i) When a pole is added to a system, the polar plot end point will shift by -90°

ii) When a zero is added to a system, the polar plot end point will shift by $+90^\circ$

Typical sketches of polar plot.

Type '0' Order '1'

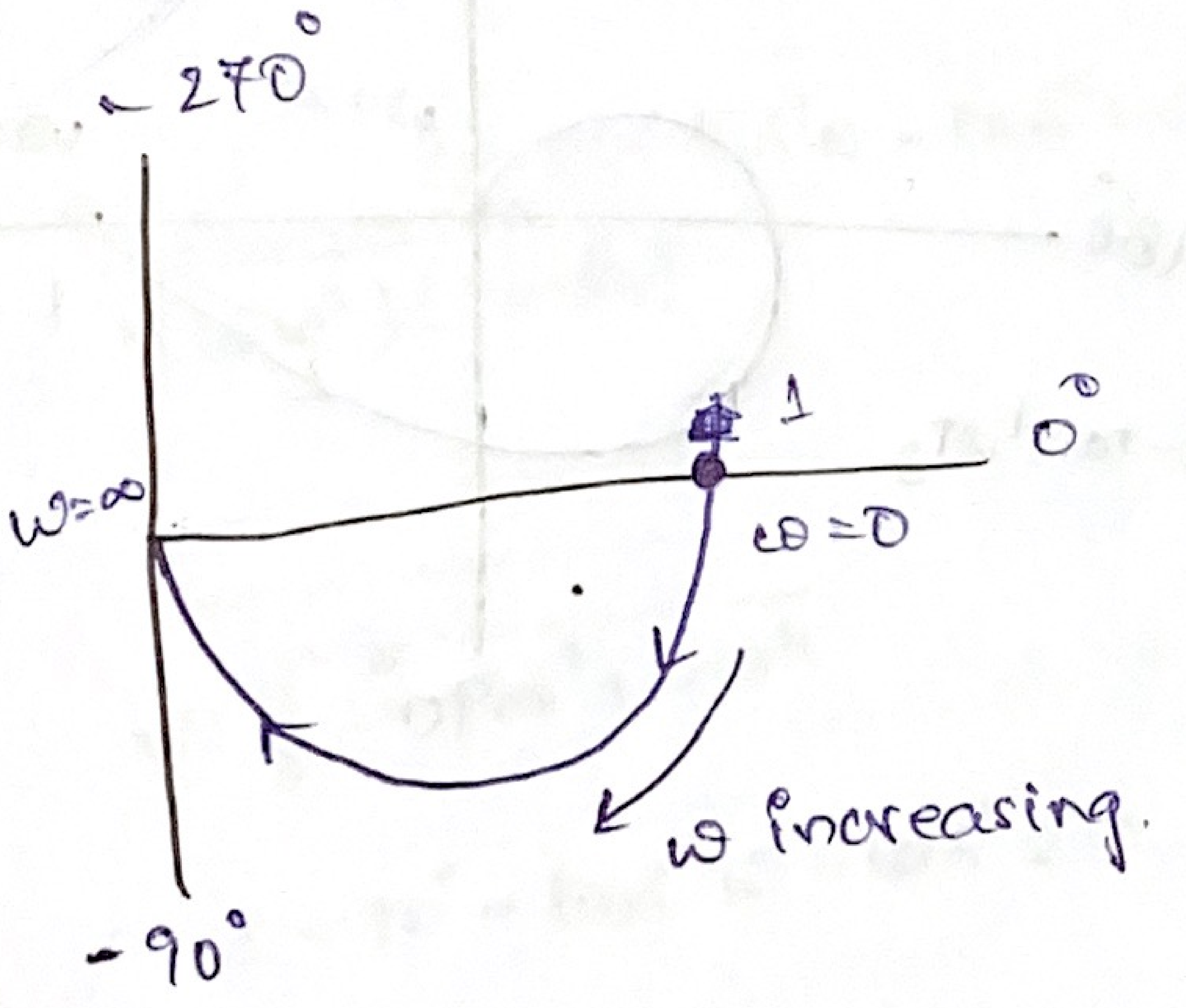
$$G(s) = \frac{1}{(1+Ts)}$$

$$G(j\omega) = \frac{1}{1+T(j\omega)} = \frac{1}{\sqrt{1+\omega^2T^2} \angle \tan^{-1}\omega T}$$

