# Tuning PID Controllers

- Controllers are designed to eliminate the need for continuous operator attention when controlling a process.
- In the automatic mode, the goal is to keep the controlled variable(or process variable) on set point.
- The controller tuning parameters determine how well the controller achieves this goal when in automatic mode.

# DISTURBANCES

- Disturbances arise from three different sources:

- Set point.
- Load.
- Noise.

- Noise is defined as a random disturbance whose frequency distribution exceeds the bandwidth of the control loop. **As such, the controller has no impact on it.**

- Set point and load changes affect the behavior of the control loop quite differently, owing to the dynamics in their path.

- A controller tuned to follow set point changes tends to respond **sluggishly to load variations**, and conversely a controller tuned to correct disturbances **tends to overshoot** when its set point is changed.

# Controller tuning

- The choice of the value of the P, I and D parameters is very much process dependent.
- In most of the cases, it is difficult to obtain the exact mathematical model of the plant.
- So, we have to rely on the experimentation for finding out the optimum settings of the controller for a particular process.
- The process of experimentation for obtaining the **optimum values of the controller parameters** with respect to a particular process is known as controller tuning

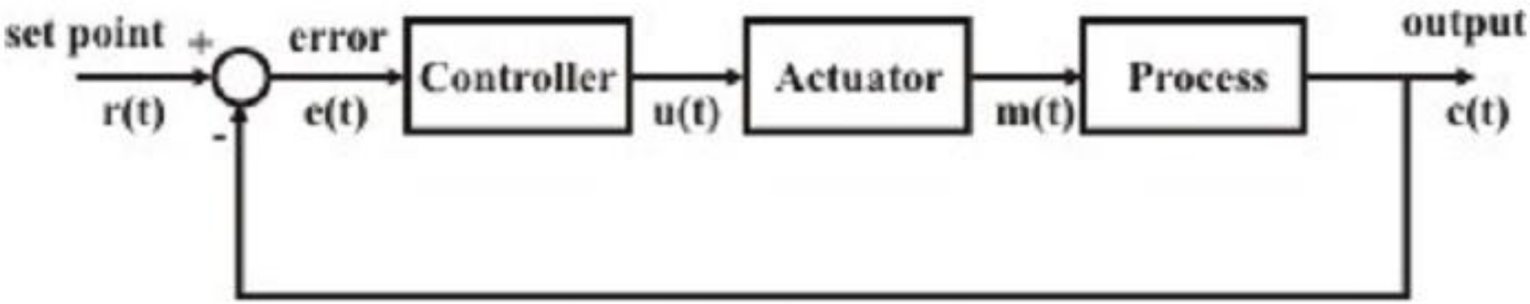


Fig. 1 Closed loop system.

The error signal is fed to the controller and the controller provides output u(t).

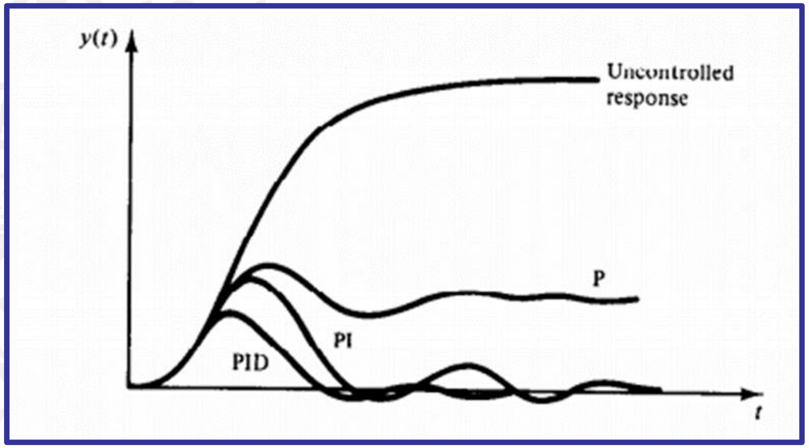
$$u(t) = K_p\left[e(t) + \tau_d \frac{de(t)}{dt} + \frac{1}{\tau_i}\int_0^t e(\tau)d\tau\right]$$

Our objective is to find out the optimum settings of the P,I,D parameters, namely *Kp* , τ*d* and τ*i*

# Performance criteria

Figure shows the response of various controller modes.
P-proportional mode
PI- Proportional-Integral Mode
PID-proportional Integral – derivative mode

- Based on the required response, one has to select the any one of controller modes.



The controller design problem is associated with three logical questions in series:

1) What type of feedback controller should be used to control a given process- P,PI,PID?

2)How do we select the best values for the adjustable parameters of a feedback controllers-$k_c$ ,$\tau_I$ ,$\tau_D$ ?

