Sub in (3)

$$43^\circ = \sin^{-1}(1 - b/x)$$

$$\sin 43^\circ = 1 - \frac{b}{x}$$

$$\Rightarrow \frac{b}{x} = 1 - \sin 43^\circ$$

$$\frac{b}{X} = 1 - 0.68 = \underline{0.318}$$

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$1.1 + 0.3i \quad 1.1 \angle 164.7^\circ$

$$-90^\circ - \tan^{-1}\omega - \tan^{-1} 0.5\omega = 165^\circ$$

$$\tan^{-1}\omega + \tan^{-1} 0.5\omega = -255^\circ$$

Apply tan on both sides

$$\frac{\omega + 0.5\omega}{1 - \omega(0.5\omega)} = -3.48.$$

$$\omega + 0.5\omega = -3.48 + 1.74\omega^2$$

$$1.74\omega^2 - 1.5\omega - 3.48 = 0$$

$$\Rightarrow \omega = \underline{1.9 \text{ rad/s}} \qquad -1.04 \text{ not consi}$$

$$1.14 \angle -165^\circ = 1 \angle -180^\circ \cdot \frac{1}{|K_N| \angle K_N}$$

$$\Rightarrow |K_N| = \frac{1}{1.14} = 0.877$$

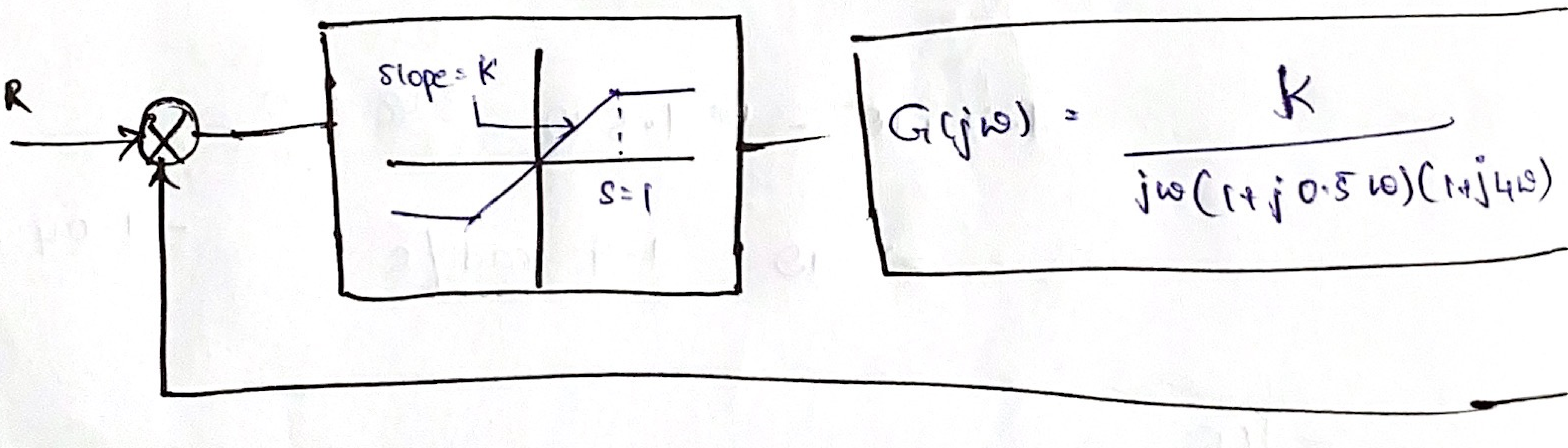
$$\angle K_N = -180^\circ + 165^\circ = -15^\circ$$

Using formulae

~~$\angle K_N$~~ · from (1)

$$\frac{\cancel{} \cos^2\beta}{\pi/2 + \beta + \frac{1}{2}\sin 2\beta} = \cancel{} 0.26 \quad \Rightarrow \frac{\pi}{2} + \beta + \frac{1}{2}\sin 2\beta = 3.73 \cos^2\beta.$$

Q. The unity feedback system shown have saturating amplifier with gain K. Determine the maximum value of K for system to stay stable. what would be the nature & frequency of limit cycle if K = 2.5.



