Deformable bodies

MECHANICS OF DEFORMABLE BODIES

LECTURE 14

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Introduction

Strength of materials or mechanics of materials involves analytical methods for determining the **strength**, **stiffness** (deformation characteristics), and **stability** of various load carrying members.

Strength is the ability of the structure to resist the influence of the external forces acting upon it.

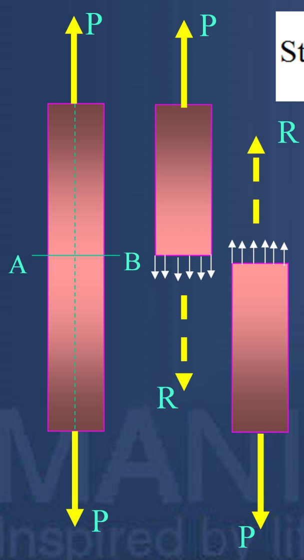
Stiffness is the ability of the structure to resist the strains caused by the external forces acting upon it.

Stability is the property of the structure to keep its initial position of equilibrium.

Mechanical properties of materials

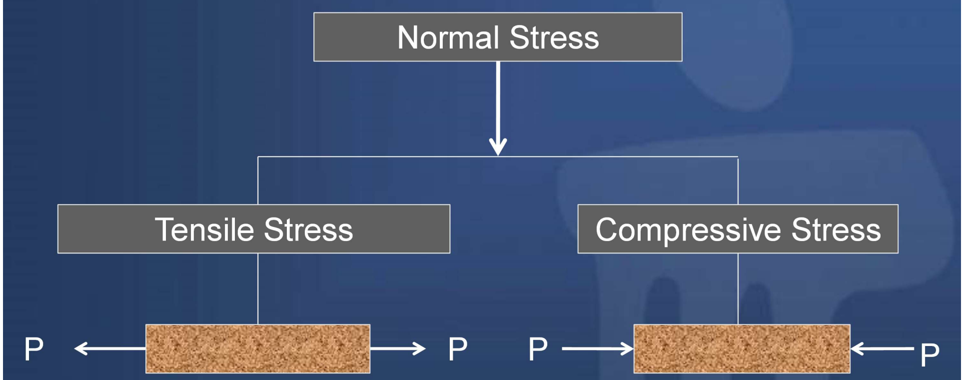
- Elasticity
- Plasticity
- Ductility
- Brittleness
- Malleability

Normal stress



$$\text{Stress} = \frac{\text{Internal resisting force}}{\text{Resisting cross sectional area}} = \frac{R}{A}$$

The body resists the deformation by developing stresses.



Units:

SI unit for stress is Pascal (Pa)

Pa= N/m²

	N/m^2	N/mm^2
1kPa	10^3	10^{-3}
1MPa	10^6	1
1GPa	10^9	10^3

Kilopascal, 1kPa= 1000 N/m^2

Megapascal , 1MPa = 1x 10^6 N/m^2

= 1x 10^6 $N/(10^6\ mm^2)$=1N/mm^2

1MPa = 1 N/mm^2

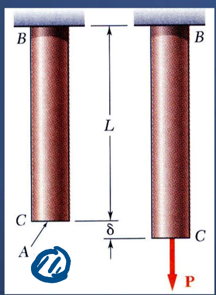
Gigapascal, 1GPa = 1x 10^9 N/m^2

= 1x 10^3 MPa

=1x 10^3 N/mm^2

STRAIN

$$\varepsilon = \frac{\delta L}{L} = \frac{\text{Change in the length}}{\text{Original length}}$$



$$\sigma = \frac{P}{A} = \text{stress}$$

$$\varepsilon = \frac{\delta}{L} = \text{Linear strain}$$

Hooks law and Modulus of elasticity

Hooks law:

$$\frac{Stress(\sigma)}{Strain(\varepsilon)} = \text{constant}$$

Modulus of elasticity:

$$\frac{Stress(\sigma)}{Strain(\varepsilon)} = \frac{\text{PL}}{\text{Adl}}$$

The following table shows modulus of elasticity of important materials:

Material	Modulus of elasticity
Steel	210 GPa
Aluminium	73Gpa
Brass	96 – 110 GPa
Cast Iron	83 – 170 GPa
Concrete	17 – 31 GPa
Rubber	0.0007 – 0.004 GPa
Tungsten	340 – 380 GPa