**3)What performance criterion should we use for the selection and tuning of the controller?**

There are variety of performance criteria we should use, such as:

- Keep the maximum deviation error as small as possible.
- Achieve short settling times.
- Minimize the integral of the errors until the process has settled to its desired set point and so on.

- The following two performance criteria are widely used:

  1) Simple performance criteria

  2)Time – Integral performance criteria

# Performance criteria depends:

## Evaluating Controller Performance

- Bioreactors can't tolerate sudden operating changes because the fragile living cell cultures could die.
  - » "good" control means PV moves ***slowly***

- Packaging/filling stations can be unreliable. Upstream process must ramp back quickly if a container filling station goes down.
  - » "good" control means PV moves ***quickly***

- The operator or engineer defines what is good or best control performance based on their knowledge of:
  - goals of production
  - capabilities of the process
  - impact on down stream units
  - desires of management

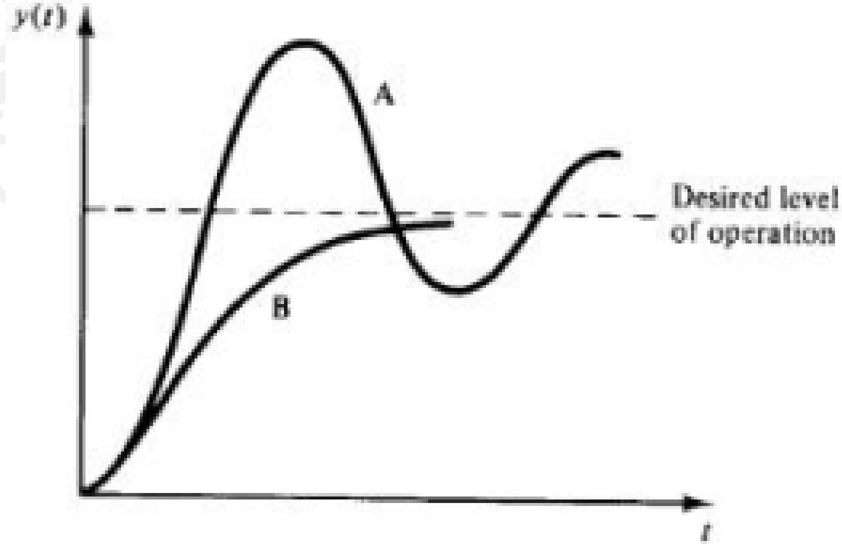
# Which controller has best performance?

- Return to the desired level of operation as soon as possible-A
- Keep the maximum deviation as small as possible

