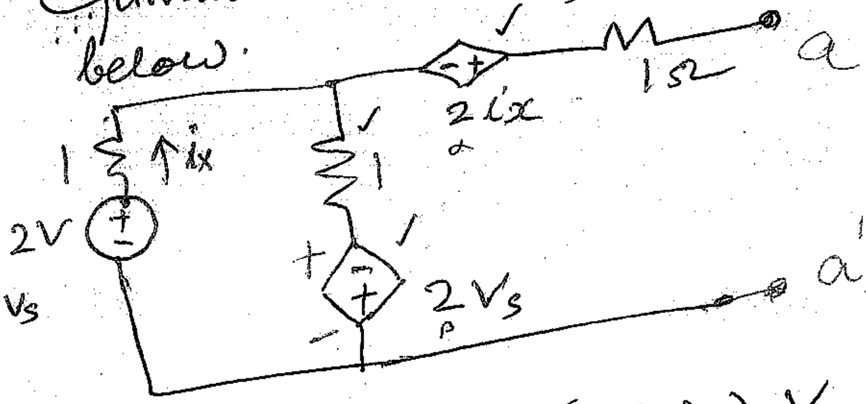
Draw the Thevenin Equivalent Circuit and Norton Equivalent Circuit for the Circuit Shown in figure below.

1A)



$$ix = \frac{V_s - 2V_s}{2} = \left(\frac{1-2}{2}\right) V_s = -\frac{1}{2} \times 2 = -1$$

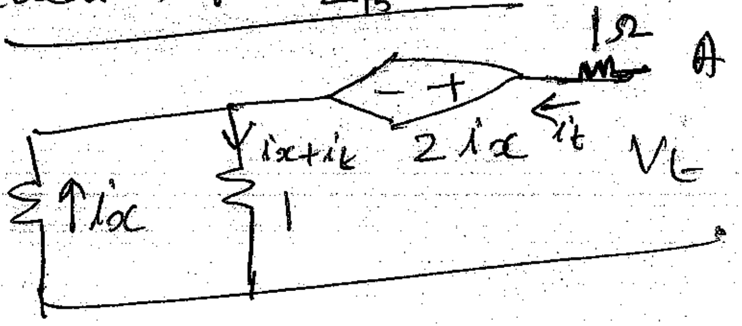
$$V_{th} = 2V_s + 2ix + 1.ix$$

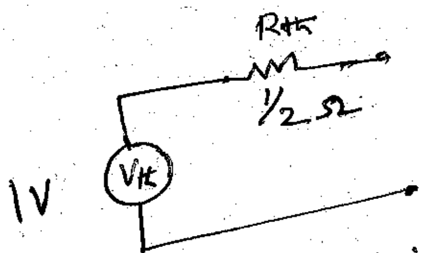
$$= 2V_s + (1+2)\,ix$$

$$= 2V_s + 3\left(-\frac{1}{2}\right)V_s$$

$$V_{th} = V_s\left[2 - \frac{3}{2}\right] = \frac{V_s}{2} = \frac{2}{2} = 1\,V$$

