega t}{2}\right]_{\alpha}^{\beta} + \frac{D}{2}\left(\cos\beta - \cos\alpha\right) + \left(S - \frac{D}{2}\right)\left(\cos\frac{\pi}{2} - \cos\beta\right)\right]$$

$$= \frac{4K}{\pi}\left[\frac{A}{2}\left[\beta - \frac{\sin 2\beta}{2} - \alpha + \frac{\sin 2\alpha}{2}\right] + \cancel{\frac{D}{2}\cos\beta} - \frac{D}{2}\cos\alpha + S\cos\beta - \cancel{\frac{D}{2}\cos\beta}\right]$$

$$= \frac{4K}{\pi}\left[\frac{A}{2}\left[\beta - \alpha - \frac{\sin 2\beta}{2} + \frac{\sin 2\alpha}{2}\right] - \frac{D}{2}\cos\alpha + S\cos\beta\right]$$

Consider at $\omega t = \alpha$, $x = D/2$

So, $A \sin \omega t = \frac{D}{2}$

$A \sin\alpha = \frac{D}{2}$ $\quad \frac{D}{2A} = \sin\alpha$

$\Rightarrow \alpha = \sin^{-1}\left(\frac{D}{2A}\right)$

at $\omega t = \beta$ $\quad x = S$

So, $A \sin \beta = S$

$\sin\beta = \frac{S}{A}$

$\Rightarrow \beta = \sin^{-1}\left(\frac{S}{A}\right)$

Replace in the above eq.

$$a_1 = \frac{4K}{\pi}\left[\frac{A}{2}\left(\beta - \alpha - \frac{\sin 2\beta}{2} + \frac{\sin 2\alpha}{2}\right) - A\sin\alpha\cos\alpha + A\sin\beta\cos\beta\right]$$

$$= \frac{4KA}{\pi}\left[\frac{\beta}{2} - \frac{\alpha}{2} - \frac{\sin 2\beta}{4} + \frac{\sin 2\alpha}{4} - \frac{\sin 2\alpha}{2} + \frac{\sin 2\beta}{2}\right]$$

$$= \frac{4KA}{\pi}\left[\frac{1}{2}(\beta-\alpha) + \frac{\sin 2\beta}{4} - \frac{\sin 2\alpha}{4}\right]$$

$$= \frac{2KA}{\pi}\left[(\beta-\alpha) + \frac{\sin 2\beta}{2} - \frac{\sin 2\alpha}{2}\right]$$

$$a_1 = \frac{KA}{\pi}\left[2(\beta-\alpha) + \sin 2\beta - \sin 2\alpha\right]$$

$$A \sin \omega t \longrightarrow \boxed{N(A,\omega)} \longrightarrow M \sin(\omega t + \phi)$$

