NETWORK THEOREMS

Superposition Theorem

- The superposition theorem states that the response in any element of a linear bilateral network containing two or more sources is the sum of the responses obtained by each source acting seperartel and all other set equal to zero.

- Apply one source and inactivate all other independent sources:
  independent voltage source: 0 V (short circuit)
  independent current source: 0 A (open circuit)

- Dependent sources are left intact

Superposition Theorem

Steps to apply superposition principle:

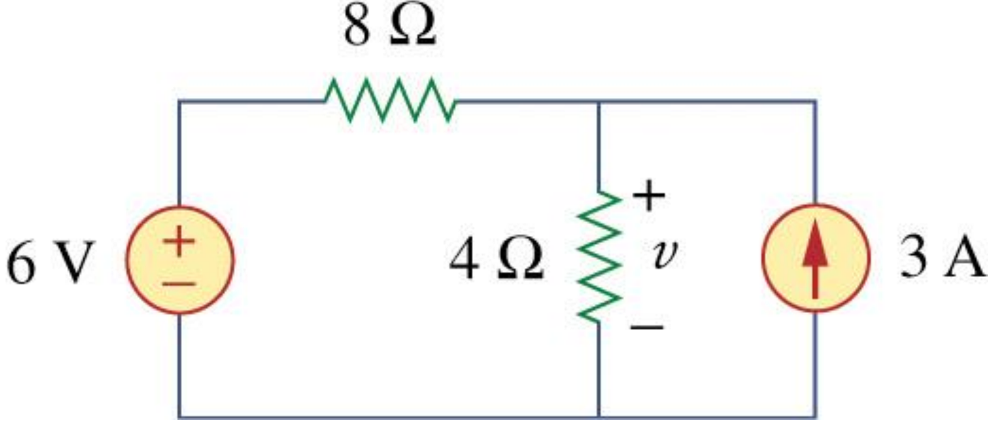
1. Turn off all independent sources except one source. Find the output (voltage or current) due to that active source using nodal or mesh analysis.

2. Repeat step 1 for each of the other independent sources.

3. Find the total contribution by adding algebraically all the contributions due to the independent sources.

NOTE: 1. Superposition involves more work but simpler circuits.
2. Superposition is not applicable to the effect on power

Use the superposition theorem to find v in the circuit



Since there are two sources,

Let $v = v_1 + v_2$

Voltage division to get

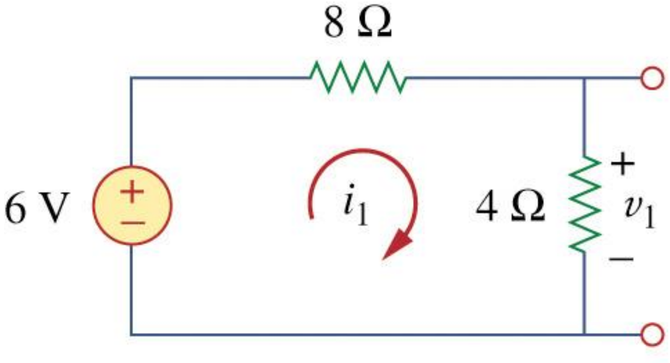
$$v_1 = \frac{4}{4+8}(6) = 2\text{V}$$

Current division, to get

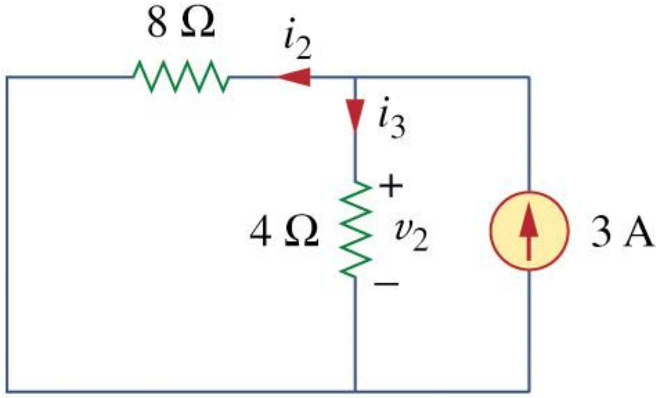
$$i_3 = \frac{8}{4+8}(3) = 2\text{A}$$

and $$v_2 = 4i_3 = 8\text{V}$$

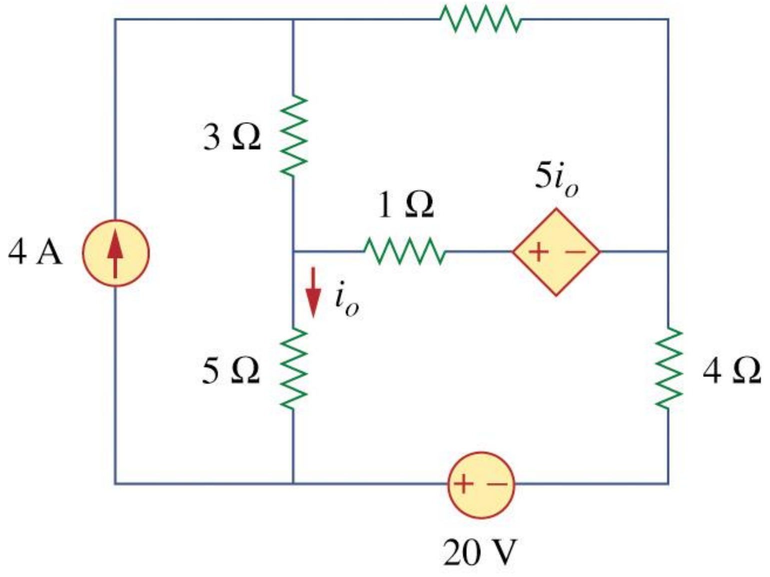
Hence $$v = v_1 + v_2 = 2 + 8 = 10\text{V}$$



(a)

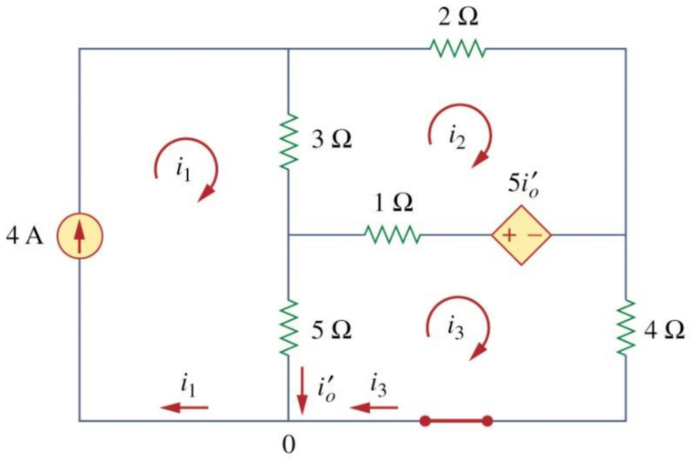


Find i_0 in the circuit using superposition.



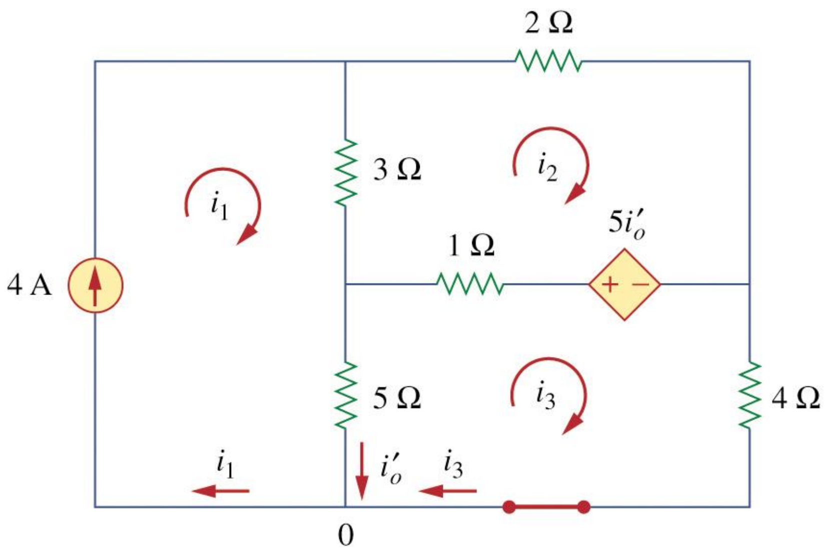
Solution: There are two independent sources

Consider 4 A current source and deactivate voltage source



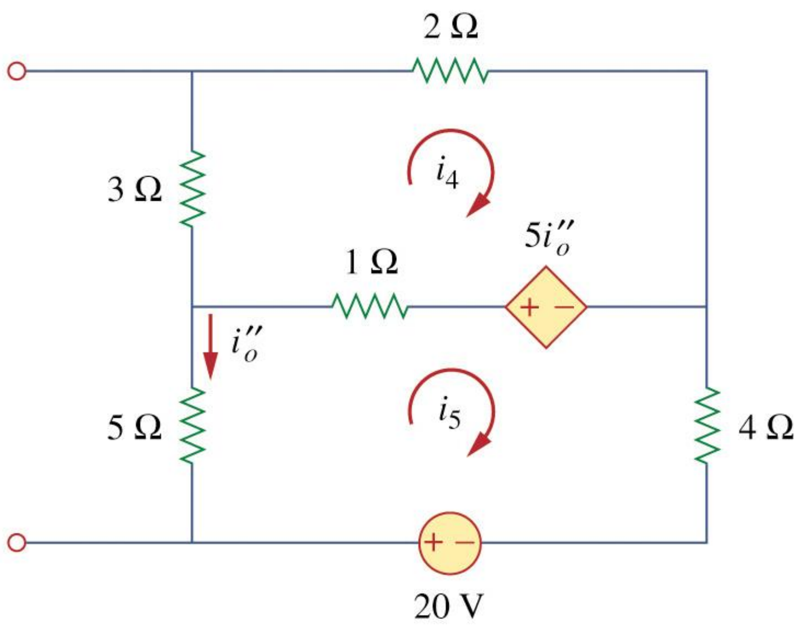
(a)

Consider 4 A current source and deactivate voltage source



(a)

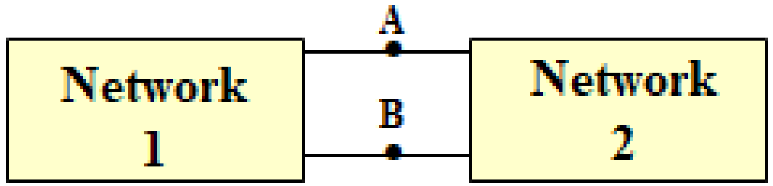
Consider 20V A voltage source and deactivate current source



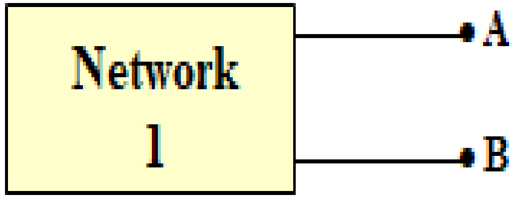
(b)

THEVENIN'S THEOREM:

Consider two Networks



Suppose Network 2 is detached from Network 1 and we focus temporarily only on Network 1.

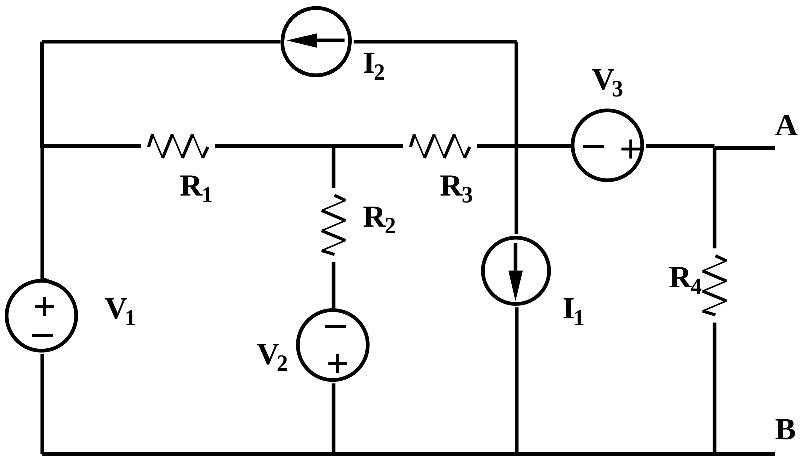


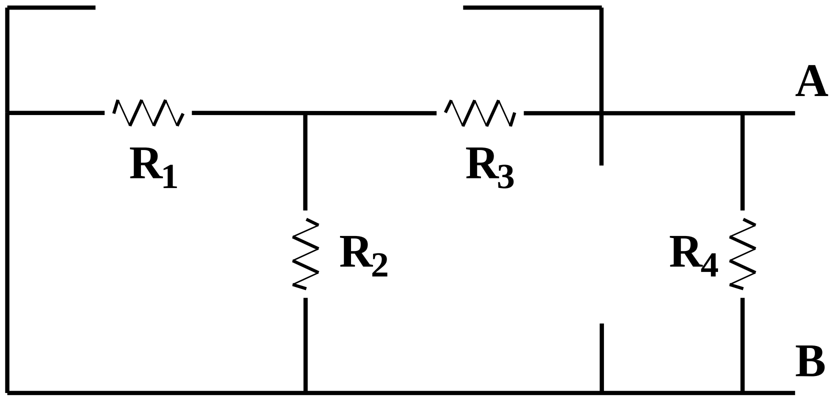
Now place a voltmeter across terminals A-B and read the voltage. We call this the open-circuit voltage. $V_{THEVENIN} = V_{TH}$

Now We deactivate all sources of Network 1.

