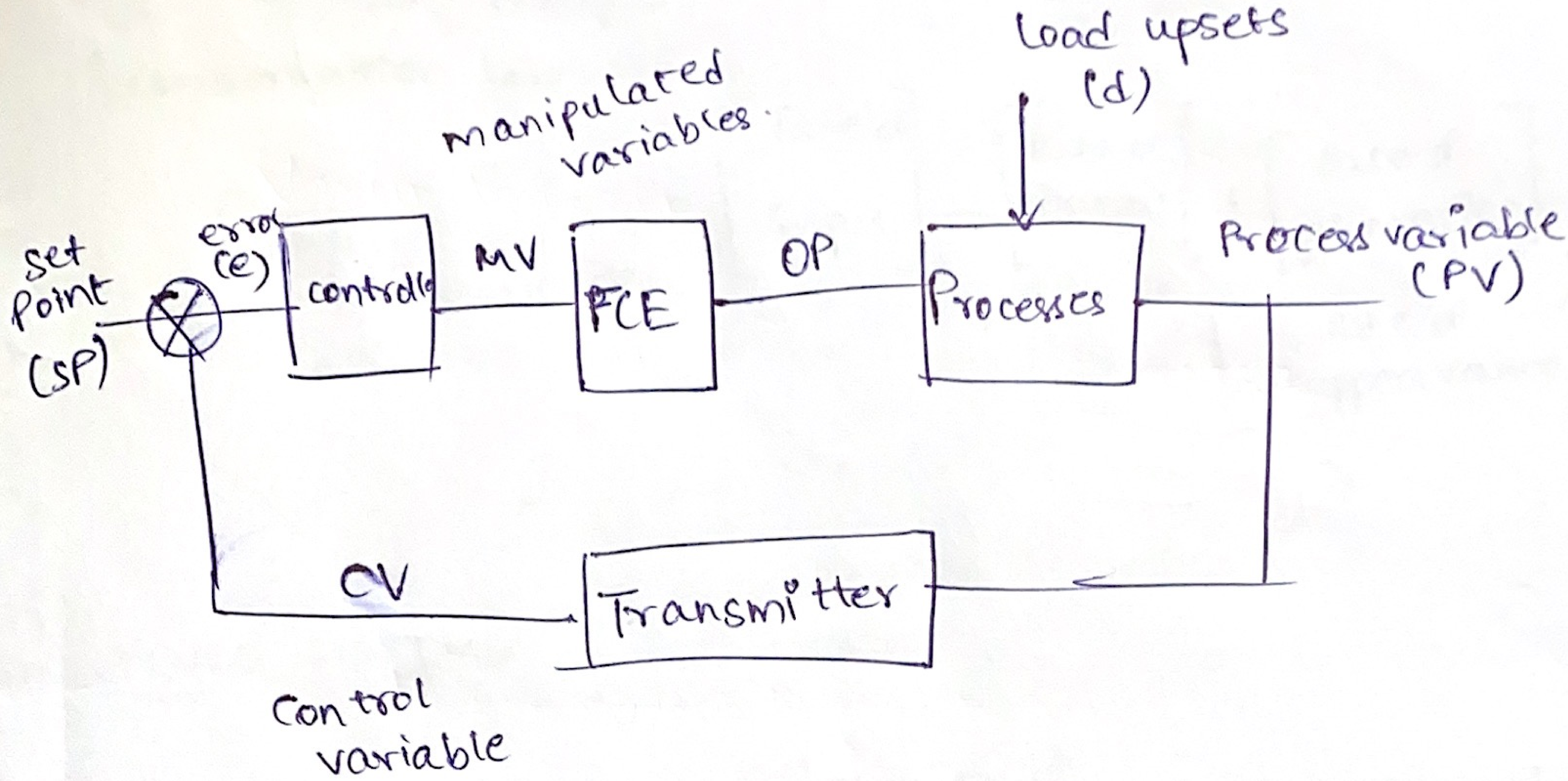

Load upsets
(d)
manipulated
variables.
error
(e)
set
Point
(SP)
controller
MV
FCE
OP
Processes
Process variable
(PV)
CV
Transmitter
Control
variable

6/8/22

Mathematical Modelling

Modelling Principles

i) Conservation law

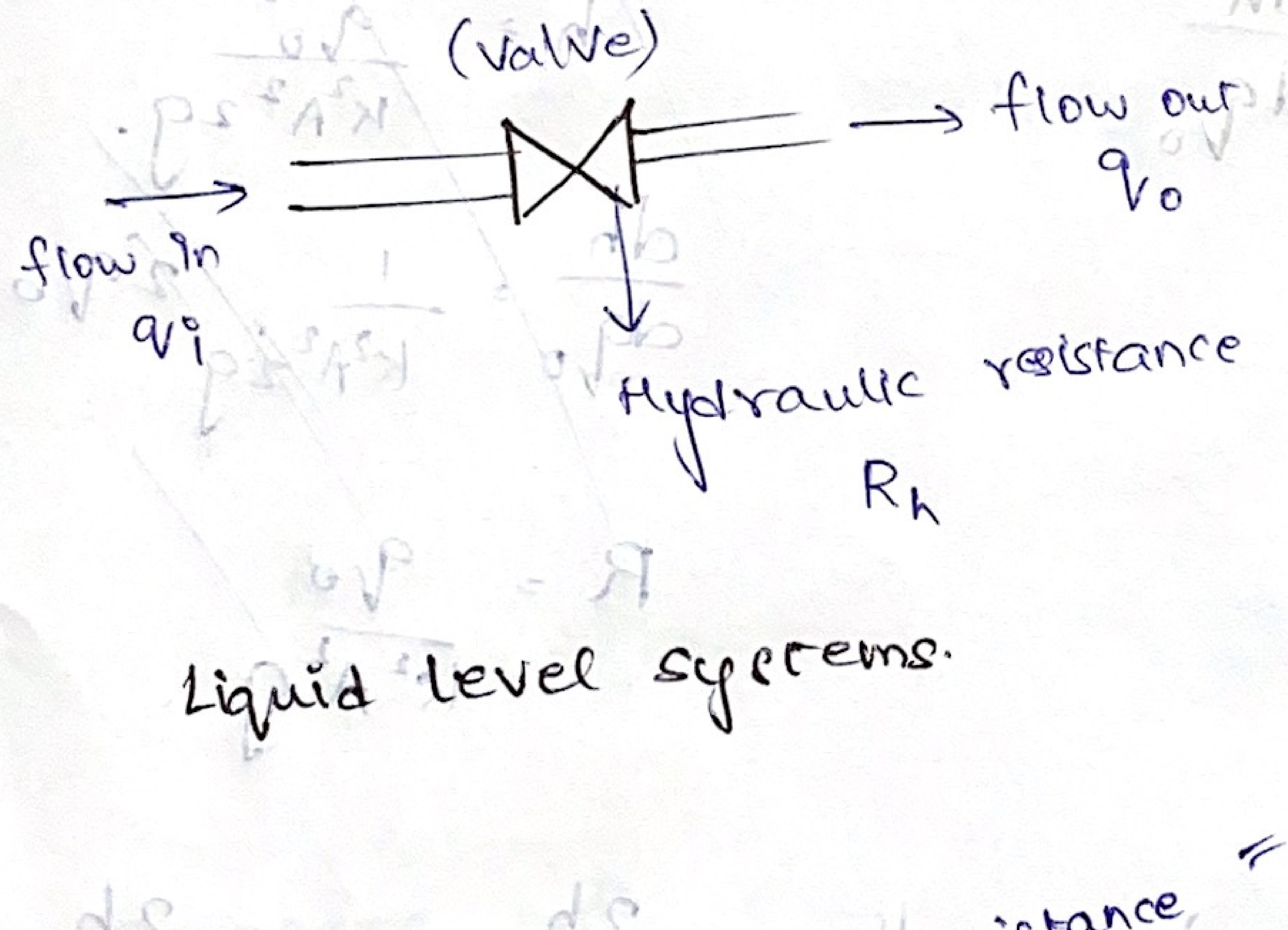
$$\left\{\text{Rate of accumulation}\right\} = \left[\text{Rate of input}\right] - \left[\text{Rate of output}\right] + \left[\text{Rate of generation}\right] - \left[\text{Rate of disappearance}\right]$$

