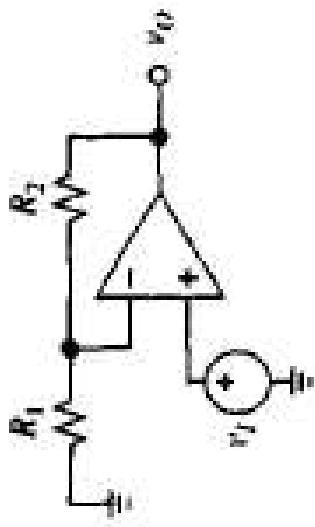


LIC: LECTURE 2

1. Voltage Follower
2. Inverting Amplifier
3. Summing Amplifier
4. Difference Amplifier
5. Integrator using Op-Amp
6. Differentiator using Op-Amp

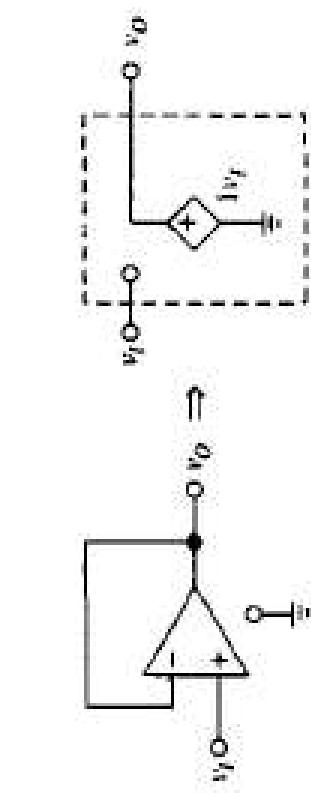
[1] Franco, Sergio. Design with operational amplifiers and analog integrated circuits. Vol. 1988. New York: McGraw-Hill.

VOLTAGE FOLLOWER USING OP-AMP

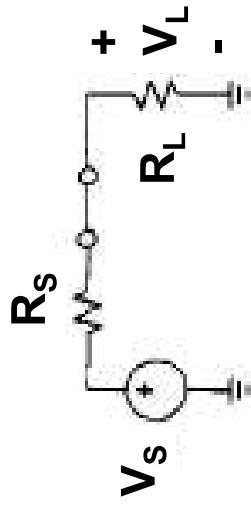
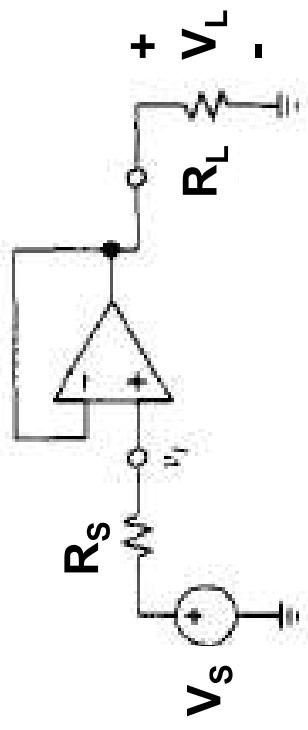


Letting $R_1 = \infty$ and $R_2 = 0$

noninverting amplifier turns it into the *unity-gain* amplifier

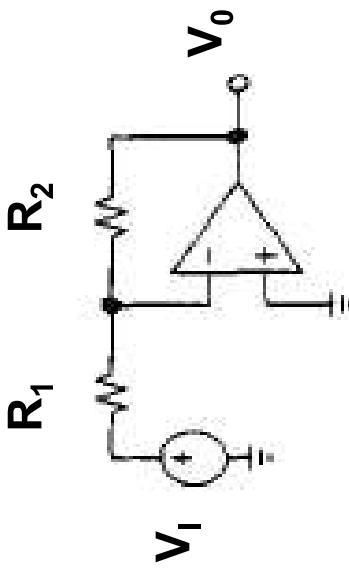


resistance transformerr, $R_i = \infty$ $R_o = 0$



INVERTING AMPLIFIER

$$V_P = 0$$



$$V_N = [R_2/(R_1 + R_2)]V_I + [R_1/(R_1 + R_2)]V_O$$

$$V_N = \frac{1}{1 + R_1/R_2} V_I + \frac{1}{1 + R_2/R_1} V_O$$

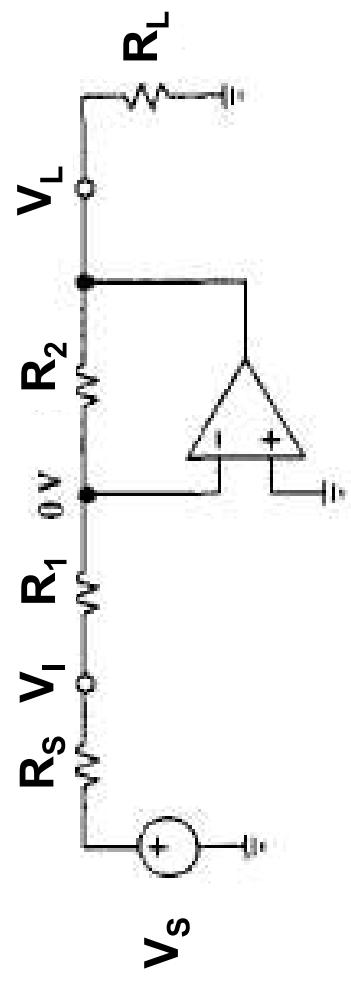
$$V_O = a(V_P - V_N) = a \left(-\frac{1}{1 + R_1/R_2} V_I - \frac{1}{1 + R_2/R_1} V_O \right)$$

$$A = \frac{V_O}{V_I} = \left(-\frac{R_2}{R_1} \right) \frac{1}{1 + (1 + R_2/R_1)/a}$$

$$A_{\text{ideal}} = \lim_{a \rightarrow \infty} A = -\frac{R_2}{R_1}$$

in the limit $a \rightarrow \infty$, V_N would be zero exactly, referred to as *virtual ground*

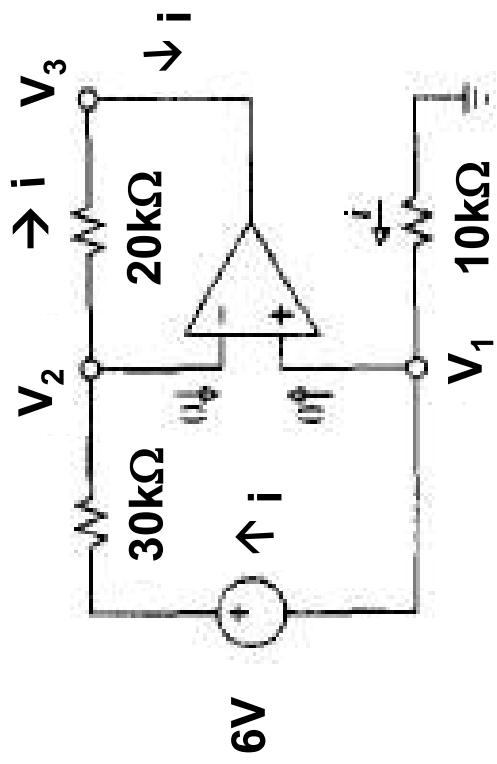
$$R_I = R_1 \quad R_o = 0$$



$$V_I = \frac{R_1}{R_s + R_1} V_S$$

$$\frac{V_L}{V_S} = -\frac{R_2}{R_s + R_1}$$

$$V_L/V_I = -R_2/R_1$$



$$V_1 = -i \times 10k$$

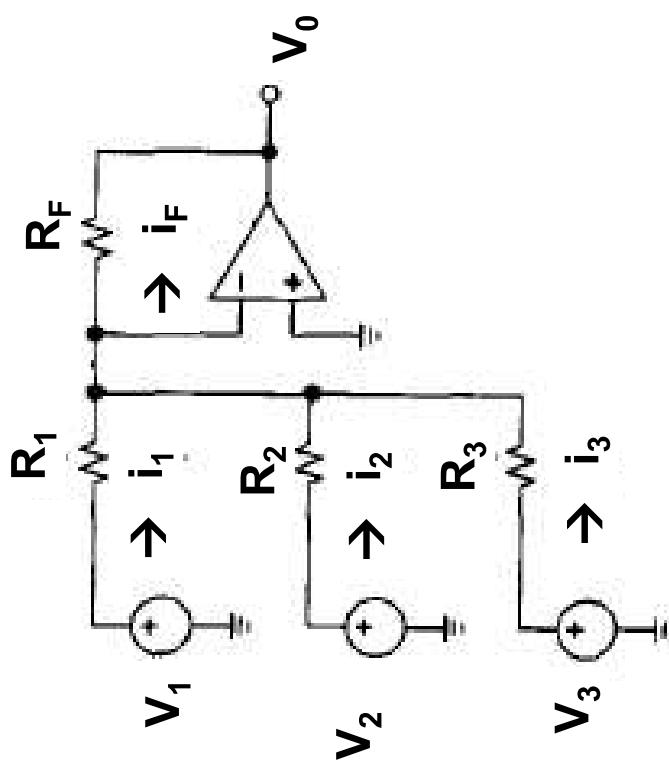
$$V_2 = -i \times 30k + 6 - i \times 10k$$

$$V_2 = V_1 \rightarrow i \times 30k = 6 \rightarrow i = 0.2mA$$

$$V_1 = -0.2mA \times 10k = -2V$$

$$V_3 = V_2 - i \times 20k = -2 - 4 = -6V$$

SUMMING AMPLIFIER



$$i_1 + i_2 + i_3 = i_F$$

$$\frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3} = -\frac{V_O}{R_F}$$

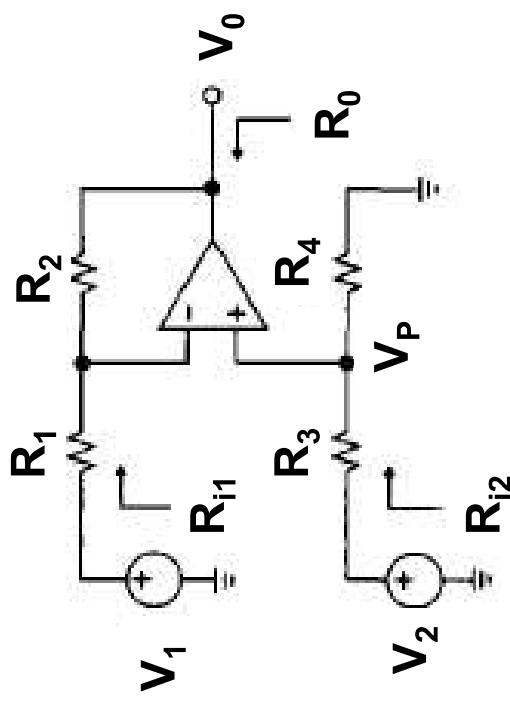
$$V_O = - \left(\frac{R_F}{R_1} V_1 + \frac{R_F}{R_2} V_2 + \frac{R_F}{R_3} V_3 \right)$$

Using standard 5% resistances, design a circuit such that $V_O = -2(3V_1 + 4V_2 + 2V_3)$

$$\frac{R_F}{R_1} = 6 \quad \frac{R_F}{R_2} = 8 \quad \frac{R_F}{R_3} = 4$$

$$\text{If } R_F = 20\text{k}\Omega, \\ R_1 = 3.33\text{k}\Omega \\ R_2 = 2.50\text{k}\Omega \\ R_3 = 5.00\text{k}\Omega$$

DIFFERENCE AMPLIFIER



$$v_{O1} = -(R_2/R_1)v_1 \text{ and } R_{i1} = R_1$$

$$v_{O2} = (1 + R_2/R_1)v_P = (1 + R_2/R_1) \times |R_4/(R_3 + R_4)|v_2$$

$$R_{i2} = R_3 + R_4.$$

$$v_O = \frac{R_2}{R_1} \left(\frac{1 + R_1/R_2}{1 + R_3/R_4} v_2 - v_1 \right)$$

$$\text{When } \frac{R_3}{R_4} = \frac{R_1}{R_2} \quad v_O = \frac{R_2}{R_1} (v_2 - v_1)$$

OP-AMP as INTEGRATOR

- till now, we have used resistors with op-amp's
- So, now suppose if we use the reactive element like inductor or capacitor along with this.