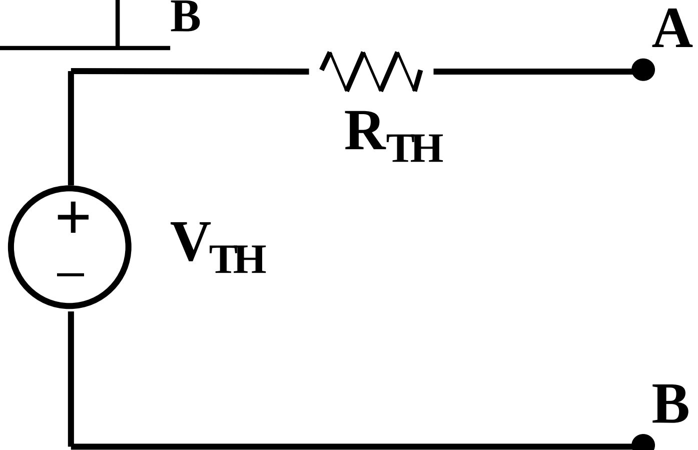
- To deactivate a voltage source, we remove the source and replace it with a short circuit.
- To deactivate a current source, we remove the source

Consider the following circuit.

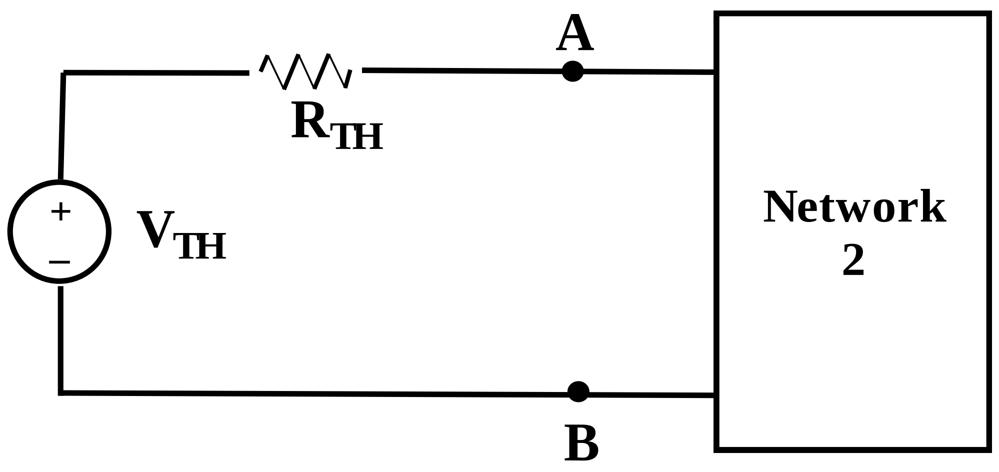




Now place an ohmmeter across A-B
and read the resistance. This is known as
Thevinins Resistance.

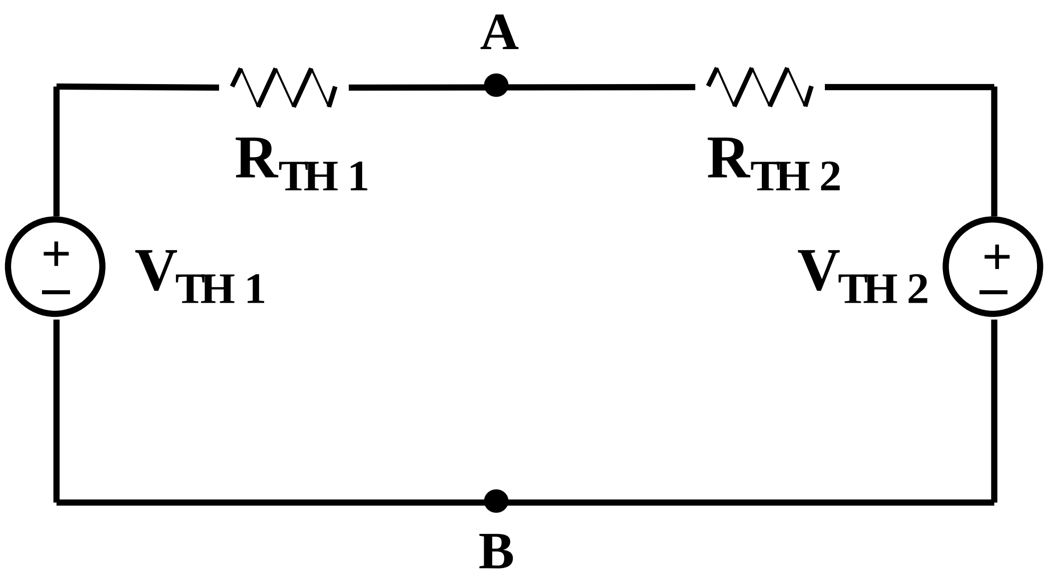


We can now tie (reconnect) Network 2 back to terminals A-B.



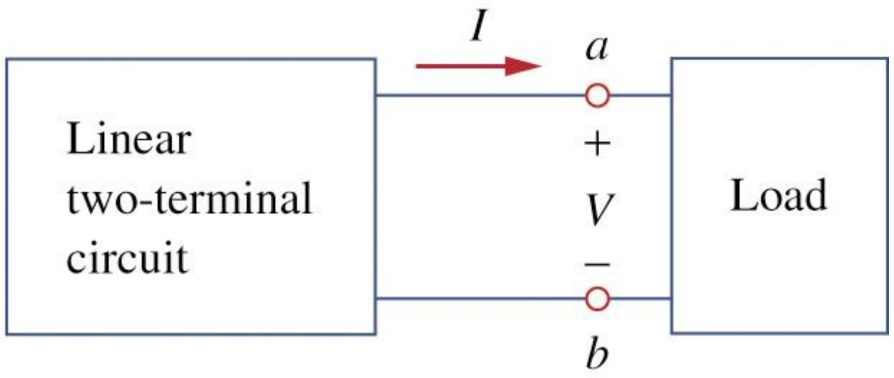
We can now make any calculations we desire within Network 2

It follows that we could also replace Network 2 with a Thevenin voltage and Thevenin resistance.

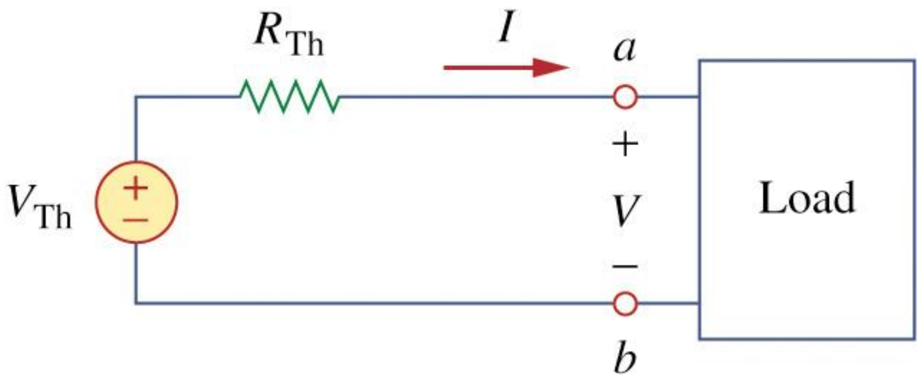


Thevenin's Theorem

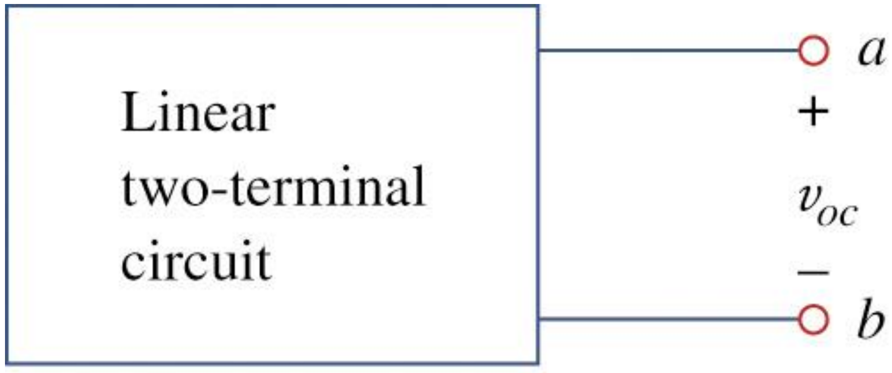
Thevenin's theorem states that a linear two-terminal circuit can be replaced by an equivalent circuit consisting of a voltage source V_{Th} in series with a resistor R_{Th} where V_{Th} is the open circuit voltage at the terminals and R_{Th} is the input or equivalent resistance at the terminals when the independent source are turn off.



(a)



How to Find Thevenin’s Voltage



$V_{Th} = v_{oc}$

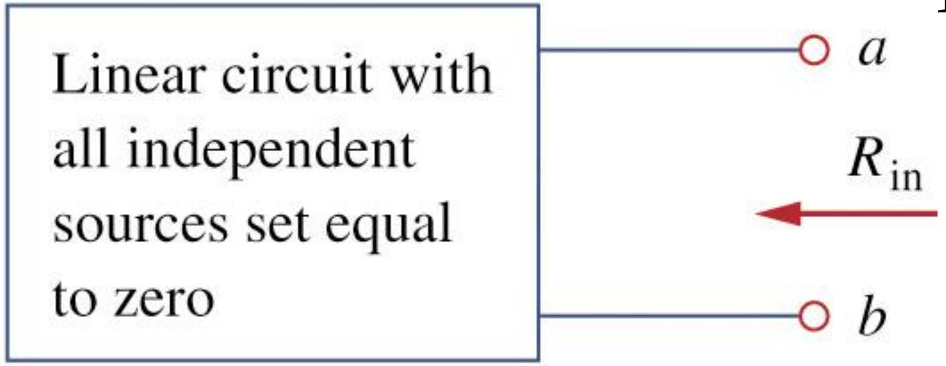
(a)

open circuit voltage at $a - b$

$V_{Th} = v_{oc}$:

1. Remove the element between the two terminals.
2. Apply suitable circuit analysis method to find out voltage across terminal.

How to Find Thevenin’s Resistance



$R_{Th} = R_{in}$

$R_{Th} = R_{in}$:

input − resistance of the dead circuit at $a - b$.

