Chapter 1

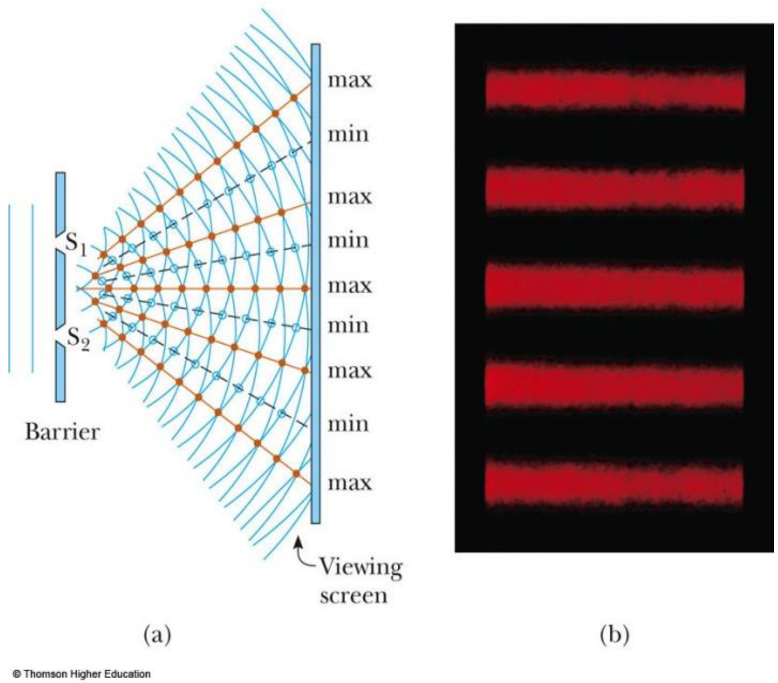
INTERFERENCE OF LIGHT WAVES

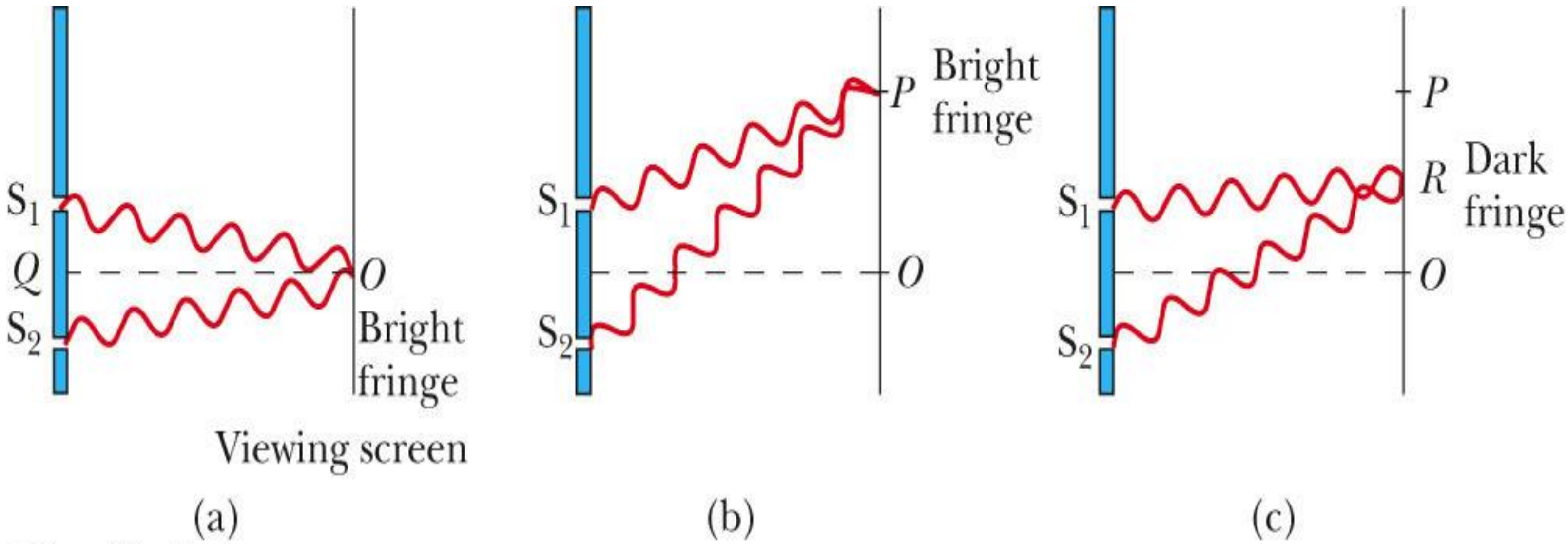
OBJECTIVES

- To understand the principles of interference.
- To explain the intensity distribution in interference under various conditions.
- To explain the interference from thin films.