Tension test on ductile and brittle material
Factor of safety
Allowable stress

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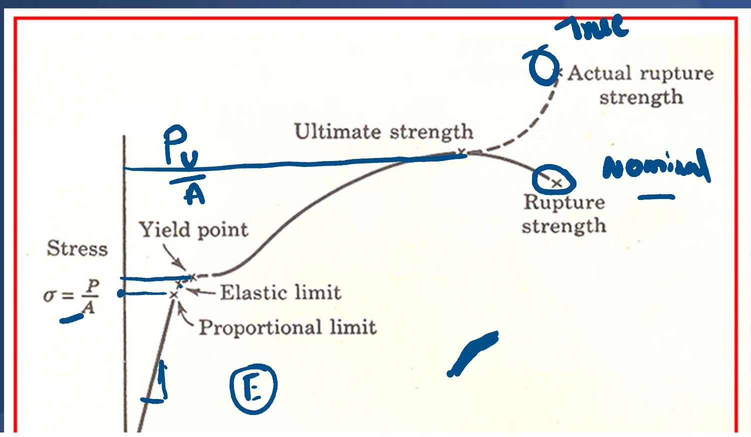
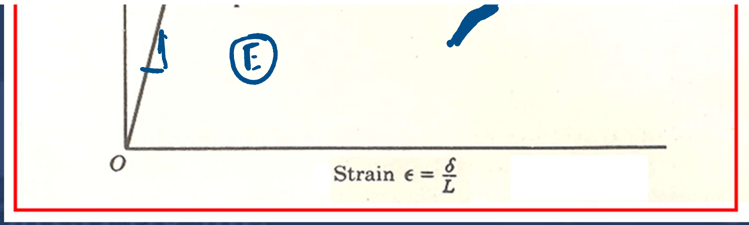
Tension test on ductile and brittle material

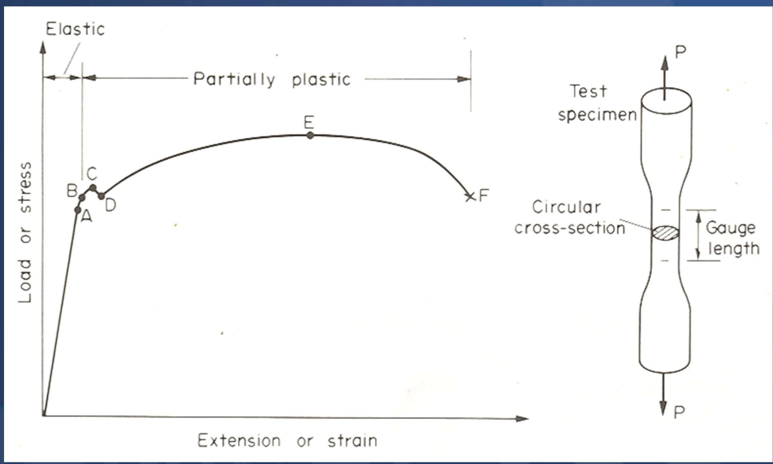
STRESS-STRAIN DIAGRAM





Typical tensile test curve for mild steel

STRESS-STRAIN DIAGRAM



Typical tensile test curve for mild steel showing upper yield point and lower yield point and also the elastic range and plastic range

Limit of Proportionality: $\sigma_P = \frac{\text{Load at proportionality limit}}{\text{Original crosssectional area}} = \frac{P_P}{A}$

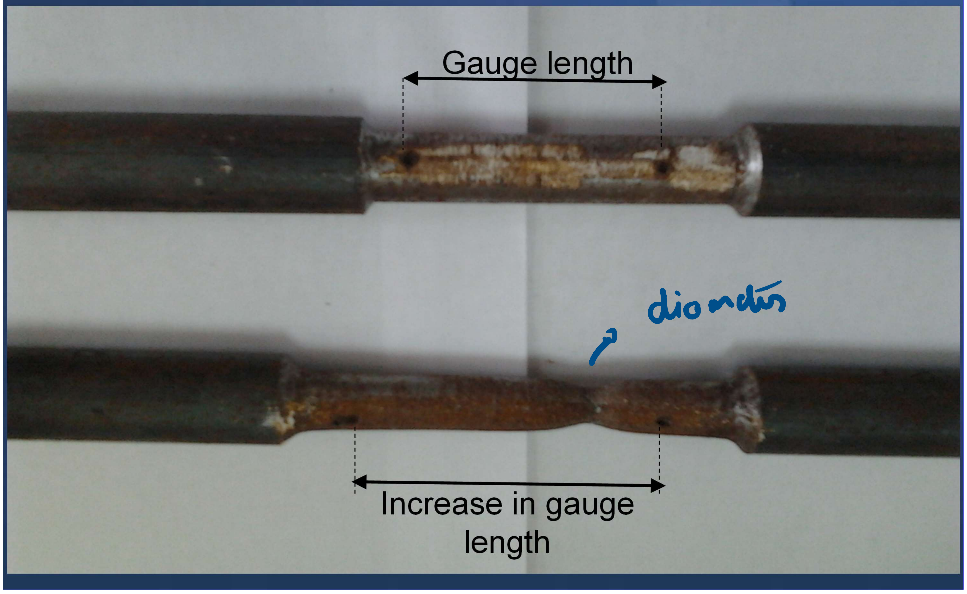
Elastic limit: $\sigma_E = \frac{\text{Load at elatic limit}}{\text{Original cross sectional area}} = \frac{P_E}{A}$

