MODERN CONTROL THEORY

Canonical Forms of state – space representation

1. Controllable canonical form

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \cdot \\ \cdot \\ \cdot \\ \dot{x}_{n-1} \\ \dot{x}_n \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \cdot & \cdot & \cdot & & \cdot \\ \cdot & \cdot & \cdot & & \cdot \\ \cdot & \cdot & \cdot & & \cdot \\ 0 & 0 & 0 & \cdots & 1 \\ -a_n & -a_{n-1} & -a_{n-2} & \cdots & -a_1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \cdot \\ \cdot \\ \cdot \\ x_{n-1} \\ x_n \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} b_n - a_n b_0 \,\vdots\, b_{n-1} - a_{n-1} b_0 \,\vdots\, \cdots \,\vdots\, b_1 - a_1 b_0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \cdot \\ \cdot \\ \cdot \\ x_n \end{bmatrix} + b_0 u$$

2. Observable canonical form

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \cdot \\ \cdot \\ \cdot \\ \dot{x}_n \end{bmatrix} = \begin{bmatrix} 0 & 0 & \cdots & 0 & -a_n \\ 1 & 0 & \cdots & 0 & -a_{n-1} \\ \cdot & \cdot & & \cdot & \cdot \\ \cdot & \cdot & & \cdot & \cdot \\ \cdot & \cdot & & \cdot & \cdot \\ 0 & 0 & \cdots & 1 & -a_1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \cdot \\ \cdot \\ \cdot \\ x_n \end{bmatrix} + \begin{bmatrix} b_n - a_n b_0 \\ b_{n-1} - a_{n-1} b_0 \\ \cdot \\ \cdot \\ \cdot \\ b_1 - a_1 b_0 \end{bmatrix} u$$

$$y = \begin{bmatrix} 0 & 0 & \cdots & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \cdot \\ \cdot \\ \cdot \\ x_{n-1} \\ x_n \end{bmatrix} + b_0 u$$

3. Diagonal canonical form

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \cdot \\ \cdot \\ \cdot \\ \dot{x}_n \end{bmatrix} = \begin{bmatrix} -p_1 & & & 0 \\ & -p_2 & & \\ & & \ddots & \\ 0 & & & -p_n \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \cdot \\ \cdot \\ \cdot \\ x_n \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ \cdot \\ \cdot \\ \cdot \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} c_1 & c_2 & \cdots & c_n \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \cdot \\ \cdot \\ \cdot \\ x_n \end{bmatrix} + b_0 u$$

4. Jordan canonical form

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \cdot \\ \cdot \\ \cdot \\ \dot{x}_n \end{bmatrix} = \left[\begin{array}{ccc|ccc} -p_1 & 1 & 0 & 0 & \cdots & 0 \\ 0 & -p_1 & 1 & \cdot & & \cdot \\ 0 & 0 & -p_1 & 0 & \cdots & 0 \\ \hline 0 & \cdots & 0 & -p_4 & & \\ \cdot & & \cdot & & \cdot & \\ \cdot & & \cdot & & & \cdot \\ \cdot & & \cdot & & & \cdot \\ 0 & \cdots & 0 & 0 & & -p_n \end{array}\right] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ \cdot \\ \cdot \\ \cdot \\ x_n \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \\ \cdot \\ \cdot \\ \cdot \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} c_1 & c_2 & \cdots & c_n \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \cdot \\ \cdot \\ \cdot \\ x_n \end{bmatrix} + b_0 u$$

<u>State transition matrix</u>

Using matrix exponential

$$e^{At} = I + At + \frac{1}{2!}A^2t^2 + \frac{1}{3!}A^3t^3 + \cdots + \frac{1}{i!}A^it^i$$

Using Laplace transform method

$$e^{At} = \emptyset(t) = L^{-1}[(\emptyset(s)] = L^{-1}[(sI - A)^{-1}]$$

<u>Solution of state equations</u>

1. Solution of state equation without input or excitation

$$x(t) = e^{At}\, X_0$$

2. Solution of state equation without input or excitation

Method 1:

$$x(t) = e^{A(t-t_0)}\, X(t_0) + \int_{t_0}^{t} e^{A(t-\tau)} B\, U(\tau)\, d\tau$$

Method 2:

$$x(t) = \emptyset(t)X(0) + L^{-1}[\emptyset(s).B.U(s)]$$

<u>**Controllability matrix:**</u> $Q_c = [\,B \;\; AB \;\; A^2B \ldots\ldots A^{n-1}B$

<u>**Observability matrix:**</u> $Q_0 = [C^T \;\; A^TC^T \;\; (A^T)^2C^T \;\; (A^T)^3C^T \ldots (A^T)^{n-1}C^T]$

Nonlinear systems

SNo.	Nonlinearity	Describing function
1	Saturation	$N = \frac{K}{\pi}[2\beta + sin2\beta]$
2	Dead Zone	$N = \frac{K}{\pi}[\pi - 2\beta - sin2\beta]$
3	Saturation with Dead zone	$N = \frac{K}{\pi}[2(\beta - \alpha) + (sin2\beta - sin2\alpha)]$
4	Ideal Relay (On-Off)	$N = \frac{4M}{\pi X}\angle 0$
5	Relay with dead zone	$N = \frac{4y}{\pi X}[cos\beta]$
6	On off with Hysteresis	$N = \frac{4M}{\pi X}\angle -\beta$
7	Coulomb and Viscous friction	$N = \frac{4\mu}{\pi X} + K$

8	Linear transfer with dead zone	$N = \frac{K}{\pi}[\pi - 2\beta + sin2\beta]$
9	Quadratic nonlinearity	$N = \frac{8X}{3\pi}$
10	Hysteresis with dead zone and Relay	$N = \frac{2M}{\pi X}[\cos\beta + cos\,\alpha + j(\sin\beta - sin\alpha)]$
11	Cubic Nonlinearity	$N = \frac{3}{4}X^2$
12	Backlash	$N = \frac{B_1 + jA_1}{X}$ Where $B_1 = \frac{KX}{\pi}\left[\frac{\pi}{2} + \beta + \frac{b(X-b)}{X^2}\sqrt{\frac{2X}{b} - 1}\right]$ $A_1 = \frac{4KX}{\pi}\left[\frac{\left(\frac{b}{2}\right)^2}{X^2} - \frac{\left(\frac{b}{2}\right)}{X}\right]$

Ackermann's formula for controller gain matrix $K = [k_1 \ \ k_2 \ \ k_3] = [0 \ \ 0 \ \ 1]Q_o^{-1}\Phi(F)$

$$L = \phi(F)Q_o^{-1}\begin{bmatrix}0\\0\\1\end{bmatrix}$$

Ackermann's formula for observer gain matrix.

Generations of Lyapunov function $V = X^T PX$.

Where P can be obtained by solving $A^T P + PA = -I$

4/4.