$$= \frac{1}{\sqrt{1+\omega^2T^2}} \angle -\tan^{-1}\omega T$$

As $\omega \to 0$ $\quad G(j\omega) \to 1\angle 0^\circ$

As $\omega \to \infty$ $\quad G(j\omega) \to 0\angle -90^\circ$

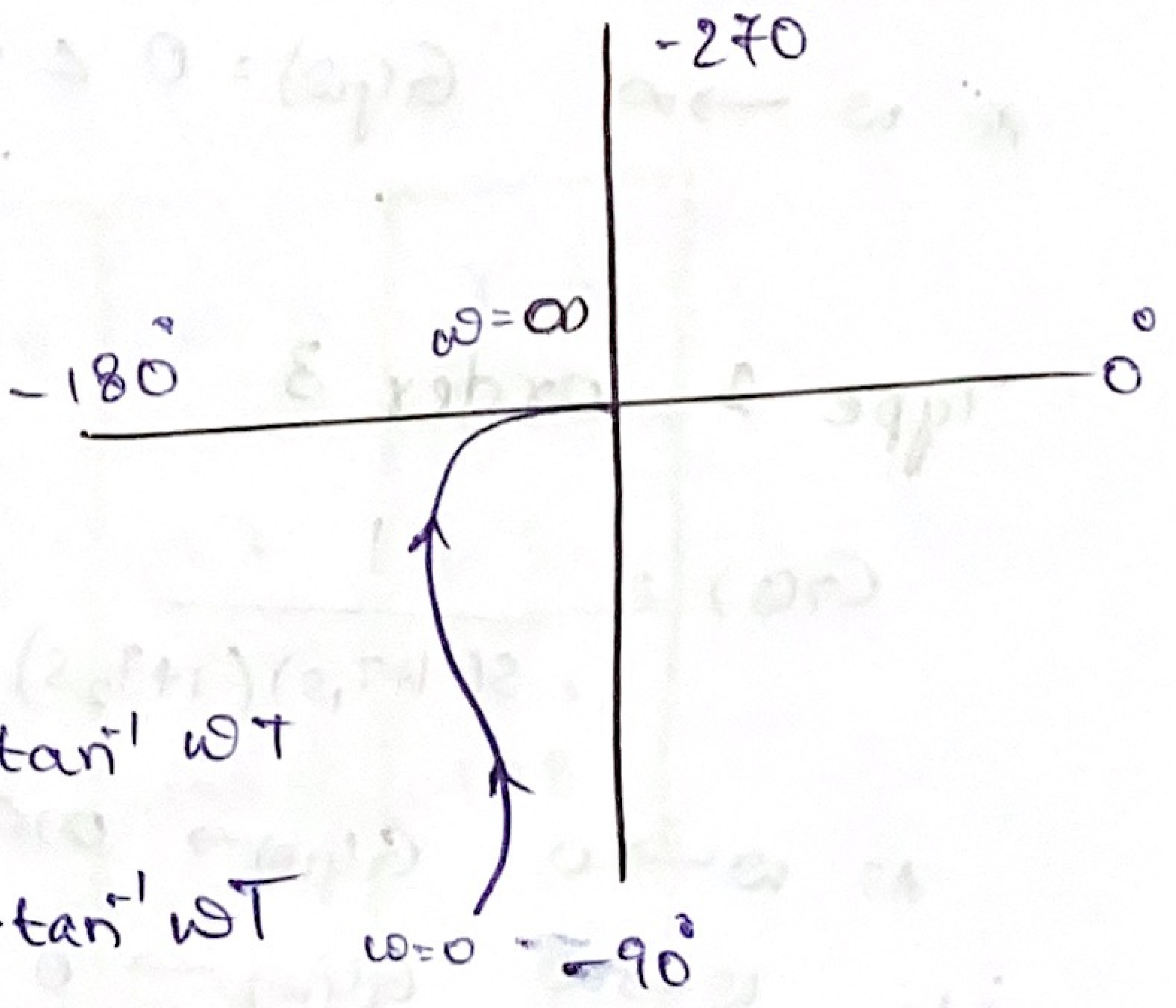


Type '1' Order '2'