Calculation for Z_{th}





Thevenin Equi: Ckt

$$Z_{th} = \frac{V_{th}}{i_t}$$

loop 1. $Ix + Ix + It = 0$

$$\therefore Ix = -\frac{it}{2}$$

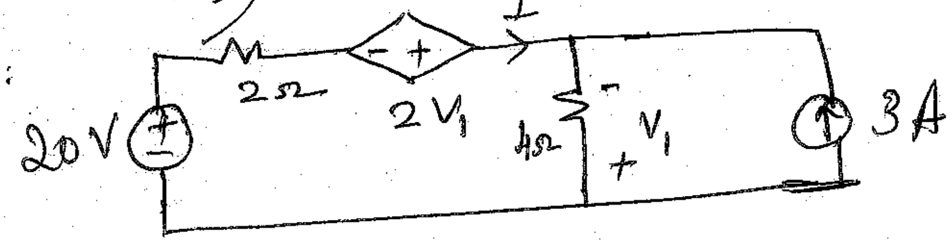
loop 2

$$V_t = 1.i_t + 2ix + ix + i_t$$

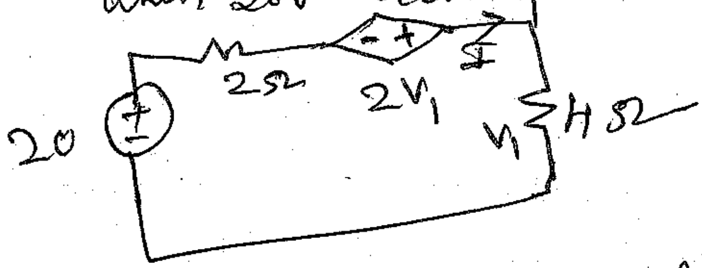
$$V_t = i_t - 2 \cdot \frac{it}{2} - \frac{it}{2} + it = \frac{it}{2}$$

$$\therefore Z_{th} = V_{th}/I_t = \frac{it/2}{it} \Rightarrow \frac{1}{2}\,\Omega \text{ Ans.}$$

IB) find I



when 20V acting



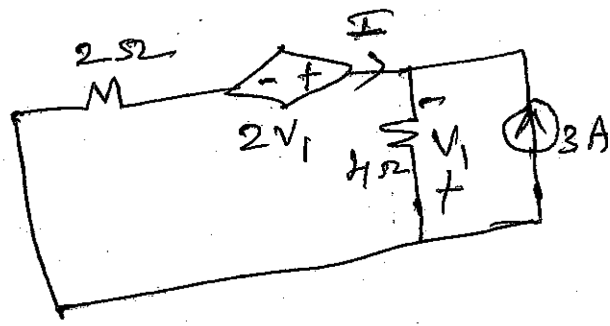
$$\cancel{6a}\quad 6I = 20 + 2V_1 \quad (1)$$

$$\text{But } V_1 = -4I \quad (2)$$

$$(1) \text{ gives } 6I + 8I = 20$$

$$I = \frac{20}{14}$$

when 3A acting



$$2I + (I+3)4 = 2V_1 \quad (3)$$

$$\text{But } V_1 = -(3I+4)4 \quad (4)$$

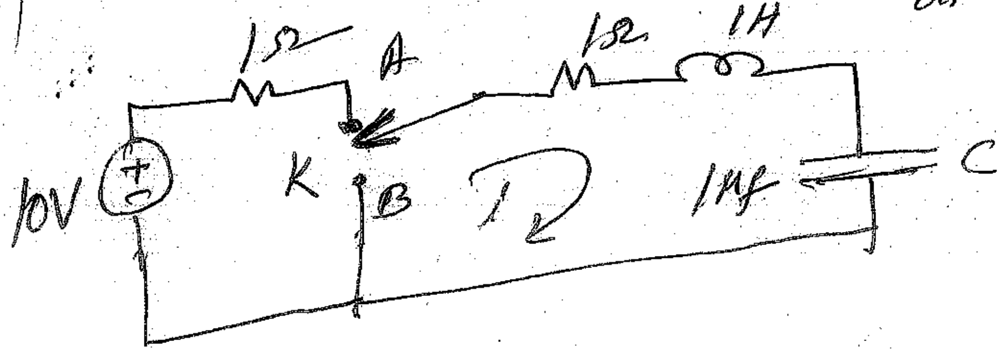
$\therefore$ (3) Gives

$$2I + 4I + 12 = -24 - 8I$$

$$\therefore I = -\frac{36}{14}$$

$$\therefore \text{Total Current} = I = \frac{20}{14} - \frac{36}{14} = -\frac{16}{14} = -\frac{8}{7}\,A$$

