ADVANTAGES OF DSP OVER ASP

Table 1.1 Comparison of analog and digital signal processing

Feature	Analog signal processing	Digital signal processing
Speed	Fast	Moderate
Cost	Low to moderate	Moderate
Flexibility	Low	High
Performance	Moderate	High
Self-calibrating	No	Yes
Data-logging capability	No	Yes
Adaptive capability	Limited	Yes

DSP APPLICATIONS

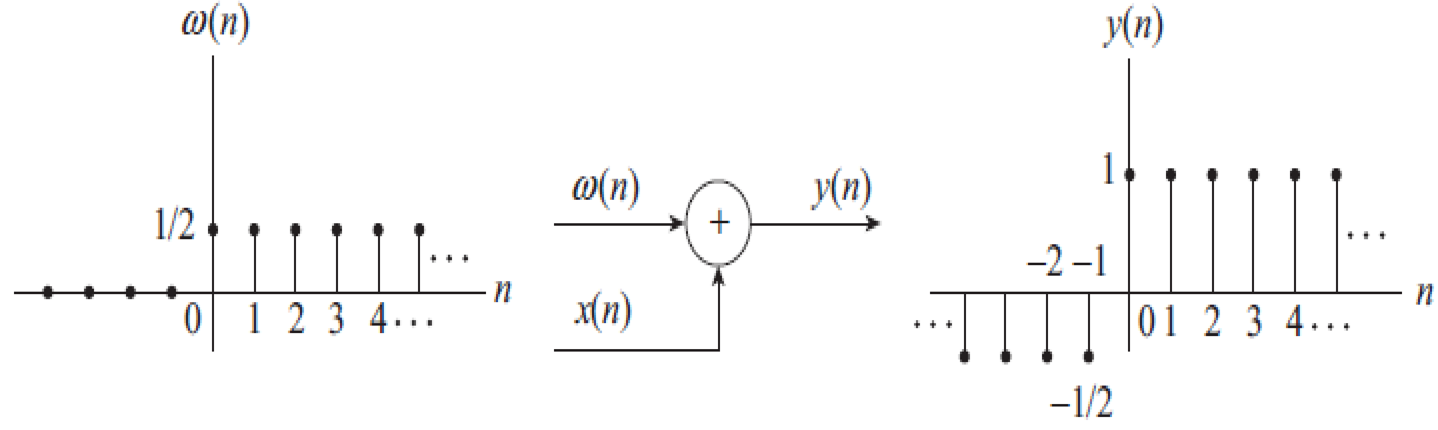
- **Speech/audio:** Speech recognition, speech synthesis, text to speech, digital audio, equalization
- **Instrumentation/control** Spectrum analysis, position and rate control, noise reduction
- **Image processing** Image enhancement, image compression, pattern recognition, satellite weather map, animation
- **Military** Secure communication, radar processing, sonar processing, missile guidance
- **Telecommunications** Echo cancellation, adaptive equalization, spread spectrum, video conferencing, data communication
- **Biomedical** EEG brain mappers, ECG analysis, X-ray storage/enhancement, scanners, patient monitoring
- **Consumer applications** Digital and, cellular mobile phones, digital television, digital cameras, internet phones, music, and video, digital answer machines, fax, and modems

BASIC OPERATIONS ON DISCRETE-TIME SIGNALS

Signal Addition

Signal addition is obtained by adding the sample values of two signals *w*(*n*) and *x* (*n*) to form a new signal *y*(*n*).

$$y(n) = w(n) + x(n)$$



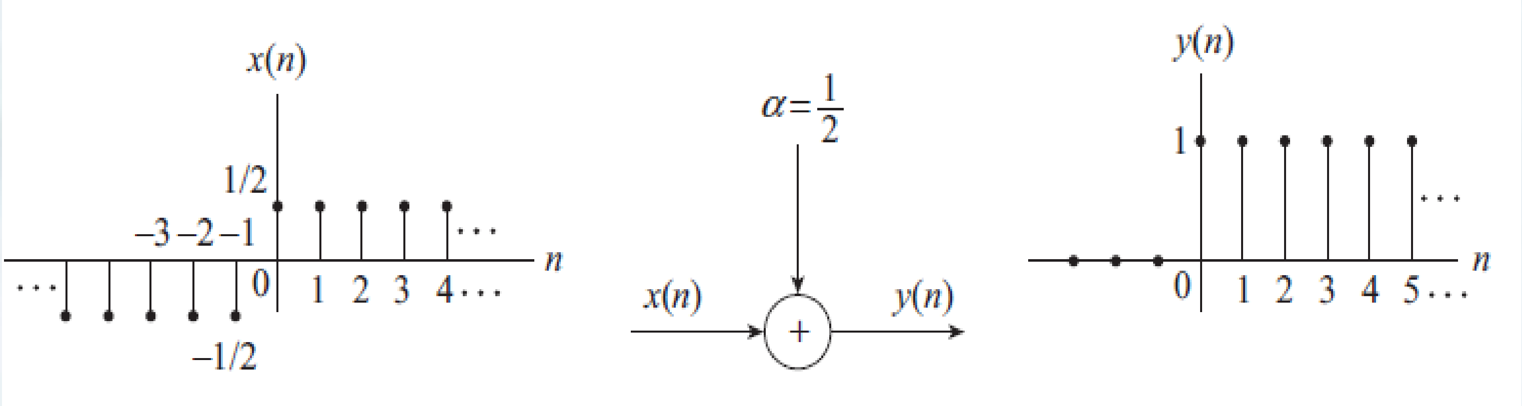
(a)

Scalar Addition

In scalar addition a new signal $y(n)$ is obtained by adding a scalar value to each sample of a signal $x(n)$,

$$y(n) = \alpha + x(n)$$

where α is a scalar.

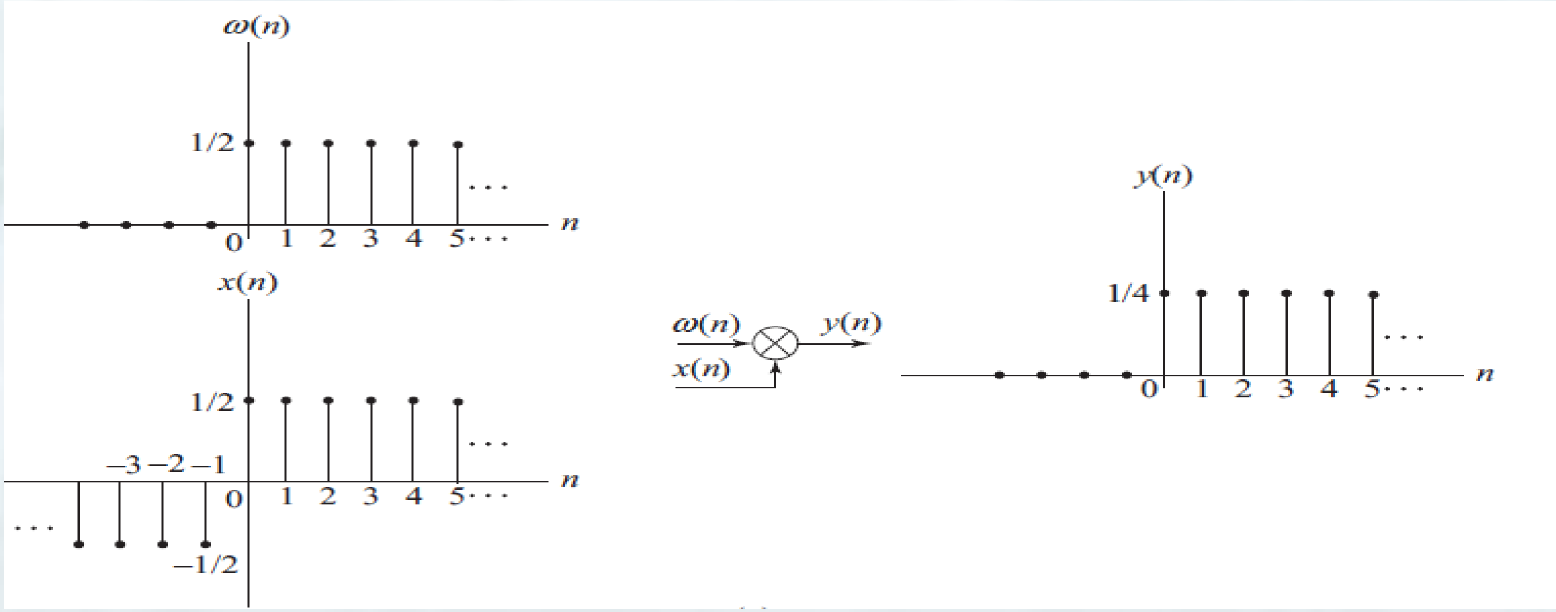


(b)

Signal Multiplication

Signal multiplication results in the product of two signals *w*(*n*) and *x* (*n*) on a sample-by-sample basis.

$$y(n) = w(n)x(n)$$



Scalar Multiplication

In scalar multiplication a new signal *y*(*n*) is obtained by multiplying a scalar value to each sample of a signal *x* (*n*),

$$y(n) = \alpha x(n)$$

where α is a scalar.

TRANSFORMATIONS OF INDEPENDENT VARIABLE (TIME)

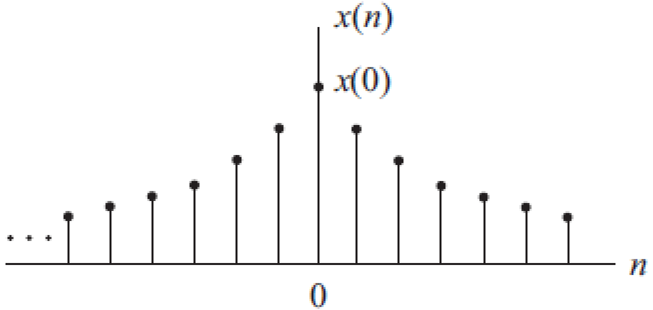
Time-shifting

- A signal $x(n)$ may be shifted in time by replacing the independent variable n by $n - n_o$, where n_o is known as the shifting factor.

- If $n_o > 0$, the signal is shifted to the right and the time shift results in a delay of the signal by n_o.

- If $n_o < 0$, the signal is shifted to the left and the time shift results in an advance of the signal by $|n_o|$.

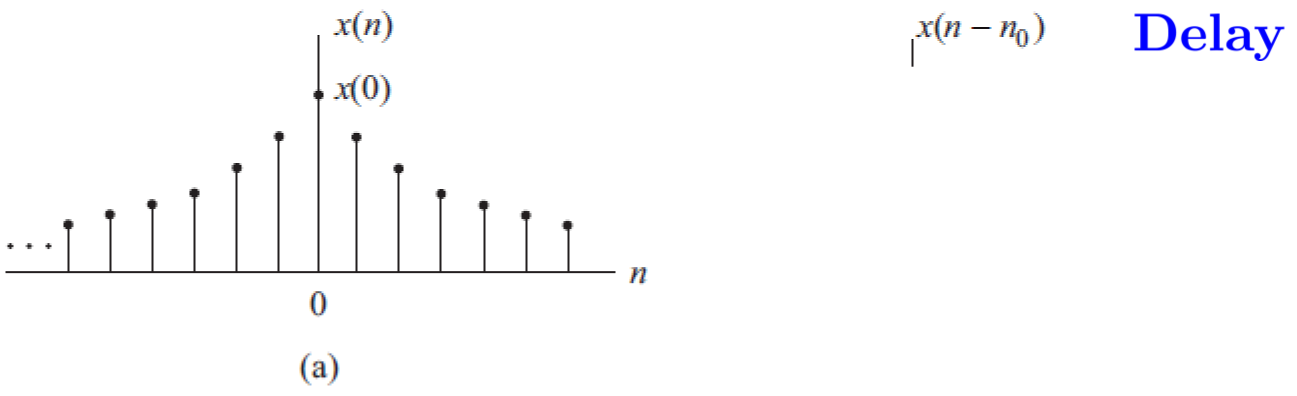
Time-shifting



(a)

Discrete-time signals related by time-shift

Time-shifting



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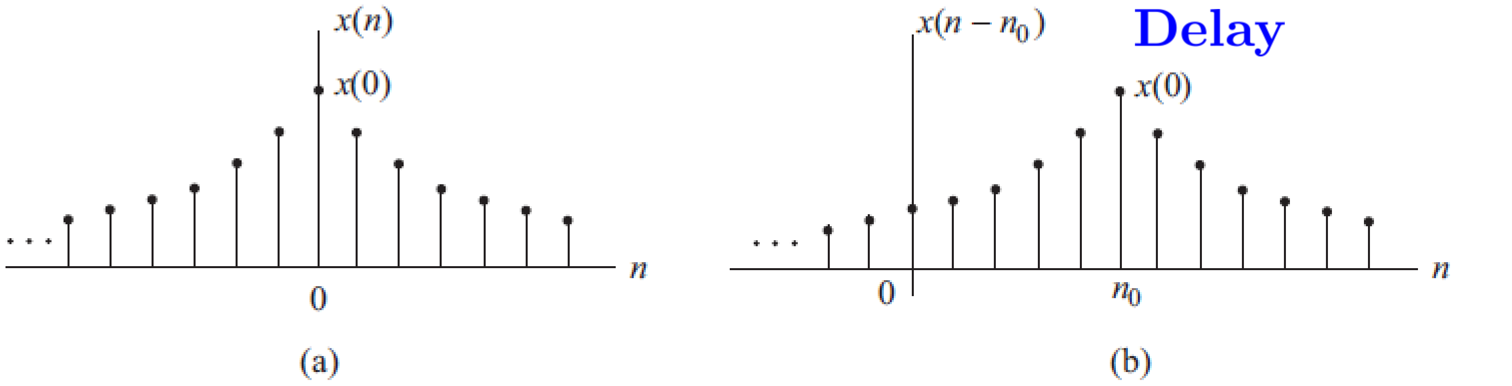

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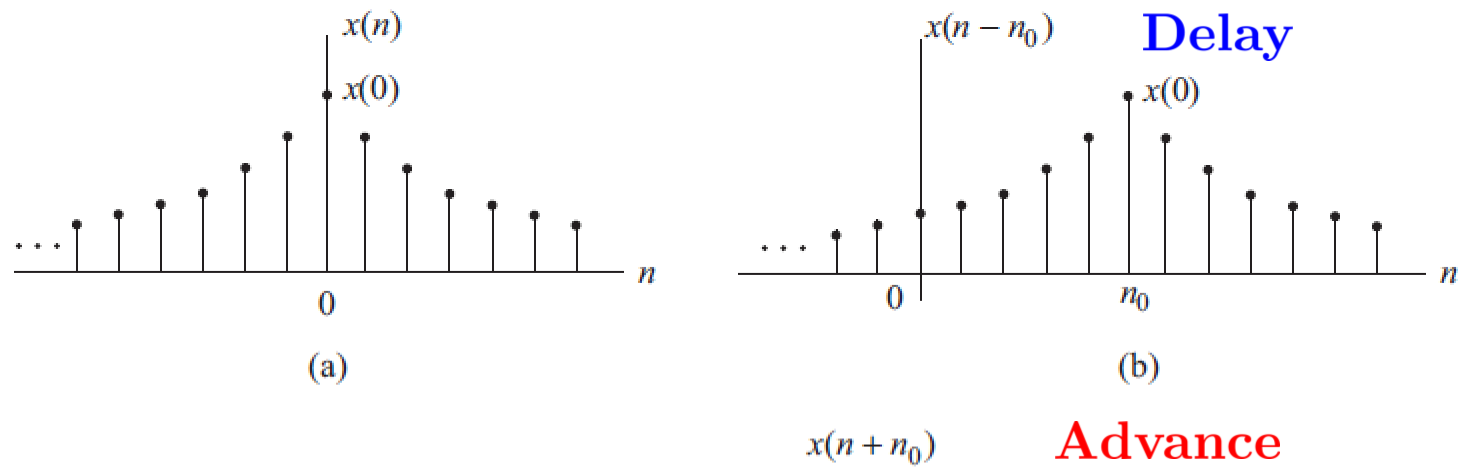
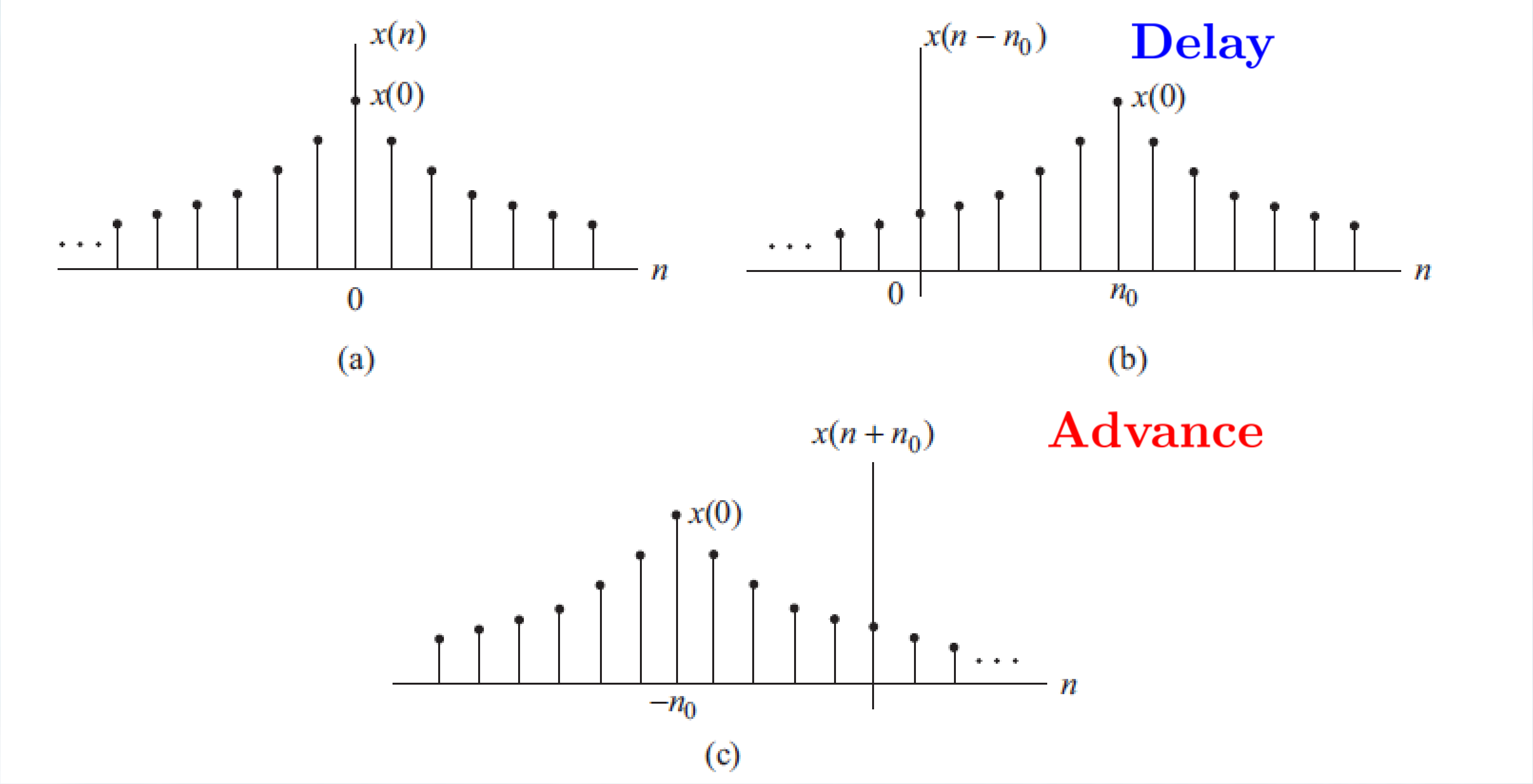


Discrete-time signals related by time-shift

Time-shifting



Discrete-time signals related by time-shift

Time-scaling

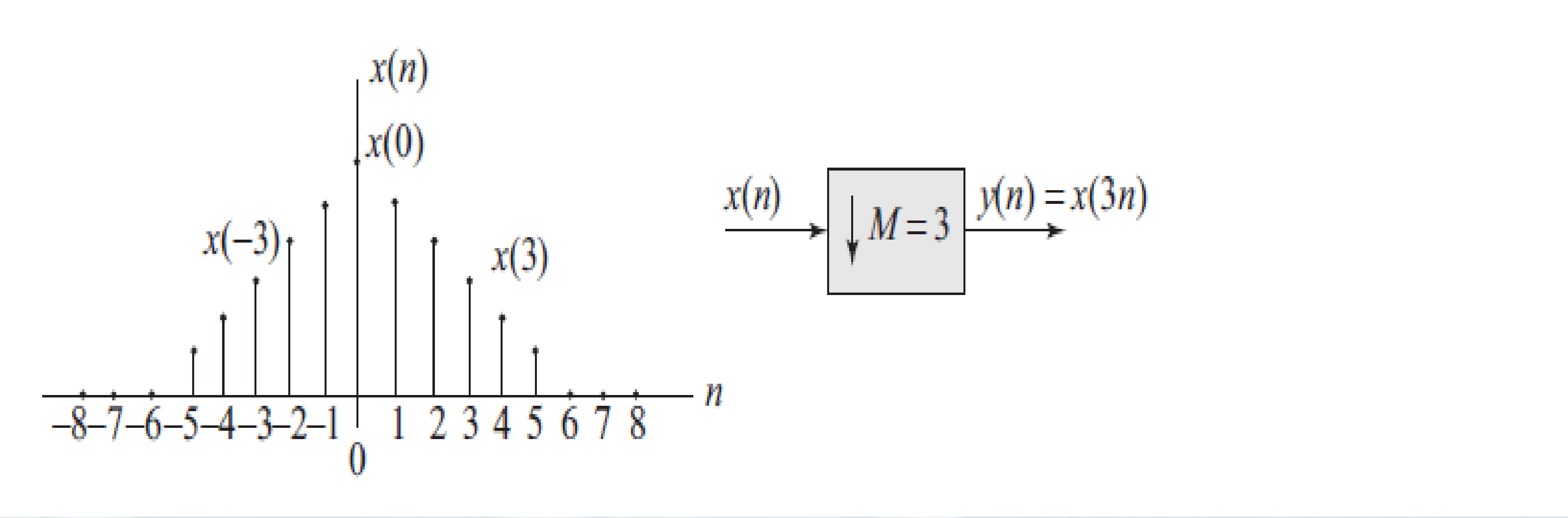
Decimation

- Decimation refers to a process of reducing the signal length by discarding signal samples.

- Thus, the decimation operation by an integer factor *M* on a signal *x* (*n*) consists of keeping every M_{th} sample of *x* (*n*) and discarding (*M* −1) samples in-between.

- Mathematically, $$y(n) = x(Mn)$$

Decimation



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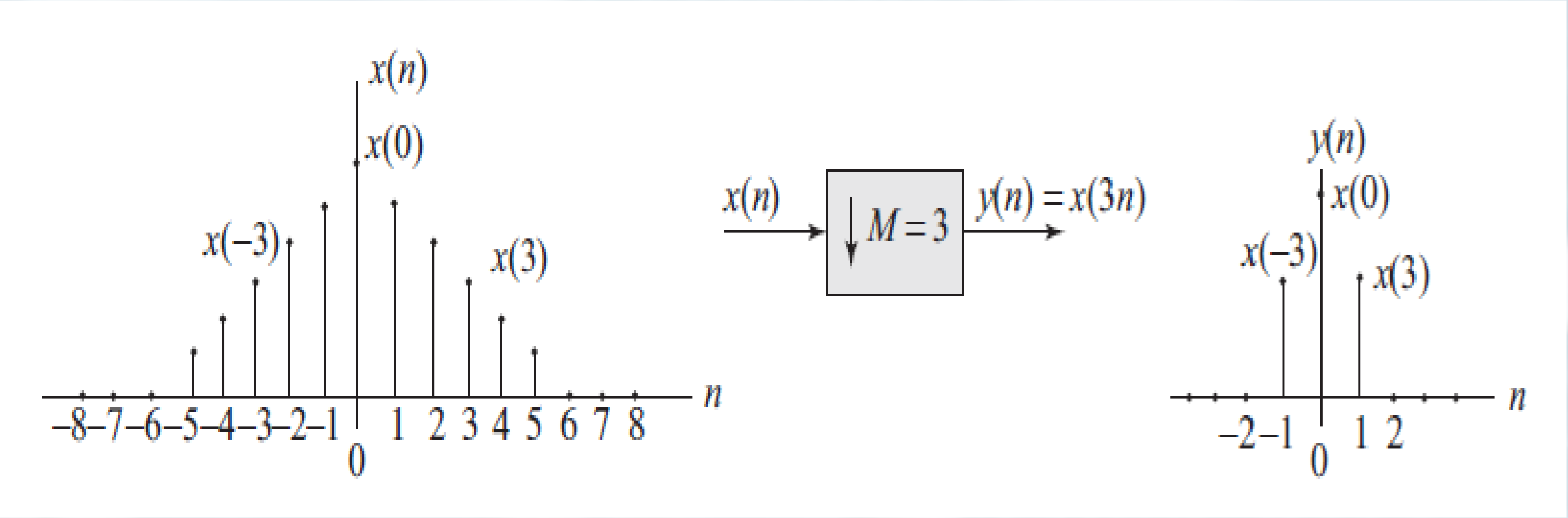
Decimation



The decimation operation by an integer factor *M* on a signal *x* (*n*) consists of keeping every M_{th} sample of *x* (*n*) and discarding (*M* −1) samples in-between

Time-scaling

Interpolation

❖ Interpolation refers to a process of increasing the signal length by inserting zeros between signal samples.

