Basic Electrical Technology

[ELE 1051]

SINGLE PHASE AC CIRCUITS

Recap

- RL, RC, RLC circuit response with AC supply
- Power associated with a series RL, RC circuits
- Loads in parallel

Corrections

- Periodic waveforms
  - Symmetrical & asymmetrical
- Obtaining Y from Z

Topics covered

- AC circuit equations and solving
- Tutorial 1

Network equations for AC circuits

KVL Equation

(Matrix form)

$$\begin{bmatrix} V_1 \\ \vdots \\ V_N \end{bmatrix} = \begin{bmatrix} Z_{11} & \cdots & Z_{1N} \\ \vdots & \ddots & \vdots \\ Z_{N1} & \cdots & Z_{NN} \end{bmatrix} \begin{bmatrix} I_1 \\ \vdots \\ I_N \end{bmatrix}$$

$$[V] = [Z][I]$$

KCL Equation

(Matrix form)

$$\begin{bmatrix} I_1 \\ \vdots \\ I_N \end{bmatrix} = \begin{bmatrix} Y_{11} & \cdots & Y_{1N} \\ \vdots & \ddots & \vdots \\ Y_{N1} & \cdots & Y_{NN} \end{bmatrix} \begin{bmatrix} V_1 \\ \vdots \\ V_N \end{bmatrix}$$

$$[I] = [Y][V]$$

All the other theorems are applicable to the AC circuits

Crammers rule

$$\begin{bmatrix} V_1 \\ \vdots \\ V_N \end{bmatrix} = \begin{bmatrix} Z_{11} & \cdots & Z_{1N} \\ \vdots & \ddots & \vdots \\ Z_{N1} & \cdots & Z_{NN} \end{bmatrix} \begin{bmatrix} I_1 \\ \vdots \\ I_N \end{bmatrix}$$

Solution for the linear simultaneous equations above is as follows

Step 1: finding the determinant

$$\Delta = \begin{vmatrix} Z_{11} & \cdots & Z_{1N} \\ \vdots & \ddots & \vdots \\ Z_{N1} & \cdots & Z_{NN} \end{vmatrix}$$

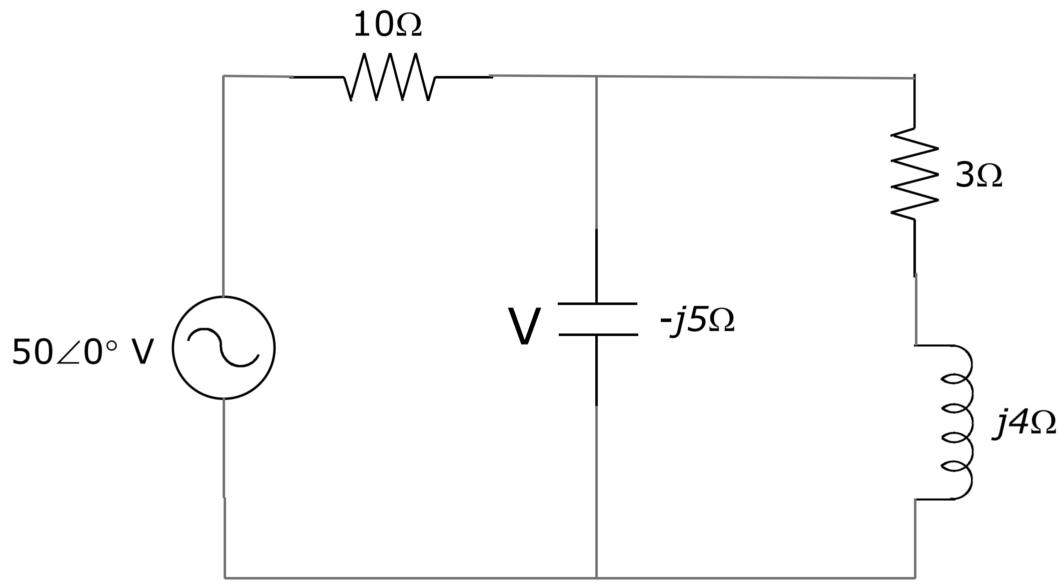
Step 2: finding the determinant after substituting first column with RHS column matrix

$$\Delta_1 = \begin{vmatrix} \boldsymbol{V_1} & \cdots & Z_{1N} \\ \vdots & \ddots & \vdots \\ \boldsymbol{V_N} & \cdots & Z_{NN} \end{vmatrix}$$