I(C) find $i(0)$, $\frac{di}{dt}(0)$, $\frac{d^2i}{dt^2}(0)$



at $t=0$, $i=0$, ∵ capacitor is already Charged to 10V and open ckted ∴ $i=0$

∴ $i(0) = 0$ Ans

at position 2

$$Ri + L\frac{di}{dt} + \frac{1}{C}\int i\,dt = 0 \quad \text{(1)}$$

$$R(0) + \frac{di}{dt} + 10 = 0$$

$$\therefore \frac{di(0)}{dt} = -10 \text{ A/sec}$$

diff Eqn (1)

$$R\frac{di}{dt} + L\frac{d^2i}{dt^2} + \frac{i}{C} = 0$$

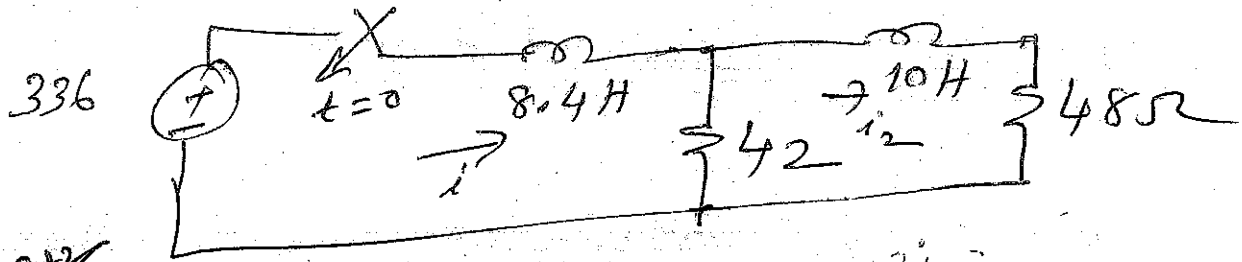
at $t=0$, $i=0$, $\frac{di}{dt}(0) = -10$

$$\therefore -10 + \frac{d^2i}{dt^2}(0) + 0 = 0$$

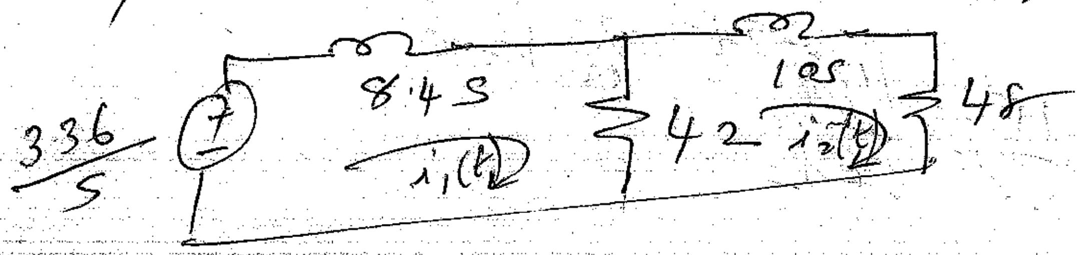
$$\therefore \frac{d^2i}{dt^2}(0) = 10 \text{ A/sec}^2$$

1A) Obtain the expression for i_1 and i_2 in the circuit shown in figure 9. when dc voltage is applied suddenly. Assume that the initial energy stored in the circuit is zero. Use transform method.

2A)



Sol:



KVL, mesh equation

$$\frac{336}{s} = (42 + 8.4s)i_1 - 42 i_2$$

$$0 = -42 i_1 + (90 + 10s) i_2$$

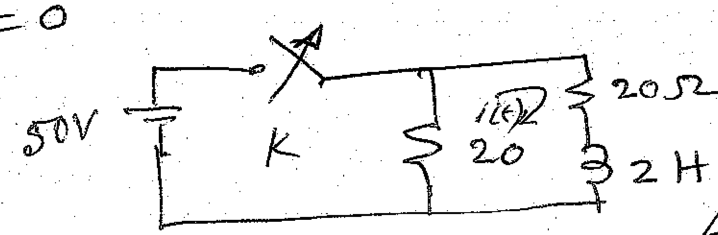
$$\Delta = \begin{vmatrix} 42+8.4s & -42 \\ -42 & 90+10s \end{vmatrix} = 0, = 84(s^2+14s+24)$$
$$= \underline{84(s+2)(s+12)}$$

$$i_1 = \frac{\begin{vmatrix} 336s & -42 \\ 0 & 90+10s \end{vmatrix}}{\Delta} = \frac{40(s+9)}{s(s+2)(s+12)}$$

$$\text{lly } i_2 = \frac{\begin{vmatrix} 42+8.4s & \frac{336}{s} \\ -42 & 0 \end{vmatrix}}{\Delta} = \frac{168}{s(s+2)(s+12)}$$

