Basic Electrical Technology

[ELE 1051]

SINGLE PHASE AC CIRCUITS

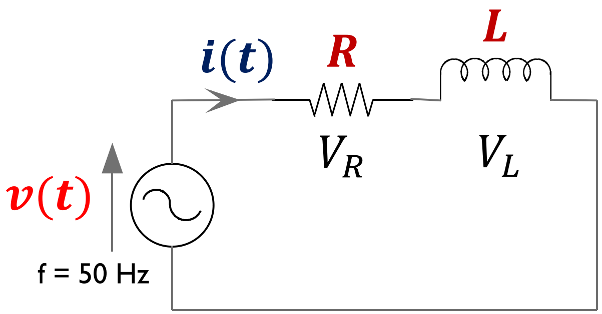
Recap

- Average value of an alternating waveform
- RMS value of an alternating waveform
- Representing AC
- R, L, C circuit response with AC supply
- Power associated with a pure R, L, C

Topics covered…

- RL, RC, RLC circuit response with AC supply
- Power associated with a series RL, RC circuits
- Loads in parallel

RL circuit analysis



Let $\bar{I}$ *be along the reference*

$$\overline{V_R} = \bar{I}R$$

$$\overline{V_L} = j\bar{I}X_L$$

$$\bar{V} = \overline{V_R} + \overline{V_L} = |V|\angle\emptyset$$

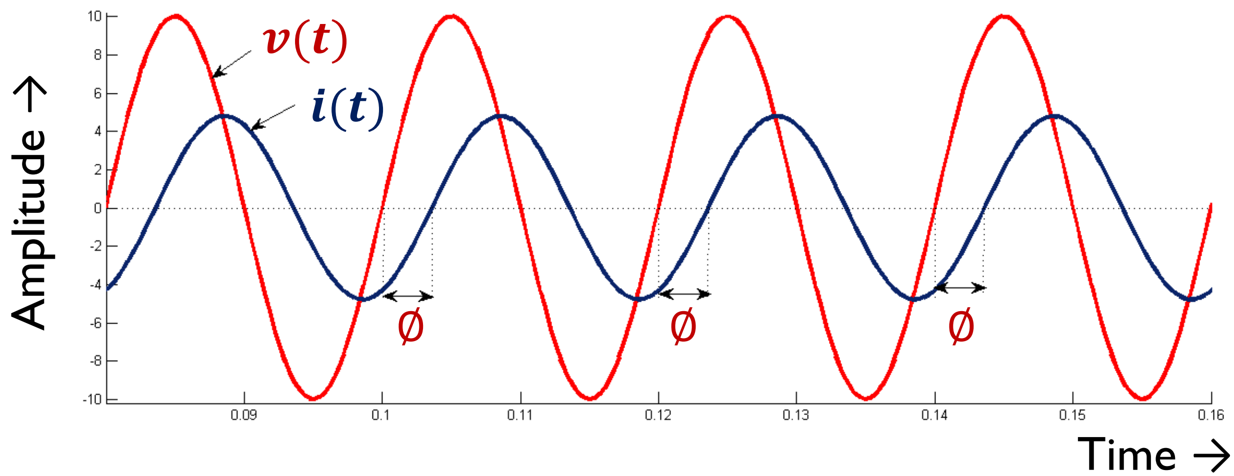
Mathematical Representation

$$i(t) = I_m \sin(\omega t)$$

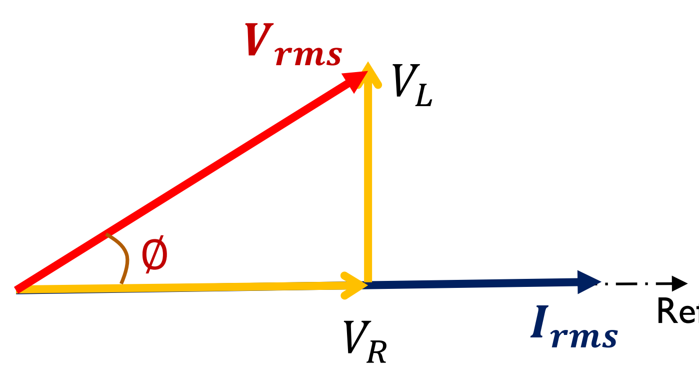
$$v(t) = V_m \sin(\omega t + \emptyset)$$