Wave optics (Physical Optics): The study of interference, diffraction, and polarization of light. These phenomena cannot be adequately explained with the ray optics.

Young's Double-Slit Experiment

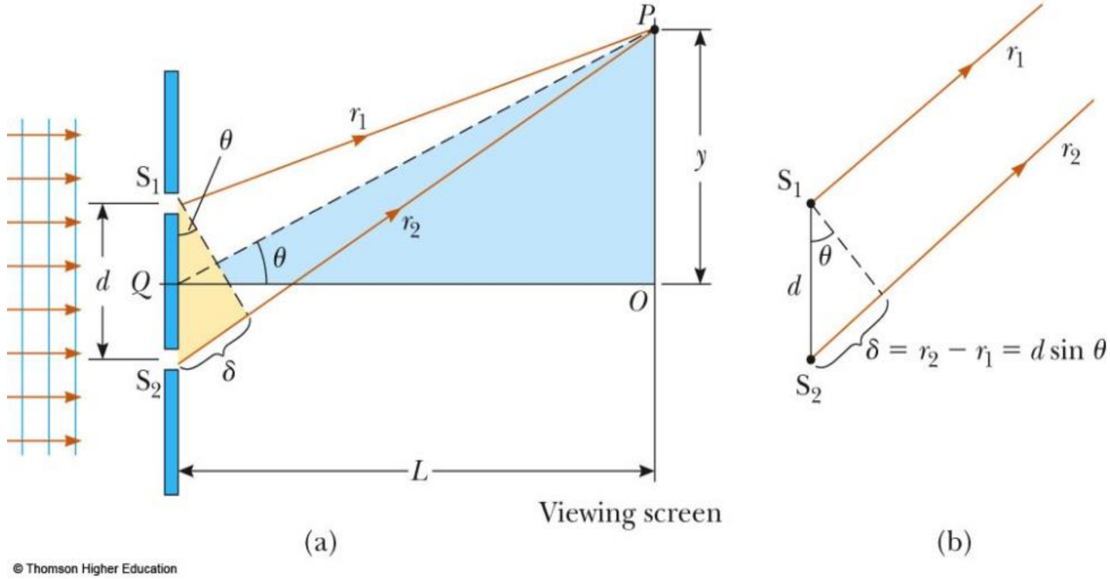




To observe interference of waves from two sources, the following conditions must be met:

- The sources must be **coherent;** that is, they must maintain a constant phase with respect to each other.
- The sources should be **monochromatic;** that is, they should be of a single wavelength.

Analysis Model: Waves in Interference



Path difference $$\delta = r_2 - r_1 = d \sin \theta$$

Condition for **constructive interference**, at point *P* is,

$$d \sin \theta_{\text{bright}} = m\lambda \quad (m = 0, \pm 1, \pm 2, \ldots)$$

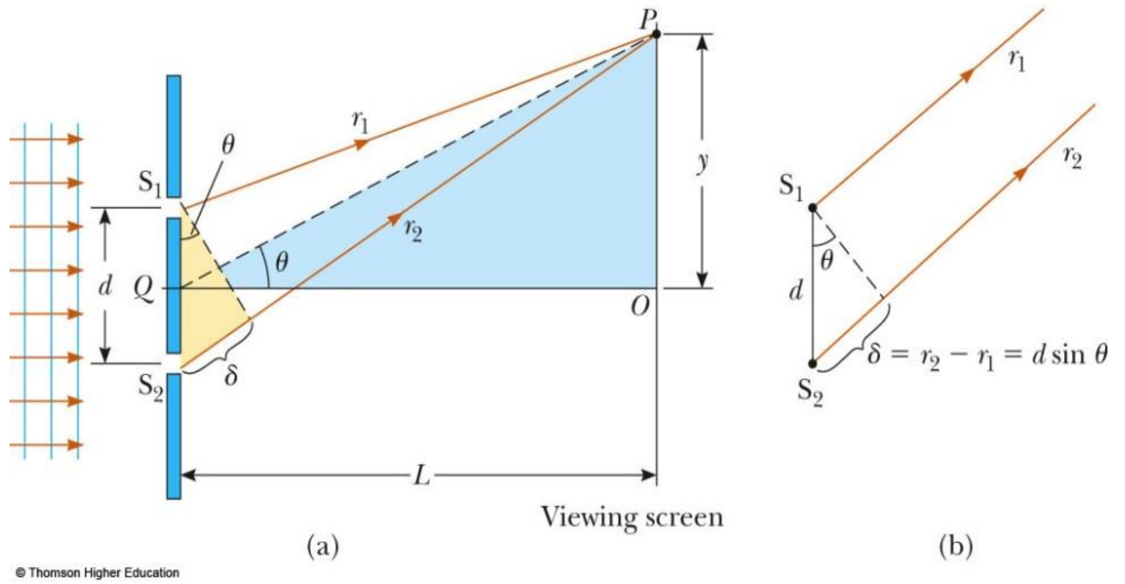
The number *m* is called the **order number**.

