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Modern Control Theory (ICE 3153)

State Space Space Modeling of Electromechanical Systems

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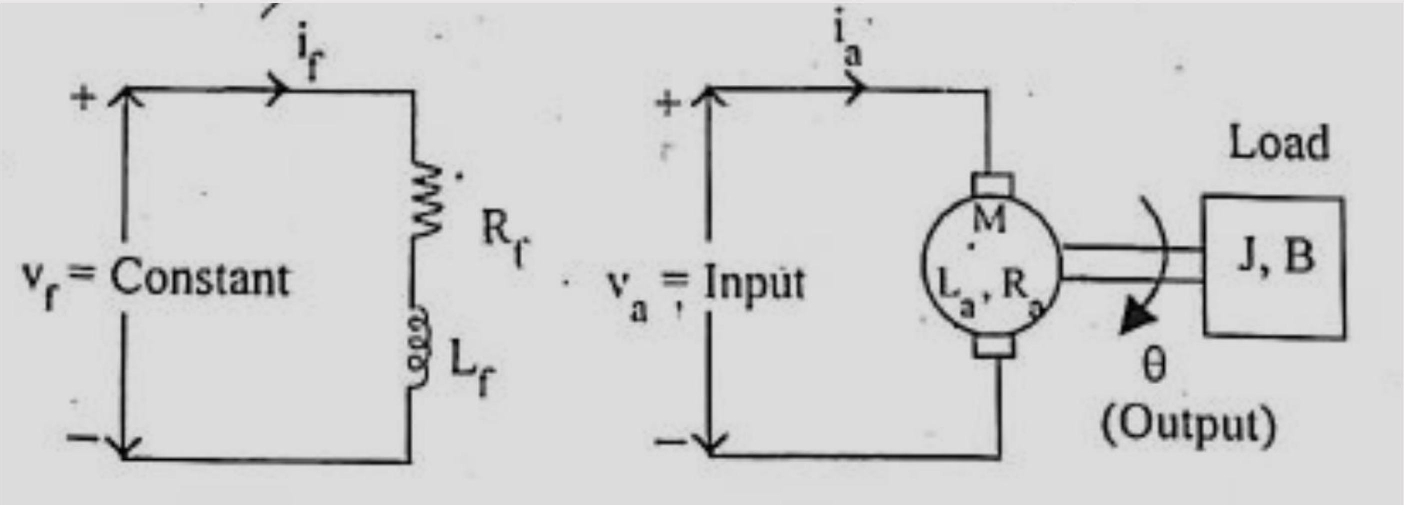
Armature Controlled DC motor



- Speed of DC motor is directly proportional to armature voltage and inversely proportional to flux.
- In armature controlled DC motor the desired speed is obtained by varying the armature voltage.
- The electrical system consist of armature and field circuits, for armature controlled DC motor, the field is excited with constant supply.
- The mechanical system consist of the rotating part of the motor and load connected to shaft.

