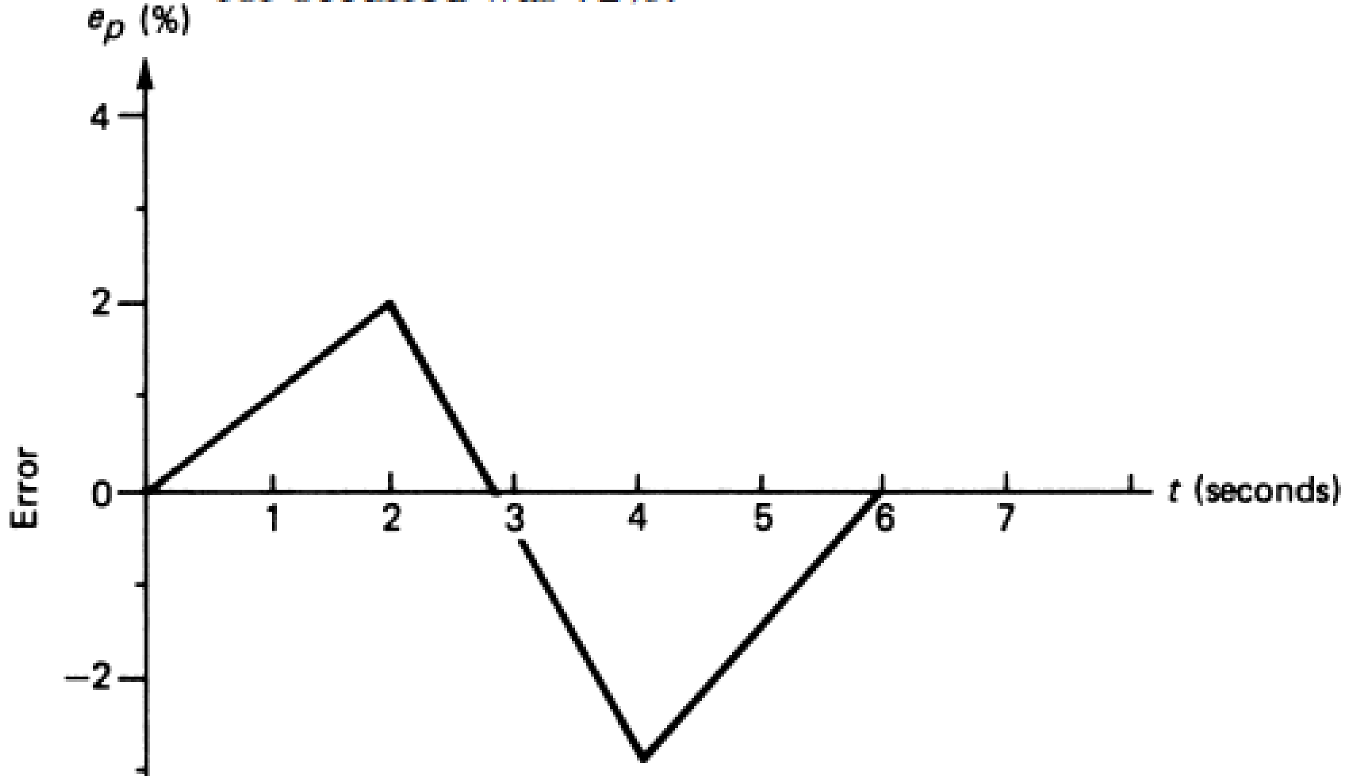
Section 6

13 A PI controller has $K_P = 2.0$, $K_I = 2.2\ \text{s}^{-1}$, and $p_I(0) = 40\%$. Plot the output for an error given by Figure 26.

14 A PD controller has $K_P = 2.0$, $K_D = 2$ s, and $p_0 = 40\%$. Plot the controller output for the error input of Figure 26.

15 A PID controller has $K_P = 2.0$, $K_I = 2.2\ \text{s}^{-1}$, $K_D = 2$ s, and $p_I(0) = 40\%$. Plot the controller output for the error of Figure 26.

16 A PI controller is reverse acting, PB = 20, 12 repeats per minute. Find (**a**) the proportional gain, (**b**) the integral gain, and (**c**) the time that the controller output will reach 0% after a constant error of −1.5% starts. The controller output when the error occurred was 72%.



Design of analog Controllers

Reference:

Process Control Instrumentation Technology

Curtis D. Johnson

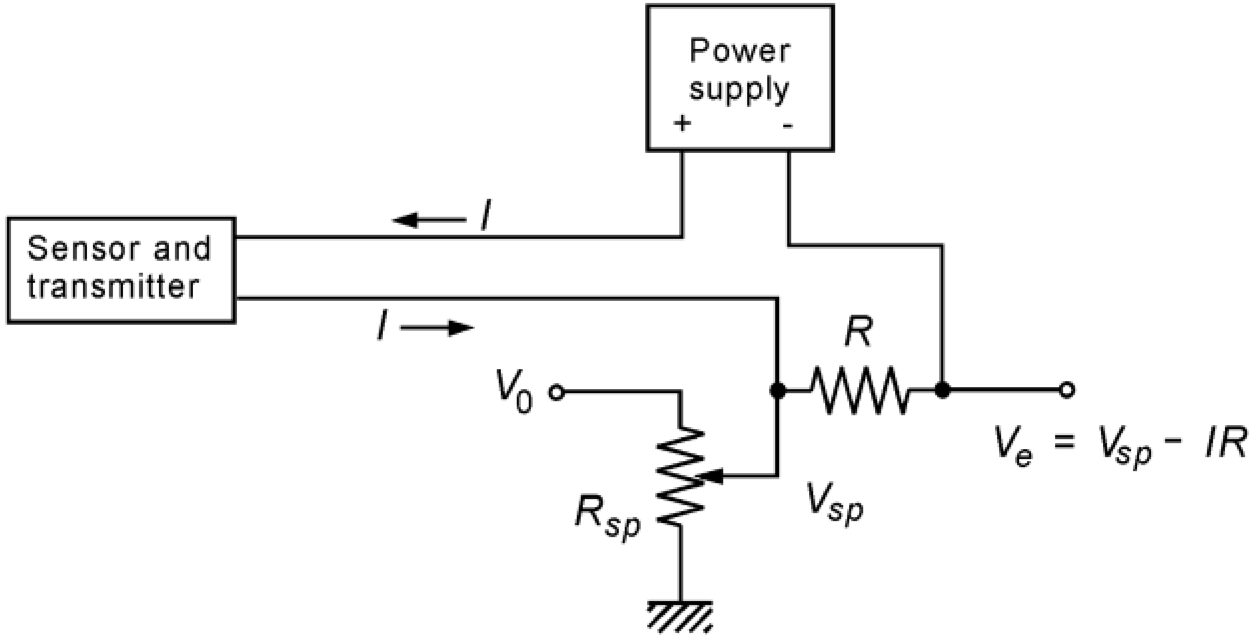
Electronic controllers

- An analog controller is a device that implements different controller modes using analog signals to represent the loop parameters.
- The analog signal may be in the form of an electric current or a pneumatic air pressure.
- The controller accepts a measurement expressed in terms of one of these signals, calculates an output for the mode being used, and outputs an analog signal of the same type.

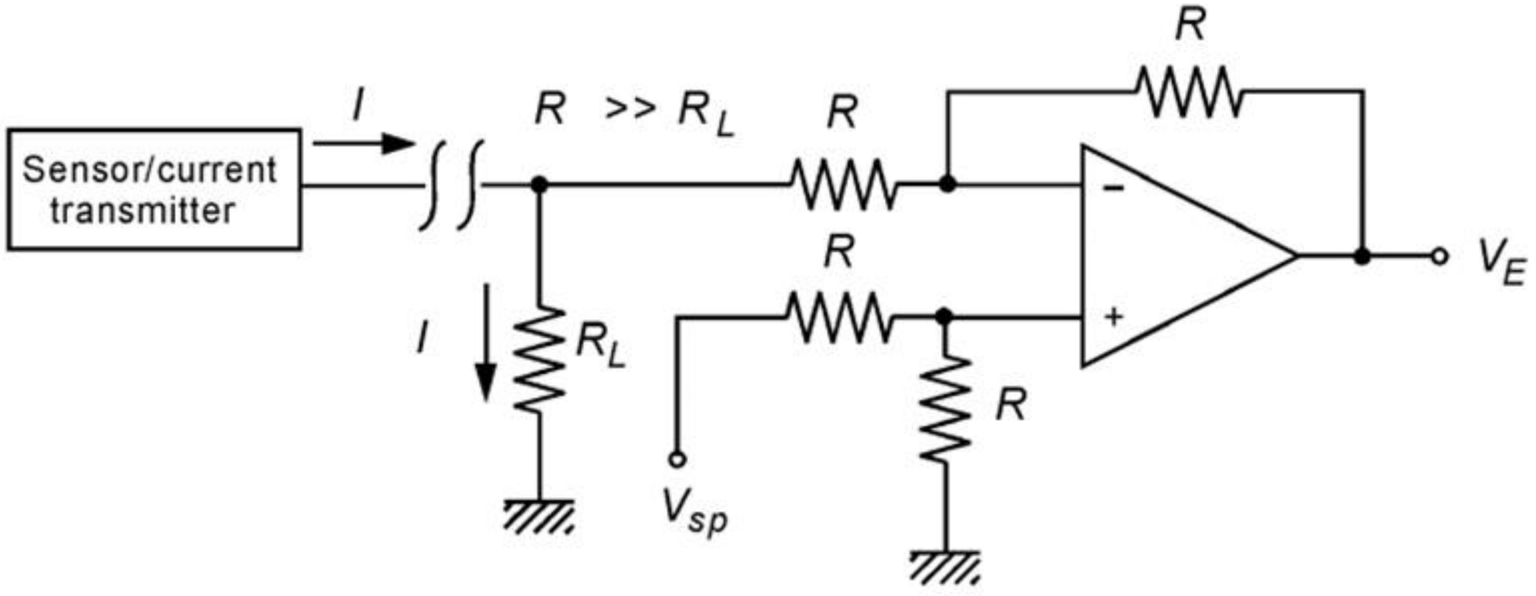
- Electronic method of realizing controller modes necessitates the use of **OPAMPs** as the primary circuit elements.
- For pneumatics the **nozzle/flapper system** forms the basis of controller mode implementation.
- Thus, electronic controllers are designed to input and output the standard 4- to 20-mA signal.
- For pneumatic controllers the signal standard is 3 to 15 psi for U.S. installations and 20 to 100 kPa in many other locations throughout the world.

Error Detector

- The detection of an error signal is done in electronic controllers by taking the difference between voltages.
- One voltage is generated by the process signal current passed through a resistor.
- The second voltage represents the set point, which is usually generated by a voltage divider using a constant voltage as a source.



Error detector using a ground-based current and a differential amplifier.

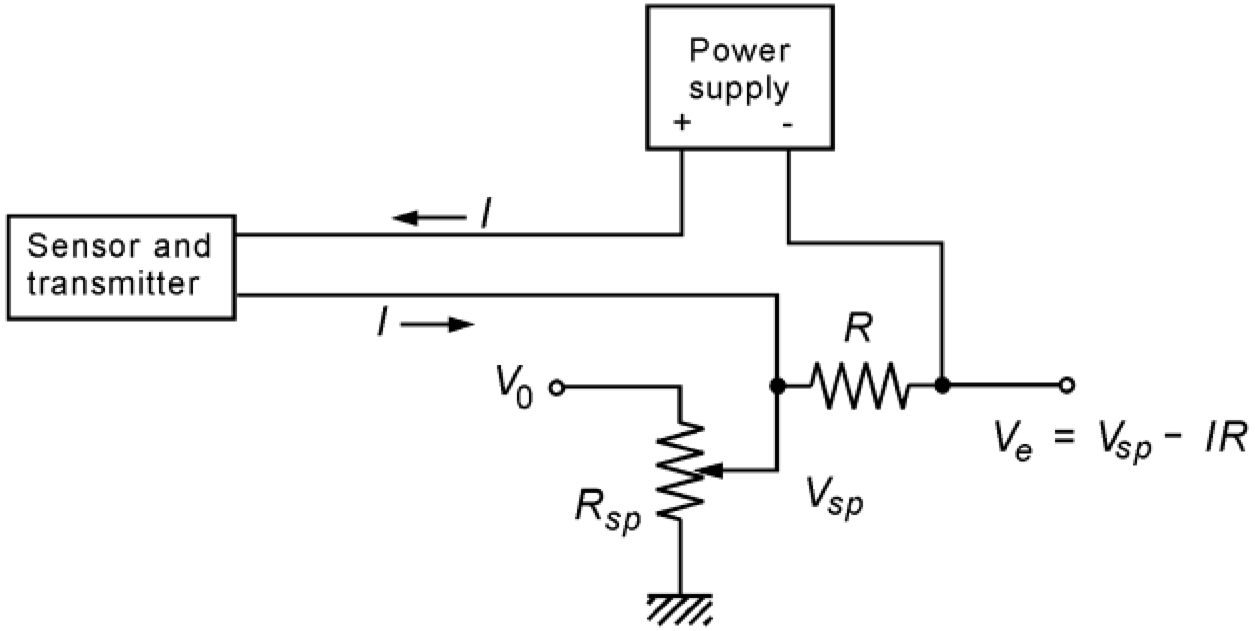


Problem #1

A sensor converts from 0 to 2.0 m in to a 4 to 20 mA current. An error detector such as shown in Fig. is used with R = 100 Ω, V_0 =5.0 V and R_{sp} = 1KΩ pot.