or

Return to the desired level of operation and stay close to in the shortest time

Then -B

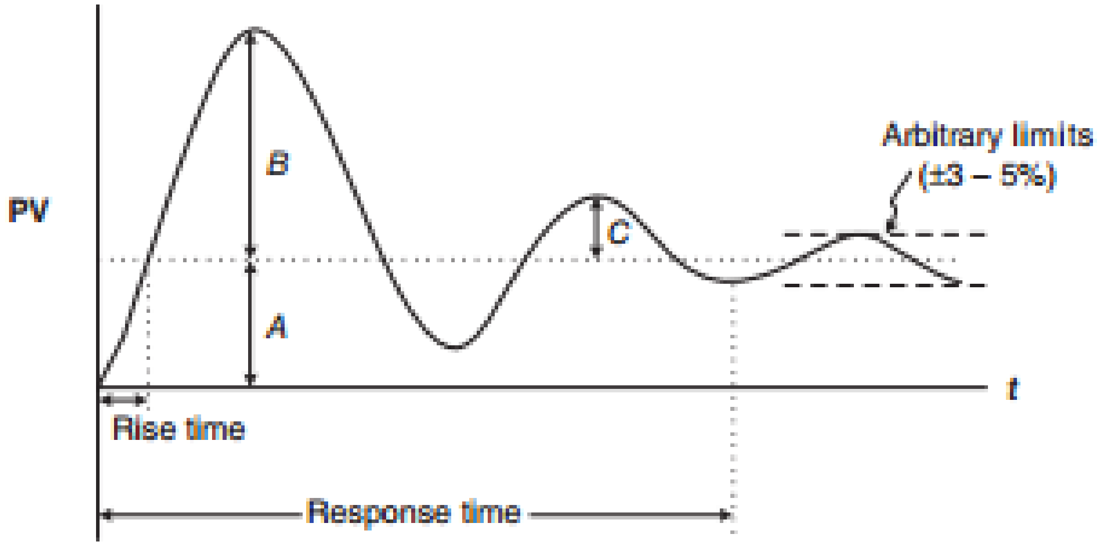


# Steady state criteria

- **Error must be Zero at steady state**

# Simple performance criteria

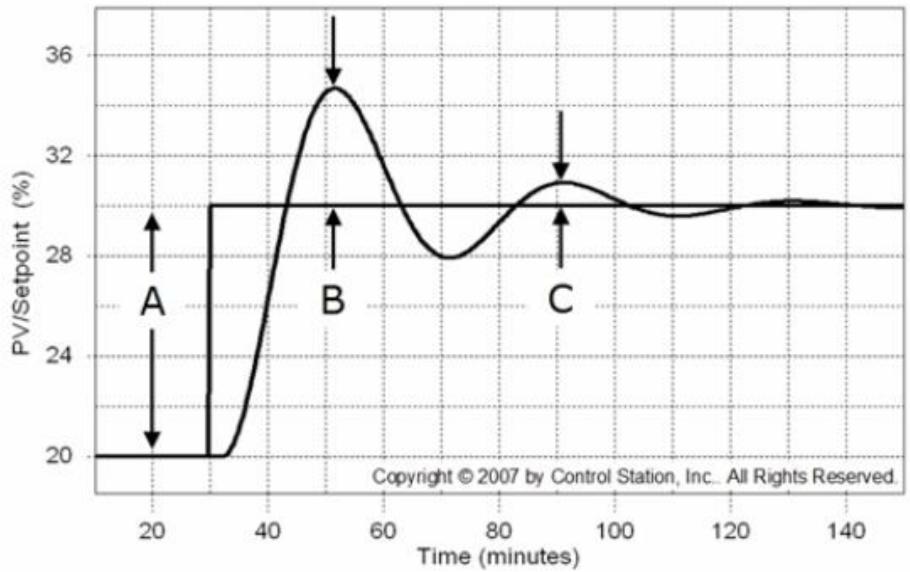
- The simple performance criteria is based on some characteristic features of the closed-loop response of a system




Closed loop response of a second or higher order system

# Simple performance criteria

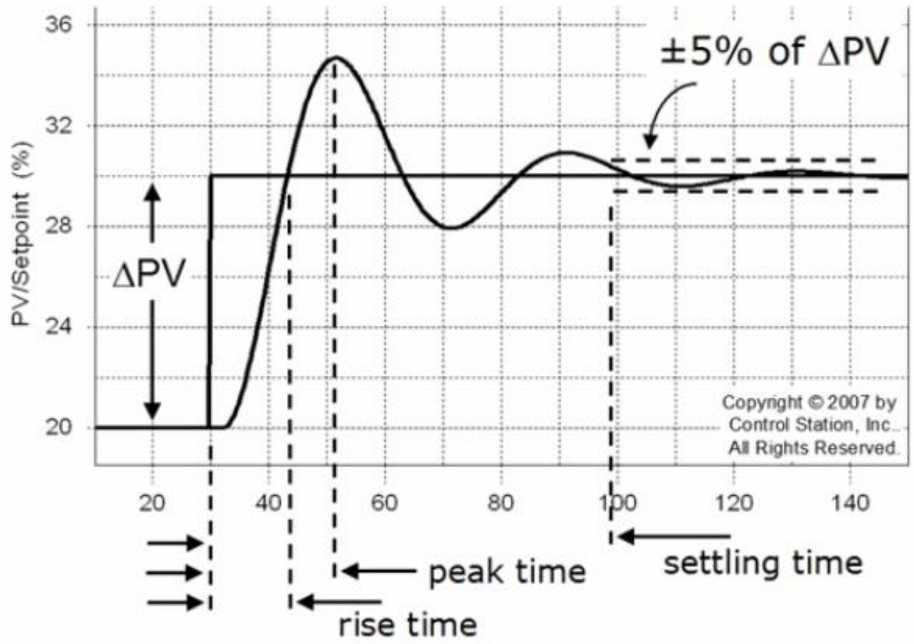
## Performance Analysis




- Rise Time = When PV first reaches SP
- Peak Time = Time of first peak
- Overshoot Ratio = B/A
- Decay Ratio = C/B
- Settling Time = Time when PV remains < 5% of SP