❖ Thus, the interpolation operation by an integer factor *L* on a signal *x* (*n*) consists of inserting *L* – 1 zero-valued samples between each two consecutive samples of *x* (*n*).

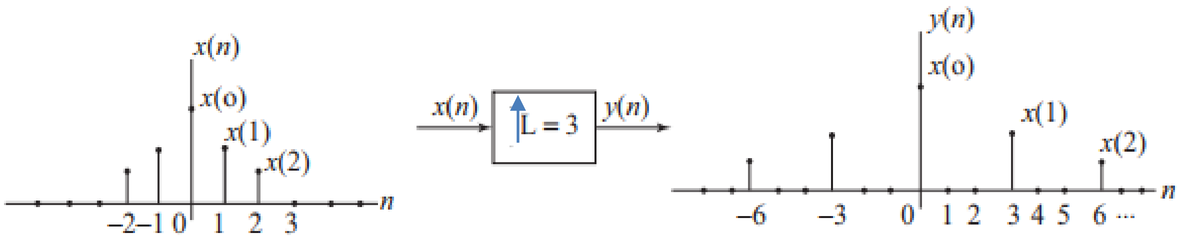
❖ Mathematically,

$$y(n) = \begin{cases} x\left(\frac{n}{L}\right) & n = 0, \pm L, \pm 2L, \cdots \\ 0 & \text{otherwise} \end{cases}$$

Interpolation

- **Interpolation (Upsampling):** Upsampling by a factor-of L. Inserting $L-1$ zero-valued samples between each two consecutive samples of $x(n)$.

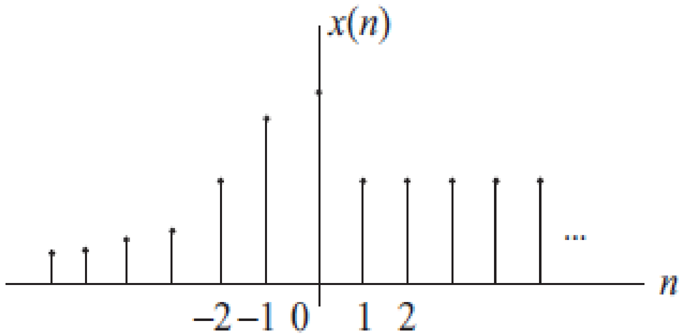
$$y(n) = \begin{cases} x\left(\frac{n}{L}\right) & n = 0, \pm L, \pm 2L, \cdots \\ 0 & \text{otherwise} \end{cases}$$



Time-reversal

- A third modification of the independent variable involves replacing n by $-n$.
- The results of this operation are *folding*, *reflection*, or *time-reversal* of the signal about the time origin $n = 0$.

Time-reversal

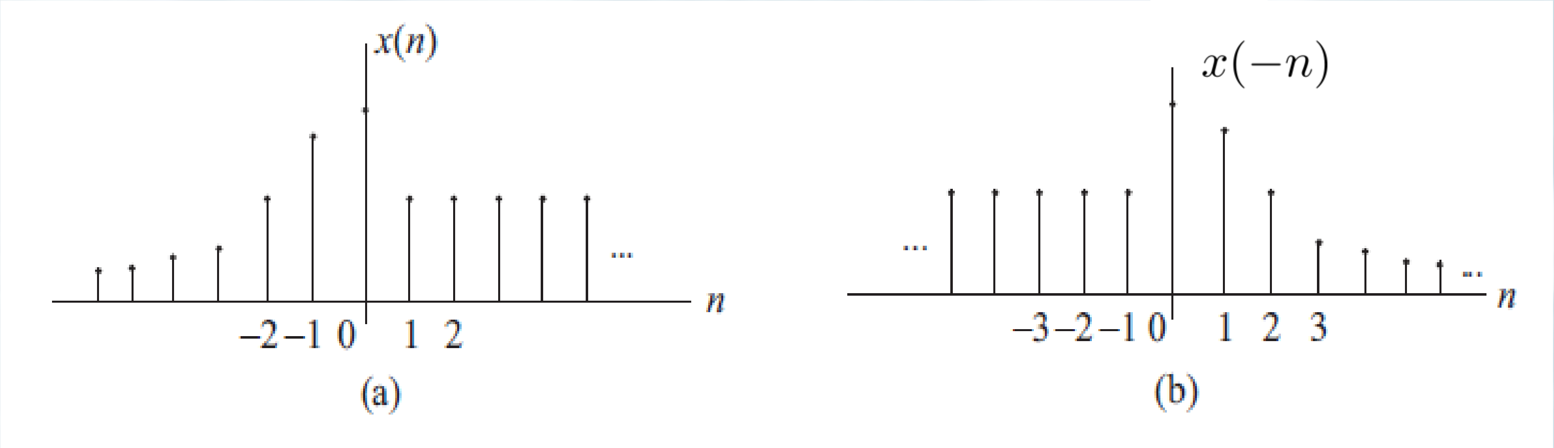


(a)

$$y(n) = x(-n)$$

As illustrated in the Figure, the signal $x(-n)$ is obtained from the signal by a reflection about $n = 0$ (i.e., by reversing the signal).

Time-reversal



As illustrated in the Figure, the signal *x* (−*n*) is obtained from the signal by a reflection about *n* = 0 (i.e., by reversing the signal).

Combined Operations

- The most general operation involving all the three operations is $x\left(-\frac{n}{m}-k\right)$, which is realized by the following sequence of operation (**time-shifting** followed by **time-scaling** followed by **time-reversal**):

$$x(n) \longrightarrow \boxed{\begin{array}{c}\text{Delay (shift right)}\\ \text{by } k\end{array}} \xrightarrow{x(n-k)} \boxed{\begin{array}{c}\text{Interpolate}\\ \text{by } m\end{array}} \xrightarrow{x\left(\frac{n}{m}-k\right)} \boxed{\text{Folding}} \xrightarrow{x\left(-\frac{n}{m}-k\right)}$$

Example

$$x(n) = (1, 2, \underset{\uparrow}{3}, 4, 5, 6)$$

Find $x(2n) = (1, \underset{\uparrow}{3}, 5)$

Find $x(n/2) = (1, 0, 2, 0, \underset{\uparrow}{3}, 0, 4, 0, 5, 0, 6)$

Example

$$x(n) = (1, -1, \underset{\uparrow}{2}, 0, 1, 1, 2)$$

Find $x(-n) = (2, 1, 1, 0, \underset{\uparrow}{2}, -1, 1)$

Find $x(n-2) = (\underset{\uparrow}{1}, -1, 2, 0, 1, 1, 2)$

Find $x(2n+1)$

First Shift $x(n+1) = (1, -1, 2, \underset{\uparrow}{0}, 1, 1, 2)$

Second Scaling $x(2n+1) = (-1, \underset{\uparrow}{0}, 1)$

Example

$$x(n) = (1, -1, 2, 0, 1, 1, 2)$$

Find $x(3-n)$

First Shift $x(n+3) = (1, -1, 2, 0, 1, 1, 2)$

Second time reversal $x(-n+3) = (2, 1, 1, 0, 2, -1, 1)$

Example

$$x(n) = (1, -1, \underset{\uparrow}{2}, 0, 1, 1, 2)$$

Find $x(4 - 2n)$

First Shift $x(n + 4) = (1, -1, 2, 0, 1, 1, \underset{\uparrow}{2})$

Second time reversal $x(-n + 4) = (\underset{\uparrow}{2}, 1, 1, 0, 2, -1, 1)$

Third time scaling $x(-2n + 4) = (\underset{\uparrow}{2}, 1, 2, 1)$

SOME BASIC DISCRETE-TIME SIGNALS

Unit Step Signal

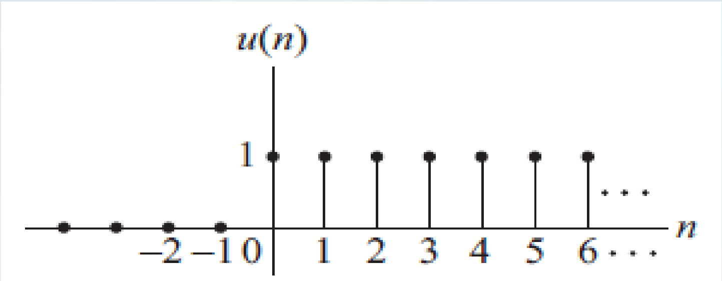
- The unit step signal is denoted by *u*(*n*) and is defined as

$$u(n) = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases}$$

$$\text{or} \qquad u(n) = \{\underset{\uparrow}{1}, 1, 1, 1, 1, \cdots\}$$

- The unit step signal is used to make an arbitrary sequence *x* (*n*) zero for *n* ≤ 0 by forming the product of the unit step signal *u*(*n*) with the sequence *x* (*n*).

$$y(n) = x(n)u(n) = \{\cdots, 0, 0, \underset{\uparrow}{x(0)}, x(1), x(2), x(3), x(4), \cdots\}$$



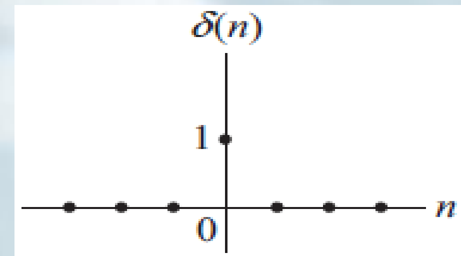
Unit Impulse (or Unit Sample) Signal

- The unit impulse signal (or unit sample signal) is denoted by $\delta(n)$ and is defined as

$$\delta(n) = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases}$$

or

$$\delta(n) = \{\cdots, 0, 0, 0, \underset{\uparrow}{1}, 0, 0, 0, \cdots\}$$



(b)

- The application of the unit impulse to linear discrete-time systems provides the system impulse response, a very useful characterization of the system.

Properties of Unit Impulse Signals

- **Time scaling:** $\delta(kn) = \delta(n)$ where *k* is an integer.

- **Unit impulse function** $\delta(n)$ and unit step function $u(n)$ are related by

$$\delta(n) = u(n) - u(n-1)$$

- **Multiplication property:** For any arbitrary signal $x(n)$

$$x(n)\delta(n-k) = x(k)\delta(n-k)$$

Properties of Unit Impulse Signals

- The unit step signal $u(n)$ may also be expressed as the **cumulative sum** of $\delta(n)$.

$$\sum_{k=-\infty}^{n} \delta(k) = \sum_{k=0}^{\infty} \delta(n-k) = u(n) = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases}$$

- **Sum property**:

$$\sum_{n=-\infty}^{\infty} \delta(n) = 1$$

- **Sifting property:**

$$\sum_{n=-\infty}^{\infty} x(n)\delta(n-k) = x(k)$$

Properties of Unit Impulse Signals

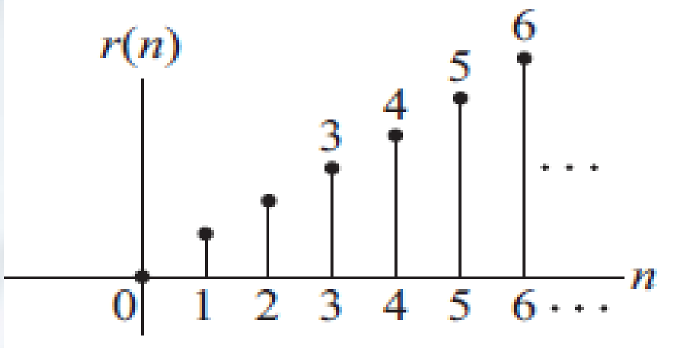
- **Signal decomposition:** Any arbitrary signal *x* (*n*) can be represented as a summation of the signal values with shifted unit impulses.

$$x(n) = \sum_{k=-\infty}^{\infty} x(k)\delta(n-k)$$

Unit Ramp Signal

- The unit ramp signal is denoted by *r* (*n*) and is defined as

$$r(n) = nu(n) = \begin{cases} n & n \geq 0 \\ 0 & n < 0 \end{cases}$$



(c)

or

$$\begin{aligned} r(n) &= \{\underset{\uparrow}{0}, 1, 2, 3, 4, \cdots\} \\ &= 0\delta(n) + \delta(n-1) + 2\delta(n-2) + 3\delta(n-3) + 4\delta(n-4) + \cdots \\ &= \sum_{k=0}^{\infty} k\delta(n-k) \end{aligned}$$

Things to Remember

- The most general operation involving all the three operations is

$$x\left(-\frac{n}{m}-k\right)$$

- which is realized by the following sequence of operation (**time-shifting** followed by **time-scaling** followed by **time-reversal**):

$$x(n) \longrightarrow \boxed{\begin{array}{c}\text{Delay (shift right)}\\ \text{by } k\end{array}} \xrightarrow{x(n-k)} \boxed{\begin{array}{c}\text{Interpolate}\\ \text{by } m\end{array}} \xrightarrow{x\left(\frac{n}{m}-k\right)} \boxed{\text{Folding}} \xrightarrow{x\left(-\frac{n}{m}-k\right)}$$

Things to Remember

Impulse Representation of Discrete-Time Signals

We can describe any discrete-time sequence $x[n]$ as:

$$x[n] = \ldots + x[-1]\delta[n+1] + x[0]\delta[n] + x[1]\delta[n-1] + x[2]\delta[n-2] + \ldots$$

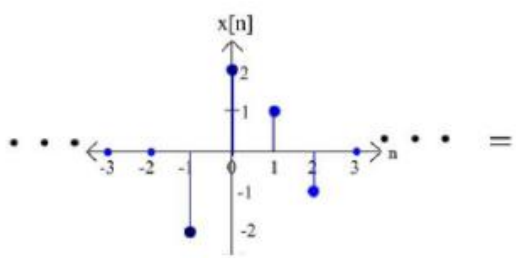
which is equivalent to the more succinct notation

$$x[n] = \sum_{k=-\infty}^{\infty} x[k]\delta[n-k]$$

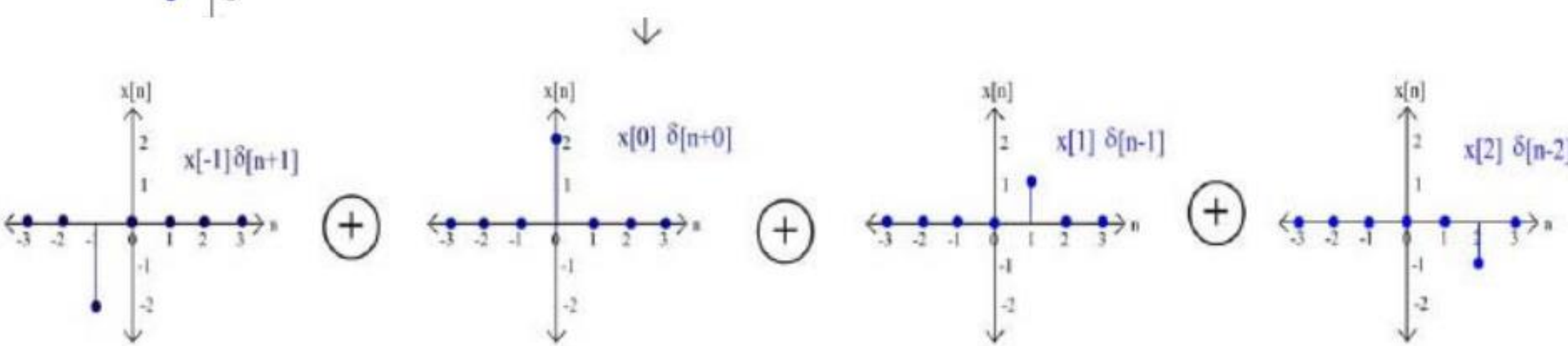
like $x(t) = \int_{-\infty}^{\infty} x(t-\tau)d\tau$

This equation expresses $x[n]$ as a series of impulse functions shifted in time, all scaled with weights $x[k]$. We will see this again when we show that the I/O relationship of a DT LTI system is a DT

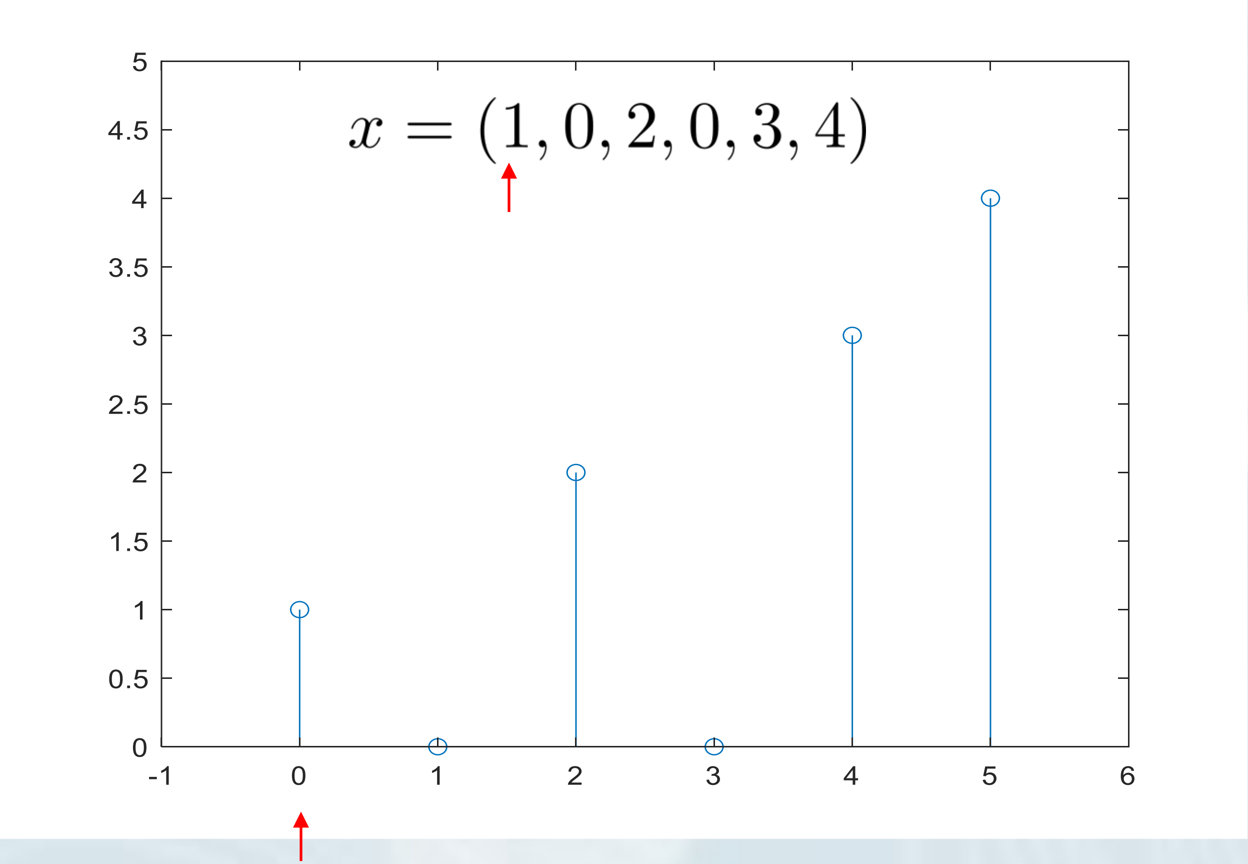
$x = [-2, 2, 1, -1]$



For each n, $x[n] = \sum_{k=-\infty}^{\infty} x[k]\delta[n-k]$

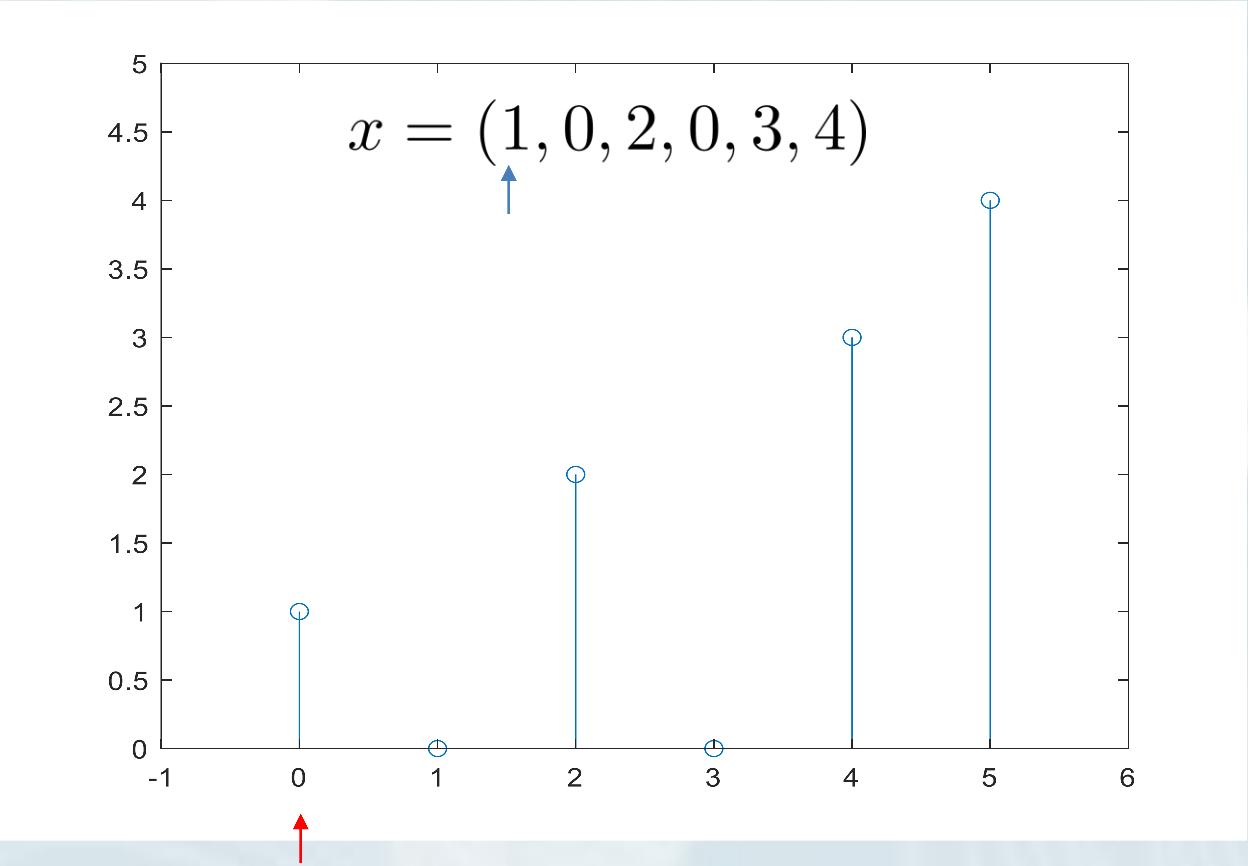


Things to Remember



Express x(n) in terms of unit impulse signal

Things to Remember



Express x(n) in terms of unit impulse signal

$$x(n) = \delta(n)x(0) + \delta(n-1)x(1) + \delta(n-2)x(2) + \delta(n-3)x(3) + \delta(n-4)x(4) + \delta(n-5)x(5)$$