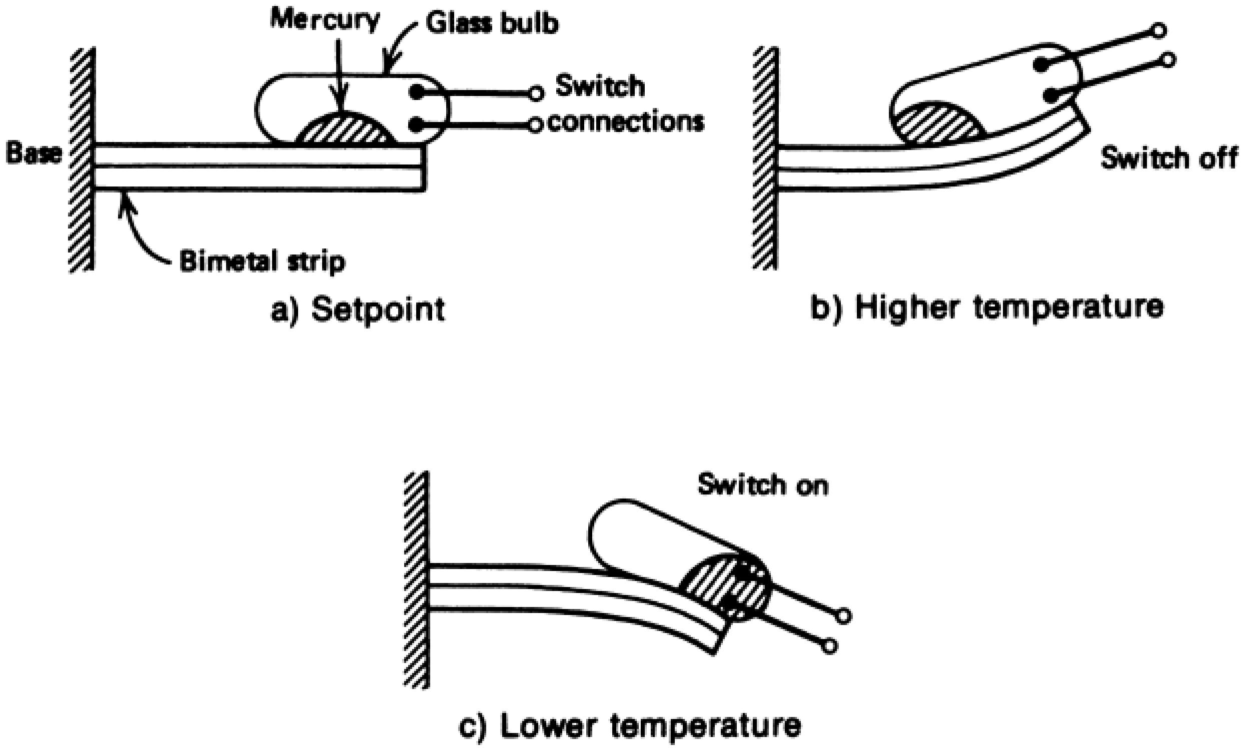
a). If the setpoint is 0.85 m what is V_{sp}?

b). If V_{sp} = 1.5 V, what is the range of error voltage as position varies from 0 to 2.0m?

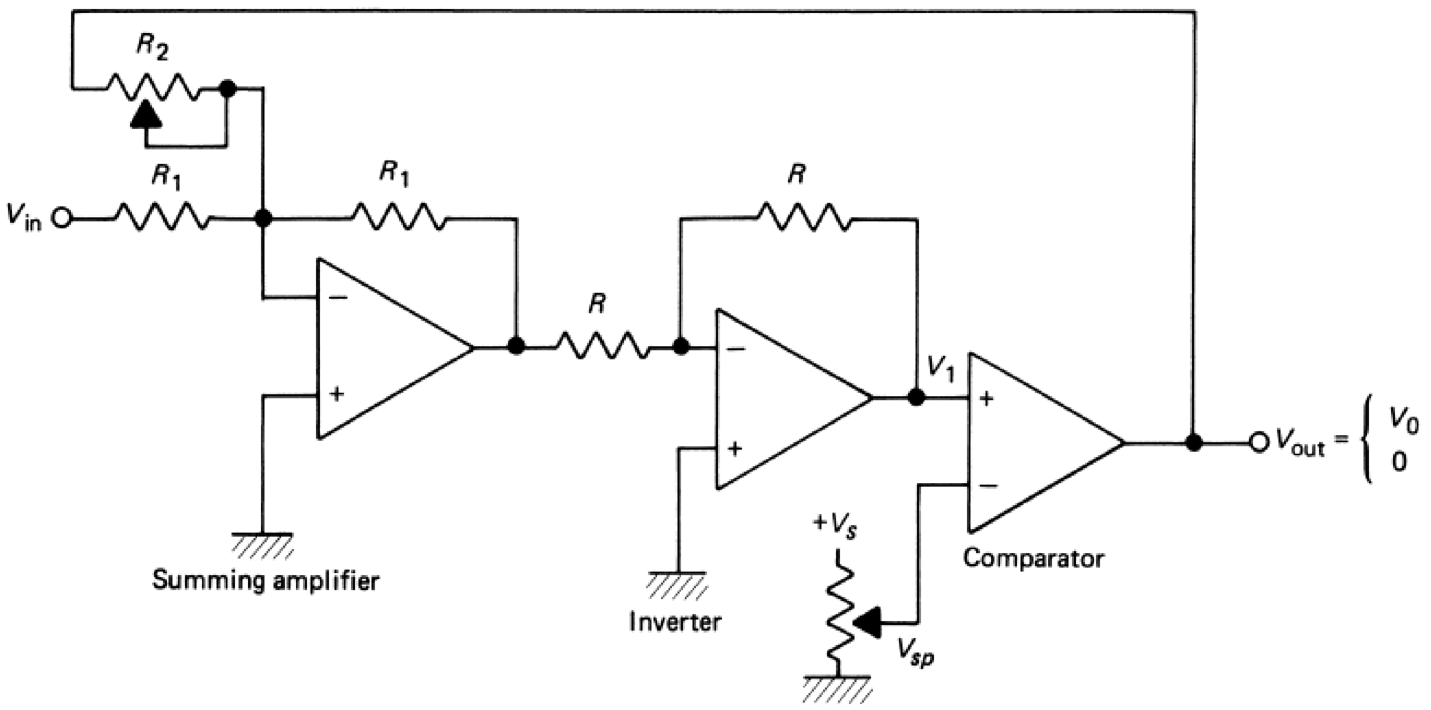


Single mode : Two position Controllers

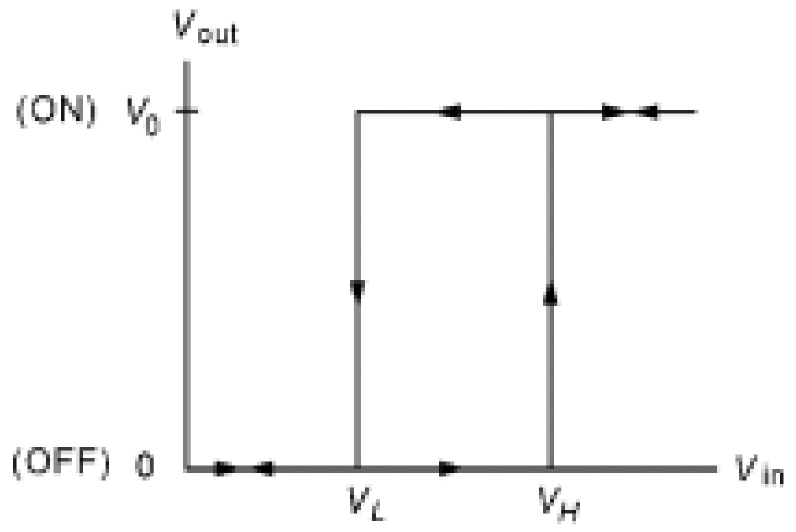
- A two position controller can be implemented by electronic and electromechanical designs.
- Many house hold air-conditioning and heating systems employ a two position controller constructed from bimetal strip and mercury switch as shown.



Op-Amp implementation of ON/OFF control:



Circuit



Output

Op-Amp implementation of ON/OFF control:

Assume that, in the beginning, the comparator is in the OFF state

$$V_{out} = 0$$

The comparator output switches states when the voltage on its input, V1 is equal to the set point value Vsp Analyzing this circuit,

$$V_1 = V_{in} + \frac{R_1}{R_2} V_{out}$$

Substituting

$$V_1 = V_{in}$$

The comparator changes to ON state when V1 = Vin = VH. Thus, the high (ON) switch voltage is

$$V_H = V_{sp}$$

$$V_{out} = V_0$$

With this V_1 changes to

$$V_1 = V_{in} + \frac{R_1}{R_2} V_0$$

If $V_{in} = V_L$ the comparator changes to OFF state, giving the relation,

$$V_1 = V_{sp} = V_L + \frac{R_1}{R_2} V_0$$

This gives the low (OFF) switching voltage of

$$V_L = V_{SP} - \frac{R_1}{R_2} V_o$$

Problem No:2

Level measurement in a sump tank is provided by a transducer scaled as 0.2 V/m. A pump is to be turned on by application of + 5 V when the sump level exceeds 2.0 m. The pump is to be turned back off when the sump level drops to 1.5 m. Develop a two position controller.

Calculations:

$V_H = (0.2\ \text{V/m}) \times (2.0\ \text{m}) = 0.4\ \text{V}$

$V_L = (0.2\ \text{V/m}) \times (1.5\ \text{m}) = 0.3\ \text{V}$

So, using the relations given in Eqs. 2.4 and 2.8,

$0.4\ \text{V} = V_{SP}$

$$0.3\text{V} = V_{SP} - \frac{R_1}{R_2} V_0$$

Therefore,

$V_{SP} = 0.4\ \text{V}$

$$0.3\text{V} = 0.4 - \frac{R_1}{R_2} 5$$

Or, $\dfrac{R_1}{R_2} = 0.02$

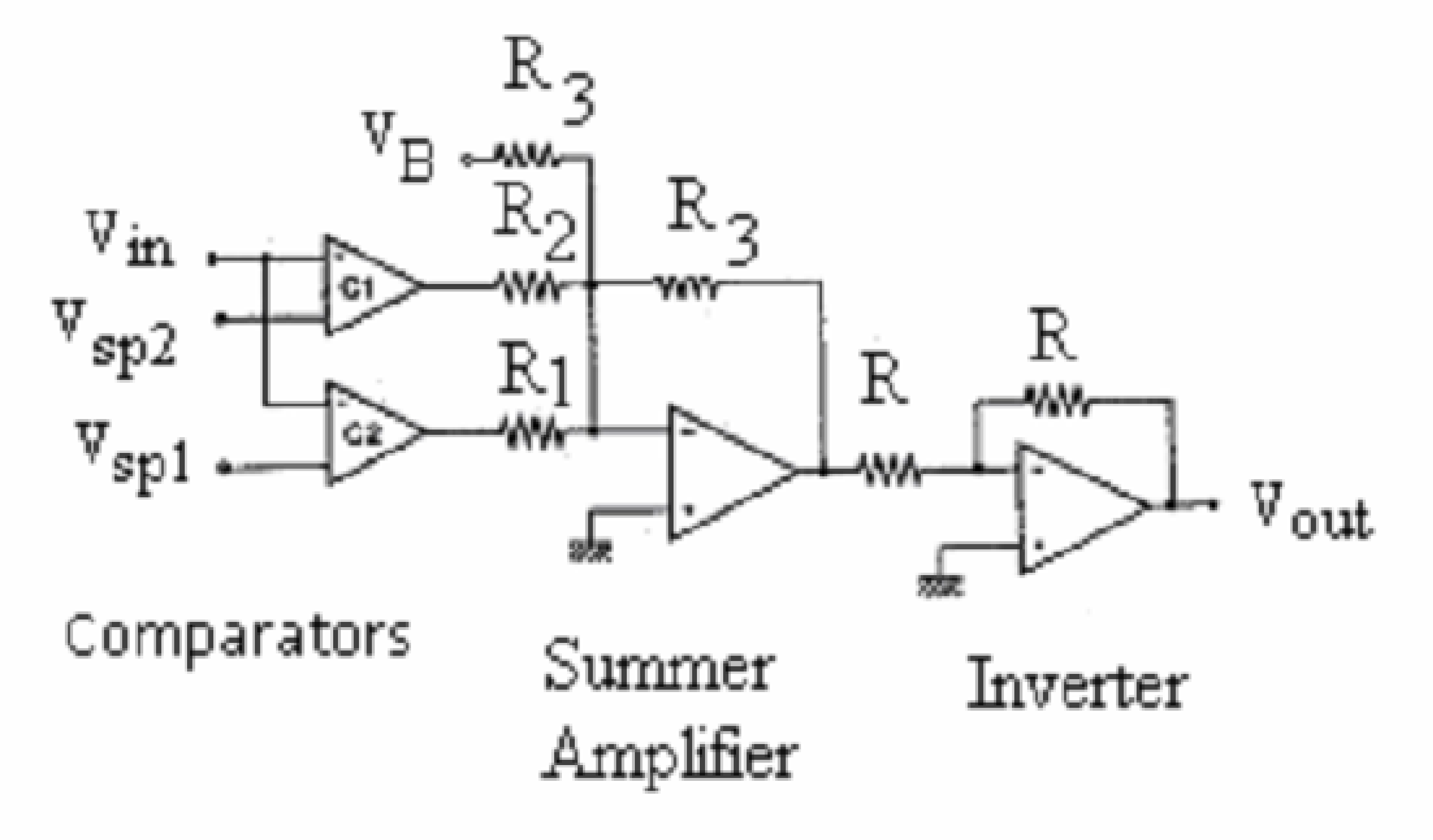
There are two unknowns and only condition to be satisfied. So, one unknown can be selected.