- $a - b$ open circuited
- Turn off all independent sources

CASE 1

If the network has no dependent sources:

- Turn off all independent source.
- R_{TH}: can be obtained via simplification of either parallel or series connection seen from a-b

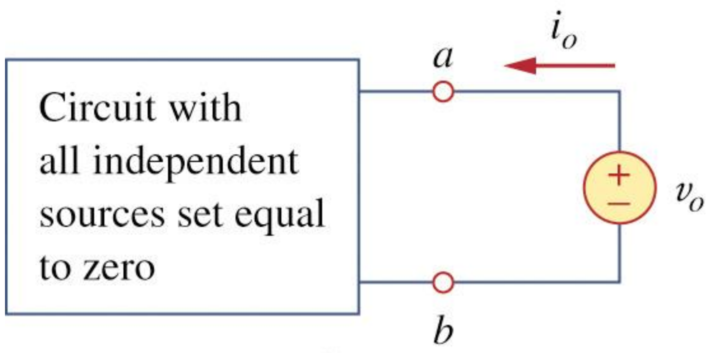
CASE 2

- If the network has dependent sources
  - Turn off all independent sources.
  - Apply a voltage source v_o at a-b

$$R_{\text{Th}} = \frac{v_o}{i_o}$$

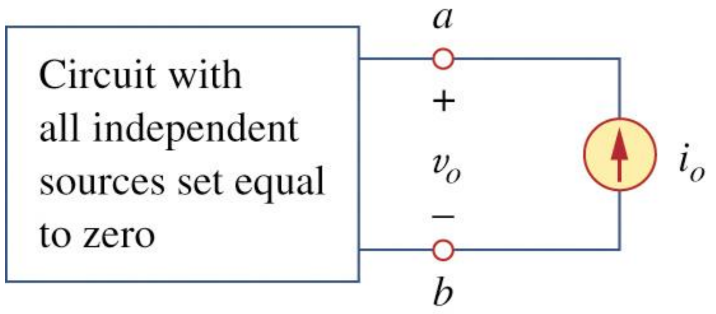
  - Alternatively, apply a current source i_o at a-b

$$R_{Th} = \frac{v_o}{i_o}$$



$$R_{\text{Th}} = \frac{v_o}{i_o}$$

(a)

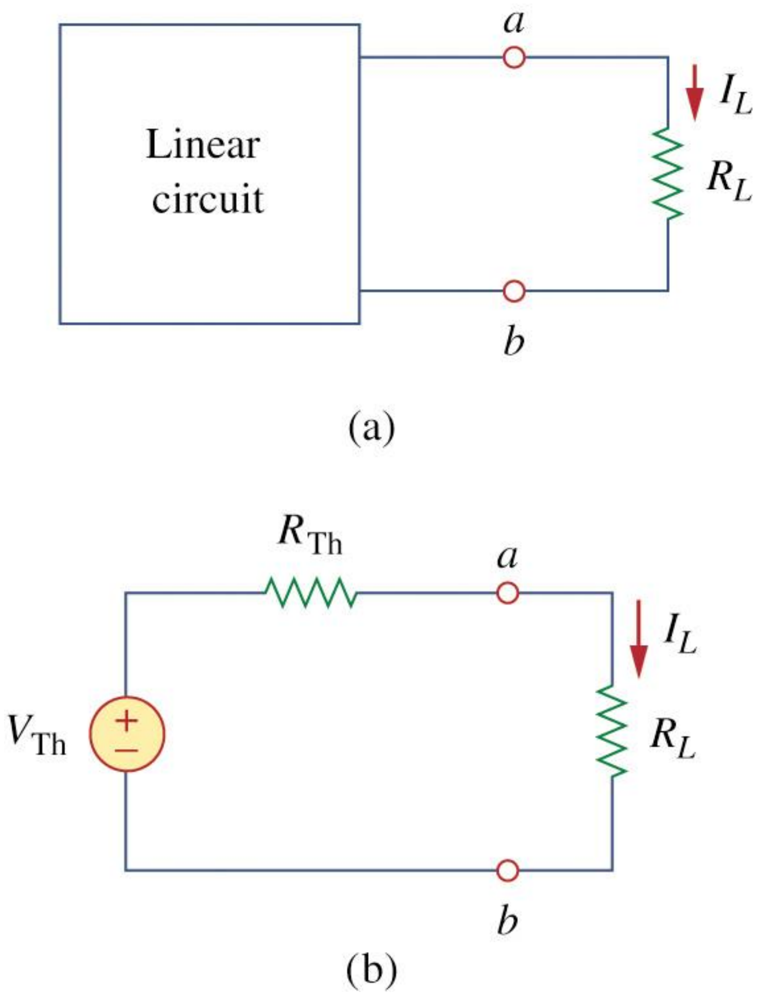


$$R_{\text{Th}} = \frac{v_o}{i_o}$$

Simplified circuit

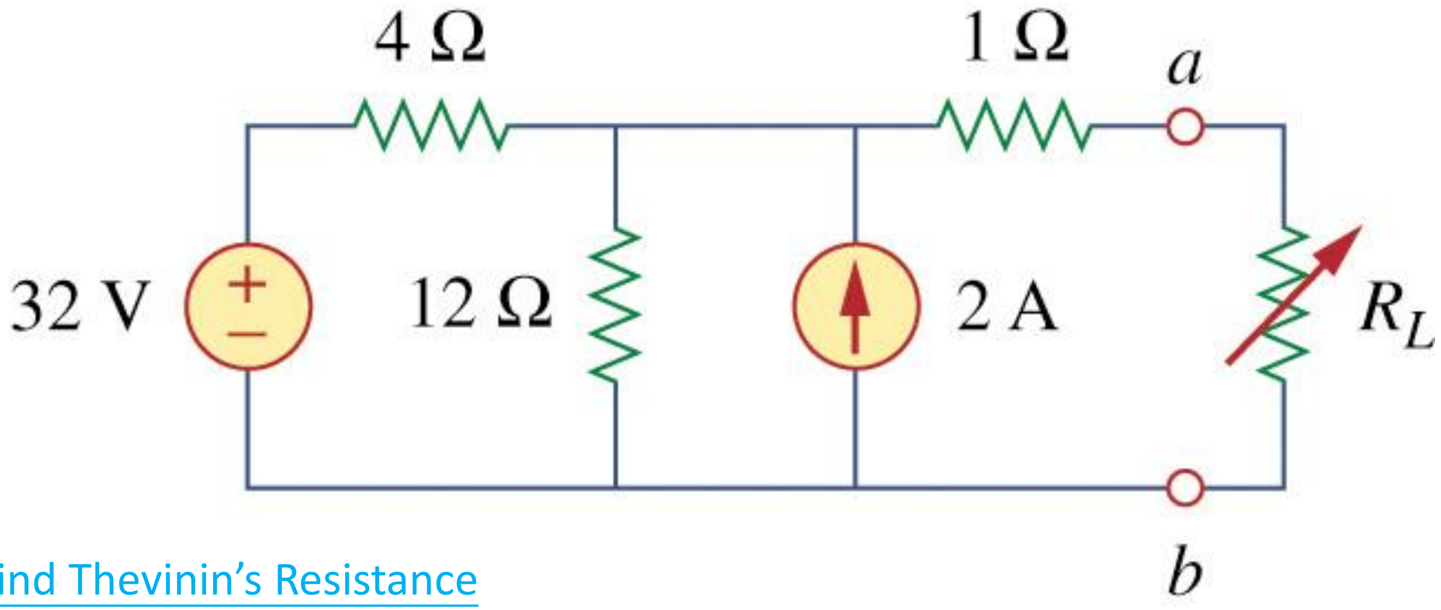
$$I_L = \frac{V_{\text{Th}}}{R_{\text{Th}} + R_L}$$

$$V_L = R_L I_L = \frac{R_L}{R_{\text{Th}} + R_L} V_{\text{Th}}$$



Example

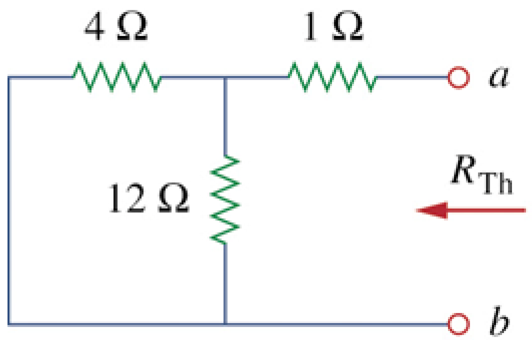
Find the Thevenin's equivalent circuit of the circuit shown in Figure to the left of the terminals *a*-*b*. Then find the current through R_L = 10,15,and 30 Ω.



To find Thevinin's Resistance

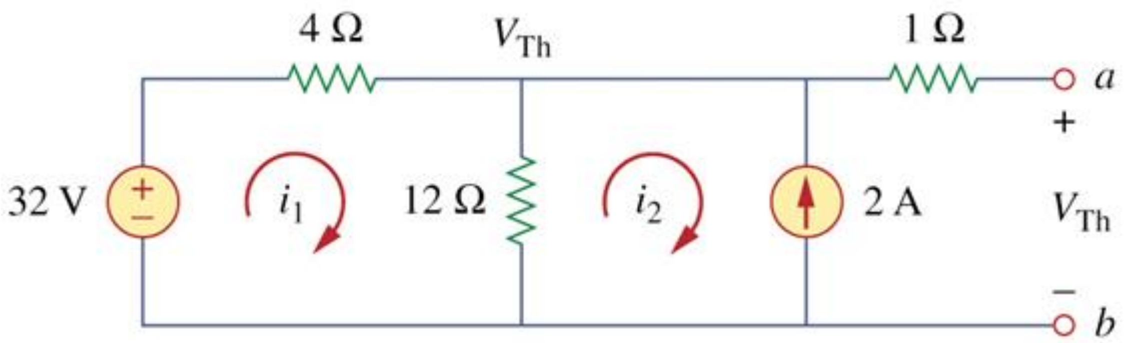
$$32\text{V voltage source} \rightarrow \text{short}$$

$$2\text{A current source} \rightarrow \text{open}$$



$$R_{\text{Th}} = 4\|12 + 1 = \frac{4\times 12}{16} + 1 = 4\Omega$$

To find Thevinin's Voltage(Open Circuit Voltag):



(1) Mesh analysis

$$-32+4i_1+12(i_1-i_2)=0\ ,\ \ i_2=-2\text{A}$$

$$\therefore i_1=0.5\text{A}$$

$$V_{\text{Th}}=12(i_1-i_2)=12(0.5+2.0)=30\text{V}$$

(2) Alternatively, Nodal Analysis

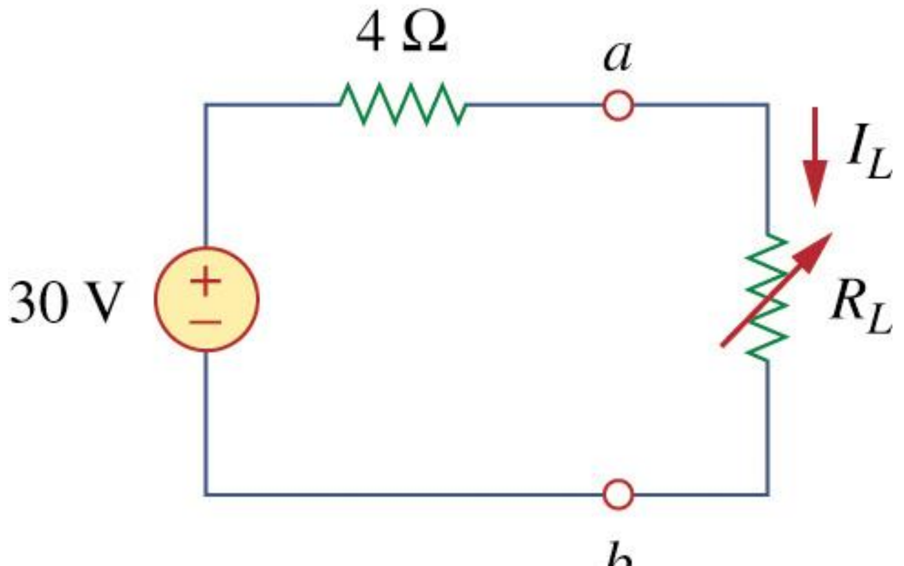
$$(32-V_{\text{Th}})/4+2=V_{\text{Th}}/12$$

$$\therefore V_{\text{Th}}=30\text{V}$$

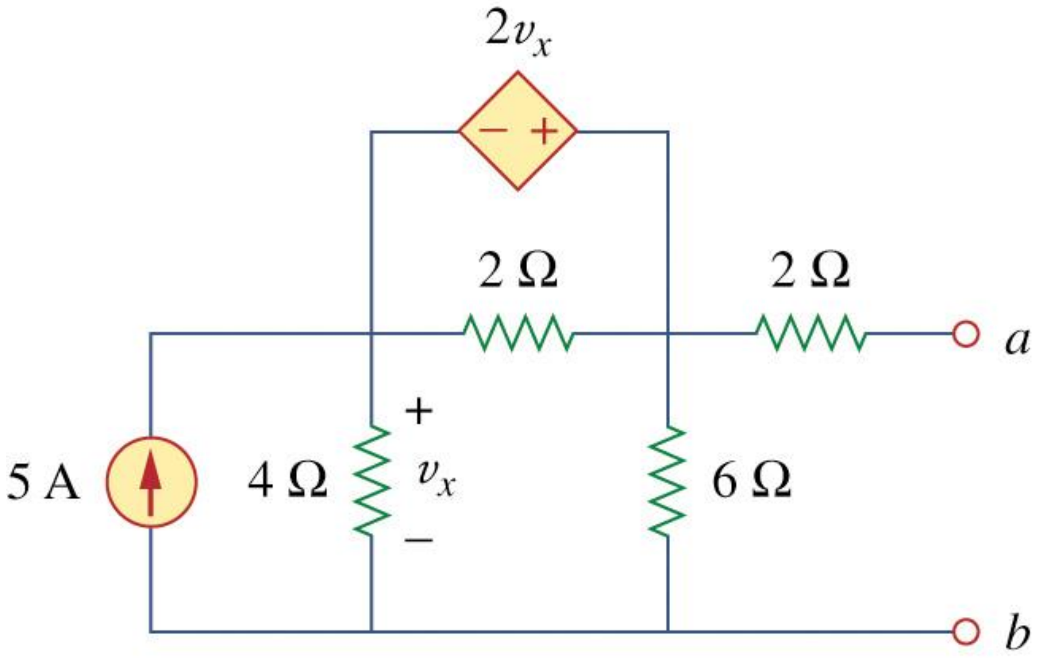
(3) Alternatively, source transform

$$\frac{32-V_{\text{TH}}}{4}+2=\frac{V_{\text{TH}}}{12}$$

$$96-3V_{\text{TH}}+24=V_{\text{TH}}\Rightarrow V_{\text{TH}}=30\text{V}$$



Find the Thevenin’s equivalent of the circuit in at terminals *a*-*b*.



