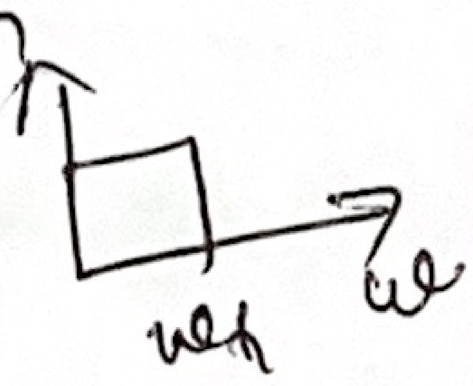
Filters: Devices (or) circuits that pass only certain frequency & block other frequencies.

Butterworth low pass filters

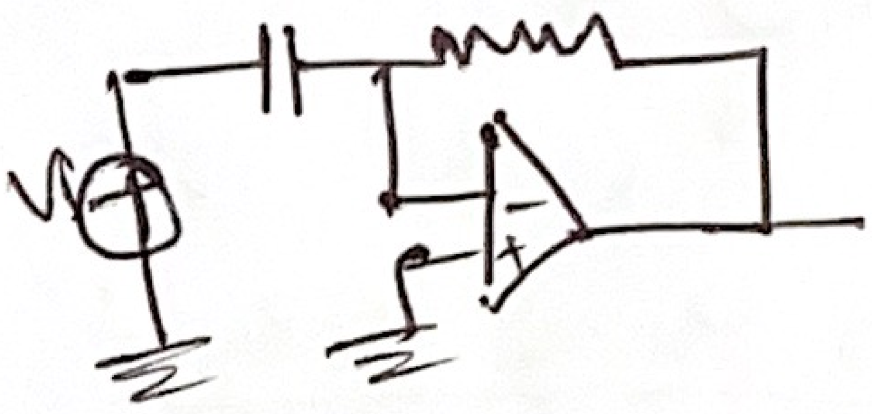
low pass filter - passes freq. passes less than certain freq.



Passband - $\omega < \omega_H$ [ω_H = half power freq.].

stopband - $\omega > \omega_H$

at $\omega = \omega_H$, gain = $A/\sqrt{2}$.

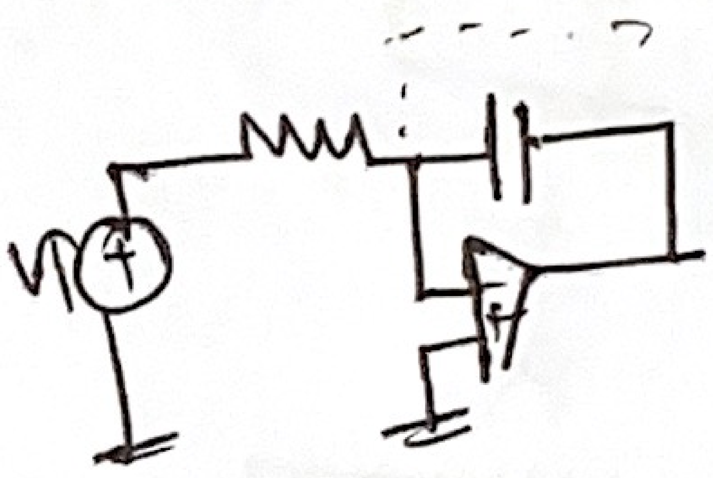


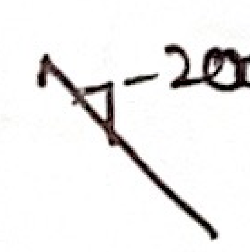
Differentiator $V_o/V_i = H(s) = -RCs$

$\omega_0 = \frac{1}{RC}$

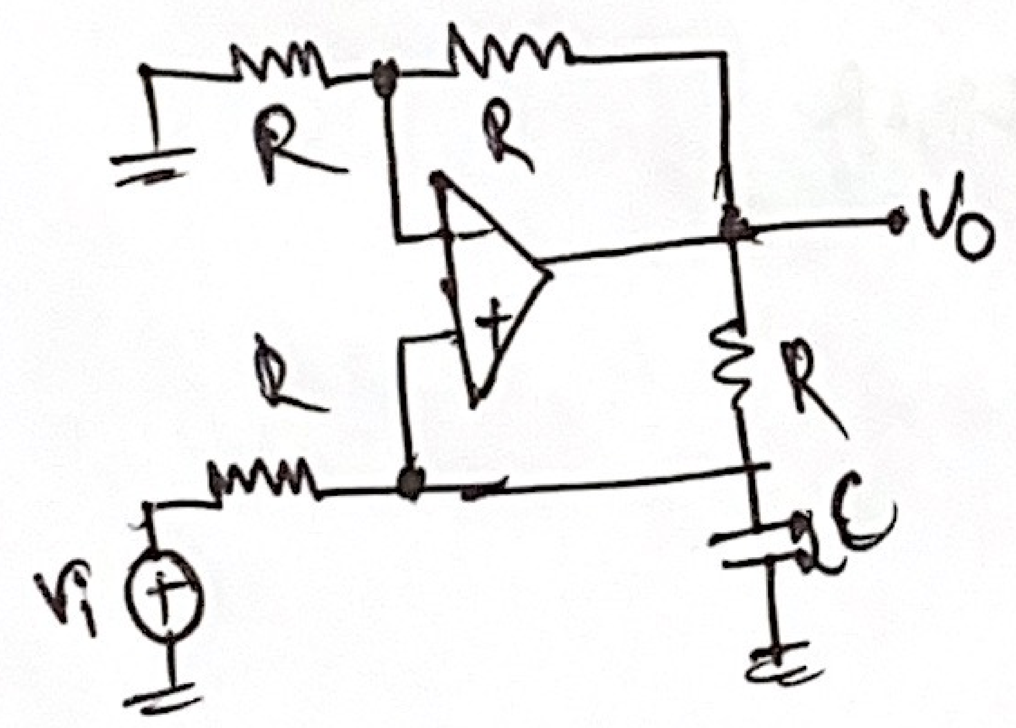
$|Z_C| = R$ - provides unity gain.

Integrator $H(s) = -\frac{1}{RCs}$.



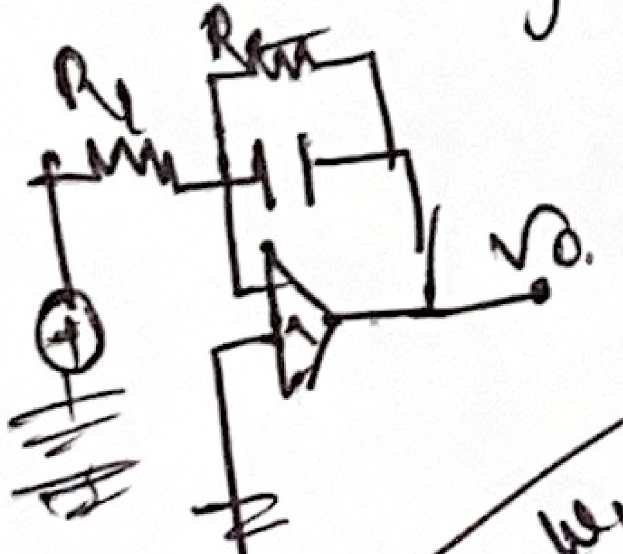


Non inverting and deboIntegrator.



$H(s) = \frac{1}{RCs}$

Low pass filter: placing a resistor parallel with feed back capacitor turns integrator into Low pass F. with gain H(s).

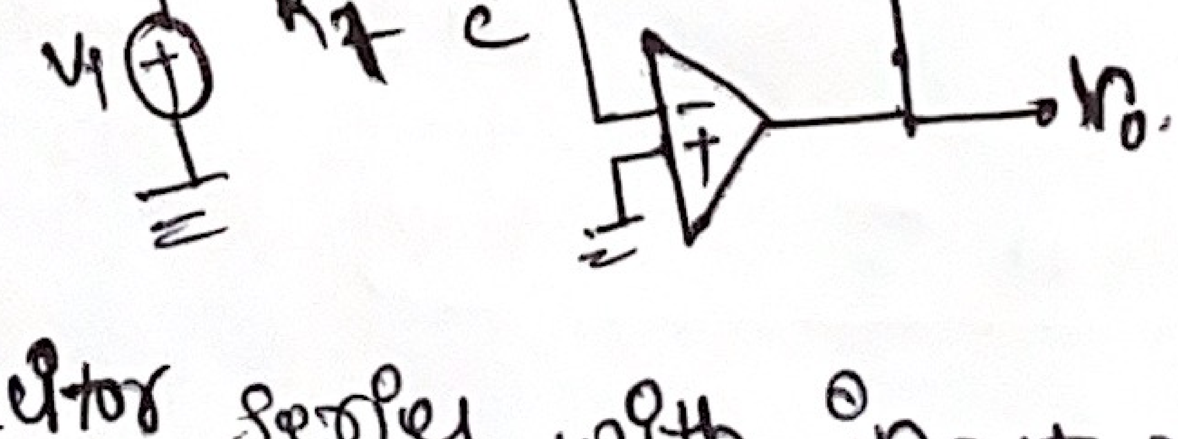


$H(s) = -\frac{R_2}{R_1} \frac{1}{R_2 C s + 1}$

$H(j\omega) = H_0 \frac{1}{1 + j\omega/\omega_0}$

$\boxed{\omega_0 = \frac{1}{R_2 C}}$ $\boxed{H_0 = -\frac{R_2}{R_1}}$

$\omega_1 = \frac{1}{R_1 C}$ unity gain freq



High pass filters placing a capacitor series with input resistor turns differentiator into HPF with gain,