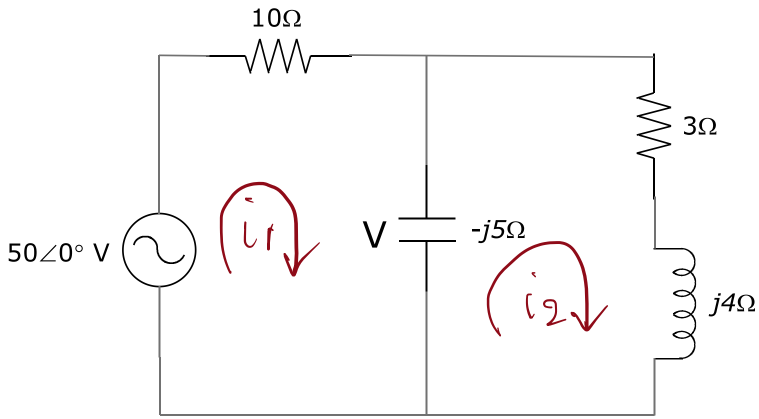
Step 3 :Solution for I_1 $\quad I_1 = \frac{\Delta_1}{\Delta}$

Illustration 1

Assigning two mesh currents, find the voltage V across the capacitor in the following circuit



$$Ans:$$
$$V = 22.36\angle - 10.30°V$$



$$\begin{bmatrix} 10 - j5 & j5 \\ j5 & 3 - j \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 50\angle 0 \\ 0 \end{bmatrix}$$

$$\Delta = 50 - j25$$

$$\Delta_{i_1} = 150 - 50j$$

$$i_1 = \frac{\Delta_{i_1}}{\Delta} = 2.8 + 0.4j \text{ A}$$
$$(2.8284 \angle 8.1301 \text{ A})$$

$$\Delta_{i_2} = -250j$$

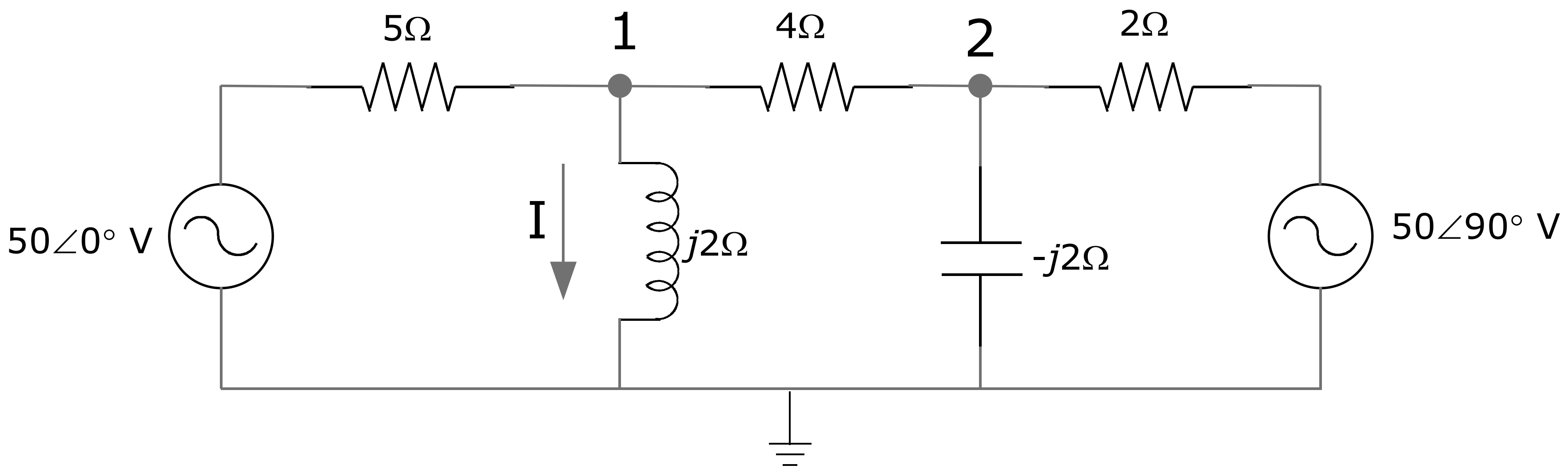
$$i_2 = \frac{\Delta_{i_2}}{\Delta} = 2 - 4j \text{ A}$$
$$(4.4721 \angle -63.43 \text{ A})$$

