$$c(t) = \int_{-\infty}^{\infty} m(\tau) \cdot r(t-\tau) \cdot d\tau$$

where τ is the dummy variable used for integration.

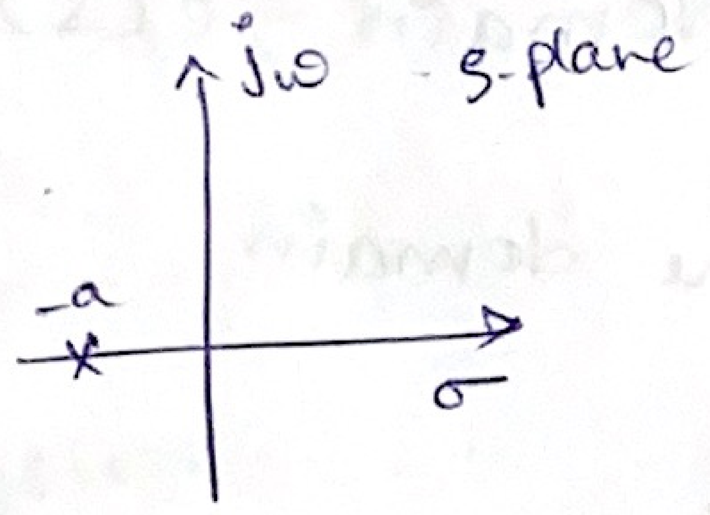
Location of poles on s plane for stability.

T.F & location of roots on s-plane.	Impulse response.

i) $M(s) = \dfrac{A}{s+a}$

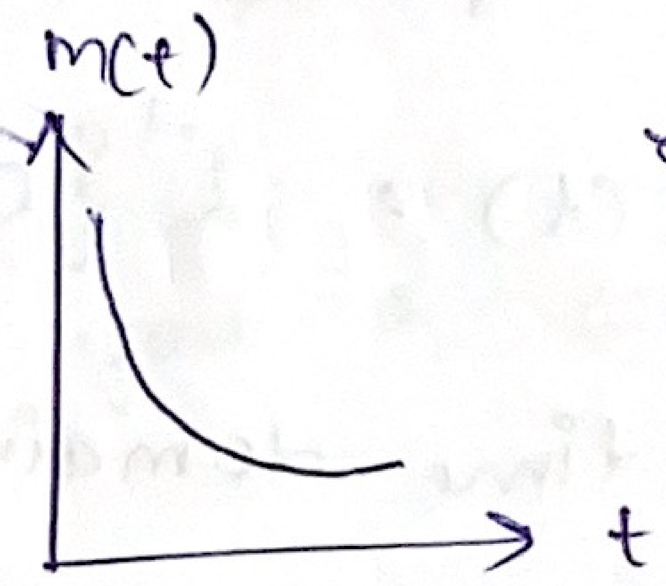
$s+a=0$

$\Rightarrow s=-a$



$m(t) = L^{-1}\{M(s)\}$

$= A \cdot e^{-at}$



exponentially decaying

Stable system.

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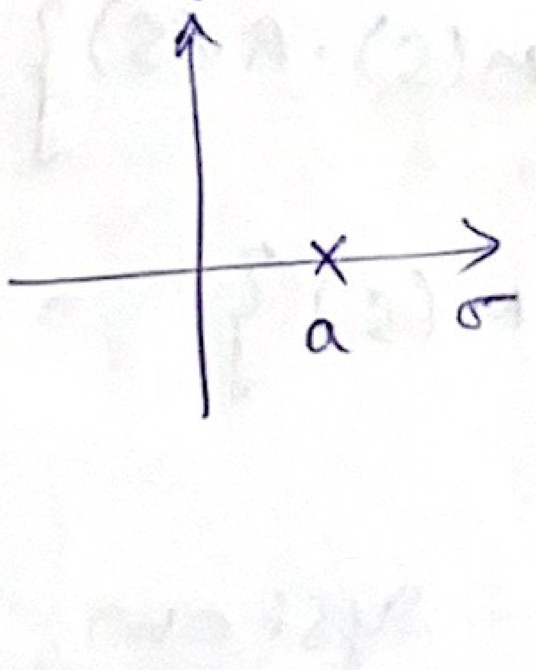
ii) $M(s) = \dfrac{A}{s-a}$

