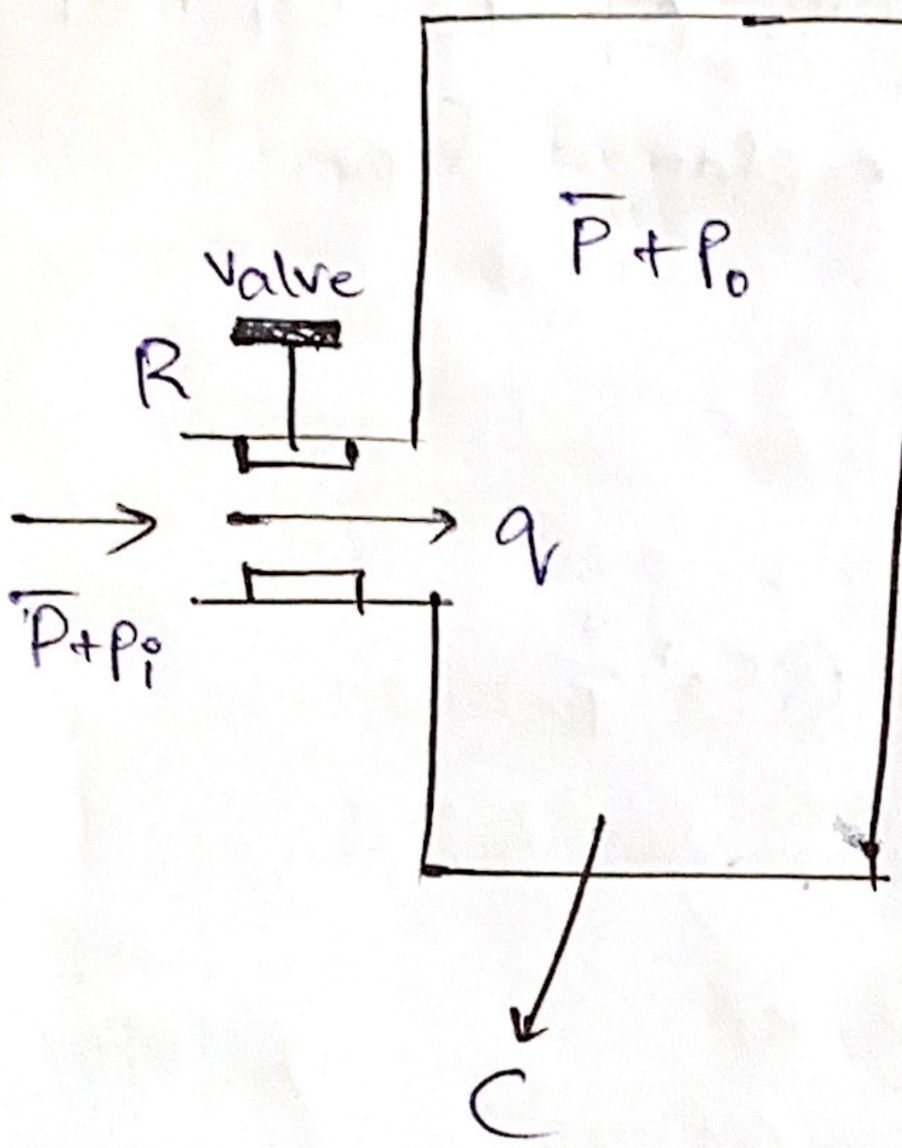
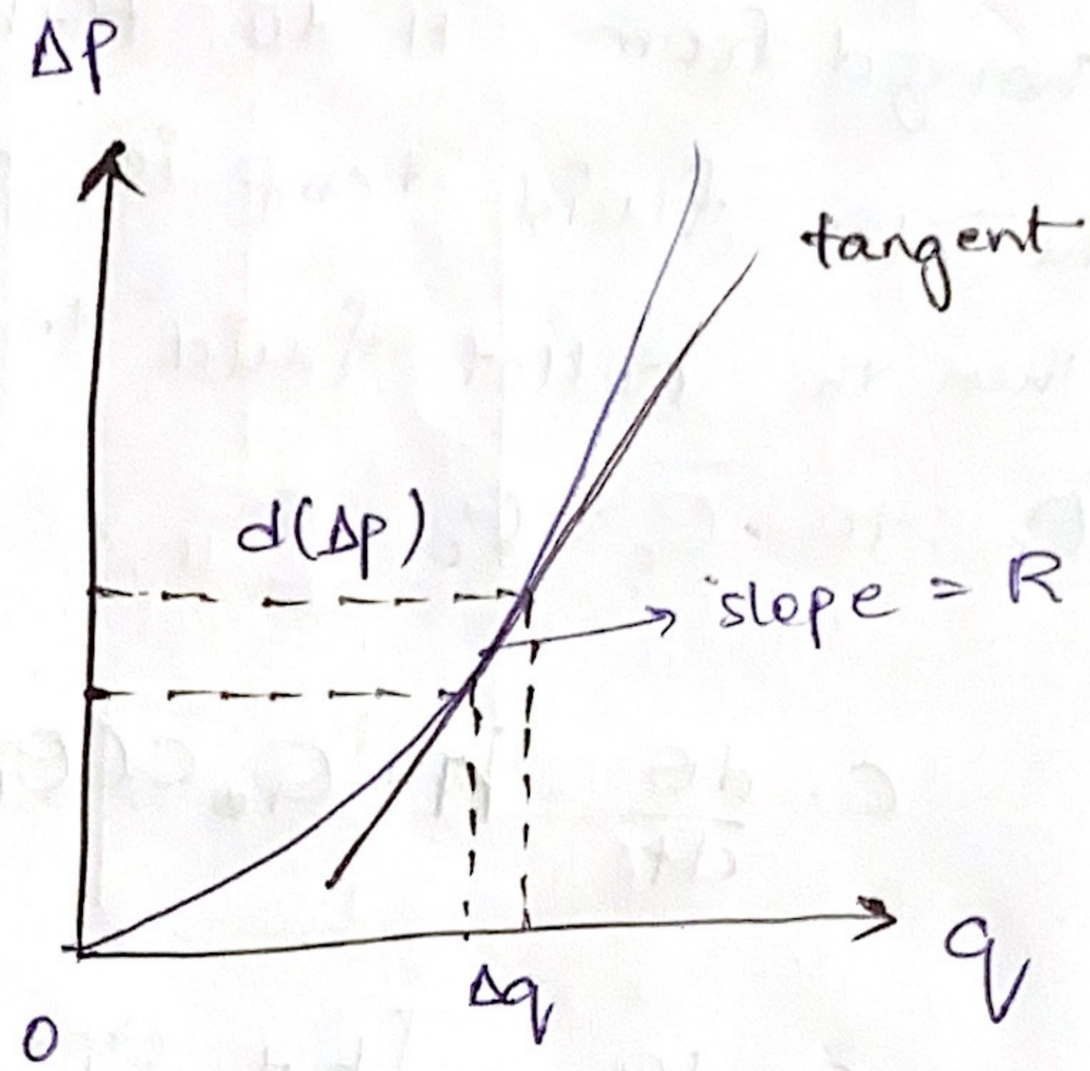
Pneumatic systems





Consider the pressure system, the gas flow through the restriction is a function of the gas pressure diff $P_i - P_o$

$$\text{Resistance, } R = \frac{\text{change in gas pressure diff}}{\text{change in gas flow rate}} \quad \frac{lbf/ft^2}{lb/sec}$$

$$= \frac{d(\Delta P)}{dq} = \frac{P_i - P_o}{q}$$

where $d(\Delta p)$ is a small change in gas pressure difference and dq is the small change in the gas flow rate.

$$\text{capacitance, } C = \frac{\text{change in gas stored}}{\text{change in gas pressure}} \quad \frac{lb}{lbf/ft^2}$$