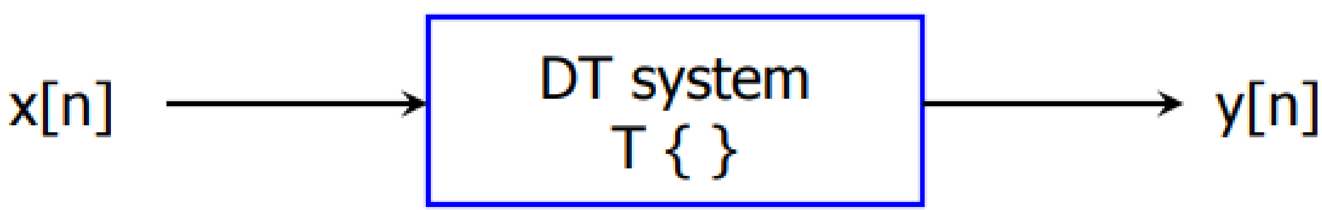
DT systems

1. **DT system**
2. DT system properties

Input-output description of DT systems

Think of a DT system as an operator on DT signals:

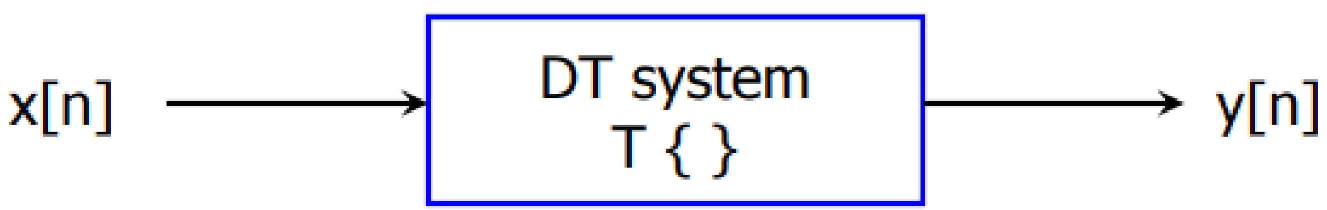
- It processes DT input signals, to produce DT output signals
- **Notation: y[n] = T{x[n]} ←→** y[n] is the response of the system T to the excitation x[n]
- Systems are assumed to be a “black box” to the user



Input-output description of DT systems

Moving average filter

$$\mathbf{y[n] = 1/M \sum_{k=0}^{M-1} x(n-k)}$$

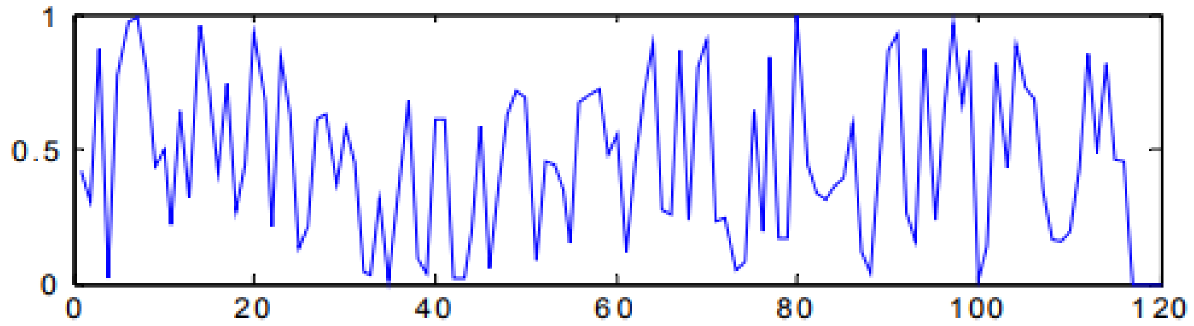


DT system example

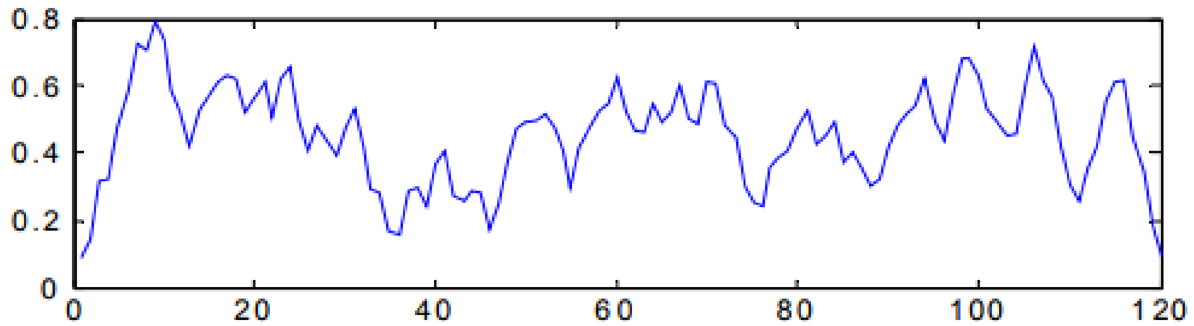
A digital low pass filter: $$\mathbf{y[n] = 1/M \sum_{k=0}^{M-1} x(n-k)}$$

y[n] = 1/5{x[n]+x[n-1]+x[n-2]+x[n-3]+x[n-4]}

Before filtering



After filtering



Classification of Discrete Time System

The discrete time systems are classified based on their characteristics. Some of the classifications of discrete time systems are,

1. Static and dynamic systems
2. Time invariant and time variant systems
3. Linear and nonlinear systems
4. Causal and noncausal systems
5. Stable and unstable systems
6. FIR and IIR systems
7. Recursive and nonrecursive systems

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Linearity

Scaling signals and adding them, then processing through the system

same as

Processing signals through system, then scaling and adding them

If $T(x_1[n]) = y_1[n]$ and $T(x_2[n]) = y_2[n]$

$\rightarrow T(ax_1[n] + bx_2[n]) = ay_1[n] + by_2[n]$

Homogeneity and superposition

Examples for linearity

Determine which of the systems below are linear,

a) $y[n] = nx[n]$

Examples for linearity

Determine which of the systems below are linear,

a) $y[n] = nx[n]$

1. Check for Homogeneity

$$\mathbf{y_1[n] = n\ a_1 x_1[n]} \qquad\qquad \mathbf{y_2[n] = n\ a_2 x_2[n]}$$

Examples for linearity

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1. Check for Homogeneity

$$\mathbf{y_1[n] = n\ a_1x_1[n]} \qquad \mathbf{y_2[n] = n\ a_2x_2[n]}$$

$$\mathbf{y_3[n] = n\ a_1x_1[n] + n\ a_2x_2[n]}$$

2. Superposition

Examples for linearity

Determine which of the systems below are linear

a) $y[n] = nx[n]$

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$$\mathbf{y_1[n] = n\ a_1 x_1[n]} \qquad \mathbf{y_2[n] = n\ a_2 x_2[n]}$$

$$\mathbf{y_3[n] = n\ a_1 x_1[n] + n\ a_2 x_2[n]}$$

2. Superposition

$$\mathbf{y_4[n] = n(a_1 x_1[n] + a_2 x_2[n])}$$

Ans. Linear as $\mathbf{y_3(n) = y_4(n)}$

Examples for linearity and time-invariance

Determine which of the systems below are linear

b) $y[n] = x^2[n]$

1. Check for Homogeneity

2. Superposition

Examples for linearity and time-invariance

Determine which of the systems below are linear

c) $y[n] = x[n^2]$

1. Check for Homogeneity

2. Superposition

Examples for linearity and time-invariance

Determine which of the systems below are linear

$$\mathbf{d)\ y[n] = m\ x(n) + c}$$

1. **Check for Homogeneity**

2. **Superposition**

Examples for linearity and time-invariance

Determine which of the systems below are linear

e) $\mathbf{y[n] = e^{x(n)}}$

1. Check for Homogeneity

2. Superposition

Time-invariance

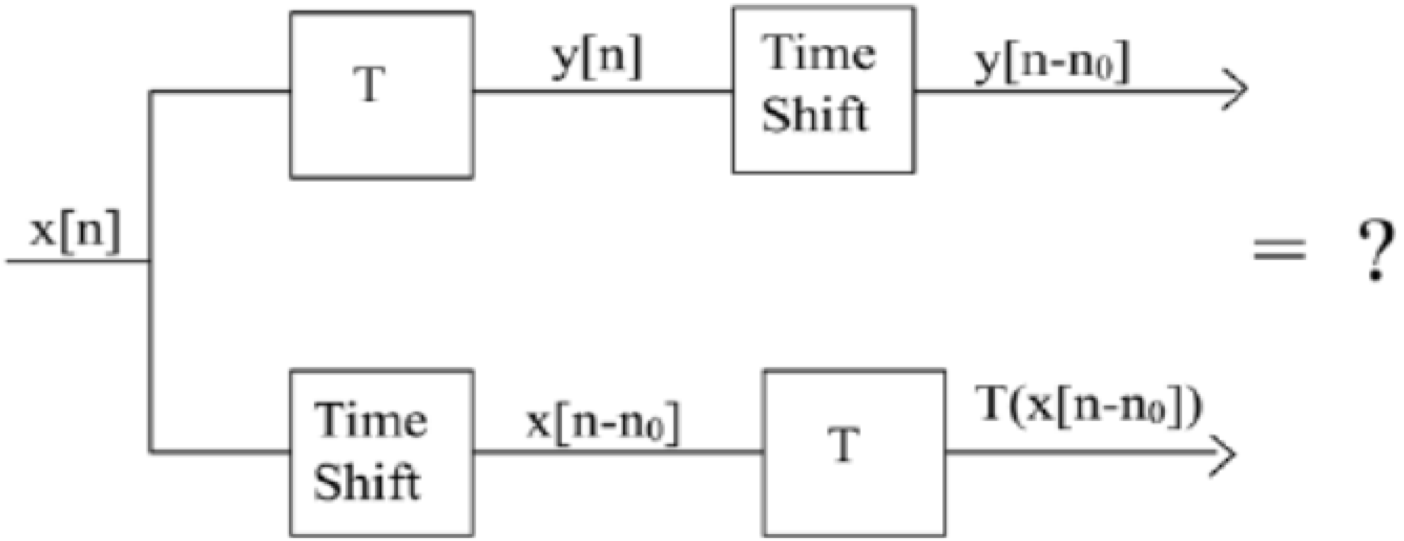
- If you time shift the input, get the **same** output, but with the **same** time shift
- The behavior of the system **doesn't change** with time

$$\text{If } T(x[n]) = y[n]$$

$$\text{then } T(x[n-n_0]) = y[n-n_0]$$

Classification of Discrete Time Signal

To test a system for time invariance, consider the following (for any and all inputs *x[n]*)



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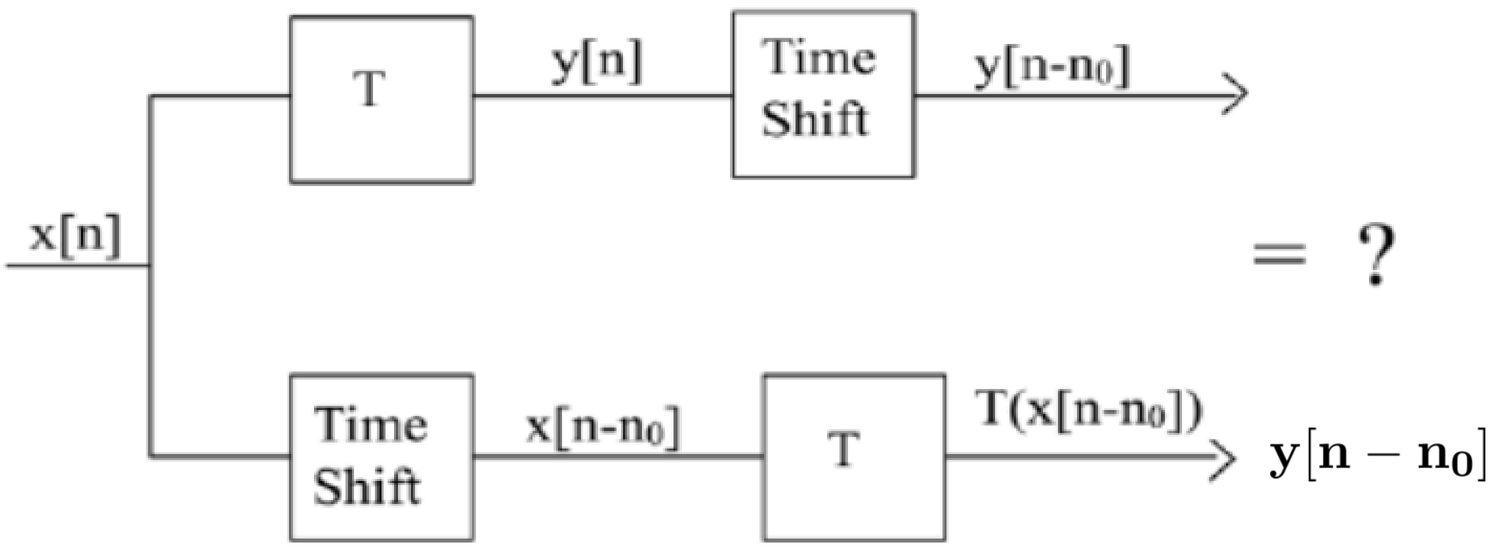


For any choice of time shift

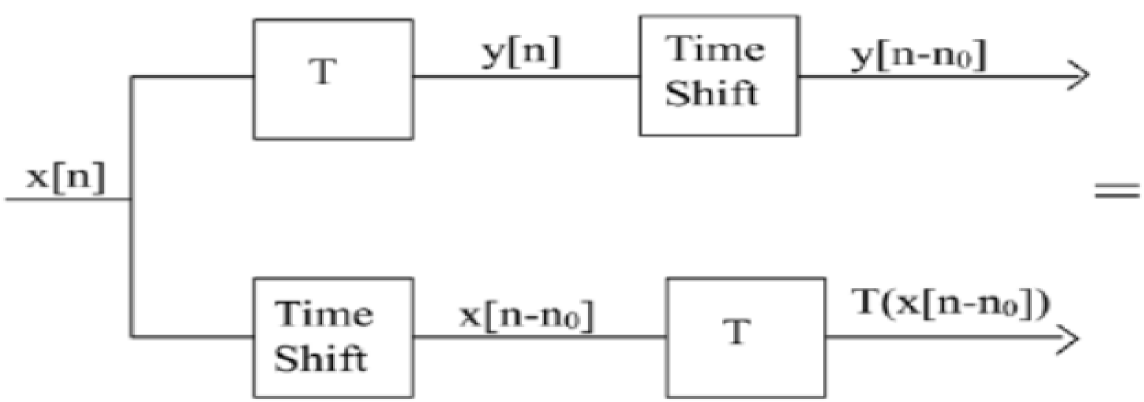
If equal, then time-invariant

Examples for linearity and time-

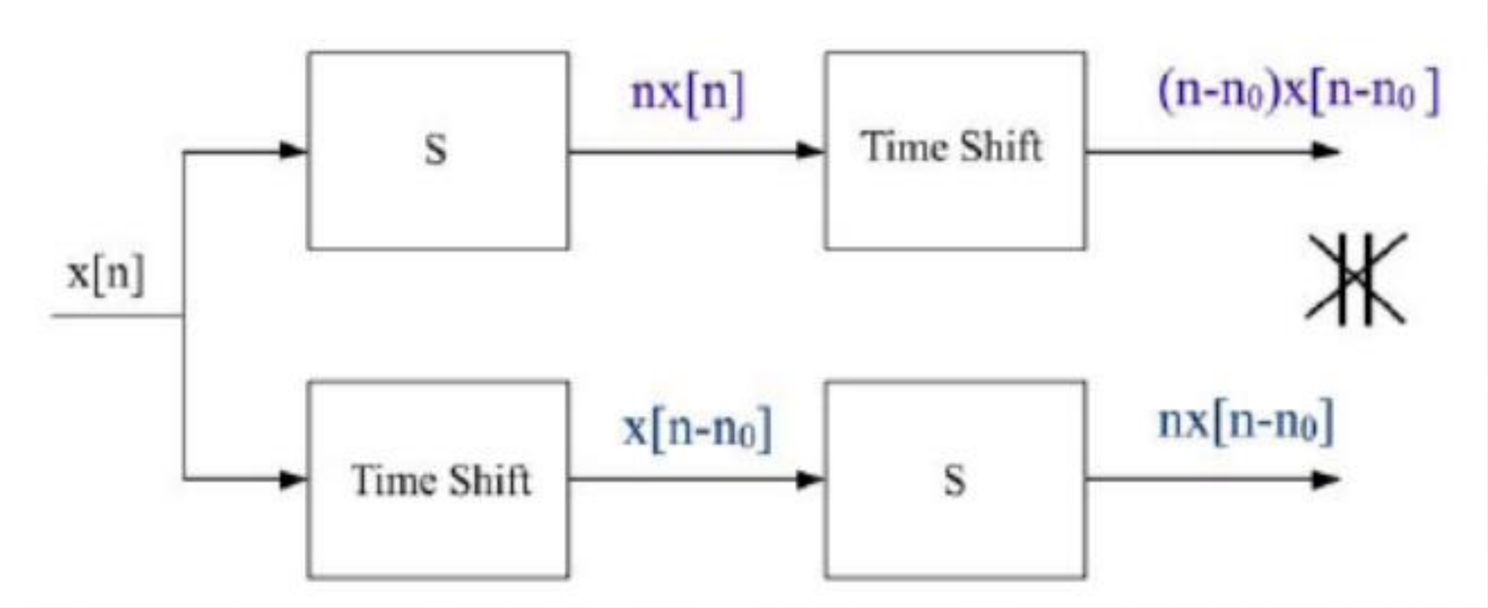
Determine which of the systems below are linear, which ones are time-invariant

a) $y[n] = nx[n]$

Steps to follow



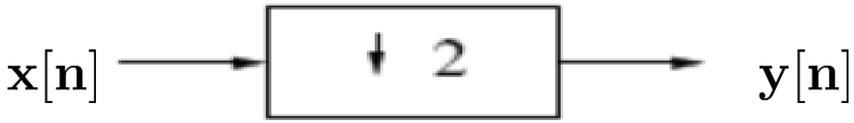
Classification of Discrete Time Signal



Classification of Discrete Time Signal

Verifying Time Invariance

a) $\mathbf{y[n] = x[2n]}$



Down Sampling

1. $y[n] = x[2n]$, ***Not* time-invariant**, Proof:

a) Do input/output system relationship first, then do time-shift second:

i) System: $v_1[n] = x[2n]$, then

ii) Time-shift: $w_1[n] = v_1[n-n_0] = \left(x[2n]\right)\Big|_{n=(n-n_0)} = x[2(n-n_0)] = x[2n-2n_0]$

b) Do time-shift first, then do input/output system relationship second:

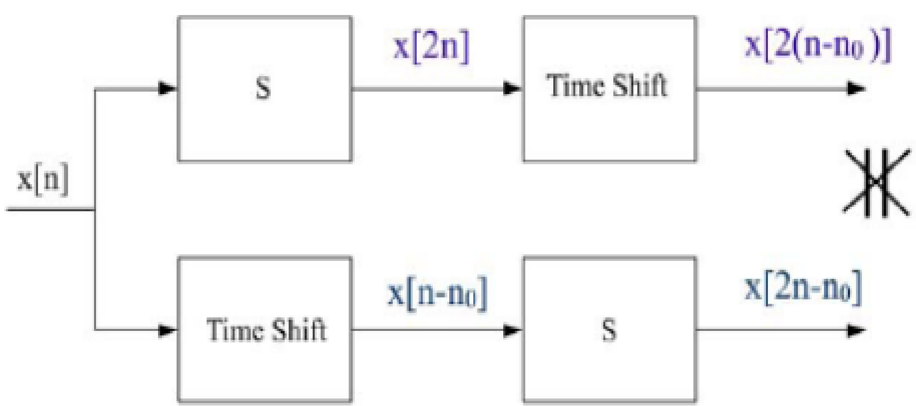
i) Time-shift: $w_2[n] = x[n]\Big|_{n=n-n_0} = x[n-n_0]$, then

ii) System: $v_2[n] = w_2[2n] = x[2n-n_0]$

c) Compare above two possible orders (a) and (b):

$w_1[n] \neq v_2[n]$, thus ***Not* time-invariant**

Time-Variant because



Classification of Discrete Time Signal

Verifying Time Invariance

$$\mathbf{x[n]} \longrightarrow \boxed{\downarrow 2} \longrightarrow \mathbf{y[n]}$$

$$\mathbf{x[n] = \{\underset{\uparrow}{1}, 2, 3, 4, 5\}}$$

First step

$$\mathbf{y[n] = x[2n] = \{\underset{\uparrow}{1}, 3, 5\}}$$

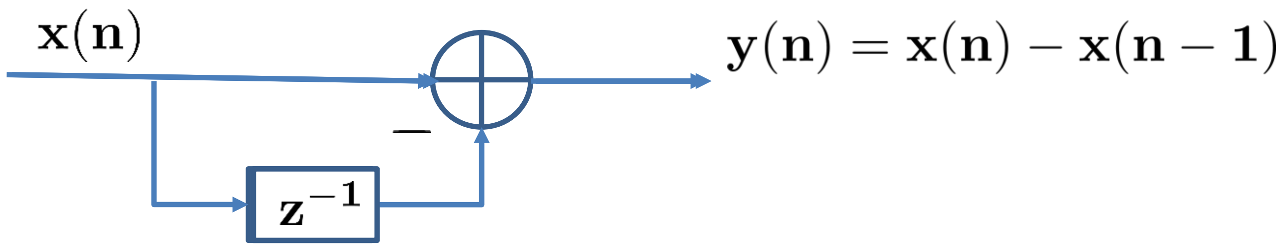
$$\mathbf{y(n-1) = x[2(n-1)] = \{\underset{\uparrow}{0}, 1, 3, 5\}}$$

Second step

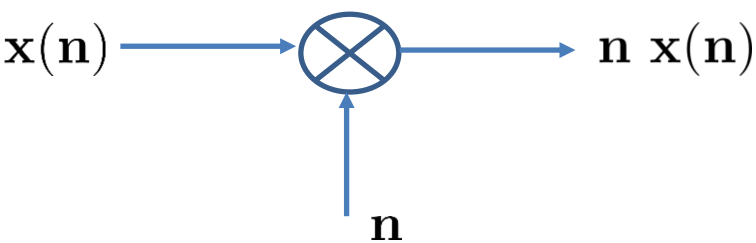
$$\mathbf{x[n-1] = \{\underset{\uparrow}{0}, 1, 2, 3, 4, 5\}}$$

$$\mathbf{x[2n-1] = \{\underset{\uparrow}{0}, 2, 4\}}$$

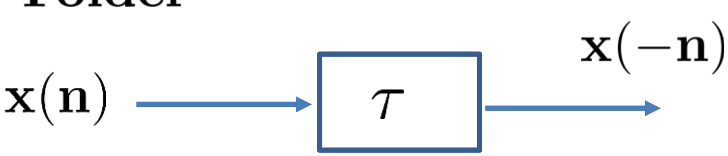
a. **”Differentation”**



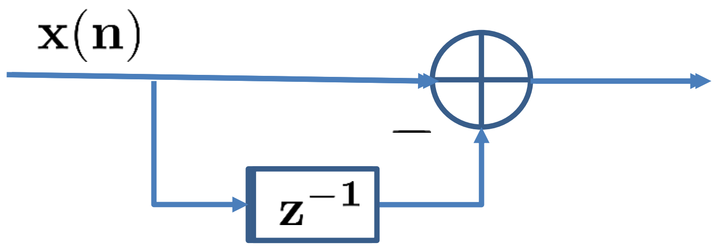
b. **”Time Multiplier”**



c. **”Folder”**



a. "Differentation"



$$\mathbf{y(n) = x(n) - x(n-1)}$$

Solution

(a) This system is described by the input–output equations

$$y(n) = T[x(n)] = x(n) - x(n-1) \tag{2.2.15}$$

Now if the input is delayed by k units in time and applied to the system, it is clear from the block diagram that the output will be

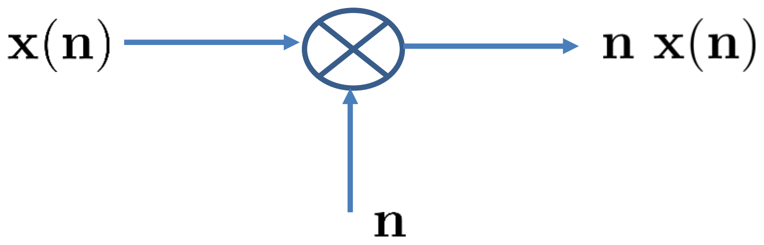
$$y(n,k) = x(n-k) - x(n-k-1) \tag{2.2.16}$$

On the other hand, from (2.2.14) we note that if we delay $y(n)$ by k units in time, we obtain

$$y(n-k) = x(n-k) - x(n-k-1) \tag{2.2.17}$$

Since the right-hand sides of (2.2.16) and (2.2.17) are identical, it follows that $y(n,k) = y(n-k)$. Therefore, the system is time invariant.

b. **"Time Multiplier"**



(b) The input–output equation for this system is

$$y(n) = T[x(n)] = nx(n) \tag{2.2.18}$$

The response of this system to $x(n-k)$ is

$$y(n, k) = nx(n-k) \tag{2.2.19}$$

Now if we delay $y(n)$ in (2.2.18) by k units in time, we obtain

$$\begin{aligned} y(n-k) &= (n-k)x(n-k) \\ &= nx(n-k) - kx(n-k) \end{aligned} \tag{2.2.20}$$

This system is time variant, since $y(n, k) \neq y(n-k)$.

c. "Folder"

$$\mathbf{x(n)} \longrightarrow \boxed{\mathcal{T}} \longrightarrow \mathbf{x(-n)}$$

(c) This system is described by the input–output relation

$$y(n) = \mathcal{T}[x(n)] = x(-n) \tag{2.2.21}$$

The response of this system to $x(n-k)$ is

$$y(n,k) = \mathcal{T}[x(n-k)] = x(-n-k) \tag{2.2.22}$$

Now, if we delay the output $y(n)$, as given by (2.2.21), by k units in time, the result will be

$$y(n-k) = x(-n+k) \tag{2.2.23}$$

Since $y(n,k) \neq y(n-k)$, the system is time variant.