Yield point: $\sigma_Y = \frac{\text{Load at yield point}}{\text{Original cross sectional area}} = \frac{P_Y}{A}$

Ultimate strength: $\sigma_U = \frac{\text{Maximum load taken by the material}}{\text{Original cross sectional area}} = \frac{P_U}{A}$

Rupture strength (Nominal Breaking stress): $\sigma_B = \frac{\text{Load at failure}}{\text{Original cross sectional area}} = \frac{P_B}{A}$

True breaking stress: $\sigma_B = \frac{\text{Load at failure}}{\text{Actual cross sectional area}} = \frac{P_B}{A}$



Ductile Materials

Percentage elongation
Percentage reduction in area

} Measures of ductility

Cup and cone fracture for a Ductile Material ►

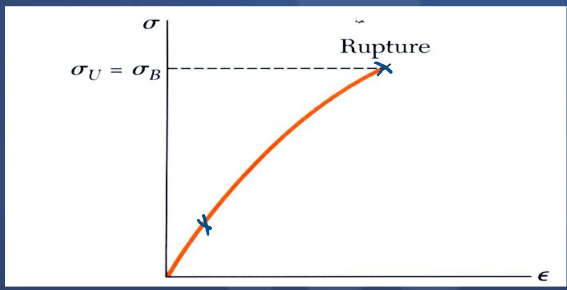
$$\text{Percentage elongation} = \frac{\text{Increase in the gauge length (upto fracture)}}{\text{Original gauge length}} \text{x}100$$

$$\text{Percentage reduction in area} = \frac{\text{Reduction in cross sectional area of neck portion (at fracture)}}{\text{Original cross sectional area}} \text{x}100$$

Example: Low carbon steel, mild steel, gold, silver, aluminum

Stress-strain Diagram

Brittle Materials :



Stress-strain diagram for a typical brittle material

Working stress & Factor of safety

Ductile Material:

Working stress = Yield Stress / Factor of Safety

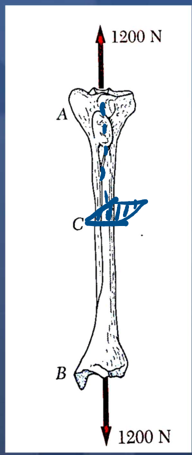
Brittle Material:

Working stress = Ultimate Stress / Factor of Safety

Factor of Safety = Maximum stress / Allowable working stress

Numerical

N1. A strain gauge located at C on the surface of bone AB indicates that the average normal stress in the bone is 3.80 MPa when the bone is subjected to two 1200 N forces as shown. Assuming the cross section of the bone at C to be annular and Knowing that its outer diameter is 25mm, determine the inner diameter of the bones cross section at C.



(N), (mm)

$\sigma = 3.8\ MPa$

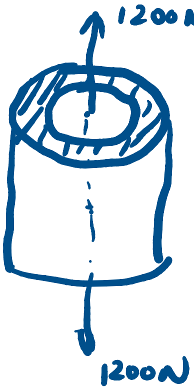
$\sigma = 3.8\ N/mm^2$

$\sigma = 3.8 \times 10^6\ N/m^2$

$P = 1200N$

$d_e = 25mm$

$d_i = ?$



$A = \frac{\pi}{4}(d_e^2 - d_i^2)$ $P = 1200N$, $\sigma = 3.8MPa$

$\sigma = \frac{P}{A} \Rightarrow 3.8N/mm^2 = \frac{1200N}{\frac{\pi}{4}(25^2 - d_i^2)}$

$\boxed{d_i = 14.93\ mm}$

T(+)

C(-)

N2 A composite bar consists of an aluminum section rigidly fastened between a bronze section and a steel section as shown in figure. Axial loads are applied at the positions indicated. Determine the stress in each section. Also determine the change in each section and the change in total length.