Sol. $G(j\omega)$ when $K = 1$

$$|G(j\omega)| = \frac{1}{\omega\sqrt{1+0.25\omega^2}\sqrt{1+16\omega^2}}$$

180°

$$\angle G(j\omega) = -90^\circ - \tan^{-1} 0.5\omega - \tan^{-1} 4\omega$$

ω rad/s	0.4	0.5	0.6	0.8	1.0	1.2
$\lvert G(j\omega)\rvert$	1.29	0.86	0.61	0.34	0.21	0.145
$\angle G(j\omega)$	-159°	-167°	-174°	-184°	-192°	-199°
	-1.21	-0.85	-0.61			

when $k = 2.5$

ω rad/s	0.6	0.65	0.75	0.8	1	1.2
	1.53	1.31	0.98	0.86	0.54	0.36
	-174	-177	-182	-184	-192	-199
	-1.52	-1.31	-0.99	-0.87	-0.53	-0.34

To find max value of K for stability.

when K is increased, the $G(j\omega)$ locus shift upwards. For a particular value of k, the $G(j\omega)$ locus crosses the starting point (i.e, $-1+0j$)

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Lyapunov stability

Lyapunov Stability.

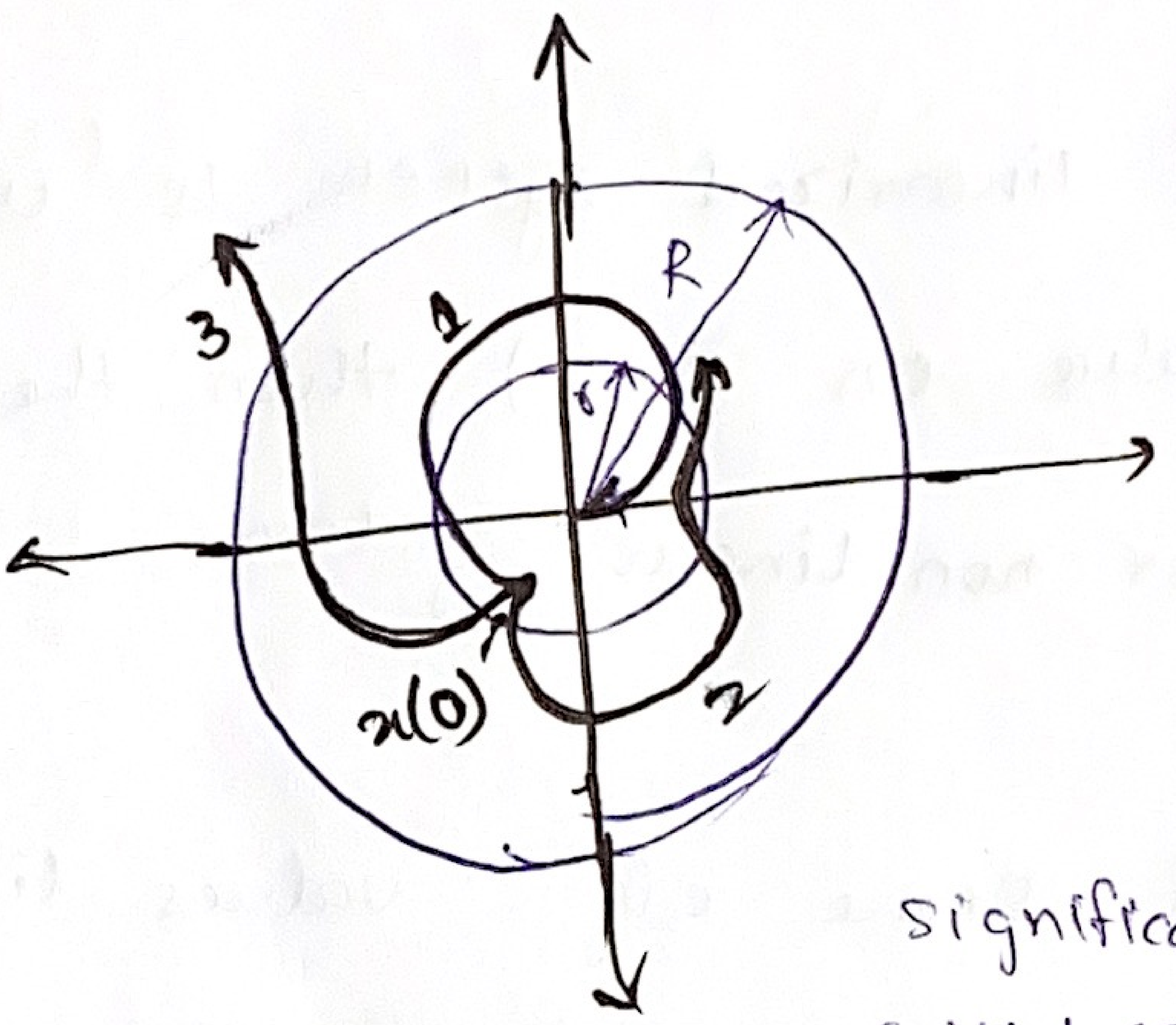
The eq. state $x = 0$ is said to be stable, if for any $R > 0$ there exists $r > 0$, such that if $\|x(0)\| < r$, then $\|x(t)\| < R$ for all $t \geq 0$. otherwise the eq. point is unstable.