$$G(s) = \frac{1}{s(1+Ts)}$$

$$G(j\omega) = \frac{1}{j\omega(1+Tj\omega)} = \frac{1}{\omega\angle 90^\circ \sqrt{1+\omega^2T^2} \angle \tan^{-1}\omega T}$$

$$= \frac{1}{\omega\sqrt{1+\omega^2T^2}} \angle -90^\circ - \tan^{-1}\omega T$$



As $\omega \to 0$ $\quad G(j\omega) \to \infty\angle -90^\circ$

$\omega \to \infty$ $\quad G(j\omega) \to 0 \angle -180^\circ$

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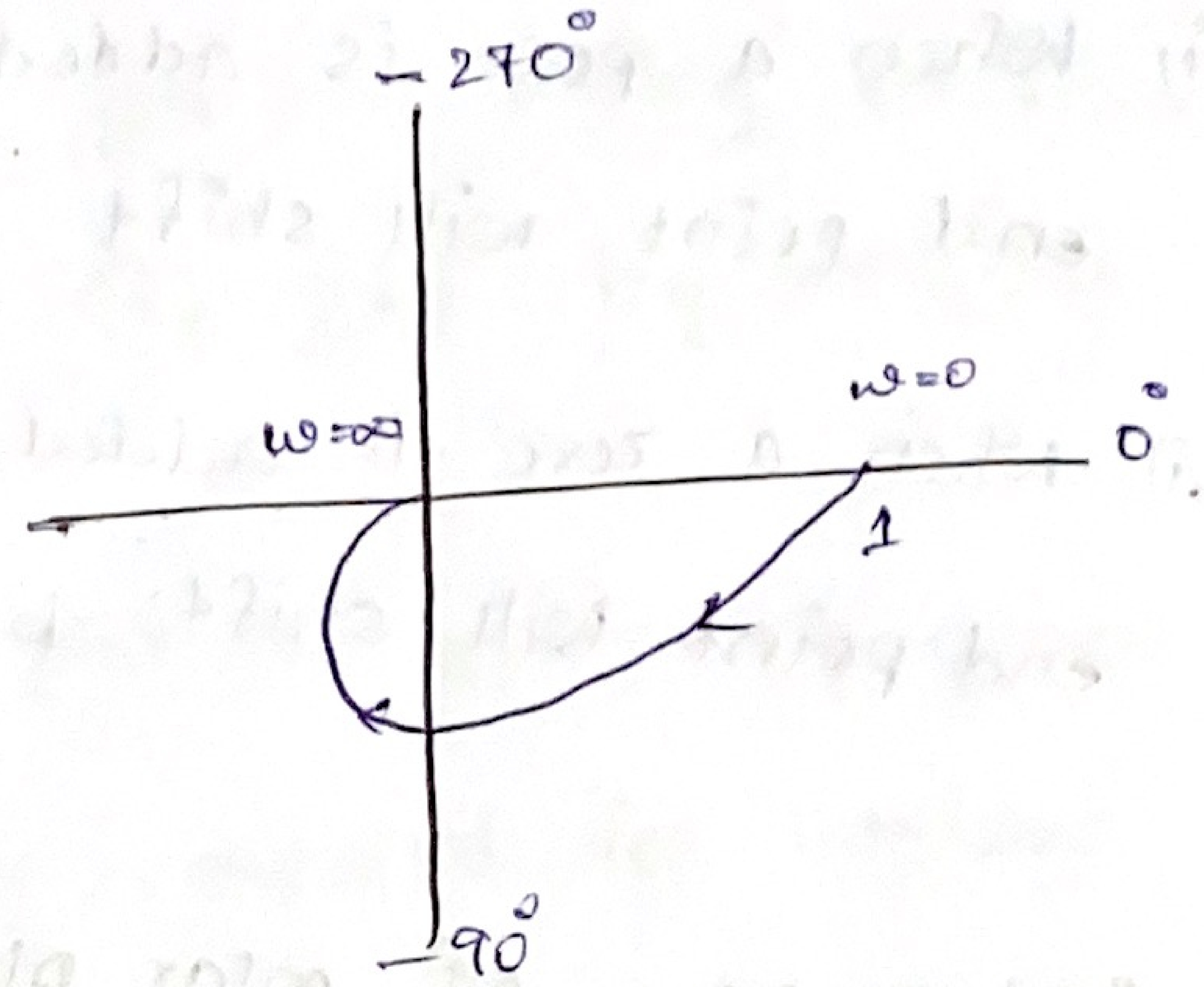
Type 0, order 2