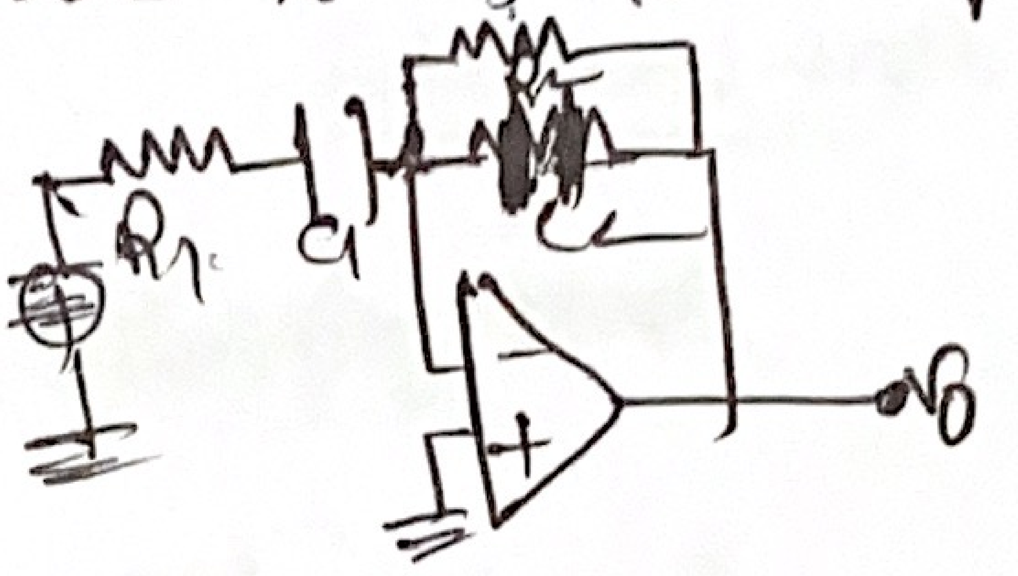
$$\boxed{H(s) = -\frac{R_2}{R_1} \frac{R_1 C s}{R_1 C s + 1}}$$

$H(j\omega) = H_0 \frac{j\omega/\omega_0}{1 + j\omega/\omega_0}$

$H_0 = -\frac{R_2}{R_1}$ $\omega_0 = \frac{1}{R_1 C}$.

wide band band pass filters combine both above circuits to bandpass

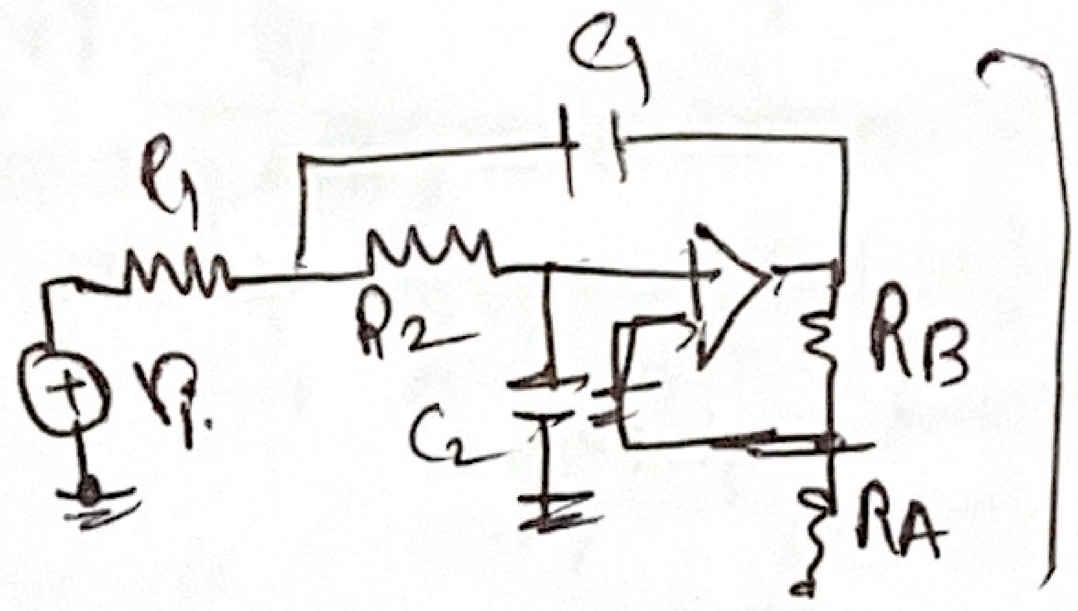
response. $H(s) = -\frac{R_2}{R_1} \cdot \frac{R_1C_1 s}{R_1C_1 s+1} \cdot \frac{1}{R_2C_2 s+1}$



$H(j\omega) = H_0 \frac{j\omega/\omega_L}{(1+j\omega/\omega_L)(1+j\omega/\omega_H)}$ H_0 = mid frequency gain.

$\boxed{H_0 = -\frac{R_2}{R_1}}$ $\omega_L = \frac{1}{R_1C_1}$ $\omega_H = \frac{1}{R_2C_2}$

Lowpass KRC filters:



$K = 1 + \frac{R_B}{R_A}$ $H(s) = \frac{1}{R_1C_1R_2C_2 s^2 + [(1-K)R_1C_1 + R_2C_2 + R_1C_2]s + 1}$

$(s = j\omega)$ $\omega_0 = \frac{1}{\sqrt{R_1R_2C_1C_2}}$ $Q = \frac{1}{(1-K)\sqrt{R_1C_1/R_2C_2} + \sqrt{R_1C_2/R_2C_1} + \sqrt{R_2C_2/R_1C_1}}$

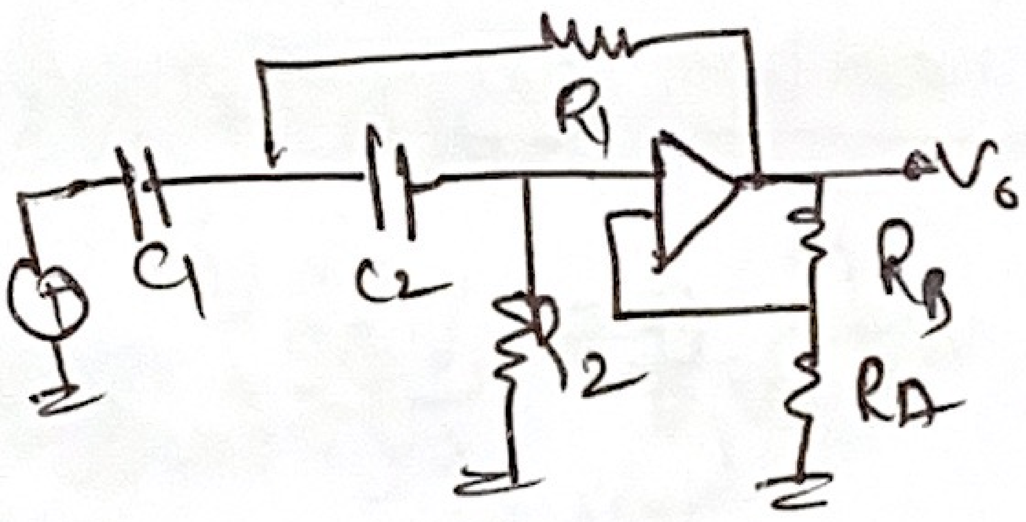
If $R_1 = R_2 = R$ $C_1 = C_2 = C$

$Q = \frac{1}{3-K}$ $\omega_0 = \frac{1}{RC}$ $R_B = (K-1)R_A$ $K = 3 - 1/Q$.

High pass KRC filter

Q = same LPF

$H_{0HP} = K$ $H(s) = H_{0HP} \cdot H_{LP}$



2nd order Lowpass filter

$$Gain(A_V) = 1 + \frac{R_A}{R_B}$$

$$f_C = \frac{1}{2\pi\sqrt{R_1R_2C_1C_2}}$$ R, C value are differ

$$f_C = \frac{1}{2\pi RC}$$ R, C values are same

active High pass

$$f_0 = \frac{1}{2\pi RC}$$

1st order High pass

$$Gain(A_V) = 1 + \frac{R_f}{R_1}$$

$$\frac{V_0}{V_i} = \frac{A_V(f/f_C)}{\sqrt{1+(f/f_C)^2}}$$ $Gain(dB) = 20\log\frac{V_0}{V_i}$

active band pass (cascading connect)