$$V = (i_2 - i_1)(-j5)$$

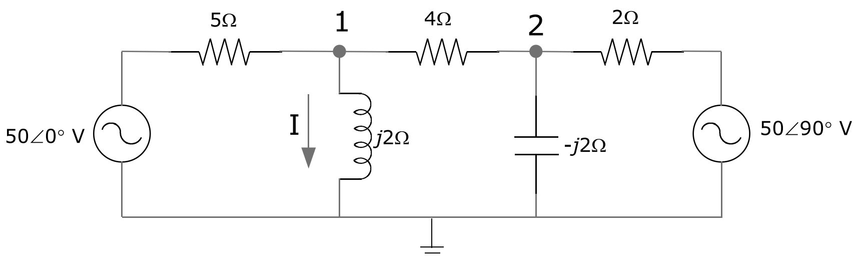
$$V = 22.36 \angle 169.69 \text{ V}$$

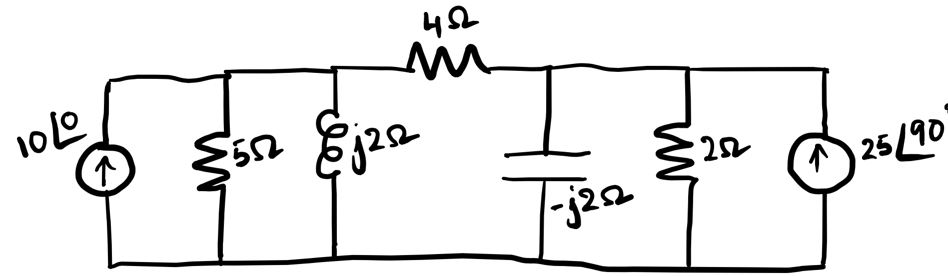
Illustration 2

Use node voltage method to obtain the current I in the network



Ans:
$I = 12.38\angle - 17.75° \, A$





$$\begin{bmatrix} \left(\frac{1}{5}+\frac{1}{2j}+\frac{1}{4}\right) & -\frac{1}{4} \\ -\frac{1}{4} & \left(\frac{1}{4}+\frac{1}{2}-\frac{1}{2j}\right) \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 10\angle 0 \\ 25\angle 90 \end{bmatrix}$$

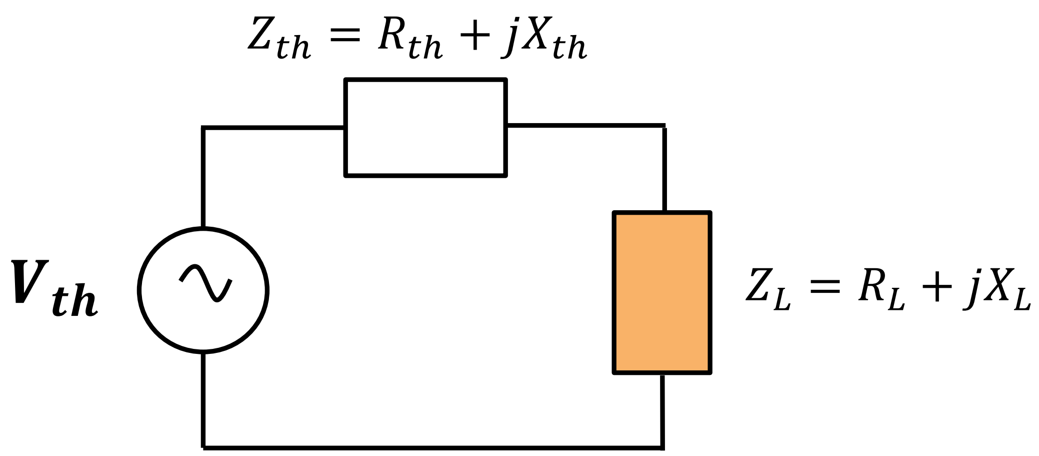
$$\Delta = 0.525 - 0.15j$$

$$\Delta_{V_1} = 7.5 + 11.25j$$

$$V_1 = \frac{\Delta_{V_1}}{\Delta} = 24.763\angle 72.25\,V$$

$$I = \frac{V_1}{2j} = 12.38\angle -17.75\,A$$

Maximum power transfer theorem



	Type of load	Condition of maximum power transfer
Case 1	Load is purely resistive	$R_L = \sqrt{R_{th}^2 + X_{th}^2}$
Case 2	Both R_L & X_L are variable	$Z_L = Z_{TH}^*$
Case 3	X_L is fixed & R_L is variable	$R_L = \sqrt{R_{th}^2 + (X_{th} + X_L)^2}$