$$C = \frac{dm}{dp}$$

$$C \cdot dP_0 = q \cdot dt$$

$$C \frac{dP_0}{dt} = q$$

$$C \frac{dP_0}{dt} = \frac{P_i - P_0}{R}$$

$$\frac{P_0(s)}{P_i(s)} = \frac{1}{RCs+1}$$

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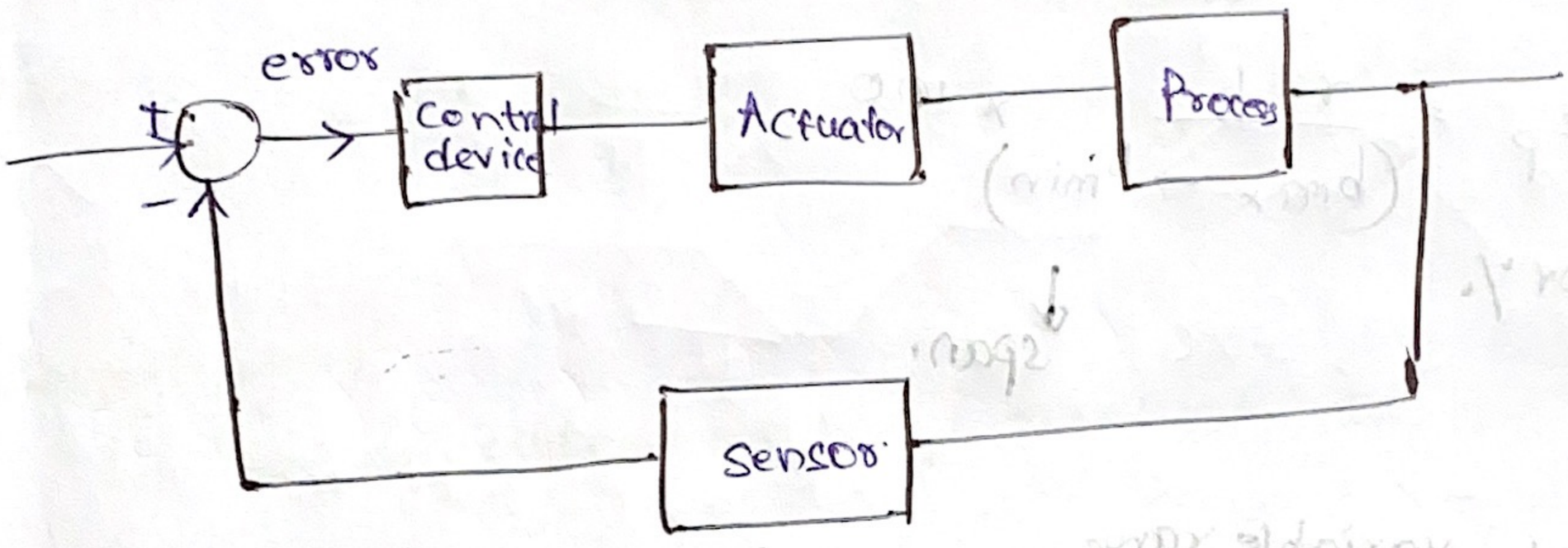
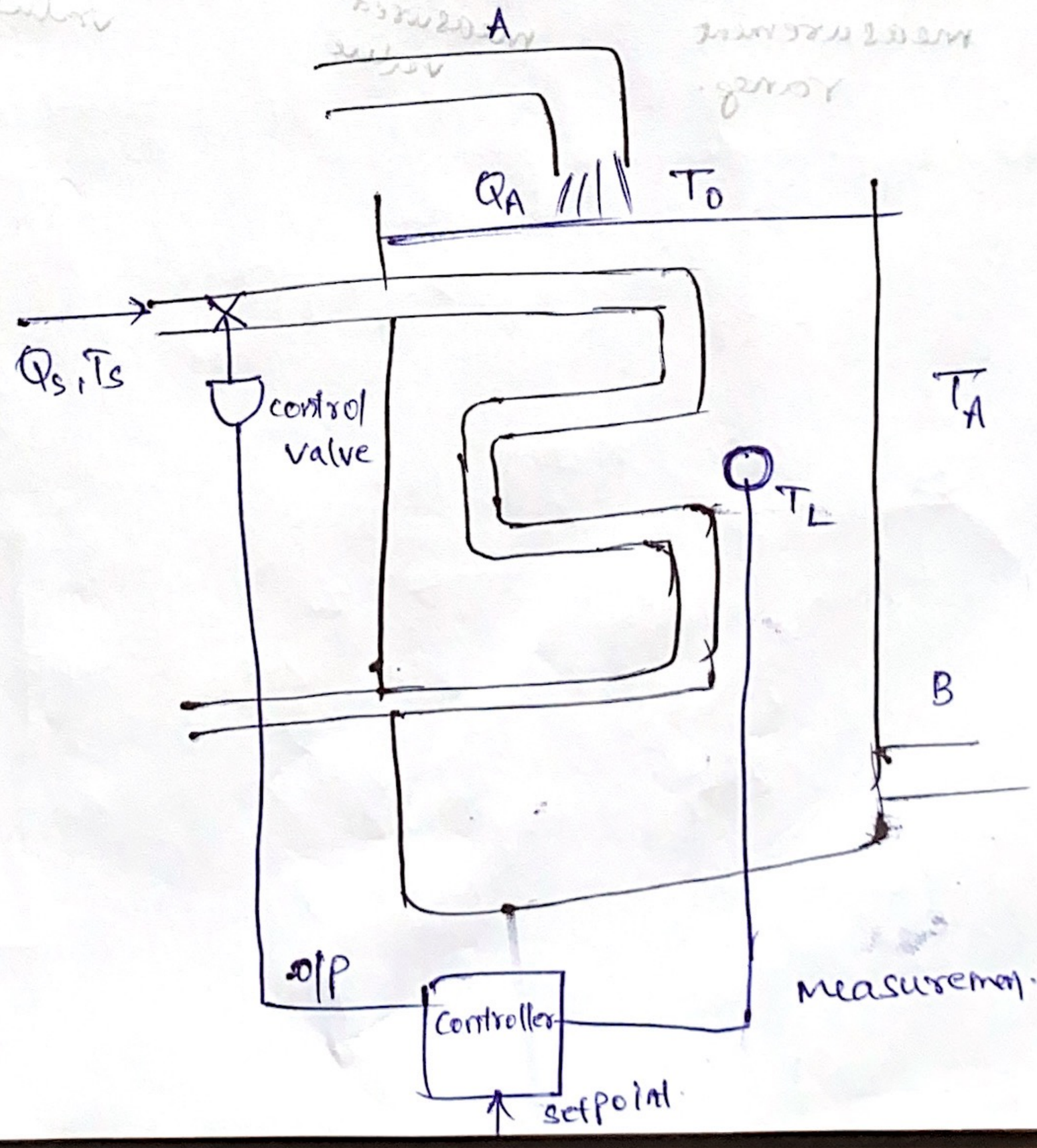


ex:-

$$T_L = F(Q_A, Q_B, Q_S, T_A, T_S, T_0)$$



arfg

arfrg

Process characteristics

i) Process equation
ii) Process Load
iii) Process Lag
iv) Self regulation.

Control system parameters

$$e = r - b$$

error

$$e_p = \frac{r - b}{(b_{max} - b_{min})} \times 100$$

error %

span.

Measured variable range.

actual measured value

$$C_p = \frac{C - C_{min}}{C_{max} - C_{min}} \times 100$$

measured value as % of measurement range.

Max of measured value

Min of measured value

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Control lag when PV gets a sudden change, then the time taken for process control loop to make necessary adjust to the final control element is called as

Dead time → time betw the instant a deviation occurs and when the corrective action first occurs.

Cycling → oscillation of dynamic variable error about zero.

Controller modes.

Direct CV ↑ CO ↑ ex. level control

Reverse CV ↑ CO ↓ ex: temp control

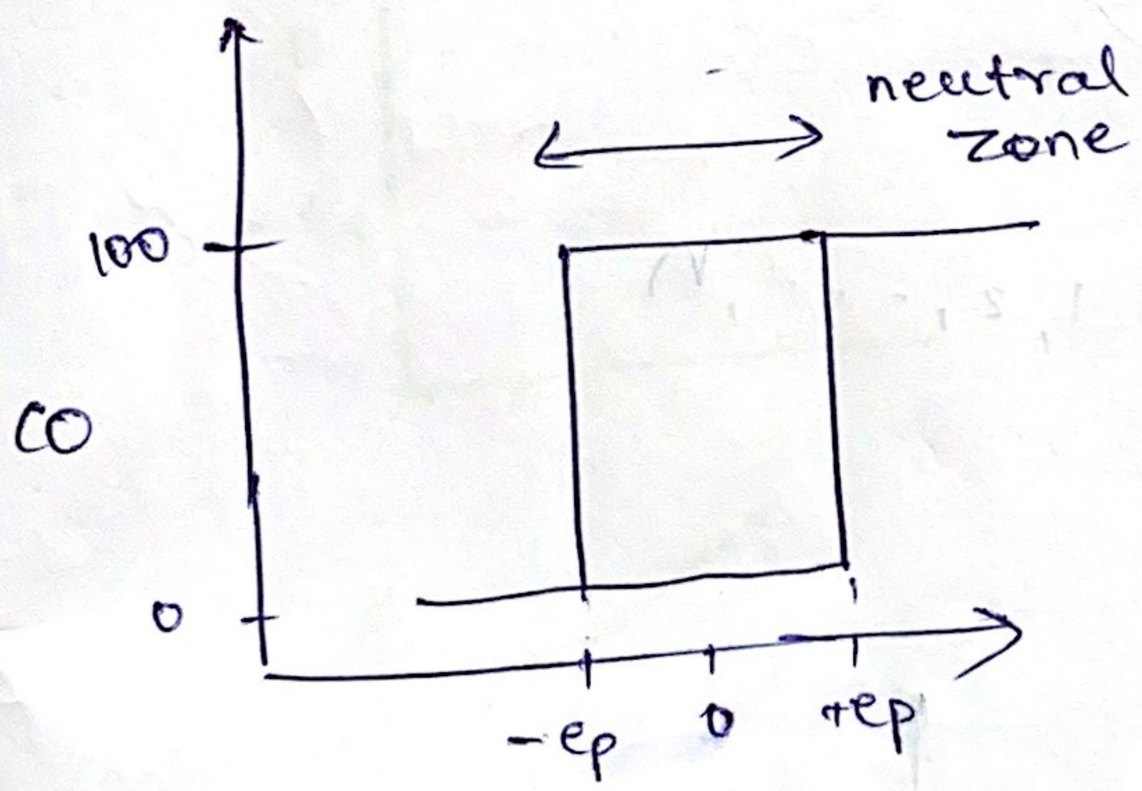
Discontinuous.

i) Two-position mode (ON/OFF).

$$P = \begin{cases} 0\% & e_p < 0 \\ 100\% & e_p > 0 \end{cases}$$

e_p = error

= setpoint − measured value



Q. A liquid level control system linearly converts a displacement of 2 to 3 m into a 4-20 mA control signal. A relay serves as the two position controller to open or close an inlet valve. The relay closes at 12mA and opens at 10mA. Find

a) relation b/w displacement level and current

Sol.

$$2 = m(4) + C$$
$$3 = m(20) + C$$
$$+$$
$$5 = 24m + 2C$$

$$(y_2 - y_1) = m(x_2 - x_1)$$
$$\frac{3-2}{20-4} = m$$
$$\frac{1}{16} m \Rightarrow m = 0.0625 \, m$$

$$5 = \frac{\cancel{24}^{\cancel{6}\,3}}{\cancel{16}_{\cancel{4}\,2}} + 2C$$

$$5 - 1.5 = 2C \Rightarrow \frac{3.5}{2} = C \Rightarrow C = 1.75 \, m$$

$$x_1 = \frac{1}{16}(12) + 1.75 = 2.5 m$$

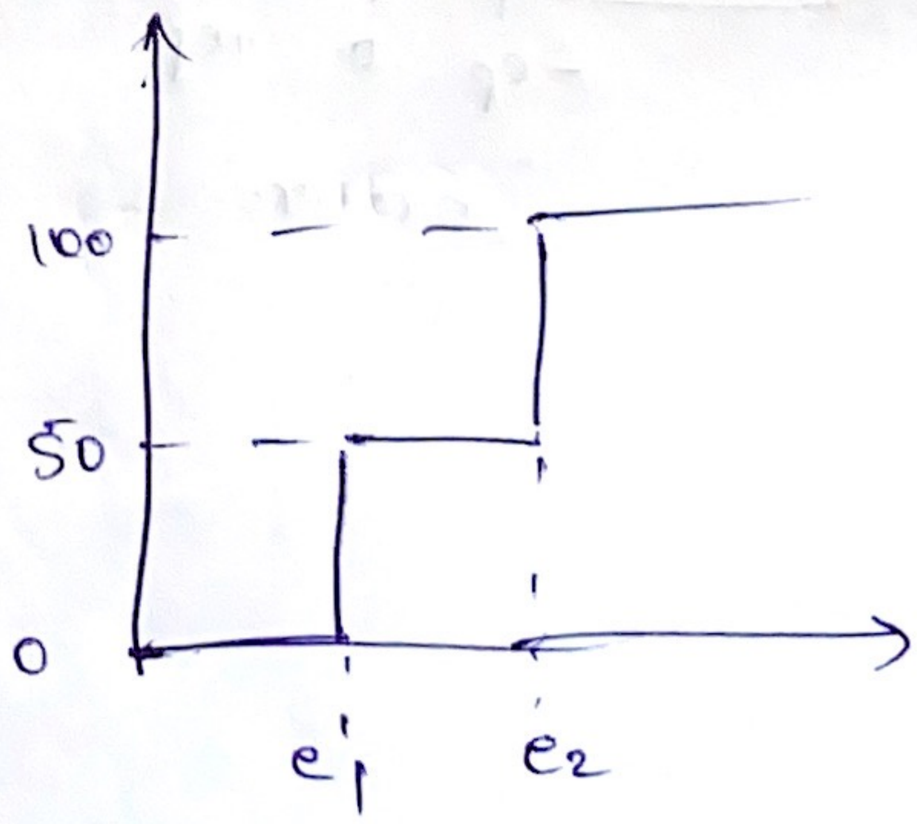
$$x_2 = \frac{1}{16}(10) + 1.75 = 2.375 \, m$$