Let R_a = Armature resistance, Ω

L_a = Armature inductance, H

i_a = Armature current, A

v_a = Armature voltage, V

e_b = Back emf, V

K_t = Torque constant, N-m/A

T = Torque developed by motor, N-m

θ = Angular displacement of shaft, rad

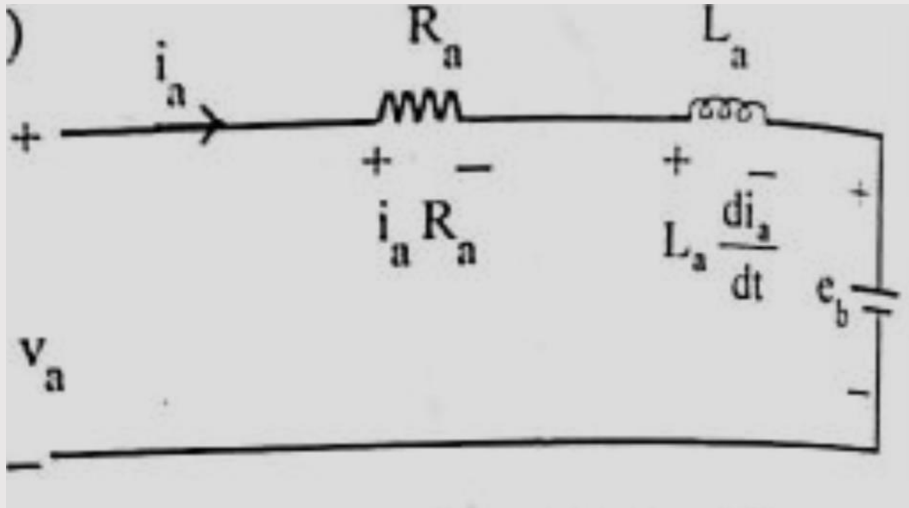
ω = $d\theta\backslash dt$ = Angular velocity of the shaft, rad/sec

J = Moment of inertia of motor and load, Kg-m^2/rad

B = Frictional coefficient of motor and load, N-m/(rad/sec)

K_b = Back emf constant, V/(rad/sec).

The equivalent circuit of the armature is shown as,



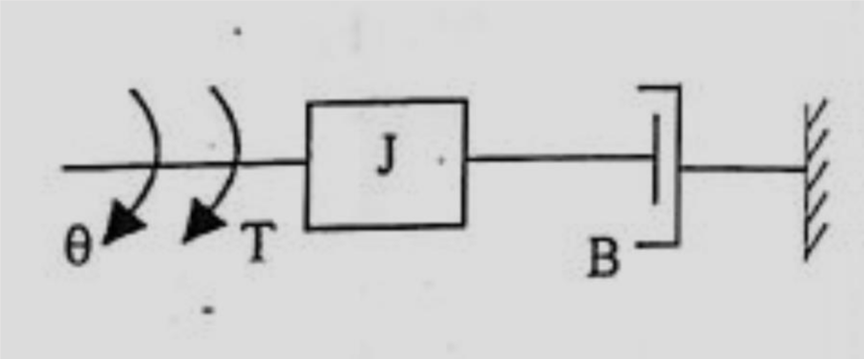
- By applying KVL,

$$i_a R_a + L_a \frac{di_a}{dt} + e_b = v_a$$

- Torque of DC motor is proportional to the product of flux and current. Since flux is constant it is proportional to current only

$$T \propto i_a \qquad \therefore \text{Torque, } T = K_t\, i_a$$

The mechanical system is shown as,



$$J\frac{d^2\theta}{dt^2} + B\frac{d\theta}{dt} = T$$

The back emf of DC machine is proportional to the spe
(angular velocity) of shaft.