linear

1 - asymptotically stable.

2 - marginally stable

3 - unstable.



significance.

r - initial condition.

R - operating region

Asymptotically stable

An eq. point 0 is asymptotically stable if it is stable and if, in addition, there exists some $r > 0$ such that $\|x(0)\| < r$ implies that $x(t) \to 0$ as $t \to \infty$

marginally stable.

An eq. point which is lyapunov stable but not asymptotically stable.

Lyapunov's linearization method

If the linearized system is strictly stable (if all the eigen values of A are on LHS of s-planee) then the eq. point is asymptotically stable for the actual non-linear system.

If linearized system is unstable (at least one eigen value on RHS) then the eq. point is also unstable for non-linear system.

If some eigen values lien of $j\omega$ axis, then we can't comment on the stability of non-linear method.

Comment on the stability using Lyapunov lin. meth.

ex:- $\dot{x}_1 = x_2^2 + x_1 \cos x_2$

$\dot{x}_2 = x_2 + (x_1+1)x_1 + x_1 \sin x_2$

Jacobian

$\dot{x}_1 = 0$ $\quad x_2^2 + x_1 \cos x_2 = 0 \Rightarrow x_2 = \sqrt{-x_1 \cos x_2}$

$\dot{x}_2 = 0$ $\quad x_2 + x_1^2 + x_1 + x_1 \sin x_2 = 0.$

$$\begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} \end{bmatrix} = \begin{bmatrix} \cos x_2 & 2x_2 + x_1 \sin x_2 \\ 2x_1 + 1 + \sin x_2 & 1 + x_1 \cos x_2 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$$

$|\lambda I - A| = 0 \quad \Rightarrow \lambda = 1, 1$

So, unstable.

Non-linear system is also unstable.