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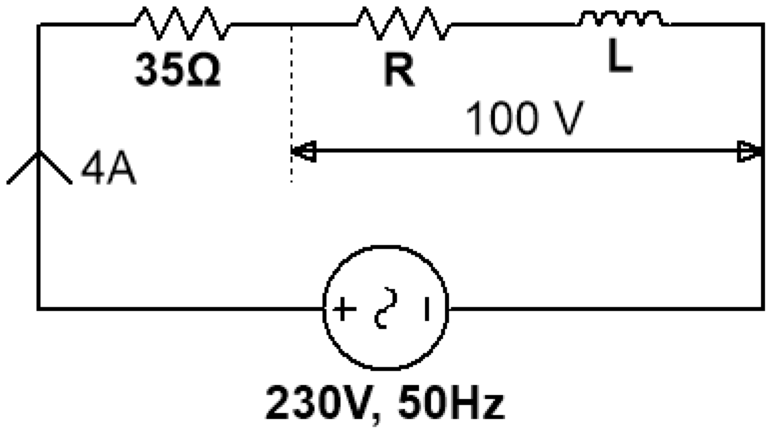
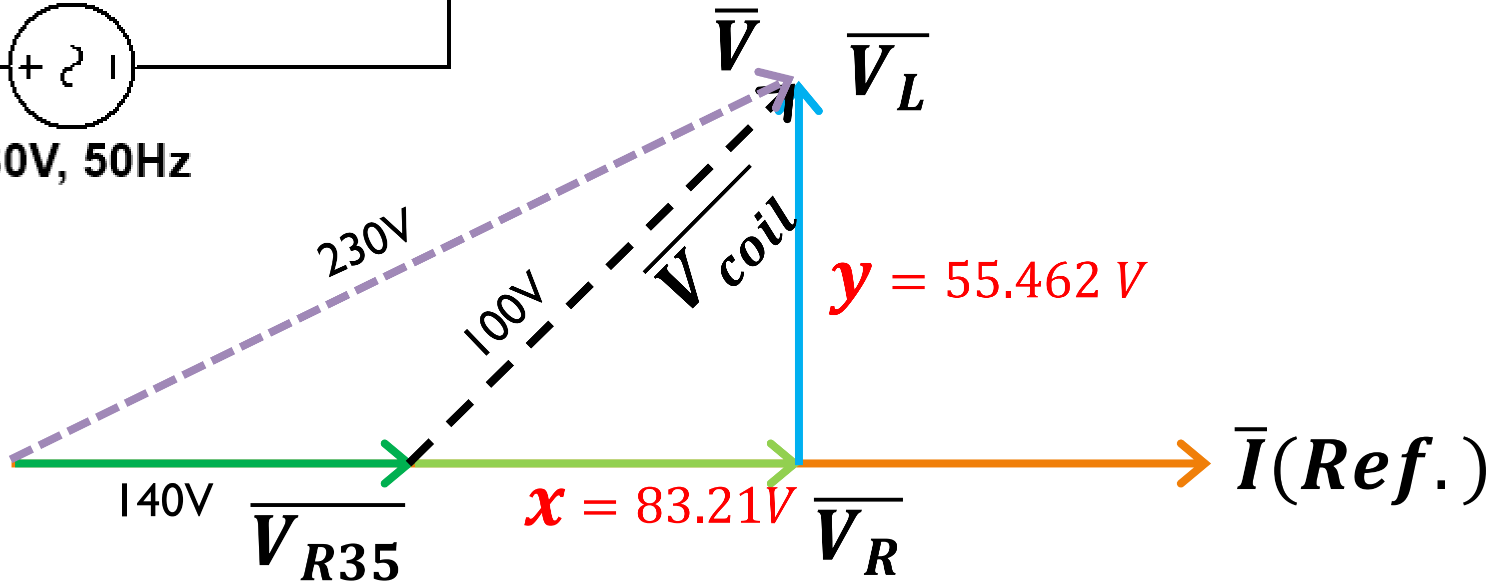
SINGLE PHASE AC CIRCUITS

Tutorial 1

Exercise 1

A resistance of 35Ω is connected in series with an inductive coil having an internal resistance 'R' and inductance 'L'. When connected across 230V, 50Hz single phase supply, voltage across the coil is 100V and the current drawn is 4 A. Find the unknowns 'R' and 'L'.





$$(140 + x)^2 + y^2 = 230^2$$

$$x^2 + y^2 = 100^2$$

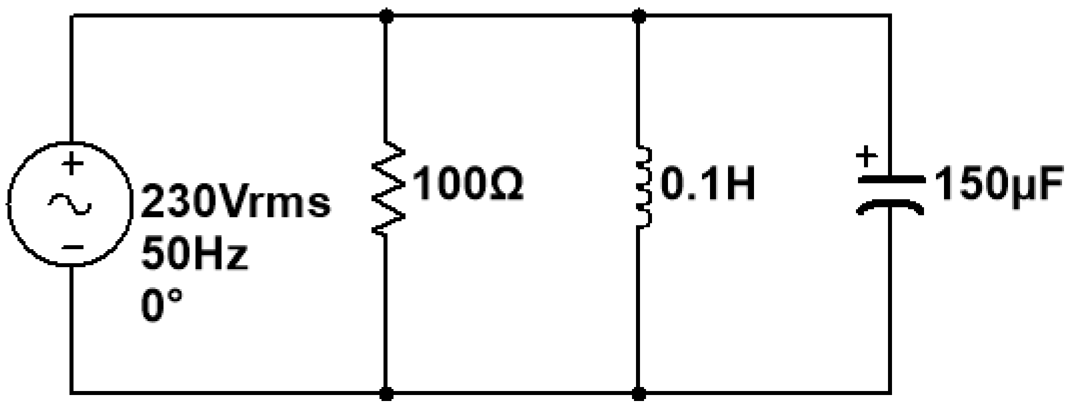
$$\boldsymbol{R} = \frac{V_R}{I} = \frac{x}{I} = \frac{83.21}{4} = \mathbf{20.80\Omega}$$