$$G(s) = \frac{1}{(1+sT_1)(1+sT_2)}$$



$$G(j\omega) = \frac{1}{(1+j\omega T_1)(1+j\omega T_2)}$$

$$= \frac{1}{\sqrt{1+\omega^2T_1^2}\angle\tan^{-1}\omega T_1 \sqrt{1+\omega^2T_2^2}\angle\tan^{-1}\omega T_2}$$

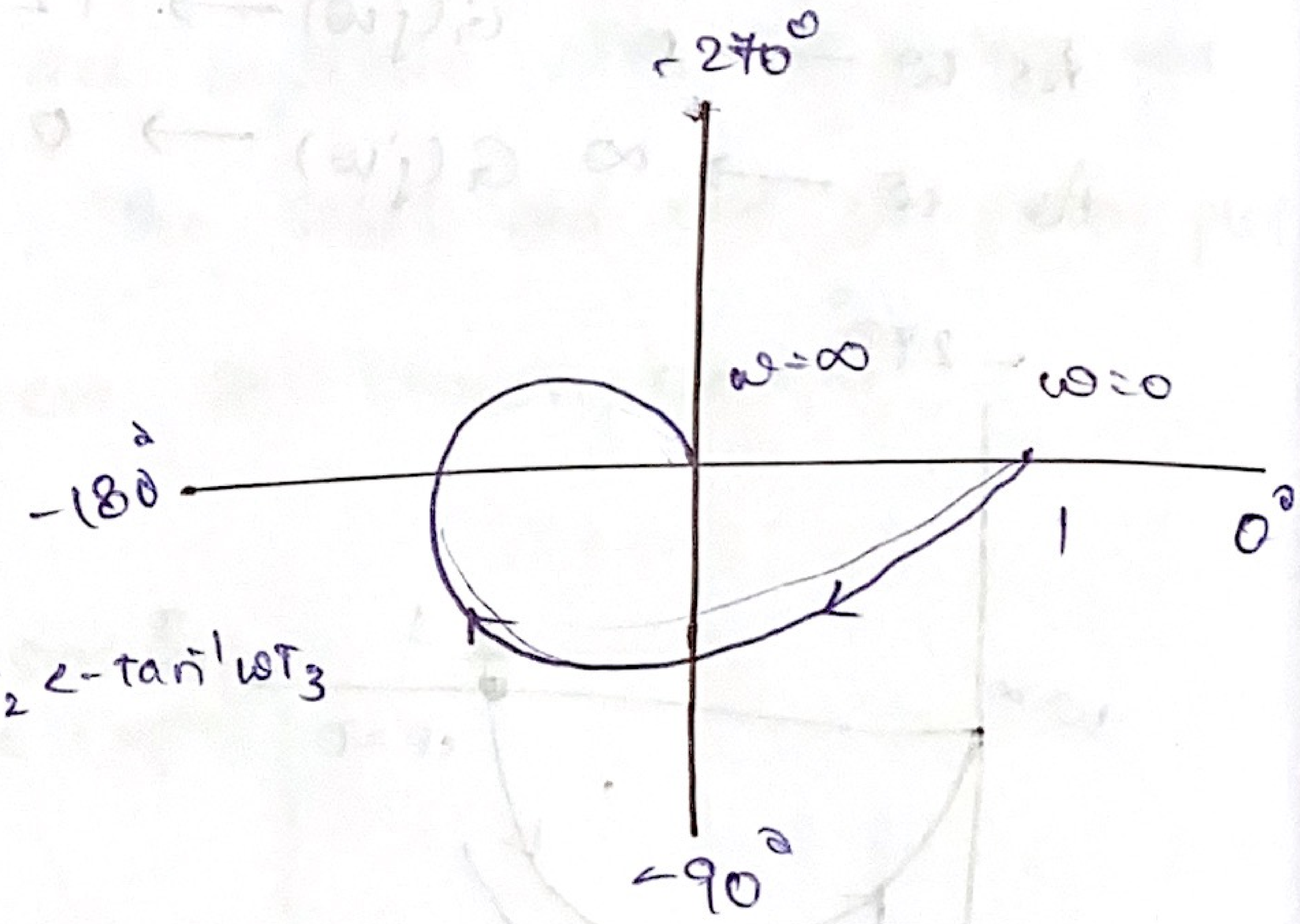
$$= \frac{1}{\sqrt{(1+\omega^2T_1^2)(1+\omega^2T_2^2)}} \angle -\tan^{-1}\omega T_1 - \tan^{-1}\omega T_2$$