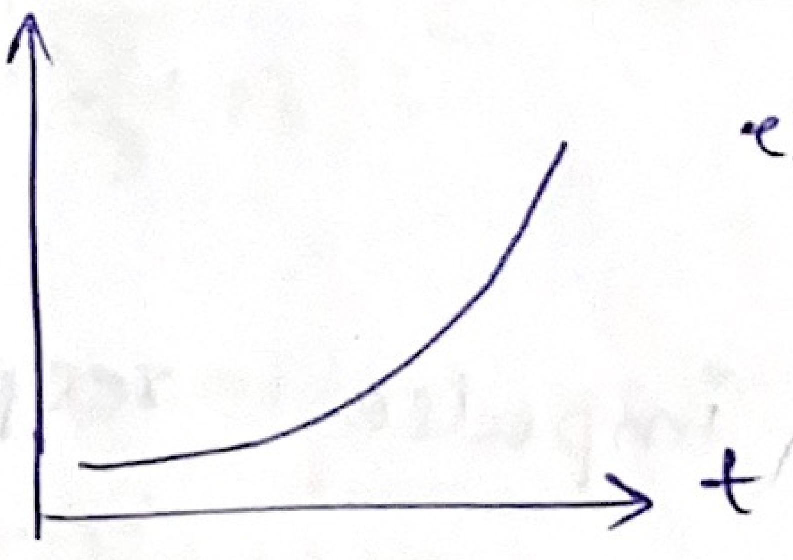
$s-a=0$

$s=+a$



$m(t) = L^{-1}\left\{\dfrac{A}{s-a}\right\}$

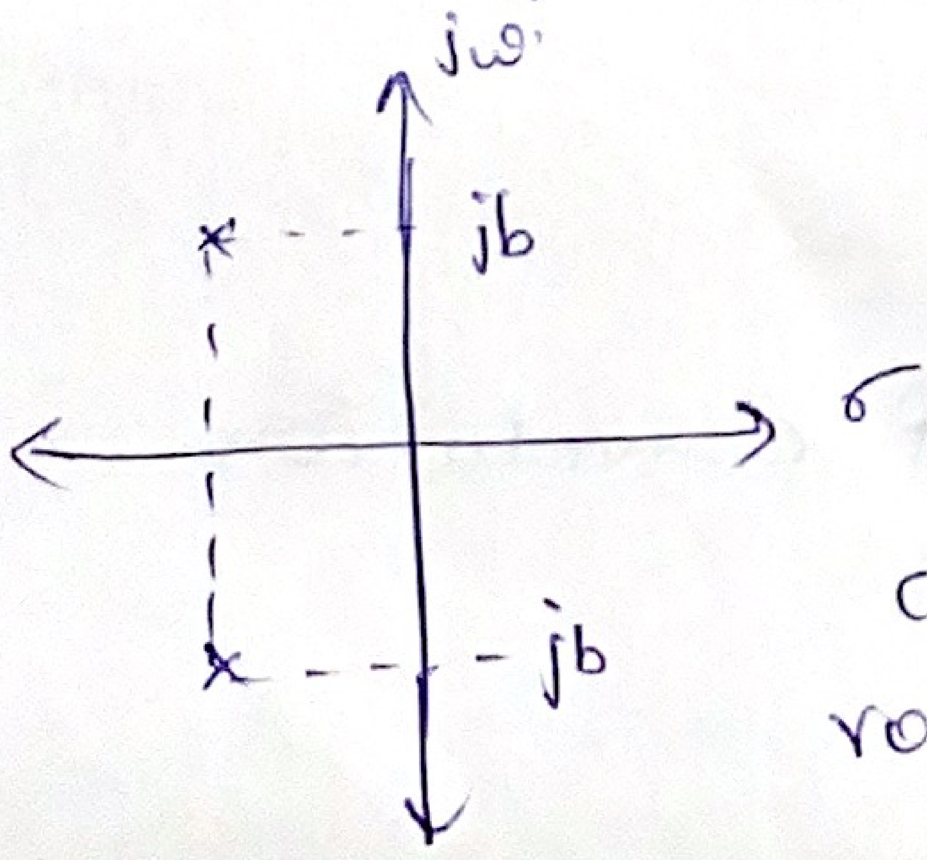
$= A \cdot e^{at}$



exponentially increasing

unstable system.

iii) $M(s) = \dfrac{A}{s+a+jb} + \dfrac{A^*}{s+a-jb}$



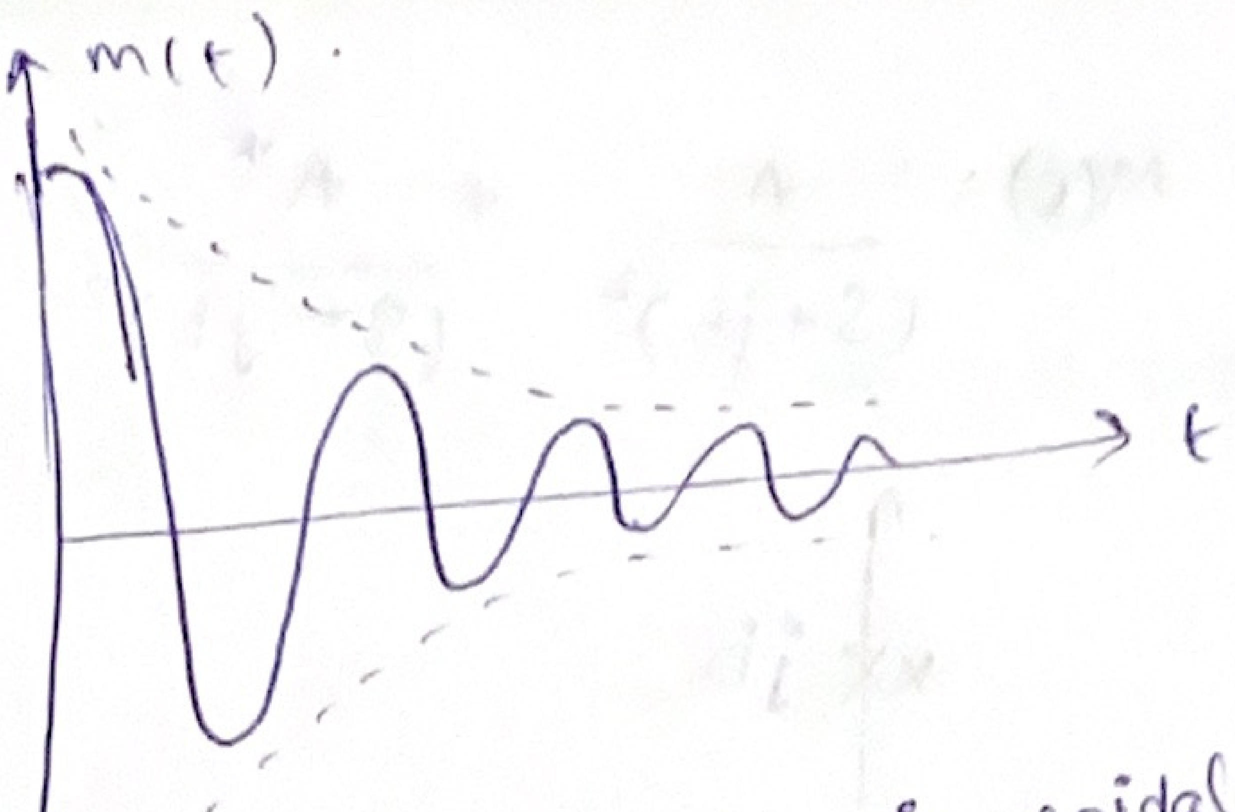
complex conjugate roots on LHS of s-plane.