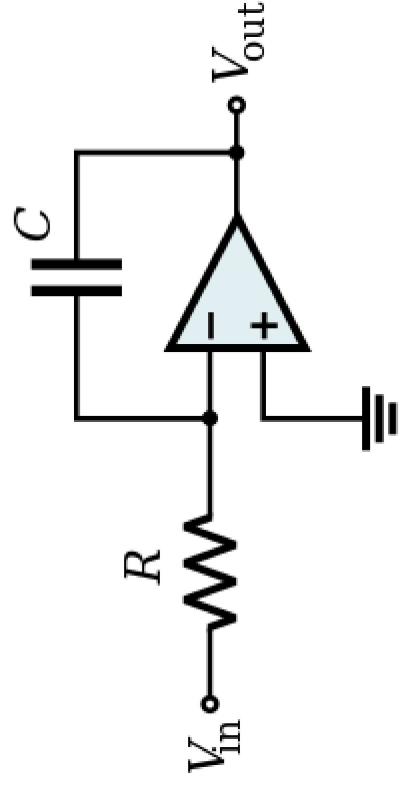
- Here, as the feedback node will act as a virtual ground.
- KCL at inverting node:

$$I_{in} = IC$$

$$\frac{V_{in}-0}{R} = IC$$

$$\begin{aligned}\frac{V_{in}}{R} &= CF \frac{dV_C}{dt} \\ &= CF \frac{d[0-V_{out}]}{dt} \\ \frac{V_{in}}{R} &= -CF \frac{dV_{out}}{dt}\end{aligned}$$

INTEGRATOR USING OP-AMP

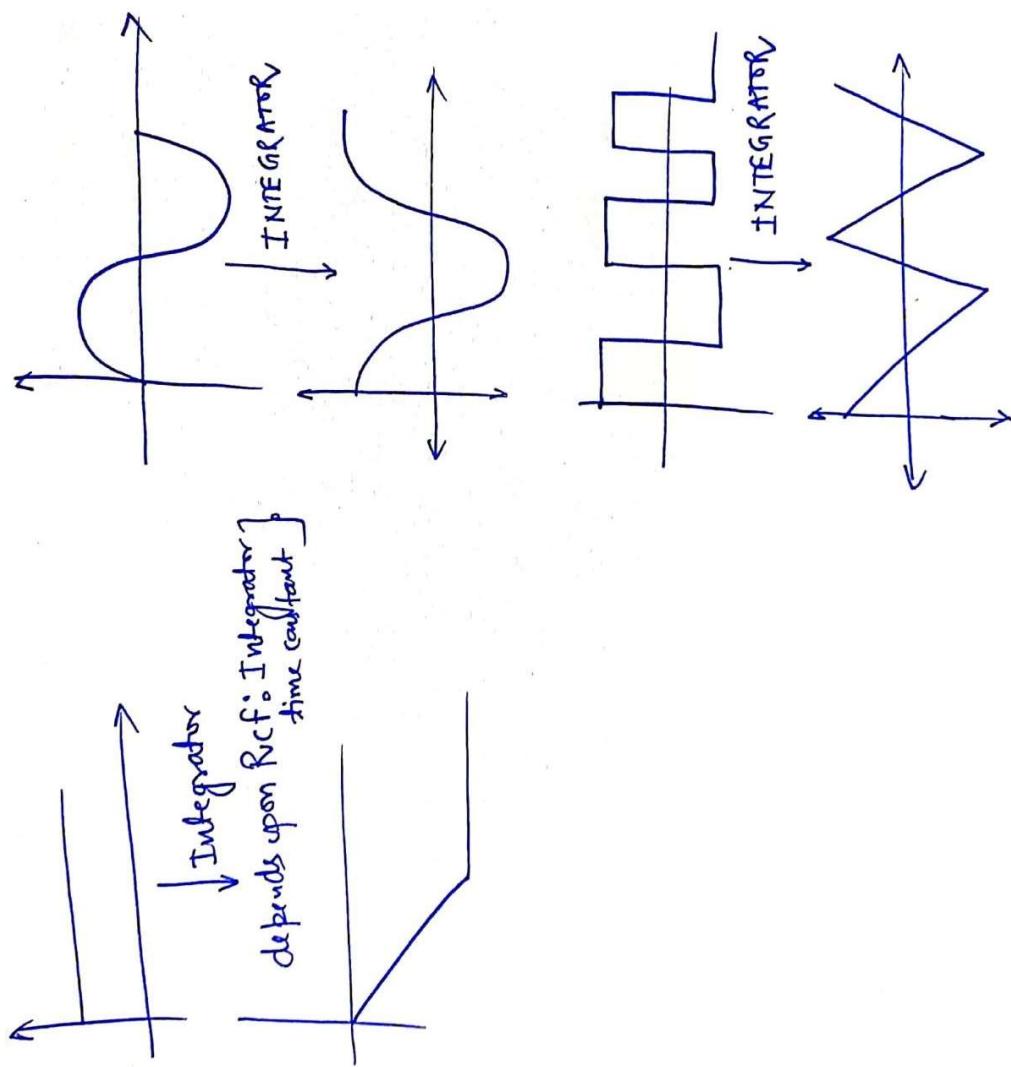


$$\frac{dV_{out}}{dt} = -\frac{1}{RC} \cdot V_{in}(t)$$

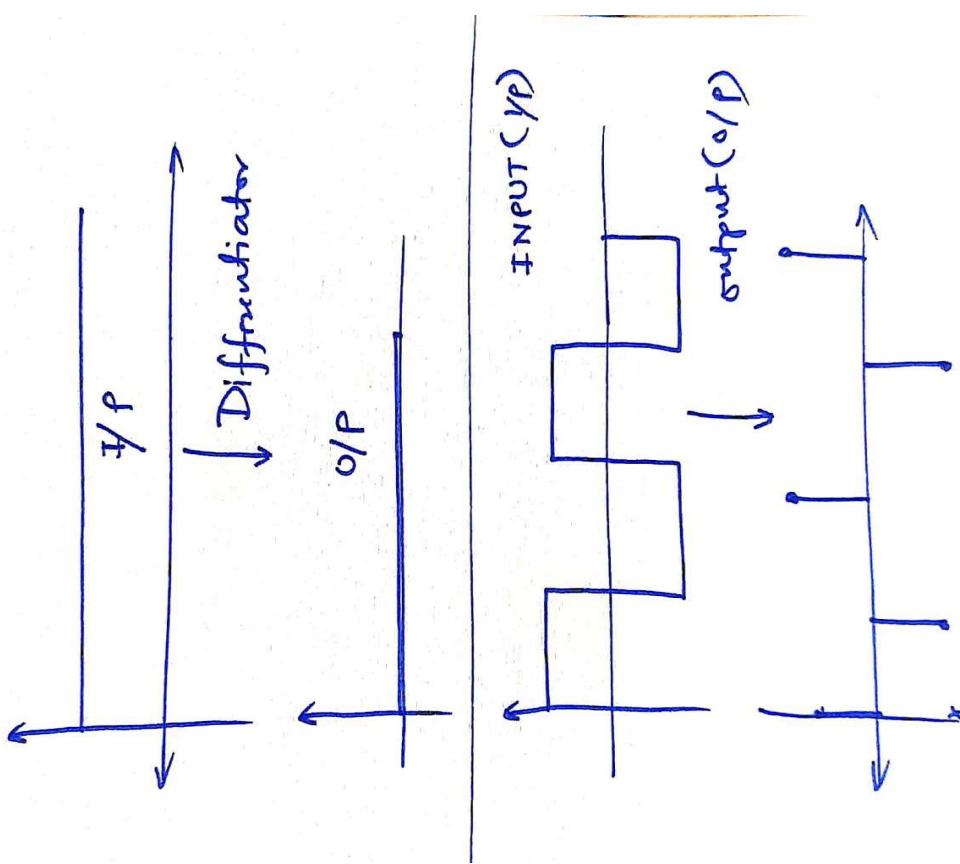
$$\boxed{\begin{aligned}V_{out}(t) &= -\frac{1}{RC} \int V_{in}(t) dt \\ \text{So } V_{out}(t) &= -\frac{1}{RC} \int V_{in}(t) dt + V_{out}(0+)\end{aligned}}$$

if initial output voltage
is $V_{out}(0+)$

EXAMPLES: INTEGRATOR USING OP-AMP



EXAMPLE: DIFFERENTIATOR USING OP-AMP



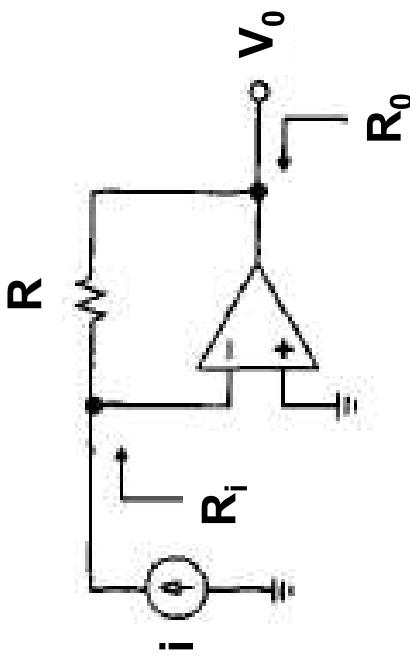
LIC: LECTURE 4

CIRCUITS WITH RESISTIVE FEEDBACK

1. Current-Voltage (I-V) Converters
2. Voltage-Current (V-I) Converters
 - (i) Floating load V-I Converter
 - (ii) Grounded load V-I Converter

Current-Voltage (I-V) Converter

$$V_0 = -iR$$

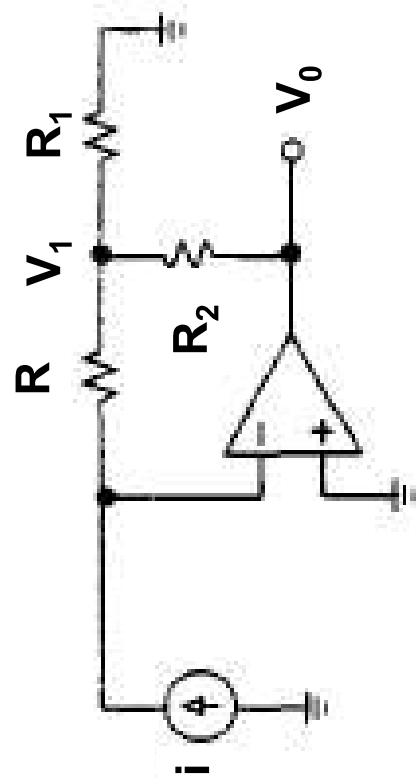


$|V_0/i| = R$ is called the sensitivity

For a sensitivity of 1V/mA, we need $R = 1k\Omega$
Sensitivity of 1V/ μ A, we need $R = 1M\Omega$

→ For large sensitivity, we need large resistors

High-Sensitivity I-V Converters



$$-V_1/R = V_1/R_1 + (V_0 - V_1)/R_2 = 0$$

$$V_1 = -Ri$$

$$i + iR/R_1 + V_0 R_2 + iR/R_2 = 0$$

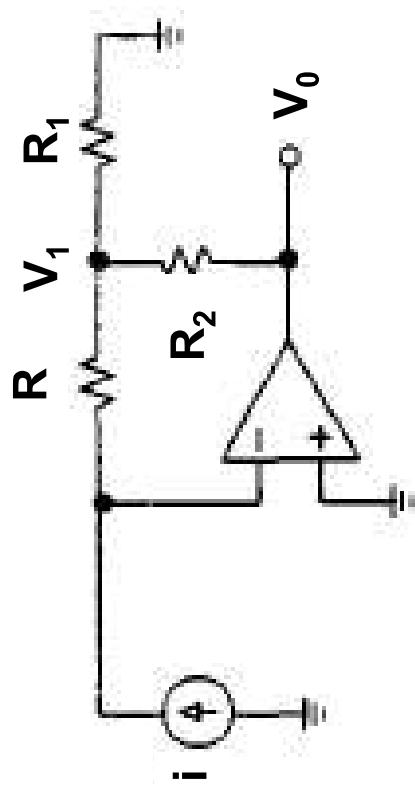
$$V_0 = -kRi$$

$$k = 1 + \frac{R_2}{R_1} + \frac{R_2}{R}$$

increases R by the multiplicative factor k

Example

Design the circuit for a sensitivity 0.1V/nA
Sensitivity $0.1\text{V}/\text{nA} \rightarrow kR = 100\text{M}\Omega$

