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Modern Control Theory
(ICE 3153)

Stability Analysis

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What is eigenvector and eigenvalue ??

Suppose $Ax=y$, where,

A is (n x n) matrix,

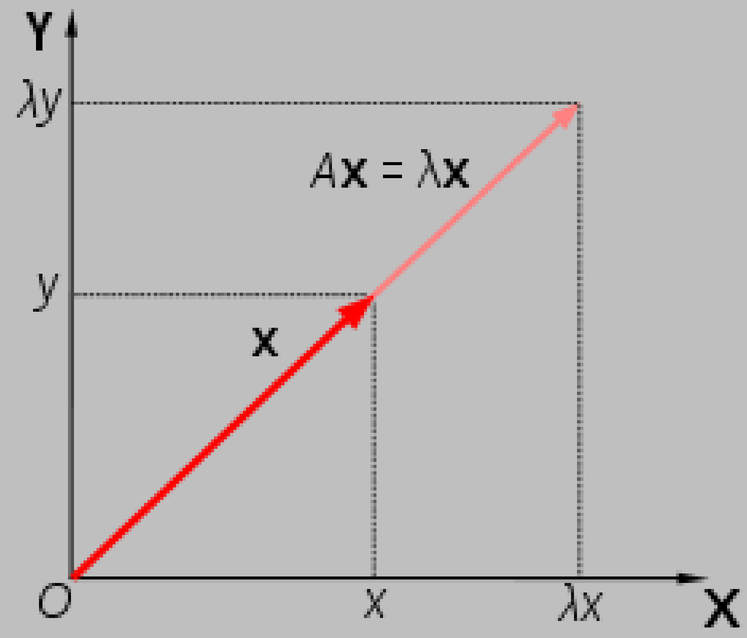
x is (n x 1) , y is (n x 1) vector

From above equation, we can say that (n x n) matrix operator A operate on (n x1) vector x, we get new transform (n x1) vector y.

- Eigenvectors of system matrix A are all vectors $x_i \neq 0$ which under the transformation of matrix A becomes multiples of multiples of themselves.

- That means Eigen Vectors are non zero.

- $Ax_i = \lambda x_i$



The eigenvector can be defined as a vector "x" such that the matrix operator (A matrix) transform it to a vector λx. This vector has the same direction in state space as vector x

Eigenvalues :

The eigenvalues of the matrix [A] are the values of λ that satisfy the equation

Condition $x_i \neq 0$

$$Ax_i = \lambda_i x_i$$

Why the eigenvalue of a system matrix is determined as $|A - \lambda I| = 0$?

Eigenvector

A nonzero column vector X is an eigenvector of a square matrix A, if there exists a scalar λ such that AX = λX, then λ is a eigenvalue (or characteristic value) of A.

An eigenvalue may be zero but the corresponding vector may not be zero.

Eigenvalue

The eigenvalues of an n*n matrix **A** are the roots of the characteristic equation:

$$|\lambda \mathbf{I} - \mathbf{A}| = 0$$

The eigenvalues are also called the characteristic roots.

Eigenvector of Matrix **A**

$$\mathbf{A}\mathbf{x} = \lambda\mathbf{x}$$

Eigenvalue of Matrix **A**

Properties of Eigenvalues

- Any general matrix

If the coefficients of the matrix [A] are real, then it is Eigenvalues are always real or complex conjugate pair.

- Real symmetric Matrix

If the matrix [A] is real, symmetric (row and column elements are same), then the Eigenvalues are always real no complex conjugate Eigenvalues.

- Eigenvalues of matrix [A] and its transpose matrix [A^T] are always the same.
- Eigenvalues of matrix [A] and it is inverse

If A has Eigen values λ_1 , i= 1,2 3…n

Then Eigen values of A invers is $A = \frac{1}{\lambda_i}$

- Eigenvalues and Determinant

The product of the Eigen values of a matrix equals the determinant of the matrix

- Trace Eigenvalues and diagonal elements

Sum of all eigenvalues of a matrix is called trace of the matrix.

Sum of diagonal elements is also called as trace of the matrix.

- Singular matrix and Eigenvalues

A matrix is singular, if and only it has zero Eigenvalues

- Zero Eigenvector and Zero Eigenvalue
- Eigenvalue can be Zero, but Eigenvector cannot be a Zero vector

- Same Eigenvector cannot be associated with different Eigenvalues.

Determination of eigenvectors

Case 1: Distinct eigenvalues

Case 2: Multiple eigenvalues

Example 1:- find the Eigen values and Eigen vectors for the given matrix

$$[A]=\begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$$

Example 2:- find the Eigen values and Eigen vectors for the given matrix

$$[A]=\begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix}$$

Concept of Stability in State Space

- The total response of a system is a sum of natural response and a forced response
- Natural response describes the way the system dissipates or
- acquires energy.
- For a control system to be useful the natural response must eventually approaches to zero or vanish, thus leaving only, the force response or some sort of oscillatory response.
- if the natural response is greater than the forced response, then system is no longer control, this condition is called instability

- **Stable:-** A linear time invariant system is stable; if the natural response approaches to zero as time approaches to infinity.

- **Unstable:-** A linear time invariant system is unstable, if the natural response grows without bound as the time approaches infinity.

- **Marginally stable:-** A linear time invariant system is marginally stable, if the natural response neither decays nor grows, but remains constant or oscillate as time approaches infinity.