$$\therefore e_b \propto \frac{d\theta}{dt}$$

$$\text{Back emf, } e_b = K_b\frac{d\theta}{dt}$$

$$i_a R_a + L_a\frac{di_a}{dt} + K_b\frac{d\theta}{dt} = v_a$$

From Torque equation and mechanical governing equation we get,

$$J\frac{d^2\theta}{dt^2} + B\frac{d\theta}{dt} = K_t i_a \ .$$

So the governing differential equations for the motor are,

$$i_a R_a + L_a \frac{di_a}{dt} + K_b \frac{d\theta}{dt} = v_a$$

$$J\frac{d^2\theta}{dt^2} + B\frac{d\theta}{dt} = K_t i_a \ .$$

Now the state variables can be selected as,

$$x_1 = i_a \ ; \ x_2 = \omega = d\theta/dt \text{ and } x_3 = \theta$$

$$\dot{x}_1 = -\frac{R_a}{L_a}x_1 - \frac{K_b}{L_a}x_2 + \frac{1}{L_a}u$$

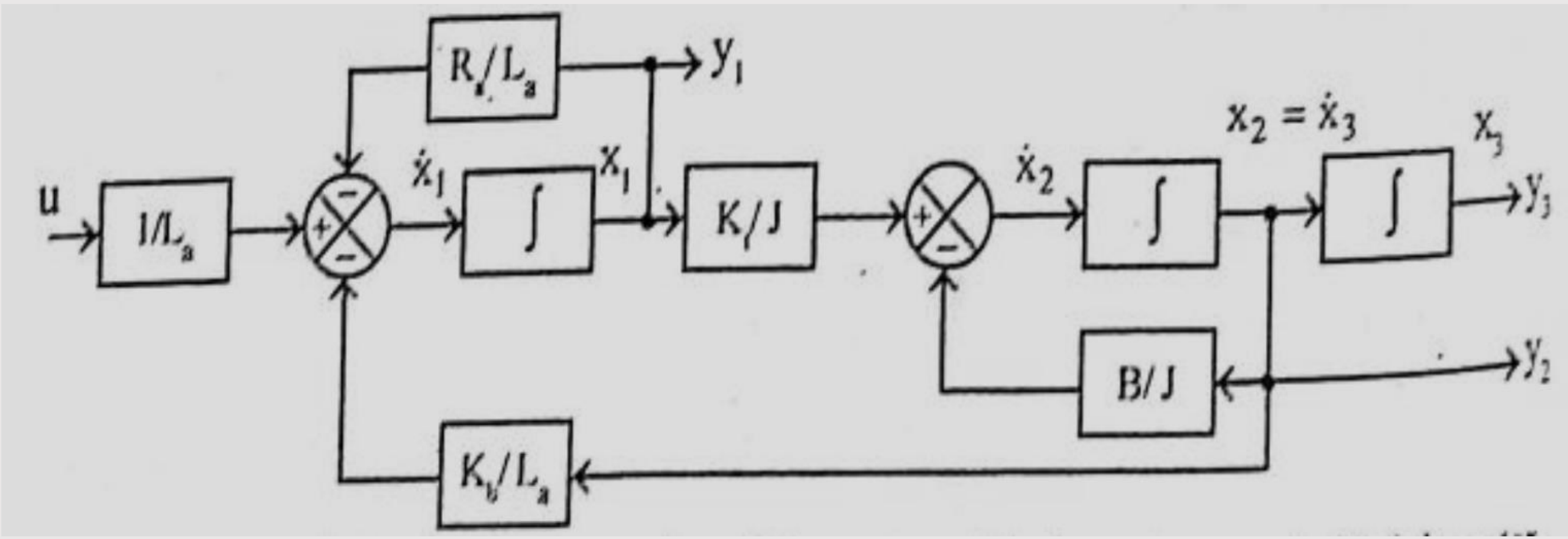
$$\dot{x}_2 = \frac{K_t}{J}x_1 - \frac{B}{J}x_2$$

$$\dot{x}_3 = x_2$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -\frac{R_a}{L_a} & -\frac{K_b}{L_a} & 0 \\ \frac{K_t}{J} & -\frac{B}{J} & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} \frac{1}{L_a} \\ 0 \\ 0 \end{bmatrix} [u]$$

$$y_1 = x_1 \quad ; \quad y_2 = x_2 \quad ; \quad y_3 = x_3$$

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$


R_a/L_a
y_1
x_2 = ẋ_3
x_3
u
1/L_a
ẋ_1
∫
x_1
K/J
ẋ_2
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∫
y_3
B/J
y_2
K_b/L_a

Field Controlled DC motor

Let R_f = Field resistance, Ω

L_f = Field inductance, H

i_f = Field current, A

v_f = Field voltage, V

θ = Angular displacement of the motor shaft, rad

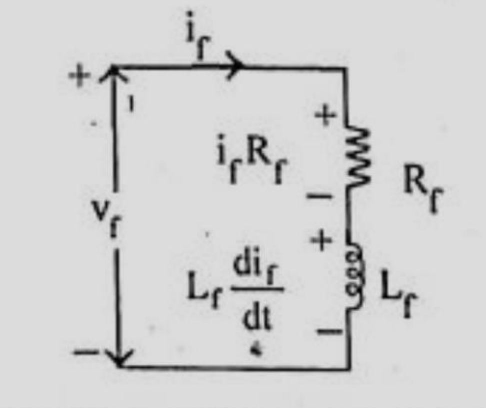
ω = $d\theta/dt$ = Angular velocity of the motor shaft, rad/sec