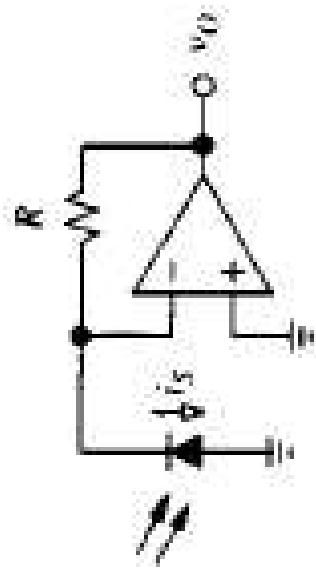


$$k = 1 + \frac{R_2}{R_1} + \frac{R_2}{R} = 100$$

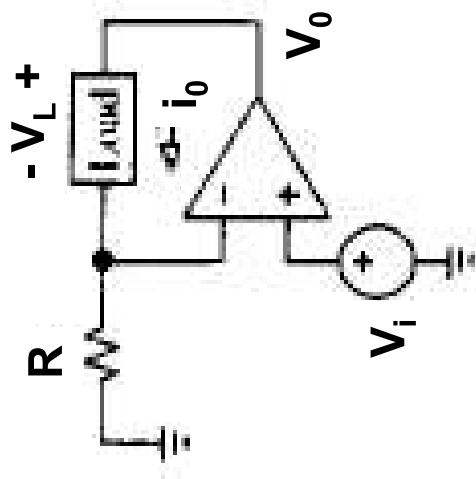
If $R = 1\text{M}\Omega$

If $R_1 = 1\text{k}\Omega$ we get, $R_2 = 99\text{k}\Omega$

Application of I-V converters as photodetectors



Voltage to Current (V-I)Converters Floating Load



Op amp must swing its output to the value $V_0 = V_i + V_L$

Will do as long as $V_{0L} < V_0 < V_{0H}$

Voltage compliance of the circuit is $(V_{0L} - V_i) < V_L < (V_{0H} - V_i)$



$$I_0 = V_i / R \quad V_0 = -V_L$$

voltage compliance is now $V_{0L} < V_L < V_{0H}$

Has higher voltage compliance

drawback i_0 must come from the source

Voltage-Current Converters (Floating Load)

FLOATING LOAD V-I CONVERTER: (Active)

GENERAL DISCUSSION:

* from circuit, $V_P = V_{in}$
 * So, by virtual ground concept
 $\boxed{V_P = V_N = V_{in}}$

- * Let current I_L flows through R_1 & current I_L flows through load R_L .
- * KCL at node V_N :

$$I_L = I_1 + I_N \quad [I_N = 0; I/P impedance of op-amp is very high]$$

$$\boxed{I_L = I_1}$$

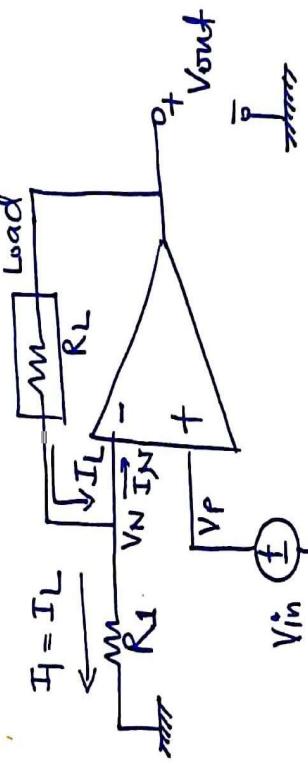
- * from circuit:

$$I_1 = \frac{V_N - 0}{R_1} = \frac{V_N}{R_1}$$

$$\boxed{I_1 = \frac{V_{in}}{R_1}}$$

$$\boxed{I_L = \frac{V_{in}}{R_1} \quad \text{or} \quad \left| \frac{I_L}{V_{in}} \right| = \frac{1}{R_1}}$$

- * $\therefore I_L = \frac{V_{in}}{R_1}$ or $\left| \frac{I_L}{V_{in}} \right| = \frac{1}{R_1}$ $\xrightarrow{\text{sensitivity of the circuit}}$ so, it is named as "transconductance ten p."
 $\boxed{I_L \propto V_{in}}$



i.e. Load current is directly proportional to I/P Voltage!

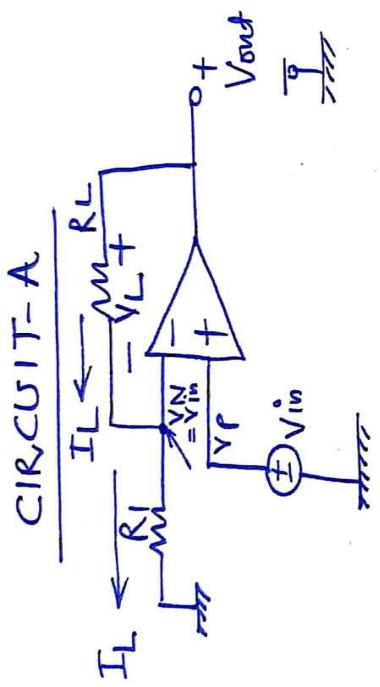
- Also, in the equation $I_L = \frac{V_{in}}{R_1}$,
 load resistance does not appear.
 - This is advantageous as compared
 to passive V-I converters.

- Here, output \rightarrow current

$\boxed{\text{Gain} = \frac{I}{V}}$

$\boxed{\text{Input - voltage} = \text{conductance}}$

Voltage-Current (V-I) Converters (Floating Load)



- In circuit A, the op-amp outputs, whatever the current I_L , it takes to make the following inverting input voltage V_{in} or make $R_1 \cdot I_L = V_{in}$

$$\text{I.e. } \boxed{\frac{I_L}{R_1} = \frac{1}{R_1} \cdot I_{in}} - (i) \quad \begin{matrix} \text{output load} & \text{Input} \\ \text{current} & \text{Voltage} \end{matrix}$$

- So, no matter what the load, the op-amp will force it to carry the current $I_L = \frac{V_{in}}{R_1}$, which depends on the control voltage V_{in} & current setting resistor R_1 , but not on the load voltage V_L .
- from circuit A, we can write

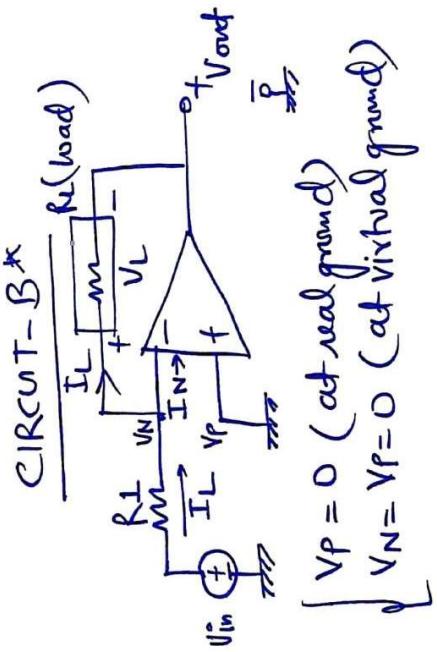
$$\boxed{V_{out} = V_L + V_{in}}$$
 where $V_L = I_L R_L$
- So to achieve this, the op-amp must swing its output to the value $V_{out} = V_{in} + V_L$

Voltage Compliance: It is the range of permissible value of V_L for which the circuit still works properly, before the onset of op-amp saturation.

- Eqn.- (i) holds regardless of the type of load; i.e. it can be linear (resistive load) or it can be non-linear (diode).

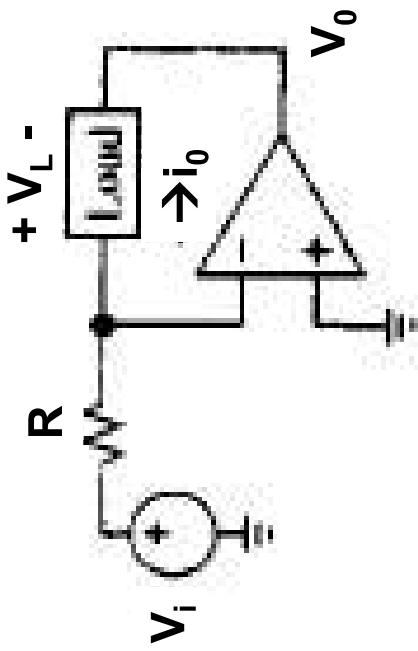
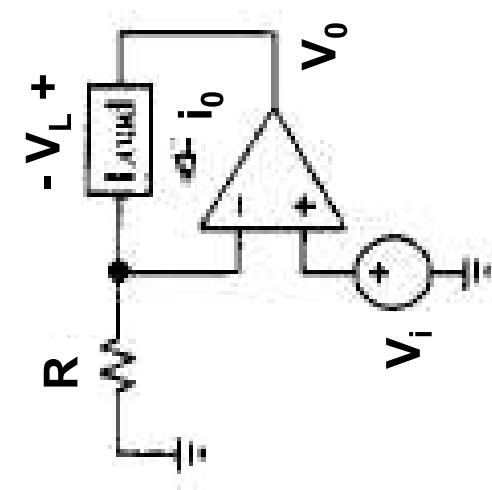
Voltage-Current Converters (V-I) Converters (Floating Load)

- OPamp's are powered with $+V_{CC}$ & $-V_{EE}$ power supplies; so a maximum output voltage achieved in op-amp will be $+V_{sat}$ & $-V_{sat}$
- So, if $+V_{CC} = 15V$ $+V_{sat} \approx 0.9V_{CC}$
 $-V_{EE} = -15V$ $\pm V_{sat} = \pm 13.5V$
- So, Circuit A: $V_{out} = V_{in} + V_L$ is valid as long as $-V_{sat} < V_{out} < +V_{sat}$
- So, voltage compliance of the circuit is $(-V_{sat} - V_{in}) < V_L < (V_{sat} - V_{in})$



- Thus it's output terminal must draw the current:
- $I_L = \frac{V_{in} - V_N}{R_1} = \frac{V_{in} - 0}{R_1} = \frac{V_{in}}{R_1}$
- & it must have the voltage swing as $V_{out} = -V_L$
- Hence, voltage compliance for circuit B:
- $-V_{sat} < V_L < V_{sat}$
- Drawback of circuit B: I_L must come from the source itself.
- OBSERVATIONS: Circuit (A & B)
 - 1) Equation $\frac{V_{in}}{R_1} = I_L$; holds for both regardless of the polarity of the source.
 - 2) The current drawn by circuit A & B, shows current for $V_{in} > 0$, making $V_{in} < 0$ will ~~not~~ simply make the direction of current inverse.
 - 3) So, two converters are thus said to be bidirectional.

In the circuit, $V_i = 5V$, $R = 10k\Omega$, $\pm V_{sat} = \pm 13V$ and a resistive load R_L
 Calculate i_0 , voltage compliance and maximum permissible value of R_L



$$I_0 = 5/10 = 0.5mA$$

$$I_0 = 5/10 = 0.5mA$$

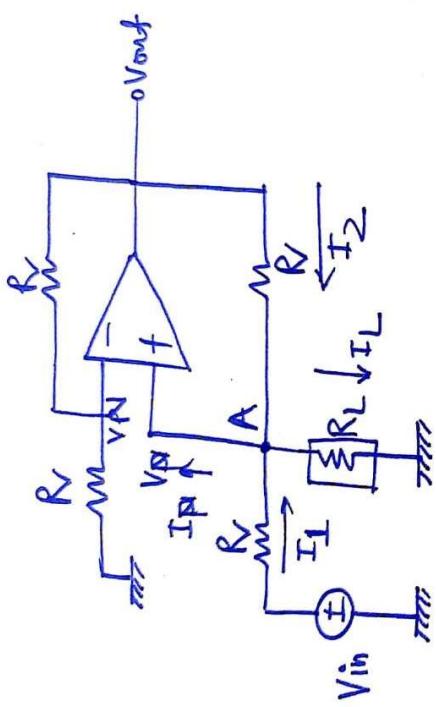
Voltage compliance
 $(-13 - 5)V < V_L < (13 - 5)V$

Voltage compliance
 $-13V < V_L < 13V$

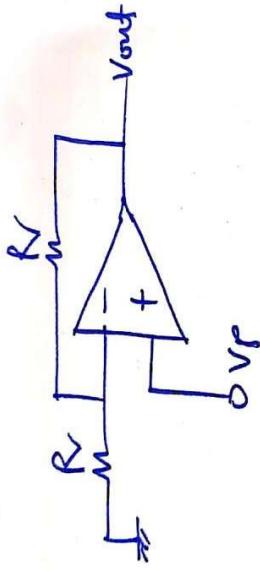
$$R_L < 8/0.5 = 16k\Omega$$

$$R_L < 13/0.5 = 26k\Omega$$

Voltage-Current (V-I) Converters (Grounded Load)



- The circuit is actually looks like a Non-inverting amplifier with input as V_P as shown below.