As $\omega \to 0$ $G(j\omega) = 1\angle 0°$

As $\omega \to \infty$ $G(j\omega) = 0\angle -180°$

Type 0, order 3

$$G(s) = \frac{1}{(1+sT_1)(1+sT_2)(1+sT_3)}$$



$$G(j\omega) = \frac{1}{(1+j\omega T_1)(1+j\omega T_2)(1+j\omega T_3)}$$

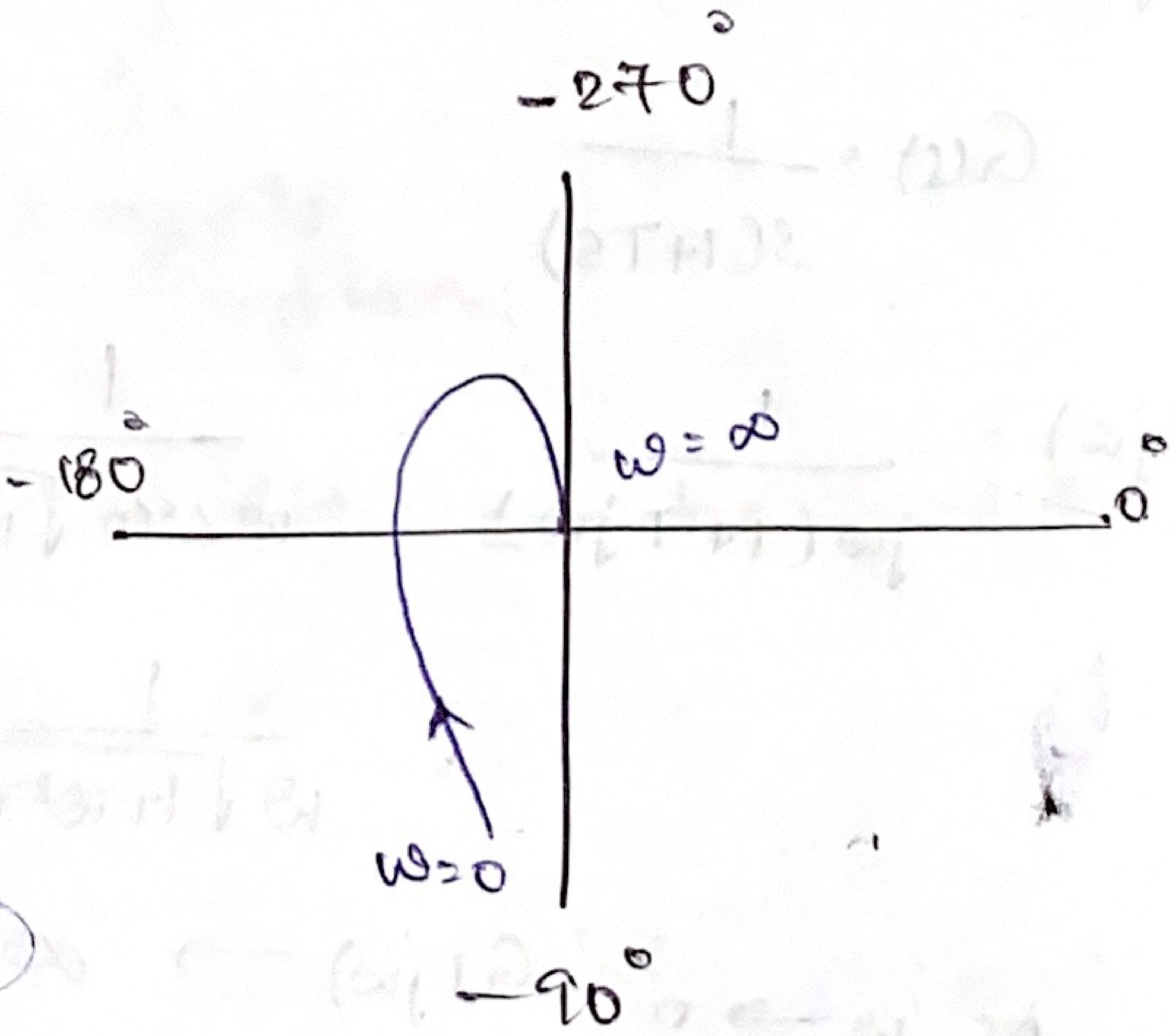
$$= \frac{1}{\sqrt{1+\omega^2T_1^2}\sqrt{1+\omega^2T_2^2}\sqrt{1+\omega^2T_3^2}} \angle -\tan^{-1}\omega T_1 \angle -\tan^{-1}\omega T_2 \angle -\tan^{-1}\omega T_3$$