Condition for **destructive interference**, at point *P* is,

$$d \sin \theta_{\text{dark}} = \left(m + \frac{1}{2}\right)\lambda \quad (m = 0, \pm 1, \pm 2, \ldots)$$

Linear positions of bright and dark fringes:

From the triangle OPQ,



$$\tan\theta = \frac{y}{L}$$

$$y_{\text{bright}} = L\tan\theta_{\text{bright}}$$

$$y_{\text{bright}} = L\frac{m\lambda}{d} \quad \text{(small angles)}$$

$$y_{\text{dark}} = L\tan\theta_{\text{dark}}$$

$$y_{\text{bright}} = L\frac{\left(m+\frac{1}{2}\right)\lambda}{d} \quad \text{(small angles)}$$

Intensity Distribution of the Double-Slit Interference Pattern

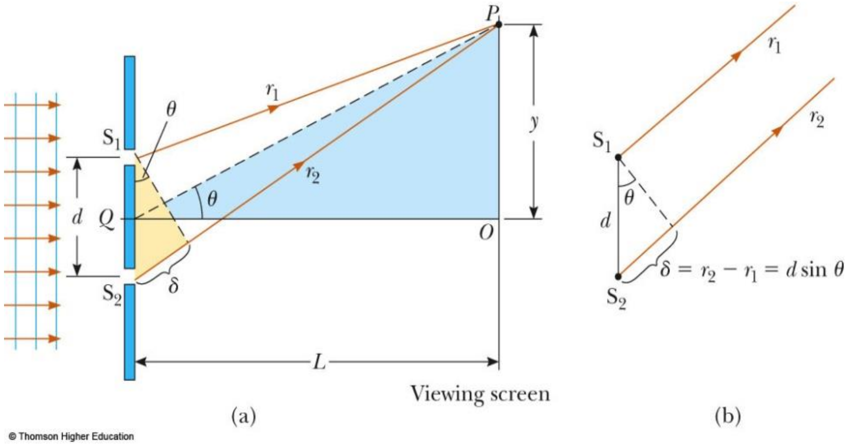
Consider two coherent sources of sinusoidal waves such that they have same angular frequency ω and phase difference ϕ.

$$E_1 = E_0 \sin \omega t \quad \text{and} \quad E_2 = E_0 \sin(\omega t + \phi)$$

$$\phi = \frac{2\pi}{\lambda}\delta = \frac{2\pi}{\lambda} d \sin\theta$$

$$E_P = E_1 + E_2 = E_0\left[\sin \omega t + \sin(\omega t + \phi)\right]$$

$$E_P = 2E_0 \cos\left(\frac{\phi}{2}\right) \sin\left(\omega t + \frac{\phi}{2}\right)$$



Intensity of a wave is proportional to the square of the resultant electric field magnitude at that point.

$$I \quad \alpha \quad E_P^2 = 4E_0^2 \cos^2\left(\frac{\phi}{2}\right)\sin^2\left(\omega t + \frac{\phi}{2}\right)$$

Most light-detecting instruments measure time-averaged light intensity, and the time averaged value of $\sin^2\left(\omega t + \frac{\phi}{2}\right)$ over one cycle is ½.

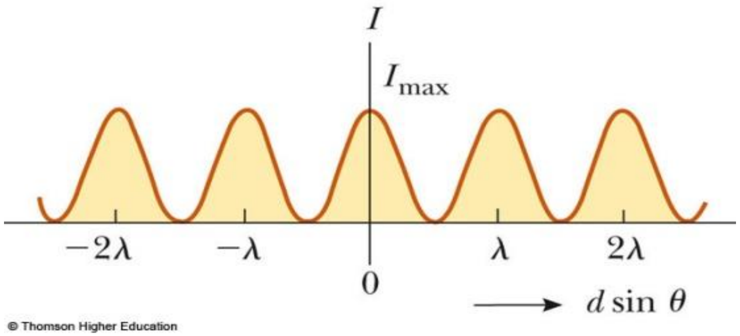
$$I = I_{\max}\cos^2\left(\frac{\phi}{2}\right)$$