# Simple performance criteria

## Performance Analysis - Time Related Criteria




- The clock for time related events begins when the SP is stepped

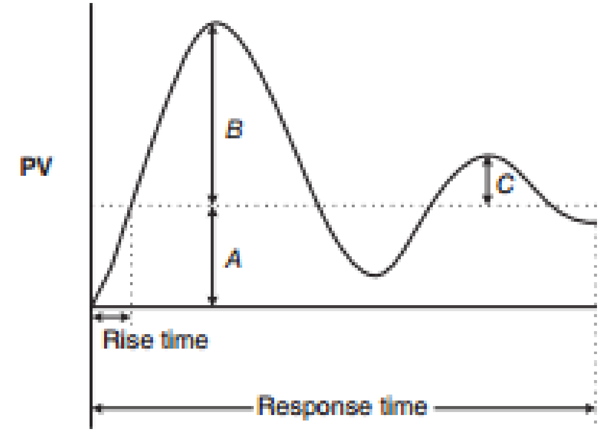
# Simple performance criteria

- Every one of this characteristics can be used by the designer as the basic criterion for selecting the controller and the values of its adjusted parameters.
- We could design the controller in order to have minimum overshoot, or minimum settling time and so on.
- One simple characteristic **does not suffice to describe the desired dynamic response** and controller designs based on multiple criteria lead to conflicting response characteristics.

# Simple performance criteria

- Hence from all the performance criteria's, the decay ratio has been the most popular by the engineers and their experience has shown that ,

**decay ratio=** $\frac{C}{B} = e^{-2\pi\varepsilon/\sqrt{1-\varepsilon^2}} = \frac{1}{4}$




is a reasonable trade-off between a fast rise time and a reasonable settling time.

- This criterion is usually known as the **One-quarter decay ratio criterion**

# Simple performance criteria

- Simple performance criteria with decay ratio of $\frac{1}{4}$ have following disadvantages:

1)Responses with $\frac{1}{4}$ decay ratios are often judged to be too oscillatory by plant operating personnel.

2) the criterion considers only two points of the closed –loop response, namely first two peaks.

An alternative approach is to develop controller design relations based on a performance index that considers the entire closed-loop response is **a time-integral performance criteria.**

# Time –integral performance criteria

The three popular performance indices are:

- **Integral of the square error(ISE)**, where

$$ISE=\int_0^{\infty} e^2(t)dt$$

e(t)=error of the response from the desired set point

- **Integral of the absolute value of the error(IAE)**, where

$$IAE=\int_0^{\infty}|e(t)|\,dt$$

# Time -integral performance criteria

- **Integral of the time -weighted absolute error(ITAE),** where

$$\text{ITAE}=\int_0^{\infty} t|e(t)|\,\mathrm{dt}$$

- Now, select the type of the controller and the values of its adjusted parameters in such a way as to minimize the ISE,IAE or ITAE of the system's response.

- The selection of any one criteria depends on the characteristics of the system we want to control.

# Time –integral performance criteria

The following are some general guidelines:

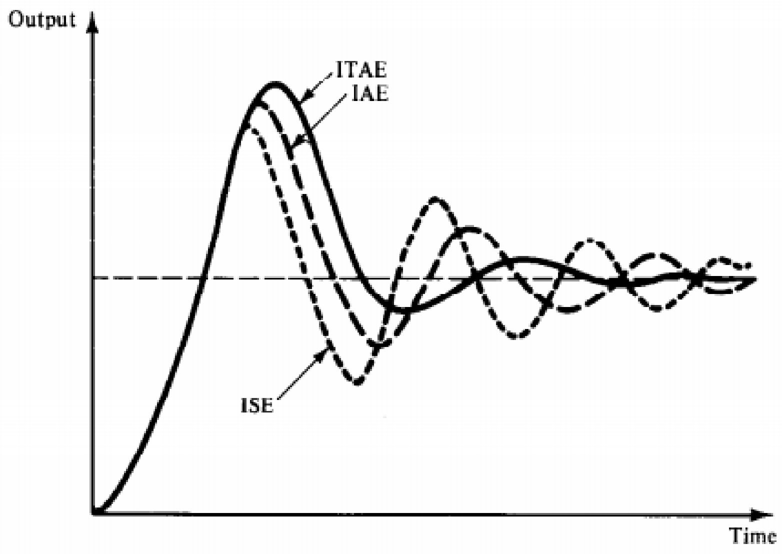
- If we want to strongly suppress large errors, ISE is better than IAE because the errors are squared and thus contribute more to the value of the integral.
- For the suppression of small errors, IAE is better than ISE because when we square small numbers(smaller than one) they become even smaller.
- To suppress errors that persist for long times, the ITAE criterion will tune the controllers better because the presence of large t amplifies the effect of even small errors in the value of the integral.

# Time –integral performance criteria

- Figure shows the shape of the expected closed-loop responses.
- When we tune the controller parameters using ISE,IAE or ITAE performance criteria, we should remember the following two points:

  1)Different criteria lead to different controller designs.

  2) For the same time integral criterion, different input changes lead to different designs.




Figure

# Selecting the type of feedback control

- The following steps should be kept in mind while selecting a feedback controller:

❖ Define an appropriate performance criterion(e.g. ISE, IAE, or ITAE).