Q. Find the current equation when the switch K is opened at $t=0$

2C)



(2 mark)

Soln: Switch K opened at $t=0$ KVL

$\frac{50}{20}$
$=2.5A$

$$20i + 20i + 2\frac{di}{dt} = 0$$

divide by 2

ch. eqn:

$$(S+20)i = 0$$

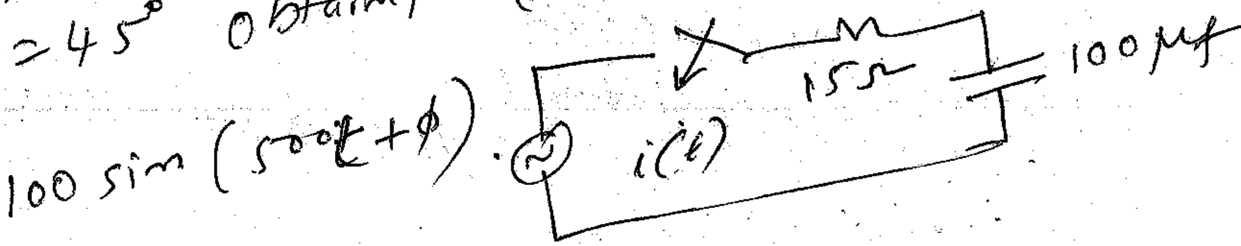
$$i = C_1 e^{-20t}$$

at $t=0$, $i = 2.5$

$$2.5 = C_1$$

$$\therefore i(t) = 2.5e^{-20t}$$

Q2B) The circuit shown in figure (Q.C) consists of series RC elements with $R = 15\ \Omega$ and $C = 100\ \mu F$. A sinusoidal voltage $V = 100\sin(500t+\phi)$ Volt is applied to the circuit at time corresponding to $\phi = 45°$. Obtain the current transient $i(t)$ (3 mark)



KVL

$$15i + \frac{1}{100\times10^{-6}}\int i\,dt = 100\sin(500t+\phi)$$

diff: $15\frac{di}{dt} + \frac{i}{100\times10^{-6}} = (100)(500)\cos(500t+\phi)$

Partial fraction

$$i_1 = \frac{15}{s} - \frac{14}{s+2} - \frac{1}{s+12}$$

$$i_2 = \frac{7}{s} - \frac{8.4}{s+2} + \frac{1.4}{s+12}$$

$\therefore$ taking $\mathcal{L}^{-1}$ transform

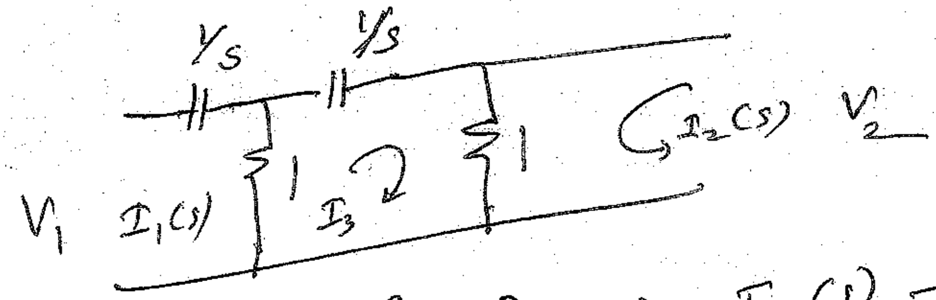
~~$i_1 = \frac{15}{s} - 14e^{-t} - \dots$~~

$$i_1 = \left(15 - 14e^{-2t} - e^{-12t}\right) A$$

$$i_2 = \left(7 - 8.4e^{-2t} + 1.4e^{-12t}\right)$$ Ans

Z parameters

3A)



mesh 1 $\quad V_1 = \left(1+\frac{1}{s}\right) I_1(s) - I_3(s)$ — (1)