$m(t) = L^{-1}\{M(s)\}$

$m(t) = Ae^{-(a+jb)t} + A^*e^{-(a-jb)t}$

$= 2Ae^{-at}\cos bt$

$= 2Ae^{-at}\sin(bt-90^\circ)$



Damped sinusoidal.

Stable system.

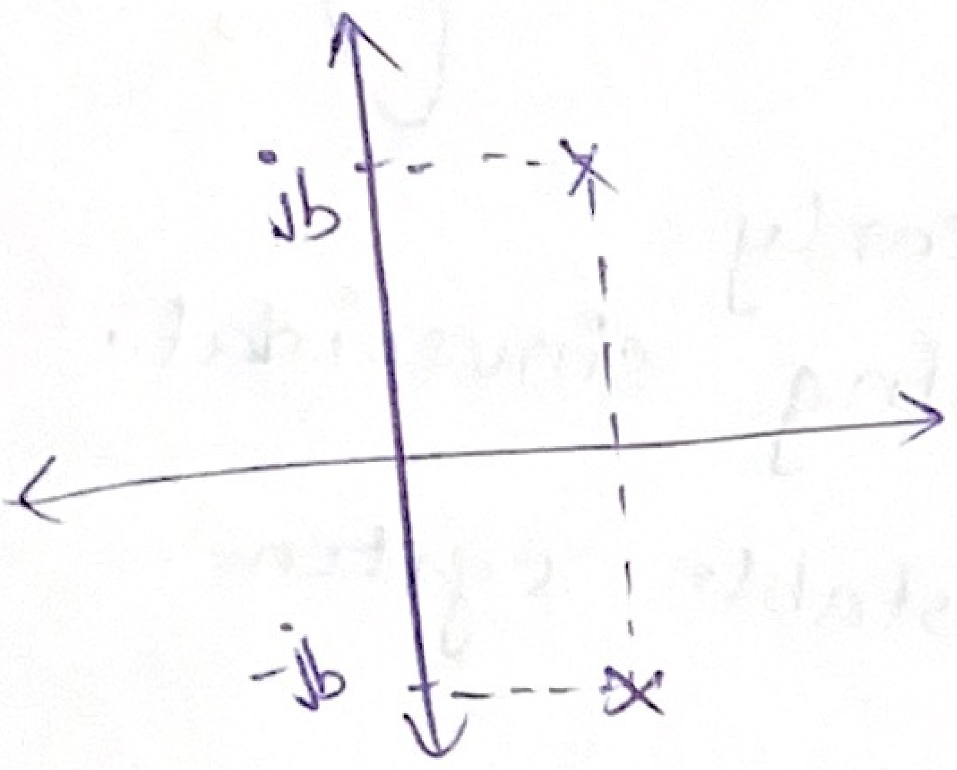
---

iv) $M(s) = \dfrac{A}{s-a+jb} + \dfrac{A^*}{s-a-jb}$

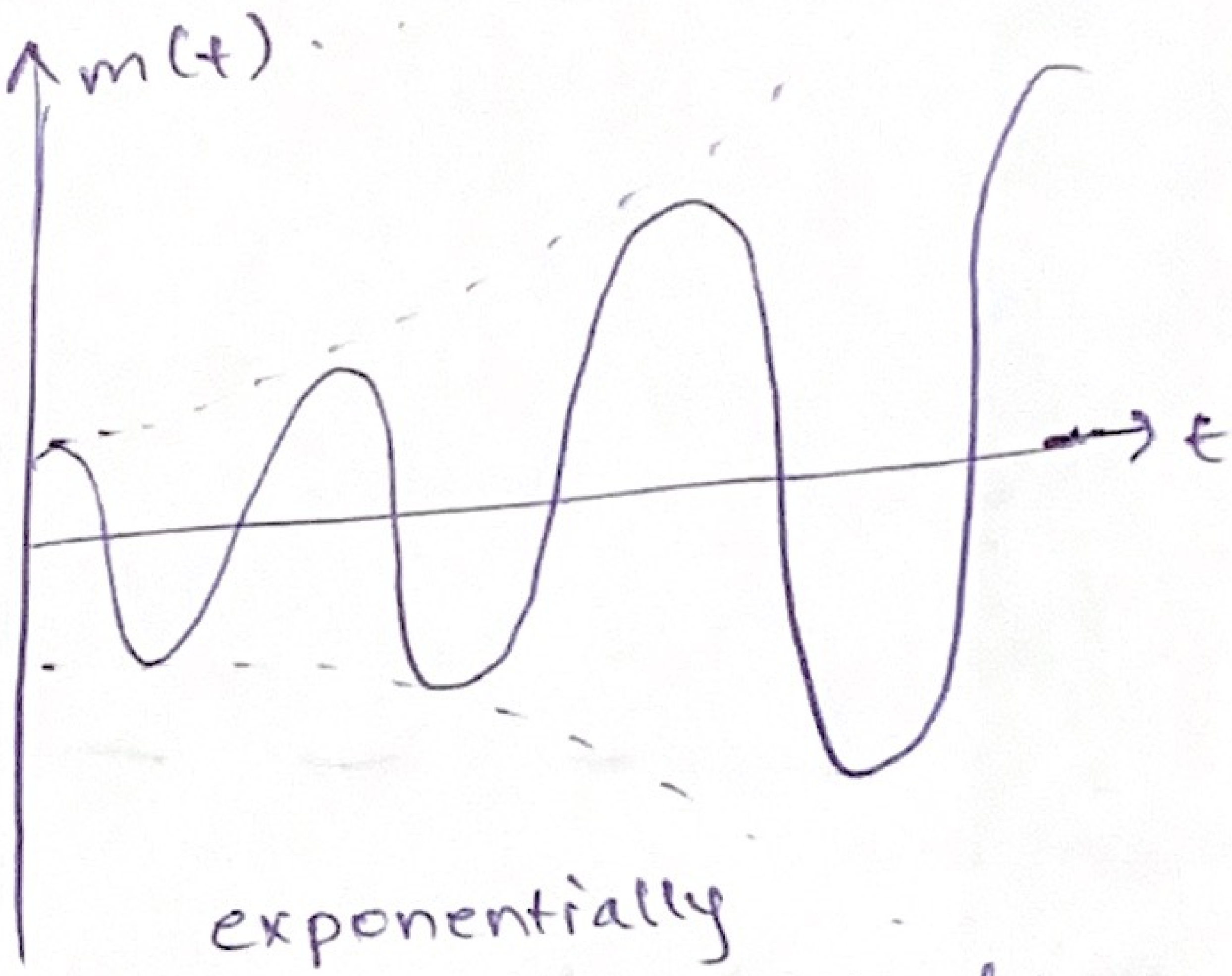
$m(t) = L^{-1}\{M(s)\}$