$$I = I_{\max}\cos^2\left(\frac{\pi d \sin\theta}{\lambda}\right)$$

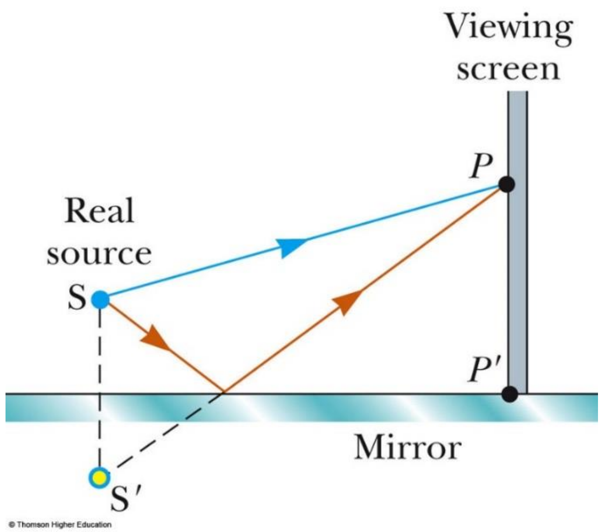
Alternatively, since $\sin\theta \approx \frac{y}{L}$

for small values of θ , we can write;

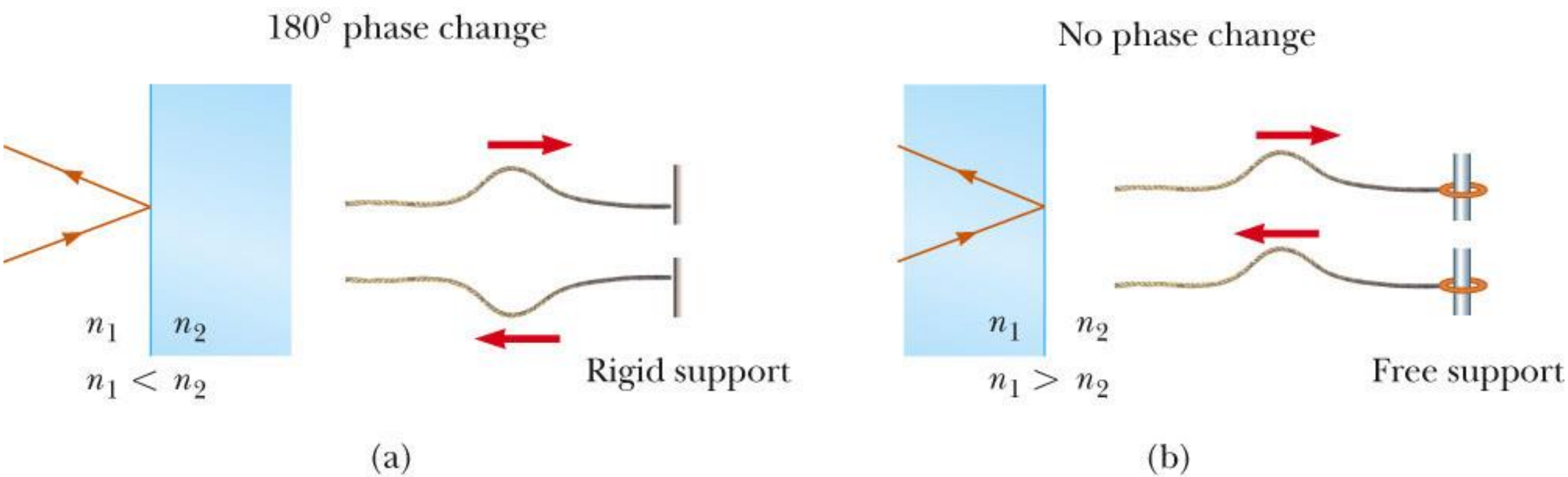
$$I = I_{\max}\cos^2\left(\frac{\pi d}{\lambda L} y\right)$$



Change of Phase Due to Reflection



Lloyd's mirror. The reflected ray undergoes a phase change of 180°.