Multiposition mode.

$$P = P_i \qquad e_p > |e_i| \quad i = 1, 2, \ldots, n$$

3- position mode.

$$P = \begin{cases} 100 & e_p > e_2 \\ 50 & e_1 < e_p < e_2 \\ 0 & e_p < e_1 \end{cases}$$



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$$\frac{1}{16} m \Rightarrow m = 0.0625 \text{ m/mA}$$

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$$5 - 1.5 = 2C \Rightarrow \frac{3.5}{2} = C \Rightarrow C = 1.75 \text{ m}$$

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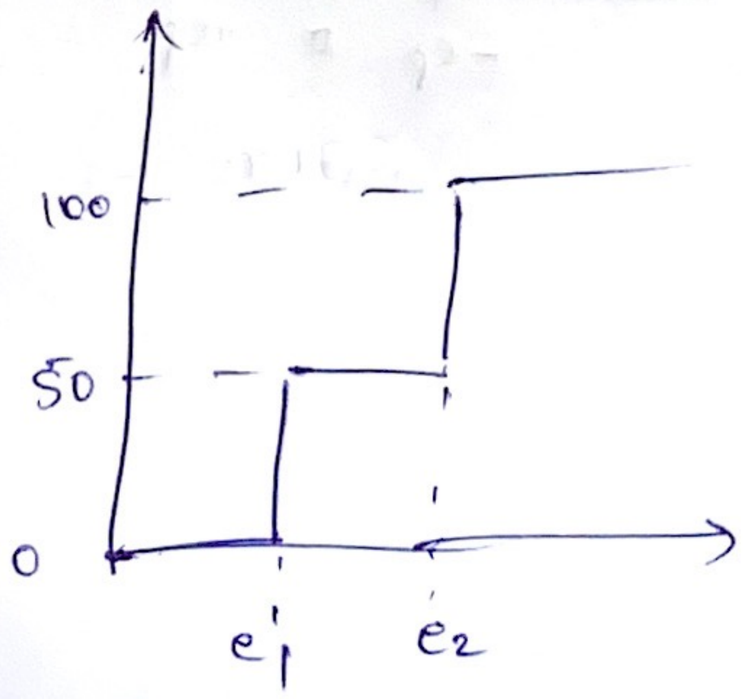
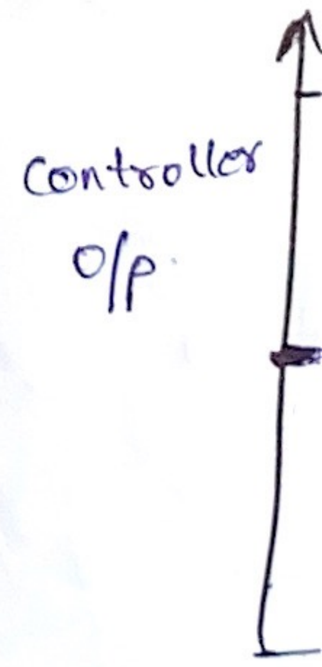
Multiposition mode.

$$P = P_i \qquad e_p > |e_i| \quad i = 1, 2, \ldots, n$$

3- position mode.

$$P = \begin{cases} 100 & e_p > e_2 \\ 50 & e_1 < e_p < e_2 \\ 0 & e_p < e_1 \end{cases}$$





Floating - control mode

i) Single speed.

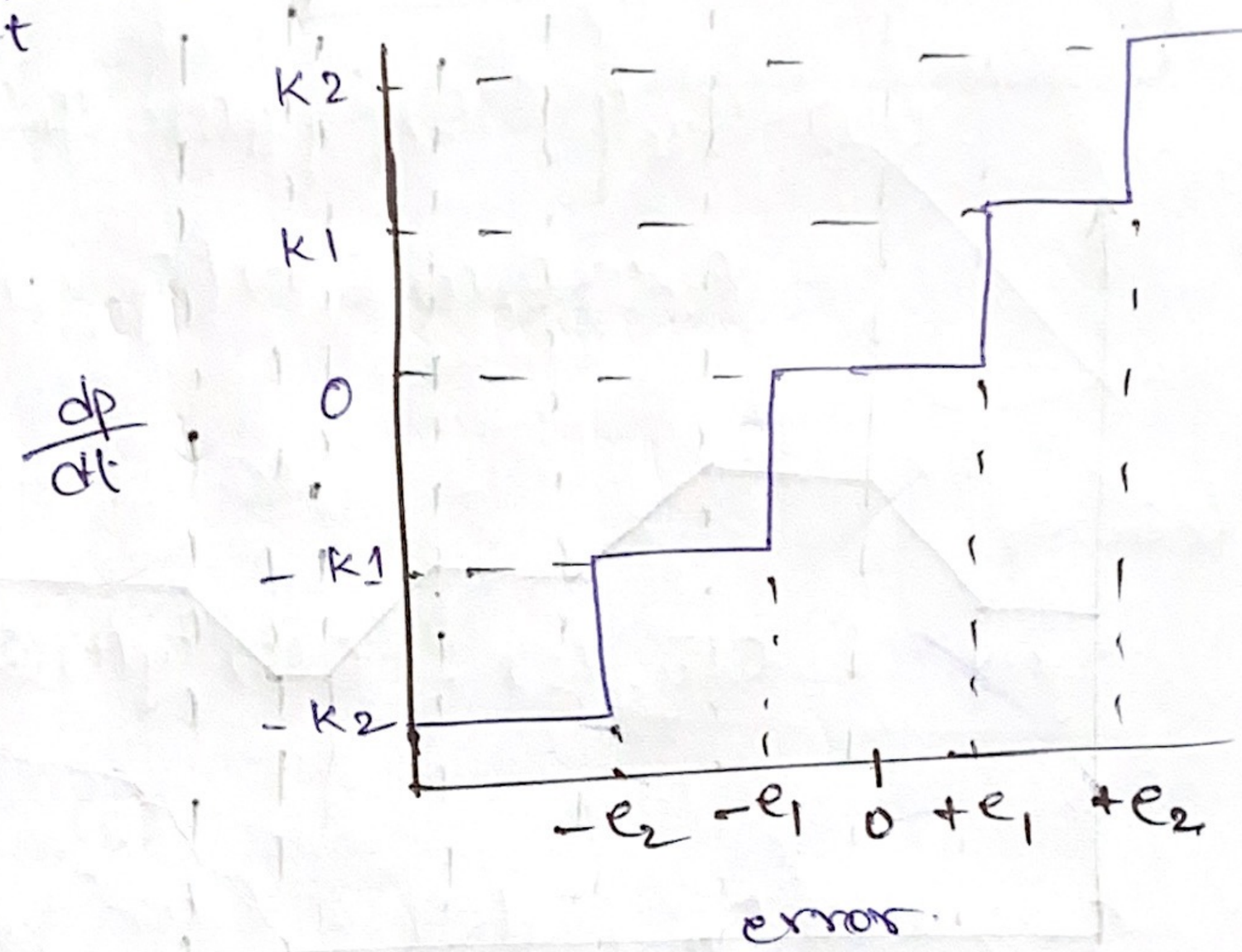
$$\frac{dp}{dt} = \pm K_F \qquad |e_p| > \Delta e_p$$

$\frac{dp}{dt}$ → co

K_F → rate constant (%/s)

ii) Multiple speed

$$\frac{dp}{dt} = \pm K_{Fi} \qquad |e_p| > e_{pi}$$



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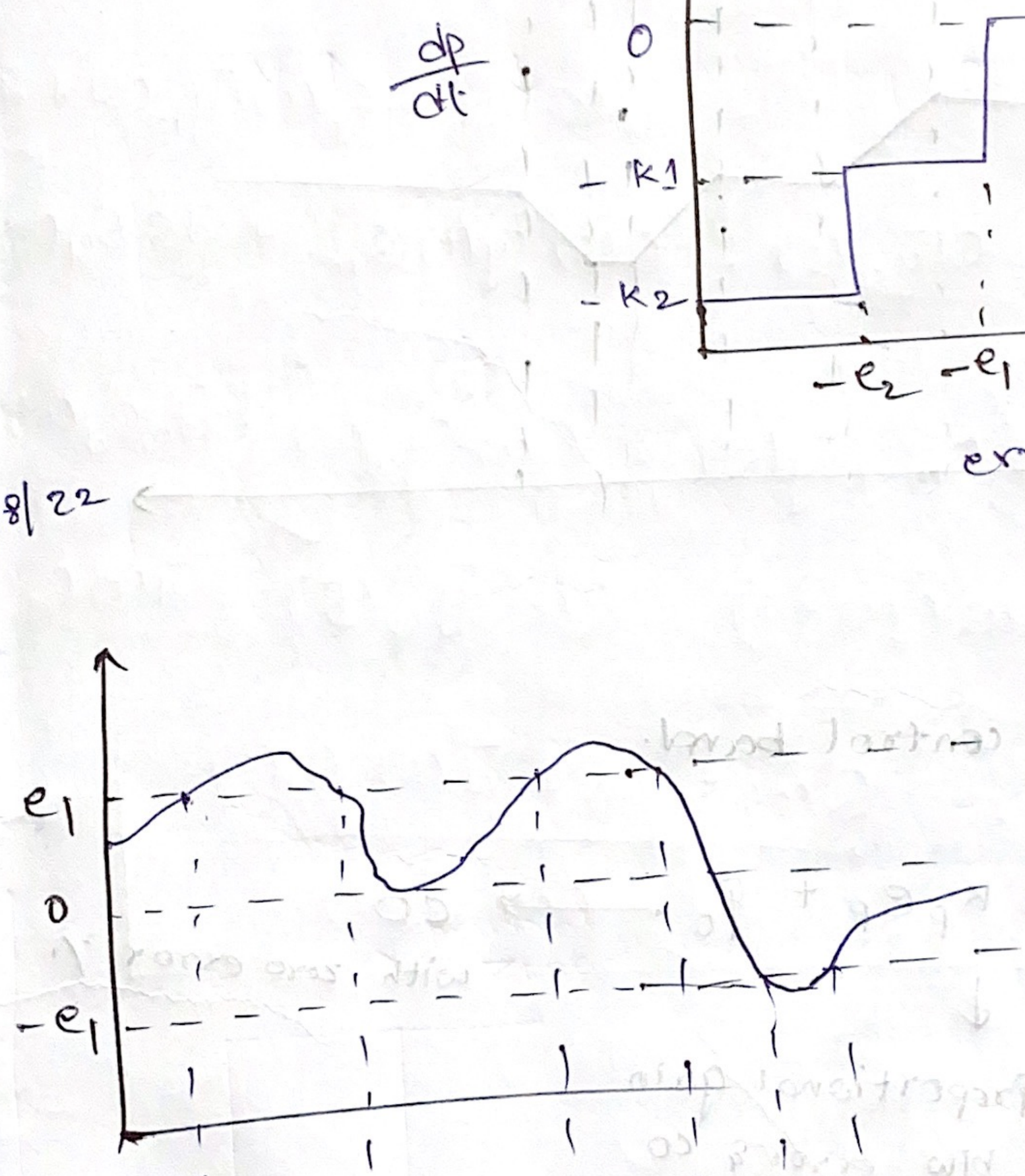