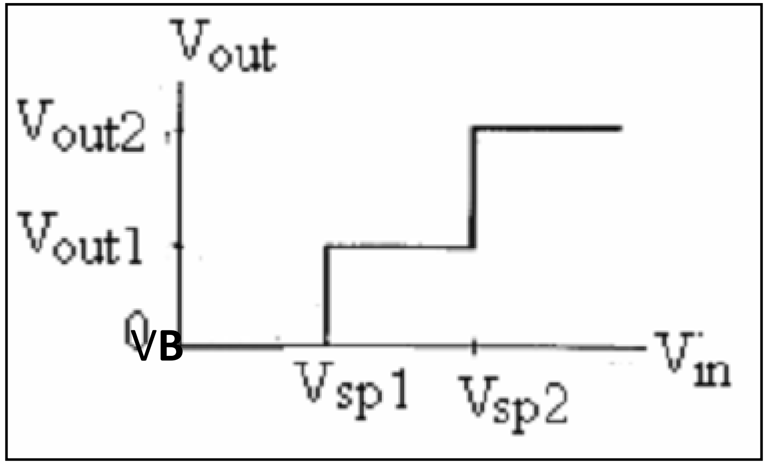
Let $R_1 = 5\ \text{K}\Omega$ then, $R_2 = 250\ \text{K}\Omega$

Problem #3

Using a system of Fig. 2.5 design a two position controller with a 0 to 10 V input and a 0 or 10 V output. The setpoint is 4.3 V and the neutral zone is to be $\pm$ 1.1 V about this setpoint.

Three position





Response with Vb=0

Three position

When $V_{in} < V_{SP1}$,

Comparator C1 is OFF, C2 is OFF (Because $V_{SP1} < V_{SP2}$)

Outputs of both comparators are 0 V. Thus,

$V_{out} = V_B$

When $V_{SP1} < V_{in} < V_{SP2}$,

Comparator C1 is OFF, C2 is ON

Outputs of comparator C1 = 0 and Output of Comparator C2 = $\frac{R_3}{R_1} V_0$ Volts. Thus,

$$V_{out} = V_B + \frac{R_3}{R_1} V_0$$

Three position

When $V_{in} > V_{SP2}$,

Comparator C1 is ON, C2 is ON

Outputs of comparator C1 = $\frac{R_3}{R_2}V_0$ and Output of Comparator C2 = $\frac{R_3}{R_1}V_0$ Volts. Thus,

$$V_{out} = V_B + \frac{R_3}{R_1}V_0 + \frac{R_3}{R_2}V_0$$

Thus,

When $V_{in} < V_{SP1}$, $V_{out} = V_B$ (2.9)

$V_{SP1} < V_{in} < V_{SP2}$, $V_{out} = V_B + \frac{R_3}{R_1}V_0$ (2.10)

$V_{in} > V_{SP2}$, $V_{out} = V_B + \frac{R_3}{R_1}V_0 + \frac{R_3}{R_2}V_0$

Proportional Mode

- Implementation of this mode requires a circuit that has a response given by

$$p = K_P e_p + p_0$$

$$\begin{aligned} p &= \text{controller output 0–100\%} \\ K_P &= \text{proportional gain} \\ e_p &= \text{error in percent of variable range} \\ p_0 &= \text{controller output with no error} \end{aligned}$$

- If we consider both the controller output and error to be expressed in terms of voltage, we see that is simply a summing amplifier with the analog electronic equation for the output voltage as

$$V_{\text{out}} = G_P V_e + V_0$$

$$\begin{aligned} V_{\text{out}} &= \text{output voltage} \\ G_P &= R_2/R_1 = \text{gain} \\ V_e &= \text{error voltage} \\ V_0 &= \text{output with zero error} \end{aligned}$$

Proportional Mode

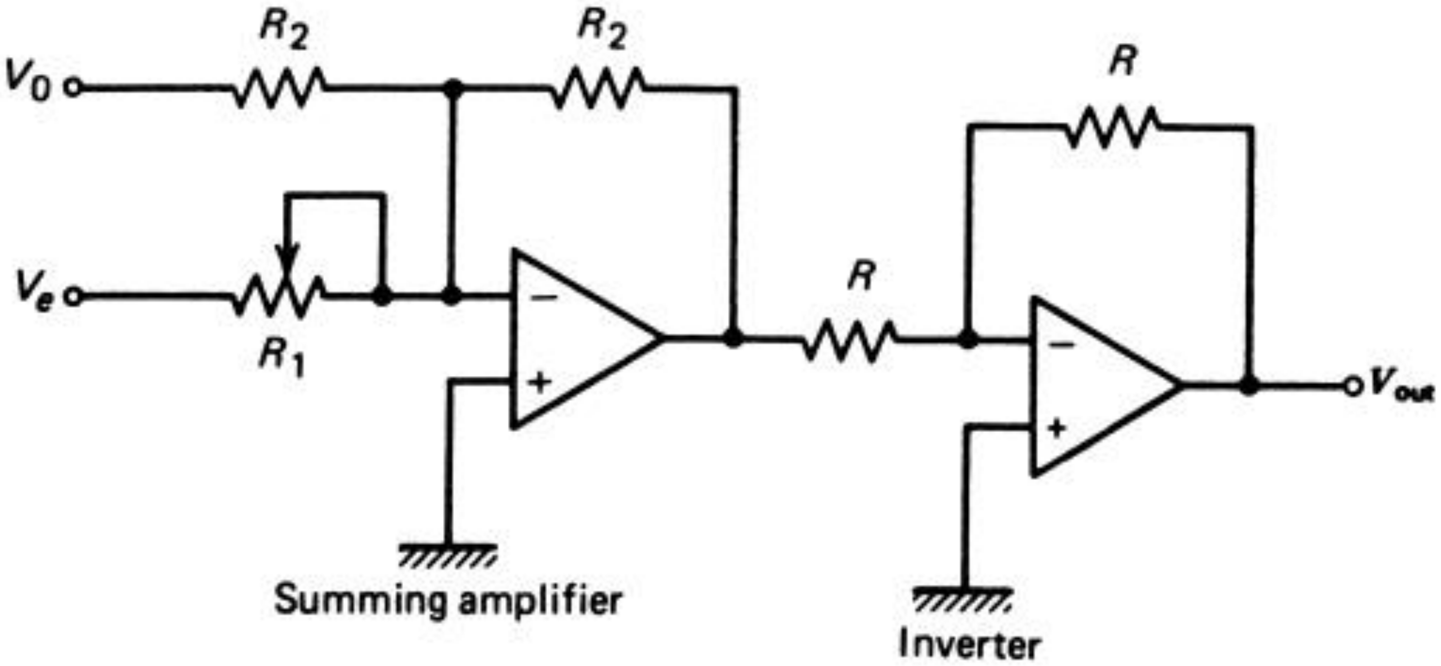


FIGURE 7
An op amp proportional-mode controller.

Design Procedure:

- The design of a proportional controller calls for specification of the proportional gain described by K_P that expresses the percent of output for an error of 1% of the measurement range.
- Alternatively it could be described as the proportional band, $PB=100/K_P$.
- This must now be expressed in terms of the voltage gain, G_P in The relationship between G_P and K_p is given by,

$$G_P = K_P \frac{\Delta V_{\mathrm{out}}}{\Delta V_{\mathrm{m}}}$$

Integral Mode

- The integral mode was characterized by an equation of the form

$$p(t) = K_I \int_0^t e_p \, dt + p(0)$$

Where

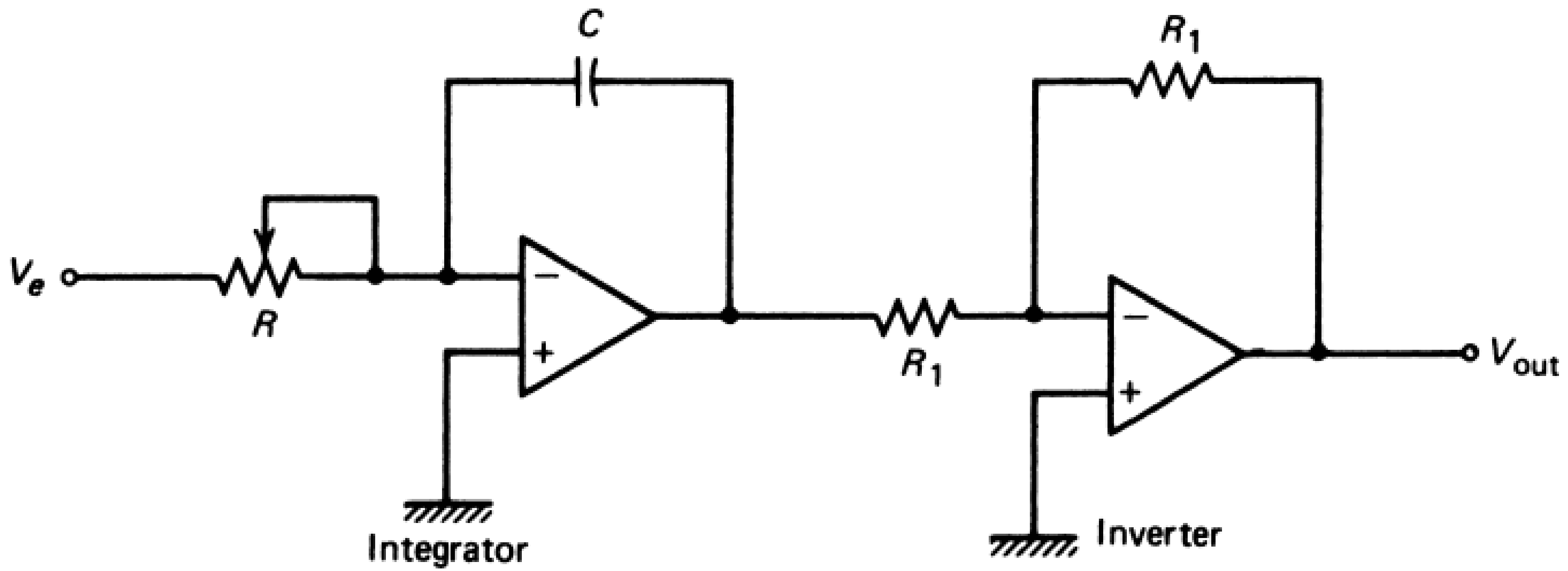
$$\begin{aligned} p(t) &= \text{controller output in percent of full scale} \\ K_I &= \text{integration gain } (\mathrm{s}^{-1}) \\ e_p &= \text{deviations in percent of full-scale variable value} \\ p(0) &= \text{controller output at } t = 0 \end{aligned}$$

- The corresponding equation relating input to output is

$$V_{\text{out}} = G_I \int_0^t V_e \, dt + V_{\text{out}}(0)$$

$$\begin{aligned} V_{\text{out}} &= \text{output voltage} \\ G_I &= 1/RC = \text{integration gain} \\ V_e &= \text{error voltage} \\ V_{\text{out}}(0) &= \text{initial output voltage} \end{aligned}$$

OpAmp integral-mode controller



Determination of G_I

- The actual value of G_I and therefore R and C, is determined from K_I and the input and output voltage ranges.