$\emptyset$ – *Phase Angle*

Graphical Representation



Phasor Representation



Impedance

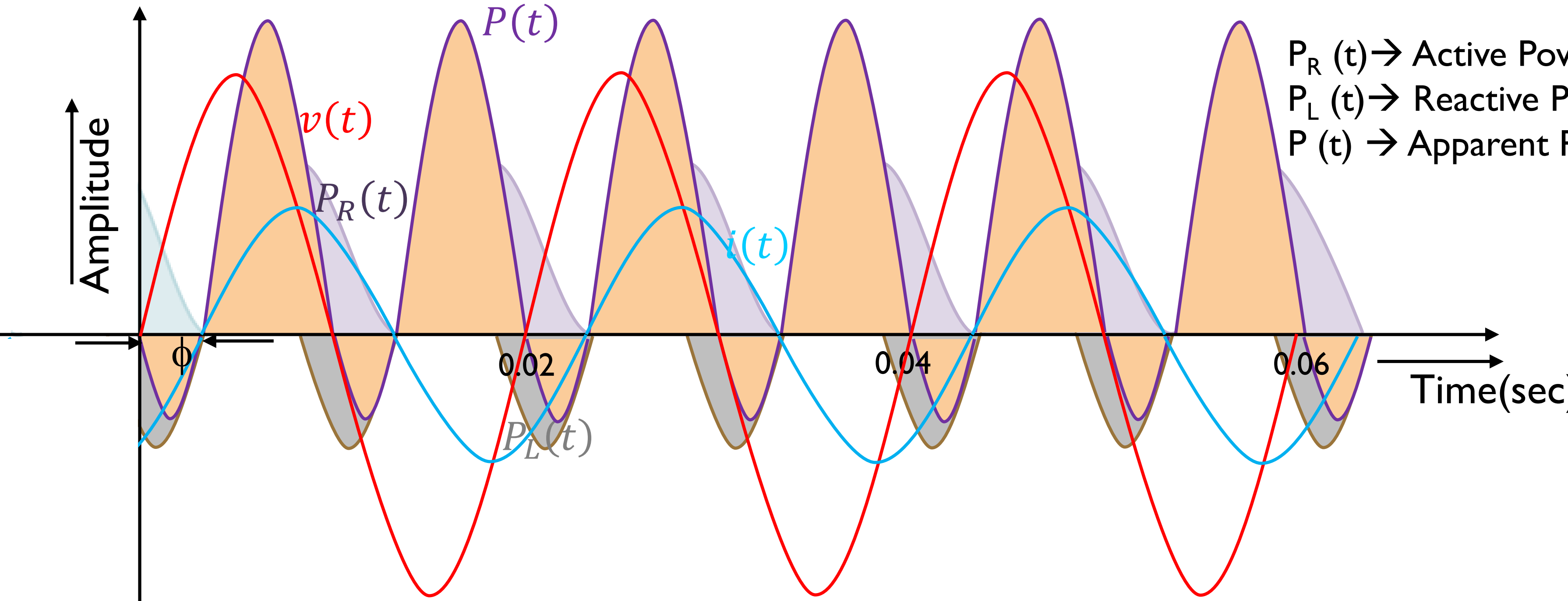
$$\frac{\bar{V}}{\bar{I}} = \frac{\bar{I}(R + jX_L)}{\bar{I}} = R + jX_L = |Z|\angle\emptyset$$