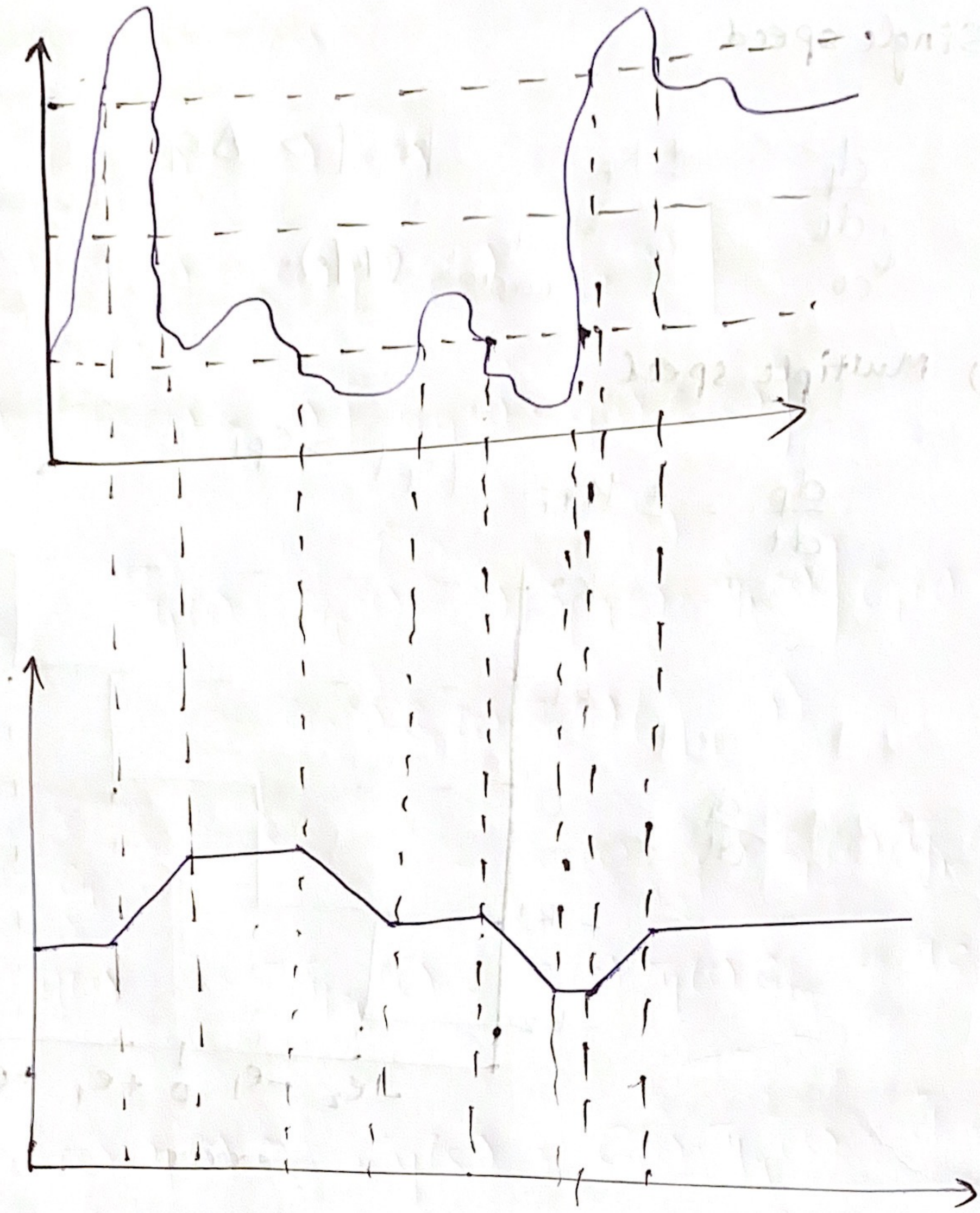
$p > -e_1 \Rightarrow 0$

$-e_1 < p < e_1 \Rightarrow 50$

$p > e_1 \Rightarrow 100$

3-position mode

floating mode

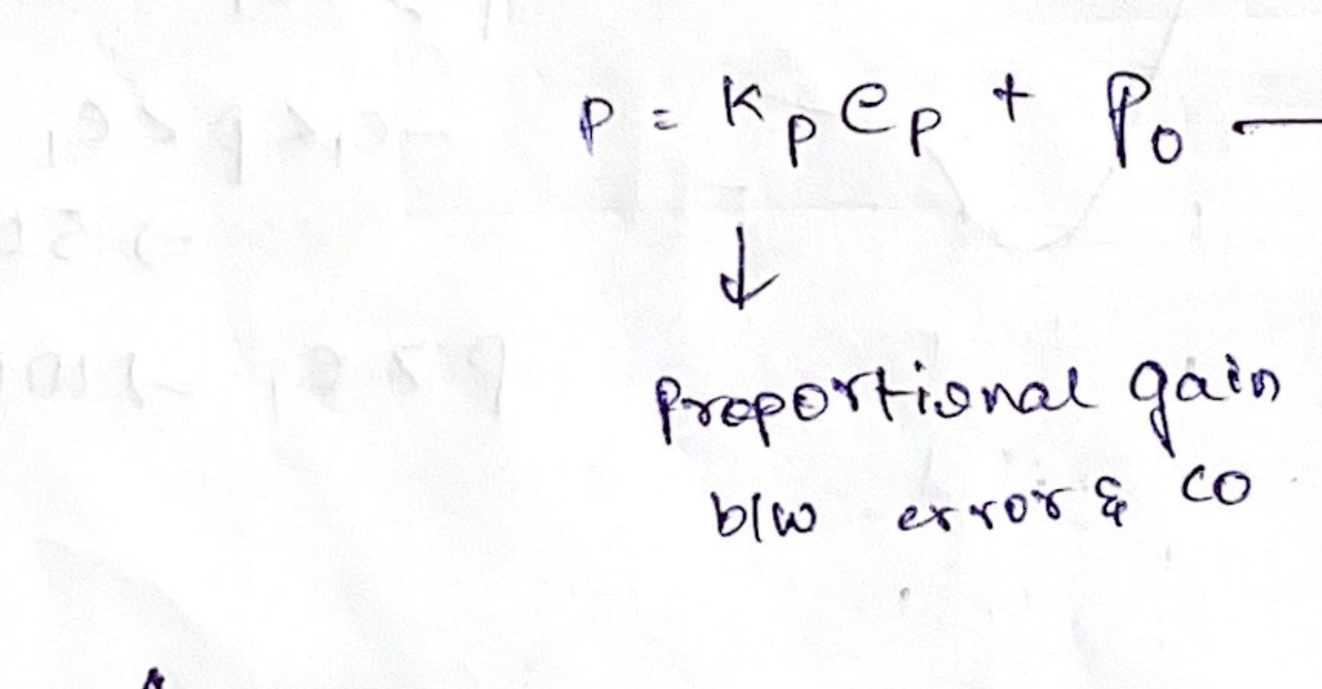


Proportional control band.