- Integral gain says that, an input error of 1 % must produce an output that changes as K_I % per second.

- Or if an error of 1 % lasts for 1 s, the output must change by K_I percent.

Steps to find out G_I value

- Find out the input range and the output range.
- Convert K_I to units of seconds
- Find the error 1% of the input for 1 s
- Find K_I percent of the output (using the seconds expression for gain).
- Integral gain = Ratio of K_I percent of the output to 1% of the input for 1 s to.
- The values of *R* and *C* can be selected from that.

Derivative Mode

- Is never used alone because it cannot provide a controller output when the error is zero.
- The control mode equation is given by

$$P(t) = K_D \frac{de_p}{dt}$$

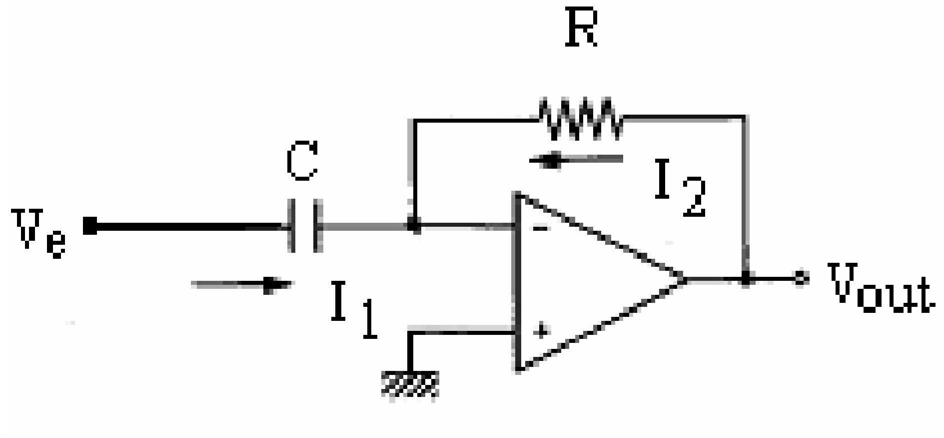
where, P = Controller output in percent of full output
K_D = Derivative time constant (s)
e_p = error in percent of full scale range

Implementation of derivative controller using OPAMP:

- Consider an OPAMP differentiator circuit shown in Fig The theoretical transfer function for this circuit will be given by:

$$V_{out} = -RC\frac{dV_e}{dt}$$

- where, the input voltage has been set equal to the controller error voltage



Implementation of derivative controller using OPAMP:

- From a practical perspective, this circuit can not be used because it tends to be unstable, that is, it may begin to exhibit spontaneous oscillations in the output voltage.
- The reason for this instability is the occurrence of very large gain at high frequencies where the derivative is very large.
- Consider the input voltage

$$V_e = V_0 \sin(2\pi ft).$$

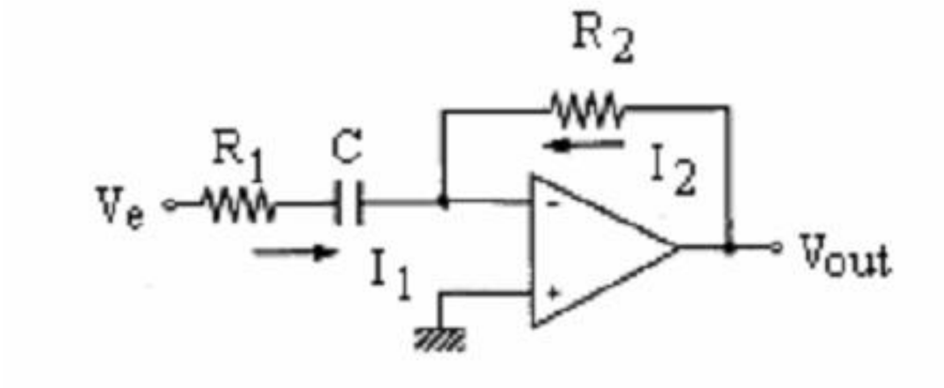
$$Vout = \frac{R}{-jX_c} V_e = -j2\pi fRCV_e$$

The magnitude of the output is expressed as,

$$|V_{out}| = 2\pi fRC|V_e|$$

Practical derivative mode op amp controller

- In order to make a practical circuit, a modification is provided that essentially clamps the gain above some frequency to a constant value.



Working principle:

- Assume a sinusoidal input voltage and use ac analysis. The input impedance is now given by:

$$X_{in} = R_1 + \frac{1}{j2\pi fC}$$

$$\text{Or,}\quad X_{in} = R_1 - \frac{j}{2\pi fC}$$

Now V_{out} is given by

$$V_{out} = -\frac{R_2}{R_1 - \frac{j}{2\pi fC}} V_e$$

$$V_{out} = -\frac{2\pi fCR_2}{2\pi fCR_1 - j} V_e$$

$$|V_{out}| = \frac{2\pi fR_2C}{\sqrt{(2\pi fR_1C)^2 + 1}} |V_e|$$

Working principle:

- For frequencies for which, $2\pi f R_1 C << 1$
  The circuit takes the derivative as required.
  But for frequencies for which, $2\pi f R_1 C >> 1$
- The response reduces to

$$\left|V_{out}\right| = \frac{R_2}{R_1}\left|V_e\right|$$

- showing no derivative action.
- Therefore when using a derivative action circuit, we must estimate the maximum physical frequency, fmax at which the system can respond.

Guidelines to design derivative mode controller

- The value of $G_D = R_2C$ is determined from K_D and knowledge of the measurement and output voltage ranges.
- For this mode, the interpretation of K_D is that, for an error change of 1% in 1 s, the output should change by K_D percent.
- Thus, G_D is found from the quotient of percent of the output voltage and 1% of the input voltage.
- K_D must be expressed in seconds

Guidelines to design derivative mode controller

- 1. Estimate the maximum frequency at which the physical system can respond, fmax.
- 2. Set $2\pi f_{max} R_1C = 0.1$ and solve for R_1 (C is found from the mode derivative gain requirement)
- 3. Equation shows that the following responses will result from this selection:
- f = 0.1 fmax — $V_{out} = 0.995\,(2\pi fR_2C)|V_e|$ — Derivative action
- f = fmax — $V_{out} = 0.707\,(2\pi fR_2C)|V_e|$ — Transition action
- f = 10fmax — $V_{out} = 0.0995\,(2\pi fR_2C)|V_e|$ — No Derivative action

Define the circuit derivative gain or derivative time in seconds as $G_D = R_2C$. G_D will be determined from the design controller derivative gain K_D.

An integral controller has an input range of 1 to 8 V and an output range of 0 to 12 V. If K_I = 12%/(%-min), find G_I, R and C.

K_I = 12%/(%/min) = K_I = [12%/(%-min)] [1min/60s)] = 0.2 s^{-1}
1 % of the input for 1 sec = (0.01)(8-1) V(1s) = 0.07 V-s
0.2 % of the output = (0.002)12V = 0.024 V
Thus the gain is, G_I = 0.024V/0.07V-s = 0.3428 s^{-1}
Or, RC = 2.92 s If C = 10 μF, R = 292 KΩ

Composite Controller Modes

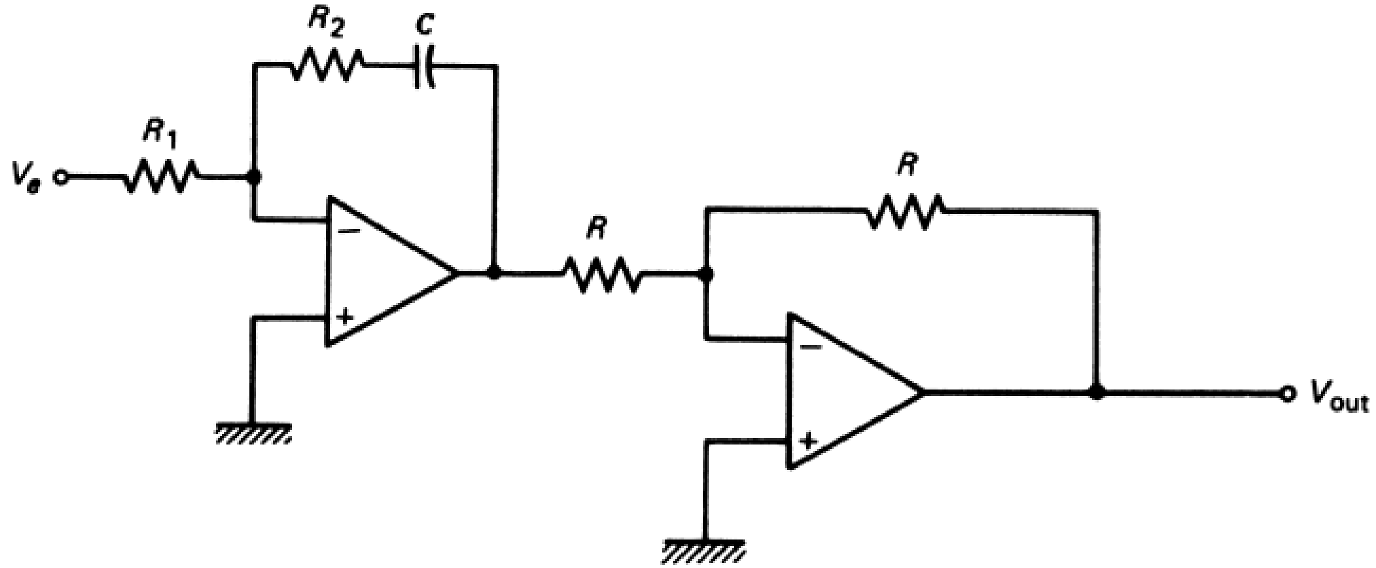
- The combination of several controller modes was found to combine the advantages of each mode and, in some cases, eliminate disadvantages.

- Composite modes are implemented easily using op amp techniques

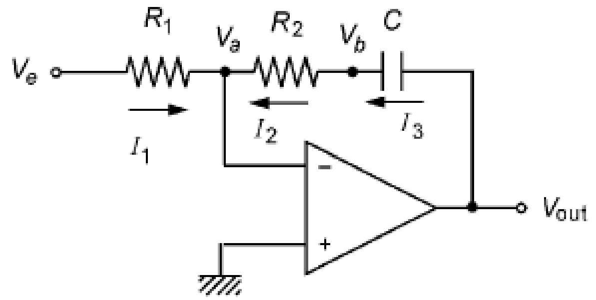
- Basically, this consists of simply combining the mode circuit

Proportional-Integral

- A simple combination of the proportional and integral circuits provides the proportional-integral mode of controller action.



$$P = K_p e_p + K_p K_I \int_0^t e_p \, dt + P_I(0)$$



$I_1 + I_2 = 0$
$I_3 - I_2 = 0$
The relationship between the voltage acros
is given by

$$I_c = C\frac{dV_C}{dt}$$

Where V_c is the voltage across the capacit

$$\frac{V_e}{R_1} + \frac{V_b}{R_2} = 0$$

$$C\frac{d}{dt}\left[V_{out_1} - V_b\right] - \frac{V_b}{R_2} = 0$$

The Eq. can be solved for V_b as:

$$V_b = -\frac{k_2}{R_1}V_e$$

Substituting this in to Eq

$$C\frac{dV_{out_1}}{dt} - C\frac{d}{dt}\left(-\frac{R_2}{R_1}V_e\right) - \frac{1}{R_2}\left(-\frac{R_2}{R_1}V_e\right.$$

$$C\frac{dV_{out_1}}{dt} + C\frac{R_2}{R_1}\frac{d}{dt}V_e + \frac{1}{R_1}V_e = 0$$

Or, $$\frac{dV_{out_1}}{dt} + \frac{R_2}{R_1}\frac{d}{dt}V_e + \frac{1}{R_1 C}V_e = 0$$

In order to solve for V_{out}, integrate this e

$$V_{out_1} = -\frac{R_2}{R_1}V_e - \frac{1}{R_1 C}\int_0^t V_e dt + V(0)$$

$$V_{out_1} = -\frac{R_2}{R_1}V_e - \frac{R_2}{R_1}\frac{1}{R_2 C}\int_0^t V_e dt + V(0)$$

- Equation has the same form as for PI mode. The adjustments of this controller are the *proportional band* through $G_P = R_2/R_1$,

$$V_{\text{out}} = \left(\frac{R_2}{R_1}\right) V_e + \frac{1}{R_1 C} \int_0^t V_e \, dt$$