$E_B = 100GPa$

$E_A = 70GPa$

$E_S = 210GPa$

$\delta l_B = ?$

$\delta l_A = ?$

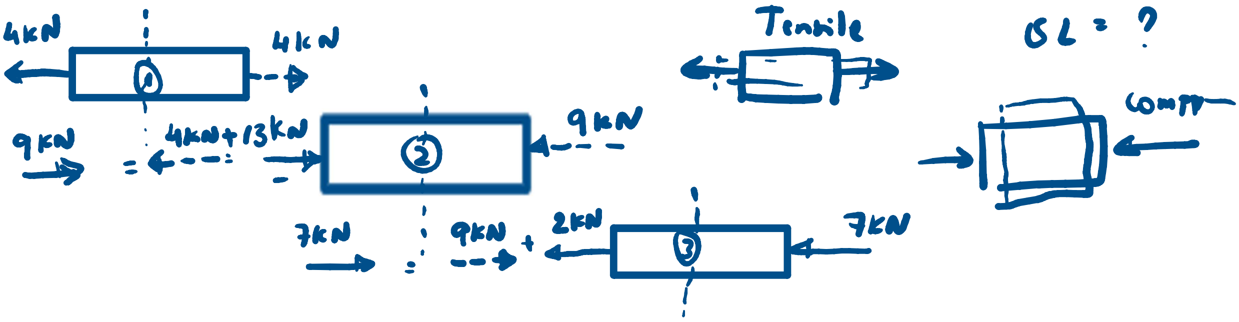
300mm 400mm 500mm

Inspired by life

$\delta l_A = ?$

$\delta l_s = ?$

$\delta L = ?$



$$\sigma_B = \frac{P_B}{A_B} = \frac{4000\,N}{120\ mm^2} = +33.33\ N/mm^2 \text{ (or) } 33.33\ MPa \text{ (Tensile)}$$

$$\sigma_A = \frac{P_A}{A_A} = \frac{-9000}{180} = -50\ N/mm^2 \text{ (or) } 50\ MPa \text{ (compressive)}$$

$$\sigma_S = \frac{P_S}{A_S} = \frac{-7000}{160} = -43.75\ N/mm^2 \text{ (or) } 43.75\ MPa \text{ (compressive)}$$

Change in length :-

$$\delta l = \frac{PL}{AE} = \frac{\sigma L}{E} \qquad \delta l_B = \frac{\sigma_B L_B}{E_B} = \frac{33.33 \times 300}{100 \times 10^3}$$

$$E = 100\ GPa = 100 \times 10^9\ N/m^2 = 100 \times 10^3 \times 10^6\ N/m^2$$
$$= 100 \times 10^3\ N/mm^2$$

$$\boxed{\delta l_B = 0.1\ mm}$$

$$\delta l_A = \frac{\overset{(-)}{\sigma_A} L_A}{E_A} = \frac{-50 \times 400}{70 \times 10^3} = -0.285\ mm$$

$$\delta l_s = \frac{\overset{(-)}{\sigma_s} l_s}{E_s} = \frac{-43.75 \times 500}{210 \times 10^3} = -0.104\ mm$$

Total change in length $\delta l = \delta l_B + \overset{(-)}{\delta l_A} + \overset{(-)}{\delta l_s} = -0.29\ mm$

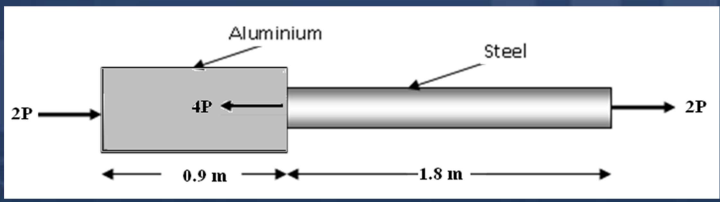
$P = ?$

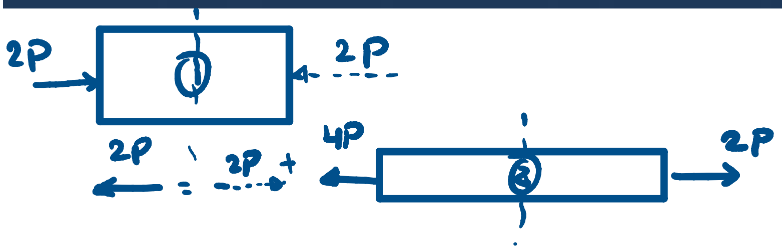
N3. An aluminum rod is fastened to a steel rod as shown. Axial loads are applied at the positions shown. The area of cross section of aluminum and steel rods are 400 mm² and 200mm² respectively. Find maximum value of P that will satisfy the following conditions.