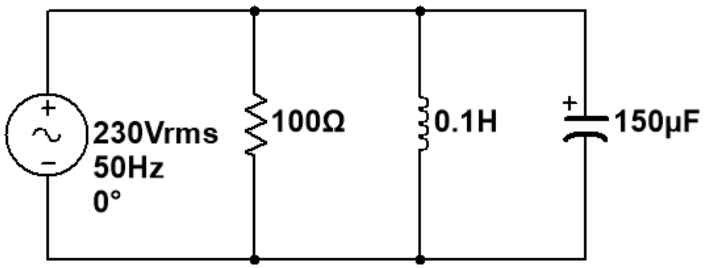
$$X_L = \frac{V_L}{I} = \frac{y}{I} = \frac{55.462}{4} = 13.8655\Omega$$

$$\therefore \boldsymbol{L} = \mathbf{0.044H}$$

Exercise 2

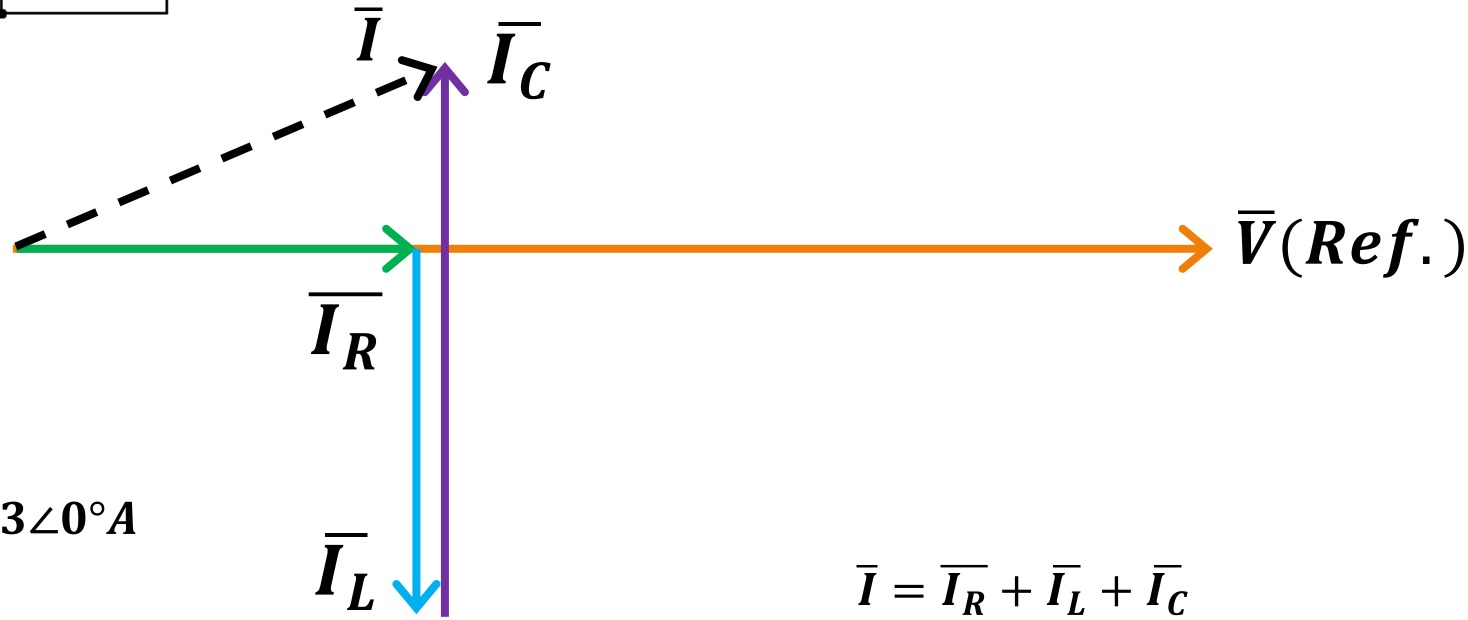
Three elements, a resistance of 100Ω, an inductance of 0.1H and a capacitance of 150μF are connected in parallel to a 230V, 50Hz supply. Calculate the current in each element and the supply current. Draw the phasor diagram.