To find R_{Th} : Fig(a)

independent source $\rightarrow 0$

dependent source $\rightarrow$ intact

$v_o = 1\text{V}, \quad R_{\text{Th}} = \dfrac{v_o}{i_o} = \dfrac{1}{i_o}$

For loop 1.

$$-2v_x + 2(i_1 - i_2) = 0 \quad \text{But } v_x = -4i_2$$

$$\therefore i_1 = -3i_2$$

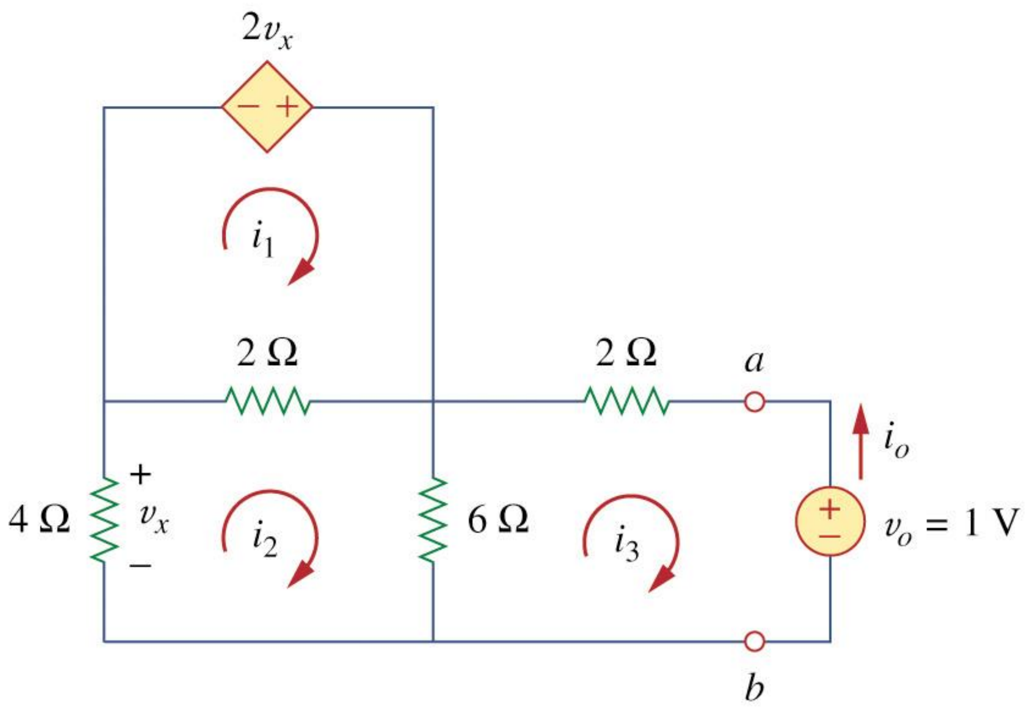
Loop 2 and 3:

$$4i_2 + 2(i_2 - i_1) + 6(i_2 - i_3) = 0$$

$$6(i_3 - i_2) + 2i_3 + 1 = 0$$

Solving these equations gives $i_3 = -1/6\text{A}$.

But $i_o = -i_3 = \dfrac{1}{6}\text{A} \qquad \therefore R_{\text{Th}} = \dfrac{1V}{i_o} = 6\Omega$



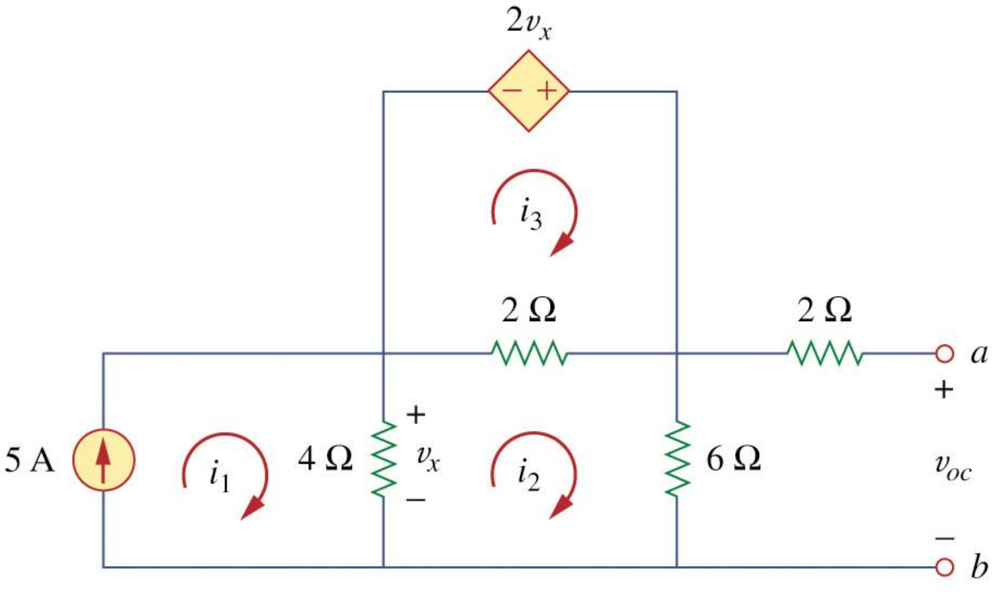
(a)

To get V_{Th} :Fig(b)

Mesh analysis

$$i_1 = 5$$

$$-2v_x + 2(i_3 - i_2) = 0 \Rightarrow \quad v_x = i_3 - i_2$$



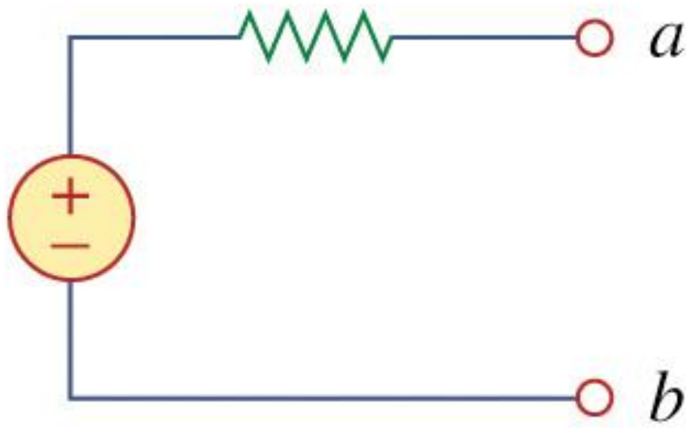
(b)

$$4(i_2 - i_1) + 2(i_2 - i_1) + 6i_2 = 0 \Rightarrow \; 12i_2 - 4i_1 - 2i_3 = 0$$

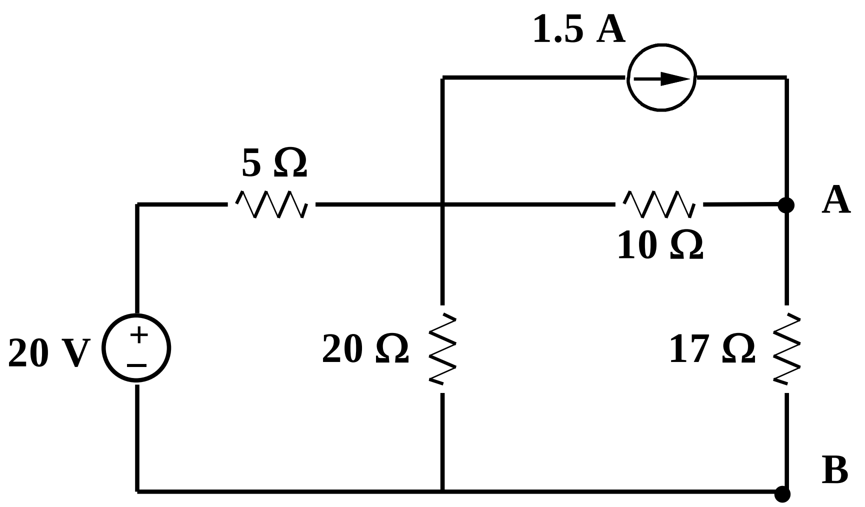
But $4(i_1 - i_2) = v_x$

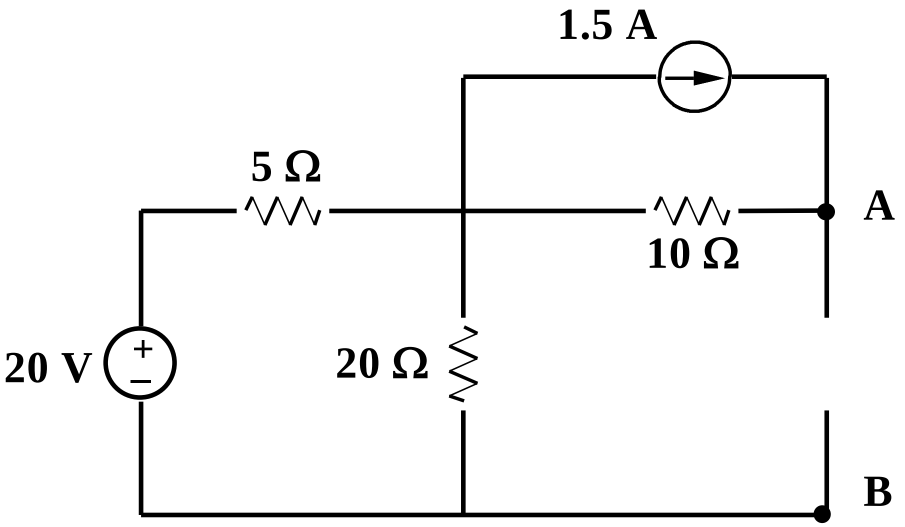
$$\therefore i_2 = 10/3.$$

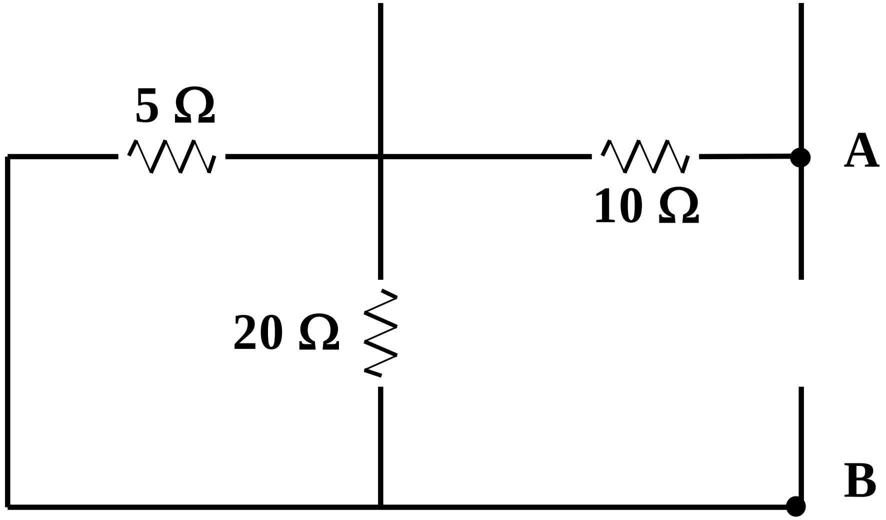
$$V_{\text{Th}} = v_{oc} = 6i_2 = 20\text{V}$$



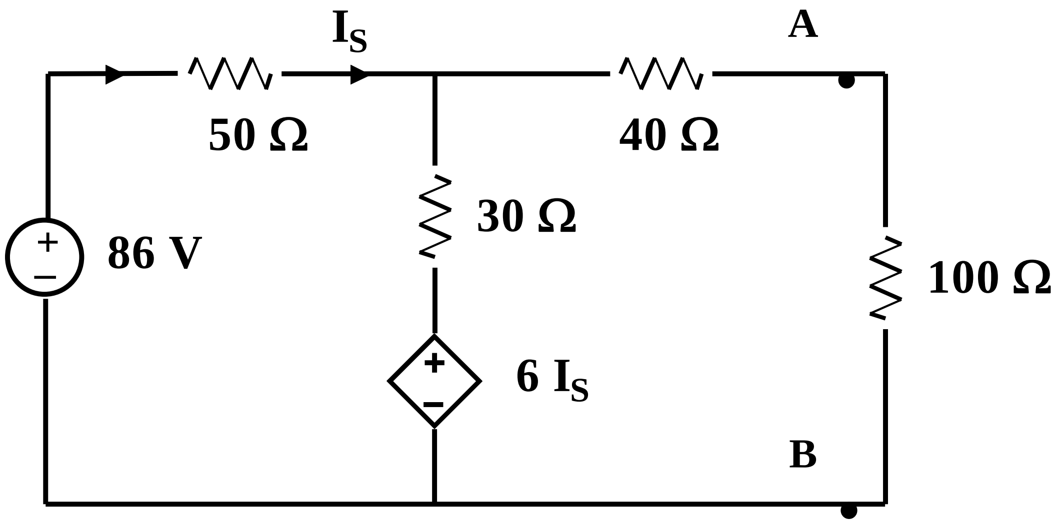
For the circuit below, find V_{AB} by first finding the Thevenin circuit to the left of terminals A-B.