FLUID SYSTEMS

Hydraulic - water or oil

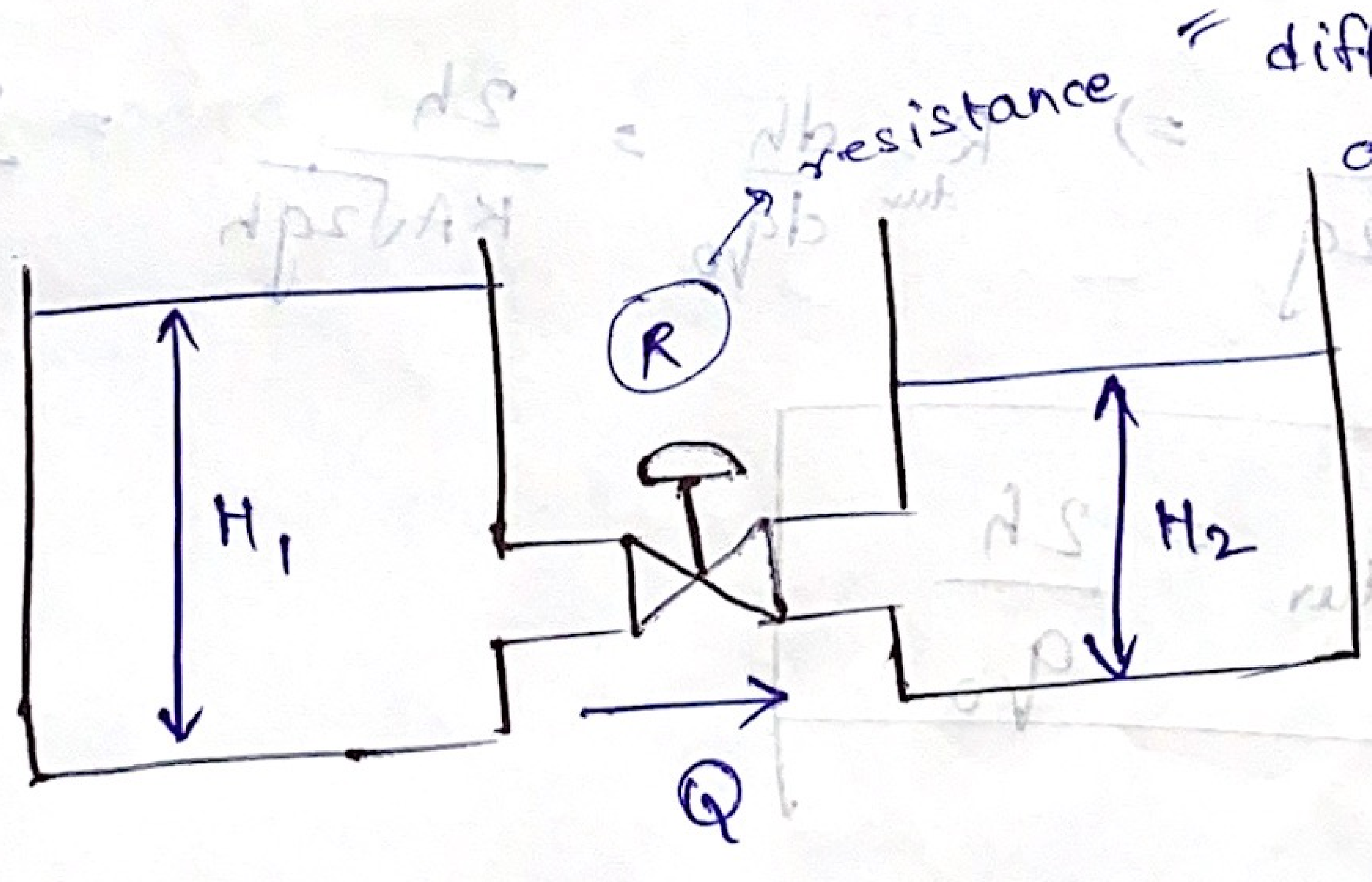
Pneumatic - air or other gases

Hydraulic resistance



Liquid level systems

Change in the level diff necessary to cause a unit change in flow rate.



$$\text{Resistance} = \frac{\text{Change in level diff}}{\text{change in flow rate}}$$

$$R = \frac{\Delta H}{\Delta Q} = \frac{H_1 - H_2}{\Delta Q} = \frac{m}{m^3/s}$$

$$R \equiv \frac{h \text{ (potential)}}{q_o \text{ (flow)}}$$

Resistance in Laminar flow

coefficient (m^2/s)

$$\boxed{Q = K_1 H}$$

steady state height. (m)

Steady state liq. flow rate (m^3/s)

$$\frac{k_1 H_1 - k_2 H_2}{Q_1 - Q_2}$$

$$R_l = \frac{dH}{dQ}$$

k_1 $\quad H = \frac{Q}{K_1}$

$$\frac{dH}{dQ} = \frac{1}{K_1}$$

Resistance in Turbulent flow

$$q_0 = KA\sqrt{2gh}$$

$$R_{hi} = \frac{dh}{dq_0}$$

$$\sqrt{h} = \frac{q_0}{KA\sqrt{2g}}$$

$$\sqrt{h} = \frac{q_0}{KA\sqrt{2g}}$$

$$h = \frac{q_0^2}{K^2A^2 2g}$$

$$\frac{dh}{dq_0} = \frac{1}{K^2A^2 \cancel{2}g} \cancel{2} q_0$$

$$R = \frac{q_0}{K^2A^2 g}$$

Diff.

$$\frac{1}{2\sqrt{h}} \cdot dh = \frac{dq_0}{KA\sqrt{2g}}$$

$$\frac{dh}{dq_0} = \frac{2\sqrt{h}}{KA\sqrt{2g}} \Rightarrow R_{tur} = \frac{dh}{dq_0} = \frac{2h}{KA\sqrt{2gh}} = \frac{2h}{q_0}$$

$$\boxed{R_{tur} = \frac{2h}{q_0}}$$

Capacitance of Liquid level systems.

It is the change in quantity of stored liquid necessary to cause a unity change in the height.

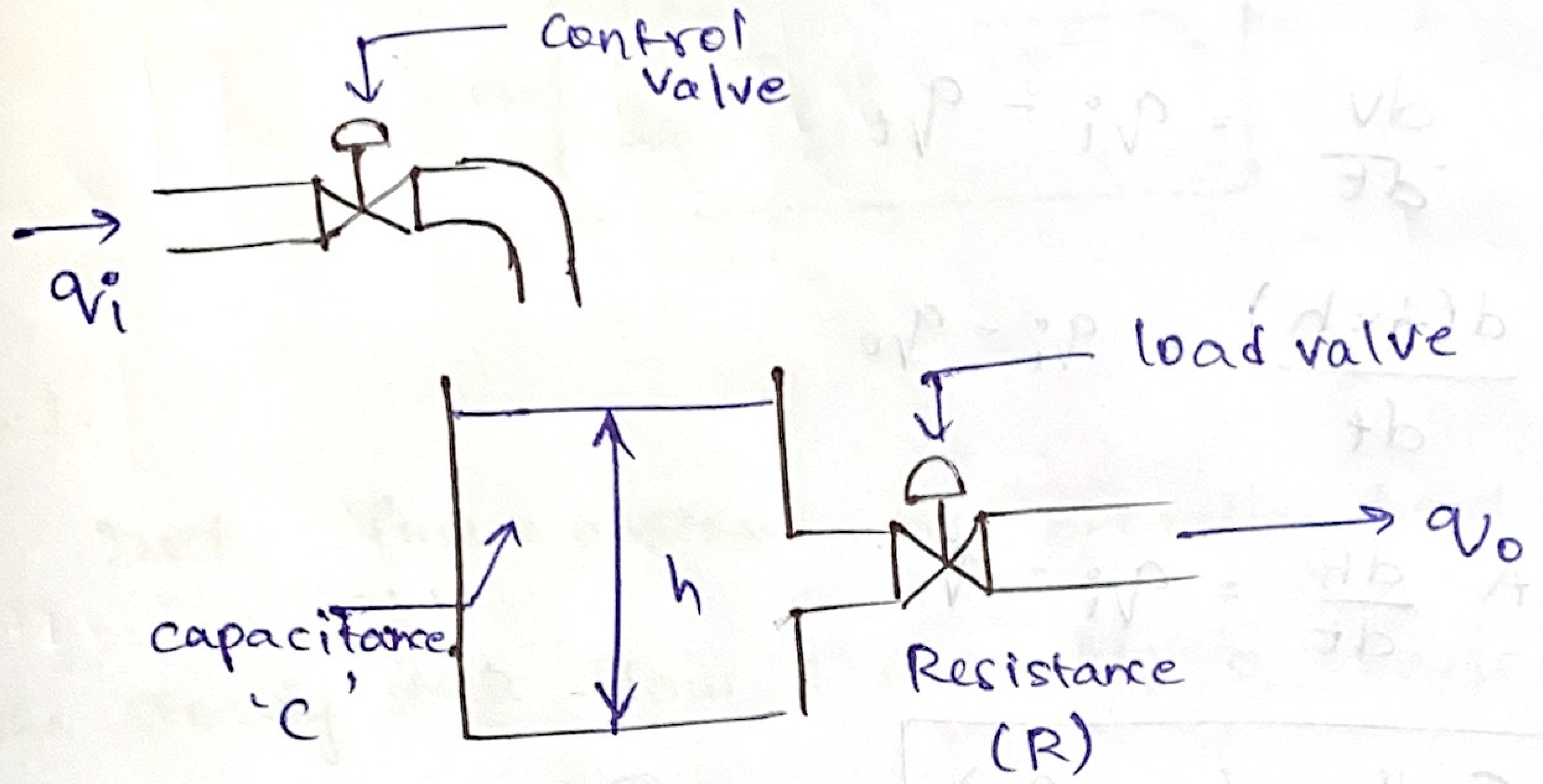


fig 1

$$\text{Capacitance} = \frac{\text{change in liquid stored}}{\text{change in height}} = \frac{m^3}{m} \text{ or } m^2$$

Both are function of time.

$$V(t) = A * h(t)$$

Volume

Area of cross section

Height

$$A = \frac{V(t)}{h(t)} = \frac{\text{quantity}}{\text{Potential.}}$$

Compare with electrical capacitance.

i.e $C = \frac{q}{V}$

So, it shows - liquid capacitance 'C_l' is simply the surface area of the liquid.

i.e $\boxed{C_L = A}$

Further,

$$\frac{dV(t)}{dt} = A \cdot \frac{dh(t)}{dt}$$

8/8/22

consider prev. diagram (fig 1)

Rate of change of fluid volume in the tank = flow in − flow out

$$\frac{dV}{dt} = q_i - q_o$$

$$\frac{d(A \times h)}{dt} = q_i - q_o$$

$$A \cdot \frac{dh}{dt} = q_i - q_o$$

$$\boxed{C \cdot \frac{dh}{dt} = (q_i - q_o)} \longrightarrow (1)$$

let $\bar{H}$ & $\bar{Q}$ be the steady state head and flow rate

h, q_i & q_o are small deviation of head and flow rate from steady state value.

$$R = \frac{dH}{dQ} = \frac{h}{q_o} \rightarrow (2) \Rightarrow q_o = \frac{h}{R} \longrightarrow (3)$$

sub (3) in (1)

$$C \frac{dh}{dt} = \left(q_i - \frac{h}{R}\right)$$

$$CR \cdot \frac{dh}{dt} = Rq_i - h$$

$$\boxed{CR \frac{dh}{dt} + h = Rq_i}$$

Apply laplace transform.

$$CR \cdot s \cdot H(s) + H(s) = R\, Q_i(s)$$

$$H(s)\{RCs + 1\} = R\, Q_i(s)$$

$$\Rightarrow \frac{H(s)}{Q_i(s)} = \frac{R}{RCs + 1} = \boxed{\frac{R}{\tau s + 1}}$$

from (3)

$$Q_o(S) = \frac{H(S)}{R}$$

Therefore,

$$\boxed{\frac{Q_o(s)}{Q_i(s)} = \frac{1}{\tau s + 1}}$$

Prob-1.