a) $\sigma_s \leq 140$ MPa
b) $\sigma_a \leq 80$ MPa
c) Total elongation ≤ 0.5 mm,

Take $E_a = 70$ GPa and $E_s = 210$ GPa

$P = ?$





a) $\sigma_s = 140$ MPa

$\frac{P_s}{A_s} = 140 \text{ N/mm}^2 \Rightarrow \frac{2P}{200} = 140 \Rightarrow \boxed{P = 14000 \text{ N}}$ (T)

14 kN (T)

b) $\sigma_a = 80$ MPa

$\boxed{P = 16000 \text{ N (C)}}$ or 16 kN (C)

c) $\delta L = 0.5$ mm

$-\delta l_a + \delta l_s = 0.5 \Rightarrow -\frac{2P(900)}{400 \times 70 \times 10^3} + \frac{2P(1800)}{200 \times 210 \times 10^3} = 0.5$

$\boxed{P = 23.36 \text{ kN}}$

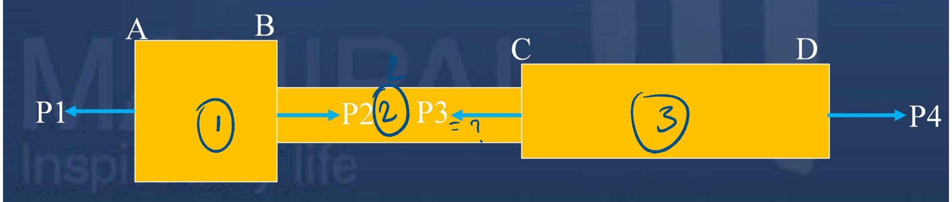
$\boxed{\text{Max P value can be 14 kN}}$

N4. A member ABCD is subjected to point loads P1, P2,P3 and P4 as shown in figure below.
Calculate the force P3 necessary for equilibrium if P1 = 120

$\xrightarrow{}$ +ve

$\Sigma F = 0$

N4. A member ABCD is subjected to point loads P1, P2, P3 and P4 as shown in figure below.
Calculate the force P3 necessary for equilibrium if P1 = 120 kN, P2 = 220 kN and P4 = 160 kN.
Determine the net change in the length of the member. Take E = 200 GN/m^2.
Given: area and length of AB: 1600 mm^2, 0.75 m ;
area and length of BC: 625 mm^2, 1.0 m;
area and length of CD: 900mm^2, 1.2 m.