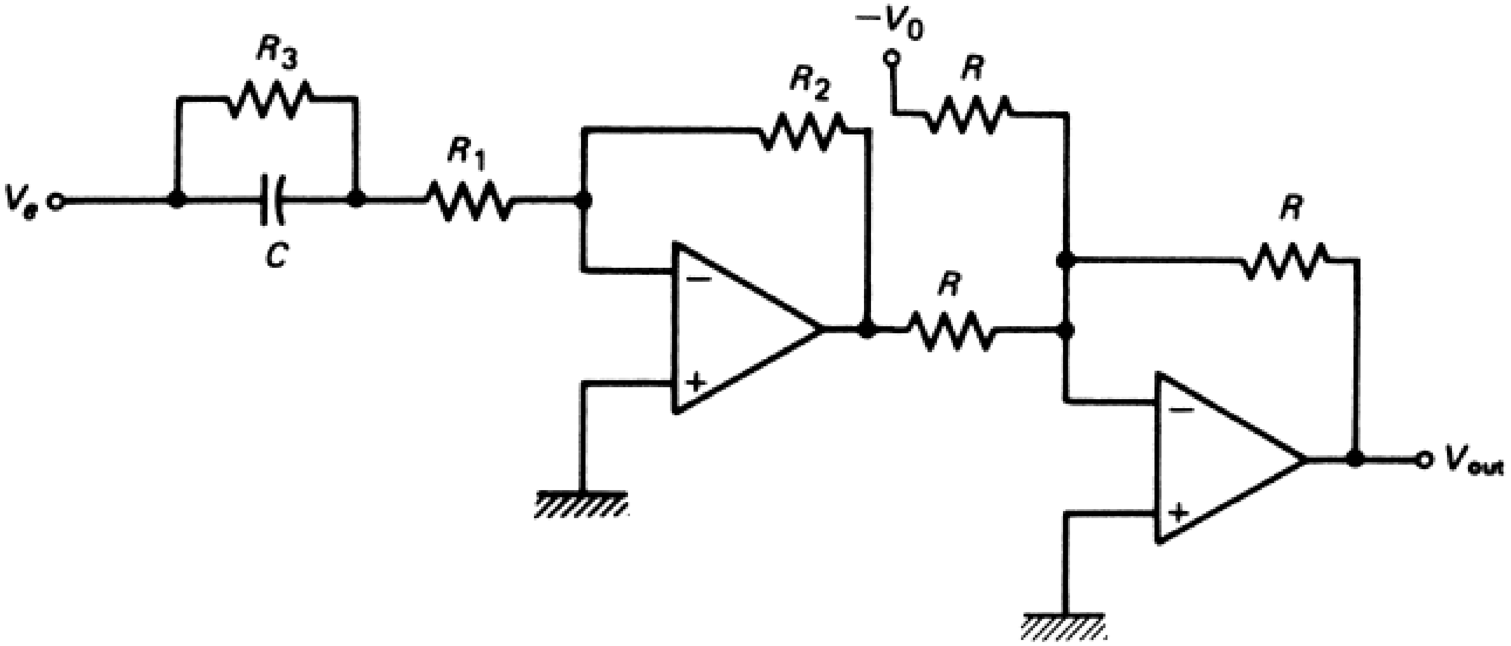
and the *integration gain* through $G_I = 1/R_2C$.

$$V_{\text{out}} = \left(\frac{R_2}{R_1}\right) V_e + \left(\frac{R_2}{R_1}\right) \frac{1}{R_2 C} \int_0^t V_e \, dt + V_{\text{out}}(0)$$

Proportional Derivative Mode of controller

- PD controller is the combination of proportional and derivative mode of controllers. The general definition of PD controller is:

$$P(t) = K_p e_p + K_p K_D \frac{de_p}{dt} + P(0)$$



$$R = \frac{R_1 R_3}{R_1 + R_3}$$

$$V_{out} = \left(\frac{R_2}{R_1 + R_3}\right) V_e + \left(\frac{R_2}{R_1 + R_3}\right) R_3 C \frac{dV_e}{dt} + V_0$$

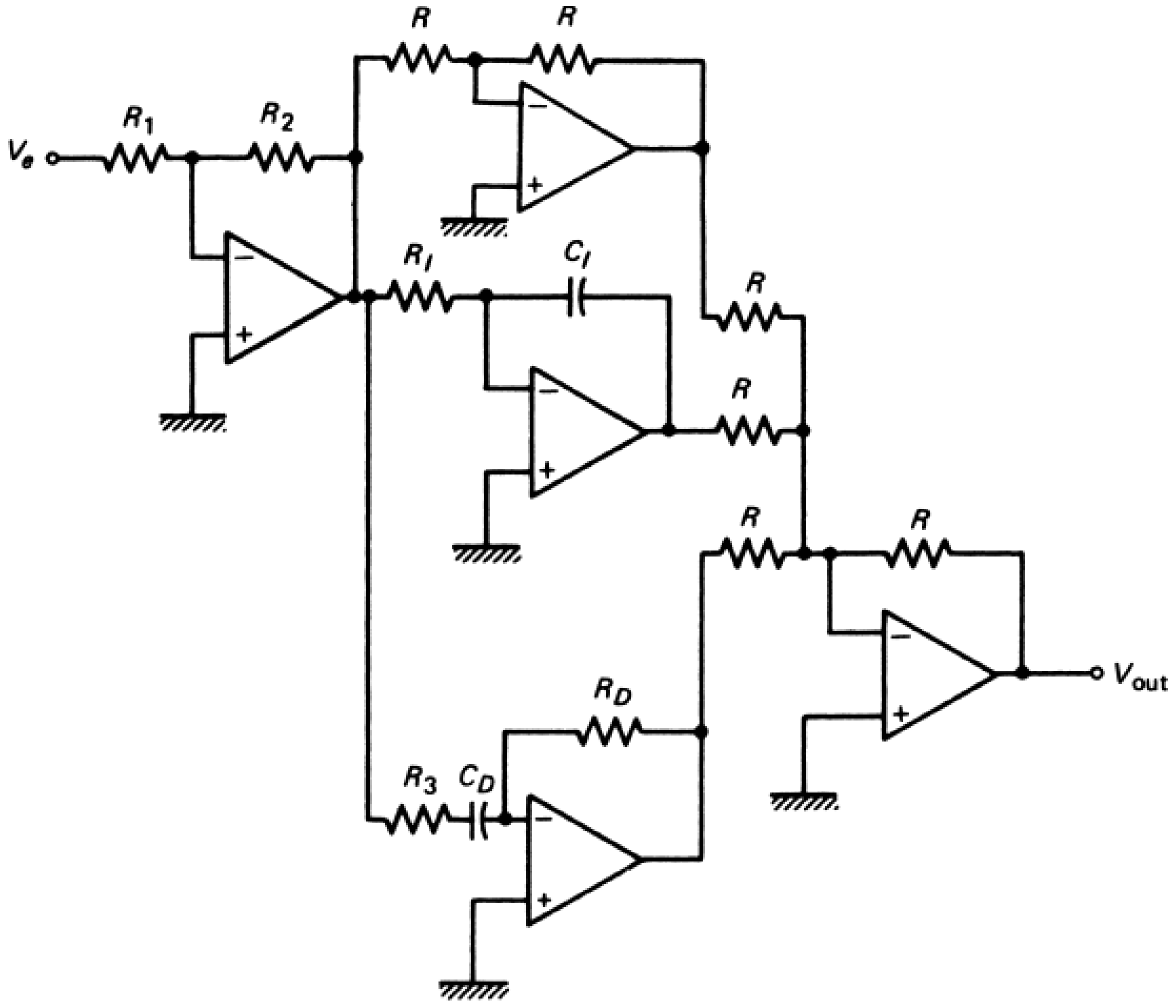
$$2\pi f_{max} RC = 0.1.$$

$$G_p = R_2/(R_1 + R_3),$$

$$G_D = R_3 C.$$

PID (Three-Mode)

$$p = K_p e_p + K_p K_I \int_0^t e_p \, dt + K_p K_D \frac{de_p}{dt} + p_I(0)$$



$$V_{p1} = -\frac{R_2}{R_1} V_e$$

$$V_p = \frac{R_2}{R_1} V_e$$

$$V_I = -\frac{1}{R_I C_I}\int_0^t V_{p1}\,dt \quad \text{or,} \quad V_I = \frac{R_2}{R_1}\frac{1}{R_I C_I}\int_0^t V_e\,dt$$

$$V_D = -R_D C_D \frac{d}{dt} V_{p1} \quad \text{or,} \quad V_D = \frac{R_2}{R_1} R_D C_D \frac{d}{dt} V_e$$

$- V_{out} = V_p + V_I + V_D$

$$-V_{out} = \left(\frac{R_2}{R_1}\right) V_e + \left(\frac{R_2}{R_1}\right)\frac{1}{R_I C_I}\int V_e\,dt + \left(\frac{R_2}{R_1}\right) R_D C_D \frac{dV_e}{dt} + V_{out}(0)$$

$$G_p = \left(\frac{R_2}{R_1}\right),\ G_I = \left(\frac{1}{R_I C_I}\right) \quad \text{and} \quad G_D = R_D C_D$$

$$-V_{out} = G_p V_e + G_p G_I \int V_e\,dt + G_p G_D \frac{dV_e}{dt} + V_{out}(0)$$

Adding an inverter at the output stage,

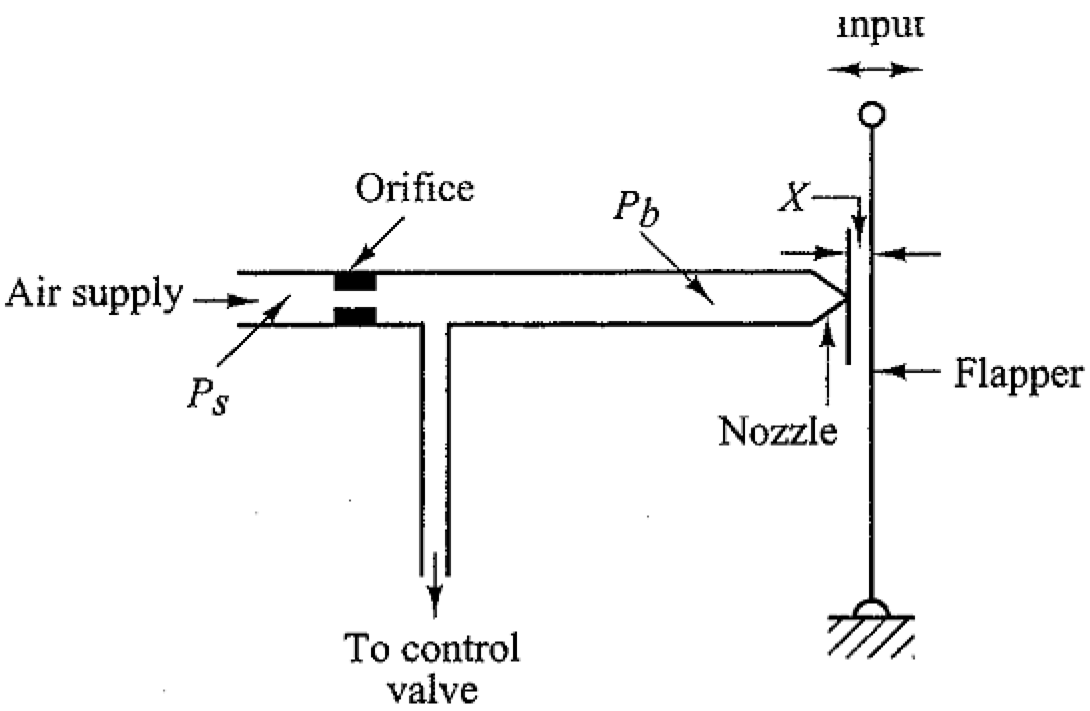
$$V_{out} = G_p V_e + G_p G_I \int V_e\,dt + G_p G_D \frac{dV_e}{dt} + V_{out}(0)$$