- Assume that current flowing through R_L is I_L .

- Applying KCL at node A; we get

$$I_1 + I_2 = I_L + I_P \quad (I_P=0)$$

$$I_L = I_1 + I_2$$

$$I_L = \frac{V_{in} - V_P}{R_V} + \frac{V_{out} - V_P}{R_V}$$

$$V_{in} + V_{out} - 2V_P = I_L \times R_V \quad (1)$$

$$\text{So; } V_{out} = \left(1 + \frac{R_V}{R_L}\right) V_P$$

- put this value in eqn.(1)

$$V_{in} + V_{out} - 2V_P = I_L \times R_V$$

$$V_{in} + 2V_P - 2V_P = I_L \times R_V$$

$$V_{in} = I_L \times R_V$$

$$\boxed{I_L = \frac{1}{R_V} V_{in} + \frac{I_L \times V_{in}}{R_V}}$$

$\Rightarrow I_L$ is independent of load resistor R_L

LIC: LECTURE 6

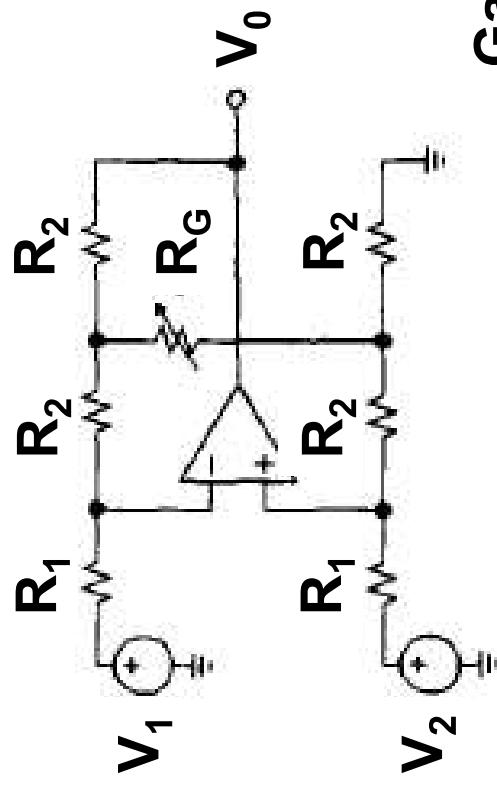
CIRCUITS WITH RESISTIVE FEEDBACK

1. Difference amplifier with variable gain
2. Instrumentation Amplifier (IA)
 - (i) Triple Op-Amp IA
 - (ii) Dual Op-Amp IA
 - (iii) Current-Output IA
3. Transducer Bridge Amplifier

When $\frac{R_4}{R_3} = \frac{R_2}{R_1}$ $V_O = \frac{R_2}{R_1} (V_2 - V_1)$

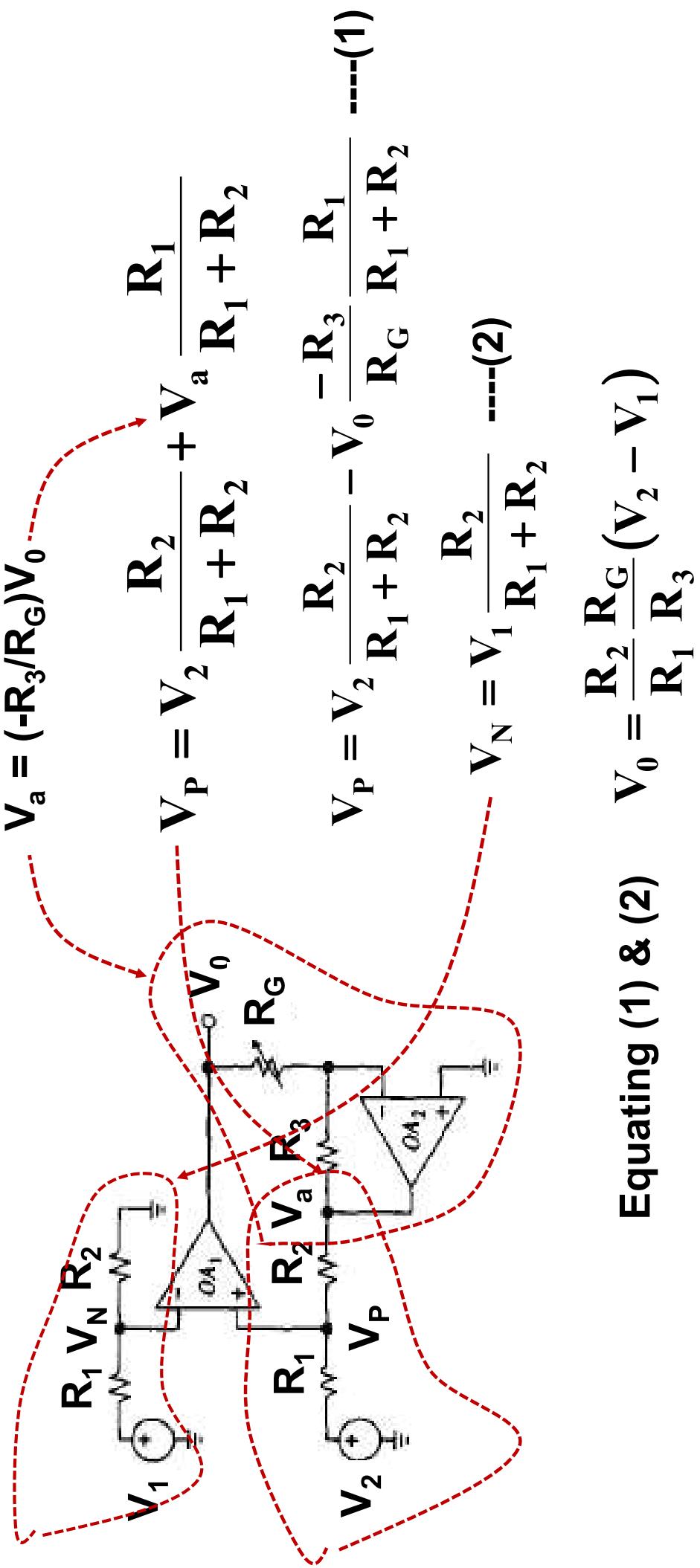
To vary gain we need to vary 2 resistors equally

DIFFERENCE AMPLIFIER WITH VARIABLE GAIN



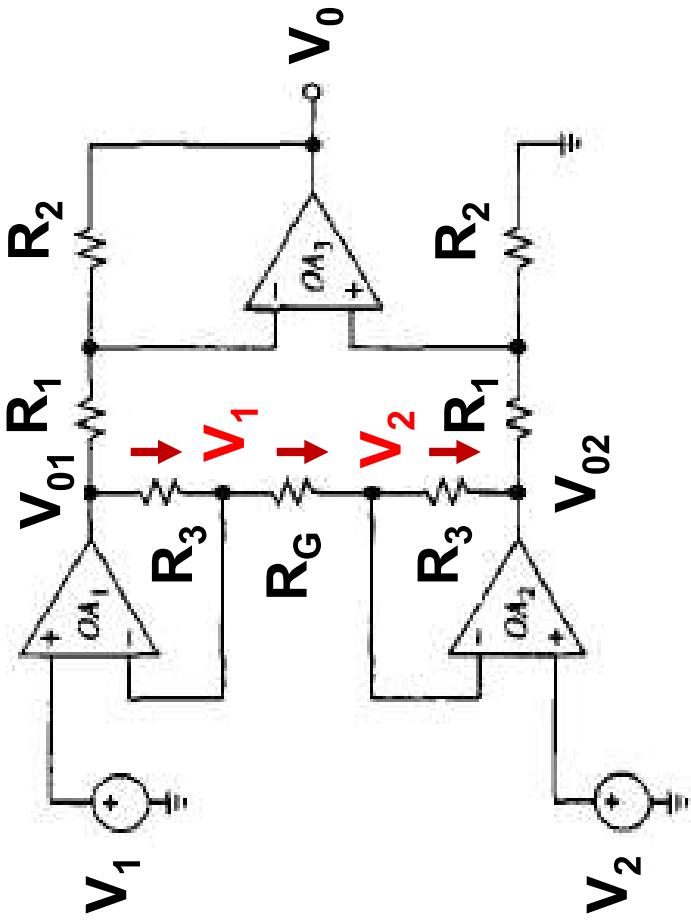
$$V_O = \frac{2R_2}{R_1} \left(1 + \frac{R_2}{R_G} \right) (V_2 - V_1)$$

Gain varies inversely with R_G



Gain varies proportionally with R_G

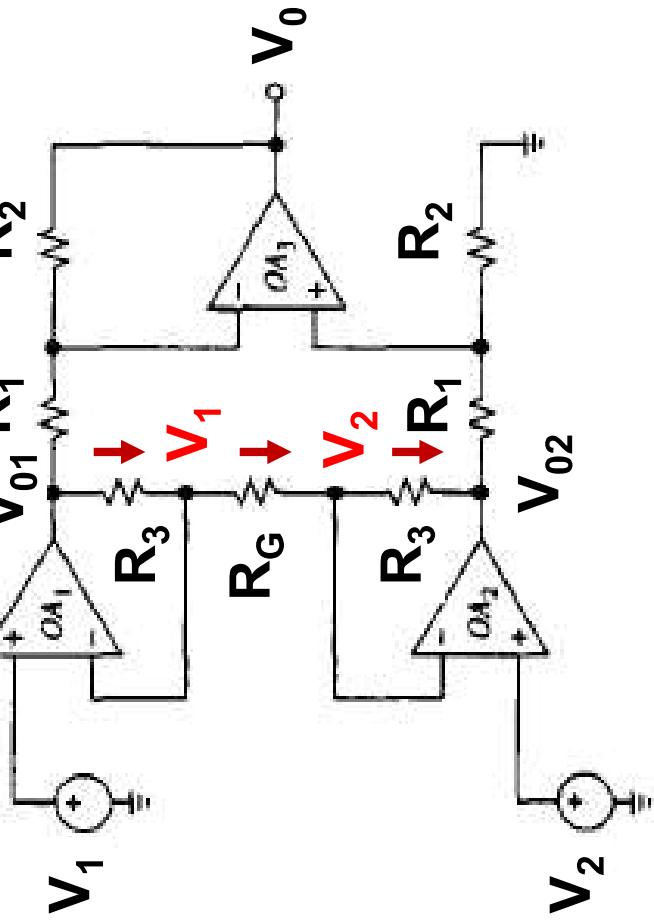
INSTRUMENTATION AMPLIFIER



- An instrumentation amplifier (IA) is a difference amplifier meeting the following specifications:
 - (a) extremely high (ideally infinite) common-mode and differential-mode input impedances;
 - (b) very low (ideally zero) output impedance;
 - (c) accurate and stable gain, typically in the range of 1 V/V to 10^3 V/V; and
 - (d) extremely high common-mode rejection ratio.
- The IA is used to accurately amplify a low-level signal in the presence of a large common-mode component, such as a transducer output in process control and biomedicine.
- For this reason, IAs find widespread application in test and measurement instrumentation—hence the name.