Z – *Impedance of the circuit*

$$\therefore R = |Z|\cos\emptyset \qquad X_L = |Z|\sin\emptyset$$

$$|Z| = \sqrt{R^2 + X_L^2} \qquad \emptyset = \tan^{-1}\frac{X_L}{R}$$

Power associated - RL circuit



P_R (t)→ Active Power
P_L (t)→ Reactive Power
P (t) → Apparent Power

Instantaneous power,

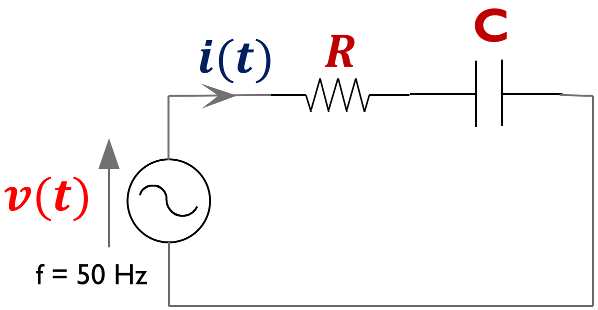
$$p(t) = v(t).\, i(t)$$
$$= V_m I_m \sin \omega t \,.\, \sin(\omega t + \emptyset)$$
$$= V_{rms} I_{rms} [\cos \emptyset - \cos(2\omega t + \emptyset)]$$

$$\boldsymbol{Average\ Power, P = \frac{1}{T}\int_0^T p(t)dt = \frac{V_m I_m}{2}\ cos\ \emptyset}$$

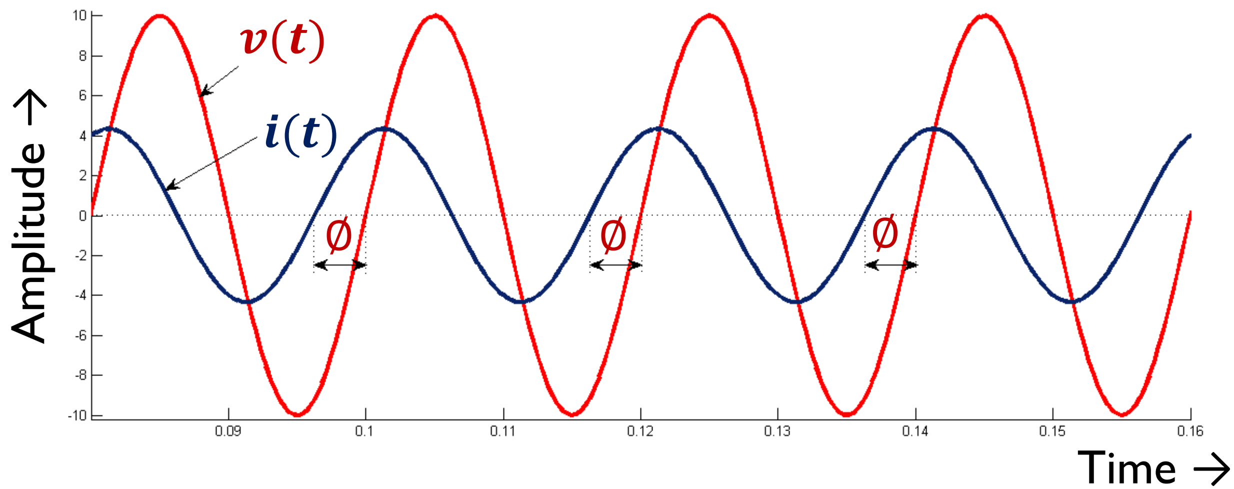
$$\boxed{\mathbf{P_{avg} = V_{rms} I_{rms} \cos \emptyset}}$$