$= A \cdot e^{-(a-jb)t} + A^* e^{-(a+jb)t}$

$= 2A e^{at} \cos t$.



complex conjugate roots on RHS of s-plane.



exponentially increasing sinusoidal.

Unstable system.

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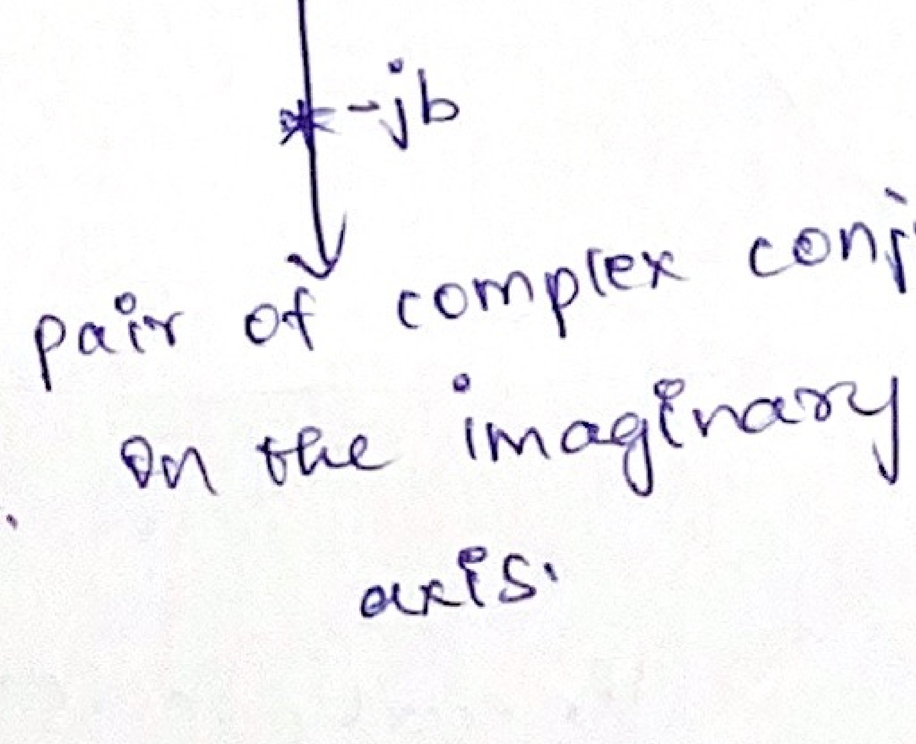
v). $M(s) = \dfrac{A}{s+jb} + \dfrac{A^*}{s-jb}$