Triple: OP-AMP
INSTRUMENTATION AMPLIFIER

INSTRUMENTATION AMPLIFIER: ANALYSIS

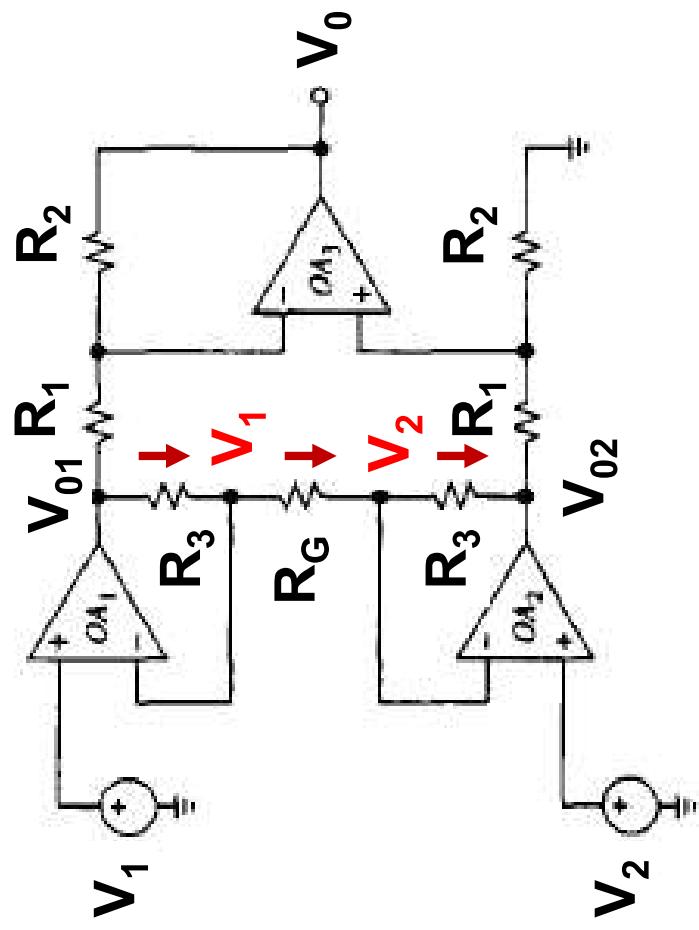


OP-AMP REQUIREMENTS:

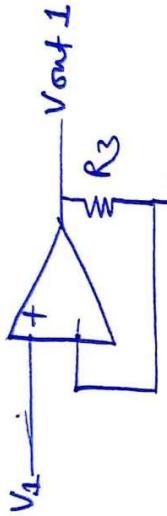
- 1- Op-amp must be provided with $\pm 15V/\pm 12V$ power supply
- 2- Op-amp must not operate in saturation region
- 3- Use of negative feedback
- 4- Using concept of virtual short
- 5- To find output expression, we need to first find V_{out1} & V_{out2}
- 6- Since op-amp is in linear region, we can use superposition theorem

Triple: OP-AMP
INSTRUMENTATION AMPLIFIER

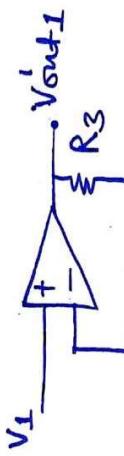
INSTRUMENTATION AMPLIFIER: ANALYSIS



STEP-1: Calculation of $V_{out\ 1}$:
To find $V_{out\ 1}$, we can use superposition theorem: So, consider op-amp-1 first *

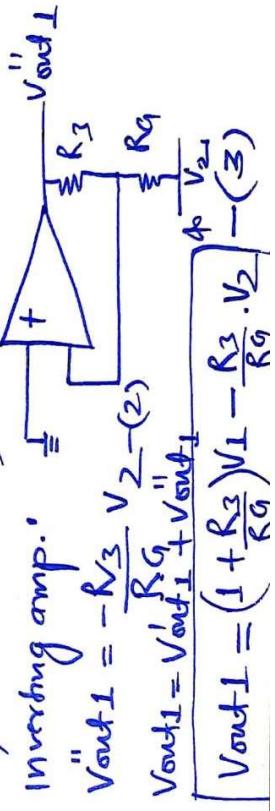


Case-1: V_1 source active, V_2 source deactivated
So, circuit becomes non-inverting amp!



$$V'_{out1} = \left(1 + \frac{R_3}{R_G}\right) V_1 - (1)$$

Case-2: V_2 source active, V_1 source deactivated
So, circuit becomes inverting amp.



Triple: OP-AMP
INSTRUMENTATION AMPLIFIER

INSTRUMENTATION AMPLIFIER: ANALYSIS

STEP-2: Calculation of V_{out2}
 * Because of similar section for op-amp 2
 we can perform the analysis & find V_{out2}
 * So, similarly :

$$V_{out2} = \left(1 + \frac{R_2}{R_1}\right) V_2 - \frac{R_3}{R_G} \cdot V_1 \quad (4)$$

STEP-3: Since op-amp-3 is a differential amplifier:

$$V_{out} = \frac{R_2}{R_1} [V_{out2} - V_{out1}]$$

$$V_{out} = \frac{R_2}{R_1} \left[\left(1 + \frac{R_2}{R_G}\right) V_2 - \frac{R_3}{R_G} V_1 - \left(1 + \frac{R_2}{R_G}\right) V_1 + \left(\frac{R_3}{R_G}\right) V_2 \right]$$

$$V_{out} = \frac{R_2}{R_1} \left[V_2 + \frac{R_2}{R_G} V_2 - \frac{R_3}{R_G} V_1 - V_1 - \frac{R_2}{R_G} V_1 + \frac{R_3}{R_G} V_2 \right]$$

$$V_{out} = \frac{R_2}{R_1} \left[(V_2 - V_1) + \frac{2R_3}{R_G} (V_2 - V_1) \right] *$$

$$V_{out} = \frac{R_2}{R_1} \left[1 + \frac{2R_3}{R_G} \right] (V_2 - V_1) *$$

- $A_d = \frac{R_2}{R_1} \left[1 + \frac{2R_3}{R_G} \right]$
 Differential Gain

$$V_{out} = A_d (V_2 - V_1)$$

$V_{out} \propto (V_2 - V_1)$ *

- \propto differential gain:

$$A_d = \frac{R_2}{R_1} \left[1 + \frac{2R_3}{R_G} \right]$$

Gain of
2nd stage 1st stage

* The gain of the instrument
 amplifier can be varied by
 adjusting variable resistance

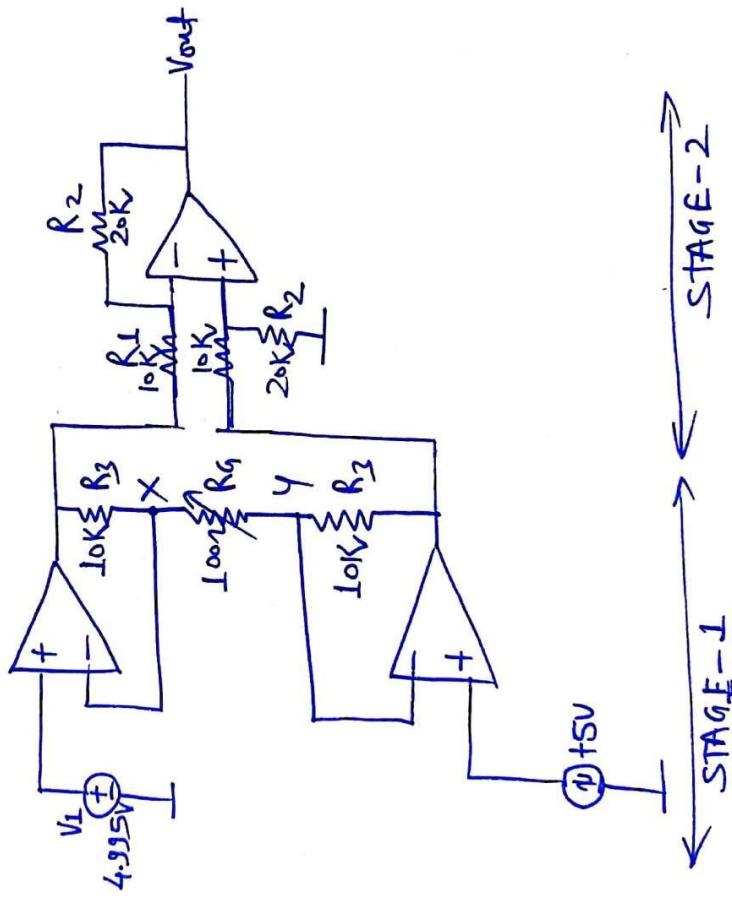
INSTRUMENTATION AMPLIFIER: PROBLEM

problem: for the given TA, determine the output voltage for the circuit given below.

$$R_3 = 10\text{ k}\Omega, R_G = 100\Omega, R_1 = 10\text{ k}\Omega$$

$$R_2 = 20\text{ k}\Omega, V_1 = 4.995\text{ V}$$

$$V_2 = 5\text{ V}$$



$$\text{Solution: } V_1 = 4.995\text{ V}, V_2 = 5\text{ V}$$

$$V_{\text{out1}} = \left(1 + \frac{R_3}{R_G}\right)V_1 - \frac{R_2}{R_G}V_2$$

$$V_{\text{out1}} = \left(1 + \frac{10\text{k}}{100}\right)V_1 - \left(\frac{10\text{k}}{100}\right)V_2$$

$$\boxed{V_{\text{out1}} \approx 4.995\text{ V}}$$

$$V_{\text{out2}} = \left(1 + \frac{R_3}{R_G}\right)V_2 - \frac{R_2}{R_G}V_1$$

$$V_{\text{out2}} = \left(1 + \frac{10\text{k}}{100}\right)V_2 - \left(\frac{10\text{k}}{100}\right)V_1$$

$$V_{\text{out2}} = \left(1 + 100\right)V_2 - 100V_1$$

$$V_{\text{out2}} = 101V_2 - 100V_1$$

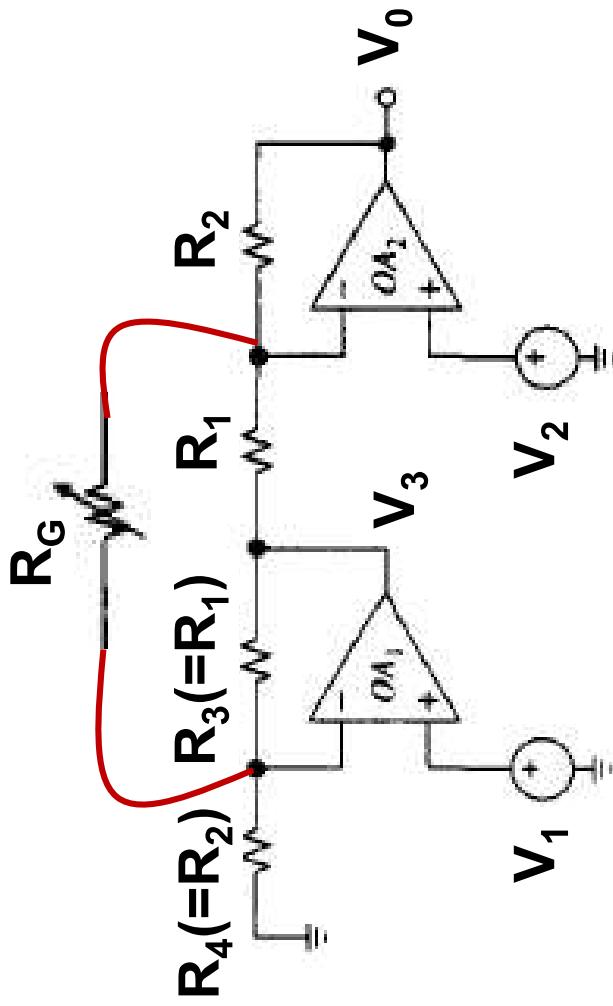
$$\boxed{V_{\text{out2}} = 5.5\text{ V}}$$

$$\text{So, } V_{\text{out}} = \frac{R_2}{R_1} (V_{\text{out2}} - V_{\text{out1}})$$

$$= \frac{20\text{k}}{10\text{k}} [5.5 - 4.995]$$

$$\boxed{V_{\text{out}} = 2.01\text{ V}}$$

DUAL OP-AMP INSTRUMENTATION AMPLIFIER



$$V_3 = \left(1 + \frac{R_3}{R_4} \right) V_1$$

$$V_0 = \left(1 + \frac{R_2}{R_1} \right) V_2 - \frac{R_2}{R_1} V_3$$

$$V_0 = \left(1 + \frac{R_2}{R_1} \right) V_2 - \frac{R_2}{R_1} \left(1 + \frac{R_3}{R_4} \right) V_1$$

$$V_0 = \left(1 + \frac{R_2}{R_1} \right) (V_2 - V_1)$$

$$\text{When } \frac{R_4}{R_3} = \frac{R_2}{R_1}$$

$$V_0 = \left(1 + \frac{R_2}{R_1} + \frac{2R_2}{R_G} \right) (V_2 - V_1)$$

- When high-quality, costlier op amps are used to achieve superior performance, it is of interest to minimize the number of devices in the circuit. • A **drawback of the dual-op-amp configuration is that it treats the inputs asymmetrically because v1 has to propagate through OA1 before catching up with v2.**

$$\text{CURRENT OUTPUT INSTRUMENTATION AMPLIFIER} \quad V_o = -\frac{R_2}{R_1} V_s$$