$\cos\emptyset$ *is called the* ***Power Factor***

RC circuit analysis



Graphical Representation



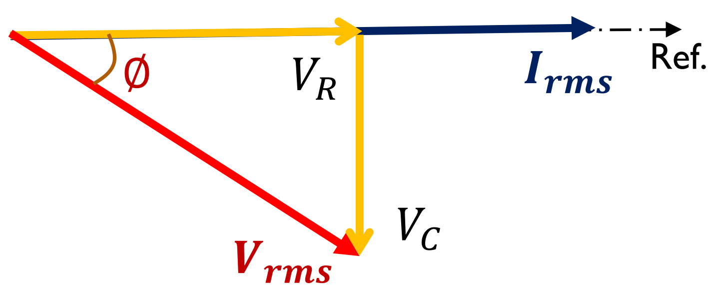
Let $\bar{I}$ *be along the reference*

$$\overline{V_R} = \bar{I}R$$

$$\overline{V_C} = -j\bar{I}X_C$$

$$\bar{V} = \overline{V_R} + \overline{V_C} = |V|\angle -\emptyset$$

Phasor Representation



Mathematical Representation

$$i(t) = I_m \sin(\omega t)$$

$$v(t) = V_m \sin(\omega t - \emptyset)$$

$\emptyset$ – *Phase Angle*

Impedance

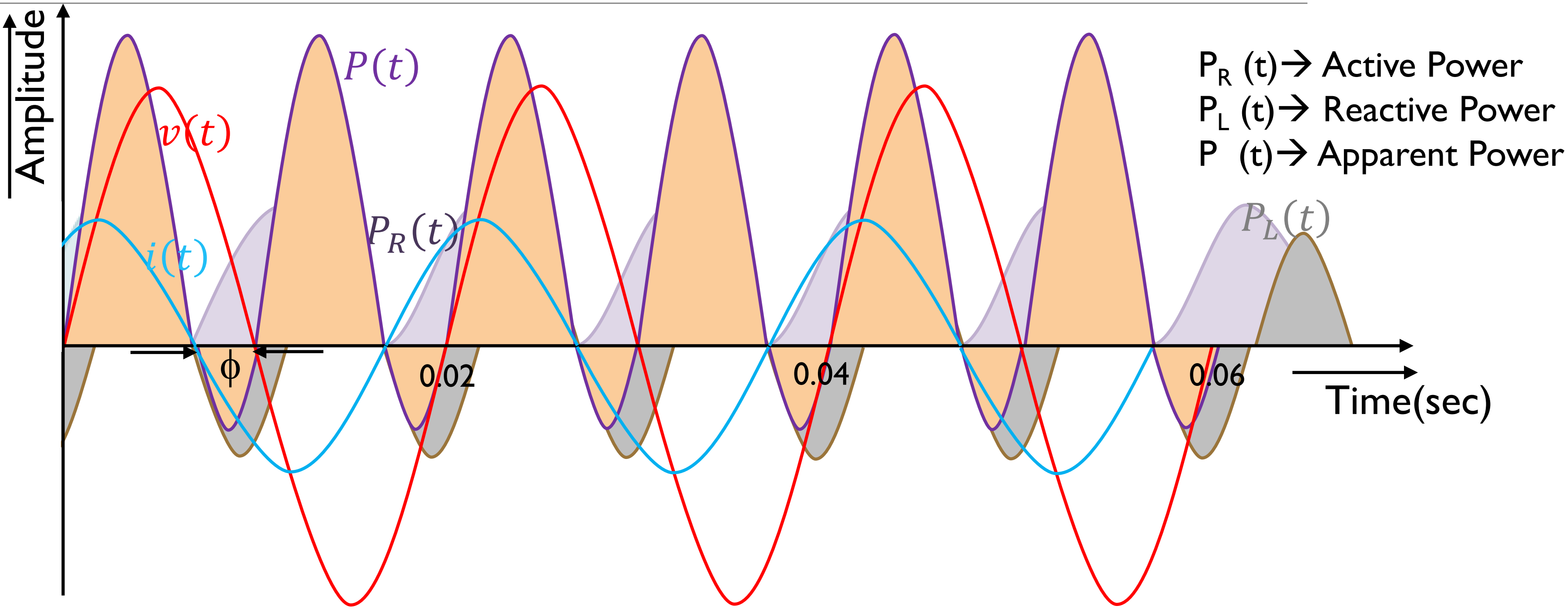
$$\frac{\bar{V}}{\bar{I}} = \frac{\bar{I}(R - jX_L)}{\bar{I}} = R - jX_L = |Z|\angle -\emptyset$$