$X_L = 31.4159\Omega$

$X_C = 21.2206\Omega$



$$\bar{I}_R = \frac{230\angle 0°}{100} = \mathbf{2.3\angle 0° A}$$

$$\bar{I}_L = \frac{\bar{V}}{jX_L} = \frac{230\angle 0°}{j31.4159} = \mathbf{7.3211\angle -90° A}$$

$$\bar{I}_C = \frac{\bar{V}}{-jX_C} = \frac{230\angle 0°}{-j21.2206} = \mathbf{10.83\angle 90° A}$$

$$\bar{\boldsymbol{I}} = \bar{\boldsymbol{I}}_R + \bar{\boldsymbol{I}}_L + \bar{\boldsymbol{I}}_C$$

$$\bar{\boldsymbol{I}} = \mathbf{4.2\angle 56.819° A}$$

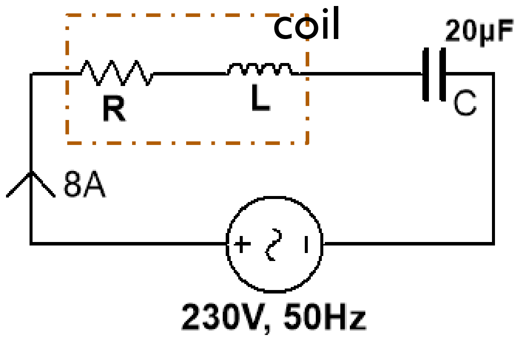
Exercise 3

A coil is in series with a 20μF capacitor across a 230 V 50 Hz supply. The current taken by the circuit is 8A and power consumed is 200W.

Calculate the inductance of the coil if the power factor of the circuit is lagging.

Calculate the inductance of the coil if the power factor of the circuit is leading.

Draw the phasor diagram.

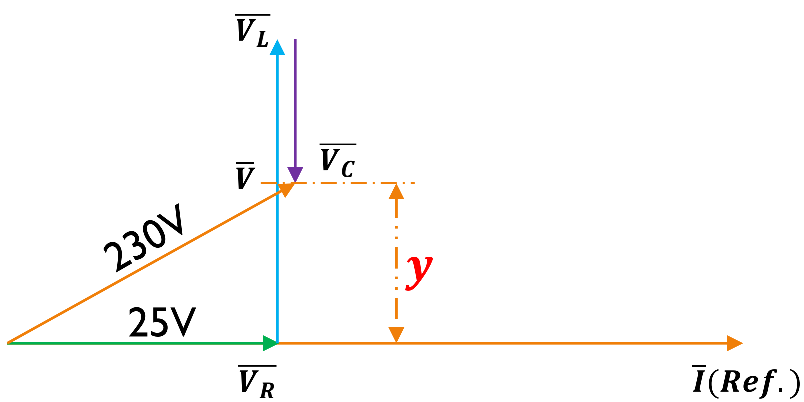


$$R = \frac{P_R}{I^2} = \frac{200}{8^2} = \mathbf{3.125\ \Omega}$$

$$\boldsymbol{V_R = 25\ V}$$

$$V_C = IX_C = \frac{8}{2\pi \times 50 \times 20\mu} = 1273.2395\ V$$

Case 1
(p.f. is lagging)



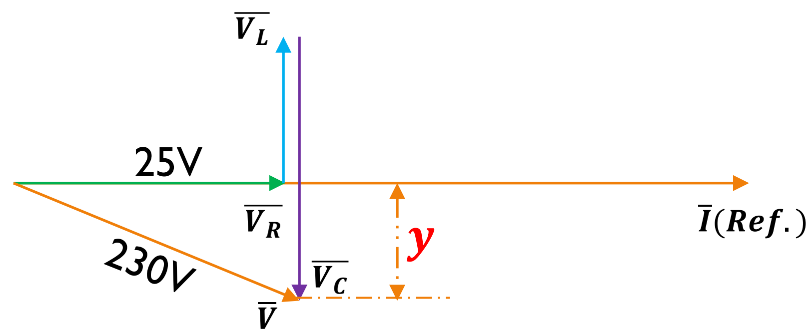
$$25^2 + y^2 = 230^2$$

$$y = 228.6372$$

$$\therefore V_L = V_C + y = 1501.8695\ V$$

$$X_L = \frac{V_L}{I} = 187.7336\Omega \qquad \boldsymbol{L = 0.5975H}$$

Case 2
(p.f. is leading)