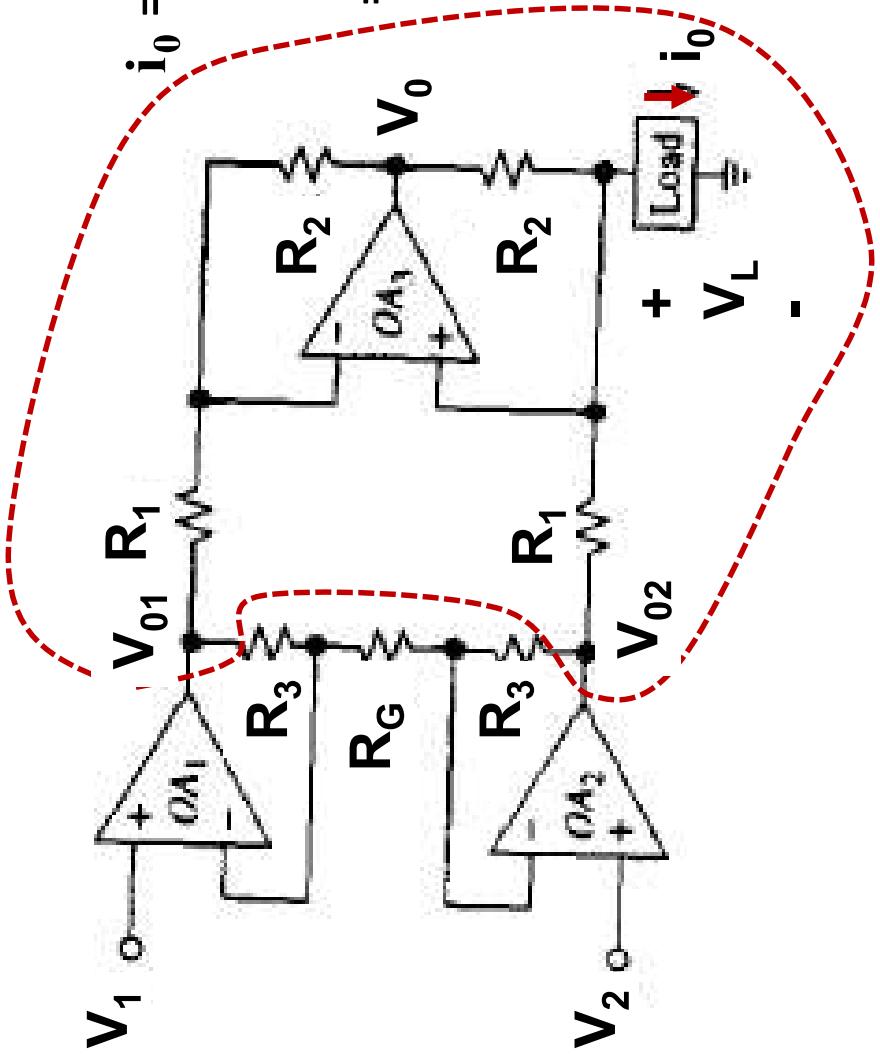
$$V_0 = -\frac{R_2}{R_1} V_{01} + \left(1 + \frac{R_2}{R_1}\right) V_L$$

$$= \frac{V_{02} - V_L}{R_1} + \frac{V_0 - V_L}{R_2} = \frac{V_{02}}{R_1} + \frac{V_0}{R_2} - \left(\frac{1}{R_1} + \frac{1}{R_2} \right) V_L$$

$$= \frac{V_{02}}{R_1} - \frac{R_2}{R_2 R_1} V_{01} - \left(\frac{1}{R_1} + \frac{1}{R_2} - \frac{1}{R_2} - \frac{R_2}{R_2 R_1} \right) V_L$$

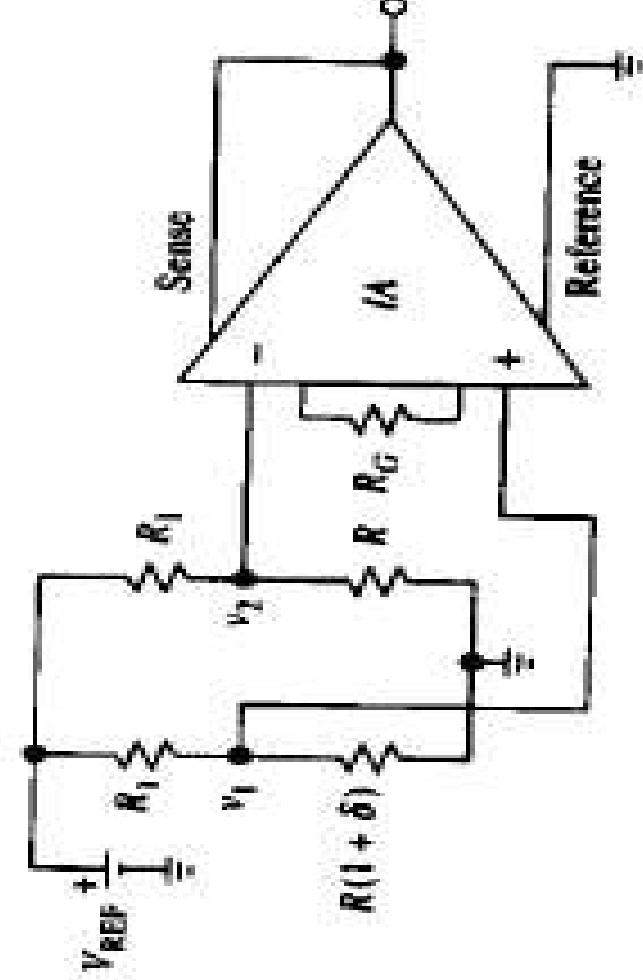
$$i_0 = \frac{1}{R_1} (V_{02} - V_{01})$$

$$i_0 = \frac{1 + \frac{2R_3}{R_G}}{R_1} (V_2 - V_1)$$

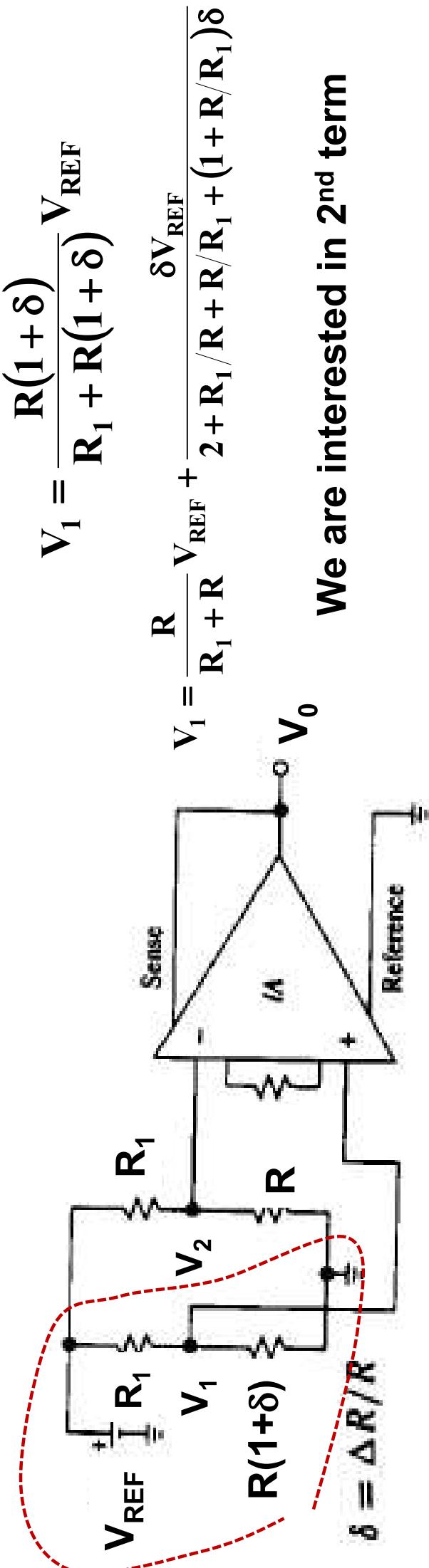


TRANSDUCER BRIDGE AMPLIFIER

- Resistive transducers are devices whose resistance varies as a consequence of some environmental condition, such as temperature (thermistors; RTDs), light (photoresistors), strain (strain gauges), and pressure (piezoresistive transducers).
- By making these devices part of a circuit, it is possible to produce an electric signal that, after suitable conditioning, can be used to monitor as well as control the physical process affecting the transducer.
- In general it is desirable that the relationship between the final signal and the original physical variable be linear, so that the former can directly be calibrated in the physical units of the latter.
- Transducers play such an important role in measurement and control instrumentation that it is worth studying transducer circuits in some detail



TRANSDUCER BRIDGE AMPLIFIER: ANALYSIS



1st term can be eliminated by second voltage divider

$$V_2 = \frac{R}{R_1 + R} V_{\text{REF}}$$

$$v_O = A(v_1 - v_2)$$

$$v_O = \frac{AV_{\text{REF}}}{1 + R_1/R + (1 + R/R_1)(1 + \delta)}$$

$$\delta \ll 1$$

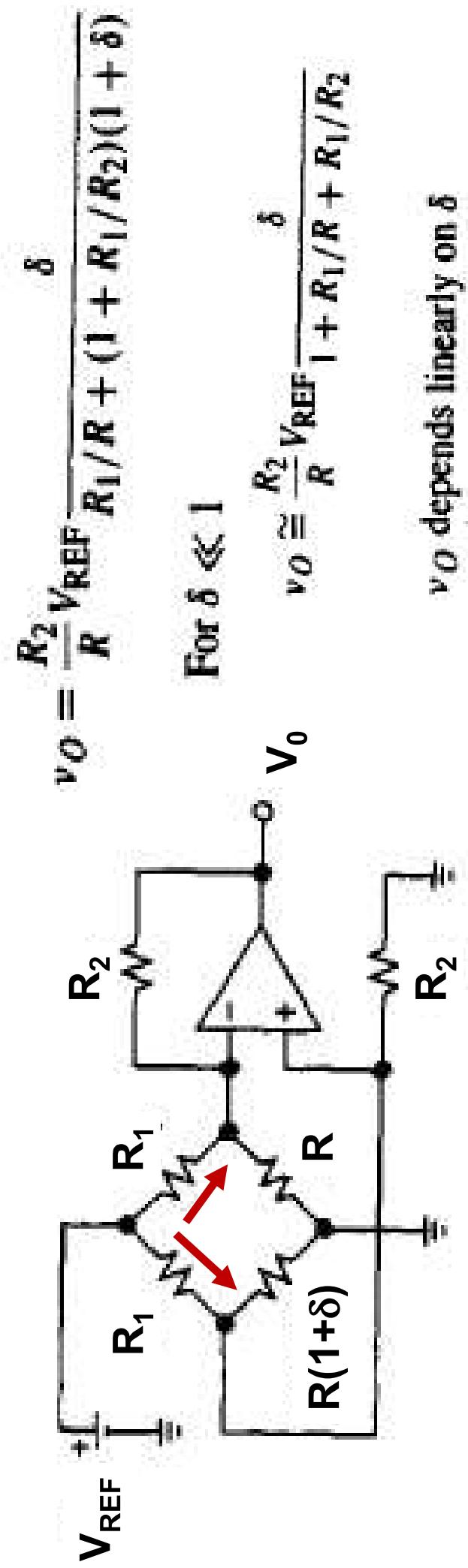
$$v_O \cong \frac{AV_{\text{REF}}}{2 + R_1/R + R/R_1}\delta$$

$$R_1 = R$$

$$v_O = \frac{AV_{\text{REF}}}{4} \frac{\delta}{1 + \delta/2}$$

$$v_O \cong \frac{AV_{\text{REF}}}{4} \delta$$

SINGLE OP-AMP TRANSDUCER BRIDGE AMPLIFIER



Response is linear as long as $\delta \ll 1$

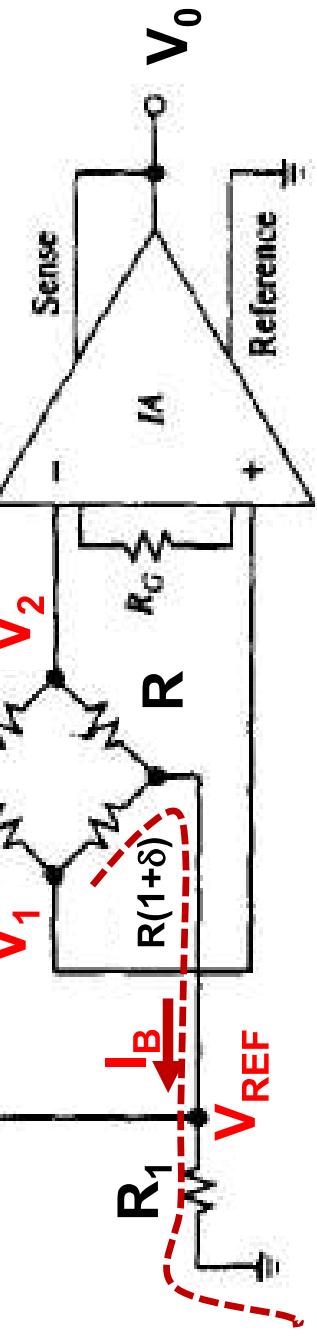
Is it possible to eliminate this limitation?