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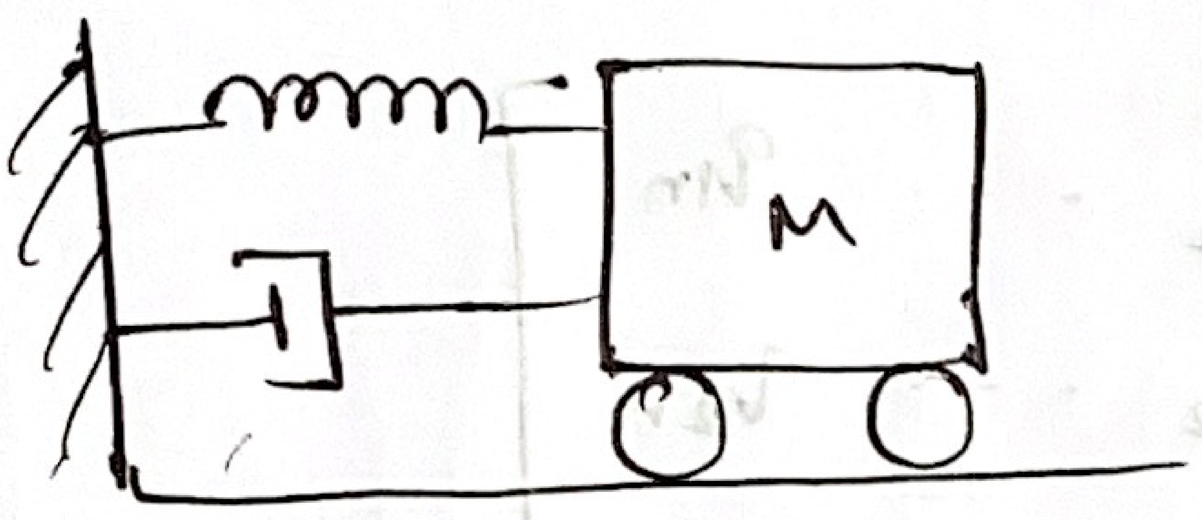
$$m\ddot{x} + b\dot{x}|\dot{x}| + k_0 x + k_1 x^3 = 0.$$

non-linear

spring damper system.



Total mechanical energy

$$V(x) = \frac{1}{2} m\dot{x}^2 + \int_0^x (k_0 x + k_1 x^3) \cdot dx.$$

$$V(x) = \frac{1}{2} m\dot{x}^2 + \frac{1}{2} k_0 x^2 + \frac{1}{4} k_1 x^4$$

Sylvester's criterion.

$V(x)$ is in quadratic form in the x_i's if $V(x)$ is in the form

$$V(x) = \sum_{i=1}^{n} \sum_{j=1}^{n} k_{ij} \cdot x_i \cdot x_j$$

$$\Rightarrow \quad V(x) = (x^T)(Q)(x).$$

where $Q = \begin{bmatrix} q_{11} & q_{12} & \cdots & q_{1n} \\ q_{21} & q_{22} & \cdots & q_{2n} \\ \vdots & & & \\ q_{n1} & q_{n2} & \cdots & q_{nn} \end{bmatrix}$

$$V(x) = [x_1, x_2 \cdots x_n] \begin{bmatrix} q_{11} & \cdots & q_{1n} \\ \vdots & & \\ q_{n1} & \cdots & q_{nn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

$q_{ij} = k_{ij}$ where $i = j$

$q_{ij} = \frac{1}{2}(k_{ij} + k_{ji}) = q_{ji}$ for $i \neq j$

A quadratic function $V(x) = x^T Q x$ is +ve definite if & only if all the principle minors are +ve.

$V(x)$ is −ve definite if $-V(x)$ is +ve definite.

Q. Check the sign definiteness.

$$V(x) = 6x_1^2 + 4x_2^2 + x_3^2 + 2x_1x_2 - 2x_2x_3 - 4x_1x_3$$

Sol. $V(x) = x^T Q x$

$$= \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix} \begin{bmatrix} q_{11} & q_{12} & q_{13} \\ q_{21} & q_{22} & q_{23} \\ q_{31} & q_{32} & q_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$Q = \begin{bmatrix} 6 & 1 & -2 \\ 1 & 4 & -1 \\ -2 & -1 & 1 \end{bmatrix}$$

$\Delta_1 = 6 > 0$ $\quad \Delta_2 = 23 > 0$ $\quad \Delta_3 > 0$

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$\because \Delta_1, \Delta_2, \Delta_3 > 0$

$V(x)$ is +ve definite.