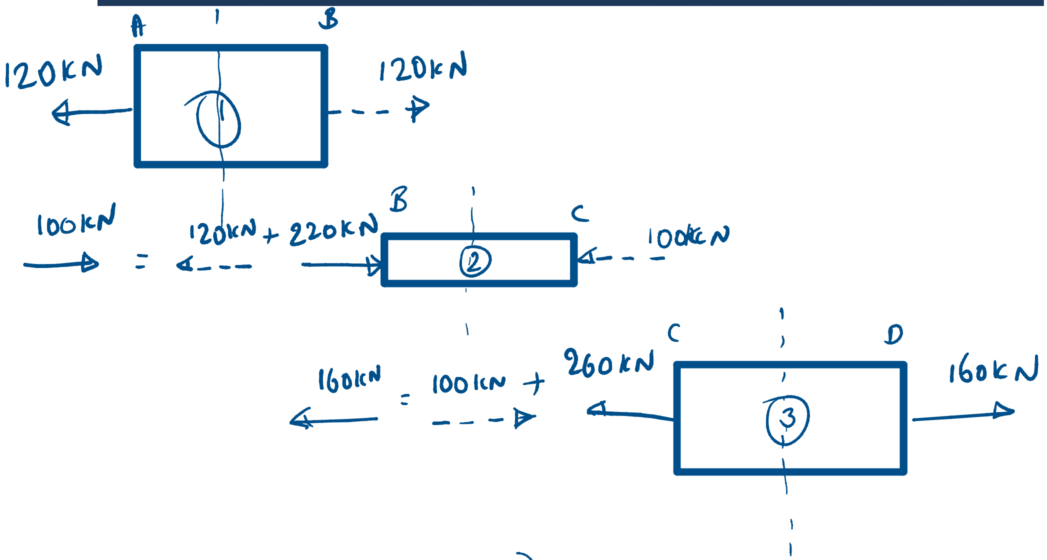


$$\sum F = 0$$

$$-P_1 + P_2 - P_3 + P_4 = 0$$

$$-120 + 220 - P_3 + 160 = 0$$

$$\boxed{P_3 = 260 \text{ kN}}$$



$$\delta L = \delta l_{AB} + \overset{(-ve)}{\delta l_{BC}} + \delta l_{CD}$$

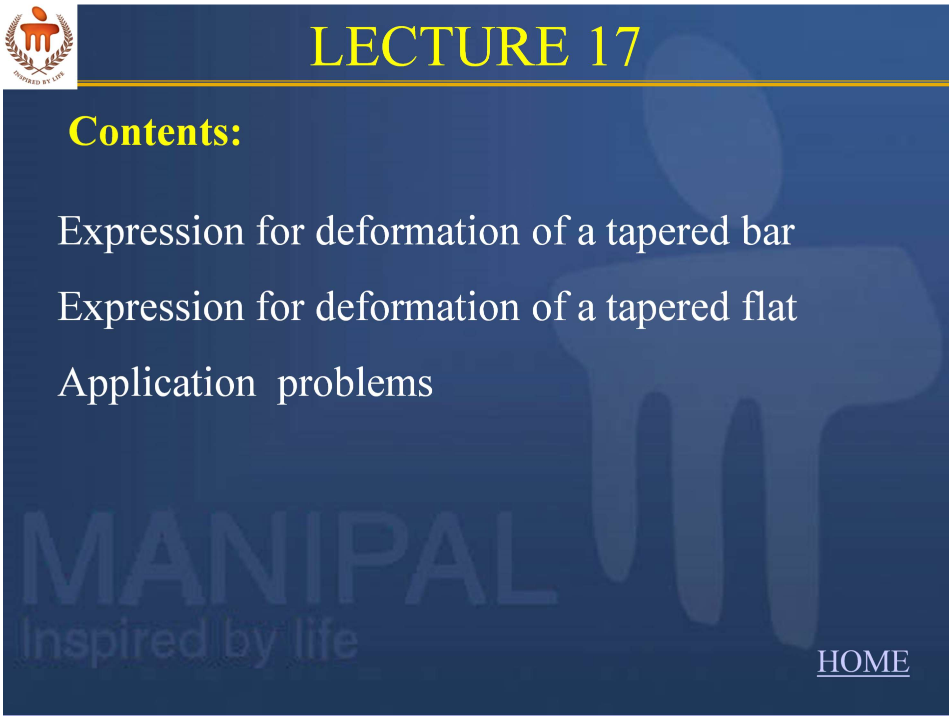
$$= \delta l_{AB} - \delta l_{BC} + \delta l_{CD}$$

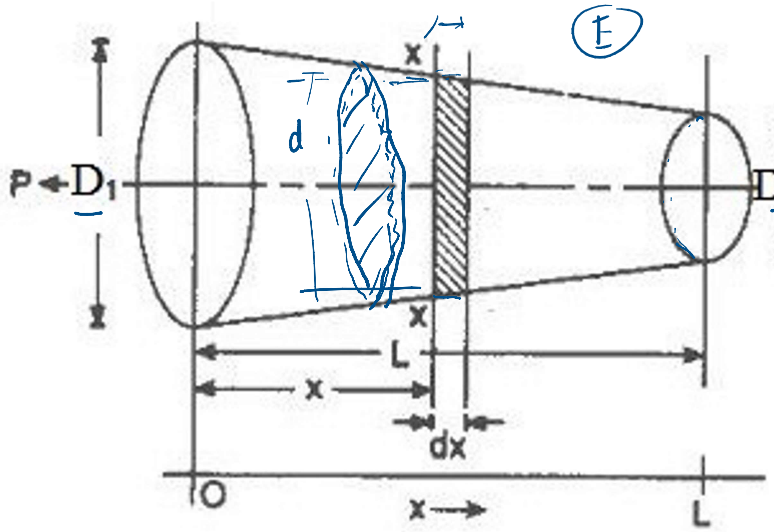
$$= \frac{\overset{(N)}{160 \times 10^3} \times \overset{(mm)}{750}}{1600 \times 200 \times 10^3} - \frac{100 \times 10^3 \times 1000}{625 \times 200 \times 10^3} + \frac{160 \times 10^3 \times 1200}{900 \times 200 \times 10^3}$$

$$\delta l = 0.28 - 0.8 + 1.07 = 0.55 \text{ mm}$$

$\boxed{\delta l = 0.55 \text{ mm}}$ (Elongation)