$m(t) = L^{-1}\{M(s)\}$

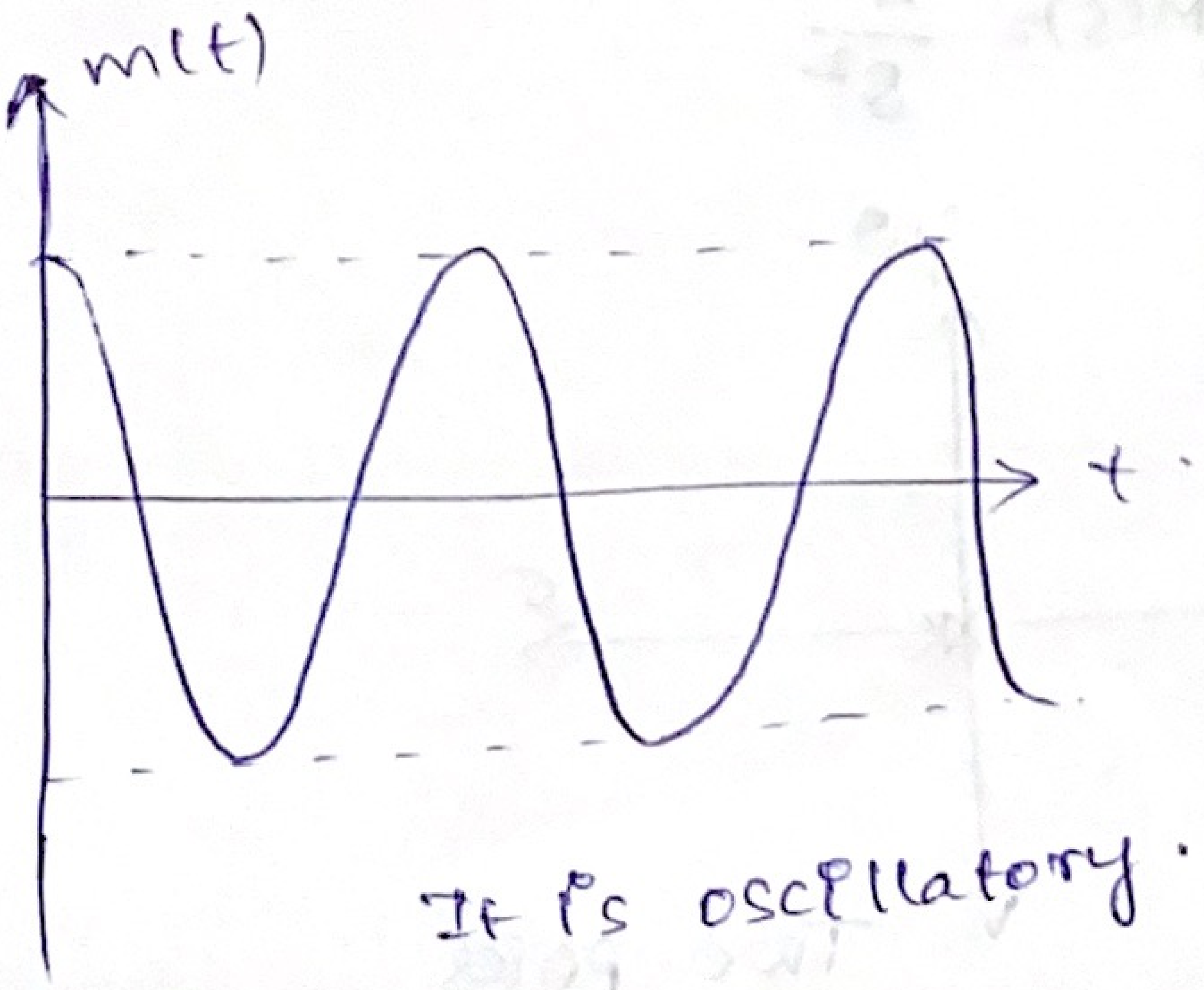
$= A e^{-jbt} + A e^{jbt}$

$= 2A \cos bt$.

$m(t) = 2A \sin(bt + 90^\circ)$



a pair of complex conjugate roots. on the imaginary axis.



It is oscillatory.

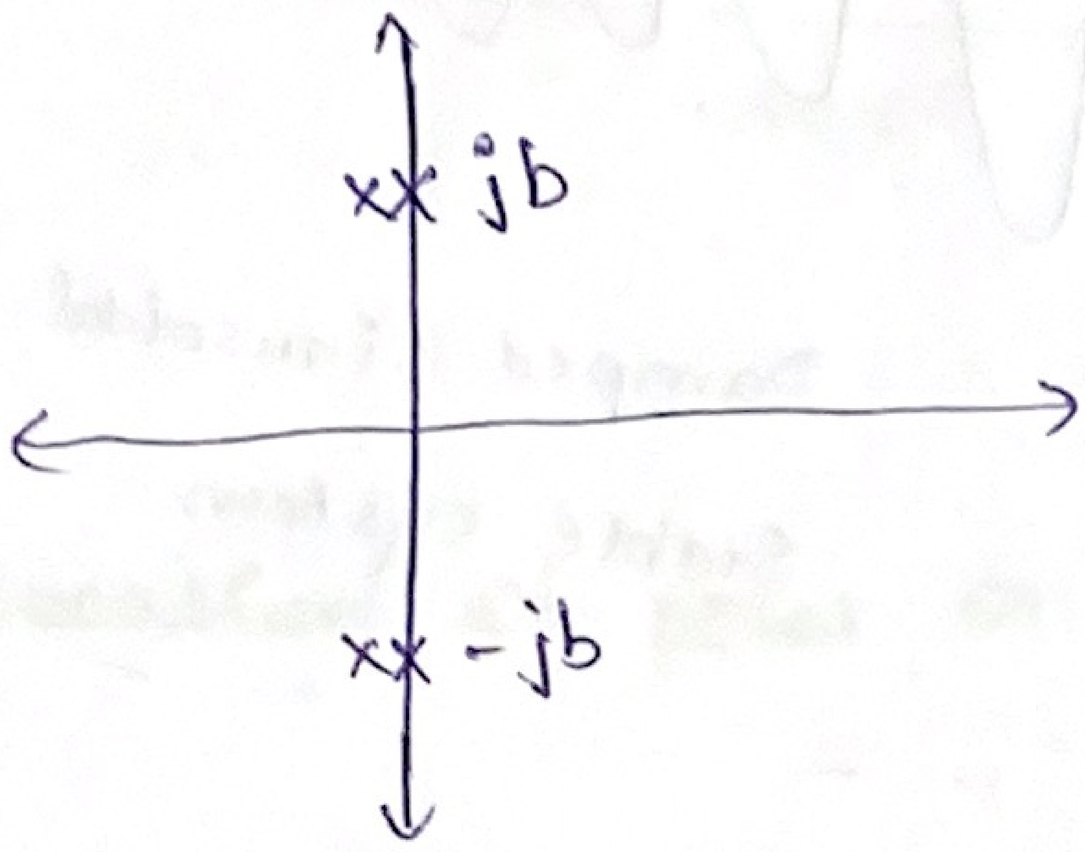
It is called marginaly stable.