Mesh 2 $\quad V_2 = \left(I_2(s) + I_3(s)\right)\cdot 1$ — (2)

mesh 3 $\quad -I_1(s) + I_2(s) + I_3(s)\left(\frac{4s+1}{2s}\right) = 0$ — (3)

from (3) $\quad I_3(s) = I_1(s) - I_2(s)\left(\frac{2s}{4s+1}\right)$ — (4)

(4) in (1)

$$V_1(s) = \frac{2s^2+5s+1}{s(4s+1)} I_1(s) + \left(\frac{2s}{4s+1}\right) I_2(s) \quad \text{— (5)}$$

(4) in (2) $\quad V_2(s) = \left(\frac{2s}{4s+1}\right) I_1(s) + \left(\frac{2s+1}{4s+1}\right) I_2(s)$ — (6)

By comparison $\quad V_1 = Z_{11} I_1 + Z_{12} I_2$

$V_2 = Z_{21} I_1 + Z_{22} I_2$

$\therefore Z_{11} = \frac{2s^2+5s+1}{s(4s+1)}$

$Z_{21} = \frac{2s}{4s+1}$

$Z_{12} = \frac{2s}{4s+1}$

$Z_{22} = \frac{2s+1}{4s+1}$

Ch: equ: $(S+666.67)i = 3333.3\cos(500t+\phi) \rightarrow (1)$

Complementary function $i_c = K_1 e^{-666.67t}$

Particular current $i_p = A\cos(500t+45) + B\sin(500t+45)$

$i_p' = -500A\sin(500t+45) + 500B\cos(500t+45)$

Sub: i_p and i_p' in equ: (1)

$-500A\sin(500t+45) + 500B\cos(500t+45) + 666.67A\cos(500t+45)$

$+666.67B\sin(500t+45) = 3333.3\cos(500t+\phi)$

$500B + 666.67A = 3333.3$

$666.67B - 500A = 0$

solving $A = 3.2,\ B = 2.4$

$\therefore i_p = 3.2\cos(500t+45) + 2.4\sin(500t+45)$

$i_p = 4\sin(500t+98.13)$

$\therefore i = i_c + i_p = K_1 e^{-666.67t} + 4\sin(500t+98.13)$

at $t=0$, $15i = 100\sin 45$

$i = \frac{100}{15}\sin 45 = 4.71$

at $t=0$, $4.71 = K_1 + 4\sin 98.13$

$\therefore K_1 = 0.75$

$\therefore i(t) = 0.75e^{-666.67t} + 4\sin(500t+98.13^\circ)$

3B. $y(t) = t^2 x(t-1) \Rightarrow$ M, C, US, L, TV & NInv

with justification $\frac{1}{2}$ $0.5 \times 6 = $ (3m)

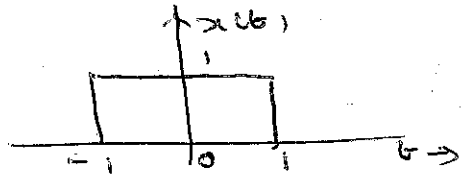
3C. (i) $x(t) = e^{jt}$ is periodic

with $\omega_0 = 1$, & $T = \frac{2\pi}{\omega_0} = 2\pi$ sec . . 1m

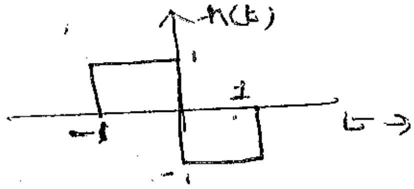
. . . . 1m } (2m)

(ii) $x(t) = \cos(3n)$ is Nonperiodic

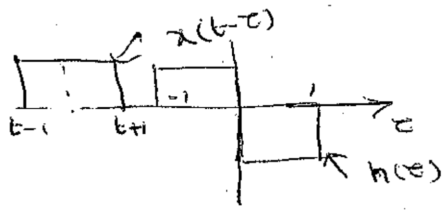
4A. $x(t) = u(t+1) - u(t-1)$



$h(t) = u(t-1) - 2u(t) + u(t-2)$



(i)



for $t+1 < -1$ ie $t < -2$

$y(t) = 0$

(1 M)