Tapered bar





Change in length in elemental strip

$$\delta l = \frac{PL}{AE}$$

$$\delta l = \frac{P\,dx}{\frac{\pi d^2}{4} \times E}$$

Total change in length

$$\delta l = \frac{4P}{\pi E}\int_0^L \frac{dx}{d^2}$$

$d = D_1 -$ Change in dia over x length

$$= D_1 - \underbrace{\left(\frac{D_1 - D_2}{L}\right)}_{K} x = D_1 - Kx$$

$\boxed{d = D_1 - Kx}$ $\quad \left(K = \frac{D_1 - D_2}{L}\right) \rightarrow$

$$\delta l = \frac{4P}{\pi E}\int_0^L \frac{dx}{(D_1 - Kx)^2}$$

$$\boxed{\delta l = \frac{4PL}{\pi D_1 D_2 E}}$$

$$\delta l = \frac{4P}{\pi E}\int_0^L (D_1 - Kx)^{-2}\,dx.$$

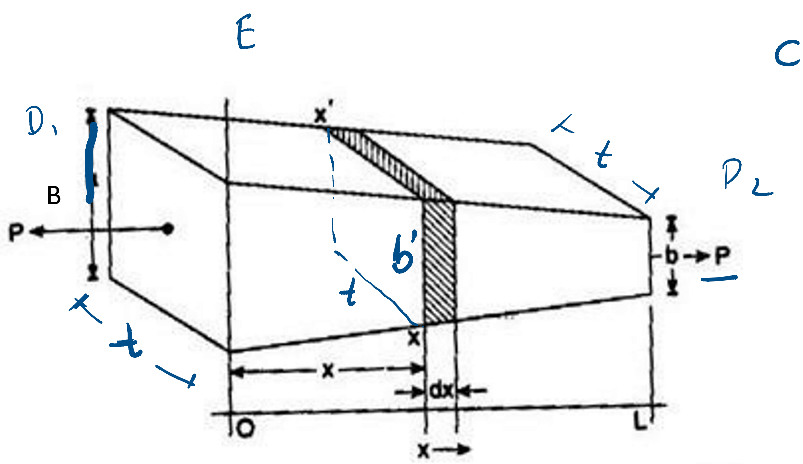
$$4P \left[(D_1 - Kx)^{-1}\right]^L$$

$$= \frac{4P}{\pi E}\left[\frac{(D_1 - kx)^{-1}}{-k(-1)}\right]_0^L$$

$$\delta L = \frac{4P}{\pi E k}\left(\frac{1}{D_1 - kx}\right)_0^L$$

$$\boxed{\delta L = \frac{4PL}{\pi E D_1 D_2}}$$

$$\delta L = \frac{PL}{\frac{\pi d^2}{4} y E}$$



Change in length in elemental area (δL)

$$\delta L = \frac{PL}{AE} = \frac{P\,dx}{b' t E}$$

Total change in length

$$\delta L = \frac{P}{tE}\int_0^L \frac{dx}{b'}$$

$b' = B -$ Change in dimension over x.

$$b' = B - \left(\frac{B-b}{L}\right)x = B - kx.$$

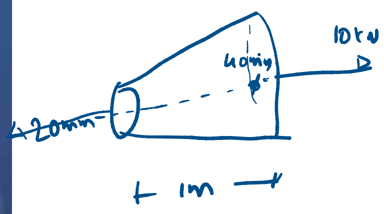
$$\delta L = \frac{P}{tE}\int_0^L \frac{dx}{(B-kx)}$$

$$\boxed{\delta L = \frac{PL}{tE(B-b)} \ln(B/b)} = \frac{2.302\,PL}{tE(B-b)} \log_{10}(B/b)$$

smallest

Largest dimension

N5. Find the modulus of elasticity of the material of a tapering bar from the following data: The bar has 20 mm diameter at one end, 40 mm diameter at the other, length 1.0 m and axial load of 10 kN. The elongation observed was 0.1 mm.



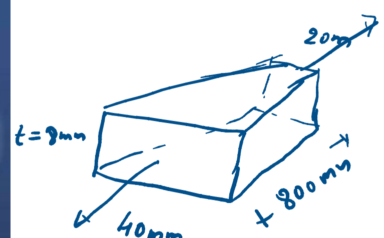
$\delta l = 0.1 mm$

$E = ?$

$$\delta l = \frac{4PL}{\pi E D_1 D_2} \Rightarrow \boxed{E = 159 \text{ GPa}}$$

$$0.1 = \frac{4 \times 10 \times 10^3 \times 1000}{\pi \times E \times 20 \times 40}$$

N6. A tapered bar of rectangular cross section is 20 mm wide at one end and 40 mm wide at the other, 8 mm thick and 800 mm long. The elongation of 0.08 mm was observed under load P. find the load P, if the modulus of elasticity of the material of the bar is 100 GPa.