$f_L < f_H$, wide band pass
$f_L > f_H$, Narrow band pass

voltage gain High pass

$$V_0/V_{in} = [A_{max} \times (f/f_C)] / \sqrt{1+(f/f_C)^2}$$

voltage gain for Lowpass

$$V_0/V_{in} = A_{max} / \sqrt{[1+(f/f_H)^2]}$$

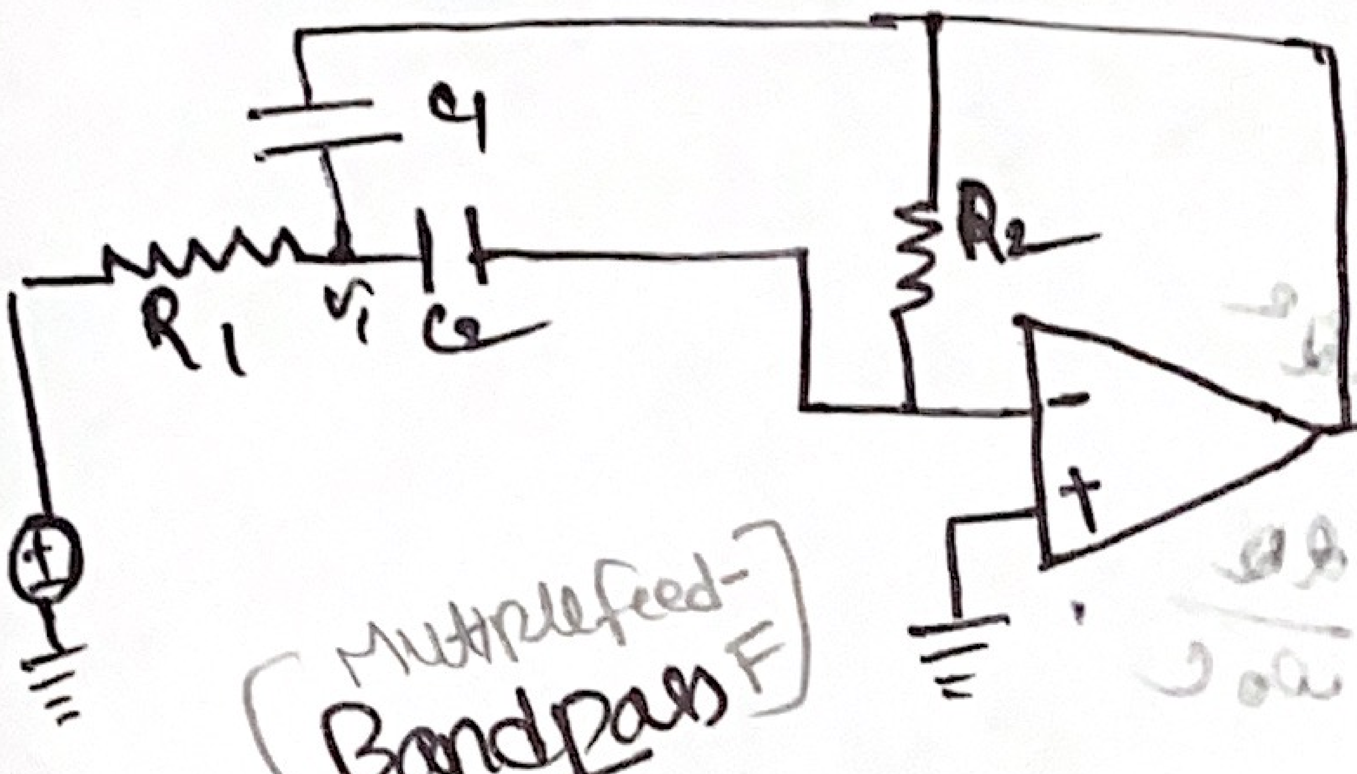
$A_{max} = A_{max1} \times A_{max2}$

$Q<0$, wide band pass
$Q>10$, Narrow band pass

Q = Quality factor

3/22

Multiple Feedback Filters.



① Inverting Amp.

$V_0/V_i = \frac{-}{①}$

op-amp acts as differentiator wrt V_i. so $[V_0 = -S R_2 C_2 V_1]$

$V_0 = -R_2 C_2 S V_1$

$V_1 = \frac{-V_0}{S R_2 C_2}$

Node eqn at V_1

$\frac{V_i - V_1}{R_1} + \frac{0 - V_1}{\frac{1}{SC_2}} = \frac{V_1 - V_0}{\frac{1}{SC_1}}$.

$\frac{V_i - V_1}{R_1} - SC_2 V_1 = SC_1(V_1 - V_0)$

$\frac{V_i}{R_1} = SC_1(V_1 - V_0) + SC_2 V_1 + V_1/R_1$

$\frac{V_i}{R_1} = -V_0\left[SC_1 - \frac{SC_1 V_1}{V_0} - \frac{SC_2 V_1}{V_0} - \frac{V_1}{R_1 V_0}\right]$.

$\frac{V_i}{\cancel{R_1}} = -V_0\left[\frac{1 + SR_1C_1 + S^2 R_1 C_1 R_2 C_2 + SR_1C_2}{SC_2 R_2 \cancel{R_1}}\right]$

$\frac{V_0}{V_i} = \frac{-SC_2 R_2}{D}$

$$\boxed{H(j\omega) = \frac{-j\omega C_2 R_2}{1 + j\omega R_1(C_1 + C_2) + -\omega^2 R_1 R_2 C_1 C_2}}$$

Take)

$\omega R_1 R_2 C_1 C_2 = \left(\frac{\omega}{\omega_0}\right)^2$

$\Rightarrow \boxed{\omega_0 = \frac{1}{\sqrt{R_1 R_2 C_1 C_2}}}$

$j\omega R_1(C_1 + C_2) = \left(\frac{j\omega}{\omega_0}\right) / Q \Rightarrow$ $\boxed{Q = \frac{\sqrt{R_2/R_1}}{\sqrt{C_1/C_2} + \sqrt{C_2/C_1}}}$

$\Rightarrow -j\omega C_2 R_2 = H_{0dB}^{BP}\left(\frac{j\omega}{\omega_0}\right)/Q$.

$H_{0dB}^{BP} = \frac{-R_2/R_1}{1 + C_1/C_2}$

$C_1 = C_2 = C \Rightarrow$

$$\omega_0 = \frac{1}{\sqrt{R_1 R_2 C^2}}$$

$$Q = 0.5\sqrt{\frac{R_2}{R_1}}, \quad H_{0dB} = -2Q^2$$

$$R_1 = \frac{1}{2\omega_0 Q C} \; ; \; R_2 = \frac{2Q}{\omega_0 C}$$

Ex: Design a multiple feedback filter with f_0 at 1KHZ, $Q = 10$, $H_0 = 20$ Db (DC gain).

$\omega_0 = \frac{1}{2\pi f}$

$\omega = \frac{1}{2\pi}$

$$R_1 = \frac{1}{2\omega_0 Q C} = \frac{2\pi \times 10^3}{2 \times 4 \times 10 \times \pi}$$

$= \pi/100$

$= 10\pi$

$= 36.34$

$$R_2 = \frac{2Q}{\omega_0 C} = \frac{1 \cdot 2 \times 10}{\frac{1}{2\pi \times 10^3}} = 2 \times \pi \times 10^3$$

$= 64 \times 10^3$

$H_0 < 2Q^2$, then

$$R_1 = \frac{Q}{H_0 \omega_0 C} \qquad R_2 = \frac{R_1}{(2Q^2 / H_{0dB}) - 1}$$

16/03/22

Filter approximations:

(i) Intro to different type of filters to Approx

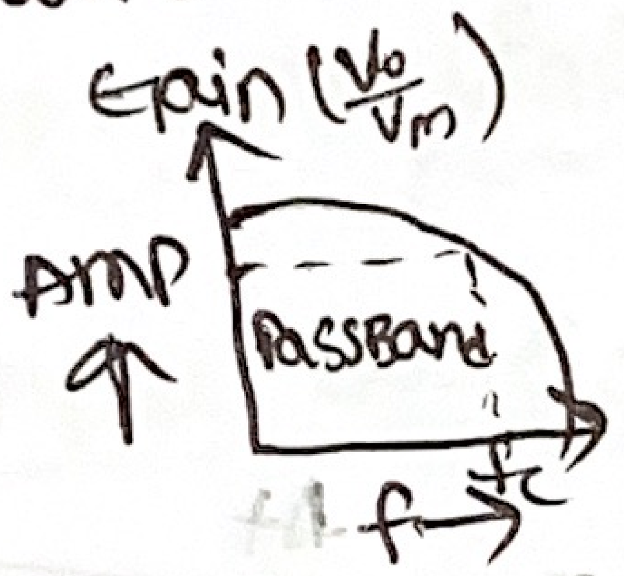
(ii) Butterwort Response.

(a) Filter design Approximation

(b) Design of first order BW filter

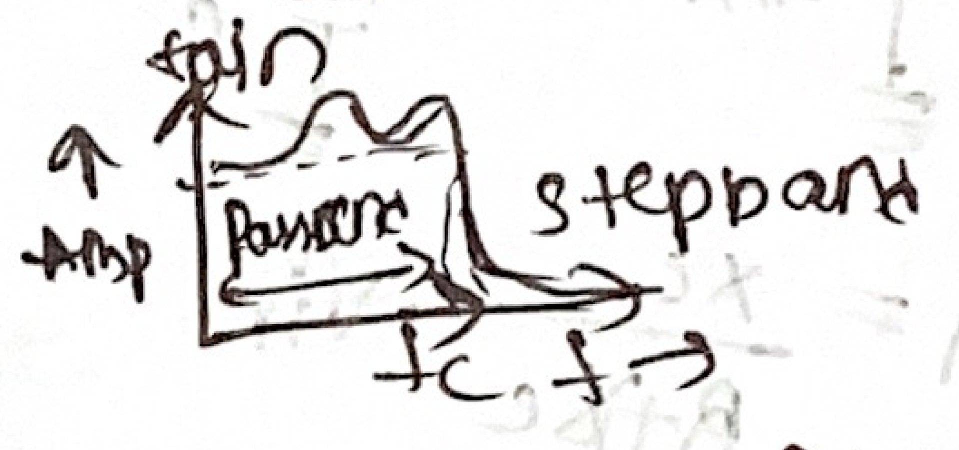
(c) Design of higher order BW Filter ex.