(ii) for $t+1 > -1$ & $t+1 < 0$ ie $-2 \leq t \leq -1$,

$$y(t) = \int_{-1}^{t+1} d\tau = \underline{\underline{t+2}}$$

(ii) for $t+1 > 0$ & $t-1 < -1$ ie $-1 \leq t \; 0$

$$y(t) = \int_{-1}^{0} d\tau - \int_{0}^{t+1} d\tau = \underline{\underline{-t}}$$

(1.5m)

(iii) for $t-1 > -1$ & $t-1 < 0$ ie $0 \leq t \leq 1$

$$y(t) = \int_{t-1}^{0} d\tau - \int_{0}^{1} d\tau = -t$$

(iv) for $t-1 > 0$ & $t-1 < 1$

ie $1 \leq t \leq 2$ $\quad y(t) = -\int_{t-1}^{1} d\tau = \underline{\underline{t-2}}$

(1.5M)

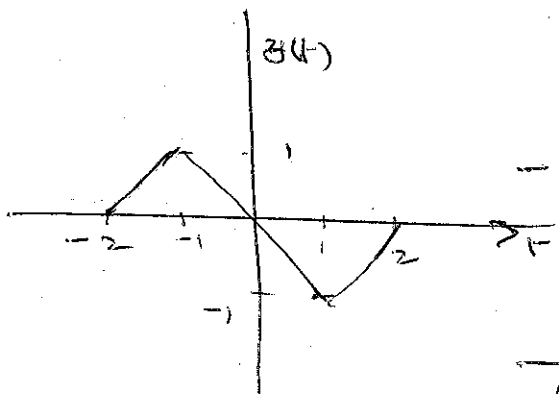
$\therefore$ $y(t) = 0 \quad t \leq -2$

$= t+2 \quad -2 \leq t < -1$

$= -t \quad -1 \leq t < 1$

$= t-2 \quad 1 \leq t \leq 2$

$= 0 \quad t \geq 2$



— (1M)

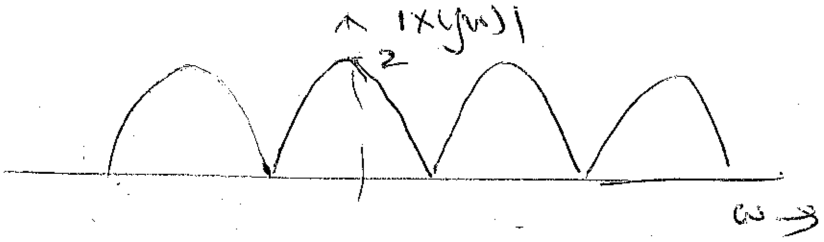
(5M)

4B. (i) $x(t) = \delta(t+1) + \delta(t-1)$ $\qquad \delta(t) \xleftrightarrow{FT} 1$

$\delta(t-t_0) \xleftrightarrow{FT} e^{-j\omega t_0}$

$$X(j\omega) = e^{j\omega} + e^{-j\omega}$$

$$= \underline{\underline{2\cos(\omega)}} \quad \text{—— 1m}$$

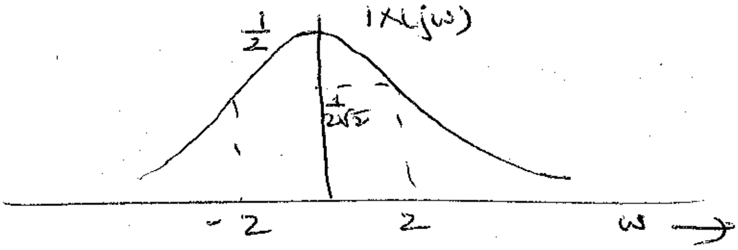


—— 0.5m

(ii) $x(t) = e^{-2t} u(t)$

$$X(j\omega) = \int_0^{\infty} e^{-2t} e^{-j\omega t}\, dt = \frac{1}{2+j\omega}$$

$$X(j\omega) = \frac{1}{\sqrt{2^2+\omega^2}} \underline{\left|-\tan^{-1}\omega/2\right.} \quad \text{—— 1m}$$