Det. time constant if operating head is 5m, the steady state flow is $0.2\ m^3/s$ & surface area is $10 m^2$

$\tau = RC$ $C = A = 10m^2$

$$\frac{H(S)}{Q_i(s)} = \frac{R}{\tau s + 1}$$

$q_o = \frac{2h}{R}$

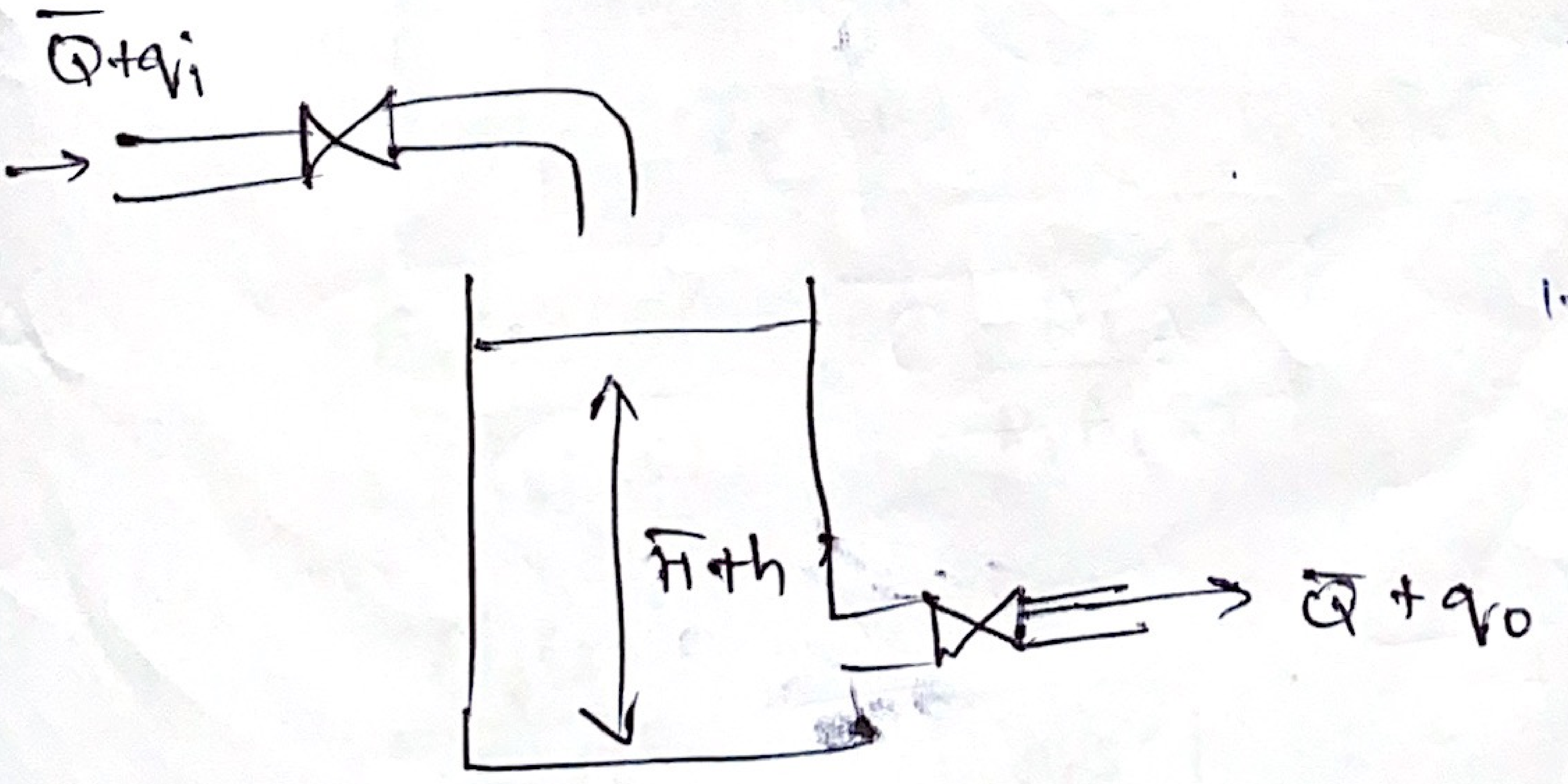
$h = 5m$ $q_o = 0.2 m^3/s$

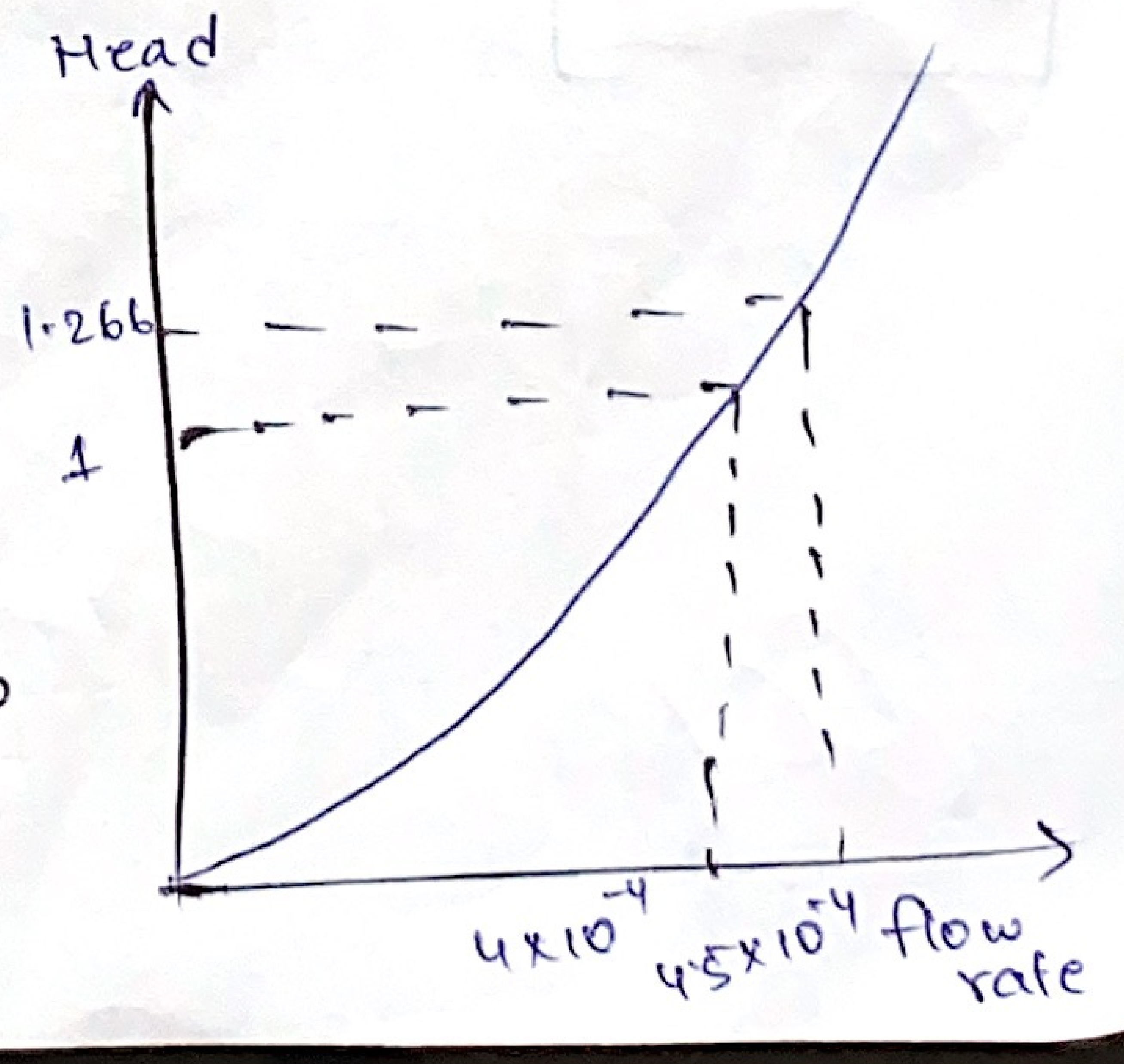
$R = \frac{2 \times 5}{0.2} = 50\ s/m^2$

$\boxed{R = \frac{2h}{q}}$ turbulent.