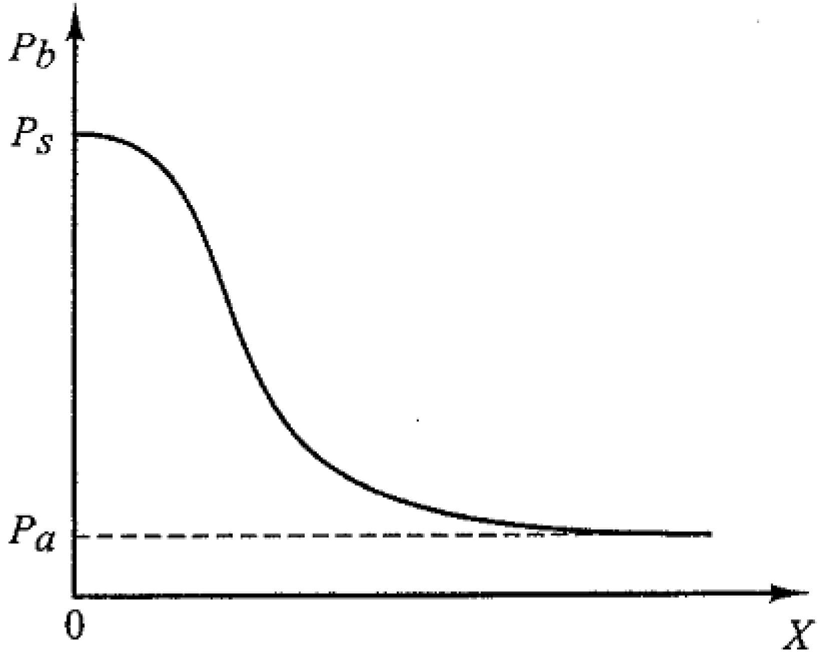
Pneumatic controllers

- Reason for using pneumatic controllers:
- 1. Competitive in cost and reliability
- 2. Safety (Danger of explosion from electrical malfunctions exists.)
- 3. Final control element is often pneumatically or hydraulically operated, which suggests that an all pneumatic process control loop might be advantageous.
- It appears that analog or digital electronic methods will eventually replace most pneumatic installations. But we will still have pneumatic equipment for many years until these are depreciated in industry

Pneumatic Nozzle-Flapper systems. (Nozzle/baffle)

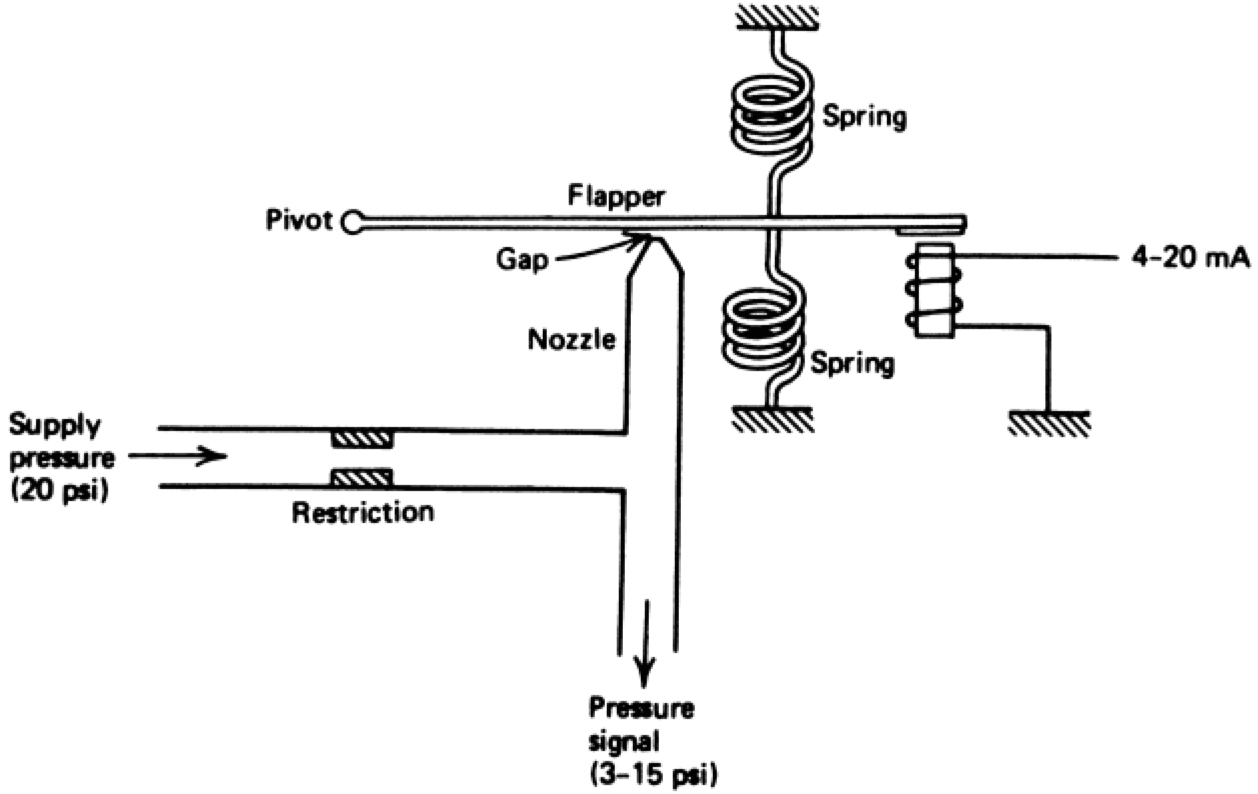
- signal conversion is from pressure to mechanical motion and vice versa





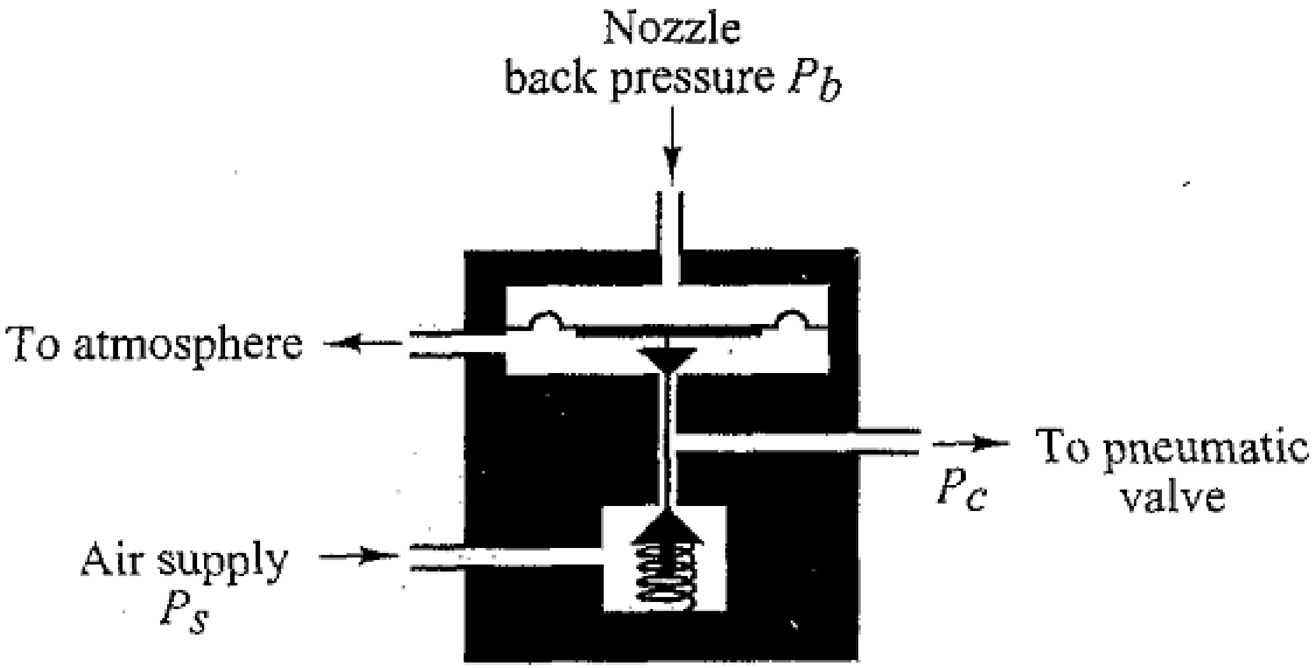
Using a nozzle/flapper for a current-to-pressure converter

- A linear way of translating the 4- to 20-mA current into a 3- to 15-psig signal

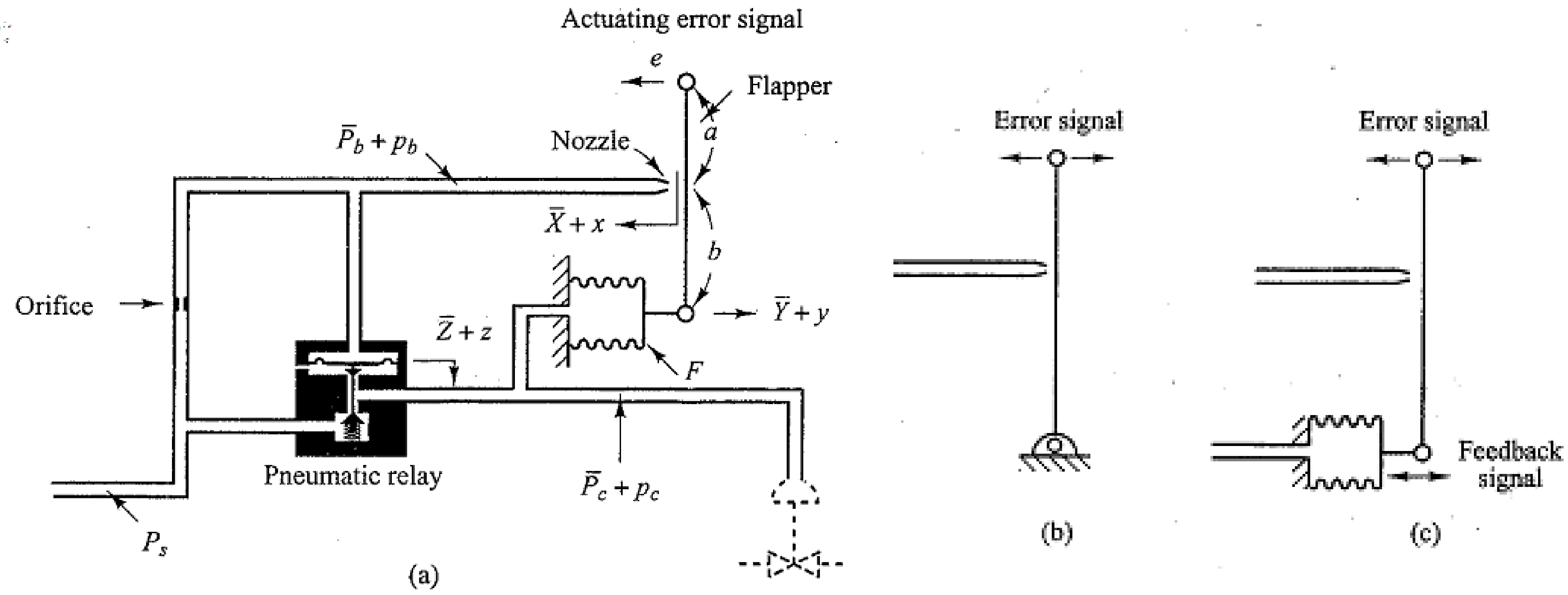


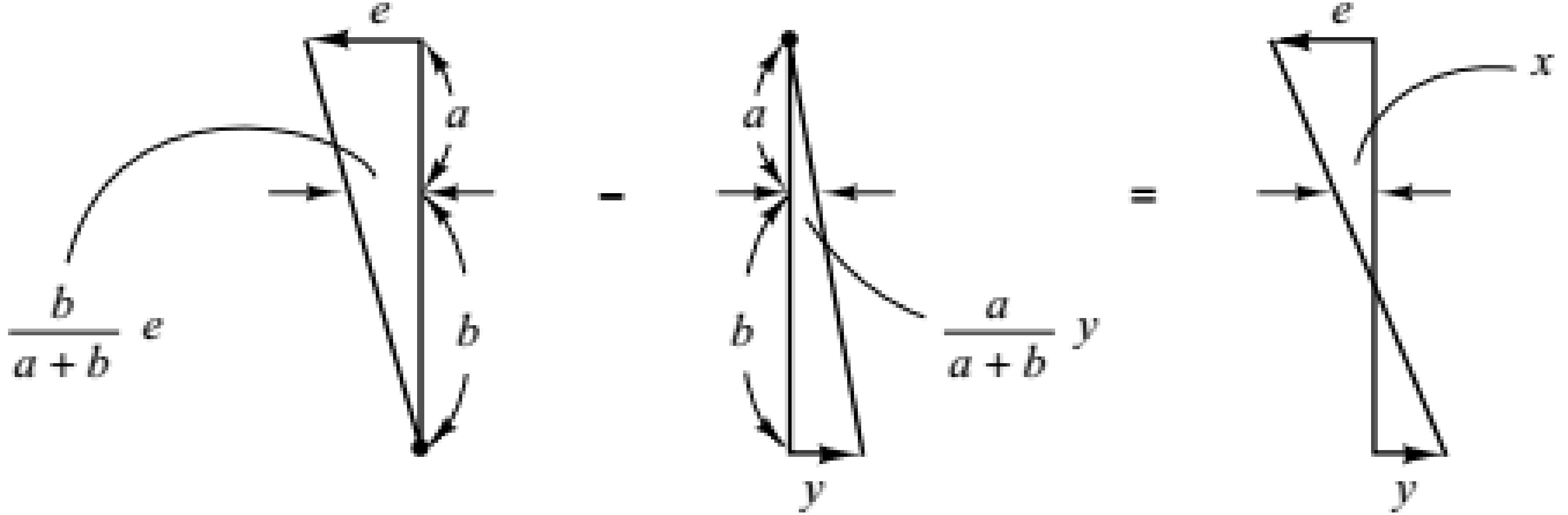
Pneumatic Relays

- In a pneumatic controller, a nozzle-flapper amplifier acts as the first-stage amplifier and a pneumatic relay as the second-stage amplifier.