[1] Butterworth Filter



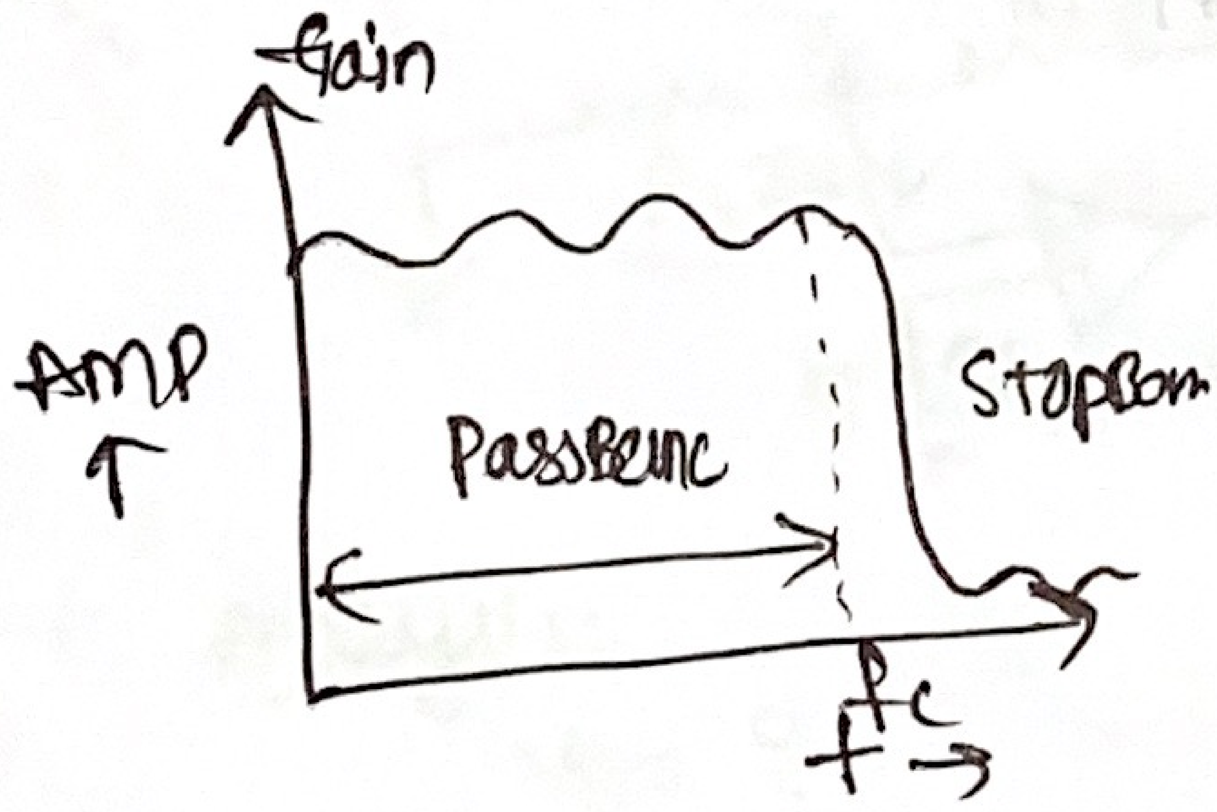
1. Maximally flat approximation
2. Flat band: flat response
3. Roll of rate ⇒ 20n dB/decade
4. Also, no ripple in stop band

[2] Cheby Filter,



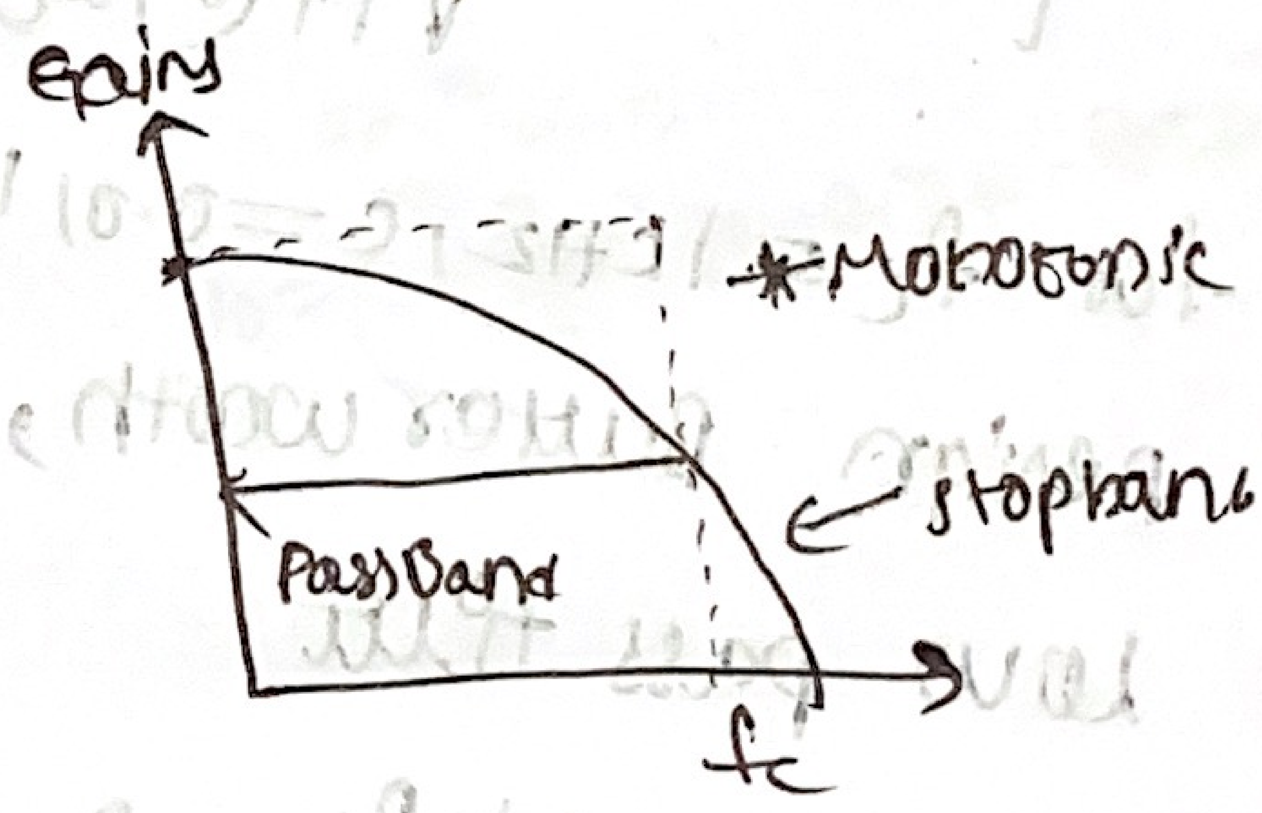
1. No. of ripple = n/2
2. Ripple pass band
3. Transition very fast.

[3] Elliptical Filter



1. Fastest transition
2. Ripple in passband & stopband

[4] Bessel filter



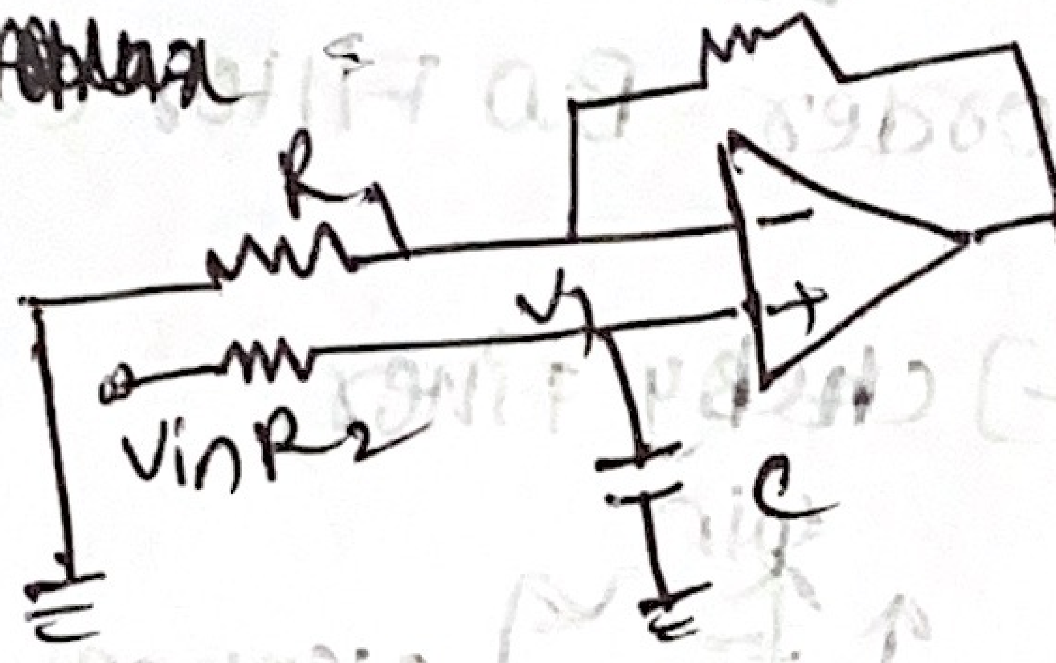
1. No ripple in pass band & stop band
2. phase frequency relation linear.
3. constant delay [slow transition].