$P = ?$

$\delta l = 0.08$

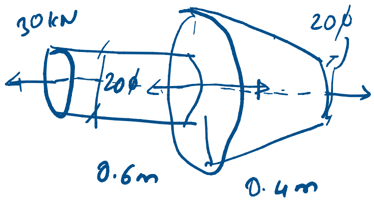
$E = 100 GPa$

$$\delta l = \frac{PL}{tE(B-b)} \ln\left(B/b\right) \Rightarrow 0.08 = \frac{P \times 800}{8 \times 100 \times 10^3 (40-20)} \ln\left(\frac{40}{20}\right)$$

$$\boxed{P = 2.308 \text{ kN}}$$

$\boxed{P = 2.308 \text{ kN}}$

N7. A uniform steel rod of diameter 20 mm is connected to an aluminium rod of diameter 60 mm at one end. The aluminium rod tapers to a diameter of 20 mm at the other end. The steel rod is 0.6 m long and is connected rigidly to 60 mm diameter end of the aluminium rod which is 0.4 m long. If E = 200 GPa for steel and 70 GPa for aluminium, find the total extension under an axial load of 30 kN.



$\delta L = ?$

$E_s = 200 \text{ GPa}$

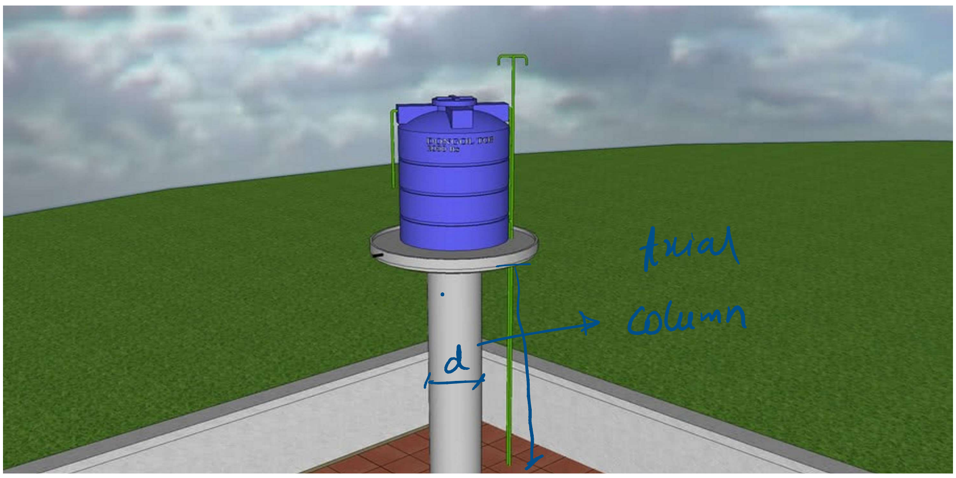
$E_a = 70 \text{ GPa}$

$$\delta L = \delta L_s + \delta L_a$$

$$= \left(\frac{PL}{AE}\right)_s + \left(\frac{4PL}{\pi E D_1 D_2}\right)_a$$

$$= \frac{30\times10^3 \times 600}{\frac{\pi (20)^2}{4} \times 200\times10^3} + \frac{4\times 30\times10^3 \times 400}{\pi \times 70\times10^3 \times 60 \times 20}$$

$$\boxed{\delta L = 0.47 \text{ mm}}$$



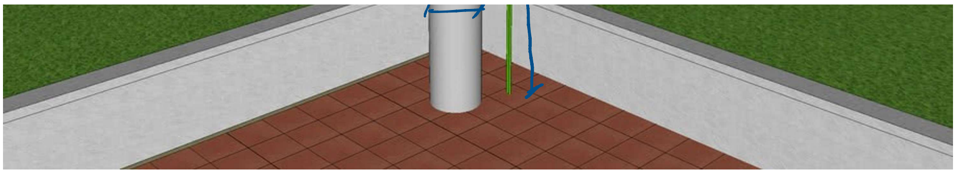
Assume

1000 litre 1000 kg

Steel

→ 250 N/mm² (Yield)

FOS = 2



$FOS = 2$

$\boxed{d = ?}$

Strength

Serviceablity

0.1mm

0.2 mm

$N \rightarrow \quad \underline{10 \times 10^3 N} = W \rightarrow \quad \underline{11 \times 10^3 N}$

Yield stress $\rightarrow$ FOS

$$\text{Allowable stress} = \frac{\text{Yield Stress}}{FOS} = \underline{125 MPa}$$

Factor of Safety

$$\sigma = \frac{P}{A}$$

$$125 = \frac{11 \times 10^3}{\frac{\pi d^2}{4}}$$

$\boxed{d = \qquad mm}$