$$M(A,\omega) = \sqrt{a_1^2 + b_1^2}$$

$$\phi(A,\omega) = \tan^{-1}\left(\frac{b_1}{a_1}\right)$$

here $b_1 = 0$

so, $M = a_1$

& $\phi = 0$

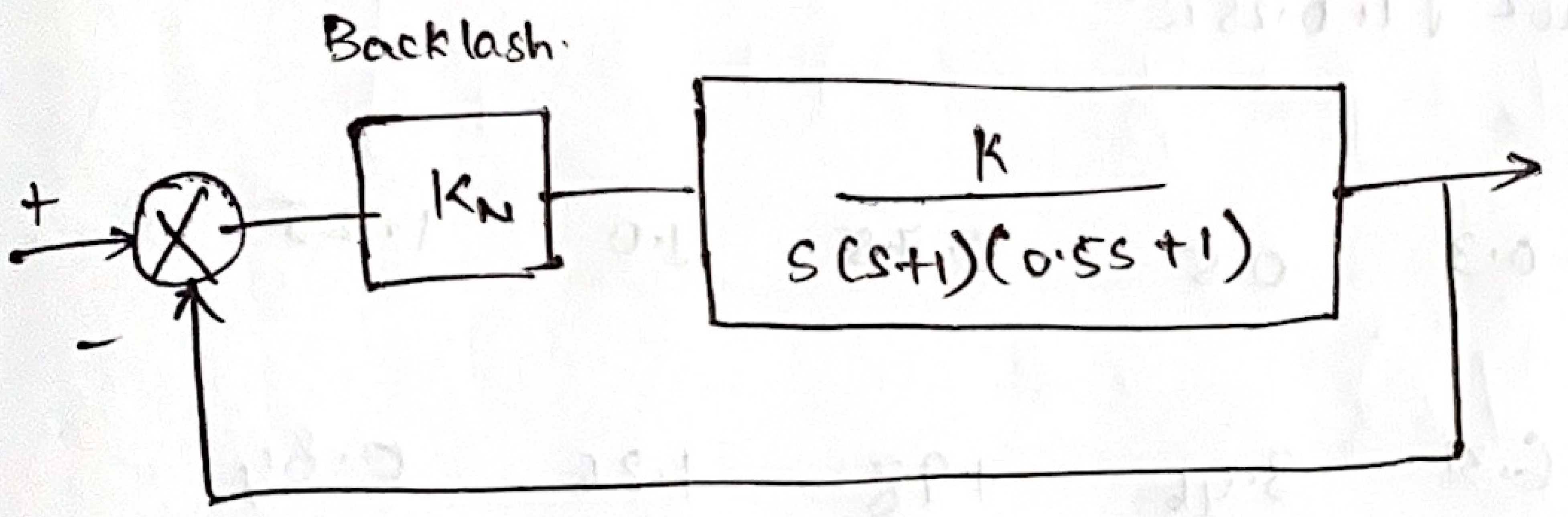
$$N(A,\omega) = \frac{M}{A} e^{i\phi} = \frac{K}{\pi}\left[2(\beta-\alpha) + \sin 2\beta - \sin 2\alpha\right]$$

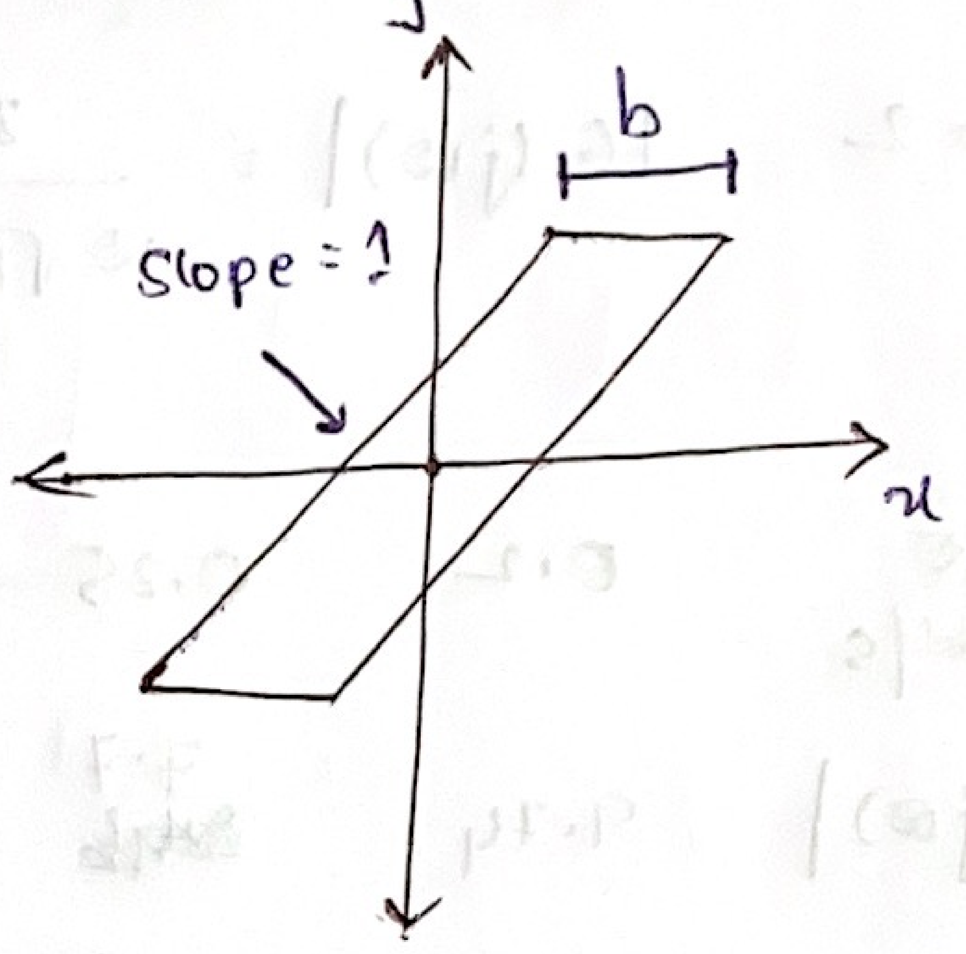
DF of Deadzone Nonlinearity.