BRIDGE LINERIZATION BY CONSTANT CURRENT DRIVE

$$\text{bridge current } I_B = V_{\text{REF}} / R_1$$

$$V_{\text{REF}} \quad \text{---} \quad I_B \rightarrow$$

$$V_1 = V_{\text{REF}} + R(1 + \delta)I_B / 2$$



$$V_2 = V_{\text{REF}} + RI_B / 2$$

$$V_1 - V_2 = R\delta I_B / 2$$

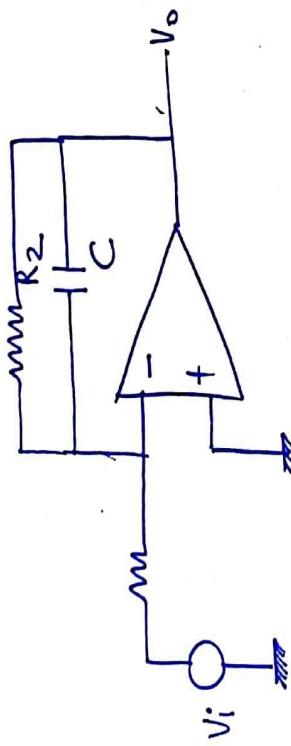
$$V_0 = \frac{ARV_{\text{REF}} \delta}{2R_1}$$

LIC: LECTURE 8

ACTIVE FILTERS

1. ACTIVE LOW PASS FILTER WITH GAIN
2. ACTIVE HIGH PASS FILTER WITH GAIN
3. ACTIVE BAND PASS FILTER WITH GAIN
4. ACTIVE SECOND ORDER FILTERS: INTRODUCTION

LOW PASS FILTER WITH GAIN



Analysis: Let $\tilde{\kappa}_2 = R_2 // \frac{1}{sC}$

$$\tilde{\kappa}_2 = \frac{(R_2 + \frac{1}{sC})}{R_2 + \frac{1}{sC}} = \frac{R_2}{1 + R_2 sC}$$

- Output of the op-amp circuit

$$V_o = -\frac{\tilde{\kappa}_2}{R_1} \cdot V_i$$

Transfer function:

$$H(s) = \frac{V_o}{V_i} = -\frac{\tilde{\kappa}_2}{R_1}$$

$$H(s) = -\frac{\left[\frac{R_2}{1 + R_2 sC}\right]}{R_1}$$

$$H(s) = \left(\frac{R_2}{R_1}\right) \left[\frac{1}{1 + sRC} \right]$$

↑ Transfer
function of Achiever
of Bandwidth

↑
①
Poles and gain
or DC gain
 $= H_{OLP}$

$$H(s) = H_{OLP} \left(\frac{1}{1 + sR_2 C} \right)$$

↑
②
Represents for Gain
action (LPF)
where
 $H_{OLP} = \frac{R_2}{R_1}$

$$H(j\omega) = H_{OLP} \left(\frac{1}{1 + j\omega R_2 C} \right)$$

$$H(j\omega) = H_{OLP} \left(\frac{1}{1 + j\omega/\omega_0} \right)$$

$$\text{where } \omega_0 = 1/R_2 C$$

$$\text{I.e. } f_0 = \frac{1}{2\pi R_2 C} \text{ - cut off freq. of LPF}$$

$$|H(j\omega)| = H_{OLP} \frac{1}{\sqrt{1 + (\omega/\omega_0)^2}}$$

↑
③
Magnitude of Transfer function

LOW PASS FILTER WITH GAIN

$$|H(j\omega)| = H_{LP} \cdot \frac{1}{\sqrt{1 + (\omega/\omega_0)^2}} \quad \text{--- Case 1: for frequencies } \omega < \omega_0$$

Case 1: for frequencies $\omega < \omega_0$
[Low freq. range]

$$\text{As } \omega < \omega_0; \frac{\omega}{\omega_0} \ll 1$$

$$\text{i.e. } \left(1 + \frac{\omega}{\omega_0}\right) \approx 1$$

$$\therefore |H(j\omega)| = H_{LP} = \text{Constant}$$

i.e. we have flat response for low frequencies.

Case 2: when $\omega = \omega_0$

$$|H(j\omega)| = H_{LP} \cdot \frac{1}{\sqrt{2}}$$

ω_0 = cut-off frequency

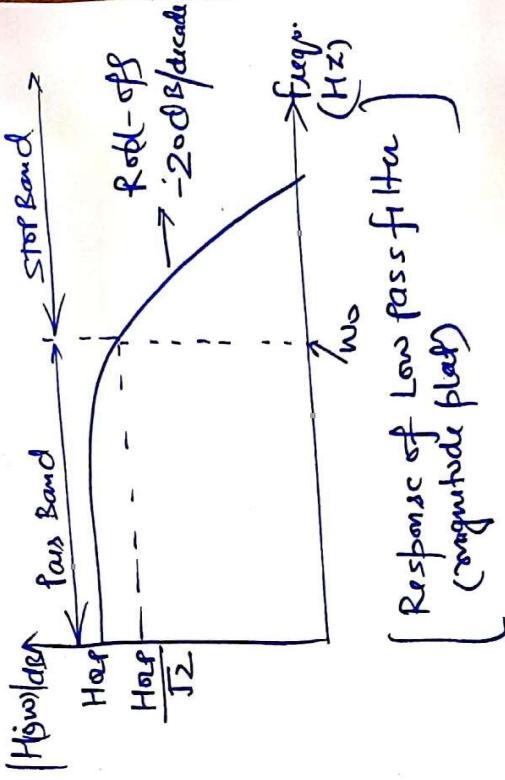
i.e. $|H(j\omega)| dB \approx 3 dB$ down of
At $\omega = \omega_0$ max value of H_{LP}

$$|H(j\omega)| = H_{LP} \cdot \frac{1}{\sqrt{1 + (\omega/\omega_0)^2}} \quad \text{--- Case 3: for frequencies } \omega > \omega_0$$

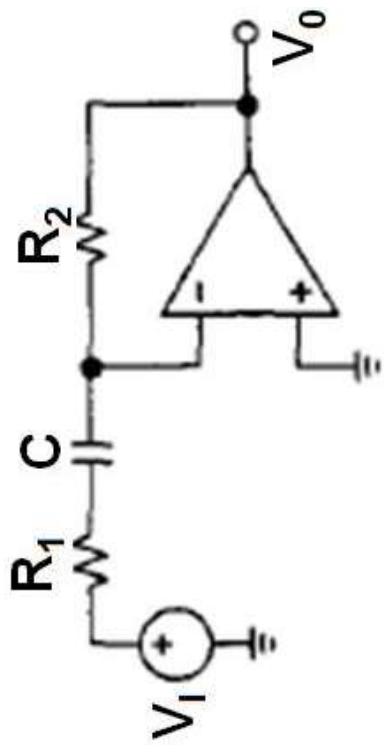
$$\left(1 + \frac{\omega}{\omega_0}\right) \approx \frac{\omega}{\omega_0}$$

$$|H(j\omega)| = H_{LP} \cdot \frac{\omega_0}{\omega}$$

- It means that as f/p frequency (ω) increases $\rightarrow |H(j\omega)|$ decreases

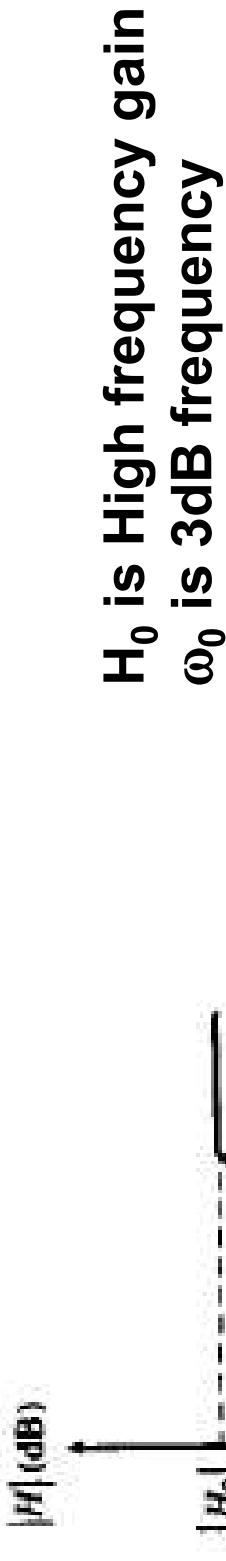


HIGH PASS FILTER WITH GAIN

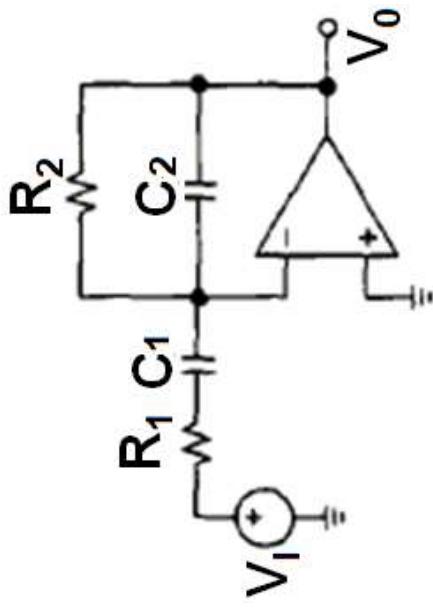


$$\frac{V_0}{V_1} = -\frac{R_2}{R_1 + 1/sC} = -\frac{sCR_2}{1 + sCR_1} = -\frac{R_2}{R_1} \frac{sCR_1}{1 + sCR_1}$$

$$H(j\omega) = H_0 \frac{j\omega/\omega_0}{1 + j\omega/\omega_0} \quad H_0 = -\frac{R_2}{R_1} \quad \omega_0 = \frac{1}{R_1 C}$$



WIDEBAND BAND PASS FILTER WITH GAIN

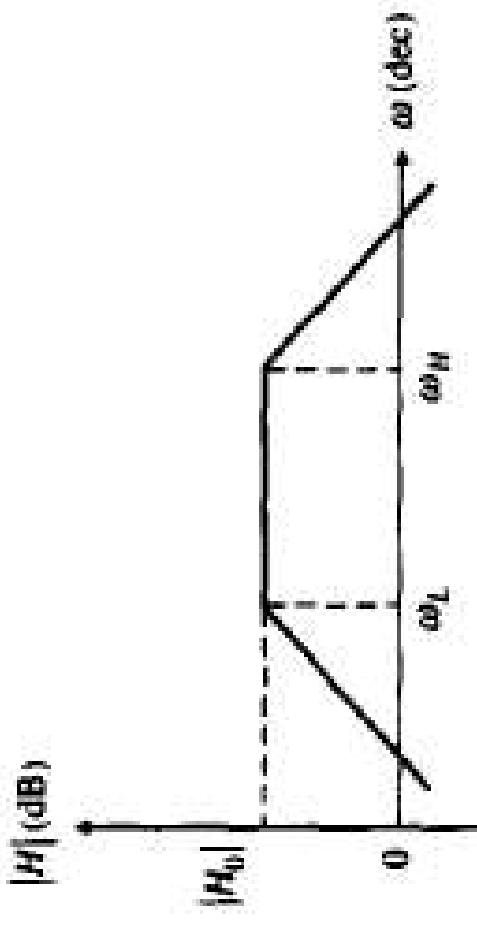


$$\frac{V_0}{V_1} = -\frac{R_2}{1+sC_2R_2} \cdot \frac{1+sC_1R_1}{sC_1} = -\frac{R_2}{R_1} \frac{1}{1+sC_2R_2} \frac{1}{1+sCR_1}$$