❖ Compute the value of the performance criterion using a P,or PI,or PID controller with the best settings for the adjusted parameters $K_c$ ,$\tau_I$ ,$\tau_D$.

❖ Select that controller which gives the best value for the performance criterion.

# Selecting the type of feedback control

There are certain drawbacks of this procedure:

- It is very tedious.
- It relies on the models(transfer functions)for the process ,sensor, and final control element which may not be known exactly.
- It incorporates certain ambiguities as to which is the most appropriate criterion and what input changes to consider.

# Effect of P,I and D control

- P-control
  - Accelerates the response of a controlled process.
  - Produces an offset
- I control
  - Eliminates any offset
  - The elimination of the offset usually comes at the expense of higher maximum deviation
  - Produces sluggish, long oscillating response
  - If we increase the gain Kc to produce faster response, the system becomes more oscillatory and may be led to instability
- D-control
  - Anticipates future errors and introduces appropriate action
  - Introduces a stabilizing effect on the closed loop response of a process.

# General prediction for selection

| Control Loop | Controller Mode | | |
|---|---|---|---|
| | Proportional | Integral | Derivative |
| Flow | Always | Usually | Never |
| Level | Always | Usually | Rarely |
| Temperature | Always | Usually | Usually |
| Analytical | Always | Usually | Sometimes |
| Pressure | Always | Usually | Sometimes |

# Controller tuning

- Controller tuning can be done by three general approaches:

- Use simple criteria such as the one-quarter decay ratio,$t_s$,$t_r$ etc

 -- This approach provides multiple solutions.

- Use time performance criteria such as ISE,IAE or ITAE.

 --This approach is cumbersome and time consuming.

- Use semi empirical rules which have been proven in practice.

 -- Open-loop transient response method (Cohen coon)

 **-Ziegler-Nicholos method**

# Tuning of controllers

1942

- J.G. Ziegler and N.B. Nichols

**by experimentation**

1953

- Modified by
  G.H. Cohen and G.A. Coon

# Three methods of tuning

- Reaction Curve Technique.(Open loop)
- Closed Loop Technique (Continuous Cycling method).
- Closed Loop Technique (Damped oscillation method)

# Reaction Curve Technique.

- This method of finding controller settings was developed by Ziegler and Nichols and later corrections were developed by Cohen and Coon.
- This method is also called as **Process reaction curve method**.
- This method can be used only for system with self-regulation.