4/11/22

Tutorial - 7

Q. A servo system is used for positioning a load has backlash characteristics as shown. The mag & phase of the DF for various values of b/x are listed below. Show that system is stable if k = 1. Also show that limit cycle exists when k = 2. Find the stability of these limit cycles & determine their freq & b/x.





backlash → b/x; Amplitude

b/x	0	0.2	0.4	1	1.4	1.6	1.8	1.9	2.0
$\lvert K_N \rvert$	1	0.954	0.882	0.592	0.367	0.248	0.125	0.064	0
$\angle K_N$	0	$-6.7°$	$-13.4°$	$-32.5°$	$-46.6°$	$-52.2°$	$-66°$	$-69.8°$	$-90°$

Sol. when $k = 1$, $G(j\omega) = \dfrac{1}{j\omega(1+j\omega)(1+0.5j\omega)}$

$j\omega$

mag = $\sqrt{0^2+\omega^2} = \omega$

angle = $90°$

$= \tan^{-1}\frac{b}{a}$

$|G(j\omega)| = \dfrac{1}{\omega(\sqrt{1^2+\omega^2})(\sqrt{1+0.25\omega^2})}$

$\angle G(j\omega) = -90° - \tan^{-1}\omega - \tan^{-1}0.5\omega$

$\therefore K=1$ $\quad |G(j\omega)| = \dfrac{1}{\omega\sqrt{1+\omega^2}\,\sqrt{1+0.25\omega^2}}$ $\quad \angle G(j\omega) = -90^\circ - \tan^{-1}\omega - \tan^{-1}0.5\omega$

ω rad/sec	0.1	0.15	0.2	0.25	0.5	0.75	1.0	1.25
$\lvert G(j\omega)\rvert$	9.93	6.57	4.87	3.85	1.73	0.97	0.63	0.42
$\angle G(j\omega)^\circ$	-99	-103	-107	-111	-131	-147	-162	-173
$G_R(j\omega)$	-1.6	-1.5	-1.4	-1.4	-1.1	-0.8	-0.6	-0.4
$G_I(j\omega)$	-9.8	-6.4	-4.7	-3.6	-1.3	-0.5	-0.2	-0.05

$K=2$ $\quad |G(j\omega)| = \dfrac{2}{\omega\sqrt{1+\omega^2}\,\sqrt{1+0.25\omega^2}}$