—— 0.5m

$\underline{\underline{3m}}$

4C. $x(t) = 1 + \cos(\pi t) + \cos(2\pi t) + \sin(5\pi t)$

periodic signal → Power signal

Use Parsevals theorem. $P = \sum_{k=-\infty}^{\infty} |a_k|^2$ —— 1m

$$x(t) = 1 + \frac{e^{j\pi t} + e^{-j\pi t}}{2} + \frac{e^{j2\pi t} + e^{-j2\pi t}}{2} + \frac{e^{j5\pi t} - e^{-j5\pi t}}{2j}$$

$a_0 = 1$, $a_1 = a_{-1} = a_2 = a_{-2} = \frac{1}{2}$, $\quad a_5 = +\frac{1}{2j}$ & $a_{-5} = -\frac{1}{2j}$

$$\therefore P = 1^2 + \frac{1}{2^2} + \frac{1}{2^2} + \frac{1}{2^2} + \frac{1}{2^2} + \frac{1}{2^2} + \frac{1}{2^2}$$

$$= \underline{\underline{2.5W}} \quad \text{—— 1m}$$

$$\underline{\underline{E = \infty}} \quad \text{——}$$

$\underline{\underline{2m}}$

5A. $\frac{d^2y(t)}{dt^2} + 5\frac{dy(t)}{dt} + 4y(t) = \frac{dx(t)}{dt} + x(t)$

FT $\quad Y(j\omega)\left[(j\omega)^2 + 5j\omega + 4\right] = X(j\omega)\left[j\omega + 1\right]$ — (0.5M)

(ii) $\frac{Y(j\omega)}{X(j\omega)} = H(j\omega) = \frac{j\omega + 1}{(j\omega)^2 + 5j\omega + 4}$

$= \frac{\cancel{(j\omega+1)}}{(j\omega+4)\cancel{(j\omega+1)}} = \frac{1}{j\omega+4}$ — (1M)

(ii) $h(t) = FT^{-1}\left\{\frac{1}{j\omega+4}\right\} = e^{-4t}u(t)$ — (1M)

(iii) for $x(t) = e^{-t}u(t)$

$y(t) = h(t) * x(t)$

$Y(j\omega) = H(j\omega) \cdot X(j\omega)$ — (1M)

$= \frac{1}{(j\omega+4)} \cdot \frac{1}{(j\omega+1)}$

$= \frac{A}{j\omega+4} + \frac{B}{j\omega+4}$

$A = \left.\frac{1}{j\omega+4}\right|_{j\omega=-1} = \frac{1}{3}$

$B = \left.\frac{1}{j\omega+1}\right|_{j\omega=-4} = -1/3$

$= \frac{1/3}{j\omega+1} - \frac{1/3}{j\omega+4}$

$\therefore \quad y(t) = \frac{1}{3}\left[e^{-t} - e^{-4t}\right]u(t)$ — (1M)

5M

5B. W.K.T $\quad e^{-t}u(t) \overset{FT}{\Longleftrightarrow} \frac{1}{j\omega+1}$ — (1M)

$t\,e^{-t}u(t) \Longleftrightarrow \frac{1}{(j\omega+1)^2}$ — (1M)

$\frac{d}{dt}\left[t\,e^{-t}u(t)\right] = \frac{j\omega}{(1+j\omega)^2}$ — (1M)

$\therefore \quad x(t) = \frac{d}{dt}\left[t\,e^{-t}u(t)\right]$

$= (1-t)\,e^{-t}u(t)$

3M

5 c. Property $x(t) \xleftrightarrow{FT} X(j\omega)$

$$X(jt) \xleftrightarrow{FT} 2\pi f(-\omega)$$

$$e^{2t} x(-t) \xleftrightarrow{FT} \frac{1}{2-j\omega} \quad - \text{1M}$$

$$\frac{1}{2-jt} \xleftrightarrow{FT} 2\pi e^{-2\omega} x(\omega) \quad - \text{1M}$$

2M

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