Formula Sheet - Laplace Tranform

1. Definition of Laplace transform of $f(t)$: $\mathcal{L}\{f(t)\} = \int\limits_0^\infty e^{-st} f(t)\, \mathrm{d}t$.

   This definition will not be provided during the quizzes/final exam.

2. $\mathcal{L}\{C\} = \dfrac{C}{s}$ for any constant C

3. $\mathcal{L}\{t^n\} = \dfrac{n!}{s^{n+1}}$ for $n = 1, 2, 3\cdots$

4. $\mathcal{L}\left\{e^{at}\right\} = \dfrac{1}{s-a}$ for any constant a

5. $\mathcal{L}\{\sin(kt)\} = \dfrac{k}{s^2+k^2}$ for any constant k

6. $\mathcal{L}\{\cos(kt)\} = \dfrac{s}{s^2+k^2}$ for any constant k

7. $\mathcal{L}\{\sinh(kt)\} = \dfrac{k}{s^2-k^2}$ for any constant k

8. $\mathcal{L}\{\cosh(kt)\} = \dfrac{s}{s^2-k^2}$ for any constant k

9. $\mathcal{L}\{f'(t)\} = sF(s) - f(0)$

10. $\mathcal{L}\{f''(t)\} = s^2F(s) - sf(0) - f'(0)$

11. $\mathcal{L}\{f'''(t)\} = s^3F(s) - s^2f(0) - sf'(0) - f''(0)$

12. $\mathcal{L}\left\{f^{(n)}(t)\right\} = s^nF(s) - s^{n-1}f(0) - s^{n-2}f'(0) - \cdots - f^{(n-1)}(0)$

13. First Translation Theorem: $\mathcal{L}\left\{e^{at}f(t)\right\} = F(s)|_{s\to s-a}$ where $F(s) = \mathcal{L}\{f(t)\}$

14. Unit Step Function: $\mathcal{U}(t-a) = \begin{cases} 0 & \text{if } 0 \le t < a \\ 1 & \text{if } t \ge a \end{cases}$

15. $f(t) = \begin{cases} g(t) & \text{if } 0 \le t < a \\ h(t) & \text{if } t \ge a \end{cases} \Rightarrow f(t) = g(t) - g(t)\mathcal{U}(t-a) + h(t)\mathcal{U}(t-a)$

    This formula will not be provided during quiz/examination.

16. $f(t) = \begin{cases} g(t) & \text{if } 0 \le t < a \\ h(t) & \text{if } a \le t < b \\ j(t) & \text{if } t \ge b \end{cases}$

    $\Rightarrow f(t) = g(t) - g(t)\mathcal{U}(t-a) + h(t)\mathcal{U}(t-a) - h(t)\mathcal{U}(t-b) + j(t)\mathcal{U}(t-b)$

    This formula will not be provided during quiz/examination.

17. Second Translation Theorem (version 1): $\mathcal{L}\left\{f(t-a)\,\mathcal{U}\,(t-a)\right\} = e^{-as}\,\mathcal{L}\left\{f(t)\right\}$

    This formula is easier to apply for finding inverse-Laplace transform.

18. Second Translation Theorem (version 2): $\mathcal{L}\left\{f(t)\,\mathcal{U}\,(t-a)\right\} = e^{-as}\,\mathcal{L}\left\{f(t+a)\right\}$

    This formula is easier to apply for finding Laplace transform.

19. $\mathcal{L}\left\{\mathcal{U}\,(t-a)\right\} = \dfrac{e^{-as}}{s}$

20. $\mathcal{L}\left\{t^n f(t)\right\} = (-1)^n \dfrac{\mathrm{d}^n}{\mathrm{d}s^n} F(s)$ where $F(s) = \mathcal{L}\left\{f(t)\right\}$

21. Definition of convolution: $f(t) * g(t) = \displaystyle\int_0^t f(\tau) g(t-\tau)\,\mathrm{d}\tau$

22. $f(t) * g(t) = g(t) * f(t)$

23. $\mathcal{L}\left\{f(t) * g(t)\right\} = \mathcal{L}\left\{f(t)\right\} \cdot \mathcal{L}\left\{g(t)\right\}$

24. $\mathcal{L}\left\{\displaystyle\int_0^t f(\tau)\,\mathrm{d}\tau\right\} = \dfrac{F(s)}{s}$

25. Let $f(t+T) = f(t)$ for all $t \geq 0$ be periodic with period $T > 0$. Then

$$\mathcal{L}\left\{f(t)\right\} = \frac{1}{1-e^{-sT}} \int_0^T e^{-st} f(t)\,\mathrm{d}t$$