Classification of Discrete Time Signal

1. $y[n] = x[2n]$, ***Not* time-invariant**, Proof:

a) Do input/output system relationship first, then do time-shift second:

i) System: $v_1[n] = x[2n]$, then

ii) Time-shift: $w_1[n] = v_1[n - n_0] = (x[2n])\big|_{n \leftarrow (n - n_0)} = x[2(n - n_0)] = x[2n - 2n_0]$

b) Do time-shift first, then do input/output system relationship second:

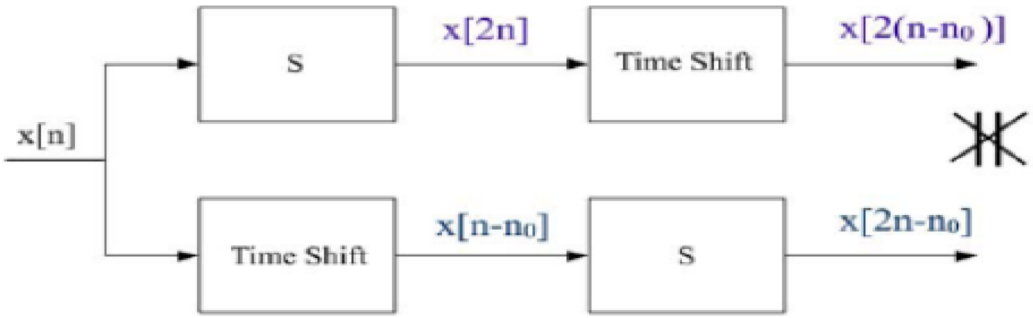
i) Time-shift: $w_2[n] = x[n]\big|_{n \leftarrow n - n_0} = x[n - n_0]$, then

ii) System: $v_2[n] = w_2[2n] = x[2n - n_0]$

c) Compare above two possible orders (a) and (b):

$w_1[n] \neq v_2[n]$, thus ***Not* time-invariant**

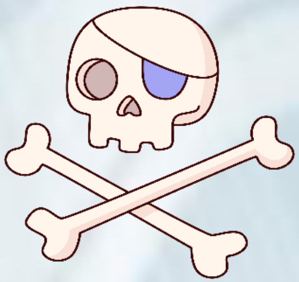
Time-Variant because

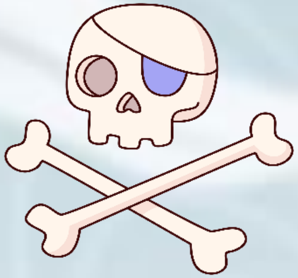


Linearity

Test the following systems for linearity.

a) $y(n) = n\,x(n)$ b) $y(n) = x(n^2)$ c) $y(n) = x^2(n)$ d) $y(n) = B\,x(n) + C$

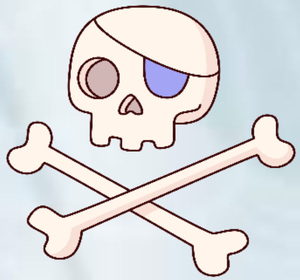


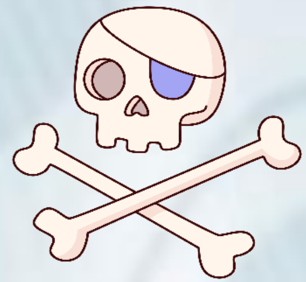


Classification of Discrete Time Signal

Test the following systems for time invariance.

a) $y(n) = x(n) + x(n-1)$ **b)** $y(n) = 2n\,x(n)$ **c)** $y(n) = x(-n)$ **d)** $y(n) = x(n) - b\,x(n-1)$



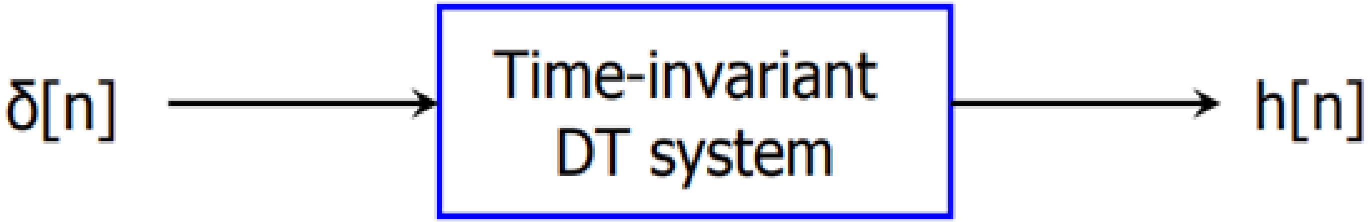


Well Done

Impulse response of DT systems

Impulse response: the output results, in response to a unit impulse

Denotation: **$h_k[n]$:** impulse response of a system, to an impulse at time *k*



Impulse response of DT systems

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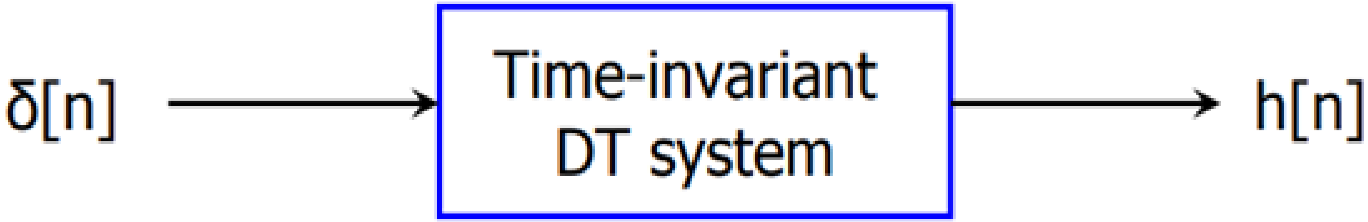
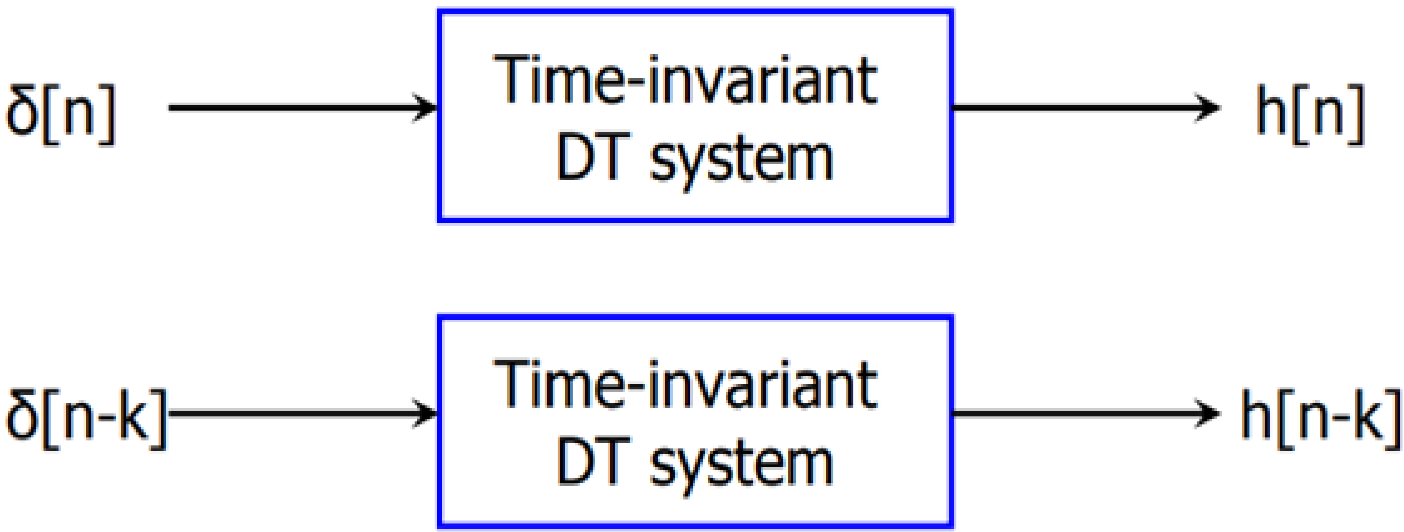


$$\mathbf{x}[\mathbf{n}] = \delta[\mathbf{n}] \longrightarrow \boxed{\mathrm{T}[\quad]} \longrightarrow \mathbf{y}[\mathbf{n}] = \mathbf{T}[\delta[\mathbf{n}]] = \mathbf{h}[\mathbf{n}]$$

Impulse response of DT systems

Impulse response: the output results, in response to a unit impulse

Denotation: **$h_k[n]$:** impulse response of a system, to an impulse at time *k*

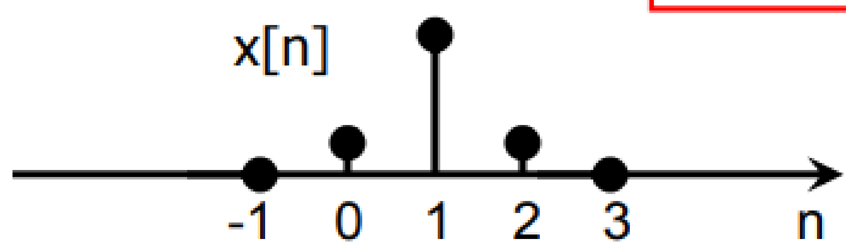


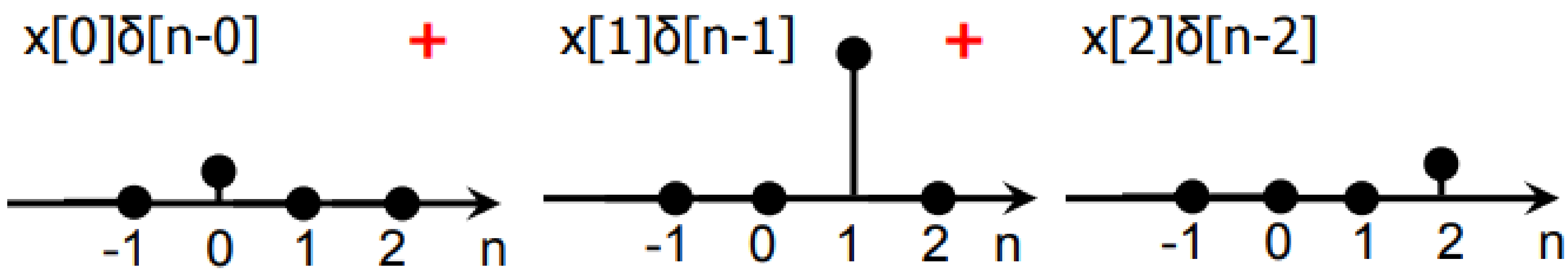
Remember: the impulse response is a sequence of values that may go on forever!!!

Impulse representation of DT signals

We can describe any DT signal x[n] as: $\boxed{x[n] = \sum_{k=-\infty}^{\infty} x[k]\delta[n-k]}$

Example:





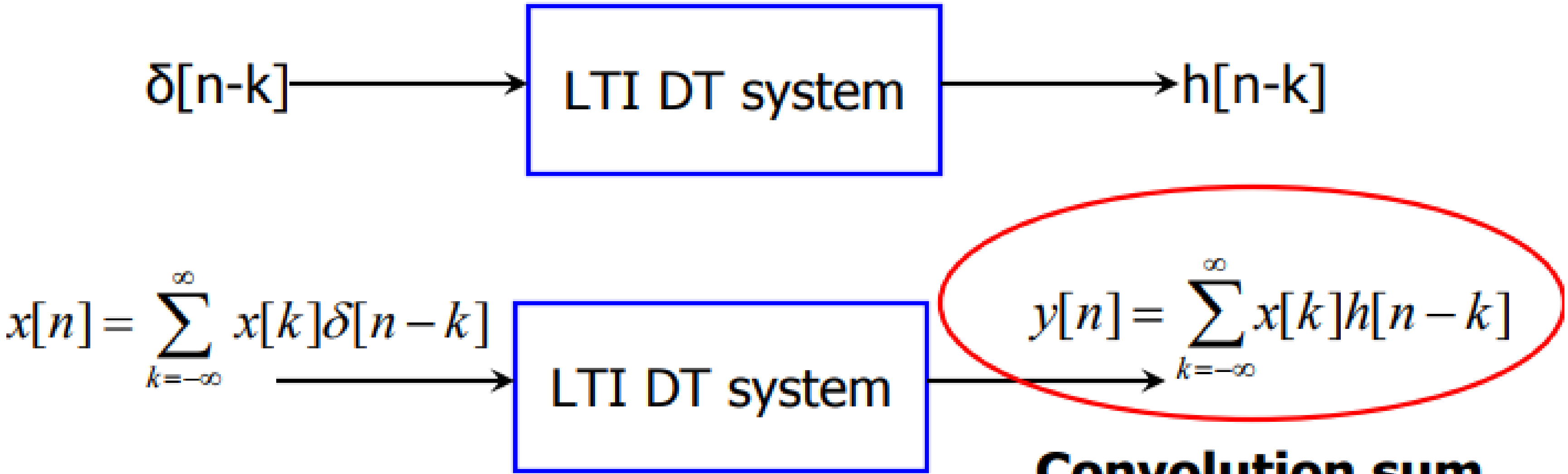
Response of LTI DT systems to arbitrary inputs

$$\delta[n-k] \longrightarrow \boxed{\text{LTI DT system}} \longrightarrow h[n-k]$$

$$\mathbf{x[n]} = \sum_{\mathbf{k=-\infty}}^{\mathbf{k=\infty}} \mathbf{x[k]}\delta\mathbf{[n-k]} \longrightarrow \boxed{\text{LTI DT system}} \longrightarrow \mathbf{y[n]} = \sum_{\mathbf{k=-\infty}}^{\mathbf{k=\infty}} \mathbf{x[k]} \mathbf{T}\left[\delta\mathbf{[n-k]}\right]$$

Notation: y[n] = x[n] * h[n]

Response of LTI DT systems to arbitrary inputs



Convolution sum

Notation: y[n] = x[n] * h[n]

Convolution sum properties

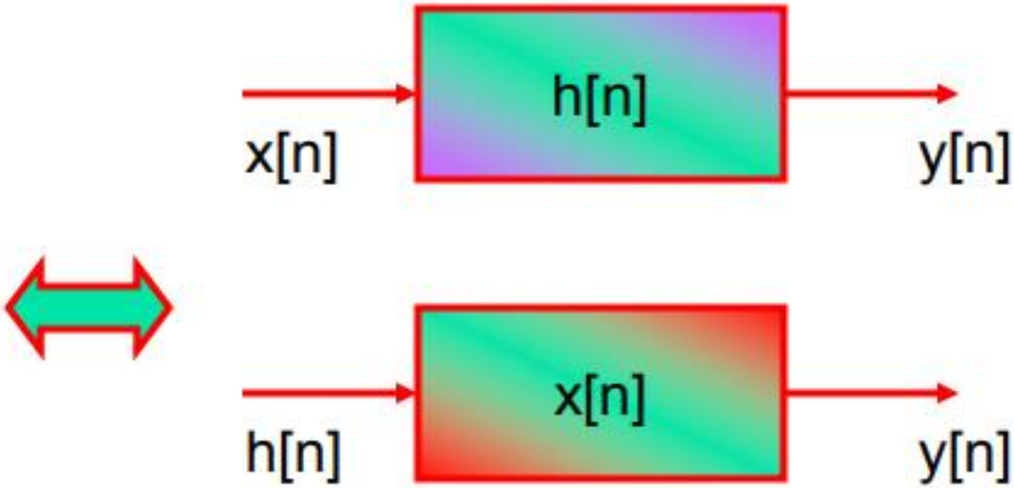
- $\delta[n] * x[n] = x[n]$

  $\delta[n-m] * x[n] = x[n-m]$

  $\delta[n] * x[n-m] = x[n-m]$
- Commutative law
- Associative law
- Distributive law

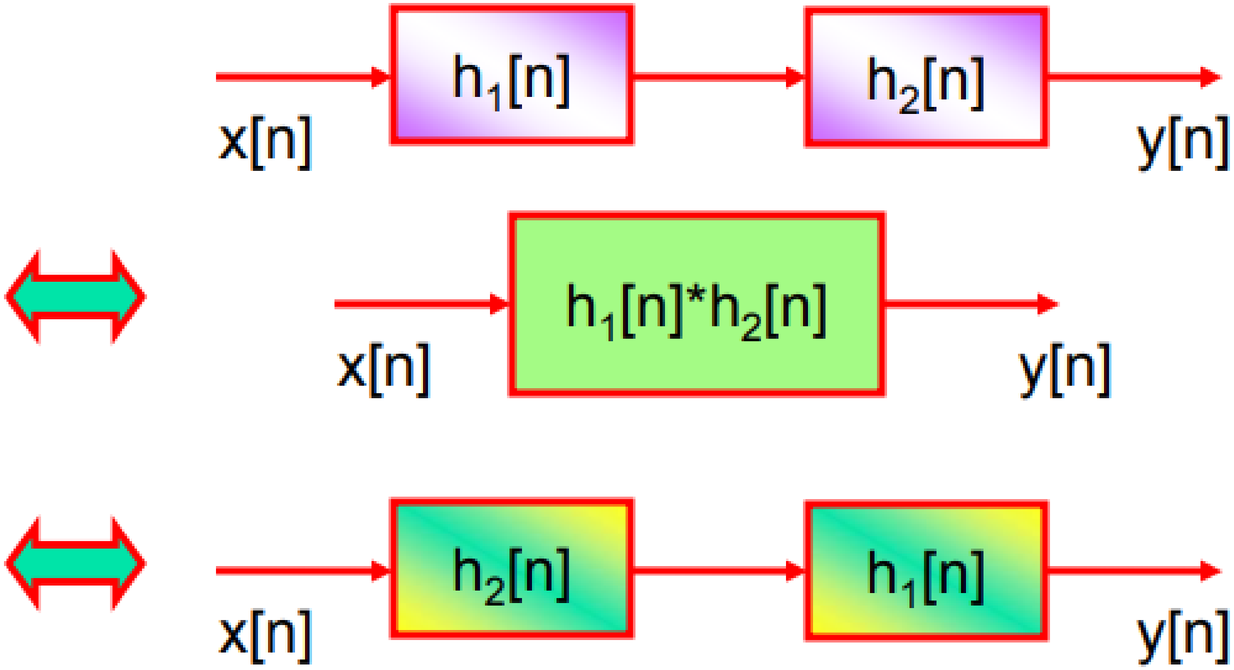
Commutative law

$$\boxed{x[n] * h[n] = h[n] * x[n]}$$



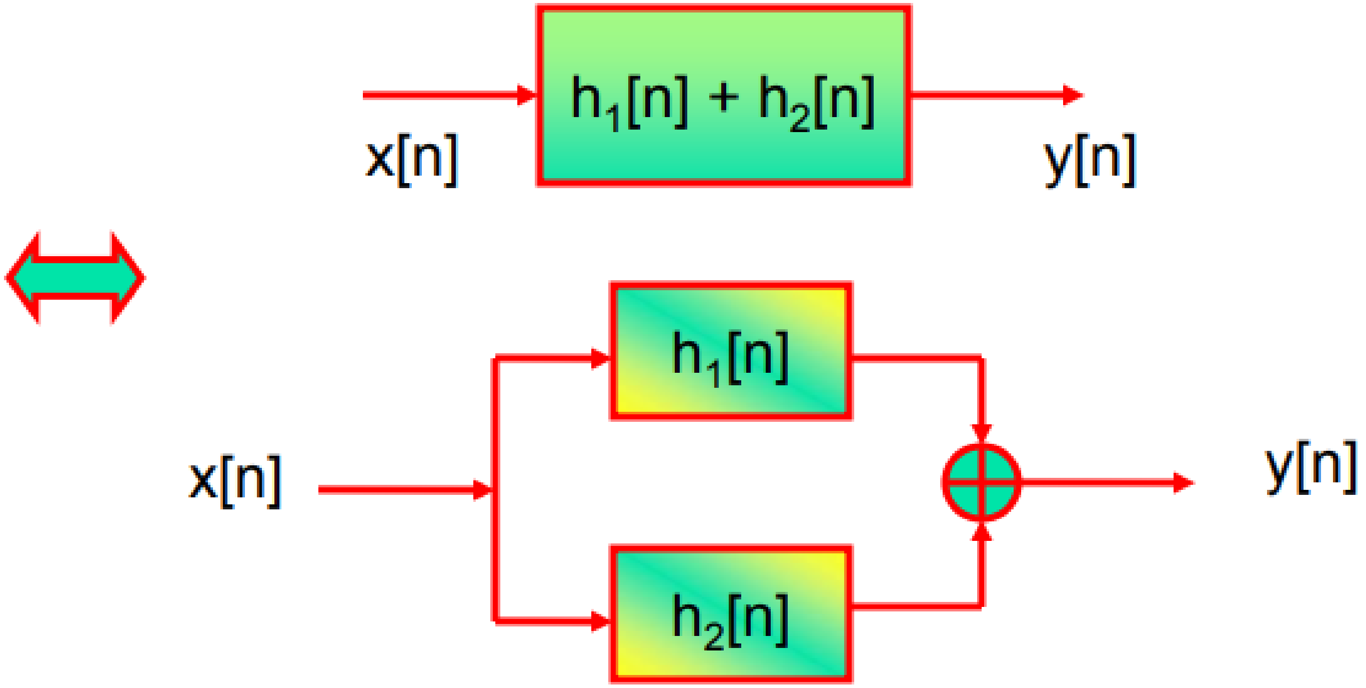
Associative law

$$\boxed{(x[n]*h_1[n])*h_2[n] = x[n]*(h_1[n]*h_2[n])}$$



Distributive law

$$x[n]*(h_1[n]+h_2[n])=(x[n]*h_1[n])+(x[n]*h_2[n])$$



Convolution of finite duration sequences

In convolution of finite duration sequences it is possible to predict the length of resultant sequence.