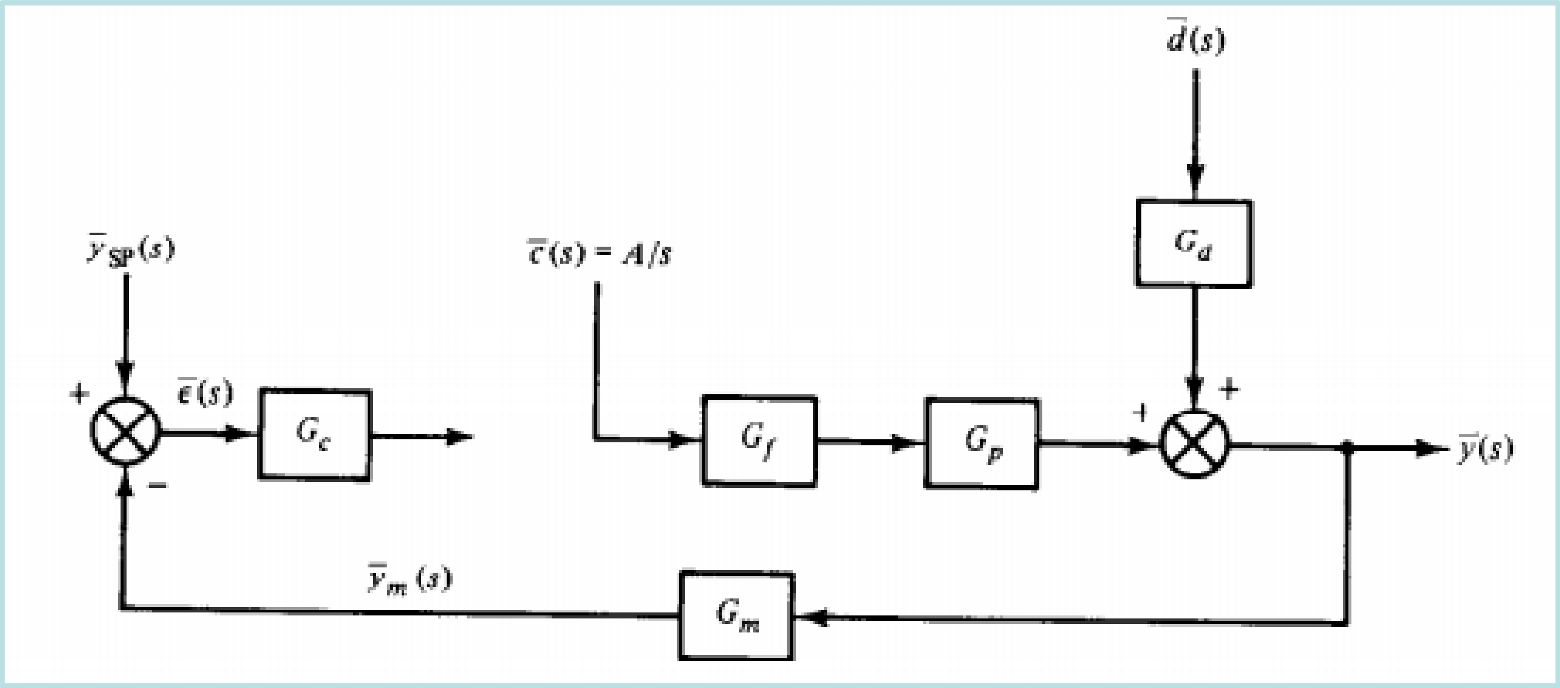



Figure 1.1

Figure :Opened control loop

- Consider the control system of figure which has been opened by disconnecting the controller from the final control element.

- Introduce a step change of magnitude A in the variable c which actuates the final control element.

- Record the value of the output with respect to time.

- This curve have a sigmoidal shape.

# Mathematical analysis

- A typical open loop controller response is shown in figure 1.2.
- A tangent line, showed as a dashed line, is drawn at the inflection point of the curve.
- The inflection point is defined as that point on the curve where the slope stops increasing and begins to decrease.
- Where the tangent line crosses the origin we get

- L=lag time in minutes or dead time..          eq(1)

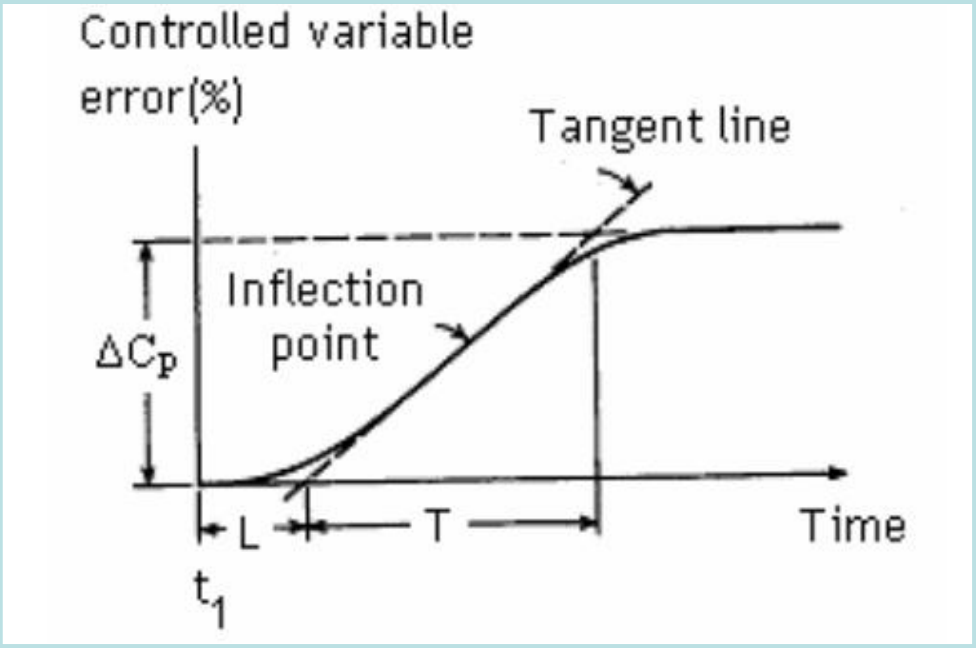



Figure 1.2

- Also from the graph ,we get T, the process reaction time, and

$$N=\frac{\Delta Cp}{T} \text{ ........eq(2)}$$

where N=reaction rate in%/min

$\Delta Cp$ =variable change in%

T= process reaction time in minute

The quantities defined by eq(1) and eq(2) are used with the controlling variable change ΔP to find controller settings.