$\tau = 50 \times 10 = 500$ sec

Prob 2. The curve of head vs flow rate. At steady state the liq flow rate is $4 \times 10^{-4}\ m^3/s$ and steady state head is 1 m. At t=0 inflow valve is opened & flow rate changed to $4.5 \times 10^{-4}\ m^3/s$. Det. the avg resista. R of the outflow. Also det the change in head as a function of time. Capacitance C of tank is $0.02 m^2$





Sol. $\bar{H} = 1\,m$ $h = 0.266\,m$ $\bar{Q} = 4\times10^{-4}\,m^3/s$ $q_i = 4.5\times10^{-4}\,m^3/s$

$$R = \frac{dH}{dQ} = \frac{0.266}{0.5\times10^{-4}} = 0.52\times10^{4}\ s/m^2$$

$$\frac{0.266}{0.5} \quad \frac{266}{500}\times10^{4} \quad \frac{266}{50}\times10^{3} \quad \frac{52}{260}\times10^{2}$$

$$V(t) = H(t)\times A$$

$$H(t) = \frac{V(t)}{A}$$

$$RC\frac{dh}{dt} + h = Rq_i$$

$$0.52\times10^{4}\times0.02\times\frac{dh}{dt} + h = 0.52\times10^{4}\times0.5\times10^{-4}$$

$$106.4\,\frac{dh}{dt} + h = 0.266$$

laplace transform.

$$(106.4)\,s\,H(s) + H(s) = \frac{0.266}{s}$$

$$H(s)\,(1+106.4s) = \frac{0.266}{s}$$

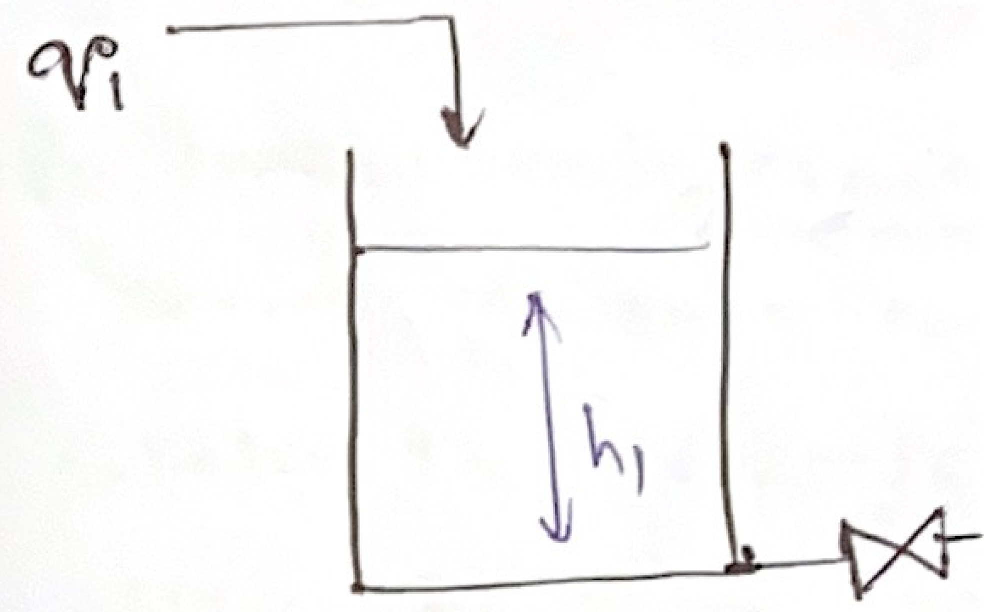
$$H(s) = \frac{0.266}{(1+106.4s)\,s}$$

$$= 0.266\left[\frac{1}{s} - \frac{1}{s+(1/106.4)}\right]$$

Inverse Laplace transform

$$h(t) = 0.266\left[1 - e^{-\frac{t}{106.4}}\right]$$

$$\boxed{\frac{1}{s+a} \rightarrow e^{-at}}$$



Mass balalance.

$A_1\frac{dh_1}{dt}$

Valve relation

Sub ②

$A_1\,d$

$A_1 s.\,H$

$H_1(S)$

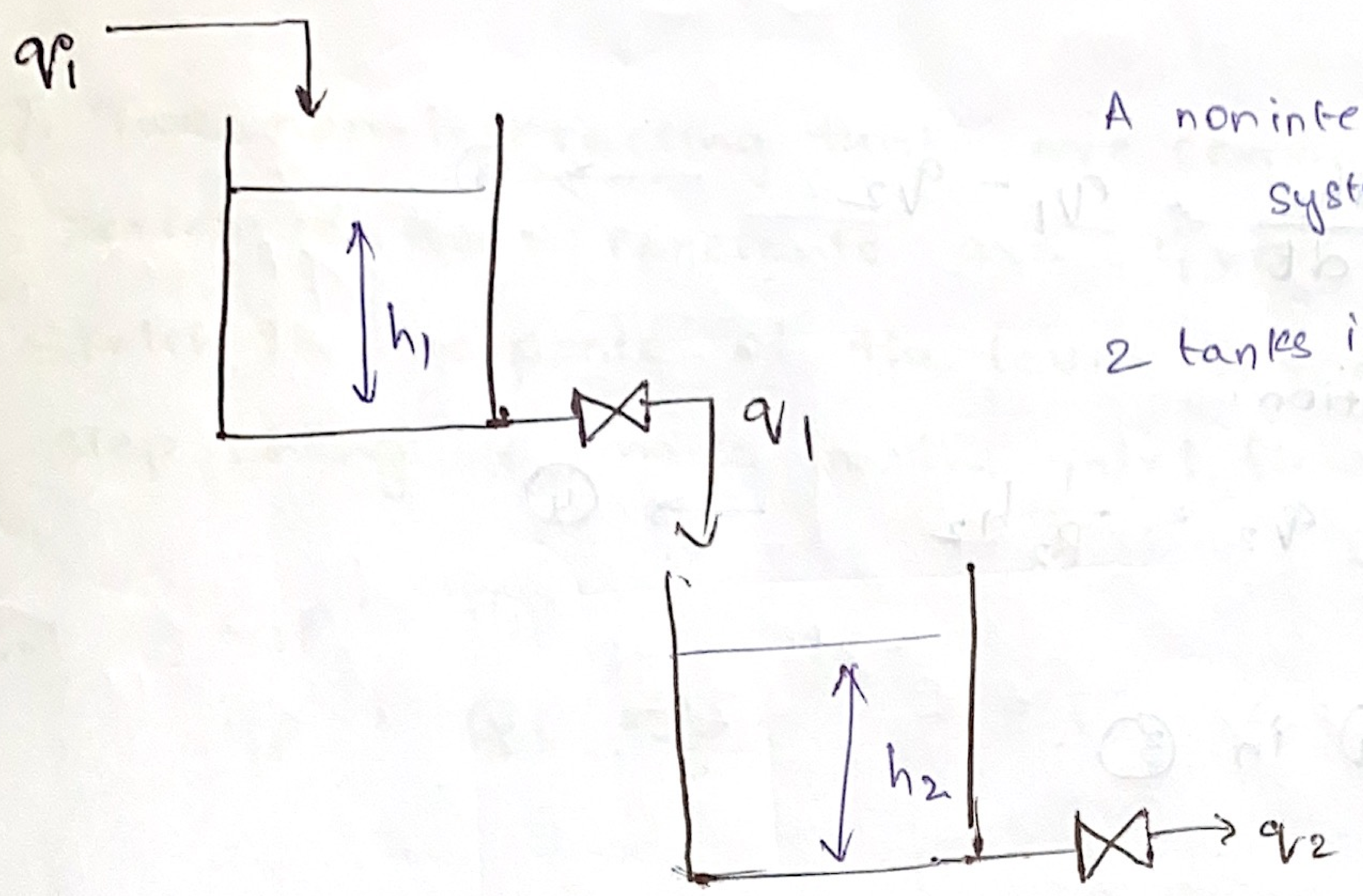
$\frac{H_1(S)}{Q_i(S)}$

Also

$H_1 = Q_\phi R$

$\frac{Q_\phi(S)\,R_1}{Q_i(S)}$

$\frac{Q_\phi(S)}{Q_i(S)}$



A noninteracting system.

2 tanks in series.

Mass balalance.

$$A_1 \frac{dh_1}{dt} = q_i - q_1 \rightarrow ①$$

Valve relation

$$q_1 = \frac{1}{R_1} h_1 \rightarrow ②$$

Sub ② in ①.

$$A_1 \frac{dh_1}{dt} = q_i - \frac{1}{R_1} h_1$$

$$A_1 s \cdot H_1(s) = Q_i(s) - \frac{1}{R_1} H_1(s).$$

$$H_1(s)\left(A_1 s + \frac{1}{R_1}\right) = Q_i(s)$$

$$\frac{H_1(s)}{Q_i(s)} = \frac{R_1}{R_1 A_1 s + 1}$$

Also

$$H_1 = Q_o R_1$$

$$\frac{Q_o(s)\cancel{R_1}}{Q_i(s)} = \frac{\cancel{R_1}}{R_1 A_1 s + 1}$$

$$\frac{Q_o(s)}{Q_i(s)} = \frac{1}{R_1 A_1 s + 1}$$

for tank 2

$$A_2 \frac{dh_2}{dt} = q_1 - q_2 \longrightarrow (3)$$

Valve relation:

$$q_2 = \frac{1}{R_2} h_2 \longrightarrow (4)$$

(4) in (3).

$$A_2 \frac{dh_2}{dt} = q_1 - \frac{h_2}{R_2}$$

Apply Laplace

$$A_2 \cdot s\, H_2(s) = Q_1(s) - \frac{1}{R_2} H_2(s)$$

$$H_2(s) \left(A_2 s + \frac{1}{R_2} \right) = Q_1(s)$$

$$\frac{H_2(s)}{Q_1(s)} = \frac{R_2}{R_2 A_2 s + 1}$$

Ilrly

$$\frac{Q_2(s)}{Q_1(s)} = \frac{1}{R_2 A_2 s + 1}$$

$$\frac{Q_2(s)}{Q_i(s)} = \frac{1}{(R_1 A_1 s + 1)(R_2 A_2 s + 1)}$$

$$Q_i(s) \longrightarrow \boxed{\frac{R_1}{\tau_1 s + 1}} \xrightarrow{H_1(s)} \boxed{\frac{1}{R_1}} \xrightarrow{Q_1(s)} \boxed{\frac{R_2}{\tau_2 s + 1}} \xrightarrow{H_2(s)} \boxed{\frac{1}{R_2}} \longrightarrow Q_2(s)$$

10/8/22

Q. Two non-interacting tanks are connected in series. The tank constants are $\tau_1 = 0.5$ $\tau_2 = 1$. $R_2 = 1$ Sketch the response of the level in tank 2, if a unit step change is made in the inlet flow rate of tank 1.