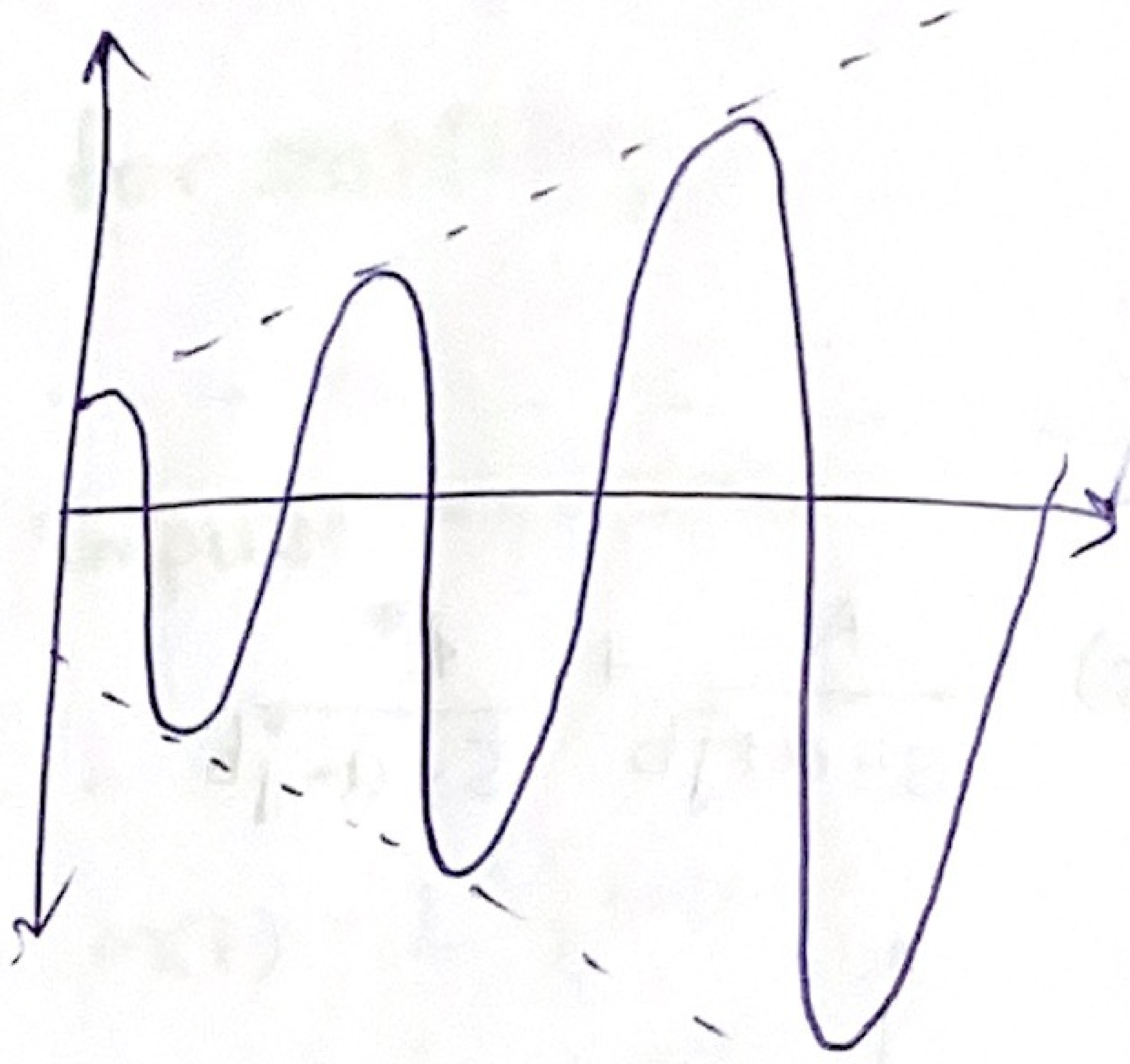
vi) $M(s) = \dfrac{A}{(s+jb)^2} + \dfrac{A^*}{(s-jb)^2}$

$m(t) = \mathcal{L}^{-1}\{M(s)\}$

$= At \cdot e^{-jbt} + A^* t \cdot e^{jbt}$

$m(t) = 2At \cos bt$





linearly increasing sinusoidal.

$\Rightarrow$ Unstable system.