As $\omega \to 0$ $G(j\omega) = 1\angle 0°$

As $\omega \to \infty$ $G(j\omega) = 0\angle -270°$

Type 1, order 3.

$$G(s) = \frac{1}{s(1+T_1s)(1+T_2s)}$$



As $\omega \to 0$ $G(j\omega) \to \infty\angle -90°$

As $\omega \to \infty$ $G(j\omega) \to 0\angle -270°$

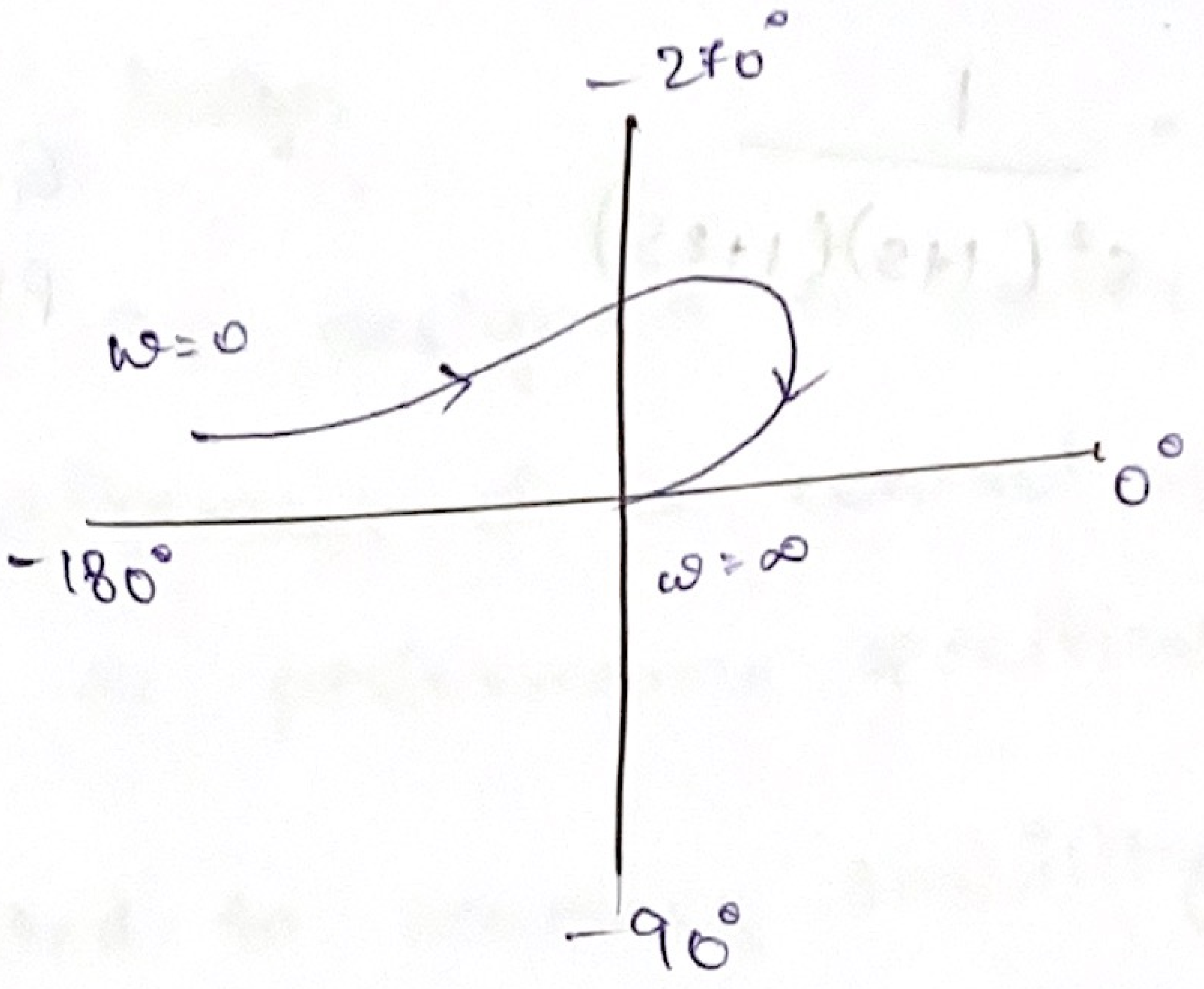
$r\angle\theta \Rightarrow a + ib$

Type 2, order 4

$$G(s) = \frac{1}{s^2(1+T_1 s)(1+T_2 s)}$$

As $\omega \rightarrow 0$ $\quad \infty \angle -180^\circ$

$\omega \rightarrow \infty$ $\quad 0 \angle -360^\circ$



Q. The open loop TF of a unity feedback system is given by $G(s) = \frac{1}{s(1+s)(1+2s)}$. Sketch the polar plot and determine PM & GM.

Sol. $G(j\omega) = \frac{1}{j\omega(1+j\omega)(1+2j\omega)}$