Higher order Butterworth filter Design:

The gain of the Butter~~worth~~worth approximation

$$\rightarrow |H(j\omega)| = \frac{1}{\sqrt{1+\epsilon^2(\omega/\omega_C)^{2n}}}$$

(ϵ = coefficient determine maximum passband variation)



$$V_0 = \left(1+\frac{R_f}{R_1}\right)V_1$$

$$V_1 = \frac{X_C}{R+X_C} \cdot V_{in}$$

$$\rightarrow \boxed{\frac{V_0}{V_{in}} = A_f / 1+j\,\omega/\omega_c} \text{ or } \left|\frac{V_0}{V_{in}}\right| = \frac{A_f}{\sqrt{1+(\omega/\omega_c)^2}}$$

$A_f = 1+R_f/R_1 \quad \omega_c = 1/RC$

As per butterworth approx ~~tercet~~;

$$|H(j\omega)| = \frac{1}{\sqrt{1+(\omega/\omega_c)^2}} \text{ for first order, } N=1$$

for $f_c = 1kHz$, $C = 0.01 \mu f$, $R\,A_f = 2$

Design Butterworth,

low pass filter

$$A_f = 1+\frac{R_f}{R_1} = 2.$$

General low pass equation transfer function.

$$\rightarrow H_{LP}(j\omega) = \frac{1}{1-(\omega/\omega_c)^2+(j\omega/\omega_c)^2}$$

2nd order Butterworth LPF with $f_c = 1kHz$

$$\frac{V_0}{V_1} = \frac{K}{(sCR)^2+sCR(3-K)+1} \quad \boxed{K = \text{gain}}$$

$$\rightarrow \boxed{H(s) = \frac{1}{1+R_1R_2C_1C_2s^2+[(1-K)R_1C_1+R_1C_2+R_2C_2]s}}$$

4^{th} order Butterworth LPF with $f_c = 1kHz$,

$$\frac{V_o}{V_i} = \frac{K}{(sCR)^2 + sCR(3-K) + 1}$$

$K = K_1 K_2 = 2.575$ (Resulting gain)

$K_1 = 2.235$

$K_2 = 1.152$.

17/03/22

Op-Amp limitations:

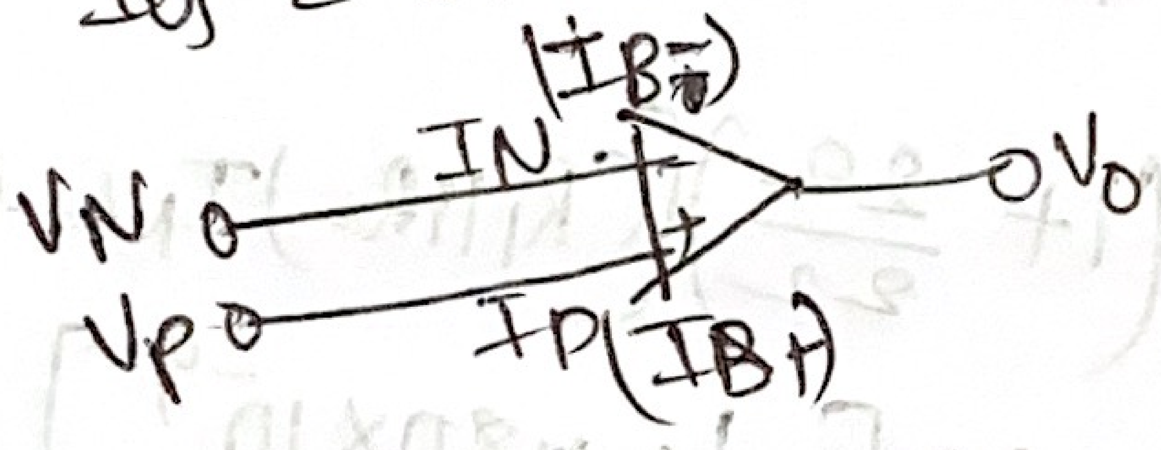
① Input bias & offset current:

I_b = Bias current

I_{os} = Offset current.

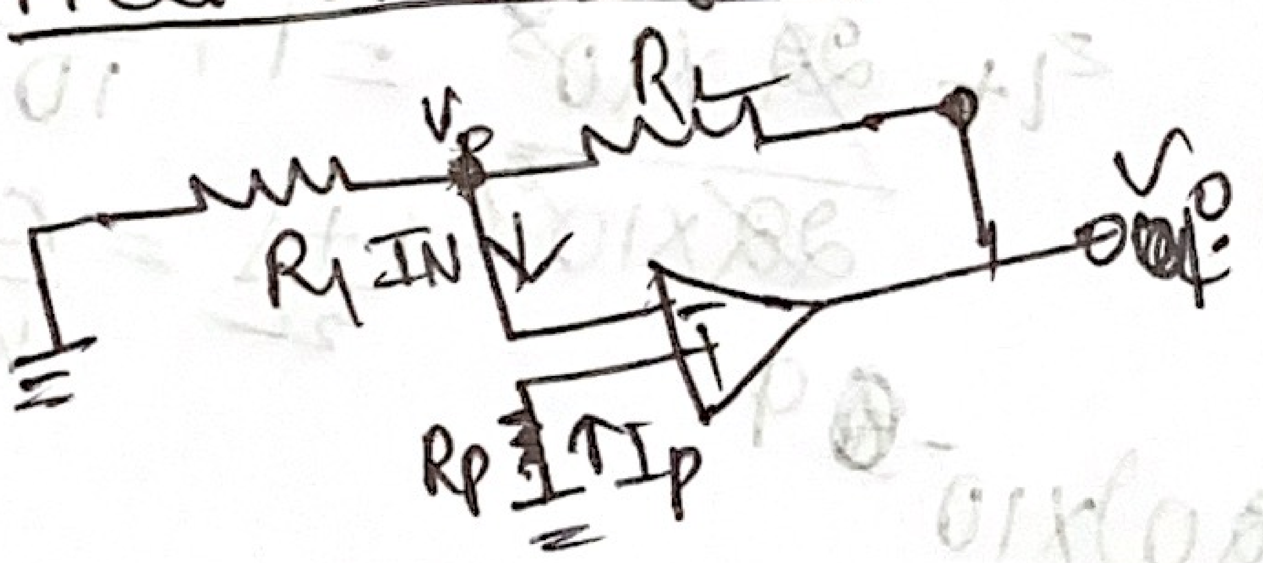
$I_B = \frac{I_P + I_N}{2}$ (average bias current)

$I_{os} = I_P - I_N$. (Difference offset current)



$\frac{I_P + I_N}{2}$

Effect of I_{os} & I_B:



$I_{os} = I_p - I_N$

$I_p = I_{os} - I_N$

$V_o = \left(1 + \frac{R_2}{R_1}\right)[(R_1 \| R_2) I_N - R_P I_P]$

Analysis:

$V_O = ?$

Apply superposition theorem.

$V_P = -R_P I_P$.

$V_O = \left(1 + \frac{R_2}{R_1}\right)V_P + I_N R_2$

$V_O = -\left(1 + \frac{R_2}{R_1}\right) I_P R_P + I_N R_2$

$V_O = \underbrace{\left(1 + \frac{R_2}{R_1}\right)}_{\text{DC noise gain}}[(R_1 \| R_2) I_N - R_P I_P]. = E_o.$

Try to minimize this value else error will be more.

$$E_0 = \left(1+\frac{R_2}{R_1}\right)\left\{\left[(R_1 \| R_2) - R_p\right] I_B - \left[(R_1 \| R_2) + R_p\right]\frac{I_{os}}{2}\right\}.$$

$R_p = R_1 \| R_2$ (If we install dummy resistance)

$$E_0 = \left(1+\frac{R_2}{R_1}\right)(R_p \cdot I_{os}).$$

(Q1) In the circuit shown in previous page, $R_1 = 22K\Omega$, $R_2 = 2.2M\Omega$, let the OPAMP rating $I_B = 80nA$, $I_{os} = 20nA$.

(a) calculate E_0 for $R_p = 0$.

(b) Repeat 'A' with $R_p = R_1 \| R_2$

(c) Repeat 'B' with all Resistances simultaneously reduced by the factor of '10'.

(d) Repeat 'c' with OPAMP replaced with $I_{os} = 3nA$.

(a) $E_0 = \left(1+\frac{R_2}{R_1}\right)(R_p \cdot I_{os}) = \left(1+\frac{22}{22}\right)\left[(R_1 \| R_2) I_N - 0\right]$

$= (2)\left[\frac{1}{11} 80 \times 10^{-9}\right]$

$1 + \frac{5.5 \times 10^3}{22 \times 10^3} = 1 + \frac{1}{10^2} = \frac{10^2+1}{10^2}$

$1+\frac{1}{22} = \frac{21}{11}$ $\quad 101 \times 10^{-9}$

(b) $E_0 = (1+1)(R_1 \| R_2)(20) \times 10^{-9}$

$= 2(11)(20) \times 10^{-9} = 440 \times 10^{-9}$

$= 0.04$.