$P = K_p e_p + P_0 \longrightarrow CO$

with zero error %

K_p → Proportional gain b/w error & CO



$PB = \frac{100}{K_p}$

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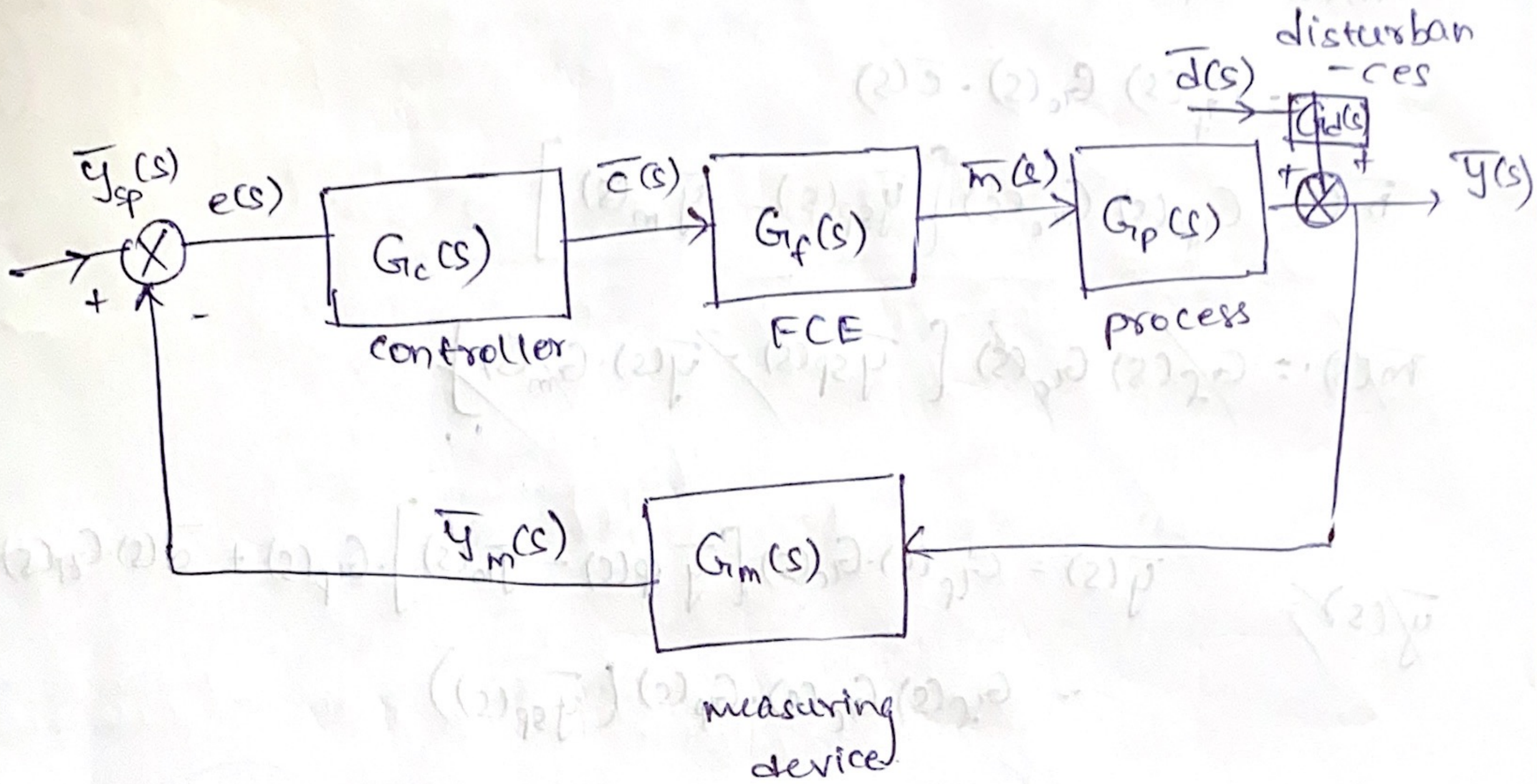


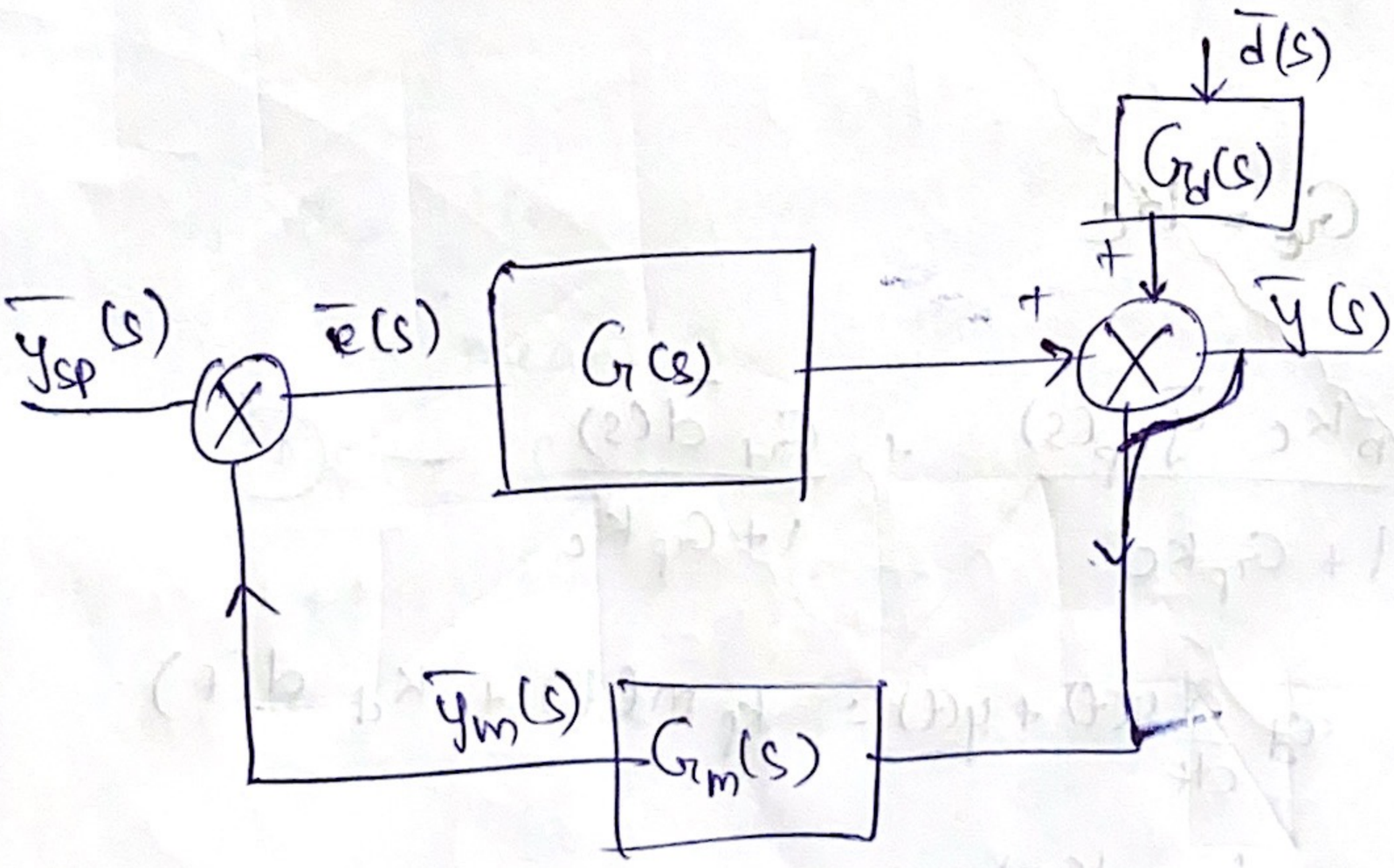
$$\bar{y}(s) = G_d(s) \cdot \bar{d}(s) + \bar{m}(s) \cdot G_p(s)$$

$$\bar{y}_m(s) = \bar{y}(s) \cdot G_m(s)$$

$$\bar{e}(s) = \bar{y}_{(sp)}(s) - \bar{y}_m(s)$$

$$\bar{c}(s) = e(s) \cdot G_c(s)$$

where G_p, G_d, G_m, G_c and G_f are the TF between the corresponding i/p's and o/p's



$G(s) = G_c(s) \cdot G_f(s) \cdot G_p(s)$.

$\bar{y}(s) = \bar{m}(s) \cdot G_p(s) + \bar{d}(s) \cdot G_d$

$\bar{m}(s) = G_f(s) \cdot \bar{e}(s)$

$= G_f(s)\, G_c(s) \cdot \bar{e}(s)$

$\bar{m}(s) = G_f(s)\, G_c(s) \cdot \left[\bar{y}_{sp}(s) - \bar{y}_m(s) \right]$

$\cancel{\bar{m}(s) = G_f(s)\, G_c(s) \left[\bar{y}_{sp}(s) - \bar{y}(s) \cdot G_m(s) \right]}$

$\cancel{\bar{y}(s) =}$ $\bar{y}(s) = G_f(s) \cdot G_c(s) \left[\bar{y}_{sp}(s) - \bar{y}_m(s) \right] \cdot G_p(s) + \bar{d}(s) \cdot G_d(s)$

$= G_f(s) \cdot G_c(s) \cdot G_p(s) \left(\bar{y}_{sp}(s) \right)$

where

$G = G_f \cdot G_c \cdot G_p$

$- G_f(s)\, G_c(s) \cdot G_p(s) \cdot \bar{y}_m(s) + \bar{d}s \cdot G_d(s)$

$\bar{y}(s) = G(s)\, \bar{y}_{sp}(s) - G(s) \cdot \bar{y}(s) \cdot G_m + \bar{d}s \cdot G_d(s)$

$G(s) \cdot \bar{y}(s) \cdot G_m + \bar{y}(s) = G(s) \cdot \bar{y}_{sp}(s) + \bar{d}(s) G_d(s)$.

$\bar{y}(s) \left(G(s) \cdot G_m(s) + 1 \right) = G(s)\, \bar{y}_{sp}(s) + \bar{d}(s)\, G_d(s)$.

$$\boxed{\bar{y}(s) = \frac{G(s)\, \bar{y}_{sp}(s) + \bar{d}(s) \cdot G_d(s)}{1 + G(s) \cdot G_m(s)}}$$

Consider

ex: $G_m = G_f = 1$ $\quad G_c = K_c$

then $\bar{y}(s) = \dfrac{G_p K_c\, \bar{y}_{sp}(s)}{1 + G_p K_c} + \dfrac{G_d\, \bar{d}(s)}{1 + G_p K_c} \longrightarrow (1)$

1st order system. $\quad \tau_d \dfrac{d}{dt} y(t) + y(t) = k_p\, m(t) + k_d\, d(t)$.

$y(s) = \dfrac{k_p\, m(s)}{1 + \tau_p s} + \dfrac{k_d\, d(s)}{1 + \tau_p s}$

$\Rightarrow \quad G_p(s) = \dfrac{k_p}{1 + \tau_p s} \qquad G_d(s) = \dfrac{k_d}{1 + \tau_p s}$

from (1).

$$\bar{y}(s) = \frac{k_p k_c \, \bar{y}_{sp}(s)}{1 + \tau_p s + k_p k_c} + \frac{k_d}{1 + \tau_p s + k_p k_c}$$

Attended 10/13

$$\bar{y}(s) = \frac{k_p' \, \bar{y}_{sp}(s)}{1 + \tau_p' s} + \frac{k_d' \, \bar{d}(s)}{1 + \tau_p' s} \longrightarrow (A)$$

$$k_p' = \frac{k_p k_c}{1 + k_p k_c}$$

$$\tau_p' = \frac{\tau_p}{1 + k_p k_c}$$

$$k_D' = \frac{k_d}{1 + k_p k_c}$$

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The parameters k_p', k_D' are known as closed loop static gains.