# Formulas for various modes as developed by Ziegler and Nichols and correction developed by Cohen and Coon

## ❖Proportional mode:

$$K_p = \frac{\Delta P}{NL}$$

- Corrections to the values ok $K_p$ are used to obtain the quarter amplitude criterion of response. One given by Cohen and Coon is shown bracket.

$$K_p = \frac{\Delta P}{NL}\left[1 + \frac{1NL}{3\Delta Cp}\right]$$

❖ **Proportional –Integral mode:**

$$K_p = 0.9 \frac{\Delta P}{NL}$$

$$T_I = 3.33L$$

If the quarter –amplitude criterion is used,the gain is

$$K_p = \frac{\Delta P}{NL} \left[0.9 + \frac{1}{12} R\right]$$

where $R = \frac{NL}{\Delta C_p}$ = log ratio(unit less)

$$T_I = \left[\frac{30+3R}{9+20R}\right] L$$

- **Three –mode**:

$$K_p = 1.2\frac{\Delta P}{NL}$$
$$T_I = 2L$$
$$T_D = 0.5L$$

If the quarter –amplitude criterion is used ,these equations are corrected by

$$K_p = \frac{\Delta P}{NL}\left[1.33 + \frac{1}{4}R\right]$$

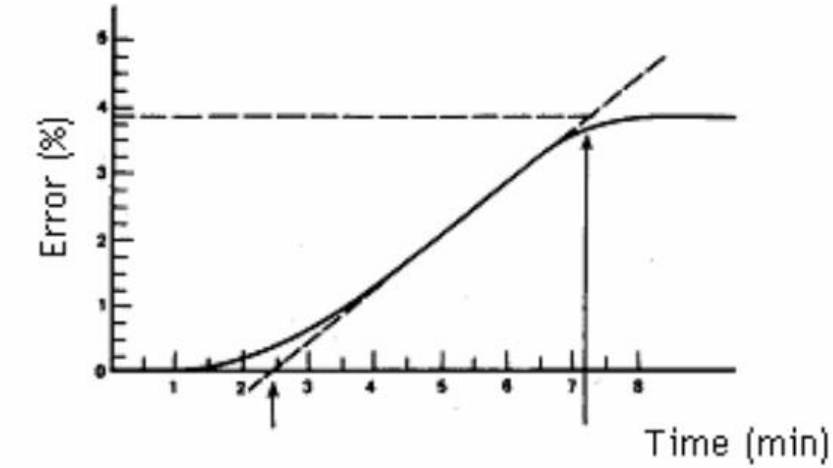
$$T_I = \left[\frac{32+6R}{13+8R}\right]L$$

$$T_D = \left[\frac{4}{11+2R}\right]L$$

# Numerical:1

- A transient disturbance test is run on a process loop. The result of a 9% controlling variable change give a process-reaction graph as shown in Figure 1.3.Find the settings for three-mode action.

Figure 1.3



# Solution :

- By drawing the inflection point tangent on the graph,we find a lag L=2.4 min and process reaction time of 4.8 min.The reaction rate is

$$N=\frac{\Delta Cp}{T}$$

$$N=\frac{\Delta Cp}{T} = 3.9\%/\ 4.8min$$

$$N=\frac{\Delta Cp}{T} =0.8125\%/min$$

$$K_p = 1.2\frac{\Delta P}{NL}$$

$$K_p = 5.54$$

$$T_I = 2L$$

$$T_I = 4.8 \text{ min}$$

$$T_D = 0.5L$$

$$T_D = 1.2\text{min}$$

- Three mode settings for a quarter-amplitude response:

$$R = \frac{NL}{\Delta C_p} = 0.50$$

$$K_p = \frac{\Delta P}{NL}\left[1.33 + \frac{1}{4}R\right]$$

$$K_p = 6.72$$

$$T_I = \left[\frac{32+6R}{13+8R}\right]L$$

$$T_I = 4.94\text{min}$$

$$T_D = \left[\frac{4}{11+2R}\right]L$$

$$T_D = 0.80\text{min}$$

# Ziegler -Nichols closed loop method

- This technique is also called as Ultimate cycle method.
- It is based on adjusting a closed loop until steady oscillations occur.
- Controller settings are then based on the conditions that generate the cycling.
- This method can be used for the systems without self regulation.

# Ziegler –Nichols closed loop method

- The method is accomplished through the following steps:

1. Reduce any integral and derivative actions to their minimum effect.($\tau_d$ =0 *and* $\tau_i = \infty$ )
2. Gradually begin to increase the proportional gain while providing periodic small disturbances to the process.
3. Note the critical gain,$K_c$ ,at which the dynamic variable just begins to exhibit steady cycling-that is ,oscillations about the setpoint.
4. Note the critical period,$T_c$ ,of these oscillations measured in minutes.

# Ziegler –Nichols closed loop method

- Now ,from the critical gain and period,the setting of the controller are assigned as follows:

1. **Proportional :**

$$K_p = 0.5K_c$$

A modification of this relation is often used when the quarter-amplitude criterion is applied.

In that case, the gain simply is adjusted until the dynamic response pattern to a step change in setpoint obeys the quarter-amplitude criterion.