Z – *Impedance of the circuit*

$$\therefore R = |Z|\cos\emptyset \qquad X_C = |Z|\sin\emptyset$$

$$|Z| = \sqrt{R^2 + X_C^2} \qquad \emptyset = \tan^{-1}\frac{X_C}{R}$$

Power associated - RC circuit



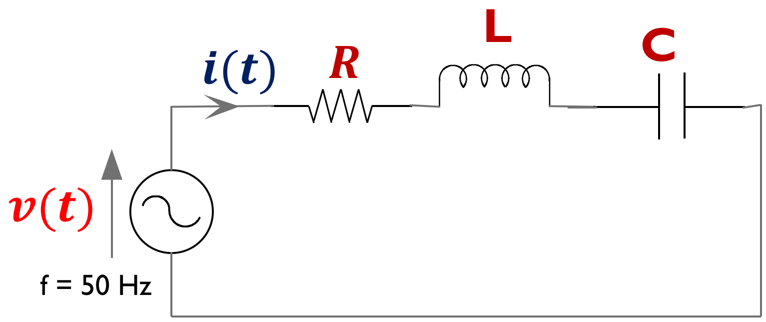
Instantaneous power,

$$p(t) = v(t).\,i(t)$$
$$= V_m I_m \sin \omega t \,.\sin(\omega t - \emptyset)$$
$$= V_{rms} I_{rms}[\cos \emptyset - \cos(2\omega t - \emptyset)]$$

$$\boldsymbol{Average\ Power, P = \frac{1}{T}\int_0^T p(t)dt = \frac{V_m I_m}{2}\cos\emptyset}$$

$$\boxed{\mathbf{P_{avg} = V_{rms} I_{rms} \cos \emptyset}}$$

RLC circuit



$Let\ i(t)\ be\ the\ reference$

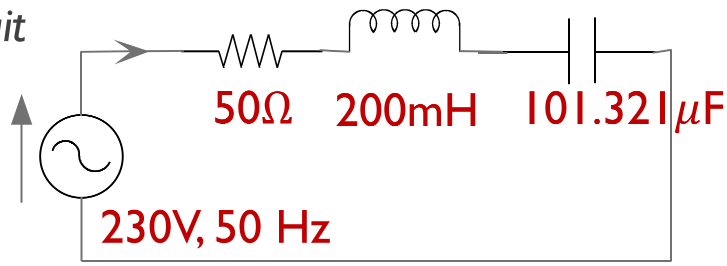
$$\boldsymbol{Impedance, Z = R + j(X_L \sim X_c)}$$

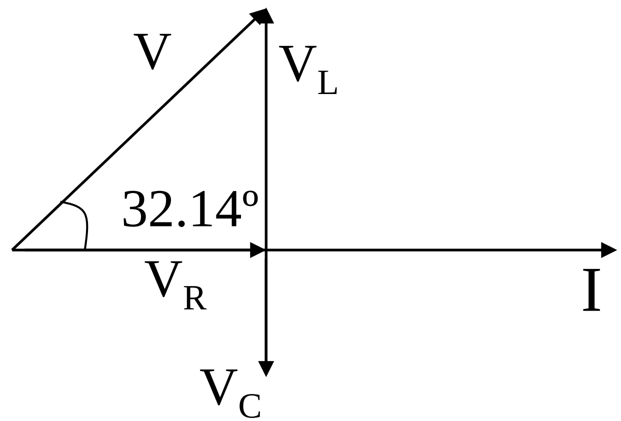
$if\ X_L = X_C$	$\Longrightarrow$	$Resistive\ circuit$ $(Resonance\ condition)$
$if\ X_L > X_C$	$\Longrightarrow$	$RL\ series\ circuit$
$if\ X_L < X_C$	$\Longrightarrow$	$RC\ series\ circuit$

Illustration 1

A resistance of 50Ω is connected in series with an inductance of 200mH and capacitance of 101.321μF across a 230V, 50 Hz, single phase AC supply. Obtain,

a) *Impedance of the circuit*
b) *Current drawn*
c) *Power factor*
d) *Power consumed*
e) *Phasor diagram*