From (A) we conclude that the closed loop response of a 1st order system has the following characteristics.

i) it remains 1st order w.r.t load and setpoint changes.

ii) the time constant has been reduced $\tau_p' < \tau_p$ which means that the closed loop response has become faster than the open loop response to changes in setpoint or load.

iii) the static gains have been decreased

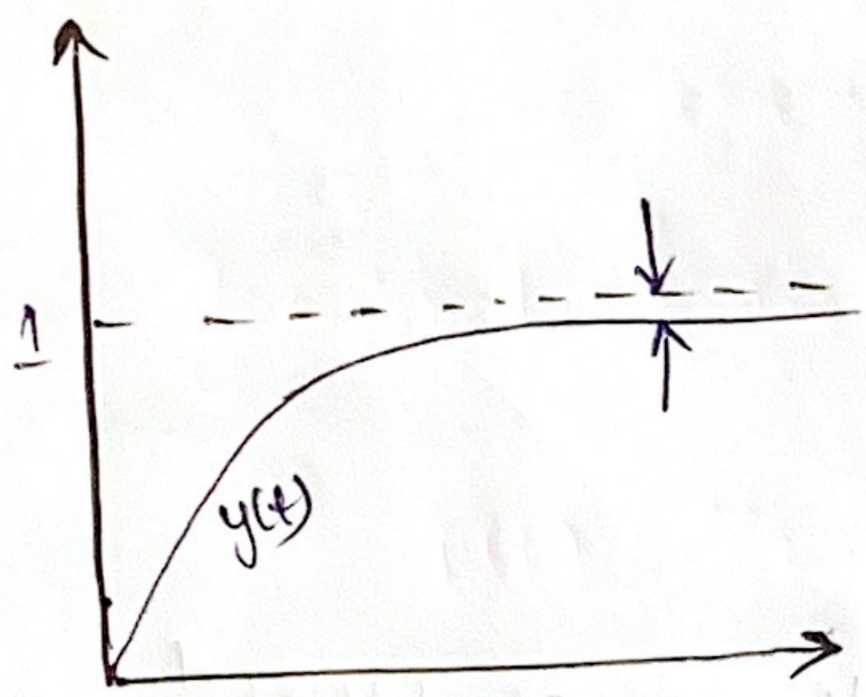
(i) Assume disturbances $= 0$ i.e $\bar{d}(s) = 0$

& setpoint change of step i.e $\bar{y}(s_p) = \frac{1}{s}$

from (A)

$$\bar{y}(s) = \frac{K_p'}{\tau_p' s + 1} \cdot \frac{1}{s}$$

$$\Rightarrow y(t) = K_p' \left(1 - e^{-t/\tau_p'}\right)$$



Step change in Setpoint.

offset = new set point − (ultimate val of the respo

$$= 1 - K_p'$$

$$= 1 - \frac{K_p K_c}{1 + K_p K_c}$$

$$\text{Offset} = \frac{1}{1 + K_p K_c}$$

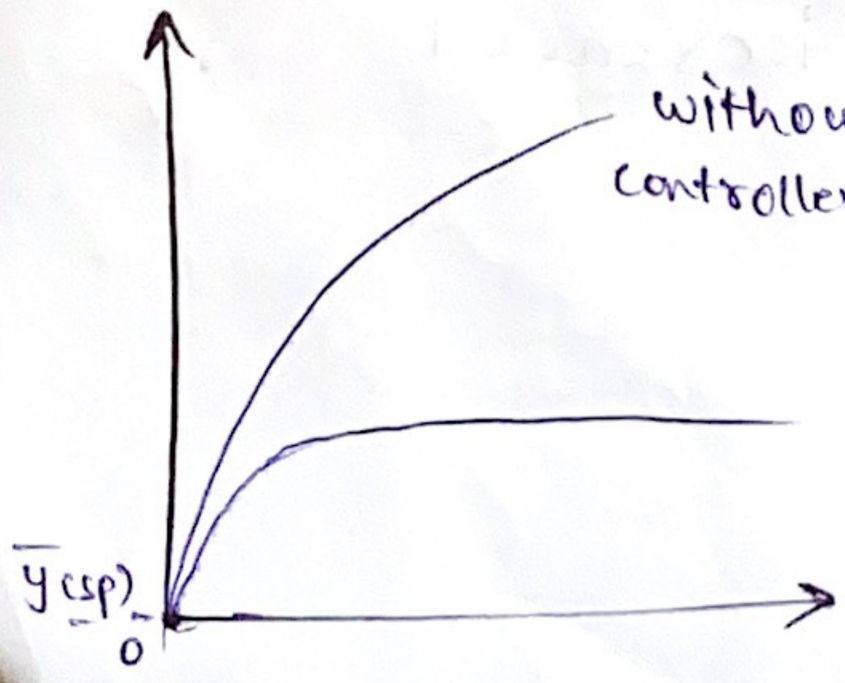
(ii) $\bar{d}(s) = \frac{1}{s}$ & $\bar{y}_{sp} = 0$

$$\bar{y}(s) = \frac{K_D'}{\tau_p' s + 1} \cdot \frac{1}{s}$$

$$\Rightarrow y(t) = K_D' \left(1 - e^{-t/\tau_p'}\right)$$

$$\text{offset} = 0 - K_D'$$

$$= 0 - \frac{K_D}{1 + K_p K_c} = \frac{-K_D}{1 + K_p K_c}$$



$K_p \to \infty$

offset $\to 0$

Remarks

i) Although the offset tends to zero. As K_c tends to ∞ we will never use extremely large values of K_c for P controller.

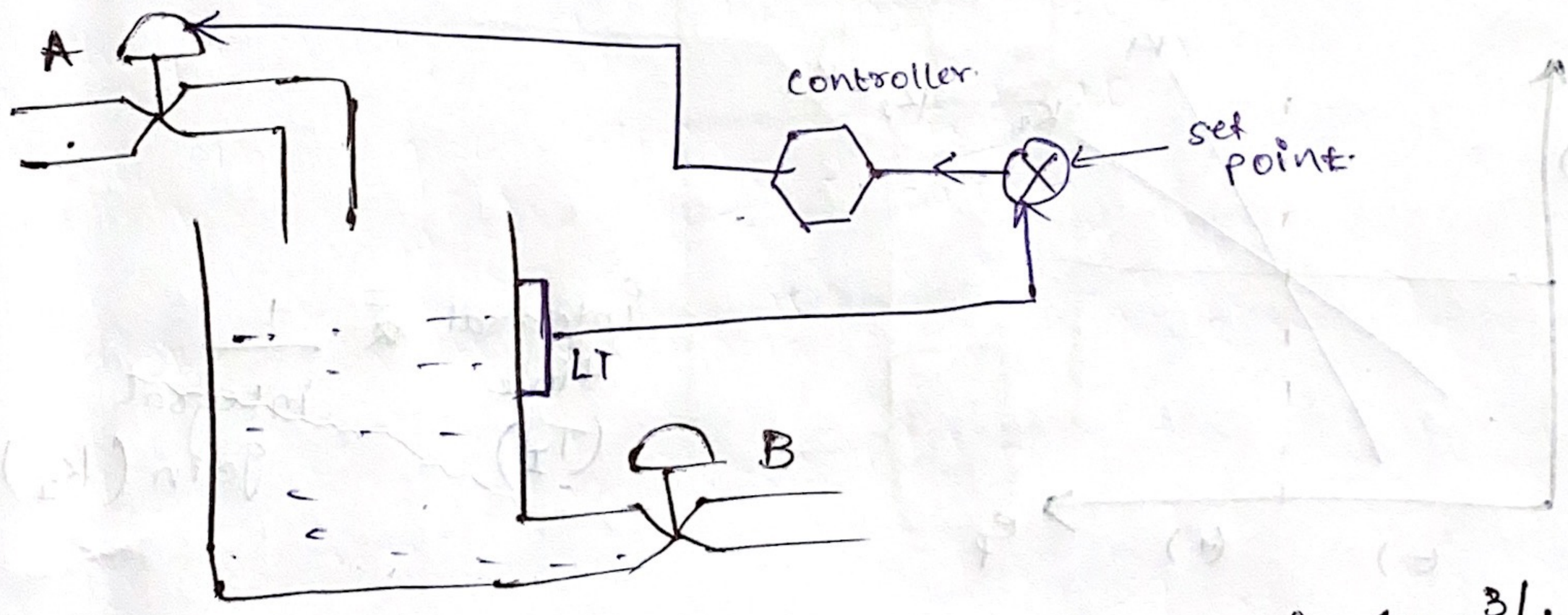
ii) If $G_m = K_m$ and $G_m = K_f$, then for unit step change in set point

$$\text{Offset} = \frac{1 - K_p K_c K_f}{1 + K_p K_c K_f K_m}$$

Step change in load

$$\text{offset} = \frac{-K_D}{1 + K_p K_c K_f K_m}$$

Q. Consider the P-mode level control system.



Valve A is linear with a flowscale of $10\ m^3/hr$ per % controller output. The controller o/p is nominally 50% with a constant of $K_p = 10\%$ per %. A load change occurs when flow through valve B changes from $500\ m^3/hr$ to $600\ m^3/hr$. Calculate the new controller o/p and offset error.

$P = K_p e_p + P_0$

$e_p = \frac{P - P_0}{K_p}$

Sol. $Q_A = \left(\frac{10\ m^3/h}{\%}\right) \times (60\%) = 600\ m^3/h$.

$= \frac{60 - 50}{10\%}$

Controller o/p

offset error, $e_p = 1\%$.