Sol. O/P = H_2

I/P : $Q_i = \frac{1}{s}$

$$\frac{H_2(s)}{Q_i(s)} = \frac{R_2}{(\tau_1 s+1)(\tau_2 s+1)}$$

$$H_2(s) = \frac{R_2\, Q_i(s)}{(\tau_1 s+1)(\tau_2 s+1)}$$

$$H_2(s) = \frac{1}{s(0.5s+1)(s+1)}$$

$$= \frac{A}{s} + \frac{B}{1+0.5s} + \frac{C}{1+s}$$

$$= (A+0.5As)(A+As) + Bs(B+Bs) \; (A + 0.5As + Bs)(1+s)$$
$$+ Cs(1+0.5s)$$

$$= A^2 + A^2 s$$

$$= (A + 0.5As + Bs)(1+s) + Cs + 0.5Cs^2$$

$$= A + 0.5As + Bs + As + 0.5As^2 + Bs^2 + Cs + 0.5Cs^2.$$

$$= s^2(0.5A + 0.5C + B) + s(0.5A + B + A + C)$$

$+ A$

$A = 1$

$A + 0.5C = 0.$

$0.5C = -1$

$C = \frac{-1}{}$

$C = -2$

$$0.5A + 0.5\cancel{B} + \cancel{B} = 0.$$

$$1.5A + \cancel{B} + C = 0$$

$$-1A - 0.5C = 0.$$

$A = 1 \quad B = 0.5 \quad C = -2$

$$H_2(s) = \frac{1}{s} + \frac{0.5}{1 + 0.5s} - \frac{2}{1+s} = \frac{1}{s} + \frac{\cancel{0.5}}{\cancel{0.5}(2+s)} - \frac{2}{1+s}$$

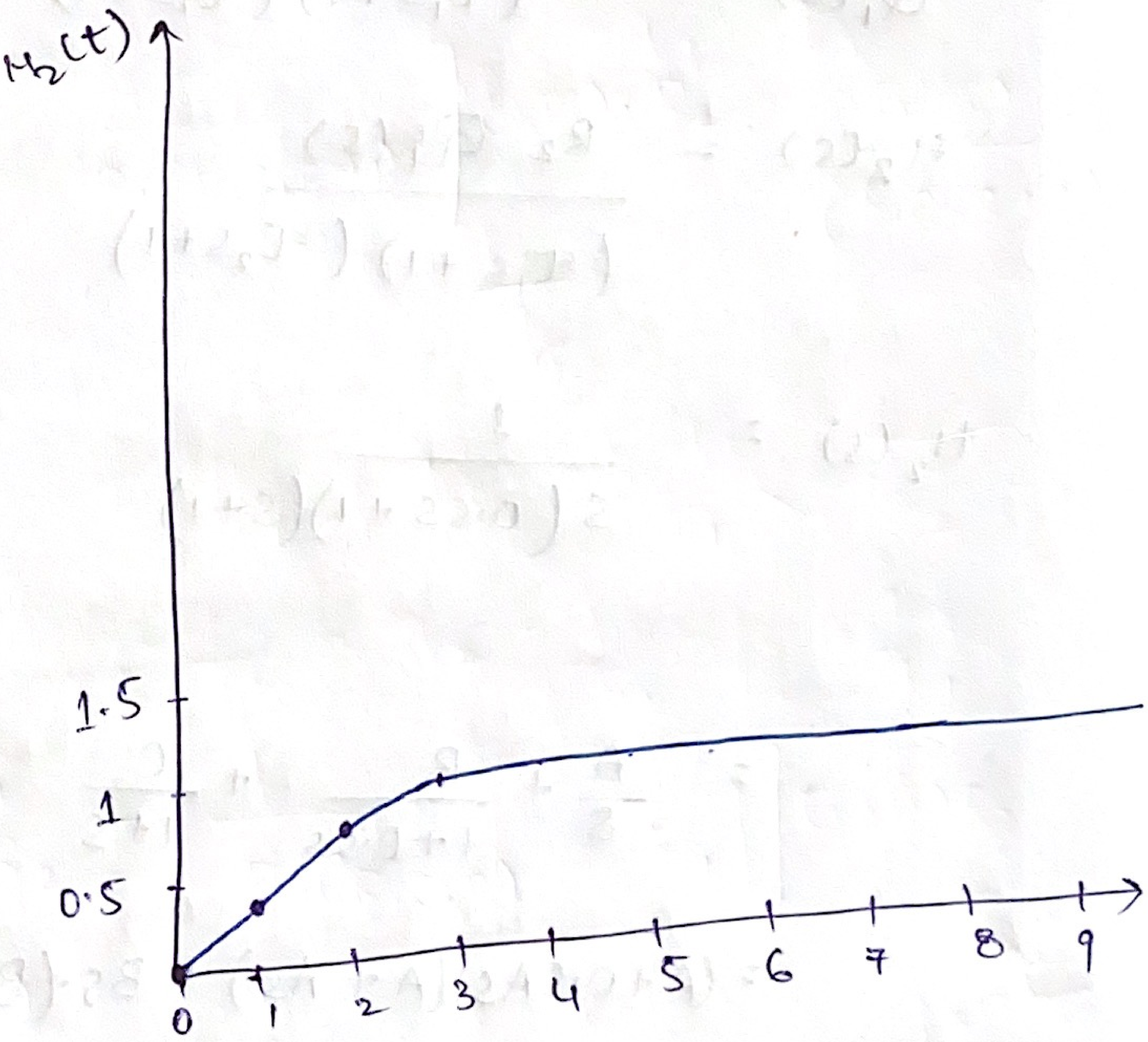
Laplace transform.

$$\cancel{H_2(t) = 1 + 0.5\, e^{-0.5t} - 2 e^{-t}}$$

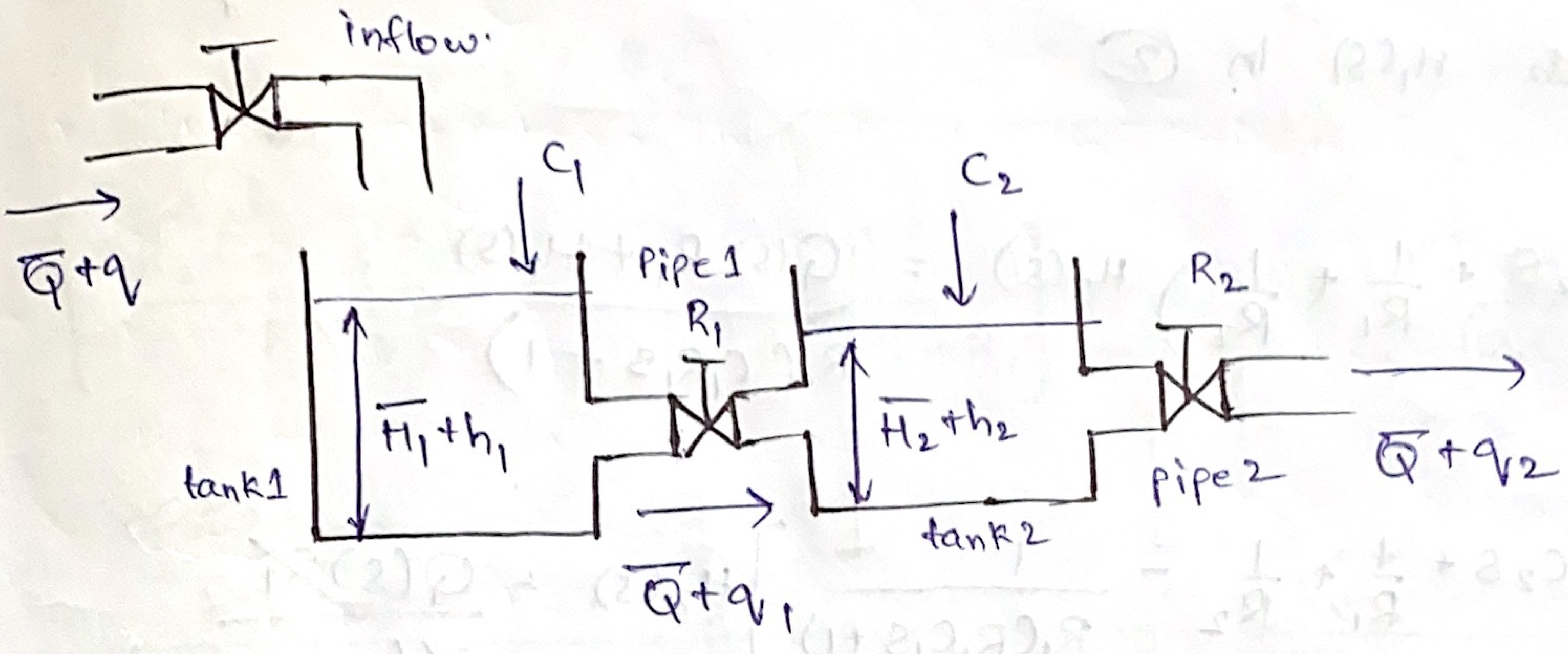
$$H_2(t) = 1 + e^{-2t} - 2 \cdot e^{-t}$$

Plot the response.