Pneumatic Proportional Controllers (Force-Distance Type)




e
a
b
$\frac{b}{a+b}e$
−
a
b
$\frac{a}{a+b}y$
y
=
e
x
y

$$p_c = K_3 z$$

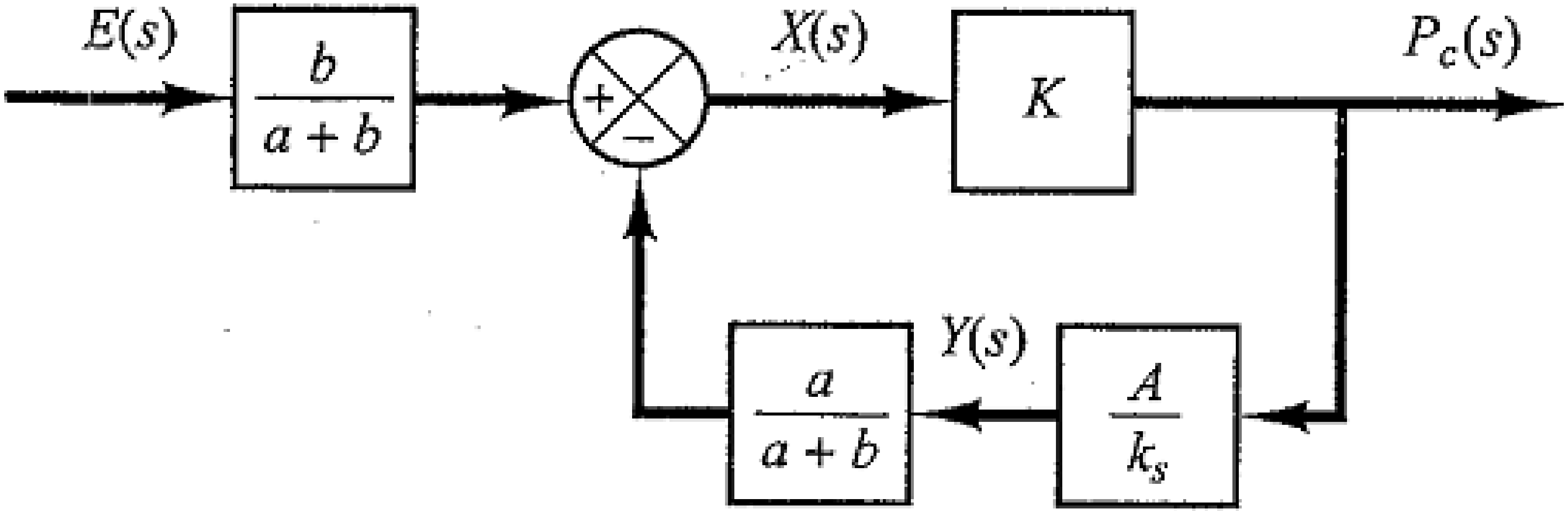
$$p_c = \frac{K_3}{K_2} p_b = \frac{K_1 K_3}{K_2} x = Kx$$

$$x = \frac{b}{a+b} e - \frac{a}{a+b} y$$

$$A p_c = k_s y$$

$$\frac{P_c(s)}{E(s)} = \frac{\frac{b}{a+b} K}{1 + K \frac{a}{a+b} \frac{A}{k_s}} = K_p$$

Block diagram for the controller



Pneumatic two-position controllers or pneumatic on–off controllers

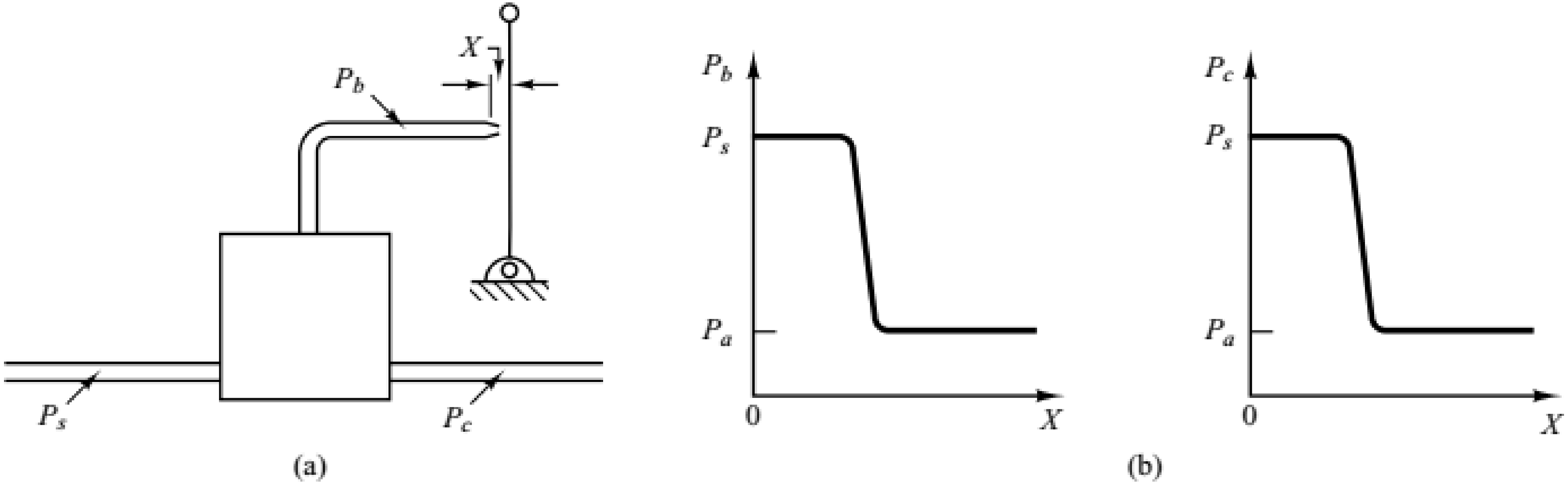
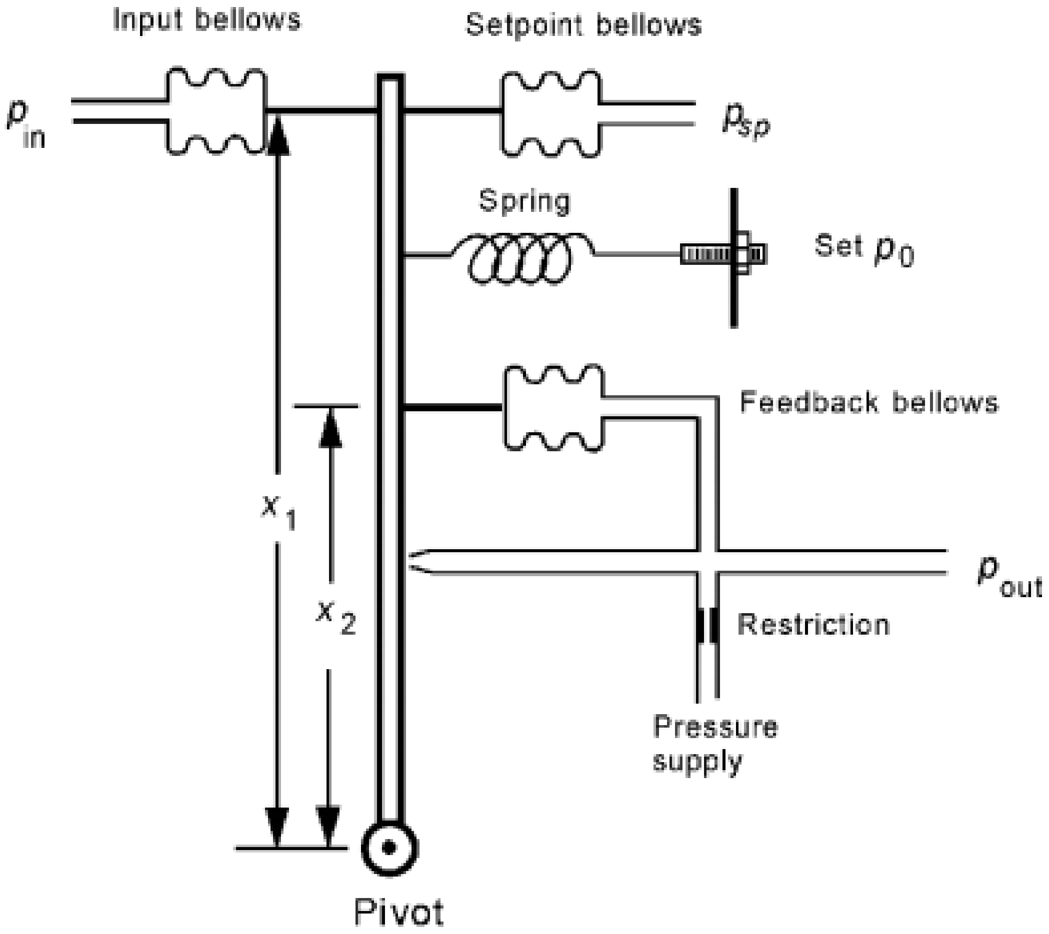


Figure 4–9
(a) Pneumatic controller without a feedback mechanism; (b) curves P_b versus X and P_c versus X.

Second implementation of **Proportional Controllers**



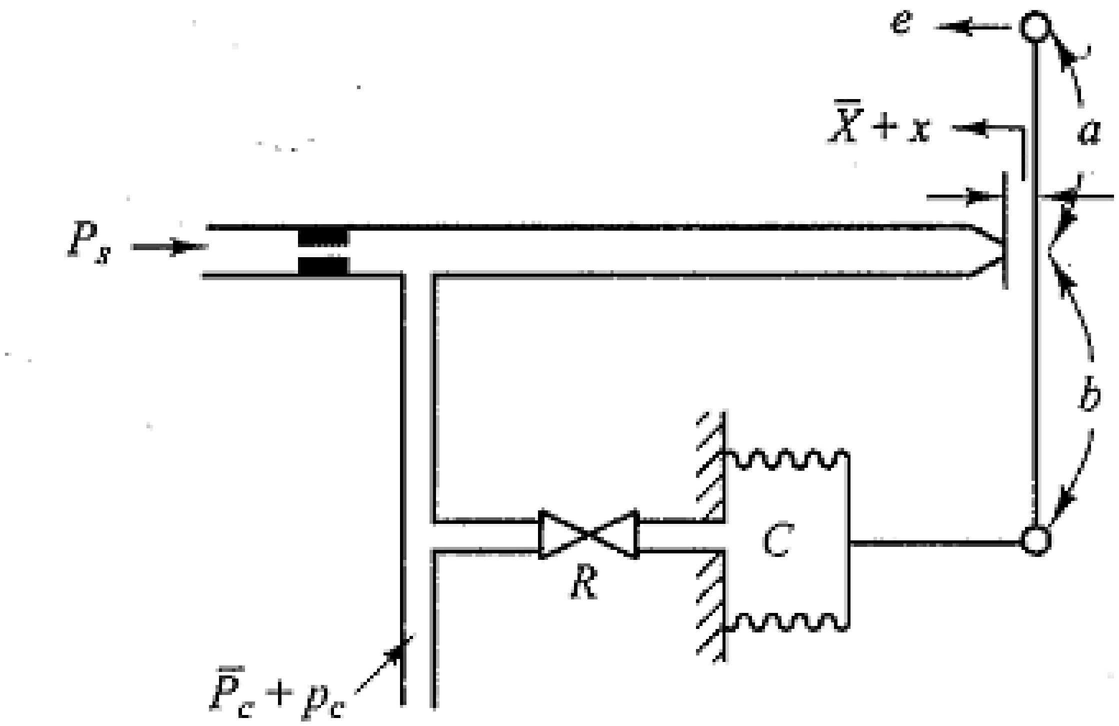
$$(p_{\text{out}} - p_0)A_2x_2 = (p_{\text{in}} - p_{sp})A_1x_1$$