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Limitations of P-controller

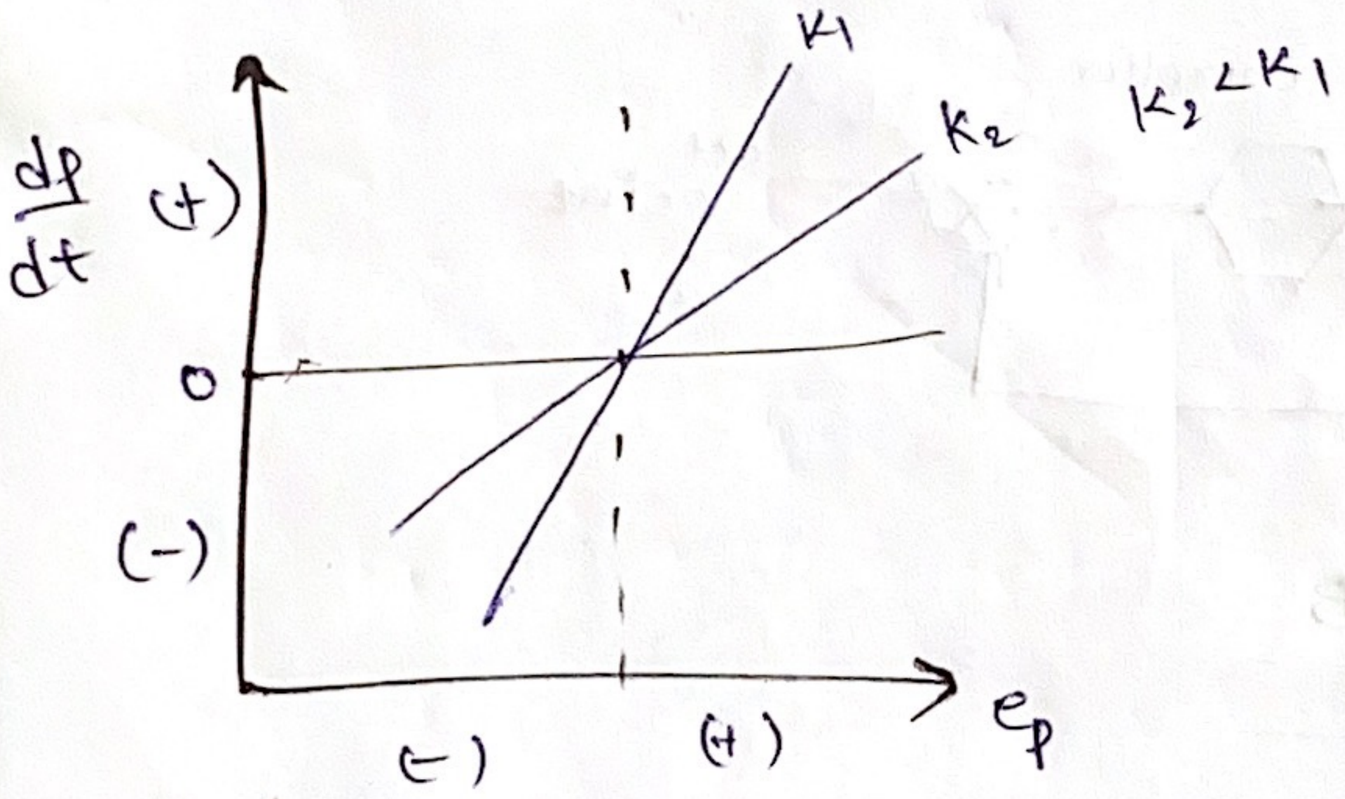
1. There is offset when a set point change is made. It can not keep the controlled variable on set point.

2. Actual process o/p will not be able to achieve the desired range.

3. Smaller gain, Larger offset

Integral controller (reset mode)

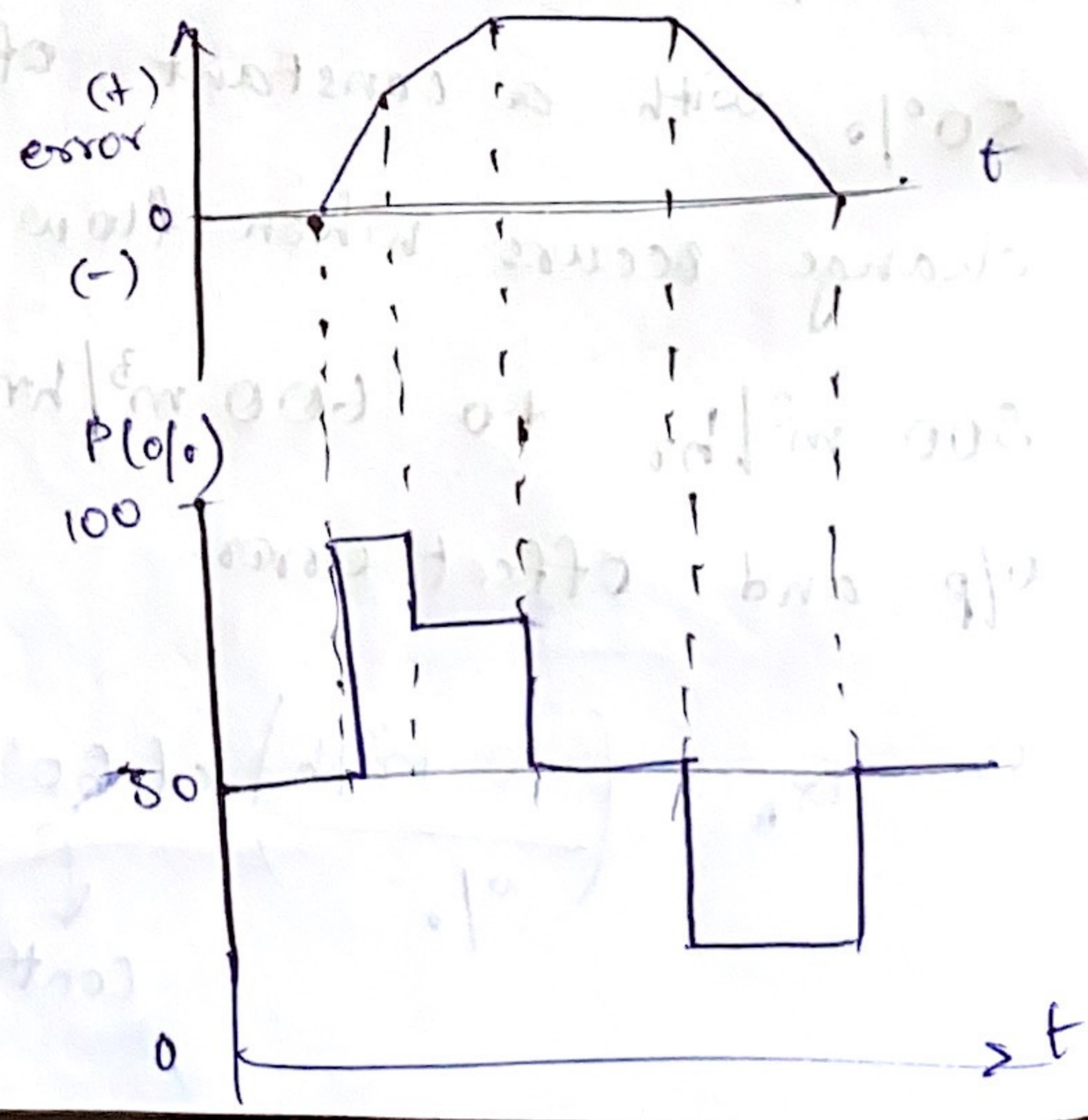
$$p(t) = K_I \int_0^t e_p \, dt + p(0) \qquad \frac{dp}{dt} = K_I e_p$$