$$X_L = 2 \times \pi \times 50 \times 0.2 = 62.8315\Omega$$

$$X_c = \frac{1}{2 \times \pi \times 50 \times 101.321\mu} = 31.4159\Omega$$

$$PF = \cos(32.14) = \boldsymbol{0.846\ lag}$$

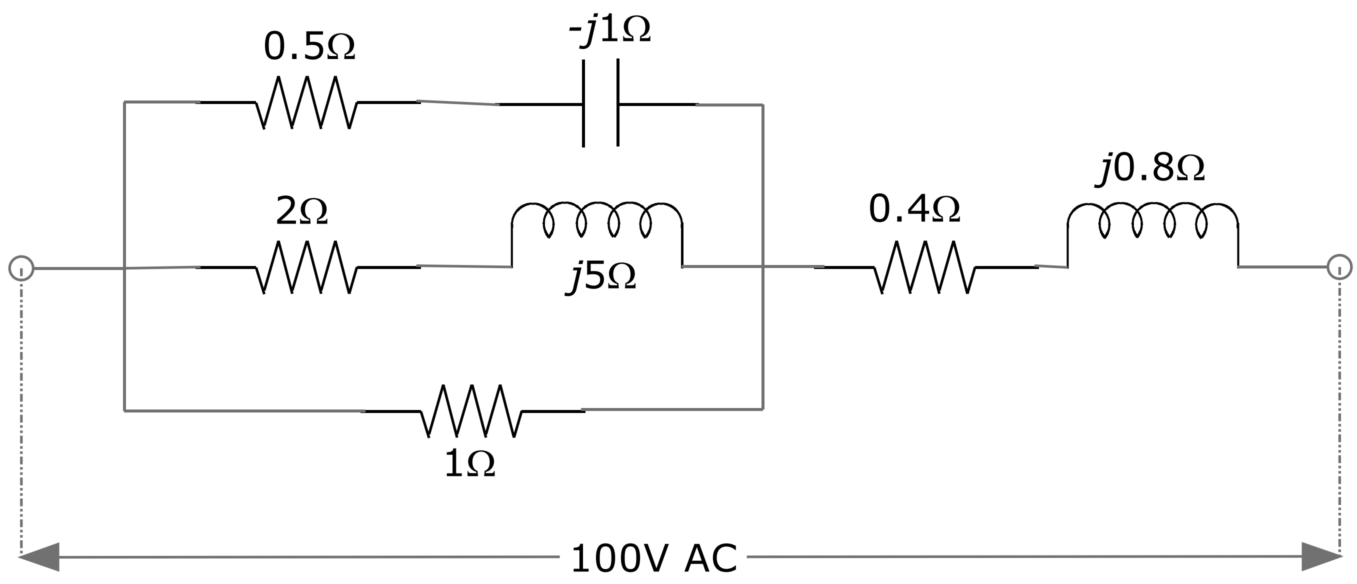
$$Z = R + jX_L - jX_c = \boldsymbol{50 + j31.4156\Omega = 59.050\angle 32.14°\ \Omega}$$

$$I = \frac{230\angle 0}{59.05\angle 32.14} = \boldsymbol{3.898\angle - 32.14°A}$$

$$P = |V_{rms}||I_{rms}|cos\emptyset$$
$$= 230 \times 3.898 \times 0.846 = \boldsymbol{759.15W}$$

Illustration 2

Determine the impedance of the circuit shown and the power consumed in each branch



$Z_1 = 0.5 - j1\Omega$ $\quad Z_3 = 1\Omega$

$Z_2 = 2 + j5\Omega$ $\quad Z_4 = 0.4 + j0.8\Omega$

$$\bar{I} = \frac{100\angle 0}{1.12\angle 29.5} = \mathbf{89.285\angle -29.5A} = \bar{I_4}$$