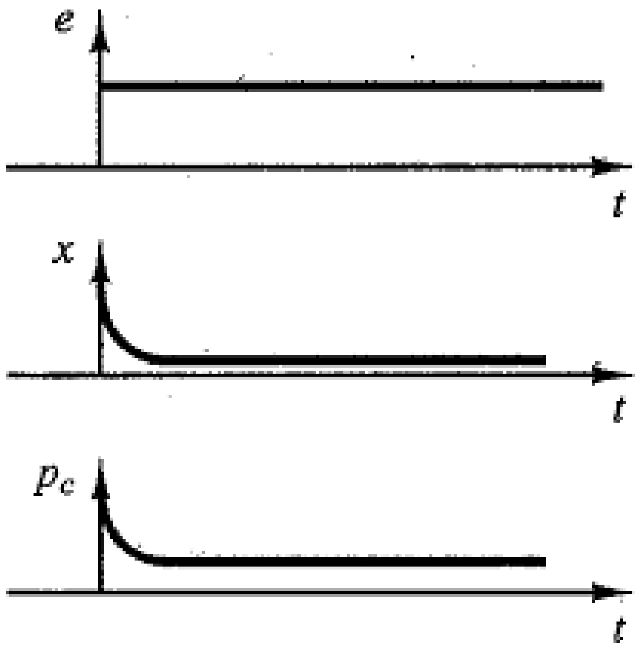
$$p_{\text{out}} = \frac{x_1}{x_2}\frac{A_1}{A_2}(p_{\text{in}} - p_{sp}) + p_0$$

p_0 = pressure with no error
p_{in} = input pressure (Pa)
A_1 = input and setpoint bellows effective are
x_1 = level arm of input (m)
p_{out} = output pressure (Pa)
A_2 = feedback bellows effective area (m^2)
x_2 = feedback lever arm (m)
p_{sp} = setpoint pressure

$$K_p = \left(\frac{x_1}{x_2}\right)\left(\frac{A_1}{A_2}\right)$$

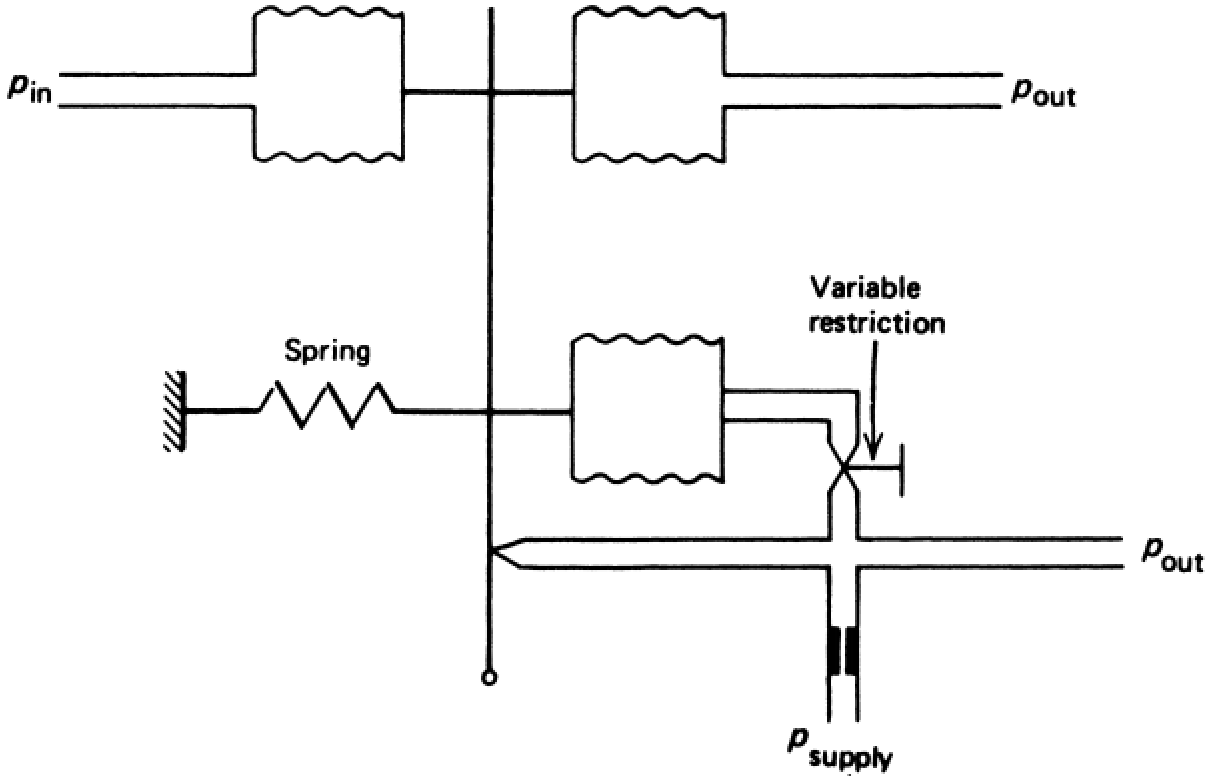
PD Mode





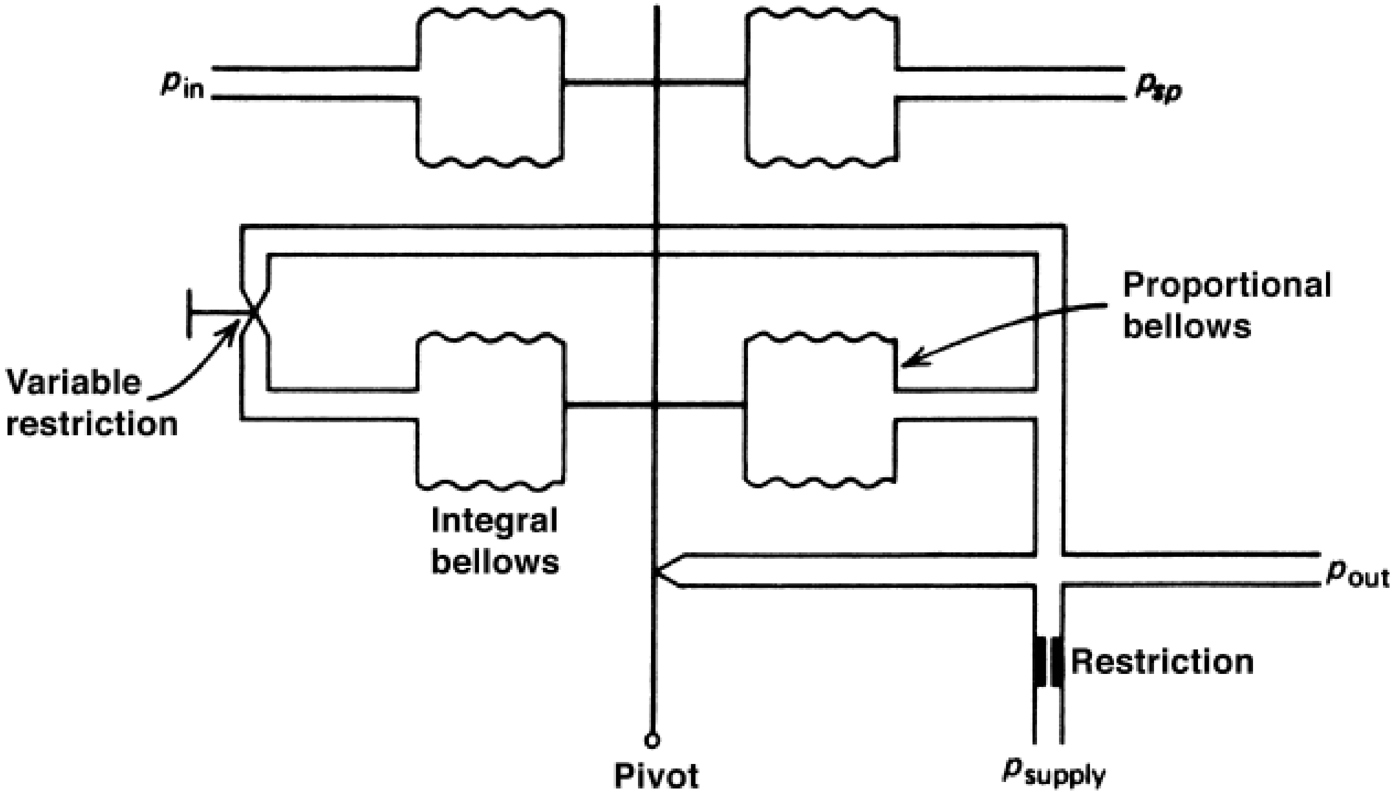
$$\frac{P_c(s)}{E(s)} = \frac{\dfrac{b}{a+b}K}{1. + \dfrac{Ka}{a+b}\dfrac{A}{k_s}\dfrac{1}{RCs+1}}$$

Pneumatic **proportional-derivative** controller.

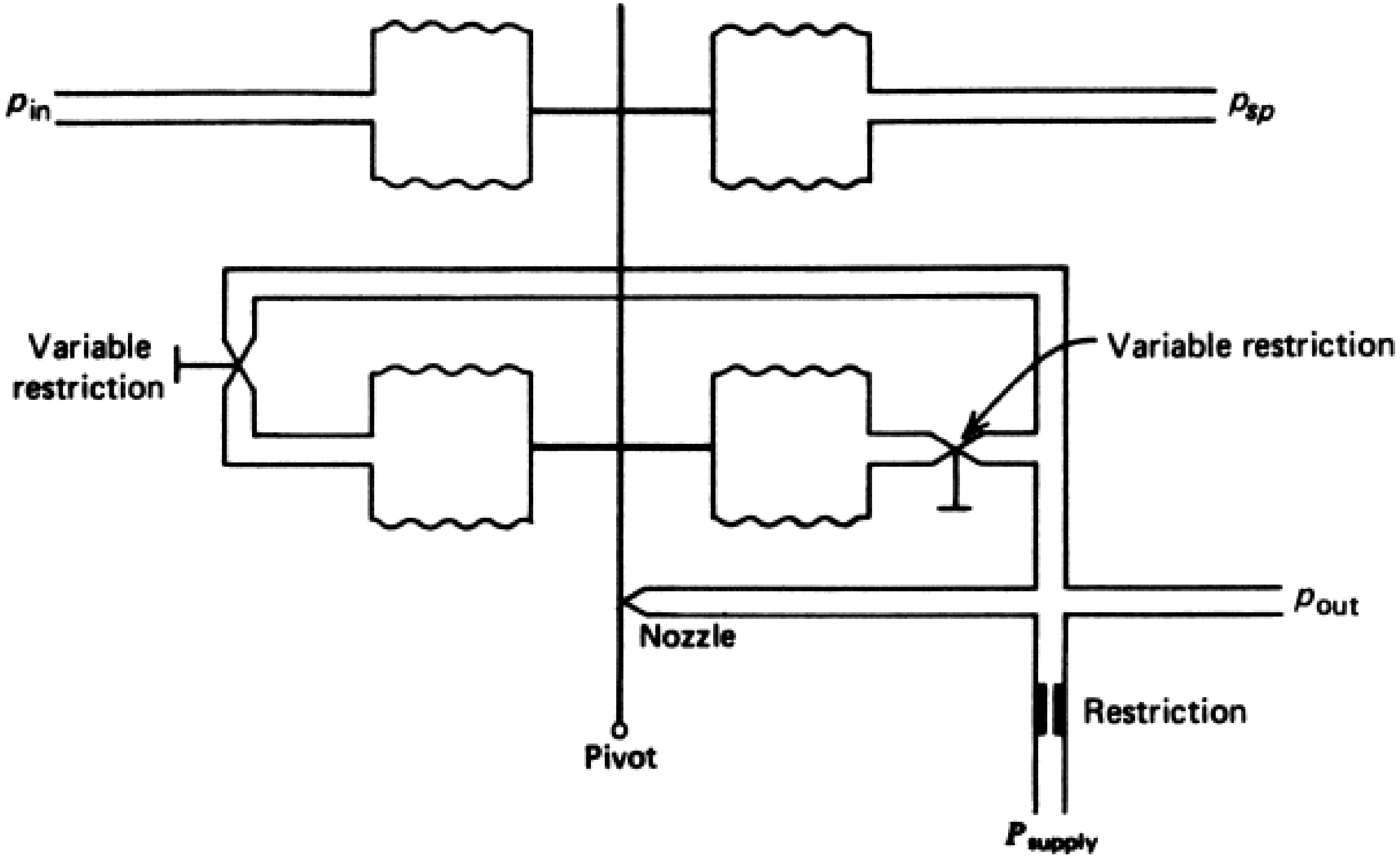


Proportional-Integral

- An extra bellows with a variable restriction is added to the proportional system.
- The integral bellows slowly moves the Flapper closer to the nozzle, thereby causing a steady increase in output pressure (as dictated by the integral mode).



Three-Mode



p_{in}
p_{sp}
Variable restriction
Variable restriction
p_{out}
Nozzle
Restriction
Pivot
P_{supply}