$$25^2 + y^2 = 230^2$$

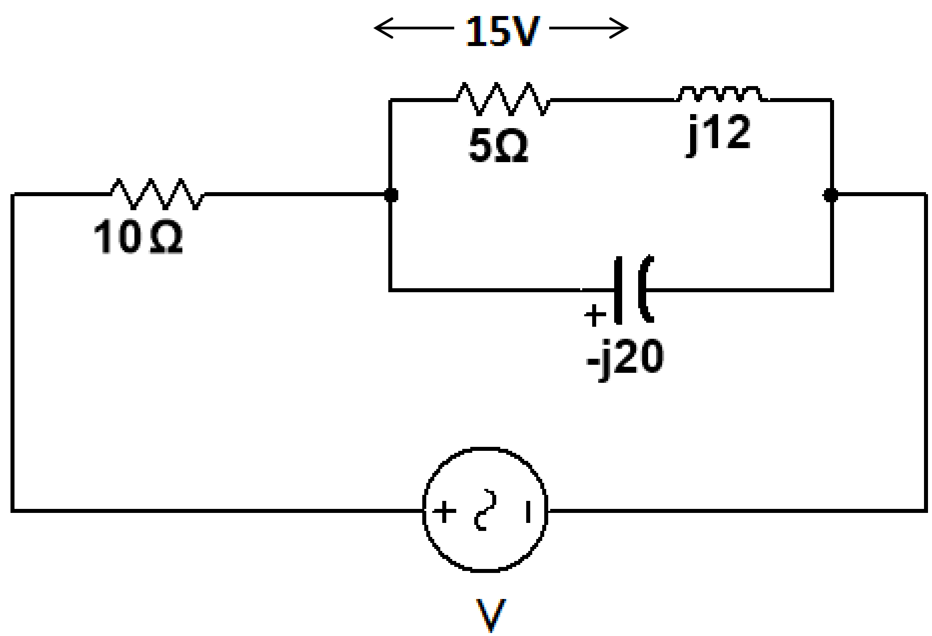
$$y = 228.6372$$

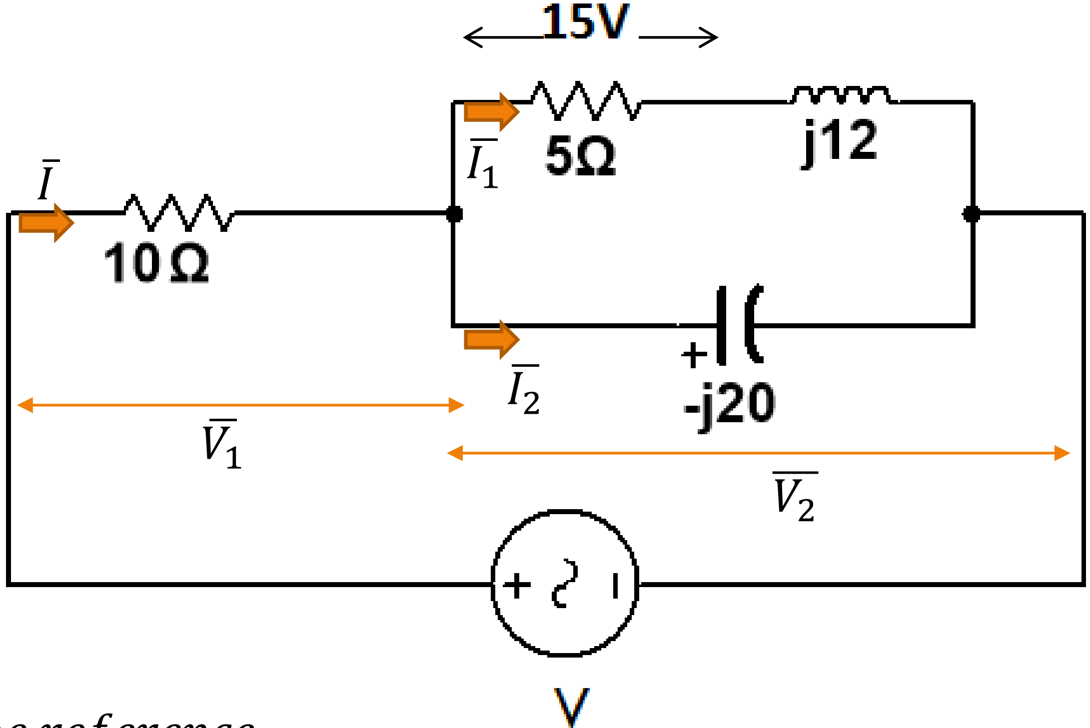
$$\therefore V_L = V_C - y = 1044.6023\ V$$

$$X_L = \frac{V_L}{I} = 130.575\Omega \qquad \boldsymbol{L = 0.4156H}$$

Exercise 4

Find the supply voltage, total current and the value of the power consumed in each arm of the series parallel circuit shown. The voltage across the 5Ω resistor is 15V.