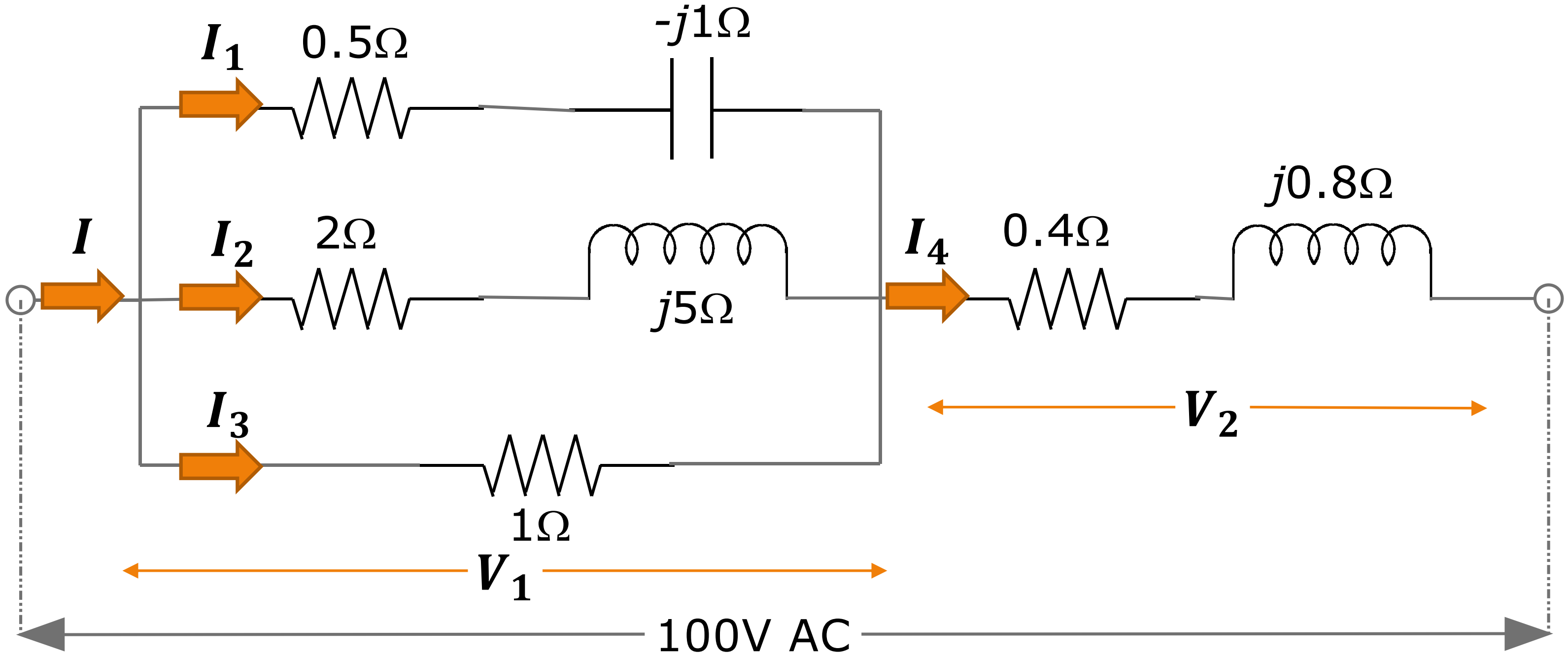
$$Z_{eq} = (Z_1||Z_2||Z_3) + Z_4 = \mathbf{1.12\angle 29.5°\Omega}$$

$$\overline{V_2} = \bar{I_4} \times Z_4 = \mathbf{79.85\angle 33.934\ V}$$

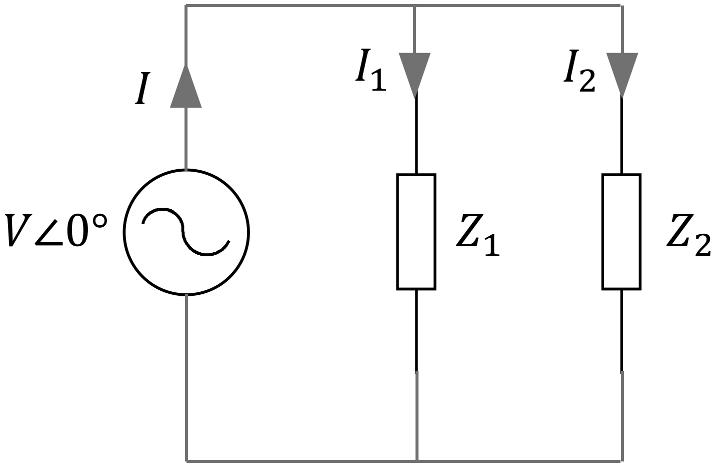
$$\bar{V} = \bar{V_1} + \bar{V_2} \qquad \overline{V_1} = \mathbf{55.91\angle -52.868\ V}$$