This also results in some gain less than $K_c$

# Ziegler –Nichols closed loop method

2. **Proportional –Integral:**

$$K_p = 0.45K_c$$

$$T_I = T_c / 1.2$$

In case the quarter-amplitude criterion is desired , we make

$$T_I = T_c$$

and adjust the gain for that necessary to obtain the quarter-amplitude response

# Ziegler -Nichols closed loop method

**Three mode(PID):**

$$K_p = 0.6K_c$$

$$T_I = T_c / 2.0$$

$$T_D = T_c / 8$$

For adjustment to give quarter-amplitude response, we set

$$T_I = T_c / 1.5$$

$$T_D = T_c / 6$$

And adjust the proportional gain for satisfaction of the quarter-amplitude response.

# Closed Loop Technique (Damped oscillation method)

- For plants are not allowed to undergo through sustained oscillations

- Initially the closed loop system is operated initially with low *gain* proportional control mode with
  $\tau d = 0$ *and* $\tau i = \infty$.

- The *gain* is increased slowly till a decay ratio (*p2/p1*) of 1/4th is obtained in the step response.

- Under this condition, the period of damped oscillation, *Td* is also noted. Let *Kd* be the proportional *gain* setting for obtaining 1/4th decay ratio.

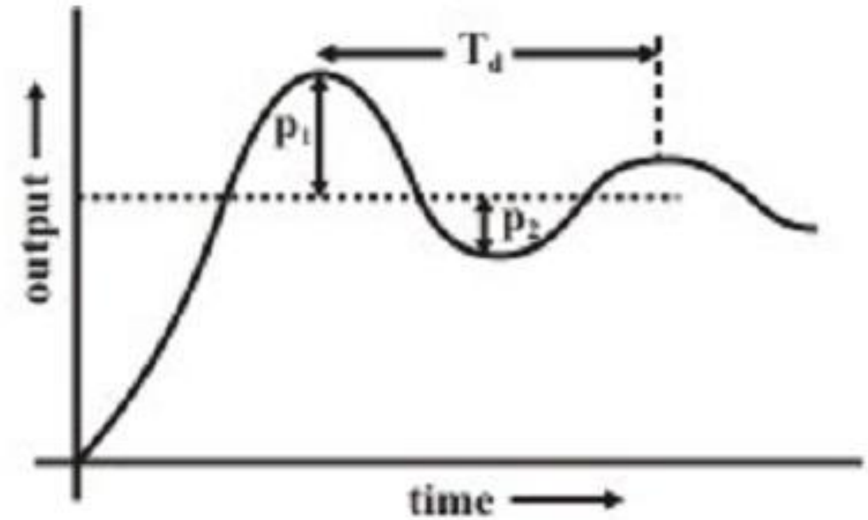



Fig. 5 Controller tuning using damped oscillation technique.

$$T_I = T_c / 1.5$$
$$T_D = T_c / 6$$

# General comments about controller tuning

- The recommended settings are empirical in nature, and obtained from extensive experimentation.
- No theoretical basis behind these selections.
- A better combination can be there.
- But with no a-priori knowledge of the system, it is always advisable to perform the experimentation and select the controller setting

- Nowadays digital computers are replacing the conventional analog controllers.

- P-I-D control actions are generated through digital computations.

- Digital outputs of the controllers are converted to analog signals before they are fed to the actuators.

- In many cases, commercial software are available for *Auto tuning* the process.

- Here the controller generates several commands those are fed to the plant. After observing the output responses, the controller parameters are selected, similar to the cases discussed above

# Frequency Response Methods

- Involves use of Bode plots for the process and control loops.

- Based on an application of the Bode plot stability criteria.

- Stability of the system is based on the **Gain and Phase Margin**.

# Stability requirement:

- If the gain is slightly less than 1 when the phase lag is , the system is stable. But if the gain is slightly greater than 1 at , the system is unstable.

- It would be well to design a system with a margin of safety from such limits to allow for variation in components and other unknown factors

# *Revised stability criteria*

1. If the phase lag is less than 140° at the unity gain frequency, the system is stable. This, then, is a 40° *phase margin* from the limiting value of 180°.

2. If the gain is 5 dB below unity (or a gain of about 0.56) when the phase lag is,the system is stable. This is a 5-dB *gain margin.*

# Tuning

- Tuning involve adjustments of the controller parameters until the stability is proved by the appropriate phase and gain margins.
- Bode plot can be determined experimentally by opening the loop and providing a variable-frequency disturbance of the controlling variable.
- The significance of the unity gain crossover in frequency is that the system can correct any disturbances of frequency less than that of the unity gain frequency.
- Any disturbance of higher frequency has little effect on the control system.