Q. $V(x) = -x_1^2 - 3x_2^2 - 11x_3^2 + 2x_1x_2 - 4x_2x_3 - 2x_1x_3$

$$Q = \begin{bmatrix} -1 & 1 & -1 \\ 1 & -3 & -2 \\ -1 & -2 & -11 \end{bmatrix}$$

$\Delta_1 < 0$

$\Delta_2 > 0$

$\Delta_3 < 0$

we can't say it is pd.

-29 -11

So, find $-V(x)$.

So, $Q = \begin{bmatrix} 1 & -1 & 1 \\ -1 & 3 & 2 \\ 1 & 2 & 11 \end{bmatrix}$

then $\Delta_1 > 0$ $\quad \Delta_2 > 0$ $\quad \Delta_3 > 0$.

So, $-V(x)$ is pd.

So, $V(x)$ is -ve definite.

14 over (86)/3

Theorem 3.2.

$V(x) \longrightarrow pd$

$\dot{V}(x) \longrightarrow nsd \longrightarrow$ stable.

$\longrightarrow nd \longrightarrow$ Asymptotically stable.

ex:- Consider nonlinear system

$$\dot{x}_1 = x_1(x_1^2 + x_2^2 - 2) - 4x_1x_2^2$$

$$\dot{x}_2 = 4x_1^2x_2 + x_2(x_1^2 + x_2^2 - 2)$$

Sol. Let $V(x_1, x_2) = x_1^2 + x_2^2 \longrightarrow$ this is strictly +ve definite.

$$\dot{V} = 2x_1 \cdot \dot{x}_1 + 2x_2 \cdot \dot{x}_2$$

$$= 2x_1(\dot{x}_1) + 2x_2(\dot{x}_2)$$

$$= 2x_1(x_1^3 + x_1x_2^2 - 2x_1 - 4x_1x_2^2) + 2x_2(4x_1^2x_2 + x_2x_1^2 + x_2^3 - 2x_2)$$

$$= 2x_1^4 + 2x_1^2x_2^2 - 4x_1^2 - \cancel{8x_1^2x_2^2} + \cancel{8x_1^2x_2^2} + 2x_2^2x_1^2 + 2x_2^4 - 4x_2^2$$

$$= 2(x_1^4 + x_2^4 - 2x_1^2 - 2x_2^2 + 4x_1^2x_2^2)$$

$$\dot{V} = 2(x_1^2 + x_2^2)(x_1^2 + x_2^2 - 2)$$

$$= (2x_1^2 + 2x_2^2)(x_1^2 + x_2^2 - 2) \quad -4x_1^2 + 2x_2^4 - 4x_2^2$$

$$= 2x_1^4 + 2x_1^2x_2^2 - 2x_1^2 - 2x_2^2 + 2x_1^2x_2^2$$

$$\dot{V} = 2(x_1^2 + x_2^2)(x_1^2 + x_2^2 - 2)$$

locally -ve definite in

$$B_2 = \{ (x_1, x_2) \mid x_1^2 + x_2^2 < 2 \}$$

the origin is asymptotically stable

Q. Consider a 2nd order linear system.

Find a Lyapunov function.

$$A = \begin{bmatrix} 0 & 4 \\ -8 & -12 \end{bmatrix}$$

Sol. P should by symmetric.

So, $P = I$

$$-Q = PA + A^T P$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & 4 \\ -8 & -12 \end{bmatrix} + \begin{bmatrix} 0 & -8 \\ 4 & -12 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$-Q = \begin{bmatrix} 0 & -4 \\ -4 & -24 \end{bmatrix}$$

we can't decide the stability based of principle minors.