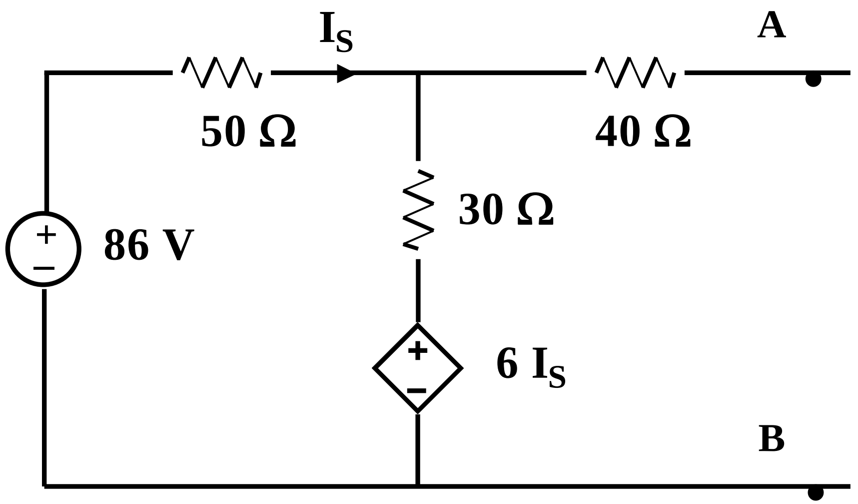


Find the voltage across the 100 Ω load resistor by first finding the Thevenin circuit to the left of terminals A-B.



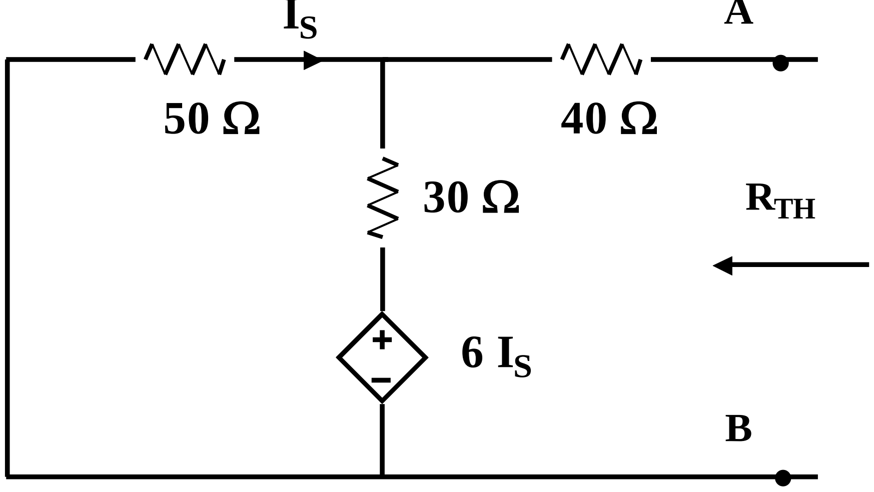
Solution:

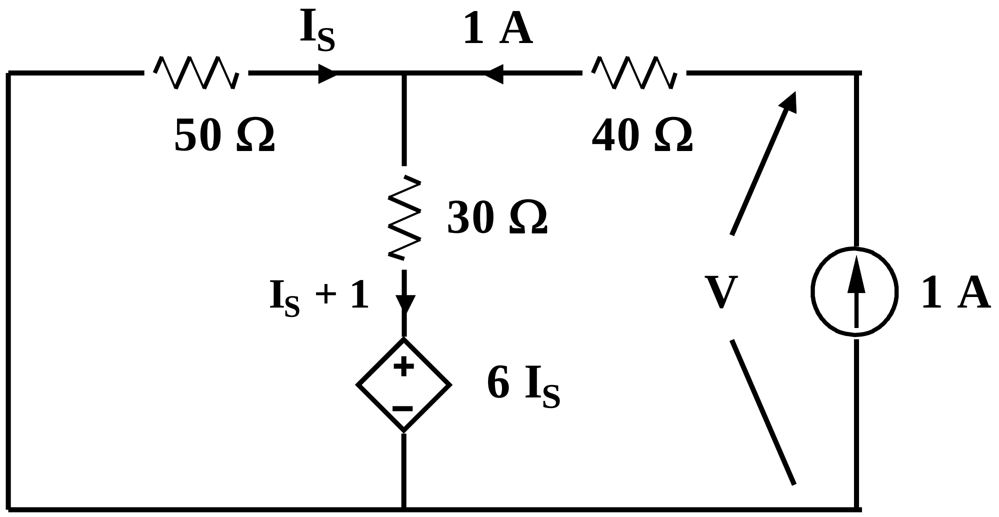
Thevenin Voltage



$$-86+80I_S+6I_S=0 \rightarrow I_S=1A$$

$$V_{AB}=6I_S+30I_S= \rightarrow 36V$$



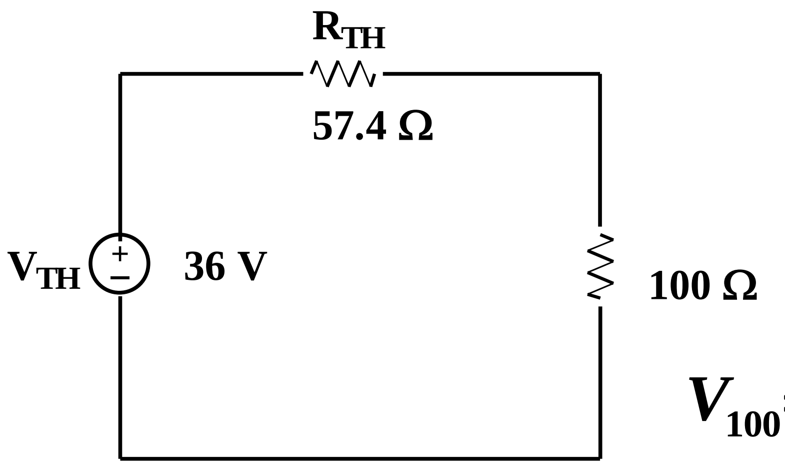


$$50I_S + 30(I_S + 1) + 6I_S = 0$$

$$I_S = \frac{-15}{43}\ A$$

$$50\left(\frac{-15}{43}\right) - 1(40) + V = 0$$

$$V = 57.4\ volts$$

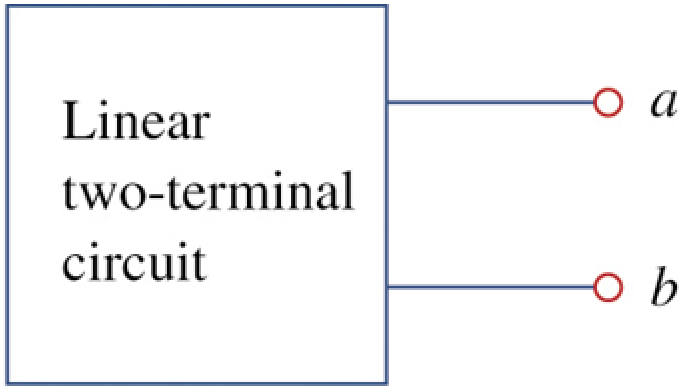


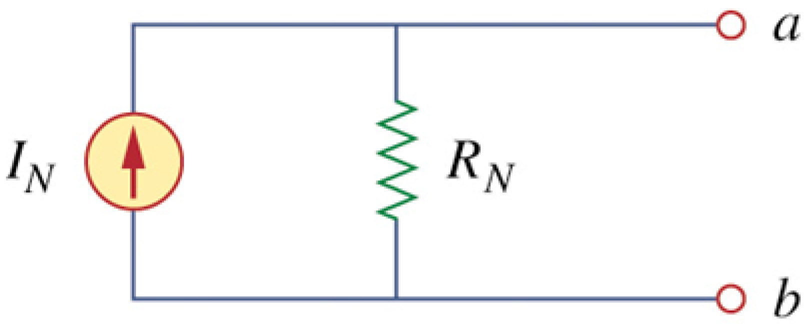
$$R_{TH} = \frac{V}{I} = \frac{V}{1} = 57.4\ \Omega$$

$$V_{100} = \frac{36x100}{57.4 + 100} = 22.9\ V$$

Norton's Theorem

Norton's theorem states that a linear two-terminal circuit can be replaced by equivalent circuit consisting of a current source I_N in parallel with a resistor R_N where I_N is the short-circuit current through the terminals and R_N is the input or equivalent resistance at the terminals when the independent source are turn off.





Thevenin and Norton resistances are equal:

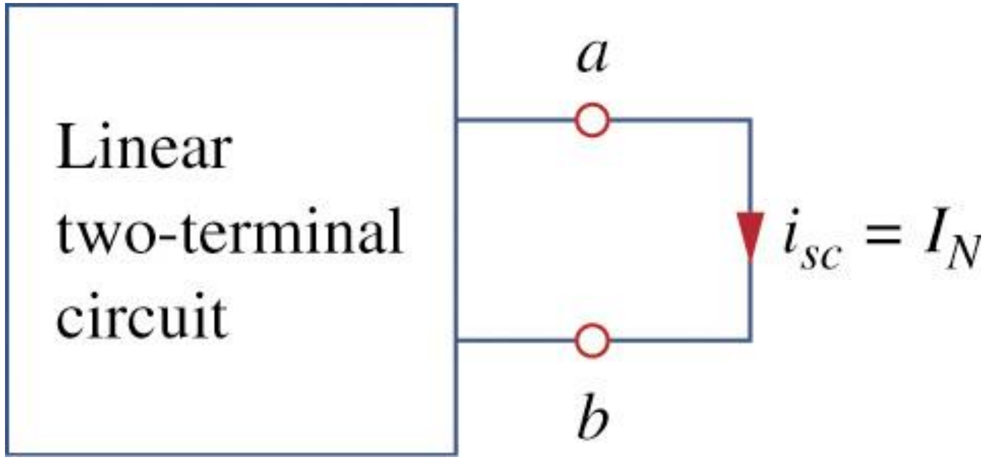
How to Find Norton Current

Thevenin and Norton resistances are equal:

$$\boxed{R_N = R_{\mathrm{Th}}}$$

Norton's Current=Short circuit current from *a* to *b*

$$I_N = i_{sc}$$



Nortons Resistance is same as that of Thevinins Resitance

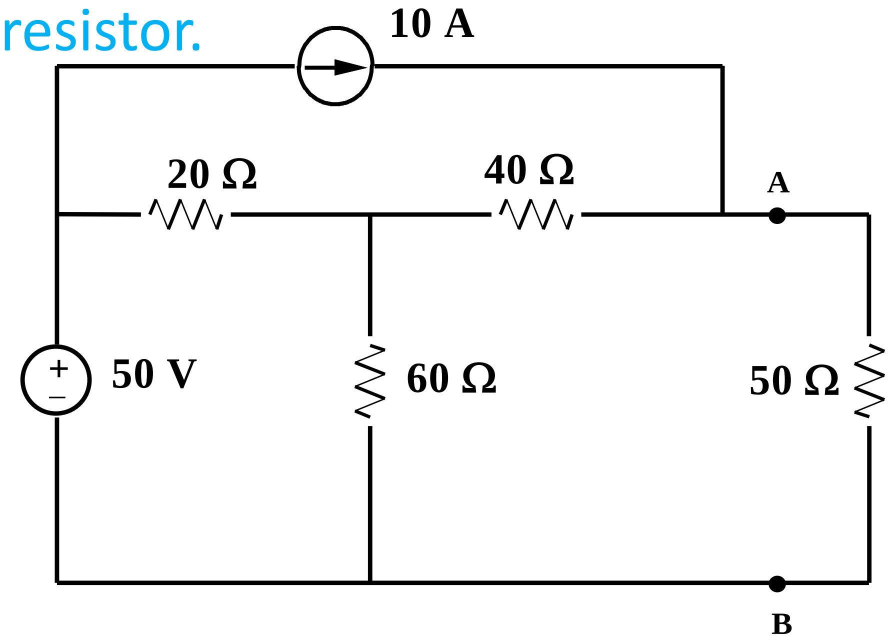
NOTE : The open circuit voltage v_{oc} across terminals *a* and *b*