If the sequence $x_1(n)$ has N_1 samples and sequence $x_2(n)$ has N_2 samples then the output sequence $x_3(n)$ will be a finite duration sequence consisting of "N_1+N_2-1" samples.

i.e., if, Length of $x_1(n) = N_1$

Length of $x_2(n) = N_2$

then, Length of $x_3(n) = N_1 + N_2 - 1$

In the convolution of finite duration sequences it is possible to predict the start and end of the resultant sequence. If $x_1(n)$ starts at $n = n_1$ and $x_2(n)$ starts at $n = n_2$ then, the initial value of n for $x_3(n)$ is "$n = n_1 + n_2$". The value of $x_1(n)$ for $n < n_1$ and the value of $x_2(n)$ for $n < n_2$ are then assumed to be zero.The final value of n for $x_3(n)$ is "$n = (n_1 + n_2) + (N_1 + N_2 - 2)$".

i.e., if, $x_1(n)$ start at $n = n_1$

$x_2(n)$ start at $n = n_2$

then, $x_3(n)$ start at $n = n_1 + n_2$

and $x_3(n)$ end at $n = (n_1 + n_2) + (N_1 + N_2 - 1) - 1$

$$= (n_1 + n_2) + (N_1 + N_2 - 2)$$

Computing the convolution sum

$$y[n]=\sum_{k=-\infty}^{\infty}x[k]h[n-k] \quad\Rightarrow\quad y[n_0]=\sum_{k=-\infty}^{\infty}x[k]h[n_0-k]$$

1. **Fold** $h[k]$ about $k = 0$, to obtain $h[-k]$

2. **Shift** $h[-k]$ by n_0 to the right (left) if n_0 is positive (negative), to obtain $h[n_0-k]$

3. **Multiply** $x[k]$ and $h[n_0-k]$ for all k, to obtain the product

$$x[k].h[n_0-k]$$

4. **Sum** up the product for all k, to obtain $y[n_0]$

Repeat from 2-4 fof all of n

Example 2.3.2

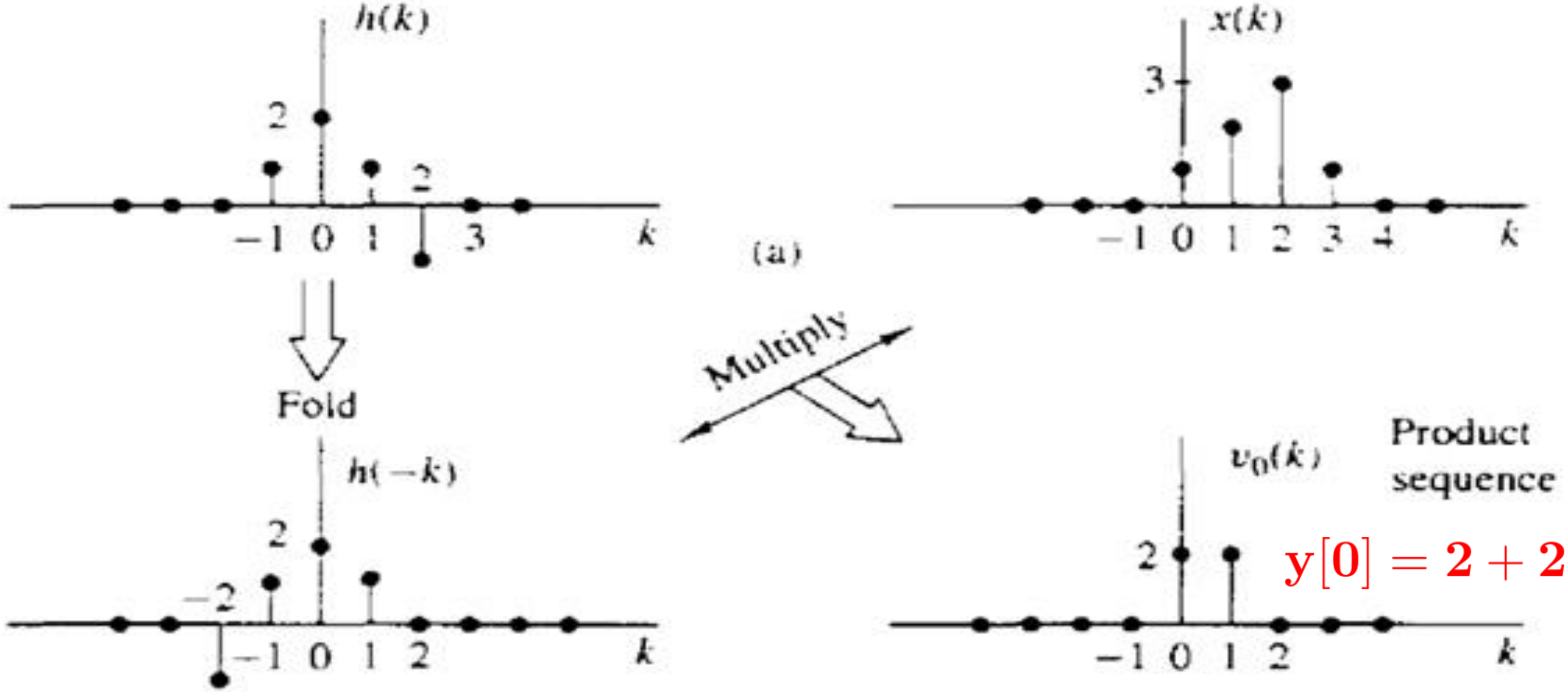
The impulse response of a linear time-invariant system is

$$h(n) = \{1, \underset{\uparrow}{2}, 1, -1\}$$

Determine the response of the system to the input signal

$$x(n) = \{\underset{\uparrow}{1}, 2, 3, 1\}$$

$$y[n] = \sum_{k=-\infty}^{k=\infty} x[k]h[n-k] \qquad \text{At } n = 0 \longrightarrow \qquad y[0] = \sum_{k=-\infty}^{k=\infty} x[k]h[0-k]$$



$$y[0] = 2 + 2 = 4$$

Example 2.3.2

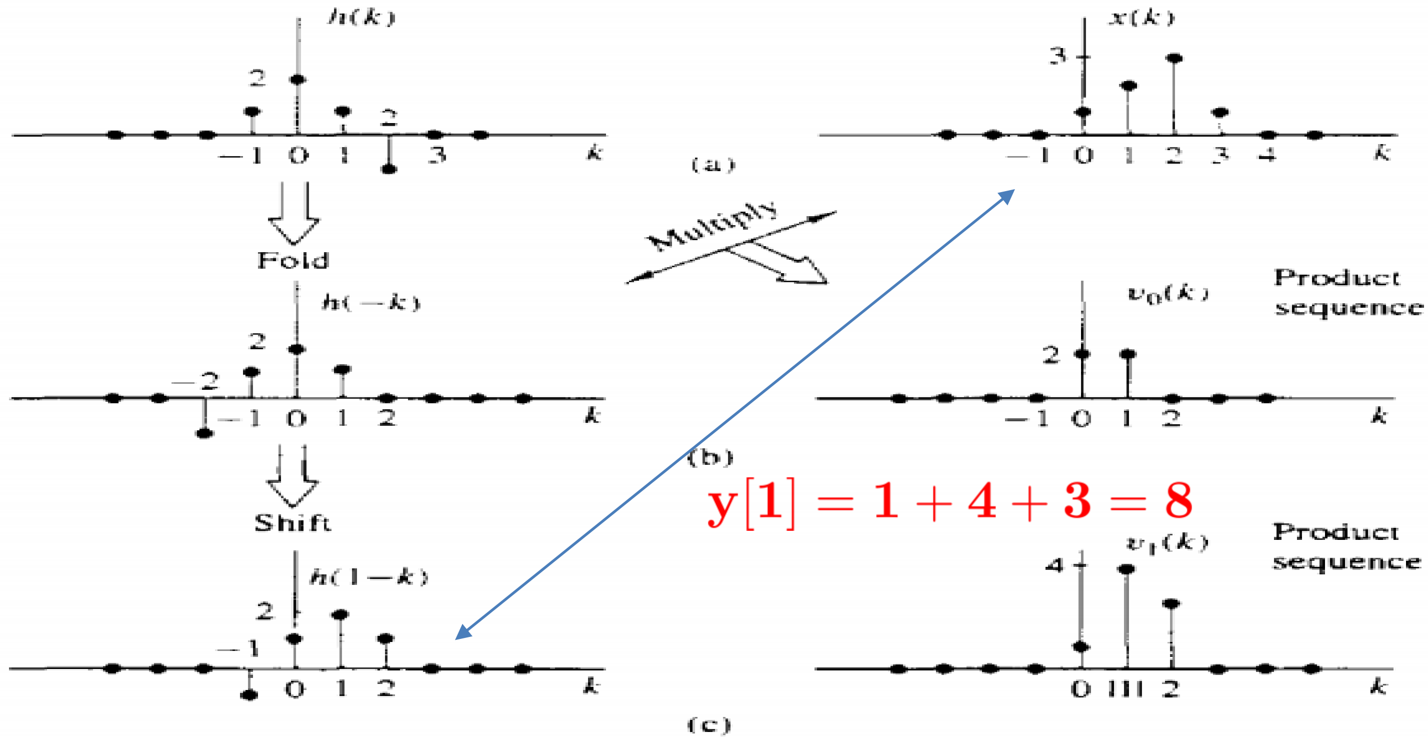
The impulse response of a linear time-invariant system is

$$h(n) = \{1, \underset{\uparrow}{2}, 1, -1\}$$

Determine the response of the system to the input signal $\mathbf{y[0] = 2 + 2 = 4}$

$$x(n) = \{\underset{\uparrow}{1}, 2, 3, 1\}$$

$$\mathbf{y[n]} = \sum_{\mathbf{k=-\infty}}^{\mathbf{k=\infty}} \mathbf{x[k]h[n-k]} \qquad \text{At } n = 0 \longrightarrow \qquad \mathbf{y[1]} = \sum_{\mathbf{k=-\infty}}^{\mathbf{k=\infty}} \mathbf{x[k]h[1-k]}$$



$$\mathbf{y[1] = 1 + 4 + 3 = 8}$$

Convolution

Response of an LTI system to an arbitrary input.

$$x[n] \longrightarrow \boxed{\text{LTI}} \longrightarrow y[n]$$

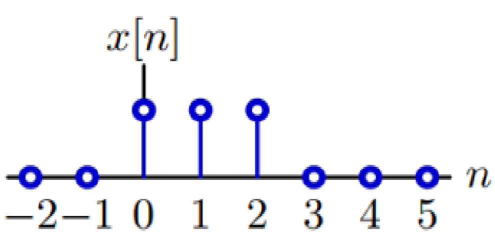
$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

Structure of Convolution

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

Starting Point: $\mathbf{n_1 + n_2 = 0}$

Length of y: $\mathbf{N_1 + N_2 - 1 = 5}$



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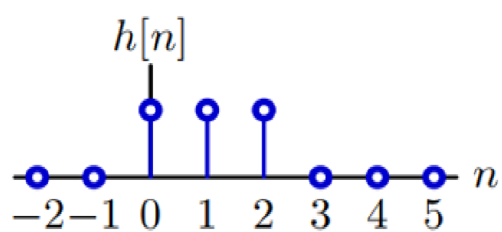
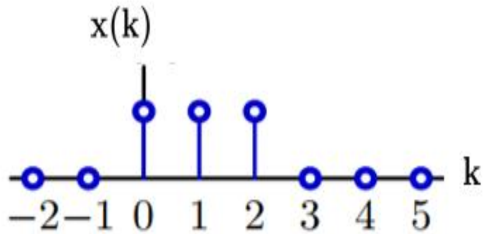


$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$



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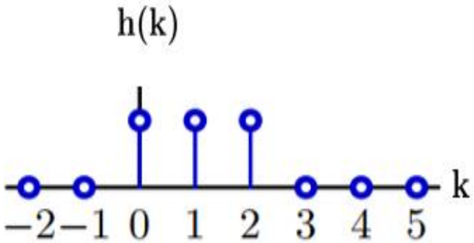
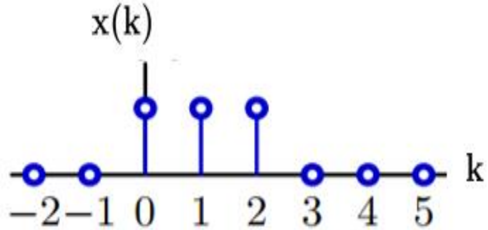
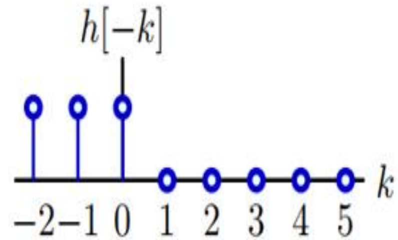
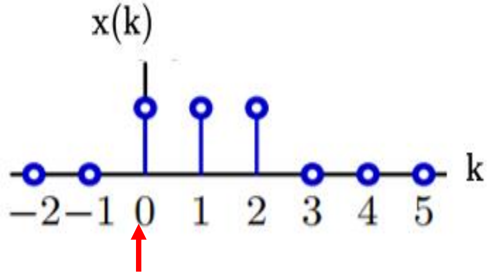


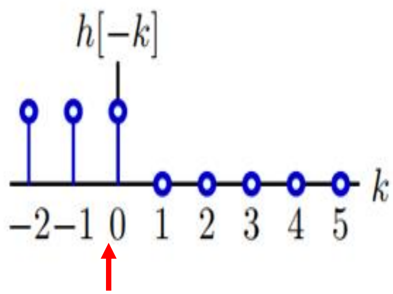
$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$





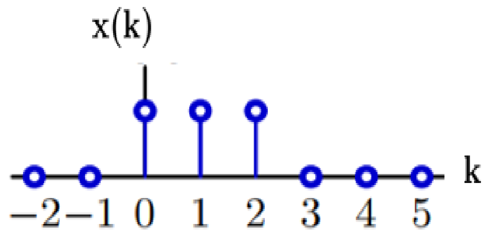
$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

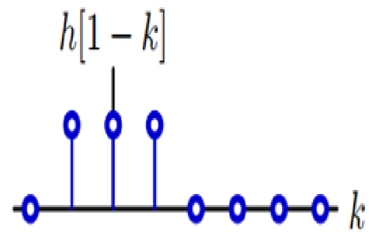




$$\mathbf{y[0] = 1}$$

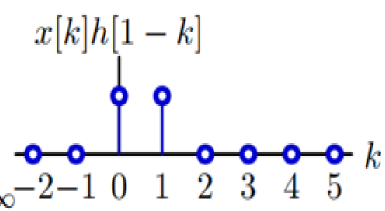
$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$





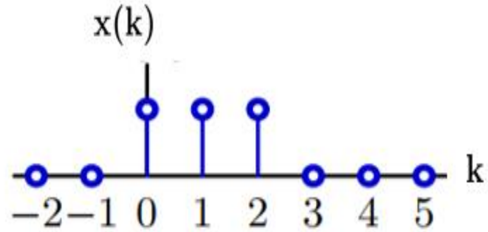
y(1) = h(0)x(0) + h(1)x(1)=2

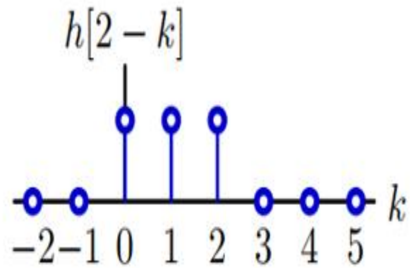
$$y(1) = \sum_{k=-\infty}^{\infty}$$



$= 2$

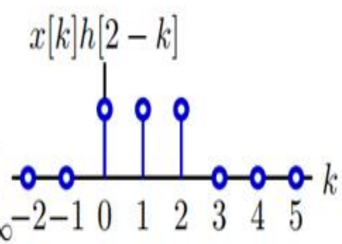
$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$





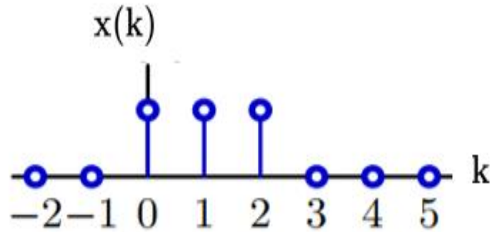
y(2) = h(0)x(0) + h(1)x(1)+h(2)x(2)=3

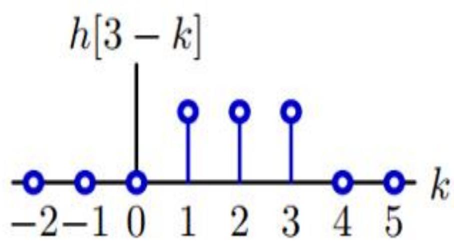
y(2) = $\sum_{k=-\infty}^{\infty}$



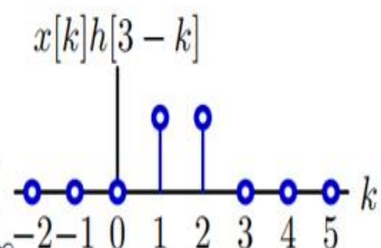
= 3

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$



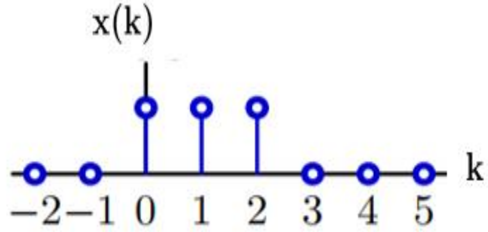


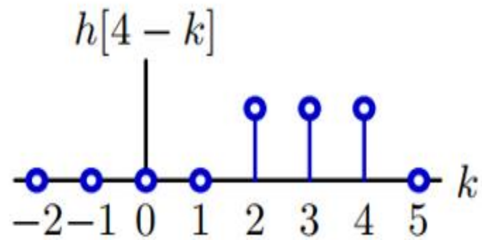
$$y(3) = \sum_{k=-\infty}^{\infty}$$



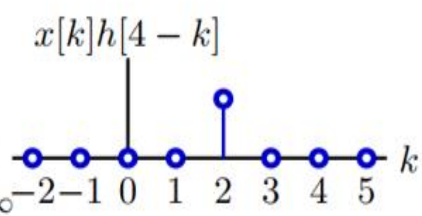
$$= 2$$

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$



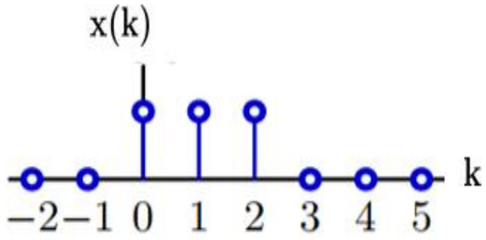


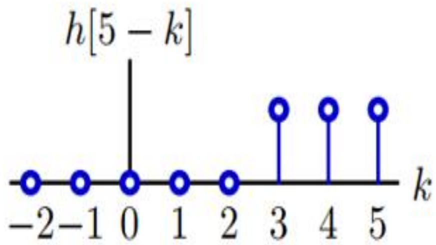
$$y(4) = \sum_{k=-\infty}^{\infty}$$



$= 1$

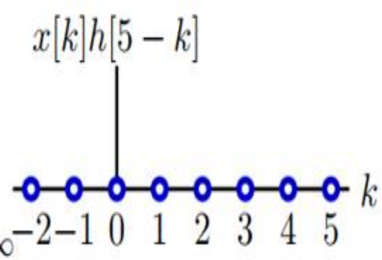
$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$





$$\mathbf{y[n] = [1, 2, 3, 2, 1, 0]}$$

$y(5) = \sum_{k=-\infty}^{\infty}$



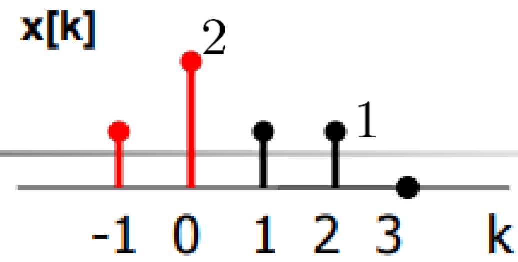
$= 0$

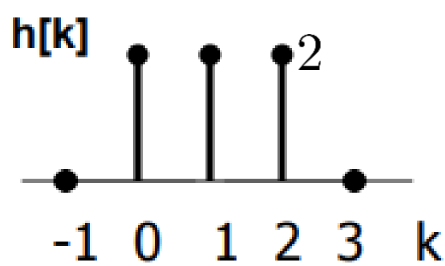
Example 1

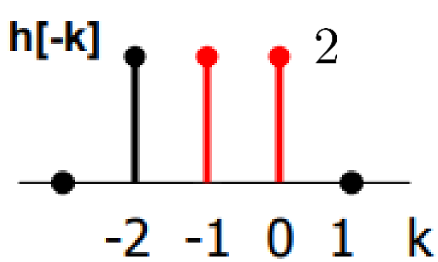
Find y[n] = x[n]*h[n] where

$$x[n] = u[n+1] - u[n-3] + \delta[n] \qquad h[n] = 2\left(u[n] - u[n-3]\right)$$

Ex1 (cont)

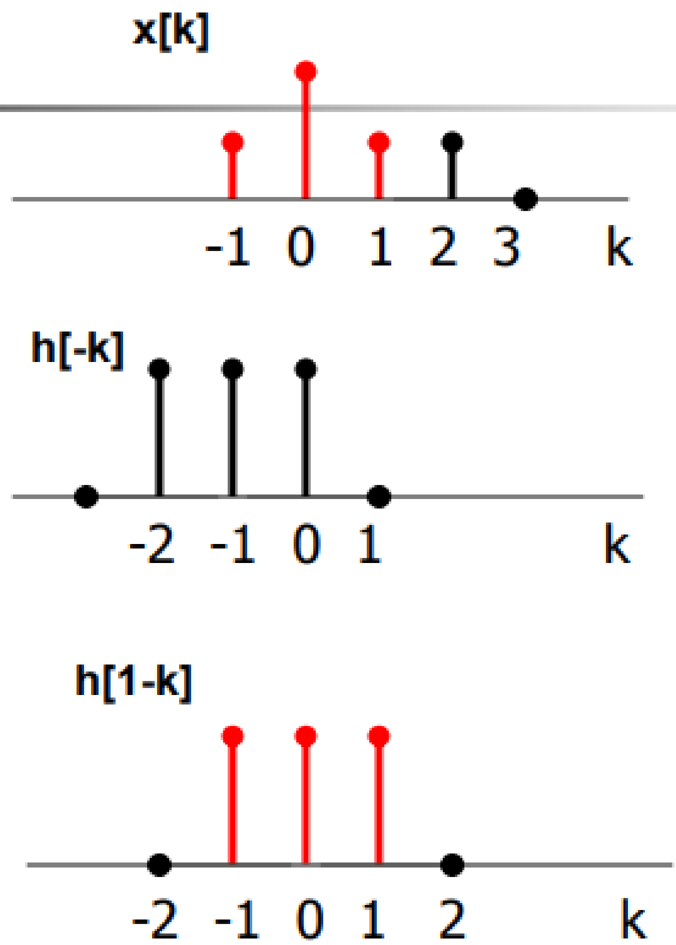






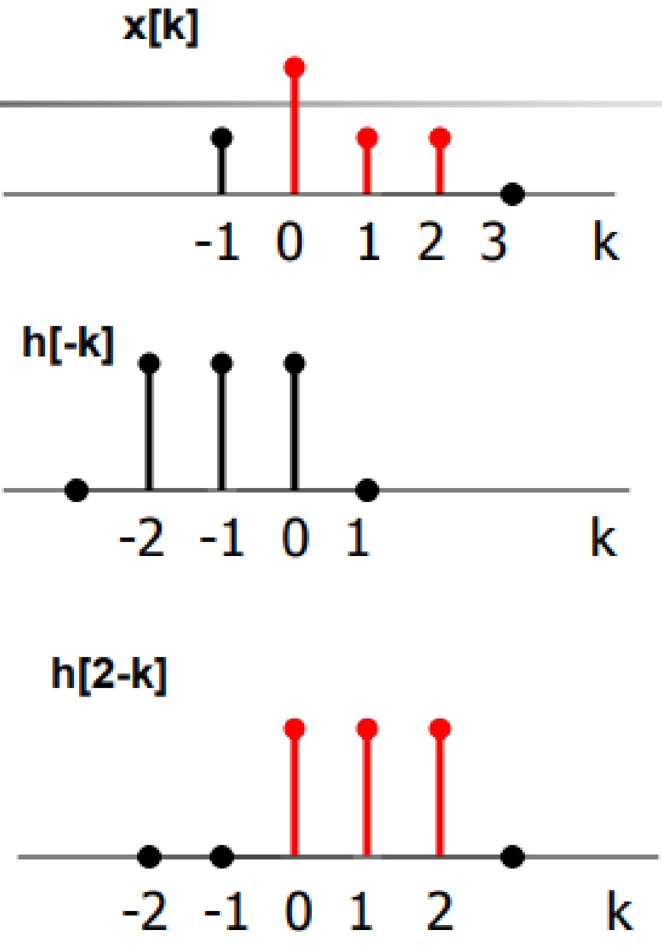
$y[0] = 6;$

Ex1 (cont)