$R_1 = 2.2K$, $R_2 = 0.22M$

(c) $\left(1+\frac{1}{10^2}\right)\left(\frac{1}{11}\right)(20 \times 10^{-9}) = 0.000364$

$(101)(22)(20 \times 10)$

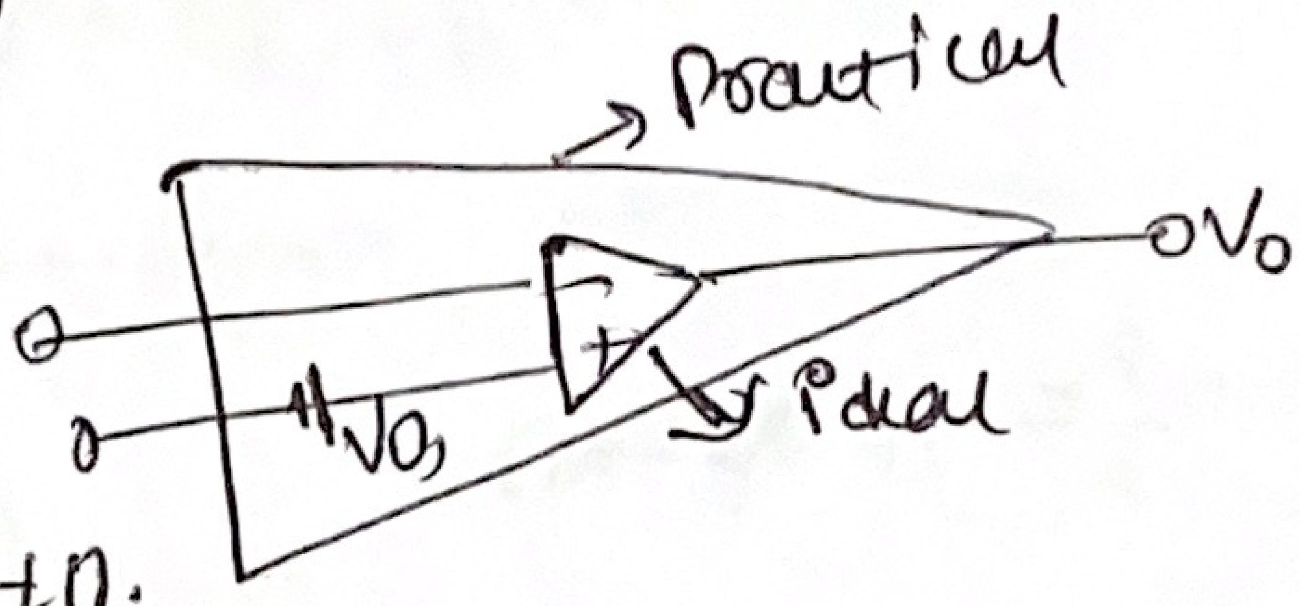
$= 44 mV$.

op-Amp limitations

1. Super beta Input op-amp
2. Fet Input op-amps.
3. Input bias current cancellation.

} I_{os} & I_B

Input offset voltage:



$V_o = a(V_p - V_N)$

$V_p = V_N \Rightarrow V_o = 0.$

V_{os}, $V_p \neq V_N$ (or) $V_o \neq 0.$

$V_o = a(V_p + V_{os} - V_N) = 0.$

$V_N = V_p + V_{os}$,

$V_o = V_{os}(1 + R_2/R_1)$
↳ E_{os}.

Power supply Rejection Ratio:

$V_s + \Delta V_s \Rightarrow V_o$,

$\frac{1}{PSRR} = \frac{\delta V_{os}}{\delta V_s}$ $\mu V/V$

Input offset error & compensation Technique:

$I_{os} \Rightarrow E_o =$

$V_{os} \Rightarrow E_o =$

$E_o = \left(1 + \frac{R_2}{R_1}\right)(R_1 \| R_2) I_{os}$

$E_o = \left(1 + \frac{R_2}{R_1}\right) V_{os}.$

$$E_o = \left(1 + \frac{R_2}{R_1}\right)\underbrace{\left[V_{os} - (R_1 \| R_2) I_{os}\right]}_{E_i}.$$