The short circuit current i_{sc} at terminals *a* and *b*

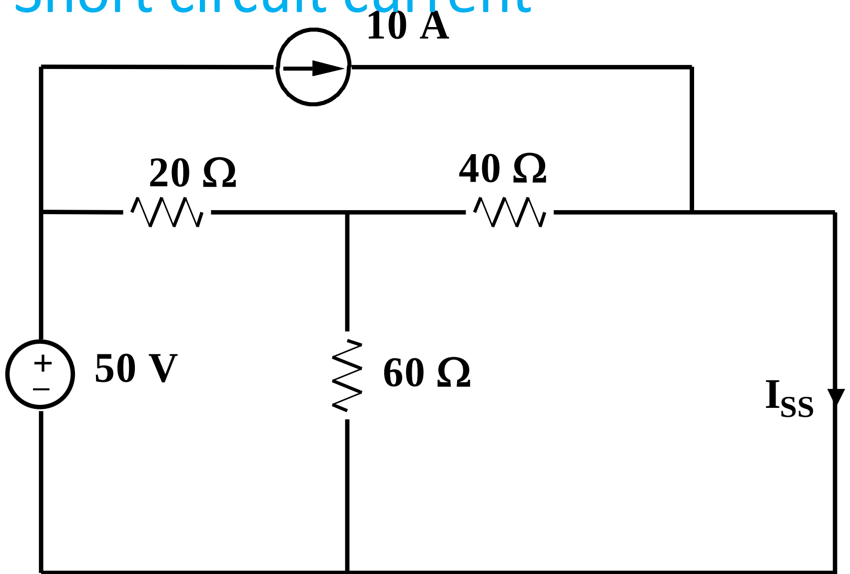
The equivalent or input resistance R_{in} at terminals *a* and *b* when all independent source are turn off.

$$V_{Th} = v_{oc} \qquad I_N = i_{sc} \qquad R_{\mathrm{Th}} = \frac{V_{\mathrm{Th}}}{I_N}$$

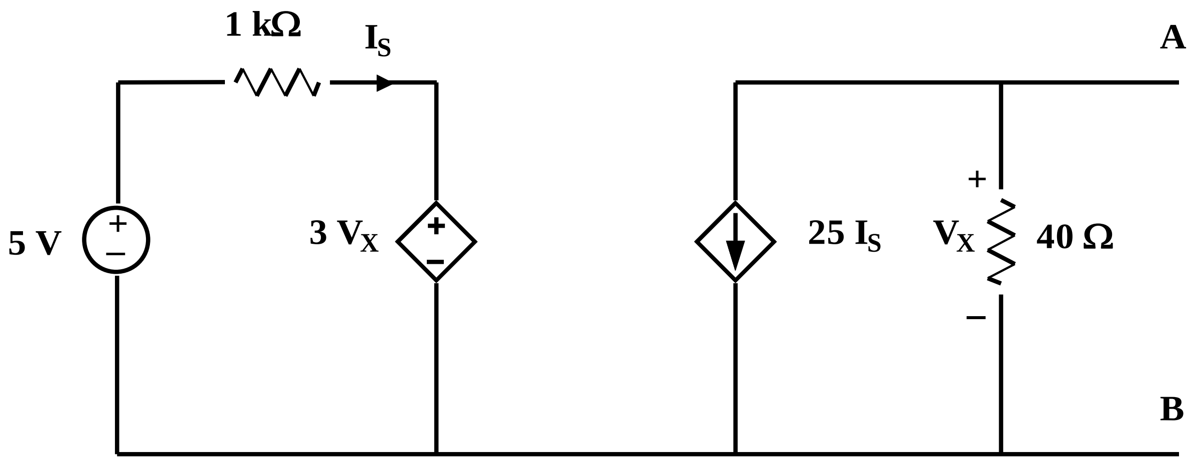
Find the Norton equivalent circuit to the left of terminals A-B and find the current in the 50 Ω resistor.



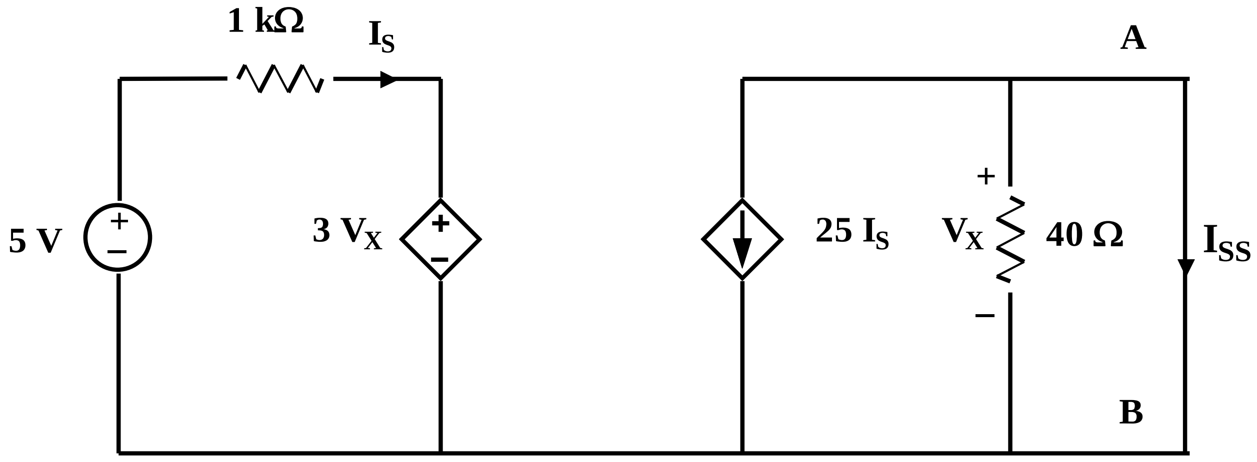
Short circuit current



For the circuit shown below, find the Norton equivalent circuit to the left of terminals A-B.



$$V_{TH} = V_X = (-25I_S)(40) = -1000I_S$$

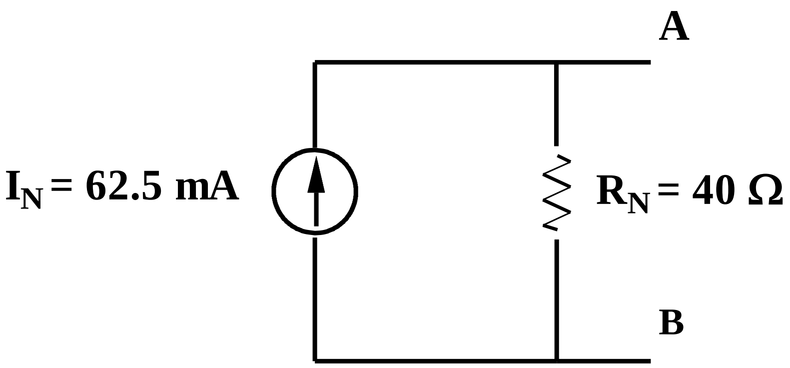


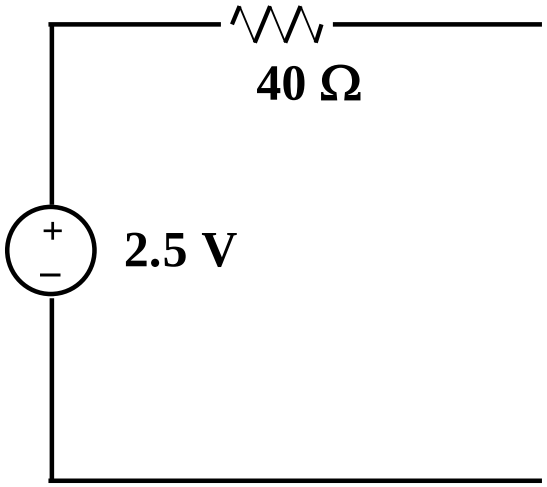
We note that $I_{SS} = -25I_S$.