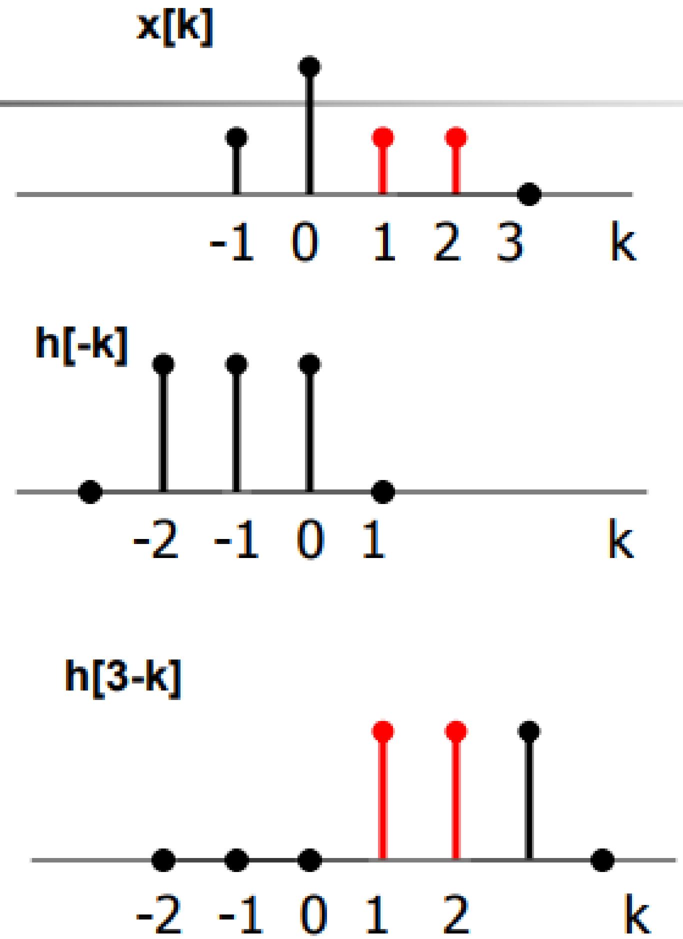
$y[1] = 8;$

Ex1 (cont)



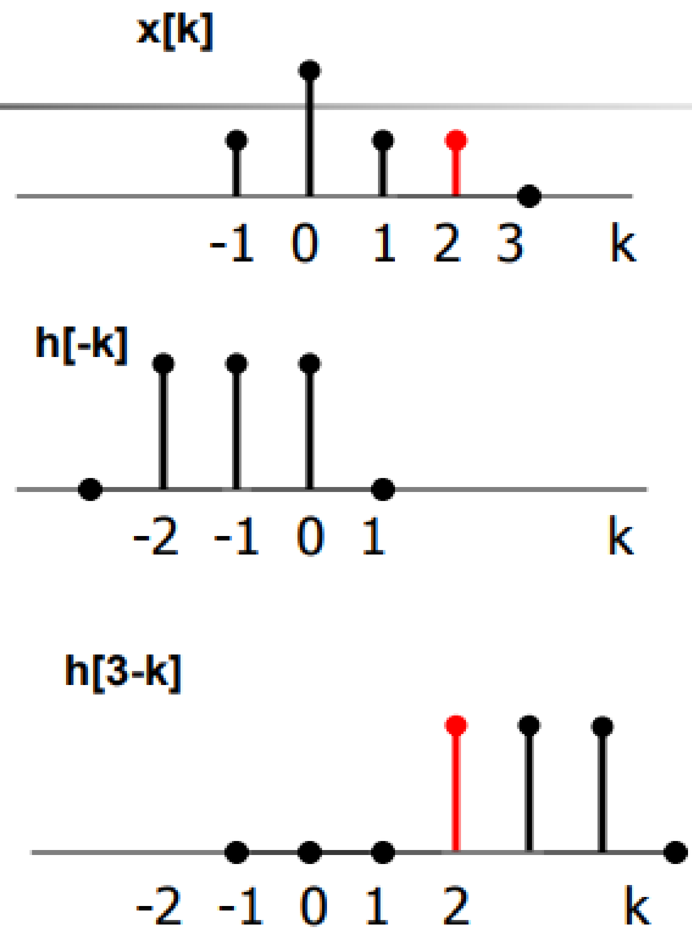
y[2] = 8;

Ex1 (cont)



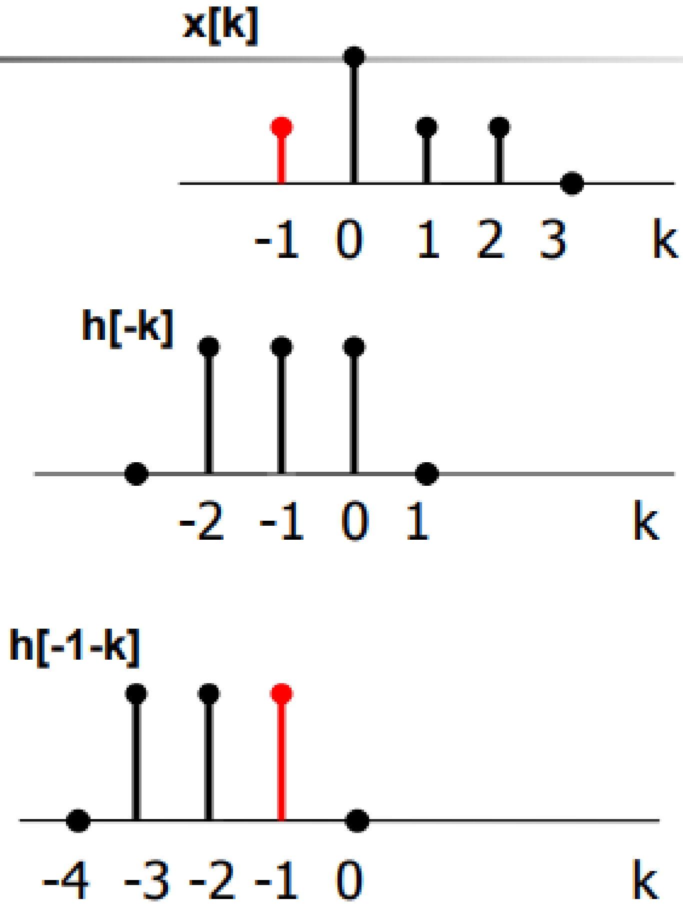
$y[3] = 4;$

Ex1 (cont)

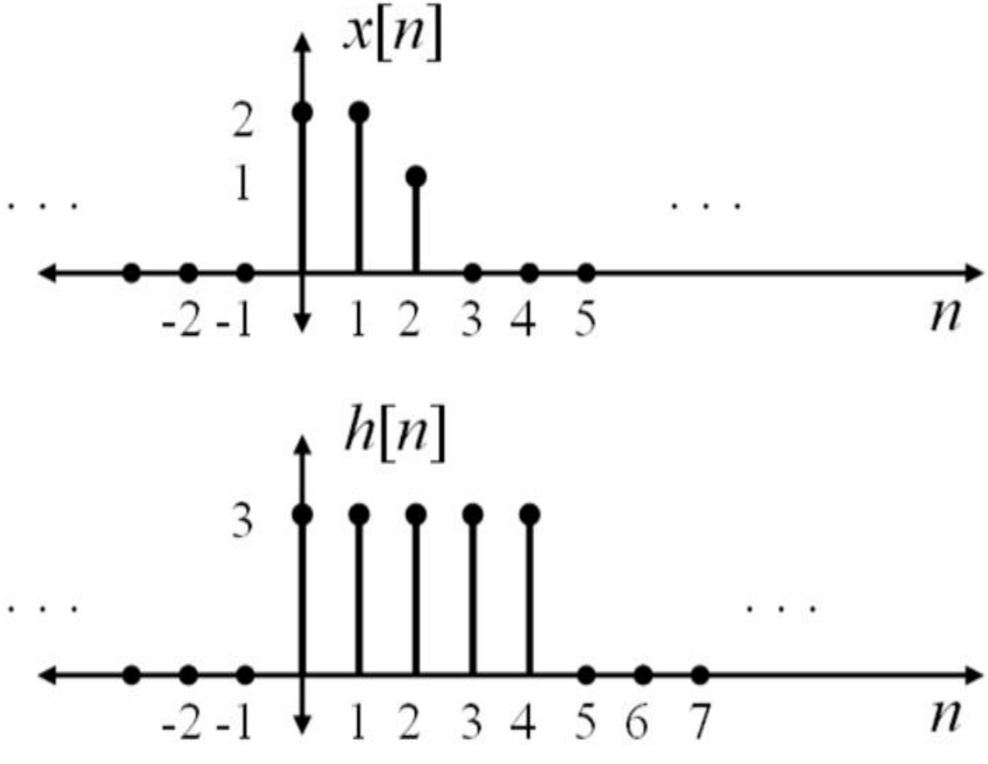


$y[4] = 2;$

Ex1 (cont)



y[-1] = 2;



x[n]
2
1
-2 -1 1 2 3 4 5
n
h[n]
3
-2 -1 1 2 3 4 5 6 7
n

Discrete-Time Convolution (Tabular Method)

1. $x_1[n] = \{1, \underset{\uparrow}{2}, -2\}$

   $x_2[n] = \{2, 0, \underset{\uparrow}{1}\}$

   $y[n] = x_1[n] * x_2[n] = ?$

[redacted]

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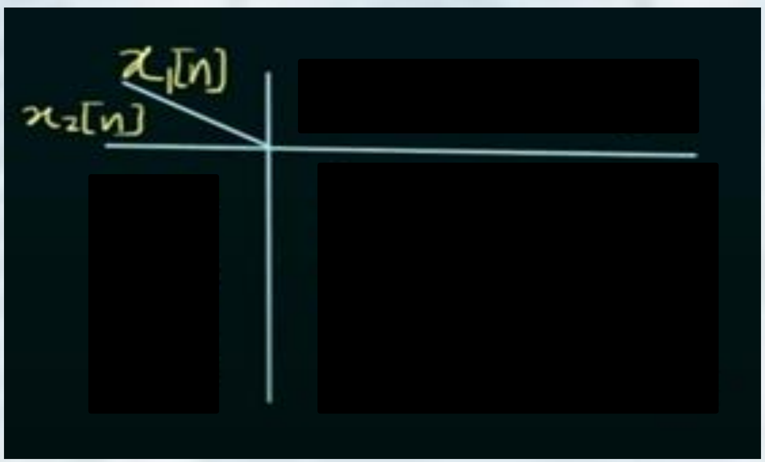

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[redacted]

$x_2[n]$ \ $x_1[n]$	1	2	−2
[redacted]	[redacted]		

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[redacted] 2	[redacted]		
0			
1			

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[redacted]	2	2	[redacted]	
	0			
	1			

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	1			

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1			

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	0	0	[redacted]	
	1	[redacted]		

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	0	0	0	[redacted]
	1	[redacted]		

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$y(t)=[2,\ 4,\ -3,\ \underset{\uparrow}{2},\ -2]$

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2. $x_1[n] = \{-1, \underset{\uparrow}{2}, 0, 1\}$

   $x_2[n] = \{3, 1, \underset{\uparrow}{0}, -1\}$

   $y[n] = x_1[n] * x_2[n] = ?$

$x_2[n]$ \ $x_1[n]$	1	2	−2
→ 2	2	4	−4
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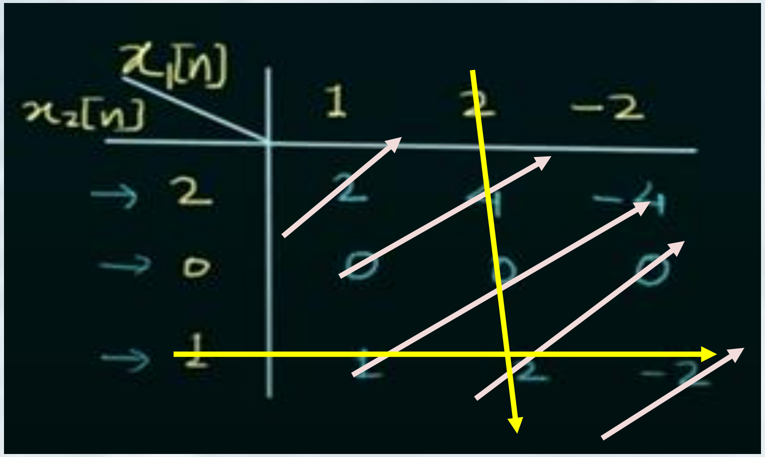
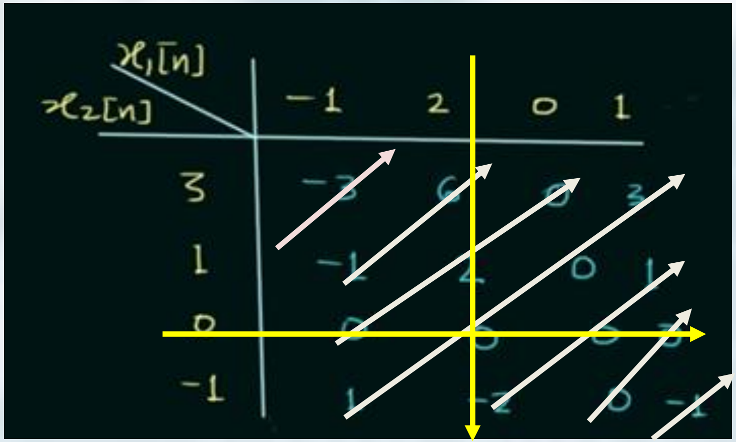


$y(t) = [2, 4, -3, \underset{\uparrow}{2}, -2]$



$y(t) = [-3, -5, 2, \underset{\uparrow}{4}, -1, 0, -1]$

Summing a Geometric Series

To sum these:

$$a + ar + ar^2 + \ldots + ar^{(n-1)}$$

(Each term is ar^k, where k starts at 0 and goes up to n-1)

We can use this handy formula:

$$\sum_{k=0}^{n-1}(ar^k) = a\left(\frac{1-r^n}{1-r}\right)$$

a is the first term

r is the **"common ratio"** between terms

n is the number of terms

EXAMPLE 2.1 Find the convolution of two finite duration sequences:

$$h(n) = a^n u(n) \quad \text{for all } n$$
$$x(n) = b^n u(n) \quad \text{for all } n$$

(i) When $a \neq b$

(ii) When $a = b$

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(ii) When $a = b$

Solution: The impulse response $h(n)$ and the input $x(n)$ are zero for $n < 0$, i.e. both $h(n)$ and $x(n)$ are causal.

$$\therefore \qquad y(n) = \sum_{k=0}^{n} x(k)h(n-k)$$

$$= \sum_{k=0}^{n} b^k a^{(n-k)} = a^n \sum_{k=0}^{n} \left(\frac{b}{a}\right)^k$$

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When $a = b$

$$y(n) = a^n[1 + 1 + 1 + \cdots + n + 1 \text{ terms}] = a^n(n + 1)$$

EXAMPLE 2.2 Find $y(n)$ if $x(n) = n + 3 \quad \text{for } 0 \leq n \leq 2$

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$$y(n) = \sum_{k=-\infty}^{\infty} x(k)h(n-k)$$

Given

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$h(n) = 0$ for $n < 0$, so the system is causal. $x(n)$ is a causal finite duration sequence whose value is zero for $n > 2$. Therefore,

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$$y(n) = \sum_{k=0}^{2} x(k)h(n-k)$$

$$= \sum_{k=0}^{2} (k+3)a^{n-k}u(n-k)$$

$$= 3a^n u(n) + 4a^{n-1}u(n-1) + 5a^{n-2}u(n-2)$$

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A causal signal is applied to a causal system

$$\therefore \qquad y(n) = \sum_{k=0}^{n} x(k)h(n-k) = \sum_{k=0}^{n} 3^k \left(\frac{1}{3}\right)^{n-k}$$

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$$= \left(\frac{1}{3}\right)^n \sum_{k=0}^{n} (3^2)^k$$

$$= \left(\frac{1}{3}\right)^n \left[\frac{1-(3^2)^{n+1}}{1-3^2}\right]$$

$$= \left(\frac{1}{3}\right)^n \left[\frac{9^{n+1}-1}{9-1}\right] = \left(\frac{1}{3}\right)^n \left[\frac{9^{n+1}-1}{8}\right]$$

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Since both $x(n)$ and $h(n)$ are causal, we have

$$y(n) = x(n) * h(n) = \sum_{k=-\infty}^{\infty} x(k)h(n-k) = \sum_{k=0}^{n} x(k)h(n-k)$$

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$$= 2^n \left[\frac{\left(\frac{3}{2}\right)^{n+1} - 1}{\frac{1}{2}}\right] = 2^{n+1}\left[\left(\frac{3}{2}\right)^{n+1} - 1\right]$$

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$$= \sum_{k=-\infty}^{\infty} (-1)^k u(k) \left(\frac{1}{2}\right)^{n-k} u(n-k)$$

$$= \sum_{k=0}^{n} (-1)^k \left(\frac{1}{2}\right)^{n-k} = \left(\frac{1}{2}\right)^n \sum_{k=0}^{n} (-1)^k (2)^k$$

$$= \left(\frac{1}{2}\right)^n \sum_{k=0}^{n} (-2)^k = \left(\frac{1}{2}\right)^n \frac{1-(-2)^{n+1}}{1-(-2)}$$

$$= \left(\frac{1}{2}\right)^n \left[\frac{1+2(-2)^n}{3}\right] = \frac{1}{3}\left(\frac{1}{2}\right)^n + \frac{2}{3}(-1)^n \quad \text{for } n > 0$$

$$= \frac{1}{3}\left(\frac{1}{2}\right)^n u(n) + \frac{2}{3}(-1)^n u(n)$$

EXAMPLE 2.6 Find the convolution of

$$x(n) = u(n), \quad h(n) = u(n-3)$$

Solution: Given $x(n) = u(n), \quad h(n) = u(n-3)$

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$$= \sum_{k=0}^{n-3} (1)(1) = n-2$$

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Circular Convolution

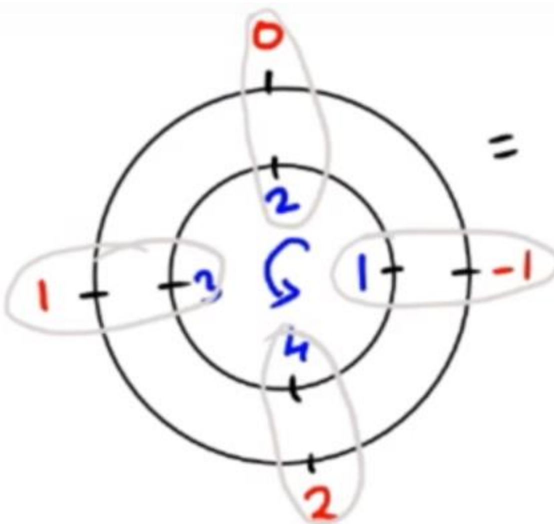
Circular Convolution as Linear Convolution with Aliasing