	$H_2(t)$
$t=0$	0
$t=1$	0.39
$t=2$	0.74
$t=3$	0.90
$t=4$	0.96
$t=5$	0.98
$t=6$	0.995
$t=7$	0.998
$t=8$	0.9993
$t=9$	0.9997



Interacting type



tank 1

$$C_1 \frac{dh_1}{dt} = q - q_1 \qquad R_1 = \frac{h_1 - h_2}{q_1} \Rightarrow q_1 = \frac{h_1 - h_2}{R_1}$$

tank 2

$$C_2 \frac{dh_2}{dt} = q_1 - q_2 \qquad R_2 = \frac{h_2}{q_2} \Rightarrow q_2 = \frac{h_2}{R_2}$$

Sub both values.

$$C_1 \frac{dh_1}{dt} = q - \frac{(h_1 - h_2)}{R_1} \Rightarrow C_1 \frac{dh_1}{dt} + \frac{h_1}{R_1} = q + \frac{h_2}{R_1}$$

laplace transform.

$$\cancel{C_2 \frac{dh_2}{dt} = q_1 - \frac{h_2}{R_2}}$$

$$\left(C_1 s + \frac{1}{R_1}\right) H_1(s) = Q(s) + \frac{H_2(s)}{R_1} \rightarrow ①$$

$$C_2 \frac{dh_2}{dt} = \frac{h_1 - h_2}{R_1} - \frac{h_2}{R_2}$$

from ①

$$H_1(s) = \frac{Q(s) R_1 + H_2(s)}{C_1 R_1 s + 1}$$

$$\Rightarrow C_2 \frac{dh_2}{dt} + \frac{h_2}{R_1} + \frac{h_2}{R_2} = \frac{h_1}{R_1}$$

laplace tra

$$\left(C_2 s + \frac{1}{R_1} + \frac{1}{R_2}\right) H_2(s) = \frac{H_1(s)}{R_1} \rightarrow ②$$

we need $Q_2(s) / Q(s)$

Sub $H_1(S)$ in (2)

$$\left(C_2 S + \frac{1}{R_1} + \frac{1}{R_2}\right) H_2(S) = \frac{Q(S) R_1 + H_2(S)}{R_1 (C_1 R_1 S + 1)}$$

$$\left[\cancel{B}.C_2 S + \frac{1}{R_1} + \frac{1}{R_2} - \frac{1}{R_1 (R_1 C_1 S + 1)}\right] H_2(S) = \frac{Q(S) \cancel{R_1}}{\cancel{R_1}(C_1 R_1 S + 1)}$$

WKT

$$H_2(S) = R_2 Q_2(S)$$

$$\left[\frac{R_1 R_2 C_2 S + (R_2 + R_1)}{R_1 R_2} - \frac{1}{R_1 (R_1 C_1 S + 1)}\right] Q_2(S) \cdot R_2 = \frac{Q(S)}{(R_1 C_1 S + 1)}$$

$$\frac{Q_2(S)}{Q(S)} =$$

$$\left[C_2 S + \cancel{\frac{1}{R_1}} + \frac{1}{R_2} - \frac{1}{R_1 (R_1 C_1 S)} - \cancel{\frac{1}{R_1}}\right] Q_2(S) R_2 = \frac{Q(s)}{R_1 C_1 S + 1}$$

$$\left[\frac{R_2 C_2 S + 1}{\cancel{R_2}} - \frac{R_2}{R_1 (R_1 C_1 S)}\right] Q_2(S) = \frac{Q(S)}{R_1 C_1 S + 1}$$

$$\frac{Q_2(S)}{Q(S)} = \frac{1}{(R_1 C_1 S + 1)\left[R_2 C_2 S + 1 - \frac{R_2}{R_1 (R_1 C_1 S)}\right]}$$

$$= \frac{R_1^2 C_1 S}{(R_1 C_1 S + 1)\left[R_1^2 C_1 S R_2 C_2 S + R_1^2 C_1 S - R\right.}$$

$$= \frac{R_1 \tau_1 S}{(\tau_1 S + 1)(R_1 \tau_1 \tau_2 S^2 + R_1 \tau_1 S - R_2)}$$

$x - R_1 \quad c_1 - p \quad s \quad Q_2(s) - u$

$y - R_2 \quad c_2 - q \quad Q(s) - v$

$= \cancel{R_1 \tau_1 s}$

$$\left[\frac{R_1R_2C_2s + R_2 + R_1}{R_1R_2} - \frac{1}{R_1^2C_1s + R_1}\right] Q_2(s) \cdot R_2 = \frac{Q(s)}{(R_1C_1s + 1)}$$

$\cancel{C_2 s + \frac{1}{R_2}} +$

---

for tank 1

$$C_1 \frac{dh_1}{dt} = q - q_1 \qquad \& \qquad q_1 = \frac{h_1 - h_2}{R_1}$$

$$C_1 \frac{dh_1}{dt} = q - \frac{h_1 - h_2}{R_1}$$

$$C_1R_1 \frac{dh_1}{dt} = R_1 q - h_1 + h_2$$

$$\tau_1 = C_1R_1 \qquad \Rightarrow \tau_1 \frac{dh_1}{dt} + h_1 - h_2 = R_1 q$$

laplace.

$$s\tau_1 h_1(s) + h_1(s) - h_2(s) = R_1 q(s).$$

$$h_1(s) = \frac{R_1 q(s) + h_2(s)}{\tau_1 s + 1} \longrightarrow (1)$$

for tank 2

$$C_2 \frac{dh_2}{dt} = q_1 - q_2 \qquad q_2 = \frac{h_2}{R_2}$$

$$C_2 \frac{dh_2}{dt} = \frac{h_1 - h_2}{R_1} - \frac{h_2}{R_2}$$

$$R_1R_2C_2 \frac{dh_2}{dt} = (h_1 - h_2)R_2 - h_2R_1$$

$$\tau_2 = C_2R_2 \qquad \tau_2 R_1 \frac{dh_2}{dt} + h_2R_2 + h_2R_1 = h_1R_2$$

$$sR_1\tau_2 h_2(s) + h_2(s)R_2 + h_2(s)R_1 = h_1(s)R_2$$

$$(R_1\tau_2 s + R_2 + R_1) h_2(s) = h_1(s) R_2$$

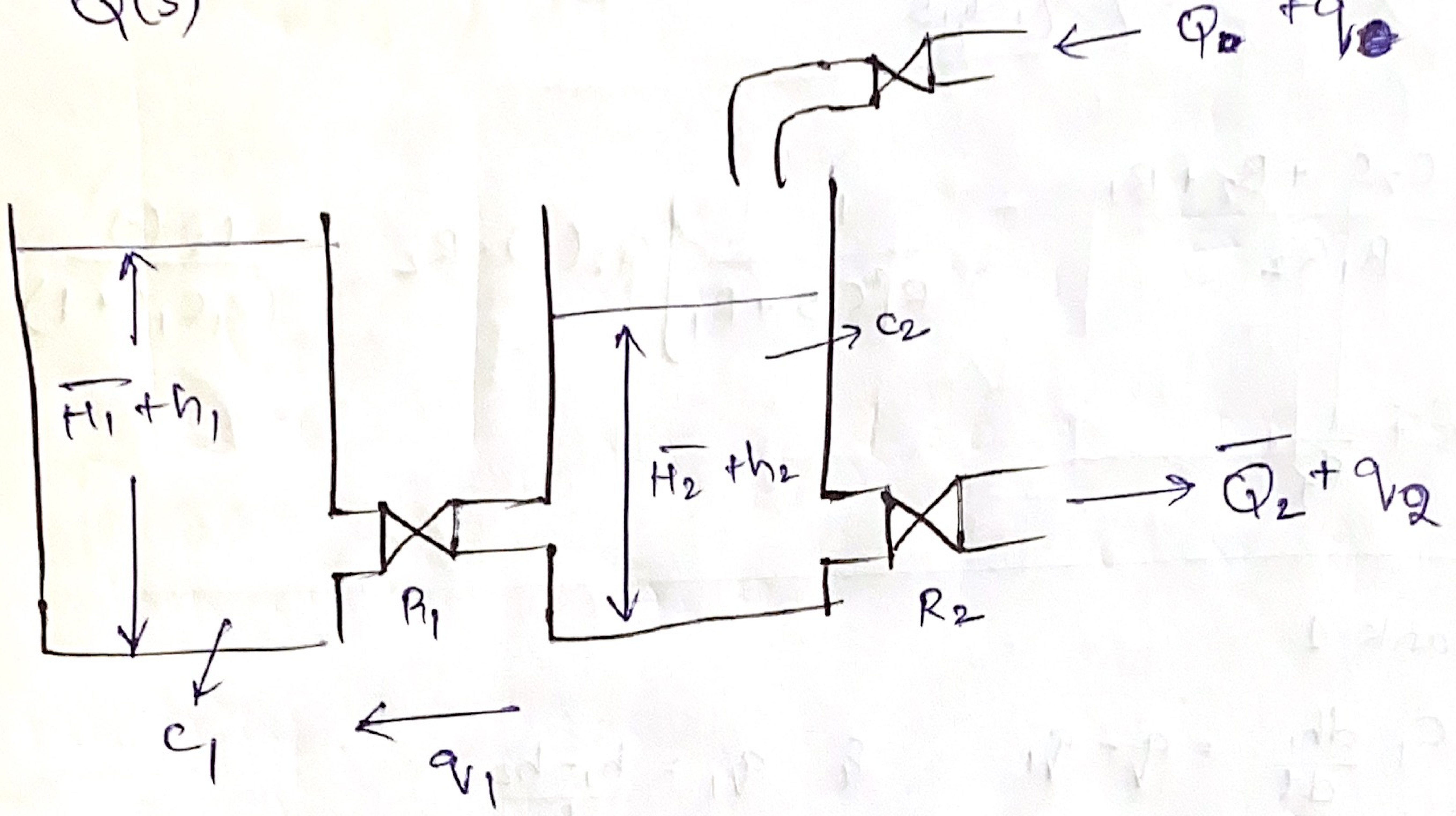
$$(R_1\tau_2 s + R_2 + R_1) Q_2(s) = h_1(s). \longrightarrow (2)$$