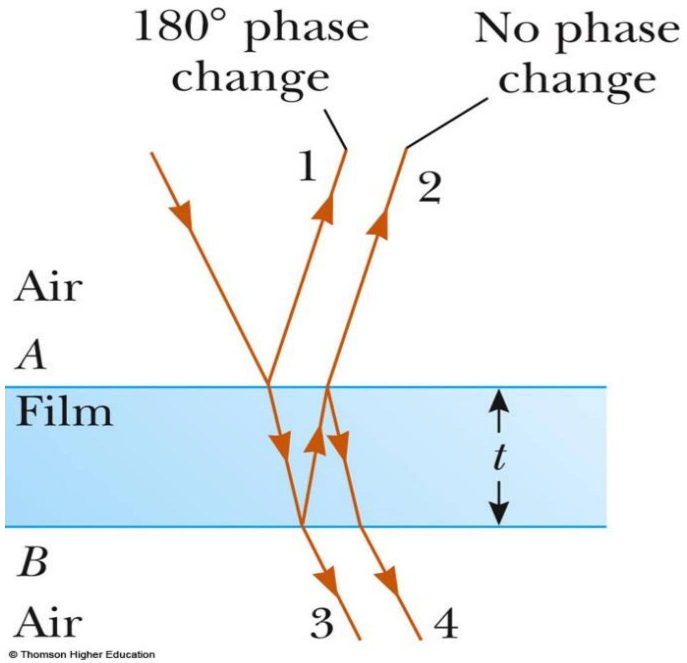
Interference in Thin Films

Consider a film of uniform thickness t and index of refraction n. Assume light rays traveling in air are nearly normal to the two surfaces of the film. If λ is the wavelength of the light in free space and n is the index of refraction of the film material, then the wavelength of light in the film is $\lambda_n = \dfrac{\lambda}{\text{n}}$

The condition for ***constructive*** interference in thin films is,

$$2t = \left(m + \frac{1}{2}\right)\lambda_n \quad (m = 0, 1, 2, \ldots)$$

$$2nt = \left(m + \frac{1}{2}\right)\lambda \quad (m = 0, 1, 2, \ldots)$$

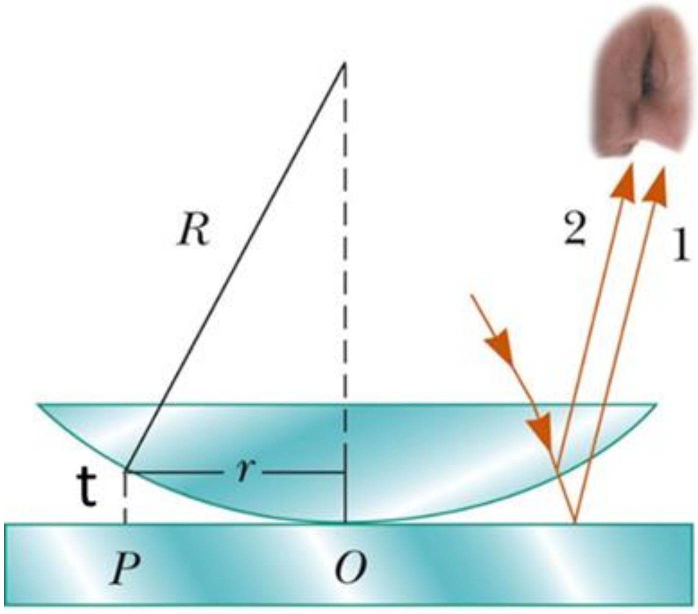


The condition for ***destructive*** interference in thin films is,

$$2nt = m\lambda \quad (m = 0, 1, 2, \ldots)$$

Newton's Rings

Expressions for radii of the dark rings:



(a)

Consider the dark rings (destructive interference)

$$2nt = m\lambda, \qquad m = 0, 1, 2, 3\ldots$$

For air film, $n \approx 1$

$$\therefore \quad 2t = m\lambda$$

From the above figure, $t = R - \sqrt{R^2 - r^2}$

$$t = R - R\left[1 - \left(\frac{r}{R}\right)^2\right]^{1/2}$$

Binomial theorem is, $(1+y)^n = 1 + ny + \frac{n(n-1)}{2!}y^2 + \ldots\ldots.$

If $r / R << 1,$

$$t = R - R\left[1 - \frac{1}{2}\left(\frac{r}{R}\right)^2 + \ldots\ldots..\right] \approx \frac{r^2}{2R}$$

$$r_{dark} \approx \sqrt{mR\lambda} \qquad (m = 0, 1, 2, \ldots)$$

Michelson Interferometer

The **interferometer,** invented by A. A. Michelson, splits a light beam into two parts and then recombines the parts to form an interference pattern.

The interference condition for the two rays is determined by the difference in their path length. When the two mirrors are exactly perpendicular to each other, the interference pattern is a target pattern of bright and dark circular fringes.

If a dark circle appears at the center of the target pattern and M_1 is then moved a distance $\lambda/4$ toward M_0, the path difference changes by $\lambda/2$. This replaces dark circle at center by bright circle. Therefore, the fringe pattern shifts by one-half fringe each time M_1 is moved a distance $\lambda/4$.