$$R_N = \frac{V_{TH}}{I_{SS}} = \frac{-1000I_S}{-25I_S} = 40\ \Omega$$

$$\boldsymbol{-5 + 1000I_S + 3(-1000I_S) = 0}$$

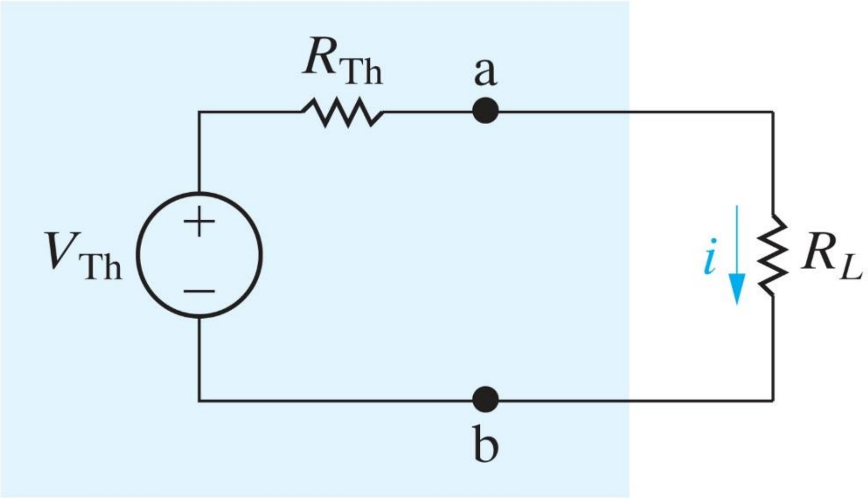
$$\boldsymbol{I_S = -2.5\ mA}$$


A
$I_N = 62.5$ mA
$R_N = 40\ \Omega$
B

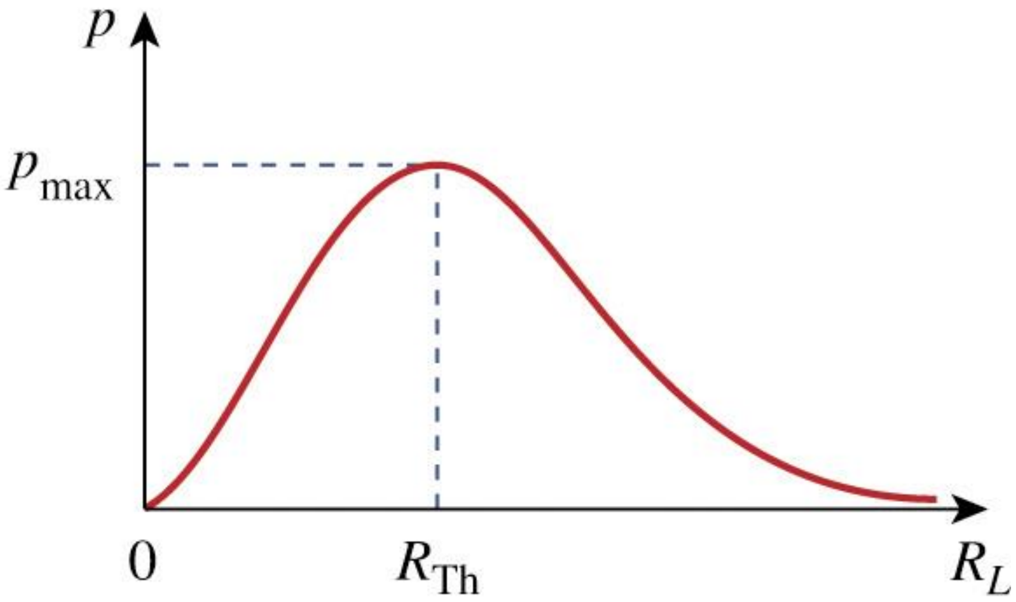

40 Ω
+
−
2.5 V

Maximum Power Transfer Theorem

Consider following circuit



R_L	$P = i^2R_L$
20	1.38
40	2.04
60	2.34
80	2.46
100	2.5
120	2.47
140	2.43
160	2.36
180	2.29



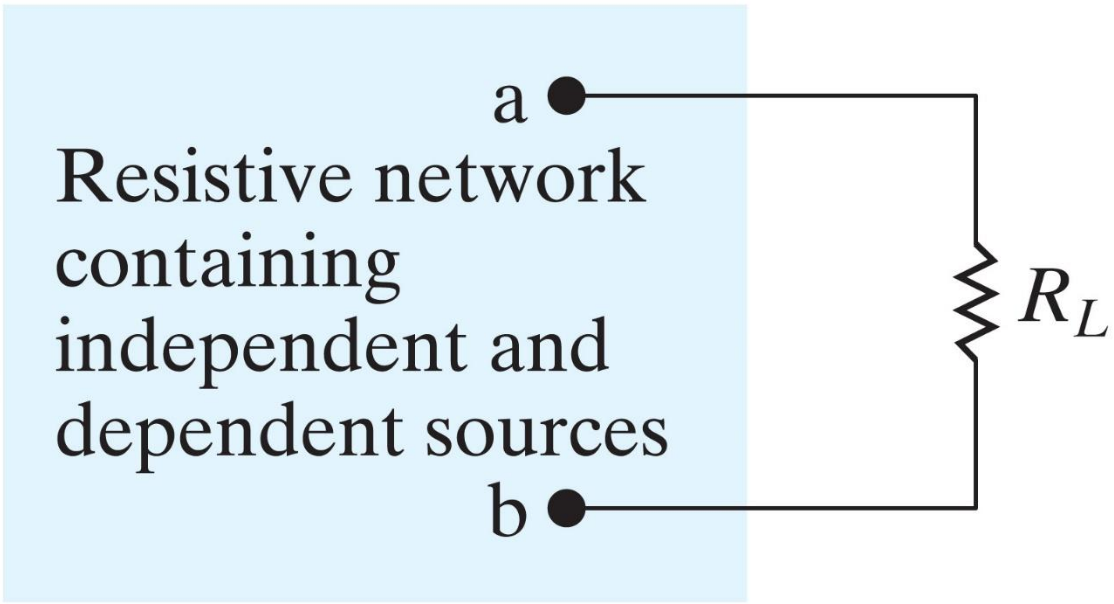
Maximum power is transferred to the load when the load resistance equals the Thevenin resistance as seen the load ($R_L = R_{TH}$).

Maximum Power Transfer Theorem

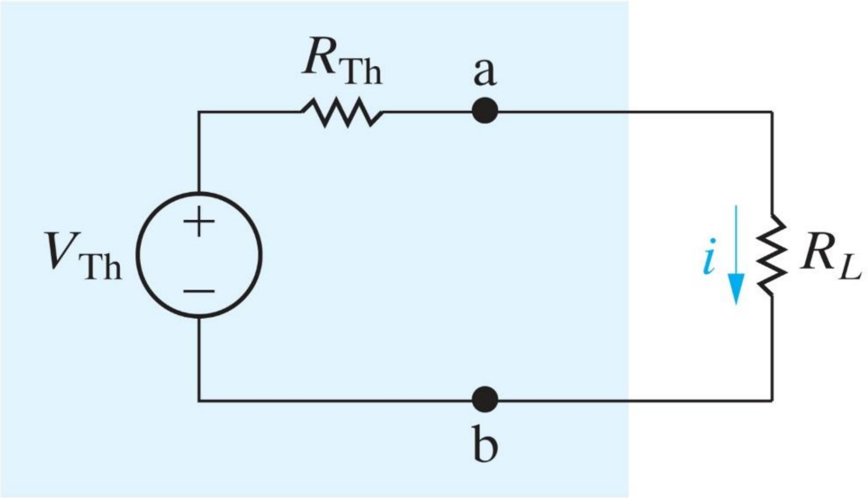
Maximize the power delivered to a resistive load

Consider the General Case

A resistive network contains independent and dependent sources.
A load is connected to a pair of terminals labeled a – b.
What value of load resistance permits maximum power delivery to the load?



Take the Thevenin equivalent of the circuit



power developed in the load

$$p = i^2 R_L = \left(\frac{V_{Th}}{R_{Th} + R_L} \right)^2 R_L$$

Find the value of R_{load} that maximizes power

$$\frac{dp}{dR_L} = V_{Th}^2 \left(\frac{(R_{Th} + R_L)^2 - 2R_L (R_{Th} + R_L)}{(R_{Th} + R_L)^4} \right) = 0$$

$$(R_{Th} + R_L)^2 = 2R_{load} (R_{Th} + R_L)$$

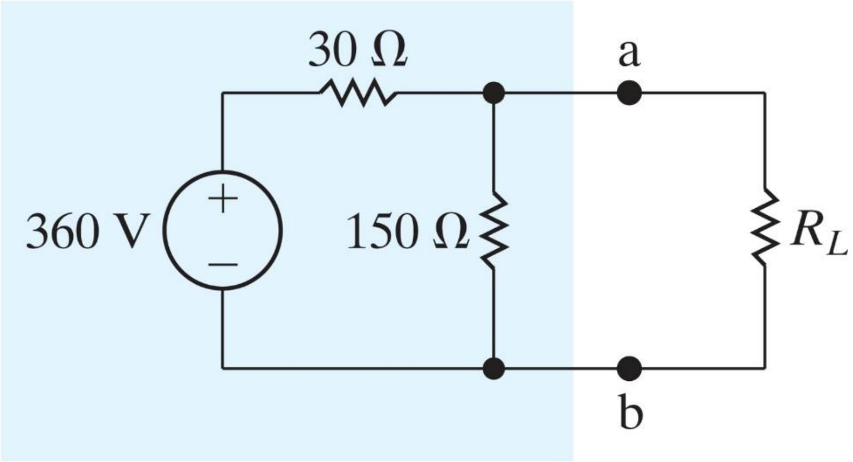
$$R_L = R_{Th}$$

The maximum power delivered to the load

$$p_{max} = I^2 R_L = \frac{V_{Th}^2}{(2R_L)^2} R_L$$

$$p_{max} = \frac{V_{Th}^2}{4R_L}$$

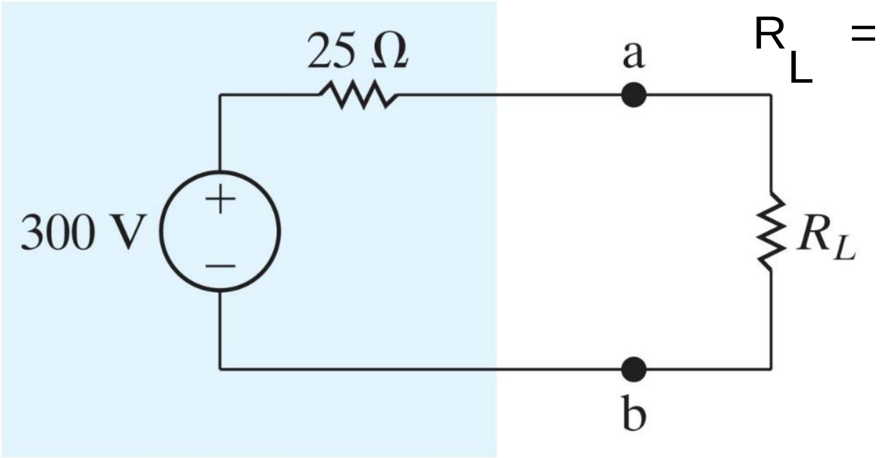
Find the value of R_L for maximum power transfer to R_L.



$$V_{Th} = \frac{150}{180}(360) = 300V$$

$$R_{Th} = \frac{(150)(30)}{150 + 30} = 25\Omega$$

Equivalent Circuit

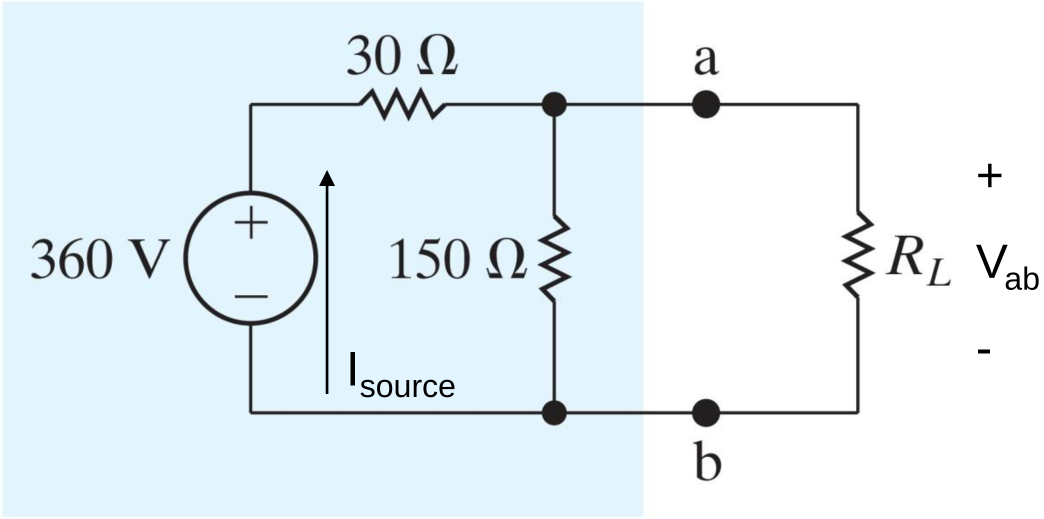


$R_L = 25\Omega$

$$p = i^2 R_L = \left(\frac{300}{50}\right)^2 (25)$$

$$p = 900W$$

- What percentage of the power delivered by the 360 V source reaches R_L?



$$V_{ab} = 150V$$

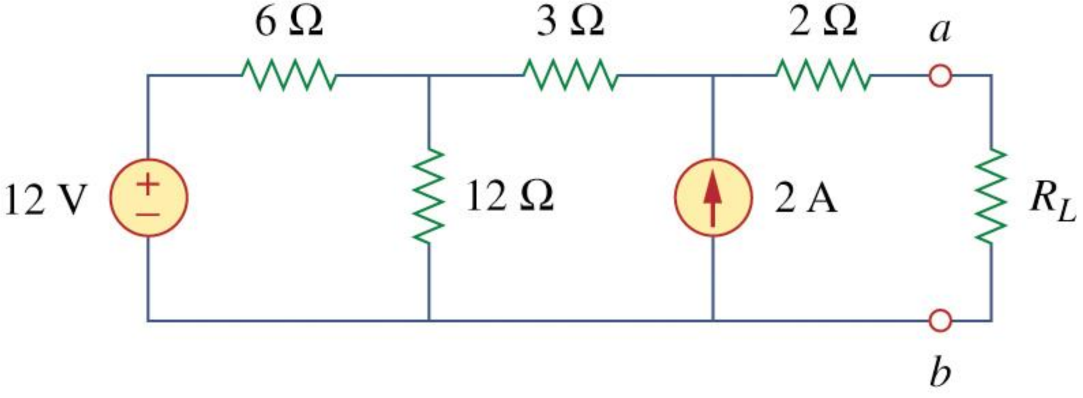
$$I_{source} = \frac{360 - 150}{30} = 7A$$