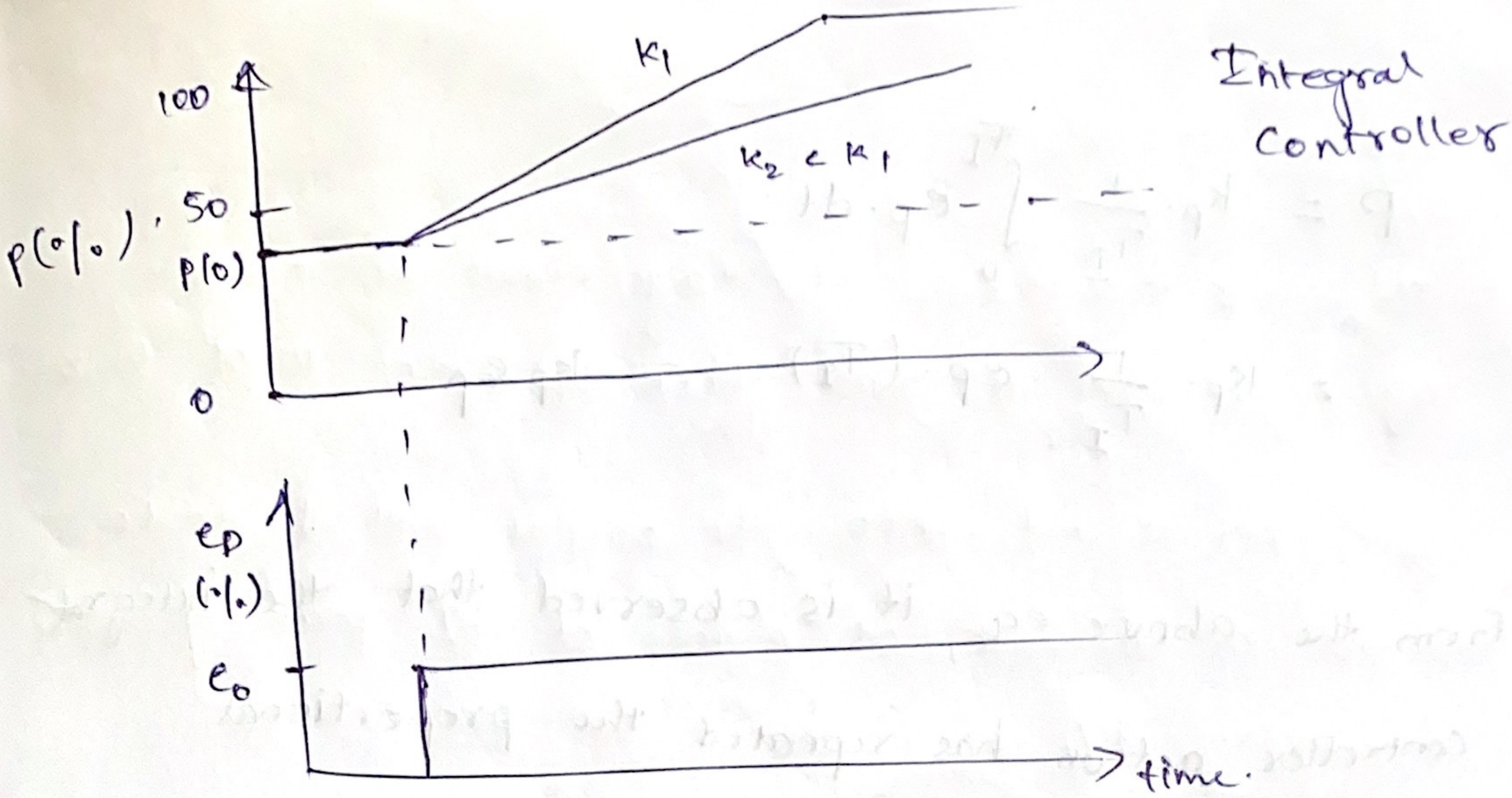


$$\text{integral time } (T_I) = \frac{1}{\text{integral gain } (K_I)}$$

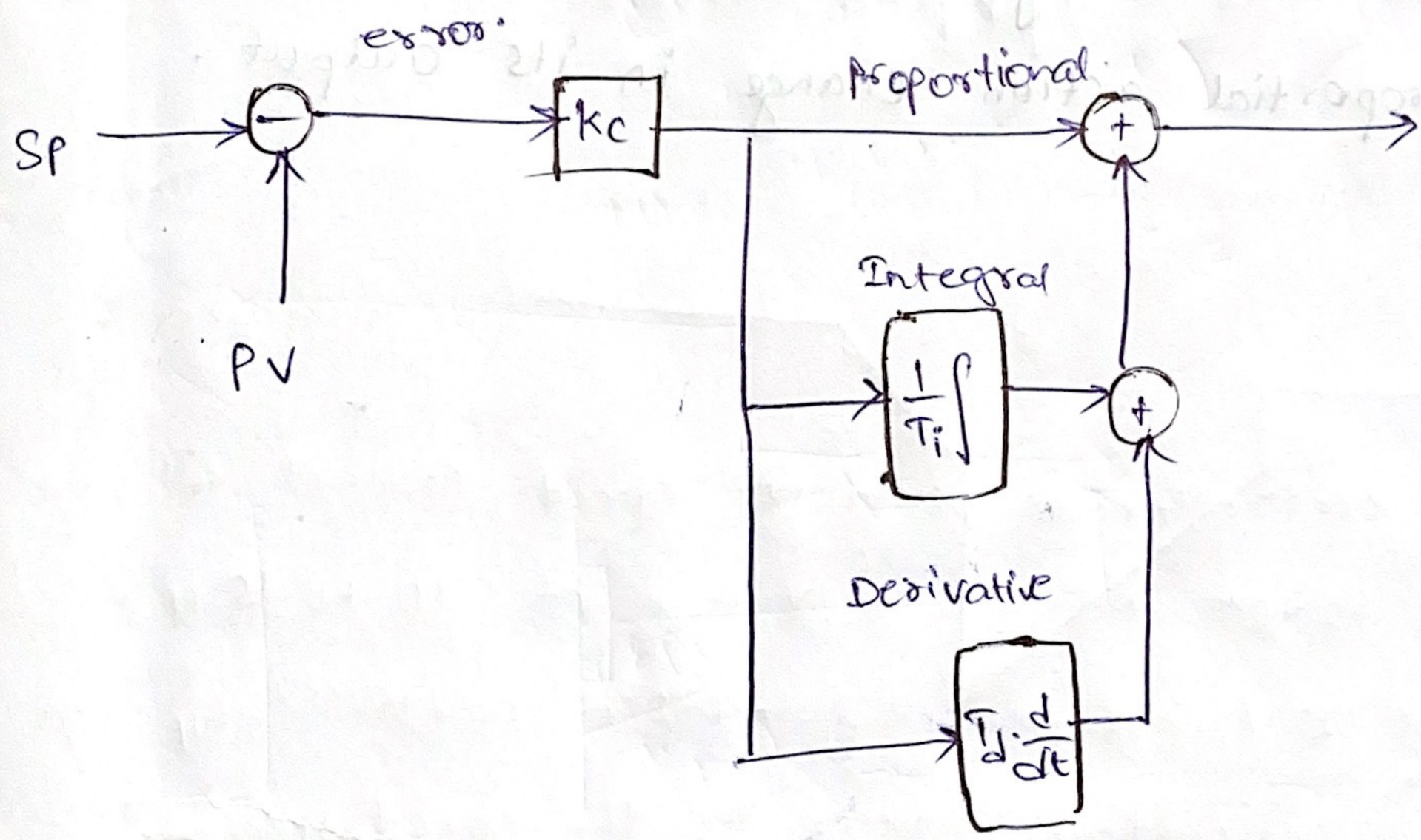
Derivative Controller

$$p(t) = K_D \cdot \frac{de_p}{dt}$$





Composite modes.



$$CO = K_c \left[E + \frac{1}{T_i}\int E \cdot dt + T_d \cdot \frac{dE}{dt} \right]$$

PI controller

$$P = K_p e_p + K_p K_I \int_0^t e_p dt + P_I(0).$$

$P_I(0)$ = integral term value at $t=0$.

$$P = k_p \cdot \frac{1}{T_I} \cdot \int_0^{T_I} e_p \cdot dt$$

$$= k_p \cdot \frac{1}{T_I} \cdot e_p \cdot (T_I) = k_p e_p$$

From the above eq, it is observed that the integral controller action has repeated the proportional controller action. This happens for every T_I min. So, it is named as reset time. Therefore reset time is the time needed by the controller to repeat the initial proportial action change in its output.

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(Rough notes)

Integral controller

1. The error is zero, the O/P stays fixed at a value equal to value it was when the error went to zero.

2. When error is not zero, the output will begin to increase or decrease at a rate of K_I percent / second for every 1% of error.

3. The integral gain, is often represented by the inverse which is called the integral time, or the reset rotation.

$$T_i = 1 / K_I \text{ in minutes.}$$

4. An integral controller is used for speed control with setpoint of 15 rpm within a range of 10 to 15 rpm. The CO is 25% initially. $K_I = 0.5$% controller op per second per percentage error. If the speed jumps to 13.5 rpm. Calculate the controller O/P after 2 sec for a constant e_p.

Sol: $e_p = \frac{15 - 13.5}{15 - 10} \times 100 = 30\%$

$$p(t) = K_I \int_0^t e_p dt + p(0)$$

$$p(t) = 0.5 \times (-30) \times (2) + 25$$

$$= 31\%$$

Integral controller

1. If error is zero, the o/p stays fixed at a value equal to what it was when the error went to zero

2. If error is not zero, the output will begin to increase or decrease at a rate of K_I percent / second for every 1% of error.

3. The integral gain, is often represented by the inverse, which is called the integral time, or the reset action.

$$T_i = 1/K_i \text{ in minutes.}$$

Q. An integral controller is used for speed control with setpoint of 12 rpm within a range of 10 to 15 rpm. The CO is 22% initiall. $K_I = -0.15\%$ controller o/p per second per percentage error. If the speed jumps to 13.5 rpm. Calculate the controller o/p after 2 sec for a constant e_p.

Sol. $$e_p = \frac{12 - 13.5}{15 - 10} \times 100 = -30\%$$

$$p(t) = K_I \int_0^t e_p \, dt + p(0)$$

$$p(t) = K_I e_p t + p(0)$$

$$p(t) = -0.15 \times (-30) \times (2) + 22$$

$$= 31\%$$

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PI controller

$$P = k_p e_p + k_p k_I \int_0^t e_p \, dt + P_I(0)$$

$P_I(0)$ = integral term value at $t=0$.

Ideal form of PI controller.

$$CO = CO_{bias} + k_c e(t) + \frac{k_c}{\tau_I} \int e(t) \cdot dt$$

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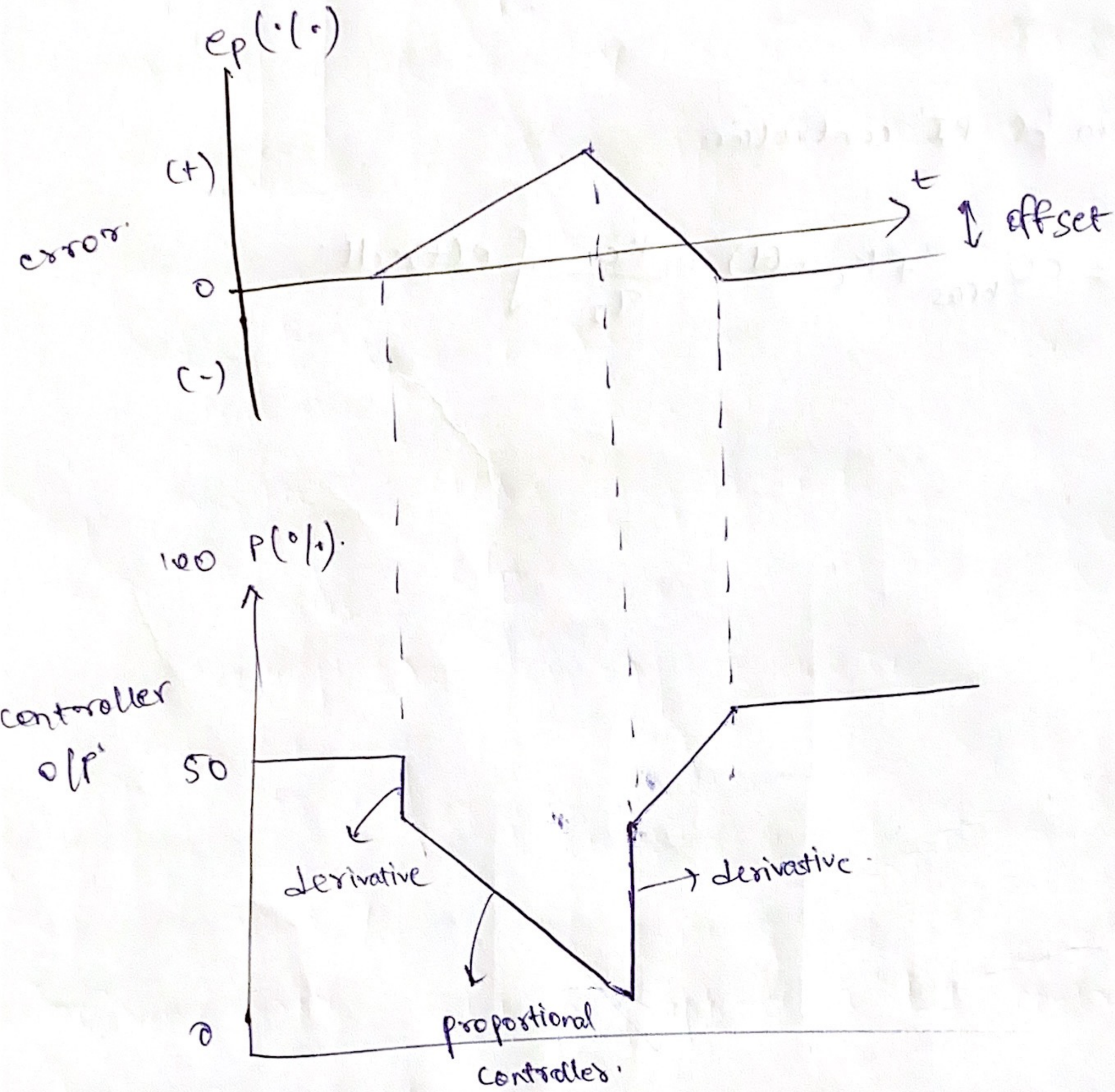
P-D controller

$$p = K_p e_p + K_p K_D \cdot \frac{de_p}{dt} + P_0$$



PID Controller

$$p = k_p e_p + k_p K_I \int_0^t e_p \, dt + k_p k_D \frac{de_p}{dt} + P_I(0)$$