(1) in (2)

$$(R_1\tau_2 s + R_2 + R_1) Q_2(s) = \frac{R_1 q(s) + h_2(s)}{\tau_1 s + 1}$$

$$\frac{Q_2(s)}{Q(s)} = \frac{1}{R_1C_1R_2C_2 s^2 + (R_1C_1 + R_2C_2 + R_2C_1)s + 1}$$

$$\frac{H_2(s)}{Q(s)}$$



$$C_2 \frac{dh_2}{dt} = q_0 - q_1 - q_2$$

$$C_1 \frac{dh_1}{dt} = q_1$$

$$R_2 = \frac{h_2}{q_0}$$

$$R_1 = \frac{h_2 - h_1}{q_1}$$

$$C_2 \frac{dh_2}{dt} = q_0 - \frac{h_2 - h_1}{R_1} - \frac{h_2}{R_2}$$

$$C_1 \frac{dh_1}{dt} = \frac{h_2 - h_1}{R_1} \quad (1)$$

$$C_2 \frac{dh_2}{dt} = q_0 - \frac{h_2}{R_1} - \frac{h_2}{R_2} + \frac{h_1}{R_1}$$

$$\cancel{C_1 \frac{dh_1}{dt} + \frac{h_1}{R_1} = \frac{h_2}{R_1}}$$

$$C_2 \frac{dh_2}{dt} + h_2\left[\frac{1}{R_2} + \frac{1}{R_1}\right] = q_0 + \frac{h_1}{R_1}$$

$$T_1 \frac{dh_1}{dt} + h_1 = h_2$$

$$(T_1 s + 1) H_1(s) = H_2(s)$$

$$C_2 \frac{dh_2}{dt} + \frac{h_2 (R_1 + R_2)}{R_1 R_2} = \frac{R_1 q_0 + h_1}{R_1}$$

$$\Rightarrow H_1(s) = \frac{H_2(s)}{(T_1 s + 1)} \rightarrow (1)$$

$$\frac{R_1 T_2 \frac{dh_2}{dt} + h_2 (R_1 + R_2)}{\cancel{R_1} R_2} = \frac{R_1 q_0 + h_1}{\cancel{R_1}}$$

$$R_1 T_2 s H_2(s) + H_2(s)(R_1 + R_2) = R_1 Q_0(s) + H_1(s)$$

$$H_2(s)\left[R_1 T_2 s + R_1 + R_2\right] = R_1 Q_0(s) + H_1(s) \quad (2)$$

(1) in (2).

$$H_2(s)\left[\cancel{\frac{1}{T_1 s + 1}}\; R_1 T_2 s + R_1 + R_2 - \frac{1}{T_1 s + 1}\right] = R_1 Q(s)$$

$$\frac{H_2(s)}{Q(s)} = \frac{R_1}{T_1 s + 1}\left[R_1 T_1 T_2 s^2 + R_1 T_1 s + R_2 T_1 s + R_1 T_2 s + R_1 + R_2 - 1\right]$$

13/8/22

Mathematical modelling of Thermal/Heat systems:

Conduction

$$q = K \Delta\theta$$

q = heat flow rate kcal/sec.

Convection

$$q = HA\,\Delta\theta$$

$\Delta\theta$ = temp difference °C

Radiation

$$q = K_r(\theta_1^4 - \theta_2^4)$$

K = heat coefficient kcal/s°C

Conduction $K = \frac{kA}{\Delta x}$

k = thermal conductivity Kcal/m sec °C

Convection $K = HA$

A = Area normal to heat flow m^2.

H = Convection coeff Kcal/m^2s°C

Δx = Thickness of conductor; m

Thermal resistance

$$\boxed{R = \frac{(\text{change in temp difference})\,°C}{\text{change in heat flow rate kcal/sec}} = \frac{d(\Delta\theta)}{dq} = \frac{1}{K}}$$

Thermal Capacitance

$$\boxed{C = \frac{\text{change in heat Stored Kcal}}{\text{change in temp °C}}} = \text{Kcal/°C}$$

Also,

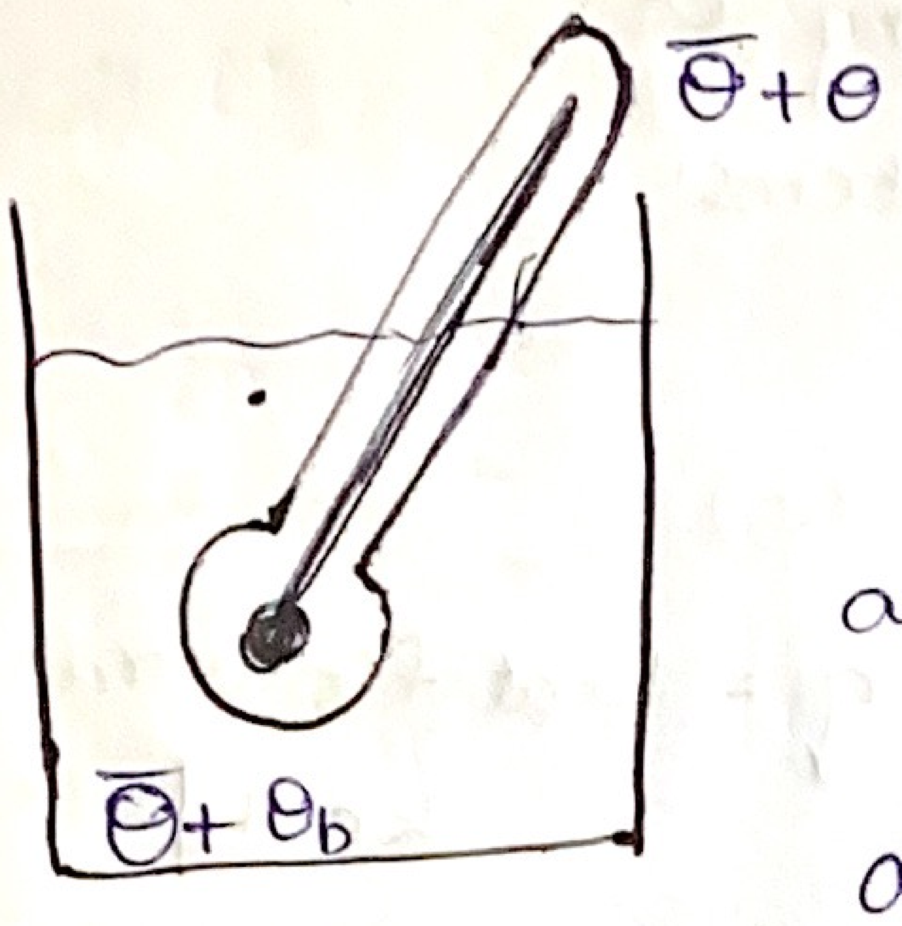
C = product of the specific heat and the mass of the material

$$\boxed{C = mc_s}$$

m = mass kg

c_s = Specific heat of substance kcal/kg°C

$\ominus + \theta_b$



Assume that, the thermometer is at uniform temp $\bar{\theta}$ °C (ambient temp) and at $t=0$, it is immersed in a bath of temp $\bar{\theta}+\theta_b$ in °C. where θ_b is the bath temp.