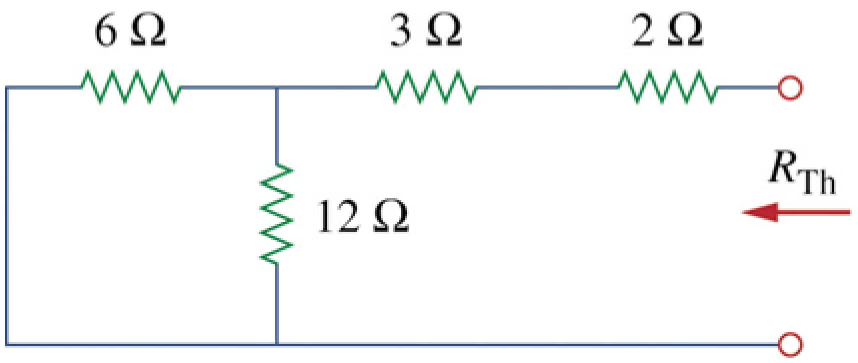
$$p_s = -I_s(360) = -2520W$$

Percentage of source power delivered to the load

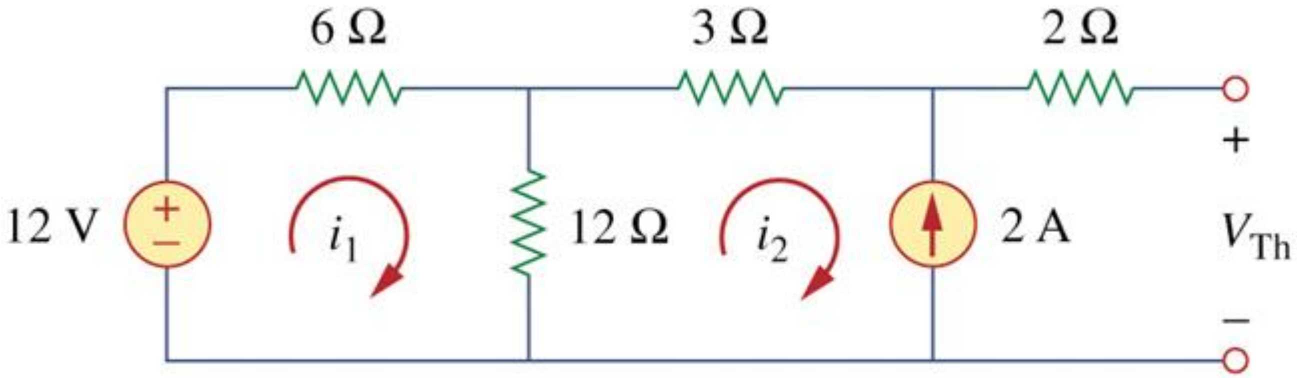
$$\frac{900}{2520} \times 100\% = 35.71\%$$

- Find the value of R_L for maximum power transfer in the circuit. Also find the maximum power.





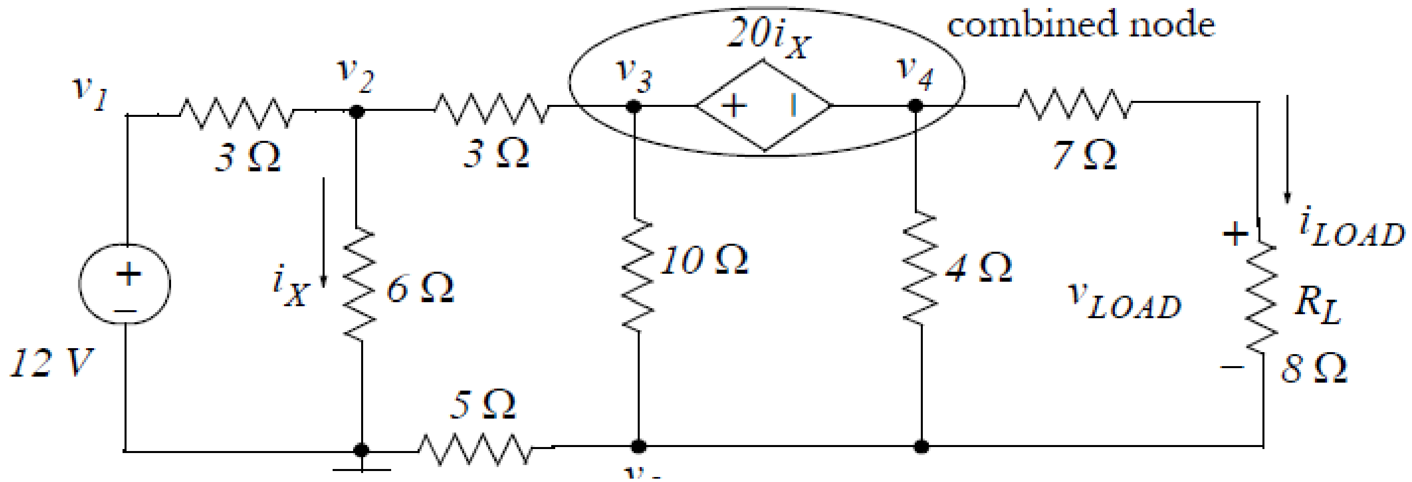
$$R_{TH} = 2+3+6\|12 = 5+\frac{6\times 12}{18} = 9\Omega$$



$$-12+18i_1-12i_2,\quad i_2=-2A$$

$$-12+6i_i1+3i_2+2(0)+V_{TH}=0 \Rightarrow V_{TH}=22V$$

$$p_{\max} = \frac{V_{TH}^2}{4R_L} = \frac{22^2}{4\times 9} = 13.44W$$

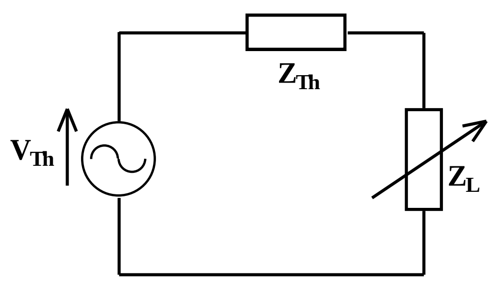


combined node
$20i_X$
v_1
v_2
v_3
v_4
3 Ω
3 Ω
7 Ω
+
−
12 V
i_X
6 Ω
10 Ω
4 Ω
v_{LOAD}
+
−
i_{LOAD}
R_L
8 Ω
5 Ω

Max Power Transfer in AC Circuits

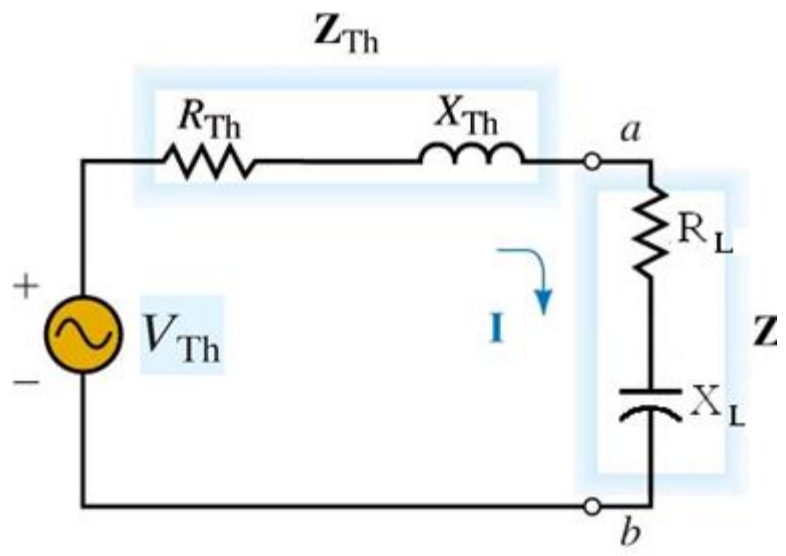
AC Circuit

(1)both load resistance and reactance are variable $Z_L = Z_{Th}^*$



(2)load being purely resistive

$R_L = |Z_L| = |Z_{Th}|$



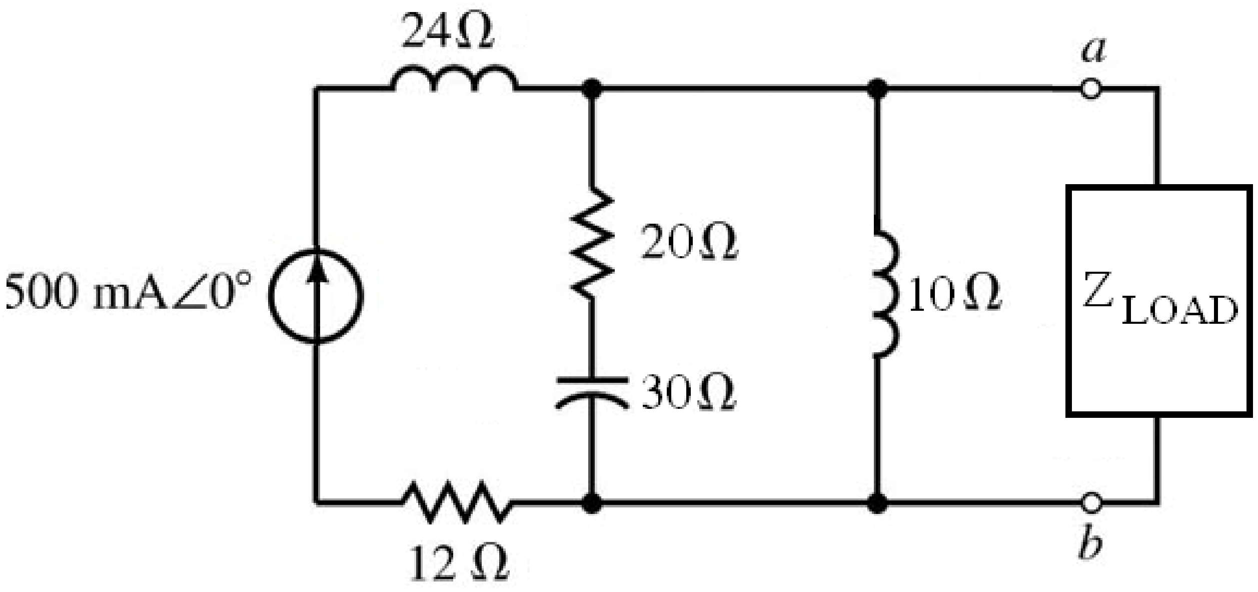
(3) load being variable resistance and fixed reactance

$Z_{Th}' = Z_{Th} \pm jX_L$

$R_L = |Z_{Th}|$

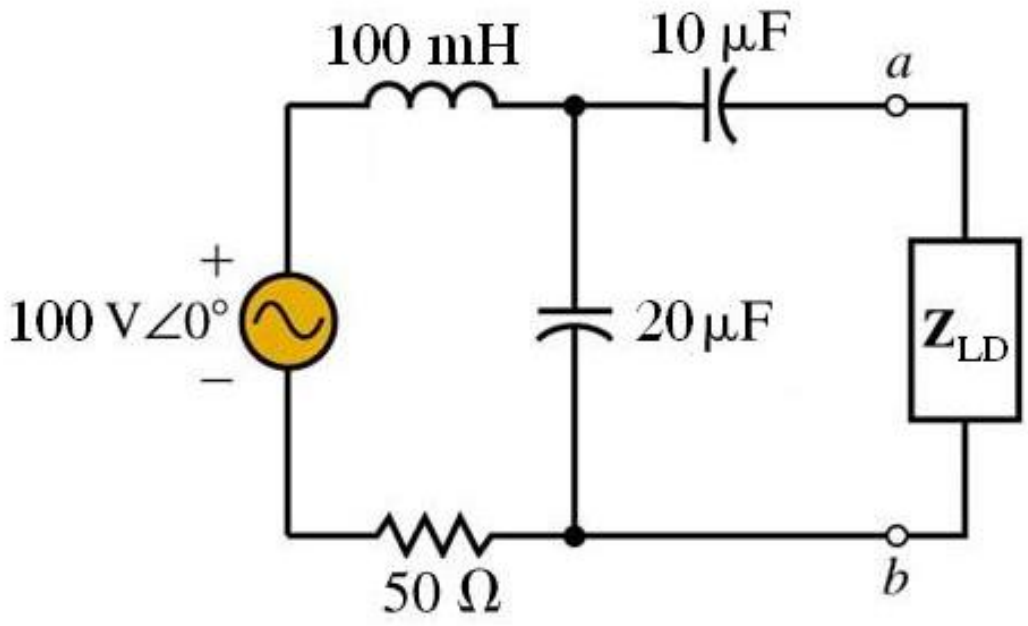
Example Problem 4

Determine the load $\mathbf{Z}_{LOAD}$ that will allow maximum power to be delivered to the load the circuit below. Find the power dissipated by the load.



Example Problem 5

Determine the load $\mathbf{Z}_{LD}$ that will allow maximum power to be delivered to the load the circuit below. Frequency is 191.15 Hz. Find the maximum power. What will happen to power if the frequency is changed to 95.575?



References:

- http://pongsak.ee.engr.tu.ac.th/le325/NetworkTheorem.pdf
- http://bapirajueca1.blogspot.com/2017/03/unit-6-network-theorems-ppt.html?m=1
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- https://www.iare.ac.in/sites/default/files/PPT/IARE_EC_PPT_1.pdf