$\because$ it is not pd or nd

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$\lambda I - A$

$$\begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} - \begin{bmatrix} 0 & 4 \\ -8 & -12 \end{bmatrix}$$

but eigen values comes out to be stable system

So, instead of assuming P,

we shall

Q. $A = \begin{bmatrix} 0 & -1 \\ 1 & -1 \end{bmatrix}$

$V(x) = x^T P x.$

P is pd symmetric.

$$A^T P + PA = -Q$$

let $Q = I$ $\qquad P = \begin{bmatrix} P_{11} & P_{12} \\ P_{12} & P_{22} \end{bmatrix}$

$$PA = \begin{bmatrix} P_{11} & P_{12} \\ P_{12} & P_{22} \end{bmatrix}\begin{bmatrix} 0 & -1 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} P_{12} & -P_{11}-P_{12} \\ P_{22} & -P_{12}-P_{22} \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 \\ -1 & -1 \end{bmatrix}\begin{bmatrix} P_{11} & P_{12} \\ P_{12} & P_{22} \end{bmatrix} + \cancel{\begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} P_{11} & P_{12} \\ P_{12} & P_{22} \end{bmatrix}} = -1\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} P_{12} & P_{22} \\ -P_{11}-P_{12} & -P_{12}-P_{22} \end{bmatrix} + \cancel{\begin{bmatrix} -P_{12} & -P_{22} \\ P_{11}-P_{12} & P_{12}-P_{22} \end{bmatrix}} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\cancel{\begin{bmatrix} 0 & 0 \\ -2P_{12} & -2P_{22} \end{bmatrix}} = \cancel{\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}}$$

$\cancel{P_{12} = 0} \quad \cancel{P_{22} = 1/2}$

$$\begin{bmatrix} P_{12} + P_{12} & P_{22} - P_{11} - P_{12} \\ -P_{11} - P_{12} + P_{22} & -P_{12} - P_{22} - P_{12} - P_{22} \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

$2P_{12} = -1$ $\qquad P_{22} - P_{11} - P_{12} = 0$

$\Rightarrow \boxed{P_{12} = -0.5}$

$-P_{11} - P_{12} + P_{22} = 0$ $\qquad -2P_{12} - 2P_{22} = -1$

$\boxed{P_{11} = 1.5}$ $\qquad -/(2P_{12} + 2P_{22}) = \cancel{+}1$

$\boxed{P_{22} = 1}$

$P = \begin{bmatrix} 1.5 & -0.5 \\ -0.5 & 1 \end{bmatrix}$ $\qquad \Delta_1 > 0$, $\Delta_2 > 0$ So, P is pd symmetric

So, $V(x) = x^T P x$

$$= \begin{bmatrix} x_1 & x_2 \end{bmatrix} \begin{bmatrix} 1.5 & -0.5 \\ -0.5 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$1.5x_1 - 0.5x_2$

=

Q. For the given 2nd order non linear systems. Use a quadratic Lyapunov function to show that the origin is asymptotically stable.

a) $\dot{x}_1 = -x_1 - x_2$

$\dot{x}_2 = 2x_1 - x_2^3$

for asymp stable

$V(x) \rightarrow pd$

$\dot{V}(x) \rightarrow nd.$

let $V(x) = x_1^2 + x_2^2 \longrightarrow pd$

$\dot{V}(x) = 2x_1\dot{x}_1 + 2x_2\dot{x}_2$

$\Rightarrow \dot{V}(x) = -(2x_1^2 + 2x_2^4 - 2x_1x_2)$

We can't determine the stability.

So, let us assume. $V(x) = x_1^2 + \frac{1}{2}x_2^2$

$\dot{V}(x) = 2x_1\dot{x}_1 + x_2\dot{x}_2$

$= 2x_1(-x_1 - x_2) + x_2(2x_1 - x_2^3)$

Bipin sir

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Intro

Phase Analys

" Constr

DF.

Prob

Lyapun

Problem