$$\mathbf{y[n] = \sum_{k=-\infty}^{k=\infty} x[k]h[((n-k))N]}$$

Circular Convolution

$x(n) = [1 \quad 2 \quad 3 \quad 4]$

$h(n) = [-1 \quad 2 \quad 1 \quad 0]$

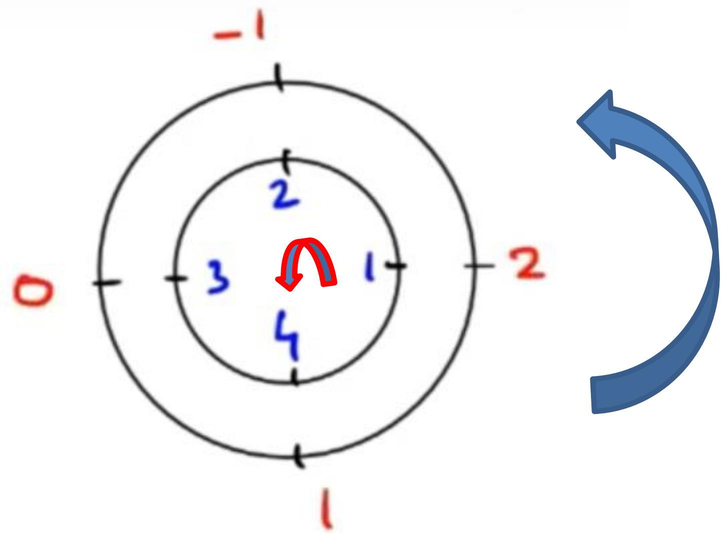


$= -1 + 0 + 3 + 8 = 10$

Circular Convolution

$x(n) = [1 \;\; 2 \;\; 3 \;\; 4]$

$h(n) = [-1 \;\; 2 \;\; 1 \;\; 0]$



$= 2 - 2 + 0 + 4 = 4$

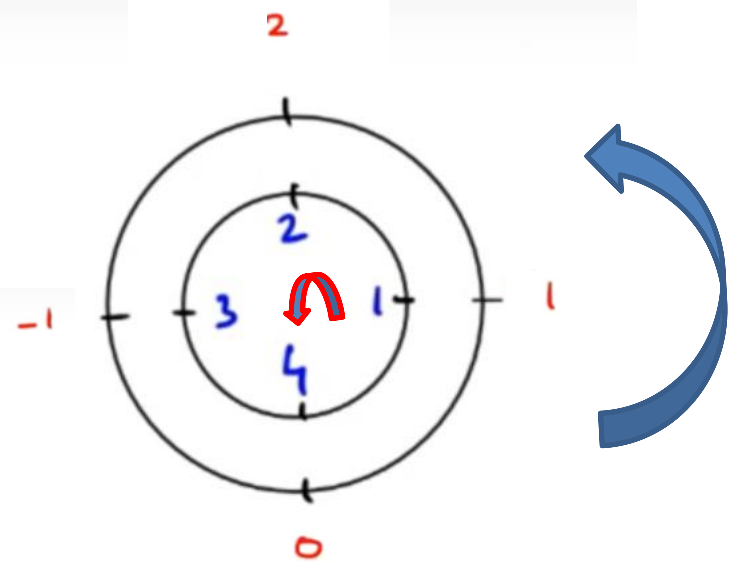
*** Inner Circle is kept as it is.**

*** Shift Outer Circle to anti clock wise direction by 1.**

Circular Convolution

$x(n) = [1 \quad 2 \quad 3 \quad 4]$

$h(n) = [-1 \quad 2 \quad 1 \quad 0]$



$= 1 + 4 - 3 + 0 = 2$

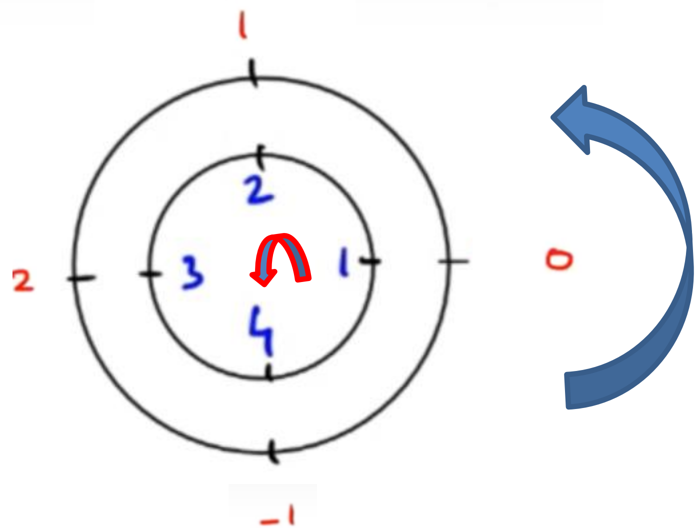
* **Inner Circle is kept as it is.**

* **Shift Outer Circle to anti clock wise direction by 1.**

Circular Convolution

$x(n) = [1 \quad 2 \quad 3 \quad 4]$

$h(n) = [-1 \quad 2 \quad 1 \quad 0]$



$= 0 + 2 + 6 - 4$
$= 4$

$$\mathbf{y(n) = [10, 4, 2, 4]}$$

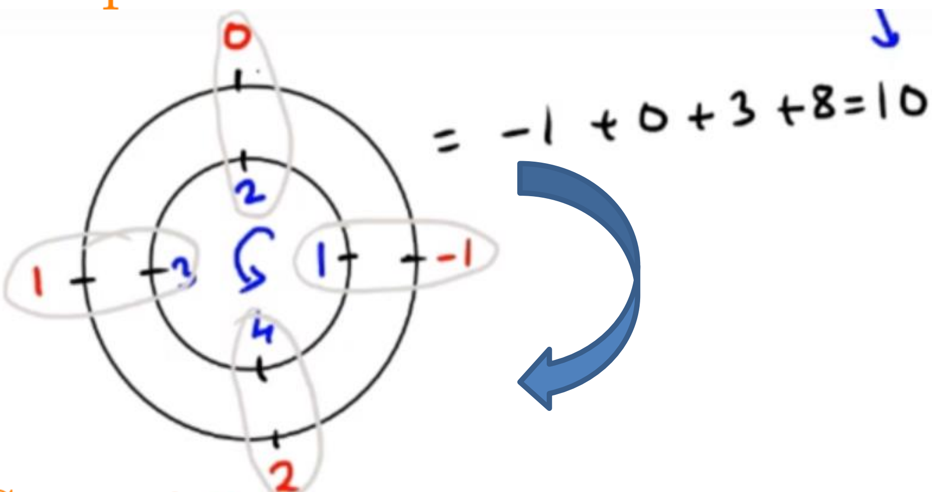
Circular Convolution

$x(n) = [1 \;\; 2 \;\; 3 \;\; 4]$

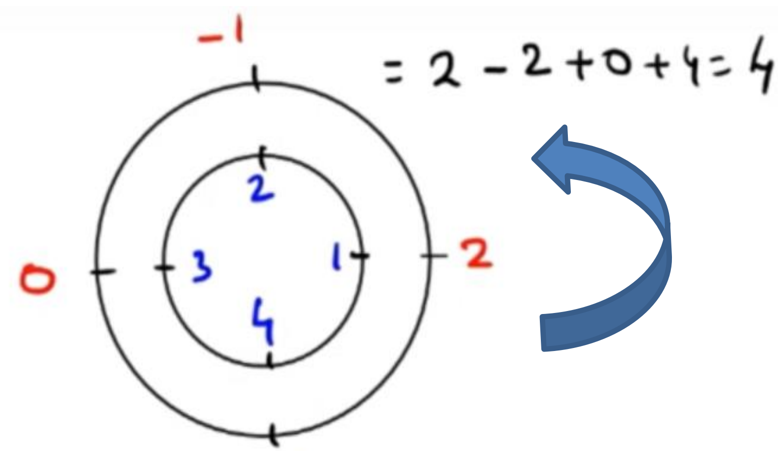
$h(n) = [-1 \;\; 2 \;\; 1 \;\; 0]$

$$\mathbf{y(n) = [10, 4, 2, 4]}$$

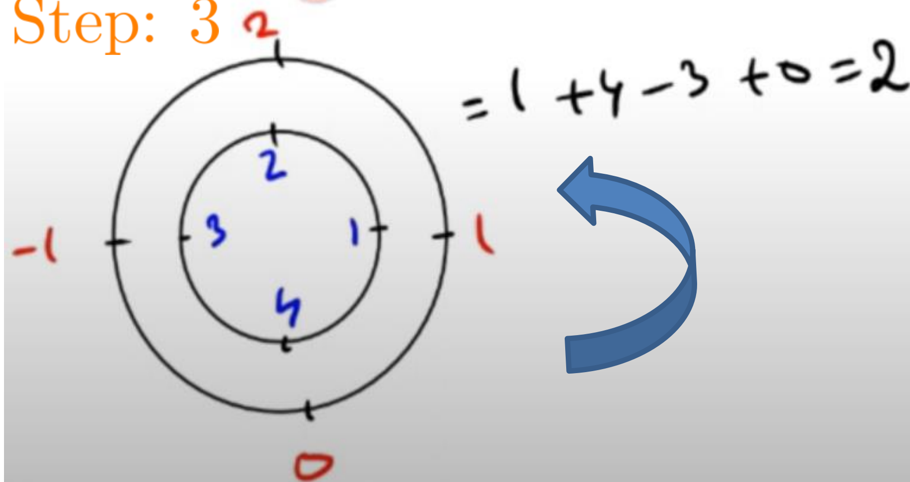
Step: 1



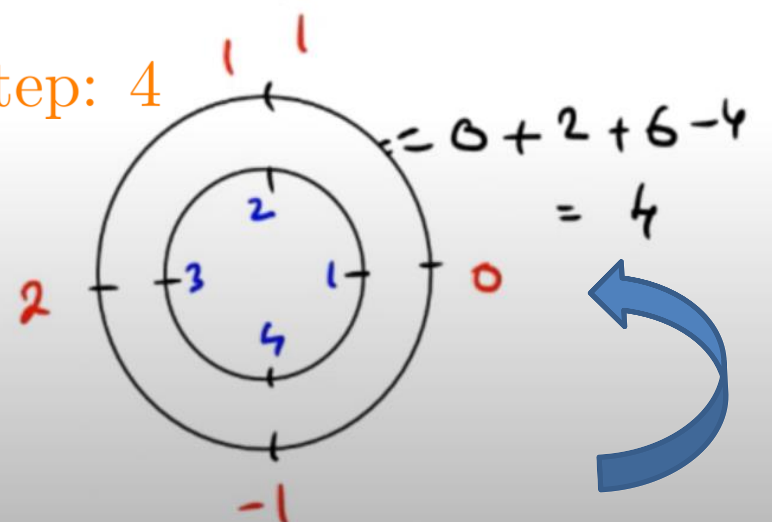
Step: 2



Step: 3



Step: 4



Perform circular convolution of the two sequences, $x_1(n) = \{2, 1, 2, -1\}$ and $x_2(n) = \{1, 2, 3, 4\}$

$$y(n) = \begin{bmatrix} 1 & 4 & 3 & 2 \\ 2 & 1 & 4 & 3 \\ 3 & 2 & 1 & 4 \\ 4 & 3 & 2 & 1 \end{bmatrix} \quad \begin{bmatrix} 2 \\ 1 \\ 2 \\ -1 \end{bmatrix}$$

Perform circular convolution of the two sequences, $x_1(n) = \{2, 1, 2, -1\}$ and $x_2(n) = \{1, 2, 3, 4\}$

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Y=[10, 10, 6, 14]

$$y[n] = \sum_{k=-\infty}^{k=\infty} x[k]h[((n-k))N]$$

$$\boxed{y[n] = f[n] \circledast h[n]}$$

Matrix Method

Ex. 3.5.5 : Determine the sequence

$$y(n) = x(n) \, \text{Ⓝ} \, h(n)$$

where $x(n) = \{1, 2, 3, 1\}$
$\uparrow$ (under 1)

and $h(n) = \{4, 3, 2, 2\}$
$\uparrow$ (under 4)

$$\mathbf{y[n]} = \sum_{\mathbf{k}=-\infty}^{\mathbf{k}=\infty} \mathbf{x[k]h[((n-k))N]}$$

$$y[n] = f[n] \circledast h[n]$$

Matrix Method

Ex. 3.5.5 : Determine the sequence

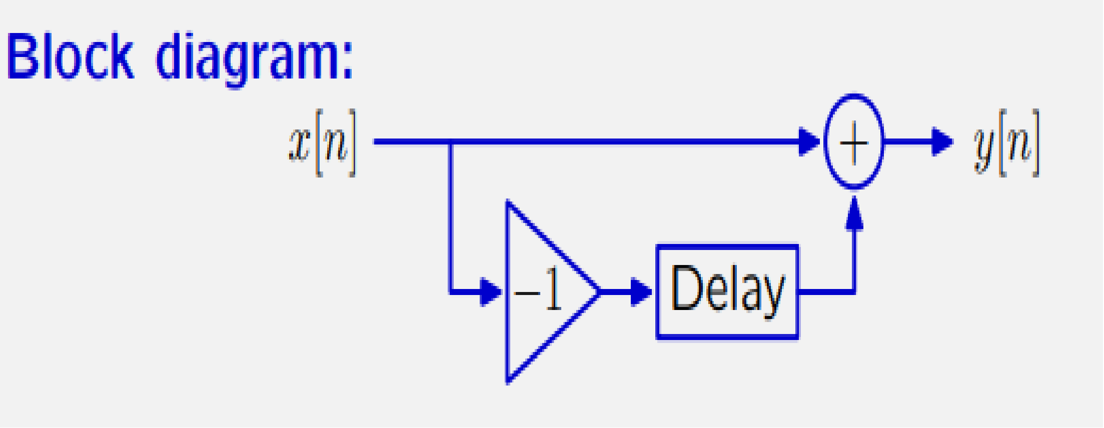
$$y(n) = x(n) \; \text{Ⓝ} \; h(n)$$

where $x(n) = \{\underset{\uparrow}{1}, 2, 3, 1\}$

and $h(n) = \{\underset{\uparrow}{4}, 3, 2, 2\}$

Ans: $\begin{bmatrix} 17 \\ 19 \\ 22 \\ 19 \end{bmatrix}$

Revision



Difference equation:

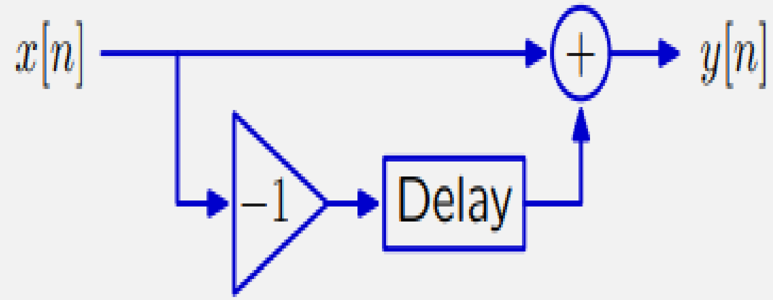
$$y[n] = x[n] - x[n-1]$$

Find impulse response of the system: h(n)=?

Find $y(n)$ **when** $x[n] = [1, 3, 0, 5, 2]$

Revision

Block diagram:



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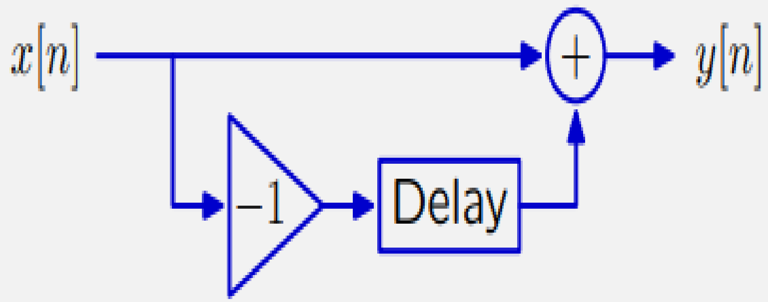
Block diagram:



Difference equation:

$$y[n] = x[n] - x[n-1]$$

h(n)=[1, -1]

y(n)=[1, 2, -3, 5, -3, -2]

Summing a Geometric Series

To sum these:

$$a + ar + ar^2 + \ldots + ar^{(n-1)}$$

(Each term is ar^k, where k starts at 0 and goes up to n-1)

We can use this handy formula:

$$\sum_{k=0}^{n-1}(ar^k) = a\left(\frac{1-r^n}{1-r}\right)$$

a is the first term

r is the **"common ratio"** between terms

n is the number of terms

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$$\sum_{k=0}^{\infty}(ar^k) = a\left(\frac{1}{1-r}\right) \quad \textbf{If r < 1}$$

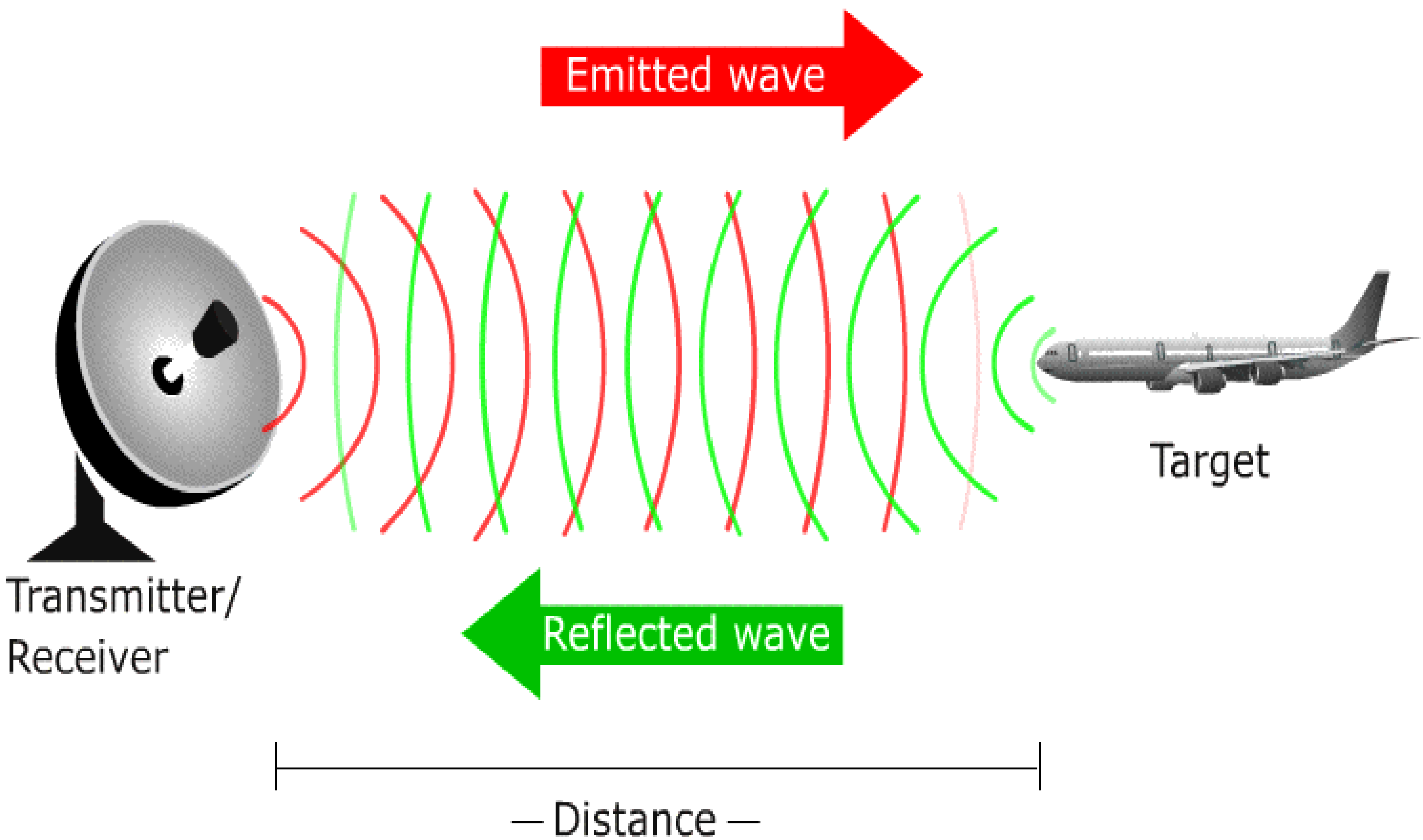
a is the first term

r is the "**common ratio**" between terms

n is the number of terms

Correlation of Discrete Time Signal

Correlation of Discrete Time Signal



Correlation of Discrete Time Signal

- **Correlation mathematically measures the similarity of signals.**

- **If similarity between tow different signal is measured it is called Cross-Correlation.**

- **If similarity between tow same signal is measured it is called Auto-Correlation.**

Correlation of Discrete Time Signal

❖ **If similarity between tow different signal is measured it is called Cross-Correlation.**

Suppose that we have two real signal sequences $x(n)$ and $y(n)$ each of which has finite energy. The *crosscorrelation* of $x(n)$ and $y(n)$ is a sequence $r_{xy}(l)$, which is defined as

$$\mathbf{r_{xy}(l) = \sum_{n=-\infty}^{\infty} x[n]y(n-l), \quad l = 0, \pm 1, \pm 2, \ldots}$$

or, equivalently, as

$$\mathbf{r_{xy}(l) = \sum_{n=-\infty}^{\infty} x[n+l]y(n), \quad l = 0, \pm 1, \pm 2, \ldots}$$

Correlation of Discrete Time Signal

- **If similarity between tow same signal is measured it is called Auto-Correlation.**

In the special case where $y(n) = x(n)$, we have the *autocorrelation* of $x(n)$, which is defined as the sequence

$$r_{xx}(l) = \sum_{n=-\infty}^{\infty} x[n]x(n-l), \quad l = 0, \pm 1, \pm 2, \ldots$$

Or equivalent

$$r_{xx}(l) = \sum_{n=-\infty}^{\infty} x[n+l]x(n), \quad l = 0, \pm 1, \pm 2, \ldots$$

Correlation of Discrete Time Signal

In dealing with finite-duration sequences, it is customary to express the autocorrelation and crosscorrelation in terms of the finite limits on the summation. In particular, if $x(n)$ and $y(n)$ are causal sequences of length N [i.e., $x(n) = y(n) = 0$ for $n < 0$ and $n \geq N$], the crosscorrelation and autocorrelation sequences may be expressed as

$$\mathbf{r_{xy}(l) = \sum_{n=i}^{N-|k|-1} x[n]y(n-l)} \qquad (2.6.11)$$

and

$$\mathbf{r_{xx}(l) = \sum_{n=i}^{N-|k|-1} x[n]x(n-l)} \qquad (2.6.12)$$

where $i = l$, $k = 0$ for $l \geq 0$, and $i = 0$, $k = l$ for $l < 0$.

Perform the correlation of the two sequences, $x(n) = \{1, 2, 3\}$ *and* $y(n) = \{2, 4, 1\}$.

Perform the correlation of the two sequences, $x(n) = \{1, 2, 3\}$ and $y(n) = \{2, 4, 1\}$.

Solution

Given that, $x(n) = \{1, 2, 3\}$ and $y(n) = \{2, 4, 1\}$. \ $y(-n) = \{1, 4, 2\}$

The sequence $x(n)$ is arranged as a column and the folded sequence $y(-n)$ is arranged as a row as shown below. The elements of the two dimensional array are obtained by multiplying the corresponding row element with column element. The sum of the diagonal elements gives the samples of the crosscorrelation sequence, $r_{xy}(m)$.

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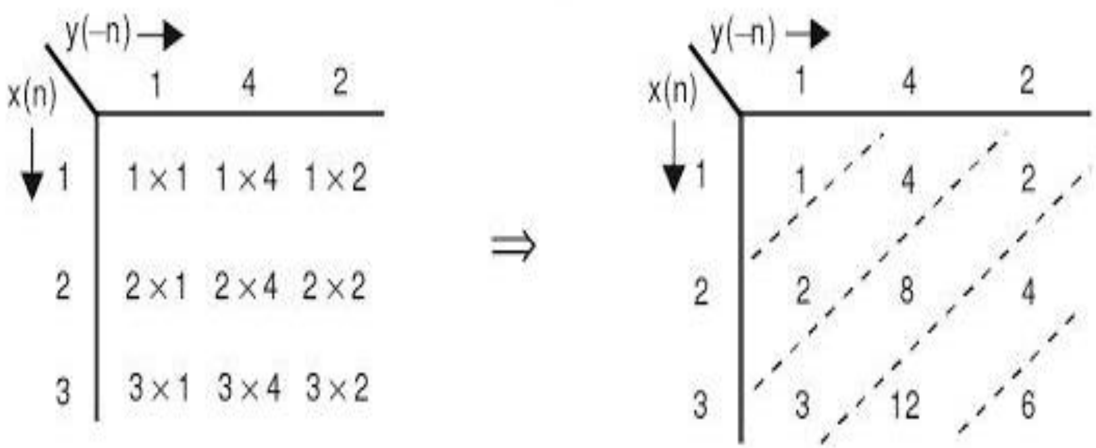


$r_{xy}(m) = \{1, 6, 13, 16, 6\}$

How do you find the origine??

Perform the circular correlation of the two sequences, $x(n) = \{1, 2, 3\}$ and $y(n) = \{2, 4, 1\}$.

Solution

Let $\bar{r}_{xy}(m)$ be the sequence obtained from circular correlation of x(n) and y(n). The sequence x(n) can be arranged as a column vector of order 3 ´1 and using the samples of y(n) a 3 ´3 matrix is formed as shown below. The product of two matrices gives the sequence $\bar{r}_{xy}(m)$.

$$\begin{bmatrix} y(0) & y(1) & y(2) \\ y(2) & y(0) & y(1) \\ y(1) & y(2) & y(0) \end{bmatrix} \begin{bmatrix} x(0) \\ x(1) \\ x(2) \end{bmatrix} = \begin{bmatrix} \bar{r}_{xy}(0) \\ \bar{r}_{xy}(1) \\ \bar{r}_{xy}(2) \end{bmatrix} \quad \Rightarrow \quad \begin{bmatrix} 2 & 4 & 1 \\ 1 & 2 & 4 \\ 4 & 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 13 \\ 17 \\ 12 \end{bmatrix}$$

$$\backslash \; \bar{r}_{xy}(m) = \{13, 17, 12\}$$

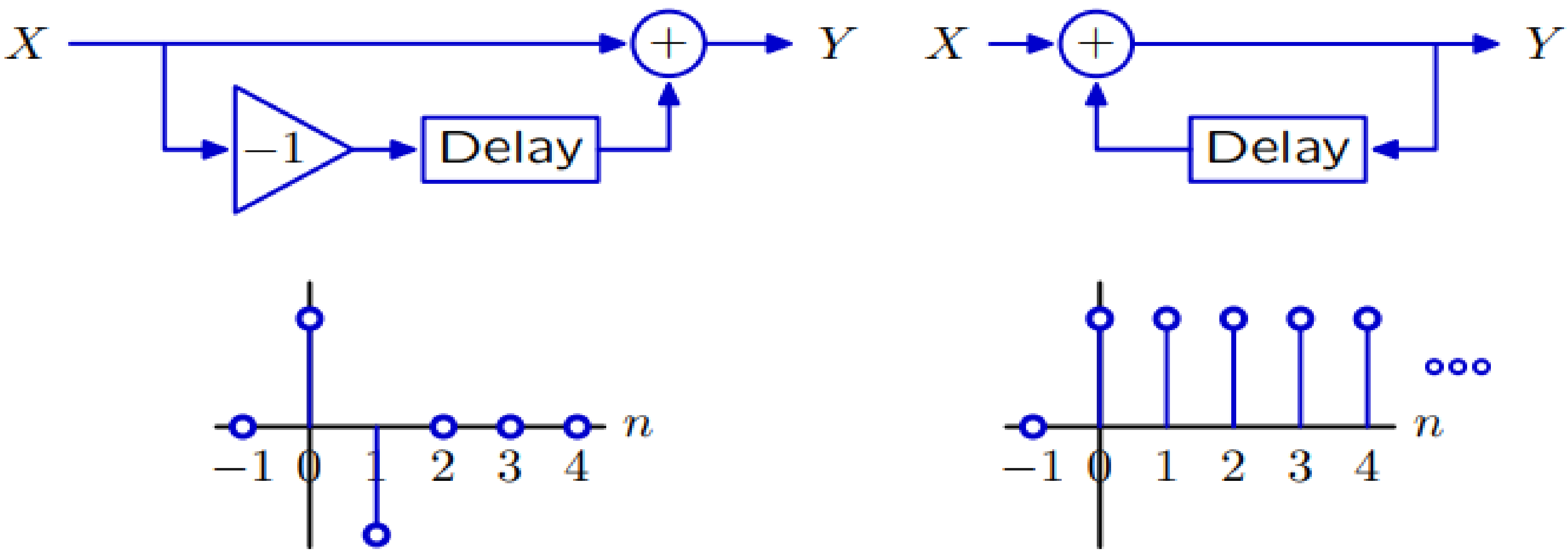
Classification of Discrete Time System

The discrete time systems are classified based on their characteristics. Some of the classifications of discrete time systems are,

1. Static and dynamic systems
2. Time invariant and time variant systems
3. Linear and nonlinear systems
4. Causal and noncausal systems
5. Stable and unstable systems
6. FIR and IIR systems
7. Recursive and nonrecursive systems

Finite and Infinite Impulse Responses

The impulse response of an acyclic system has finite duration, while that of a cyclic system can have infinite duration.