Assume $\overline{V_2}$ *as the reference*

$$|I_1| = \frac{15}{5} = 3A \qquad \tan^{-1}\frac{X_L}{R} = \tan^{-1}\frac{12}{5} = 67.38°$$

$$\therefore \boldsymbol{\overline{I_1} = 3\angle - 67.38°\ A}$$

$$\overline{V_1} = \overline{I} \times 10 = 14.15\angle - 35.3746°\ A$$

$$\overline{V_2} = \overline{I_1} \times (5 + j12) = 39\angle 0°\ V$$

$$\boldsymbol{\overline{V} = \overline{V_1} + \overline{V_2} = 51.1972\angle - 9.207°\ V}$$

$$\therefore \boldsymbol{\overline{I_2} = \frac{\overline{V_2}}{-j20} = 1.95\angle 90°\ A}$$

$$\boldsymbol{P_{5\Omega} = I_1^2 \times 5 = 45W}$$

$$\boldsymbol{\overline{I} = \overline{I_1} + \overline{I_2} = 1.415\angle - 35.3746°\ A}$$

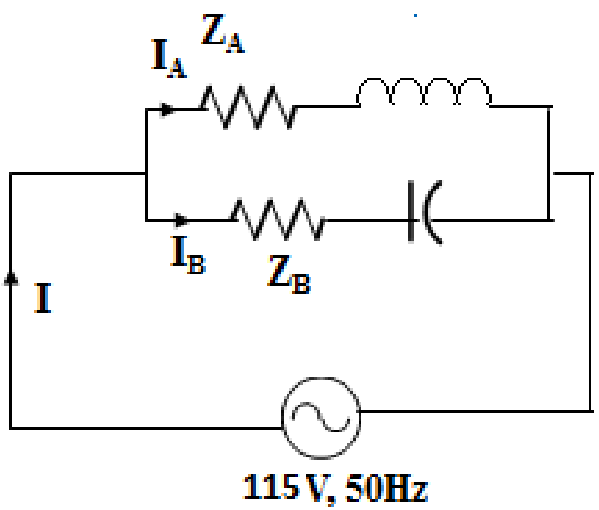
$$\boldsymbol{P_{10\Omega} = I^2 \times 10 = 20.022\ W}$$

Exercise 5

Two impedances Z_A and Z_B are connected in parallel across a 115V, 50Hz supply. The total current taken by the combination is 10A at unity p.f. Z_B has resistance of 10Ω and 200μF capacitor connected in series. Z_B consists of a resistor and inductor is series. Find

(a) The current in each branch

(b) The resistance and inductance of Z_A



Assume supply voltage as the reference $\Longrightarrow$ $\bar{V} = 115\angle 0°\ V$

Given, supply current, $\bar{I} = 10\angle 0°\ A$

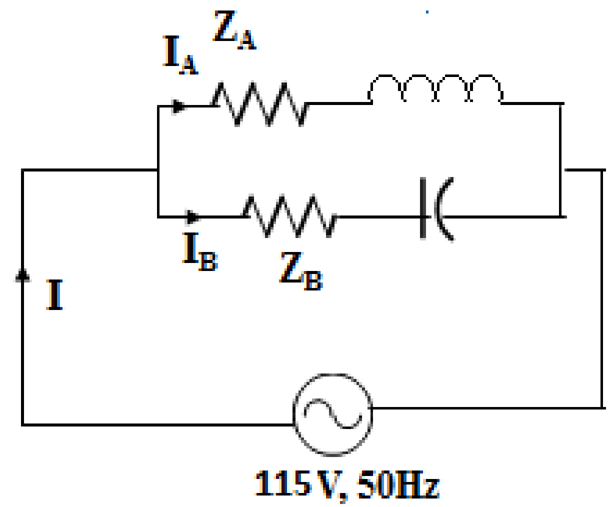
$X_C = 15.9154\ \Omega$ $\qquad\qquad Z_B = 10 - j15.9154\ \Omega$

$$\overline{\boldsymbol{I_B}} = \frac{\bar{\boldsymbol{V}}}{\boldsymbol{Z_B}} = \mathbf{6.1182\angle 57.8579°}\ \boldsymbol{A}$$

$$\bar{I} = \bar{I_A} + \bar{I_B}$$

$$\overline{\boldsymbol{I_A}} = \mathbf{8.5048\angle - 37.5261°}\ \boldsymbol{A}$$

$$Z_A = \frac{\bar{V}}{\overline{I_A}} = \underbrace{10.7237}_{\boldsymbol{R_A}} + \underbrace{j8.2364}_{\boldsymbol{jX_A}}\ \Omega$$



$$\boldsymbol{R_A} = \mathbf{10.72}\ \boldsymbol{\Omega}$$

$$\boldsymbol{L_A} = \mathbf{0.0262}\ \boldsymbol{\Omega}$$