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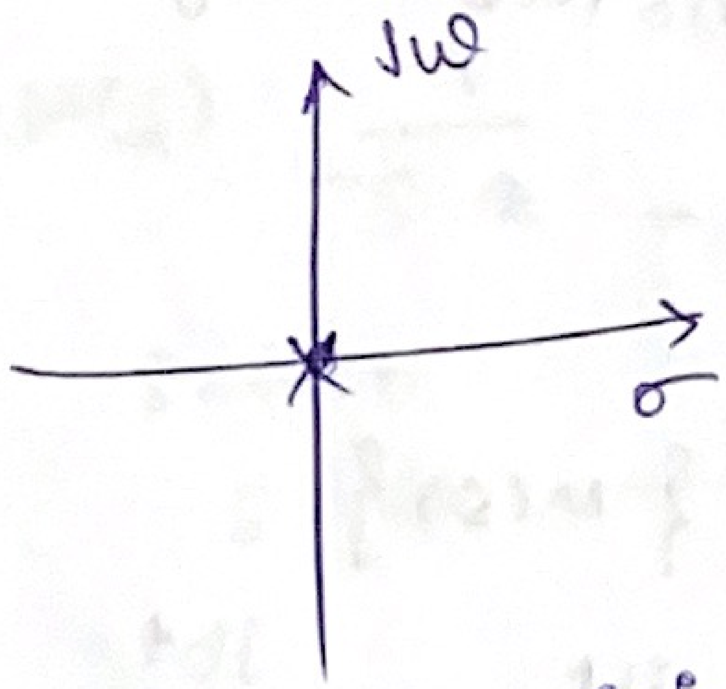
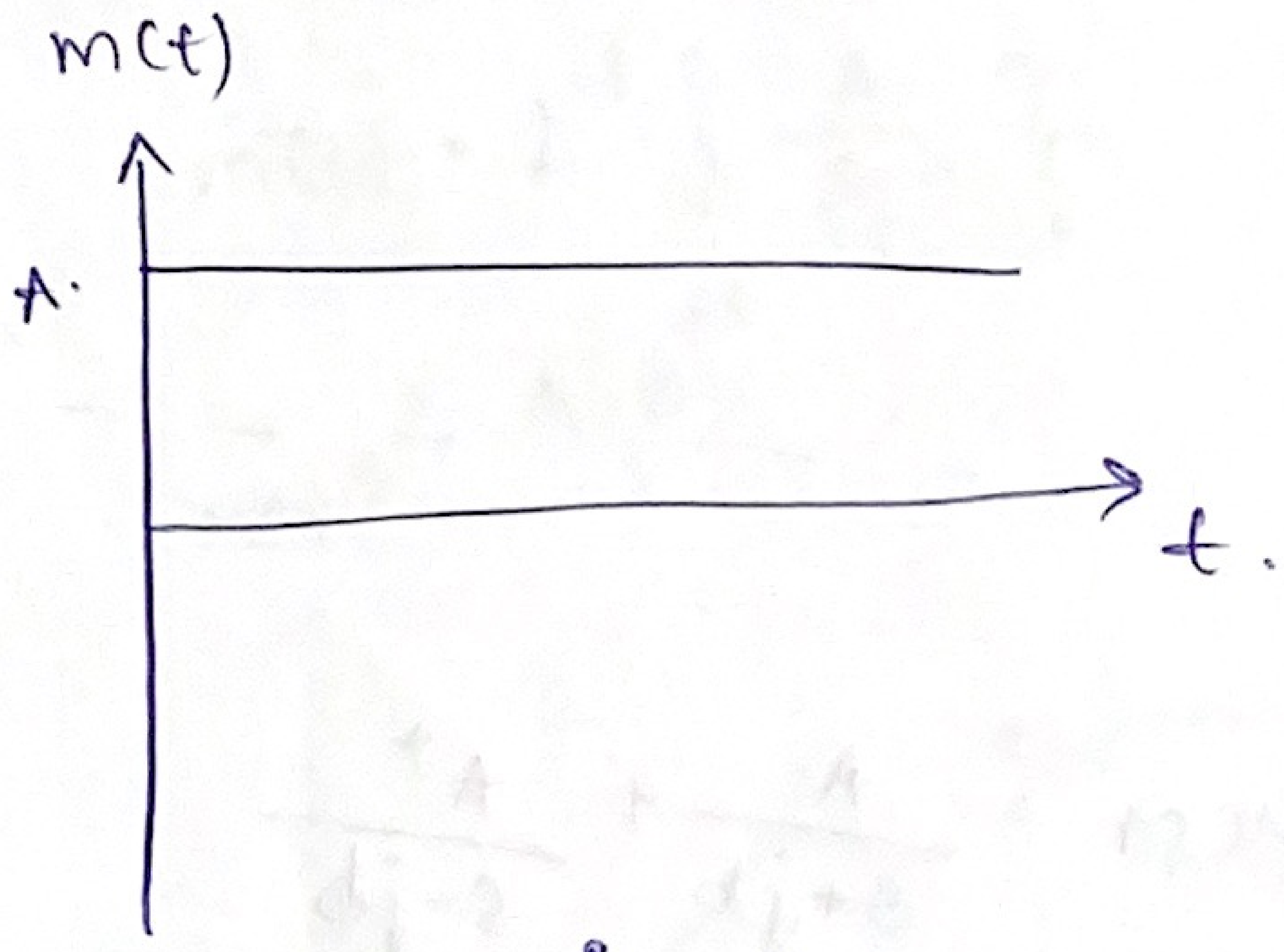
vii) $M(s) = \dfrac{A}{s}$

$m(t) = \mathcal{L}^{-1}\{A/s\} = A$



Pole at origin.



marginally stable system.