θ is the change in temp of thermometer such that $\theta_0 = 0$

Heat balance eq.

$$C\frac{d\theta}{dt} = q \longrightarrow (1)$$

$$R = \frac{d(\Delta\theta)}{dq} = \frac{\Delta\theta}{q} \Rightarrow q = \frac{\Delta\theta}{R} = \frac{(\bar{\theta}+\theta_b)-(\bar{\theta}+\theta)}{R}$$

$$q = \frac{\theta_b - \theta}{R} \longrightarrow (2)$$

Sub (2) in (1)

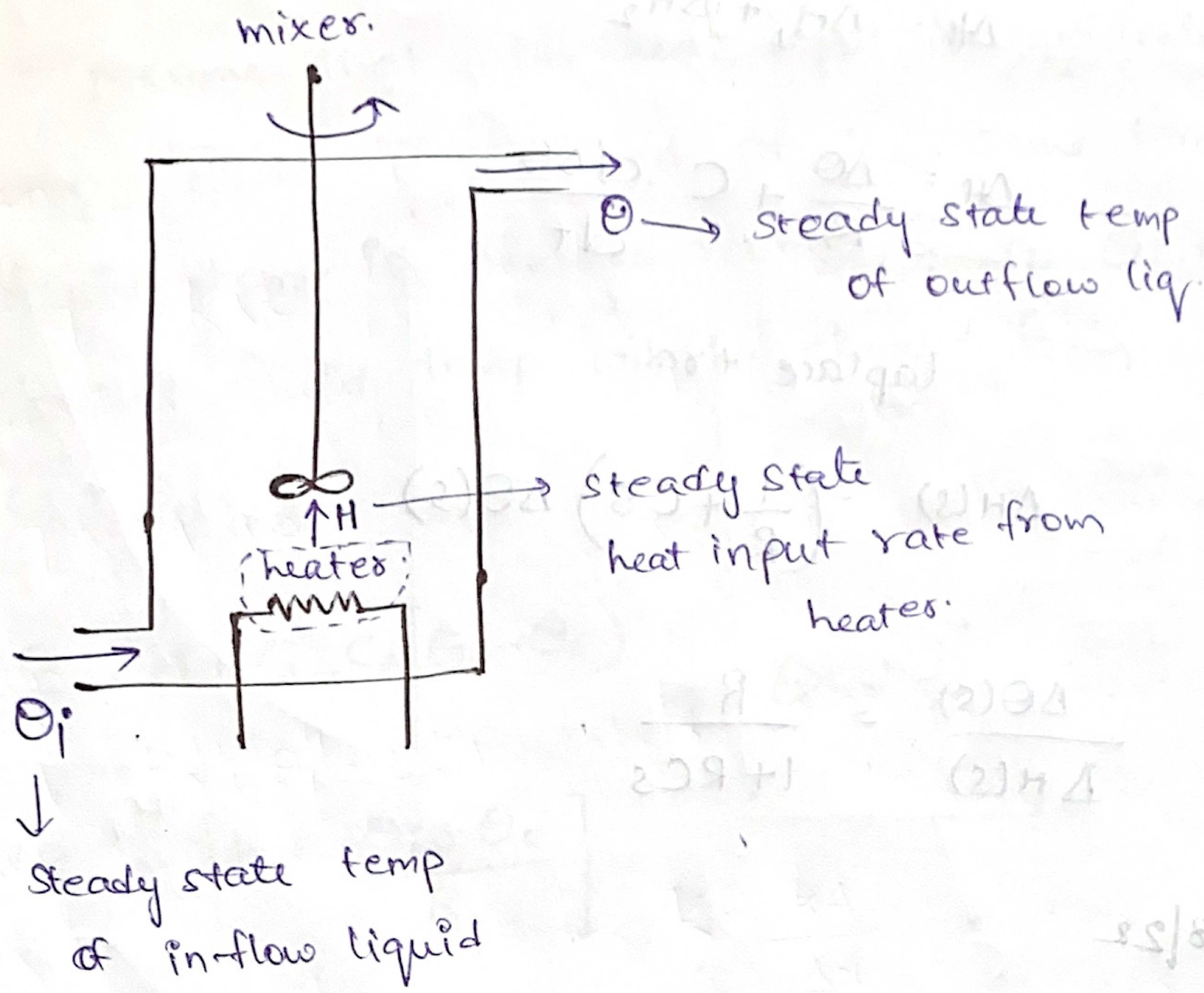
$$C\frac{d\theta}{dt} = \frac{\theta_b - \theta}{R}$$

$$C\frac{d\theta}{dt} + \frac{\theta}{R} = \frac{\theta_b}{R}$$

$$sC\,\theta(s) + \frac{1}{R}\theta(s) = \frac{\theta_b(s)}{R}$$

$RC \longrightarrow$ time constant

$$\theta(s)(1+RCs) = \theta_b(s)$$



let ΔH be a small change in the heat input rate from its steady state value. This change in H will result in

- change in heat o/p rate by ΔH_1
- Change in heat storage rate of liq in the tank by an amount ΔH_2
- Change in temp of outflowing liq by $\Delta\theta$.

$$\Delta H_1 = Q C_s \Delta\theta$$

Q - steady state liquid flow rate.

$$\Delta H_1 = \Delta\theta / R$$

wher $R = 1/QC_s$

Change in heat storage rate is

$$\Delta H_2 = m C_s \frac{d\Delta\theta}{dt}$$

where m = mass of liq in tank.

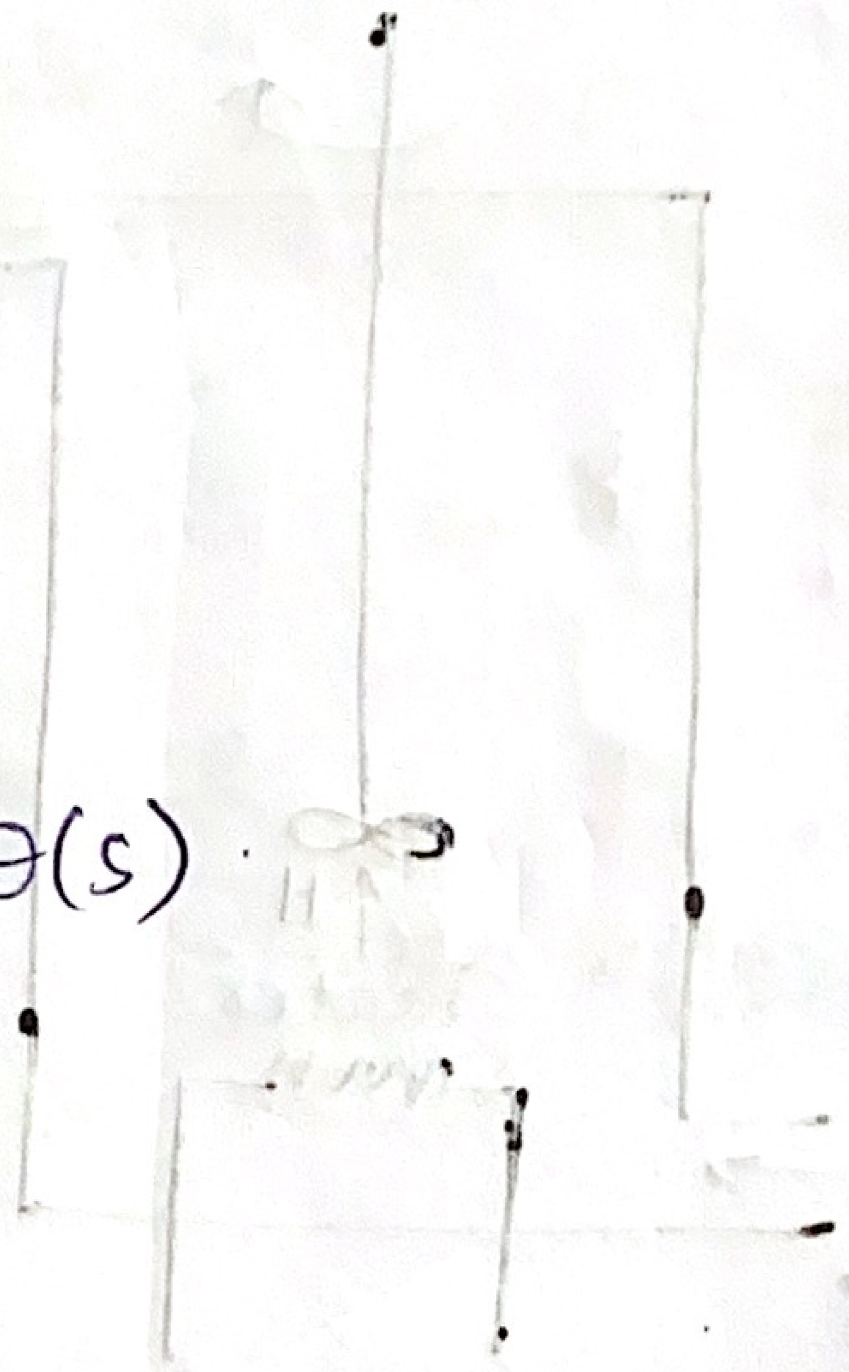
$\frac{d\theta}{dt}$ = rate of rise of temp in the tank.

$$\Rightarrow \Delta H_2 = C\, d\theta/dt$$

$C = mC_s$ = thermal capacitance.

$$\Delta H = \Delta H_1 + \Delta H_2$$

$$\Delta H = \frac{\Delta \theta}{R} + C \cdot \frac{d\Delta\theta}{dt}$$



Laplace tran

$$\Delta H(s) = \left(\frac{1}{R} + Cs\right) \Delta\theta(s)$$

$$\frac{\Delta\theta(s)}{\Delta H(s)} = \frac{R}{1+RCs}$$

17/8/22

case (ii) Change in fluid temp

case (iii)

Let us Assume that, the heat input is suddenly changed from $\bar{H}$ to $\bar{H}+h$, and at the same time the inlet fluid temp is changed from $\bar{\theta_i}$ to $\bar{\theta_i}+\theta_i$. Then the outlet fluid temp will be changed from $\bar{\theta_o}$ to $\bar{\theta_o}+\theta_o$

$$C \cdot \frac{d\theta}{dt} = H + Q C_s (\theta_i - \theta_o)$$

$$QC_s = \frac{1}{R}$$

$$C \cdot \frac{d\theta}{dt} = \left[h + \frac{\theta_i - \theta_o}{R} \right]$$

$$C \cdot \theta(s) \cdot s = H(s) + \frac{\theta_i(s) - \theta_o(s)}{R}$$

$$(RCs+1)\,\theta_o(s) = RH(s) + \theta_i(s)$$

$$\theta_o(s) = \frac{R}{RCs+1} H(s) + \frac{1}{RCs+1}\theta_i(s).$$