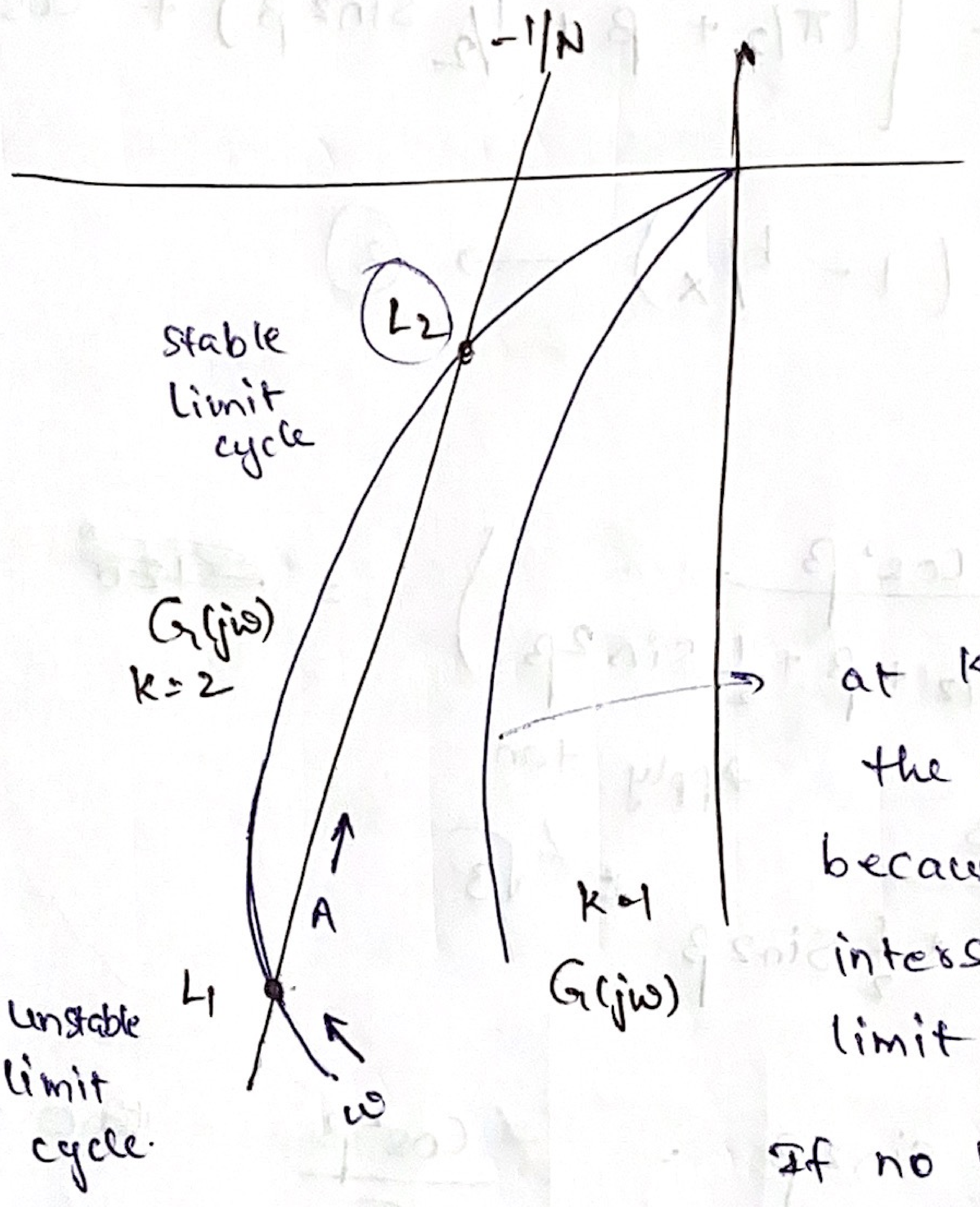
ω rad/s	0.2	0.25	0.3	0.5	0.75	1.0	1.25
$\lvert G(j\omega)\rvert$	9.74	7.7	6.31	3.46	1.98	1.26	0.84
$\angle G(j\omega)$	-107	-111	-115	-131	-147	-162	-173
$G_R(j\omega)$	-2.9	-2.8	-2.7	-2.3	-1.7	-1.2	-0.8
$G_I(j\omega)$	-9.31	-7.2	-5.7	-2.6	-1.1	-0.4	-0.1