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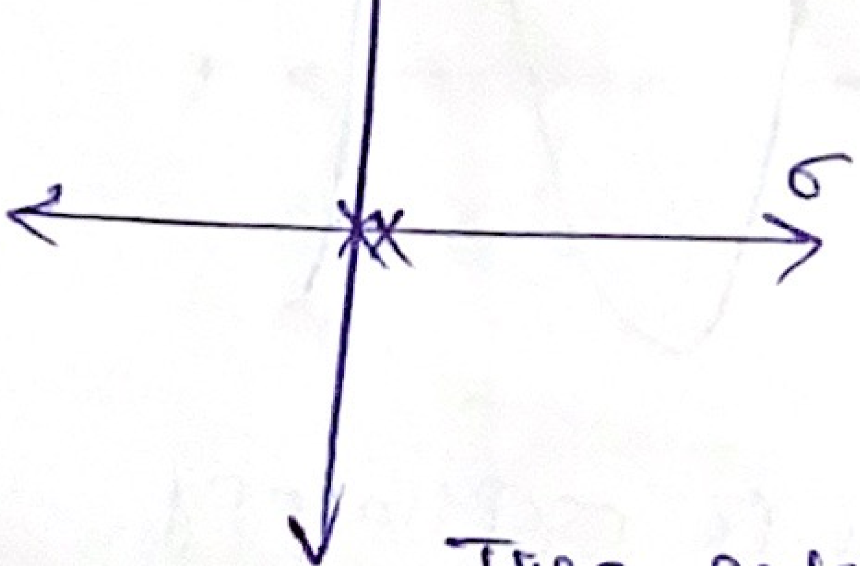
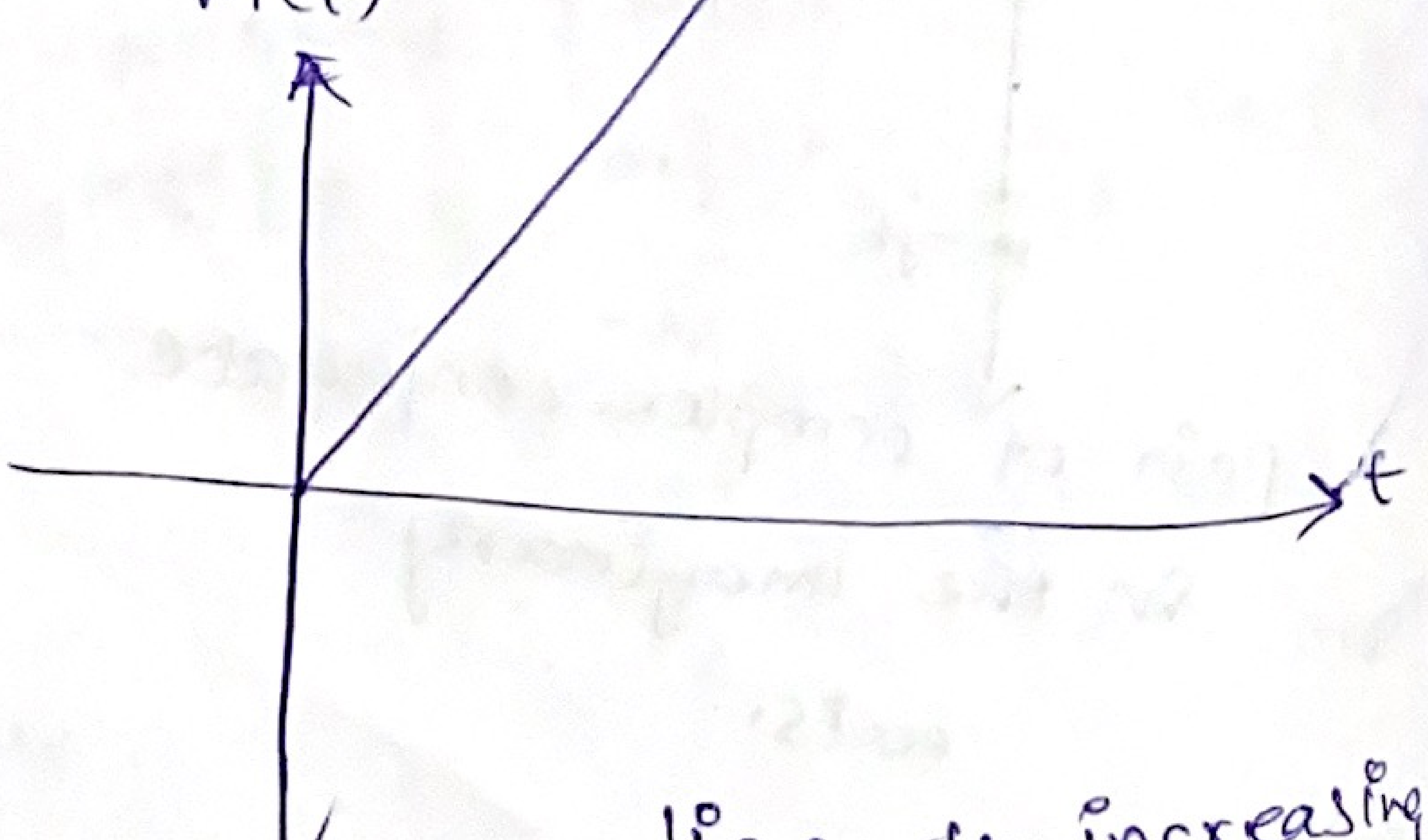
viii) $M(s) = \dfrac{A}{s^2}$

$m(t) = \mathcal{L}^{-1}\left\{\dfrac{A}{s^2}\right\} = At$



Two poles at origin



linearly increasing with time

$\Rightarrow$ Unstable system.

ROUTH HURWITZ CRITERION

$$a_0 s^n + a_1 s^{n-1} + a_2 s^{n-2} + a_3 s^{n-3} + \dots + a_{n-1} s + a_n = 0$$

where $a_0 > 0$

s^n	:	a_0	a_2	a_4	a_6	a_8
s^{n-1}	:	a_1	a_3	a_5	a_7	a_9 .....
s^{n-2}	:	b_1	b_2	b_3	b_4	b_5 ....
s^{n-3}	:	c_1	c_2	c_3	c_4	c_5 .....
.						
:						
s^1	:	g_0				
s^0	:	h_0				

ROUTH CRITERION.

"The necessary and sufficient condition for stability is that all of the element in the first column of the routh array be positive. If this condition is not met, the system is unstable and the no. of sign changes in the element of the first column of routh's array corresponds to the no. of roots of characteristic eq in the right half of s-plane."

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$$b_1 = \frac{a_1 a_2 - a_3 a_0}{a_1} \qquad c_1 = \frac{b_1 a_3 - b_2 a_1}{b_1}$$

$$b_2 = \frac{a_1 a_4 - a_5 a_0}{a_1} \qquad c_2 = \frac{b_1 a_5 - b_3 a_1}{b_1}$$

$$b_3 = \frac{a_1 a_6 - a_7 a_0}{a_1}$$