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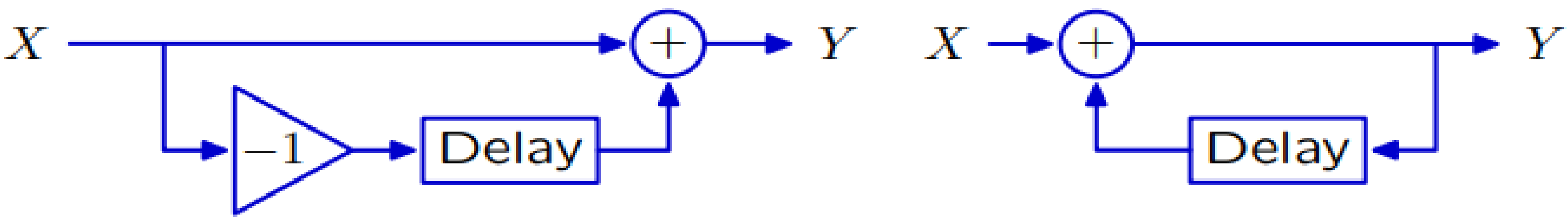
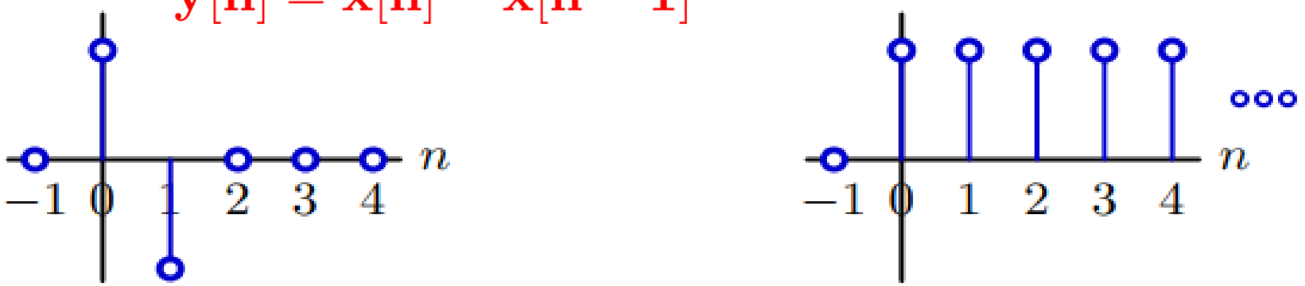


$$\mathbf{y[n] = x[n] - x[n-1]}$$



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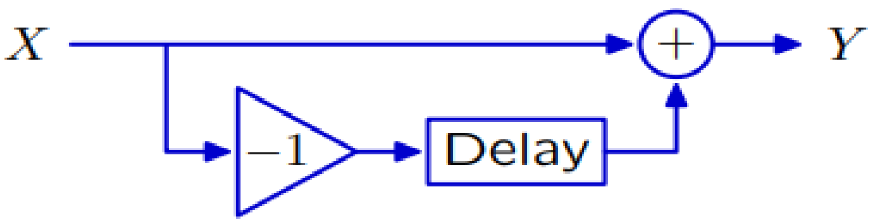
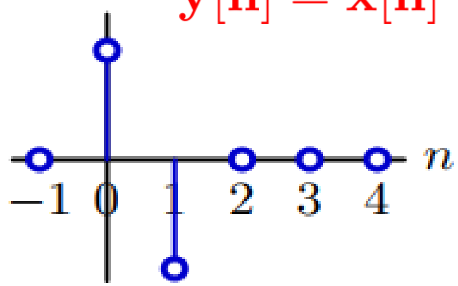
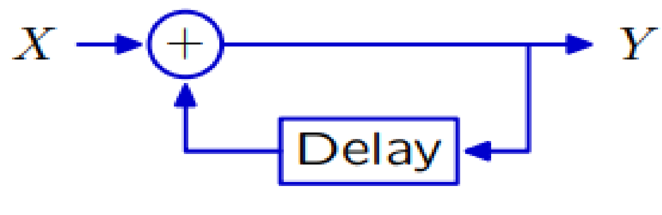
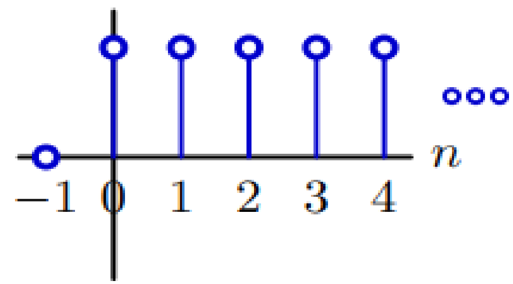




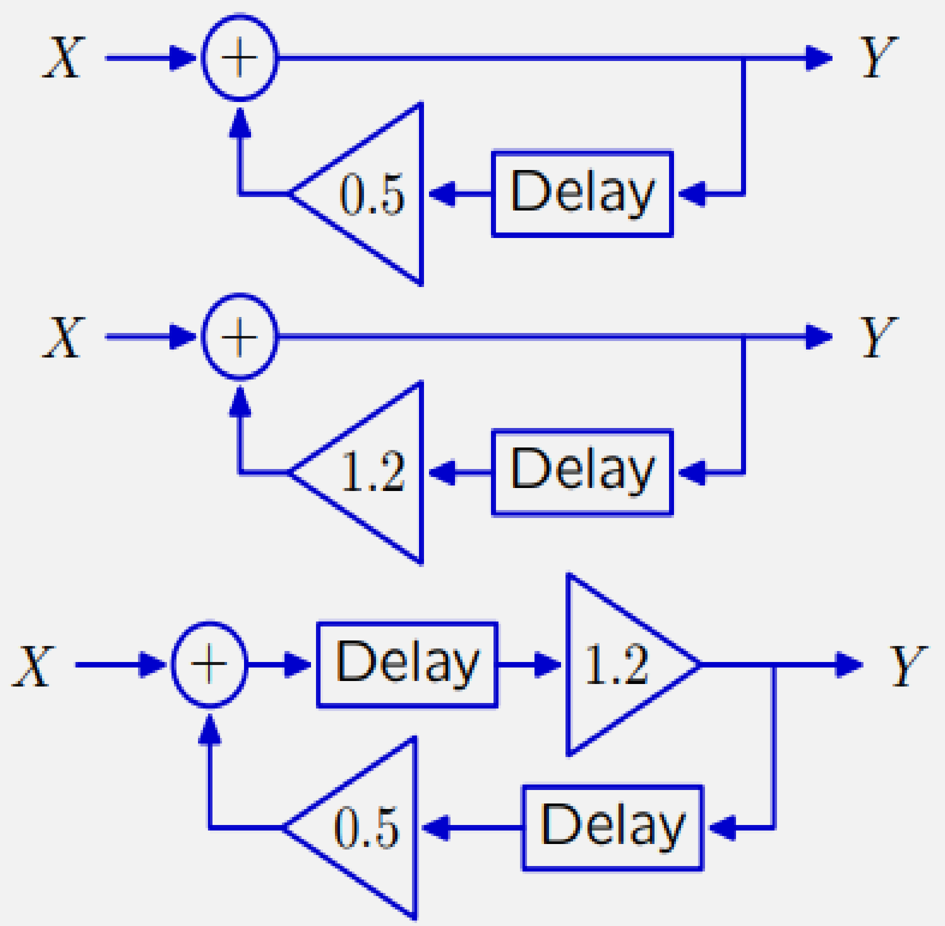
$$\mathbf{y[n] = x[n] - x[n-1]}$$

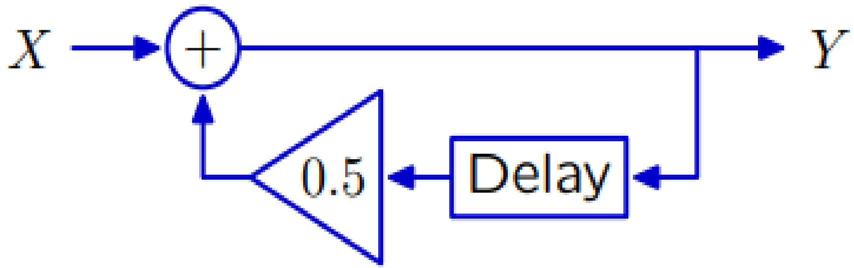


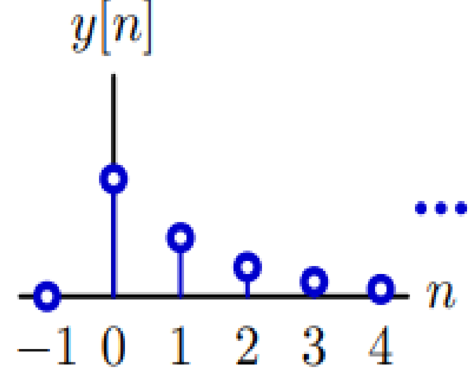


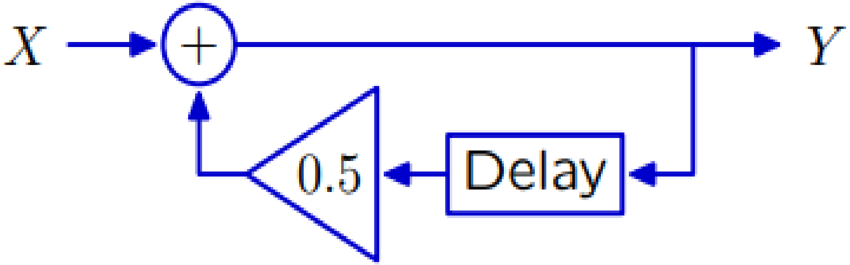
$$\mathbf{y[n] = x[n] + y[n-1]}$$

How many of these systems have divergent unit-sample responses?




X
+
Y
0.5
Delay


y[n]
n
−1
0
1
2
3
4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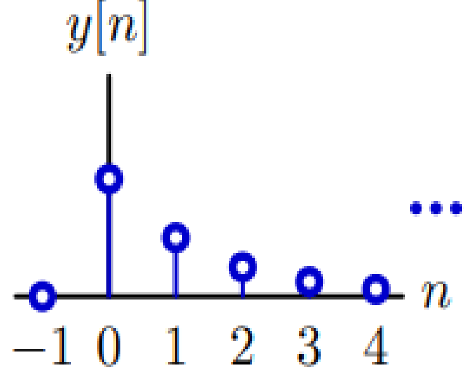
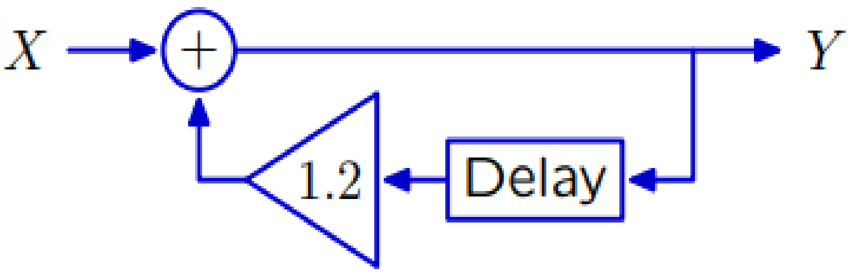

y[n]
...
n
−1 0 1 2 3 4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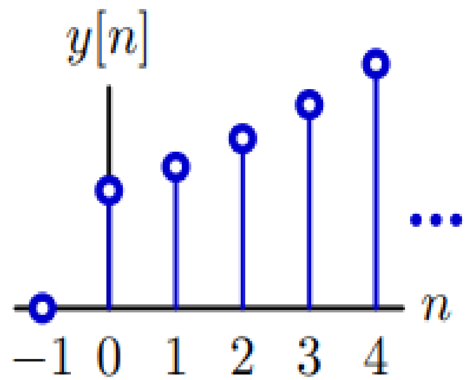
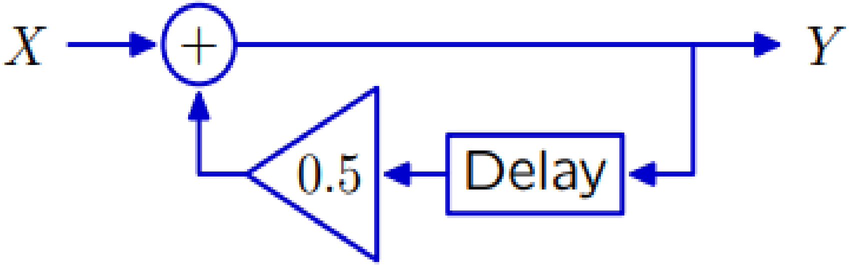
1.2
Delay
Y


y[n]
...
n
−1 0 1 2 3 4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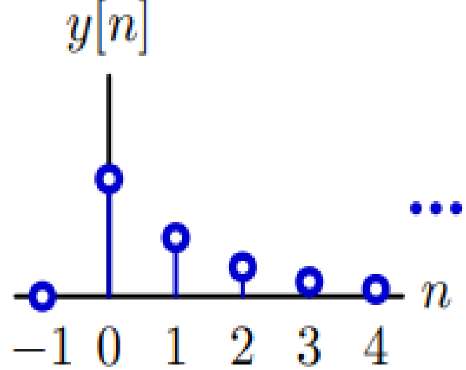
y[n]
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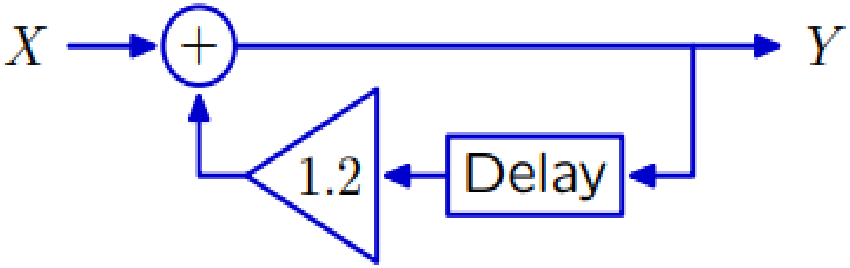
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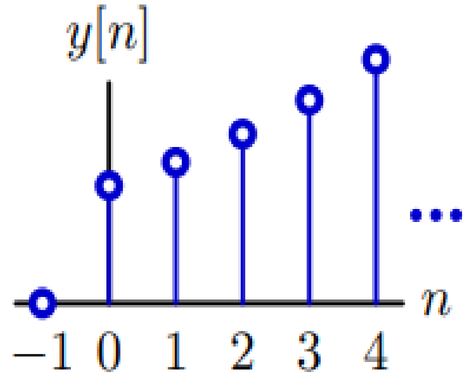
Delay


y[n]
n
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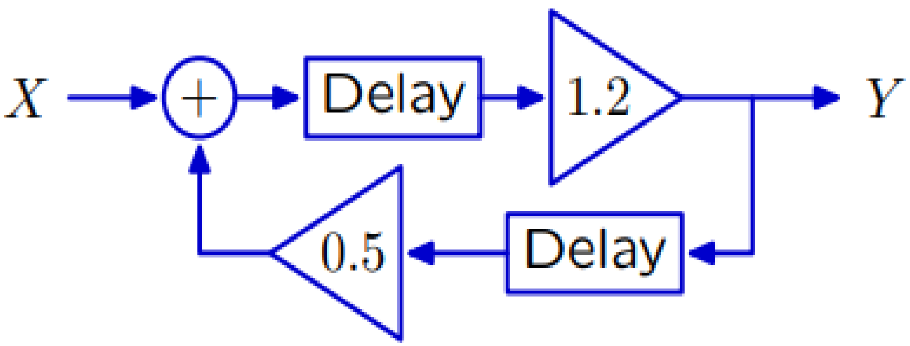
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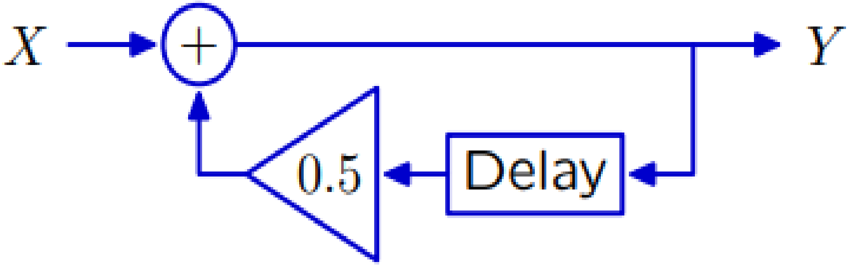
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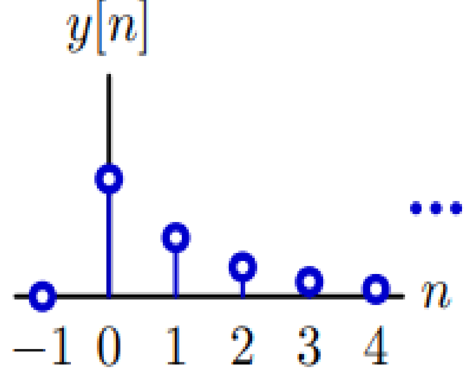
Delay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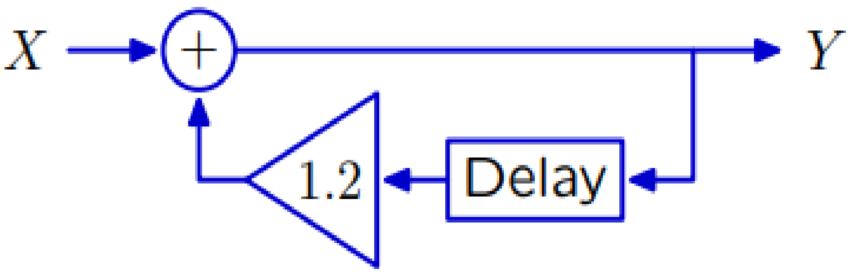
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Y
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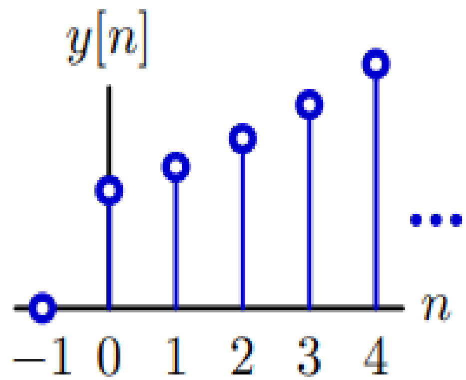
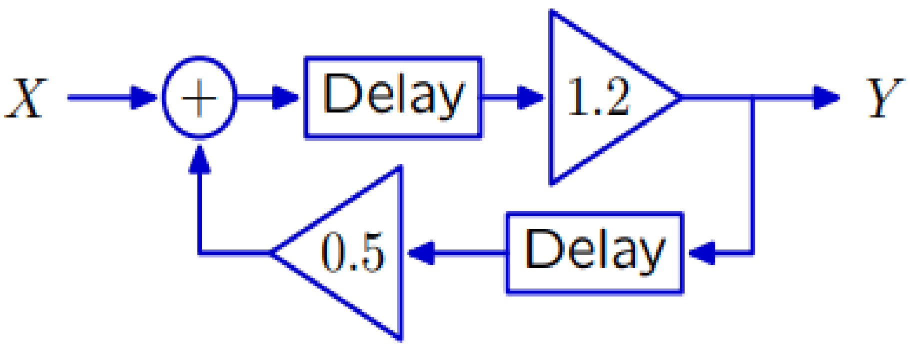
Delay


y[n]
n
−1 0 1 2 3 4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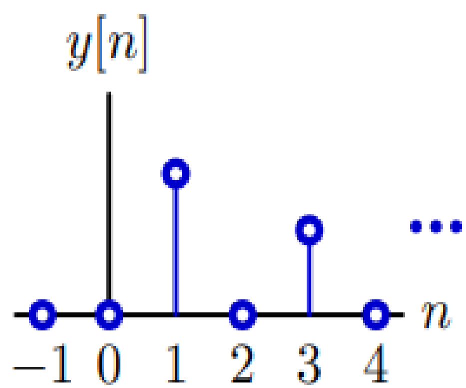
Delay
1.2
Y
0.5
Delay


y[n]
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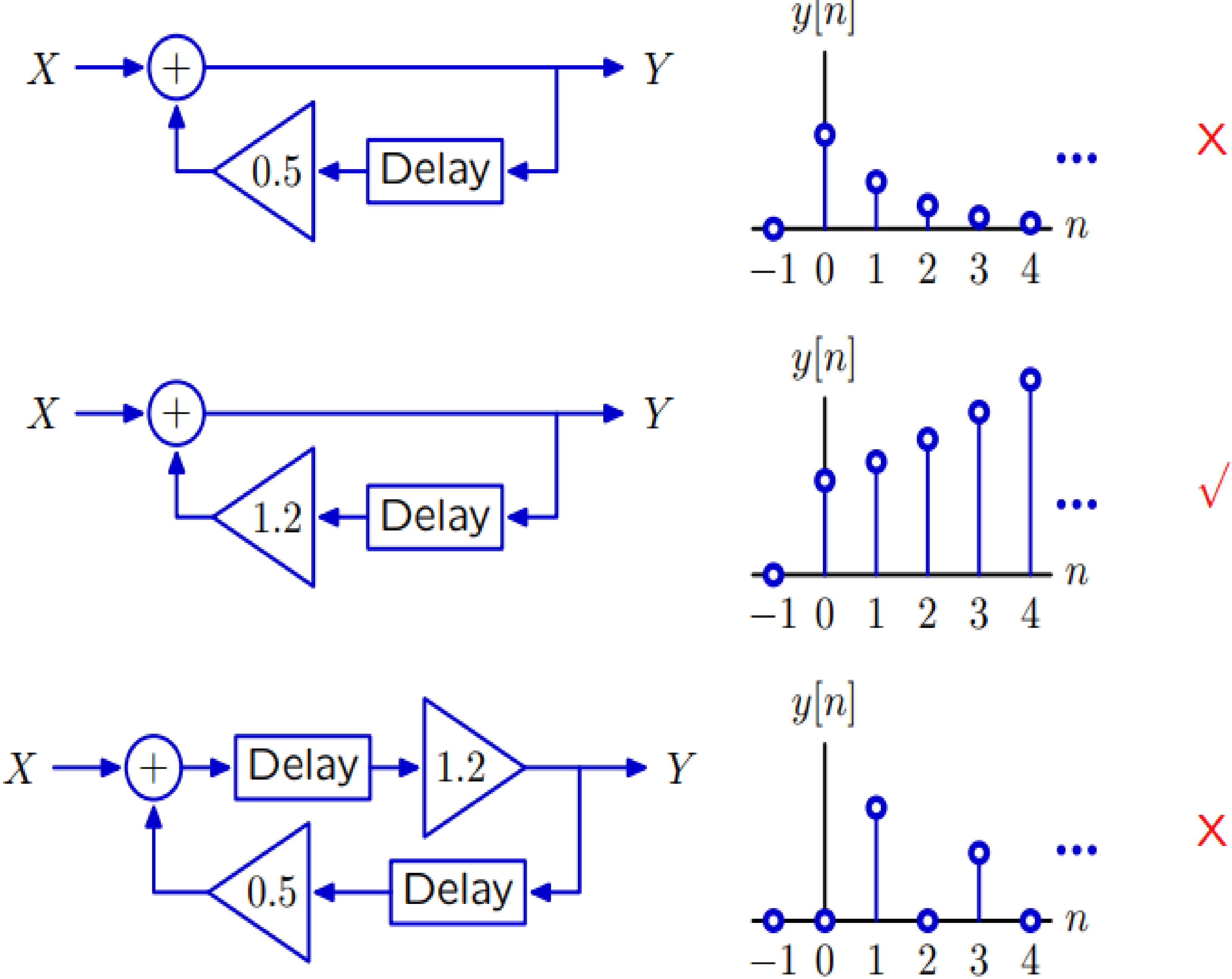
X
Y
0.5
Delay
y[n]
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X
1.2
Delay
Y
y[n]
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−1 0 1 2 3 4
√
X
Delay
1.2
Y
0.5
Delay
y[n]
n
−1 0 1 2 3 4
X