$$= \frac{1}{\omega \angle 90^\circ \sqrt{1+\omega^2} \angle -\tan^{-1}\omega \sqrt{1+4\omega^2} \angle -\tan^{-1}2\omega}$$

$$= \frac{1}{\sqrt{(1+\omega^2)(1+4\omega^2)}} \angle -90 - \tan^{-1}\omega - \tan^{-1}2\omega$$

$$|G(j\omega)| = \frac{1}{\omega\sqrt{1+5\omega^2+4\omega^4}}$$

$$\angle G(j\omega) = \angle -90^\circ - \tan^{-1}\omega - \tan^{-1}2\omega$$

ω rad/s	0.35	0.4	0.45	0.5	0.6	0.7	1
$\|G(j\omega)\|$	2.2	1.8	1.5	1.2	0.9	0.7	0.3
ϕ $\angle G(j\omega)$	-144	-150°	-156°	-162	-171°	-179.5°	198°

ω rad/s	0.35	0.4	0.45	0.5	0.6	0.7	1
$G_R(j\omega)$	-1.78	-1.56	-1.37	-1.14	-0.89	-0.7	-0.29
$G_I(j\omega)$	-1.29	-0.9	-0.65	-0.37	-0.14	0	0.09

$$G(s) = \frac{1}{s^2(1+s)(1+2s)}$$

GM = 0

PM = -90°