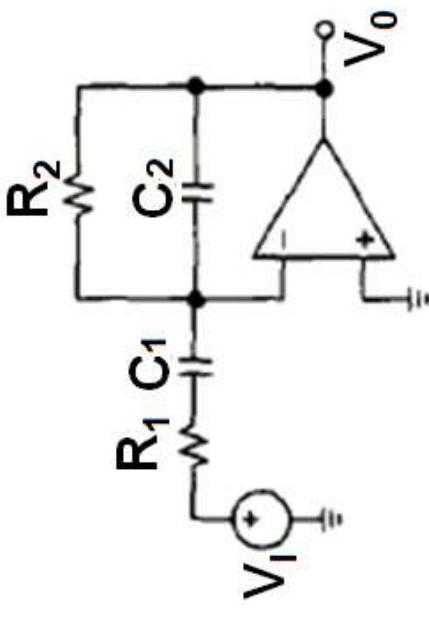
$$H(j\omega) = H_0 \frac{j\omega/\omega_L}{(1+j\omega/\omega_L)(1+j\omega/\omega_H)} \quad H_0 = -\frac{R_2}{R_1} \omega_L = \frac{1}{R_1 C_1}$$

$$\omega_H = \frac{1}{R_2 C_2}$$

H_0 is mid-frequency gain
 ω_L is lower 3dB frequency
 ω_H is higher 3dB frequency



Design a BPF for $H_0 = 20\text{dB}$ and audio frequency range



$$H_0 = 20\text{dB} \rightarrow \text{Gain} = 10$$

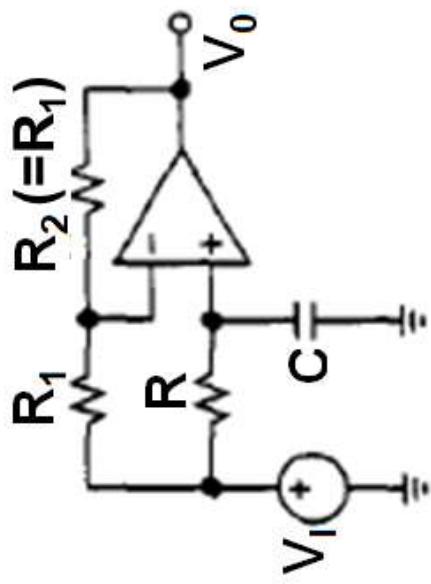
$$\text{gain} = 10 \Rightarrow \frac{R_2}{R_1} = 10 \quad \text{If } R_1 = 10\text{k}\Omega, R_2 = 100\text{k}\Omega$$

Audio frequency $\rightarrow f_L = 20\text{Hz}$ & $f_H = 20\text{kHz}$

$$\omega_L = \frac{1}{R_1 C_1} = 2\pi 20 \Rightarrow C_1 = 0.7958\mu\text{F}$$

$$\omega_H = \frac{1}{R_2 C_2} = 2\pi 20000 \Rightarrow C_2 = 39.78\text{pF}$$

Phase Shifters



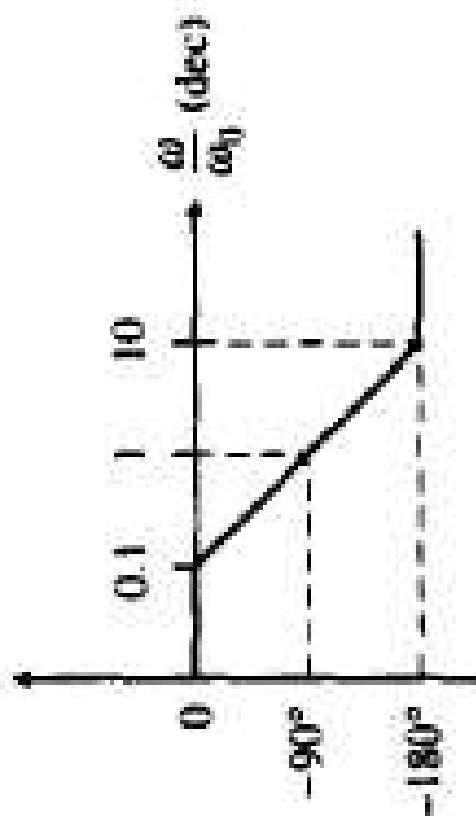
$$H(s) = 2 \frac{1}{1+sCR} - 1 = \frac{1-sCR}{1+sCR}$$

$$H(j\omega) = \frac{1-j\omega/\omega_0}{1+j\omega/\omega_0} = 1 \angle -2 \tan^{-1}(\omega/\omega_0)$$

From $|H(j\omega)|$ point, it is a all pass filter

From $\angle H(j\omega)$ point it is a phase shifter

* H



phase = 0° as $f \rightarrow 0$

phase = 90° at $f = f_0 = 1/RC$

phase = 180° as $f \rightarrow \infty$

LPF $H(j\omega) = H_0 \frac{1}{1 + j\omega/\omega_0}$

Numerator decides type of filter

HPF $H(j\omega) = H_0 \frac{j\omega/\omega_0}{1 + j\omega/\omega_0}$

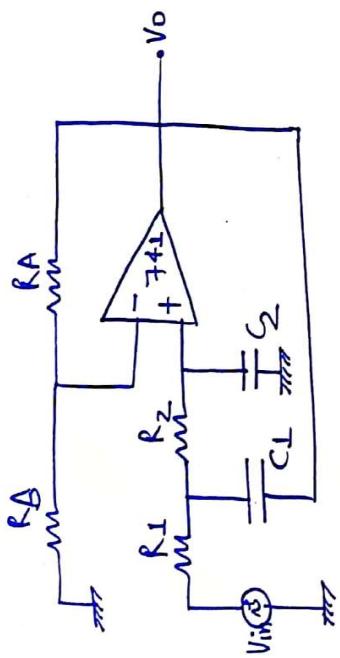
If numerator $N(j\omega) = 1 \rightarrow \text{LPF}$

APF $H(j\omega) = \frac{1 - j\omega/\omega_0}{1 + j\omega/\omega_0}$

If numerator $N(j\omega) = j\omega/\omega_0 \rightarrow \text{HPF}$
If numerator $N(j\omega) = 1 - (j\omega/\omega_0) \rightarrow \text{APF}$

H_0 is acting as scaling factor
Moves response up or down

SECOND ORDER KRC FILTER



- As input frequency increases i.e. in the high frequency region.
- Reactance $X_C = \frac{1}{2\pi f C}$ \downarrow \Rightarrow output of amplifier goes on reducing.
- So, behaves as low pass filter.

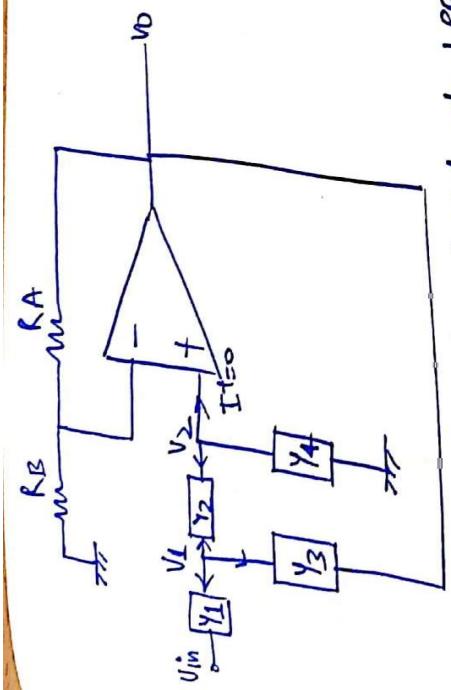
Gain of Non-inverting amplifier

$$K_v = 1 + \frac{R_A}{R_B} *$$

- for frequencies $f < f_0$, i.e. in low frequency region, reactance $(X_C = \frac{1}{2\pi f C})$ offered by both capacitors are very high. So both capacitors acts as open circuit.

- So, op-amp allows all the input to reach at the output with gain K_v below f_0 .

SECOND ORDER KRC FILTER



General structure of 2nd-order LPF:

Applying KCL at V_1 , we get:

$$(V_1 - V_{in})\gamma_1 + (V_1 - V_0)\gamma_3 + (V_1 - V_2)\gamma_2 = V_{in}\gamma_1$$

$$(\gamma_1 + \gamma_2 + \gamma_3)V_1 - V_0\gamma_3 - V_2\gamma_2 = V_{in}\gamma_1$$

$$\left(\gamma_1 + \gamma_2 + \gamma_3 \right) V_1 - \left(\frac{\gamma_2}{K} + \gamma_3 \right) V_0 = \gamma_1 \cdot V_{in} \quad \text{--- (1)}$$

$$V_0 = KV_2$$

$$[\gamma_2 + \gamma_4] V_2 = V_1 \gamma_2$$

$$\left(\gamma_2 + \gamma_4 \right) \frac{V_0}{K} = V_1 \gamma_2$$

$$\boxed{V_1 = \frac{\gamma_2 + \gamma_4}{K\gamma_2} \cdot V_0} \quad \text{--- (2)}$$

Substitute (2) in (1) we get

$$\left(\gamma_1 + \gamma_2 + \gamma_3 \right) \left(\frac{\gamma_2 + \gamma_4}{K\gamma_2} \right) V_0 - \left(\frac{\gamma_2 + \gamma_3}{K} \right) V_0$$

$$= \gamma_1 \cdot V_{in}$$

SECOND ORDER KRC FILTER

$$H(s) = \frac{V_o}{V_{in}}$$

$$H(s) = \frac{K \gamma_1 \gamma_2}{\gamma_1 \gamma_2 + (\gamma_1 + \gamma_2 + \gamma_3) s + (1-K) \gamma_2 \gamma_3} \quad (3)$$

for a LRF:

$$\gamma_1 = \frac{1}{R_1}, \quad \gamma_2 = \frac{1}{R_2}, \quad \gamma_3 = C_1 s, \quad \gamma_4 = C_2 s$$

$$H(s) = \frac{K}{1 + R_1 R_2 C_1 C_2 s^2 + ((1-K) R_1 C_1 + R_1 C_2 + R_2 C_2) s} \quad (4)$$

Comparing with 2nd order T-f.

$$H(j\omega) = \frac{H_0 P}{1 - (w/w_0)^2 + j(w/w_0) Q}$$

Comparing, we get: put $s = j\omega$ in eqn.(4)

$$\begin{cases} H_0 P = K = \frac{1 + RA}{RB} \\ \frac{1}{w_0^2} = R_1 R_2 C_1 C_2 \\ \frac{1}{w_0} = \frac{1}{J R_1 R_2 C_1 C_2} \end{cases} \quad P$$

SECOND ORDER KRC FILTER

$$\frac{1}{\omega_0 Q} = (1-K) R_1 C_1 + R_2 C_2 R_1 C_2$$

$$\& \omega_0 = \frac{1}{\sqrt{R_1 R_2 C_1 C_2}}$$

$$Q = \frac{1}{(1-K) \sqrt{\frac{R_1 C_1}{R_2 C_2}} + \sqrt{\frac{R_2 C_2}{R_1 C_1}} + \sqrt{\frac{R_1 C_2}{R_2 C_1}}} \quad \text{Quality Factor}$$

If: $R_1 = R_2 = R$, $C_1 = C_2 = C$

$$Q = \frac{1}{3 - K}$$

$\alpha \rightarrow$ damping ratio

$\boxed{\alpha = 3 - K}$; By choosing the value of K
we can get damping ratio