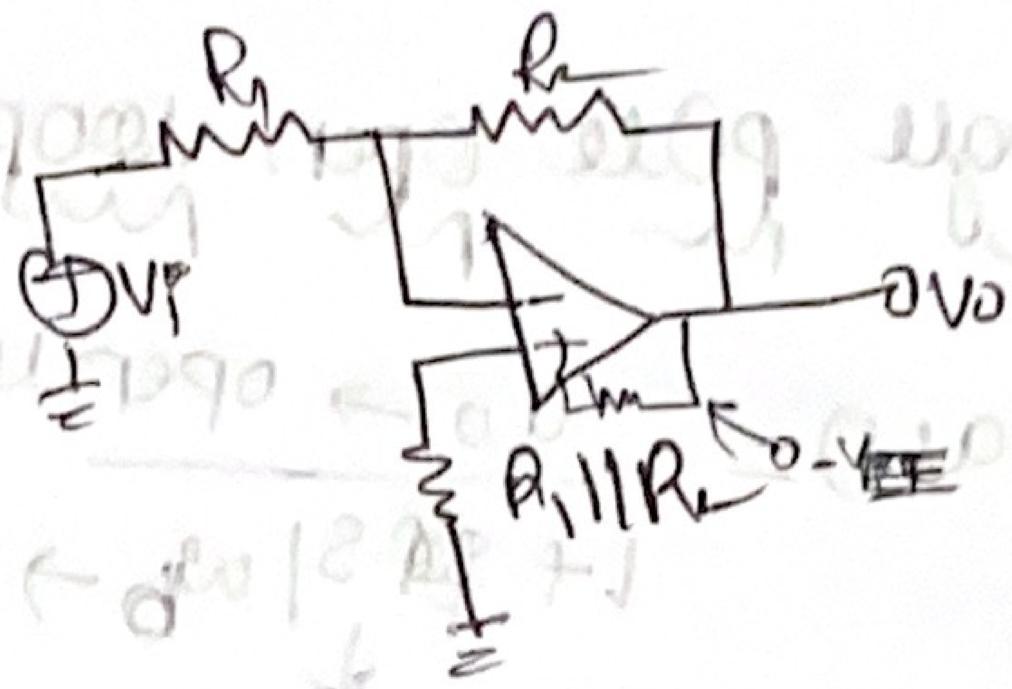
$V_o = A_f V_i + E_o$

$A_f = -R_2/R_1$

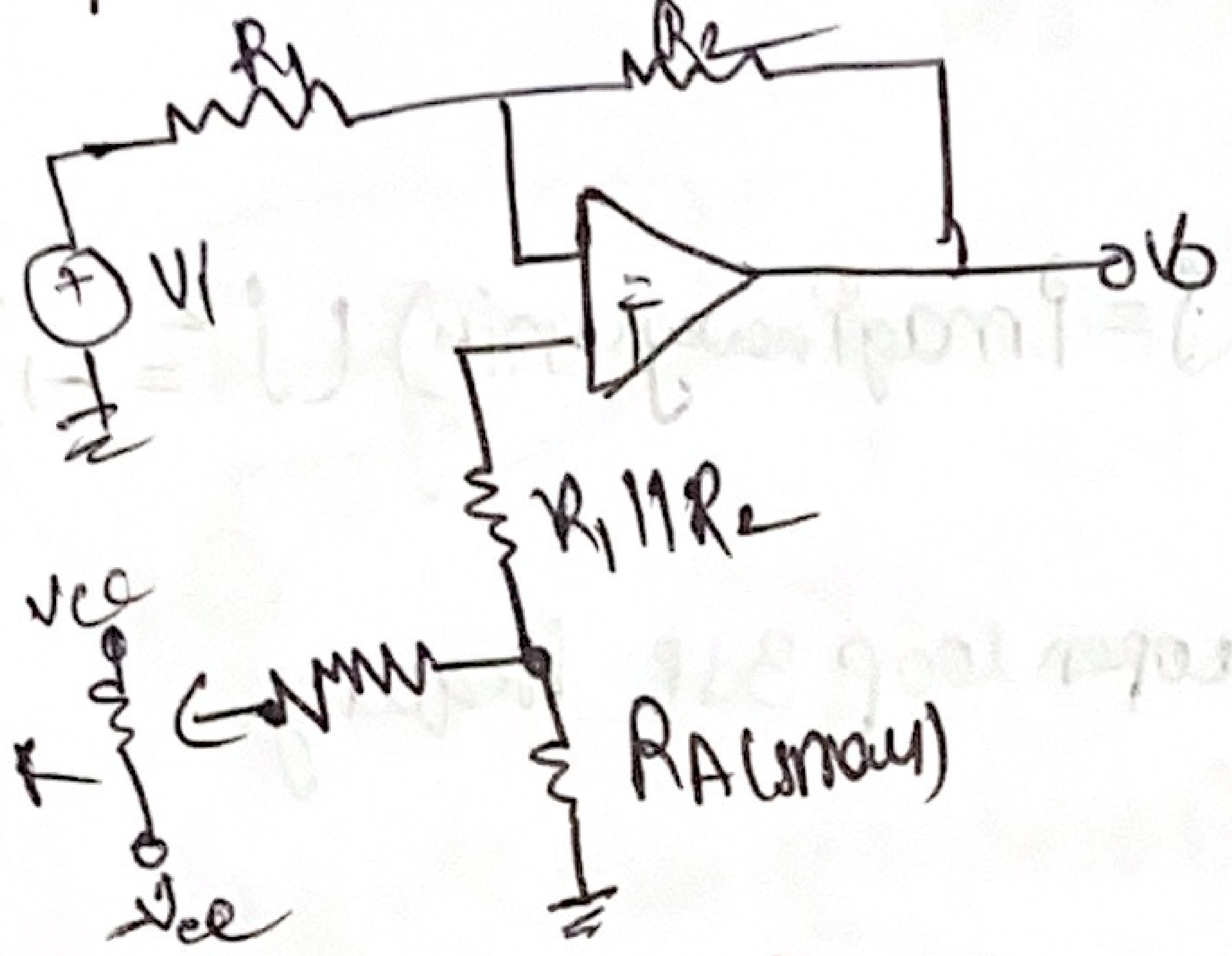
$A_f = 1 + R_2/R_1$

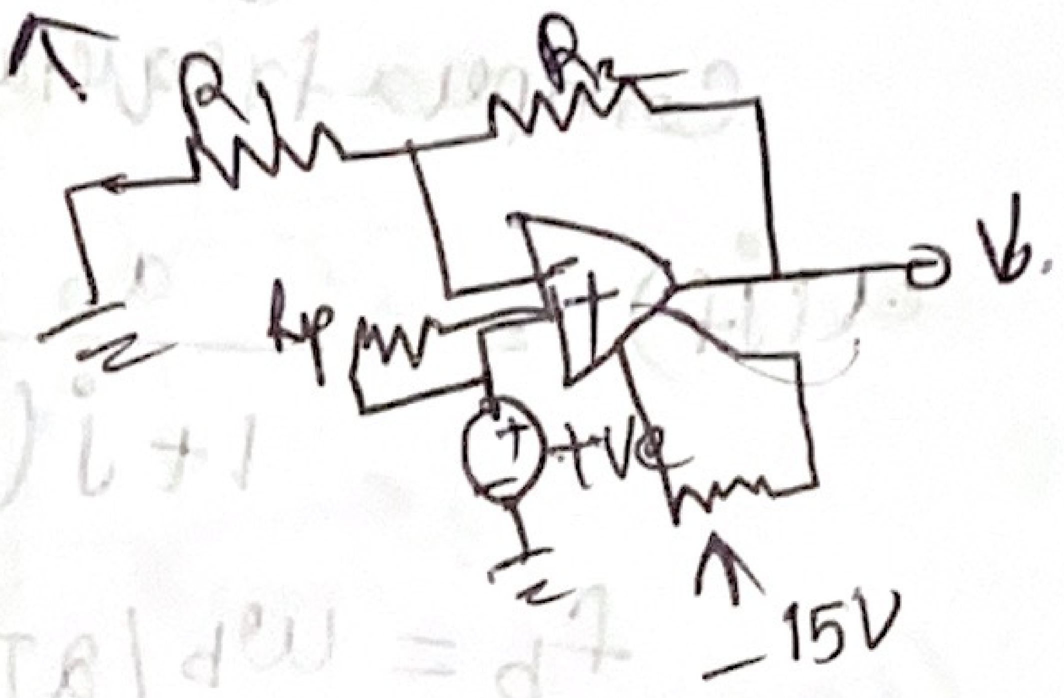
① Internal offset null:

$v_i = 0 \Rightarrow V_0 = 0.$



② external offset null:

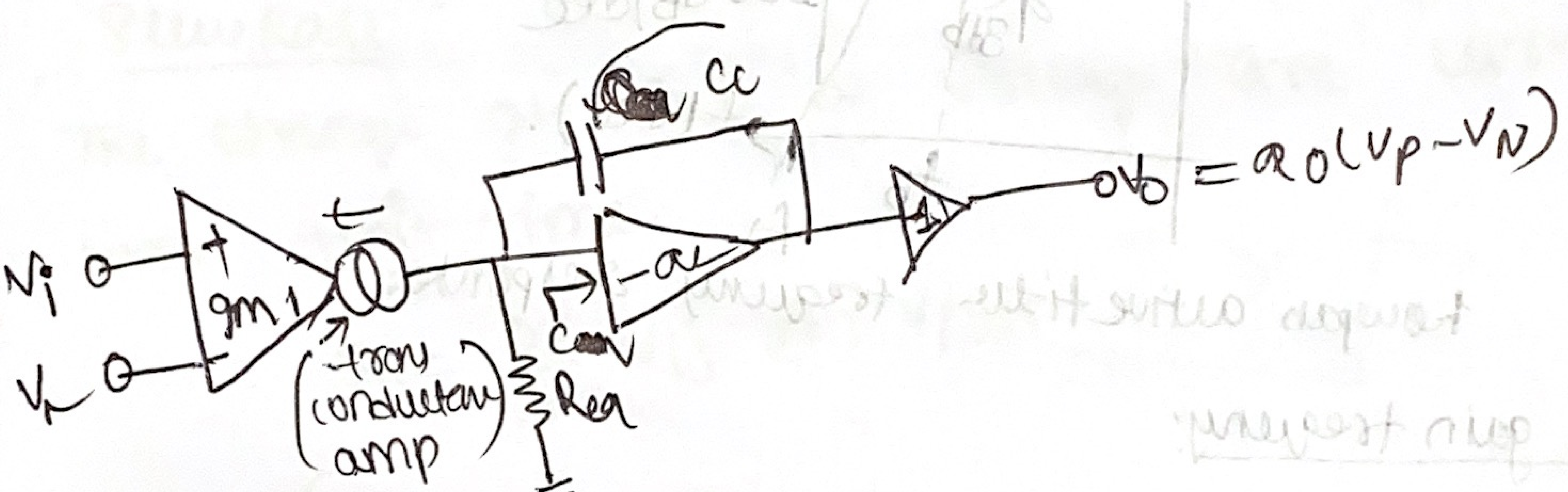




$R_B >> R_C$.

Dynamic opamp limitations:

→ Open door Response.



~~$V_0 = 1 \times -a_2 \times R_{ea} \times$~~

$V_0 = 1 \times -a_2 \times R_{ea} \times -g_{m1}$

$\Rightarrow \boxed{V_0 = a_2 R_{ea} g_{m1} (V_P - V_N)}$

$\boxed{a_0 = a_2 R_{ea} g_{m1}}$

→ $f_b = \frac{1}{2\pi C_{ea} R_{ea}}$, (Roll off frequency) (dominant pole freq)

23/3/22

Single pole open loop gain:

$$a(s) = \frac{a_0}{1 + s/\omega_b}$$

$a_0 \rightarrow$ open loop gain

$\omega_b \rightarrow$ S-plane pole location.

$s \rightarrow$ Complex frequency

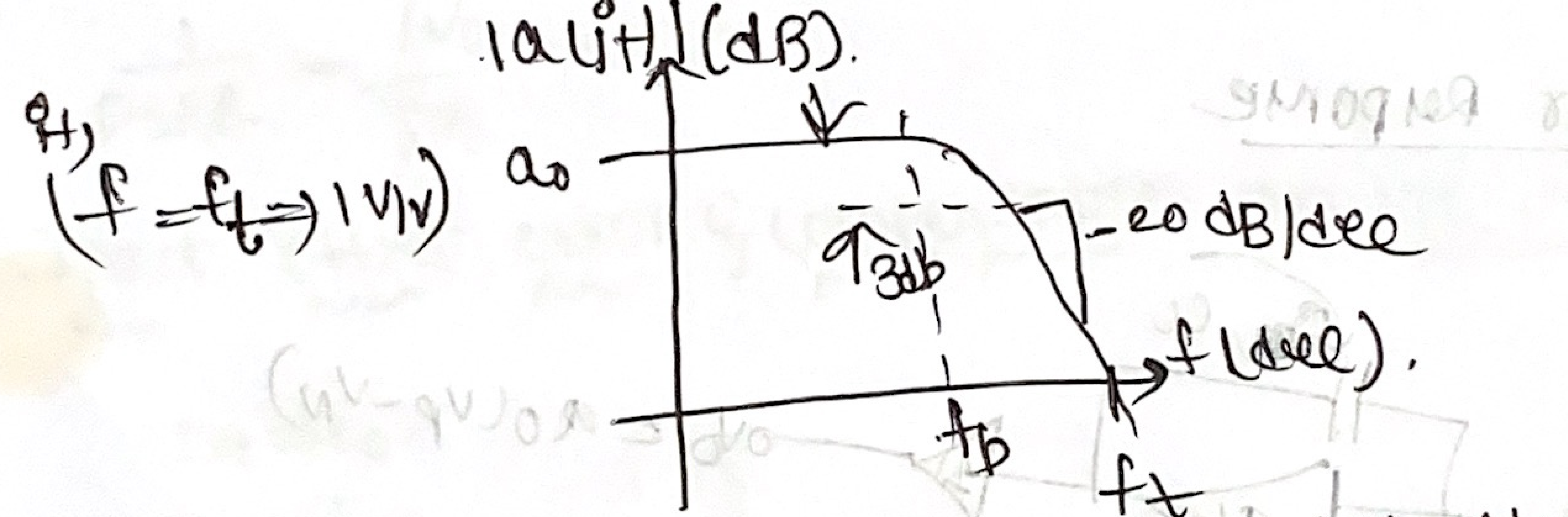
$$a(jf) = \frac{a_0}{1 + j(f/f_b)}$$ (j = imaginary unit). ($j^2 = -1$).

$f_b = \omega_b / 2\pi \rightarrow$ is the open loop 3dB frequency

Magnitude & phase:

$$|a(jf)| = \text{mag}(a(jf)) = \frac{a_0}{\sqrt{1 + (f/f_b)^2}}$$

$$\angle a(jf) = \text{ph}(a(jf)) = -\tan^{-1}(f/f_b).$$



Lowpass active filter frequency responses

Unity gain frequency:

$$1 = \frac{a_0}{\sqrt{1 + (f/f_b)^2}}$$

$f_t >> f_b$

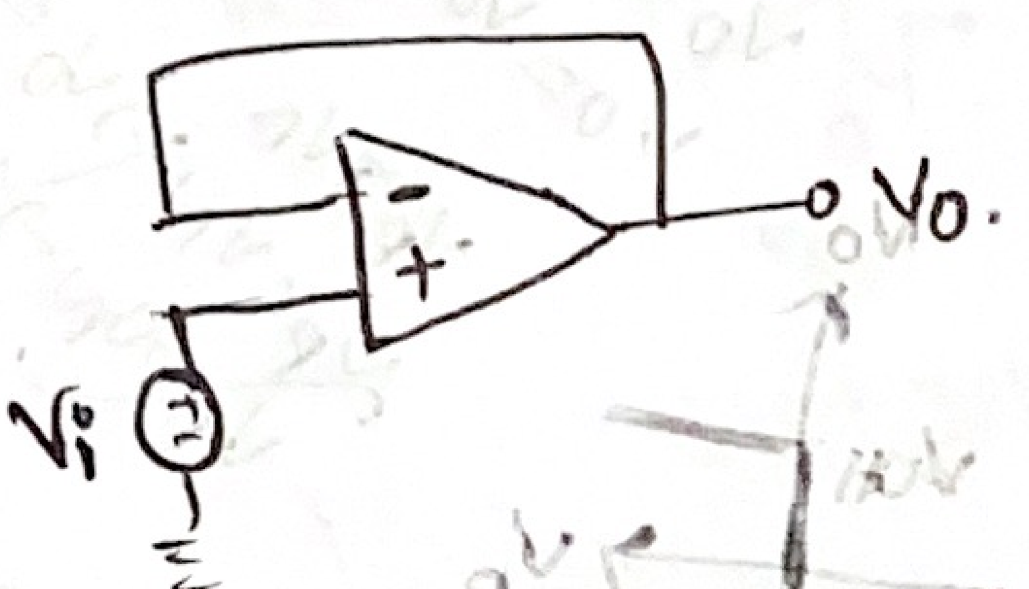
$f_t = a_0 f_b$

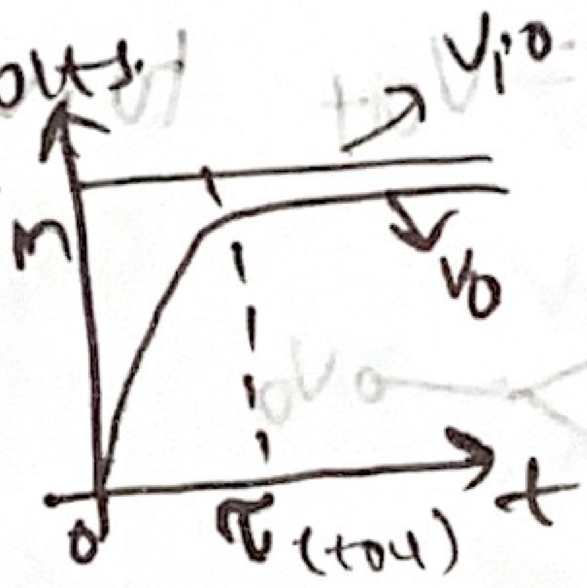
24/3/22

Transient Response:

→ Rise time - T_R

$$A(jf) = \frac{a_0}{1+j\, f/f_b}$$





$$t_R = \tau(\ln 0.9 - \ln 0.1) = \frac{1}{2\pi f_t}(\ln 0.9 - \ln 0.1).$$

$$\tau = \frac{1}{2\pi f_t} \Rightarrow V_o(t) = V_m(1 - e^{-t/\tau})$$

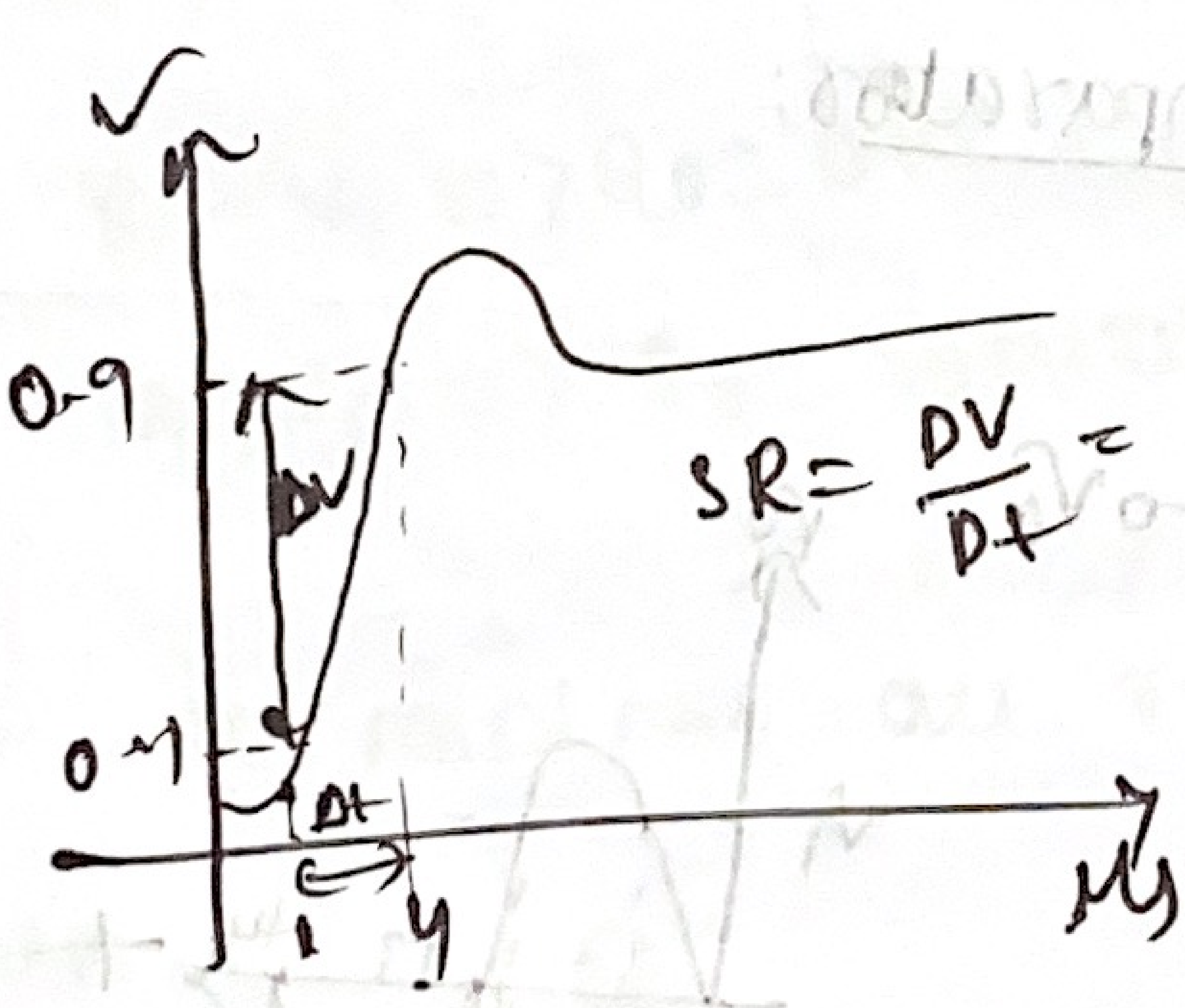
$$\boxed{t_R = \frac{0.35}{f_t}}$$

Slew Rate:

The change of Rate of voltage are current wrt unit time.

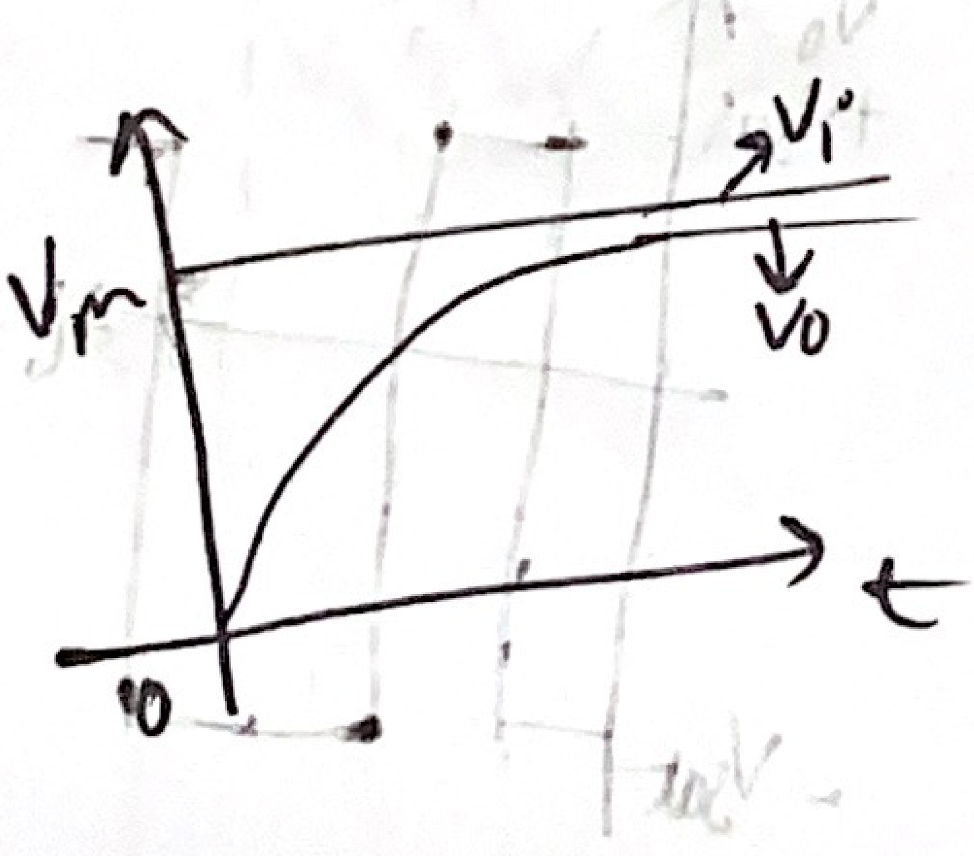
$$SR = \frac{\Delta V}{\Delta t}\ V/s \text{ (or) } V/\mu s.$$

Ex



$$SR = \frac{DV}{Dt} = \frac{0.9-0.1}{4-1} = \frac{0.8}{3}\ V/\mu s$$

Slew rate limitation



$$V_o(t) = V_m(1 - e^{(-t/\tau)})$$

$$\left.\frac{dV_o}{dt}\right|_{t=0} = \frac{V_m}{\tau} \uparrow$$