Polar plot of $\dfrac{-1}{K_N}$

$$\frac{-1}{K_N} = -1 \times \frac{1}{K_N} = 1\angle -180^\circ \times \frac{1}{|K_N|\angle K_N}$$

$$\left|\frac{-1}{K_N}\right| = \frac{1}{|K_N|} \qquad \angle(-1/K_N) = -180^\circ - \angle K_N$$

b/x	0	0.2	0.4	1	1.4	1.6	1.8	1.9	2
$\lvert K_N \rvert$	1	0.954	0.882	0.592	0.367	0.248	0.125	0.064	0
$\angle K_N$	0	-6.7°	-13.4°	-32.5°	-46.6°	-55.2°	-66°	-69.8°	-90°
$\lvert \frac{-1}{K_N} \rvert$	1	1.04	1.13	1.68	2.72	4.03	8	15.62	∞
$\angle\left(\frac{-1}{K_N}\right)$	-180°	-173.3	-166.6	-147.5	-133.4	-124.8	-114	-110.2	-90°
img	0	-0.121	-0.375	-0.90	-1.976	-3.28	-7.30	-14.64	
Real	-1	-1.032	-1.005	-1.41	-1.808	-2.28	-3.25	-5.38	



at $k=1$ the system is stable because there are no intersections, i.e no limit cycles.

If no limit cycles, the o/p of system is zero. so for 0 i/p, o/p is 0.

So, system is stable.

from the graph

$$L_1 = (-2.6 - j4.4)$$

$$= 5.11 \angle -120.57^\circ$$

$\left|\frac{-1}{K_N}\right|$ = mag = 5.11

$\angle\frac{-1}{K_N}$ angle = -120°

$$|K_N| = \frac{1}{\left|\frac{-1}{K_N}\right|} = \frac{1}{5.11} = 0.195$$

$$\angle K_N = -180^\circ - \angle\left(\frac{-1}{K_N}\right)$$

$$= -180^\circ + 120^\circ = -60^\circ$$

$$-90^\circ - \tan^{-1}\omega - \tan^{-1} 0.5\omega = -120^\circ$$

$$\tan^{-1}\omega + \tan^{-1} 0.5\omega = 30^\circ$$

Apply tan

$$\tan(\tan^{-1}\omega + \tan^{-1} 0.5\omega) = \tan(30^\circ)$$

$$\frac{\omega + 0.5\omega}{1 - \omega(0.5\omega)} = 0.577$$

$$\omega + 0.5\omega = 0.577(1 - 0.5\omega^2)$$

$$0.28\omega^2 + 1.5\omega - 0.577 = 0$$

-5.7

$$\underline{\omega = 0.36 \text{ rad/s}}$$

formulae.

$$\angle K_N = \tan^{-1}\left(\frac{-\cos^2\beta}{\pi/2 + \beta + \frac{1}{2}\sin 2\beta}\right) \longrightarrow (1)$$

$$|K_N| = \frac{1}{\pi}\left[\left(\pi/2 + \beta + 1/2 \sin 2\beta\right)^2 + \cos^4\beta\right]^{1/2} \rightarrow (2)$$

$$\beta = \sin^{-1}(1 - b/x) \longrightarrow (3)$$

$$\tan^{-1}\left(\frac{-\cos^2\beta}{\pi/2 + \beta + \frac{1}{2}\sin 2\beta}\right) = -60^\circ$$

Apply tan

$$\frac{-\cos^2\beta}{\pi/2 + \beta + \frac{1}{2}\sin 2\beta} = -\sqrt{3}$$

$$\frac{\pi}{2} + \beta + \frac{1}{2}\sin 2\beta = \frac{\cancel{-}\cos^2\beta}{\sqrt{3}} = 0.577\cos^2\beta$$

Sub in (2)

$$|K_N| = \frac{1}{\pi}\left((0.577\cos^2\beta)^2 + \cos^4\beta\right)^{1/2}$$

$$\frac{1}{5.11} = \frac{1}{\pi}\left(0.33\cos^4\beta + \cos^4\beta\right)^{1/2}$$

$$0.195 = \frac{1}{\pi}\sqrt{(1.33)\cos^4\beta}$$

$$0.195\pi = 1.15\cos^2\beta$$

$$\cos^2\beta = 0.53$$

$$\cos\beta = 0.72 \Rightarrow \beta = 43^\circ$$