T = Torque developed by motor, N-m

K_{tf} = Torque constant, N-m/A

J = Moment of inertia of rotor and load, Kg-m^2/rad

B = Frictional coefficient of rotor and load, N-m/(rad/sec).



$$R_f i_f + L_f \frac{di_f}{dt} = v_f$$

$$\therefore T \propto i_f \; ; \; \text{Torque, } T = K_{tf}\, i_f$$

$$x_1 = i_f \quad ; \quad x_2 = \omega = d\theta \backslash dt \quad ; \quad x_3 = \theta$$

$$\dot{x}_1 = -\frac{R_f}{L_f}x_1 + \frac{1}{L_f}u$$

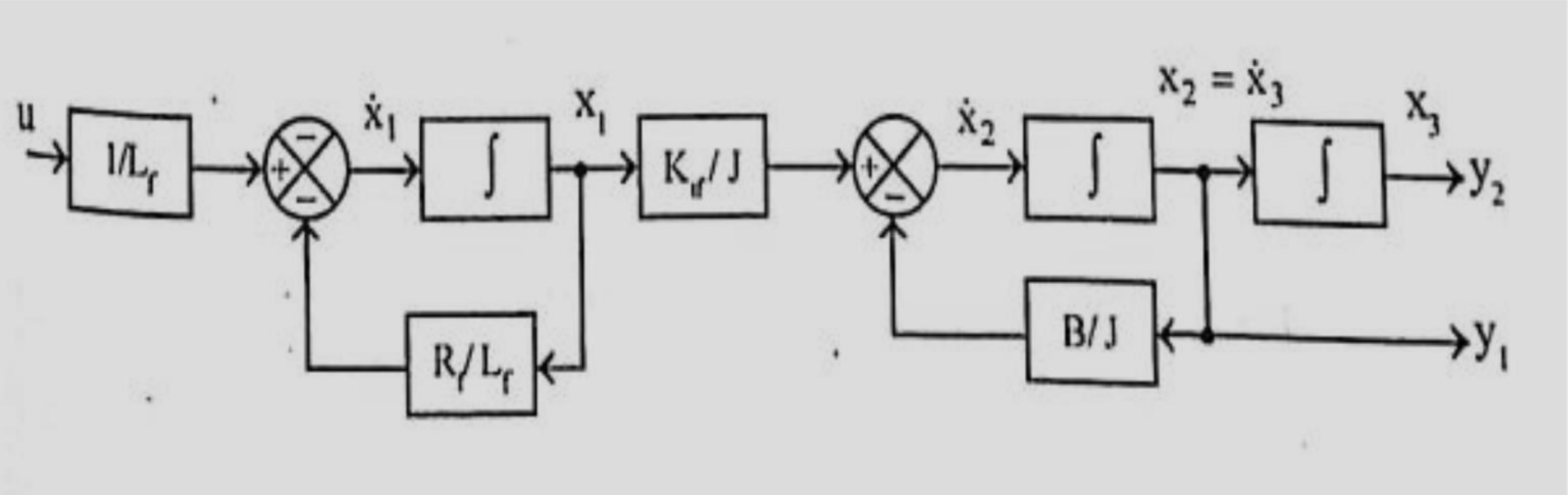
$$\dot{x}_2 = \frac{K_{tf}}{J}x_1 - \frac{B}{J}x_2$$

$$\dot{x}_3 = x_2$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -\frac{R_f}{L_f} & 0 & 0 \\ \frac{K_{tf}}{J} & -\frac{B}{J} & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} \frac{1}{L_f} \\ 0 \\ 0 \end{bmatrix} [u]$$

$$y_1 = \omega \quad ; \quad y_2 = \theta$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$



u
1/L_f
ẋ_1
∫
x_1
R_f/L_f
K_tf/J
ẋ_2
∫
x_2 = ẋ_3
∫
x_3
y_2
B/J
y_1