$$\bar{I_1} = \frac{\overline{V_1}}{Z_1} = \mathbf{50.00\angle 10.565\ A} \qquad \bar{I_2} = \mathbf{10.38\angle -121.068\ A} \qquad \bar{I_3} = \mathbf{55.91\angle -52.868\ A}$$

$$P_1 = |I_1|^2 \times R_1 = \mathbf{1.25\ kW} \qquad P_2 = 0.215\ kW \qquad P_3 = 3.125\ kW \qquad P_4 = 3.188\ kW$$



Impedance in parallel



Let $Z_1 = R_1 + jX_1$ $\quad$ $Z_2 = R_2 + jX_2$

$$\frac{1}{Z_{eq}} = \frac{1}{Z_1} + \frac{1}{Z_2} \qquad I_1 = I * \frac{Z_2}{Z_1 + Z_2} \qquad I_2 = I * \frac{Z_1}{Z_1 + Z_2}$$

$$Y_{eq} = Y_1 + Y_2 \qquad \color{red}{Y: Admittance}$$

$$Y_1 = \frac{1}{Z_1} = \frac{1}{R_1 + jX_1} = \frac{1}{(R_1 + jX_1)} * \frac{(R_1 - jX_1)}{(R_1 - jX_1)} = \frac{R_1}{(R_1^2 + X_1^2)} - j\frac{X_1}{(R_1^2 + X_1^2)} = G_1 - jB_1$$

$$Y_2 = \frac{1}{Z_2} = \frac{1}{R_2 + jX_2} = \frac{1}{(R_2 + jX_2)} * \frac{(R_2 - jX_2)}{(R_2 - jX_2)} = \frac{R_2}{(R_2^2 + X_2^2)} - j\frac{X_2}{(R_2^2 + X_2^2)} = G_2 - jB_2$$

$G: Conductance$ $\qquad$ $B: Susceptance$

$$\mathbf{Y_{eq} = (G_1 + G_2) - j(B_1 + B_2) = G_{eq} - jB_{eq}}$$

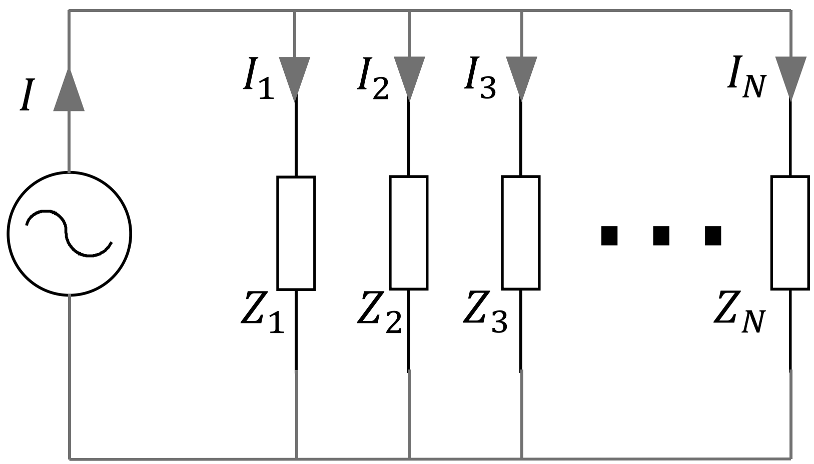
Impedance in parallel

For 'N' impedances connected in parallel,

$$\frac{1}{Z_{eq}} = \frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3} + \cdots + \frac{1}{Z_N}$$

$$Y_{eq} = Y_1 + Y_2 + Y_3 + \cdots + Y_N$$

$$\mathbf{Y_{eq} = G_{eq} \pm \mathit{j}B_{eq}}$$



$$I_1 = VY_1; I_2 = VY_2; I_3 = VY_3; \ldots \ldots I_N = VY_N$$

$$I = I_1 + I_2 + I_3 + \cdots + I_N = VY_{eq}$$