MANIPAL INSTITUTE OF TECHNOLOGY
MANIPAL
(A constituent unit of MAHE, Manipal)

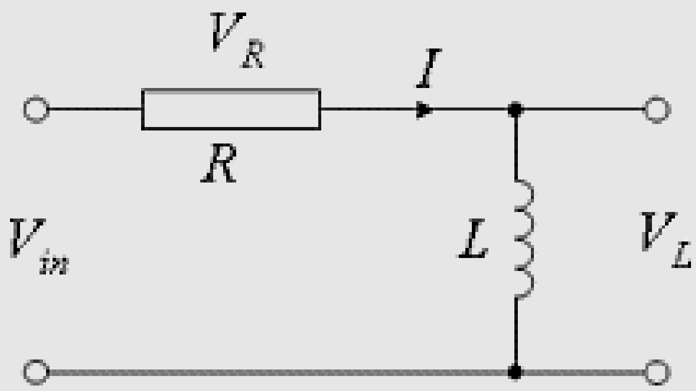
Modern Control Theory (ICE 3153)

State Space Space Modeling of Electrical Systems

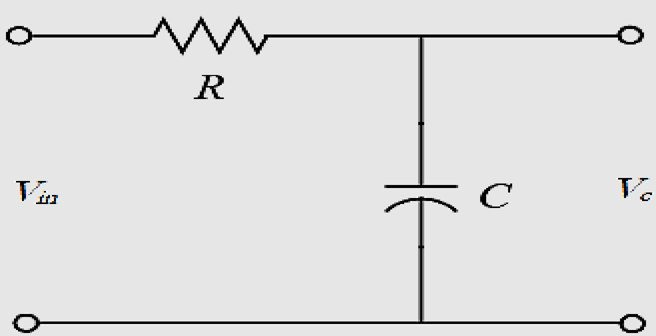
Bipin Krishna
Assistant Professor (Sr.)
ICE Department
Manipal Institute of Technology
MAHE, Karnataka, India

Question 1:

Obtain the state – space representation of a (i) RL circuit and (ii) RC circuit. V_{in} is the input voltage to the circuit, I is the current in circuit.



(i)



(ii)

- Apply KVL or KCL based on the circuit given.
- Write a differential equation in terms of variables that can be chosen as state variable or variables that need to measured as output.
- For inductor, its preferable to choose current through L as state variable and for capacitor its voltage across C.
- Once the state space model is developed, other output variables of the circuit can be obtained from the model.

RL Circuit:

Applying KVL,

Voltage balance equation → $V_i = V_R + V_L$

$$V_i = i * R + L\frac{di}{dt}$$

No. of states = 1; x_1 = i; V_i is the input and i is the output variable.

State equation can be written as:

$$\dot{x}_1 = \frac{V_i}{L} - x_1 * \frac{R}{L}$$

Output equation can be written as:

$$y = x_1$$

State – space model can be written as:

$$\dot{x}_1 = \left[-\frac{R}{L}\right][x_1] + \left[\frac{1}{L}\right][V_i]$$

$$y = [1][x_1]$$

RC Circuit:

Applying KVL,

Voltage balance equation → $V_i = V_R + V_C$

$$V_i = i * R + V_c;\ \frac{1}{C}\int i\,dt = V_c;\ i = C\frac{dV_c}{dt}$$

$$V_i = R * C\frac{dV_c}{dt} + V_c$$

No. of states = 1; $x_1 = V_c$; V_i is the input and V_c is the output variable.

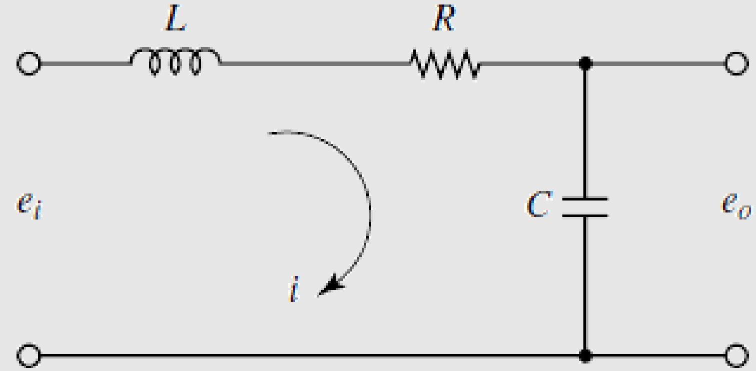
State – space model can be written as:

$$\dot{x}_1 = \left[-\frac{1}{RC}\right][x_1] + \left[\frac{1}{RC}\right][V_i]$$

$$y = [1][x_1]$$

Question 2:
Ex, MCE – 5th Edition, K. Ogata

For the RLC circuit shown in figure, e_i is the input voltage and e_o is the output voltage. Obtain a state – space representation.



Applying KVL,
Voltage balance equation → $V_i = V_R + V_c + V_L$

$$V_i = i * R + L\frac{di}{dt} + V_c\ ;\ \frac{1}{C}\int i\,dt = V_c; i = C\frac{dV_c}{dt}$$

No. of states = 2; x_1 = i; and $x_2 = V_c$,
I and V_c are the output variable.

Writing differential eqn. in terms of state variable

$$V_i = x_1 * R + L\dot{x}_1 + x_2\ ;\ x_1 = C\dot{x}_2$$

State equation can be written as:

$$\dot{x}_1 = \frac{V_i}{L} - \frac{R}{L}x_1 - \frac{x_2}{L}\ ;\ \dot{x}_2 = \frac{x_1}{C}$$

State – space model can be written as:

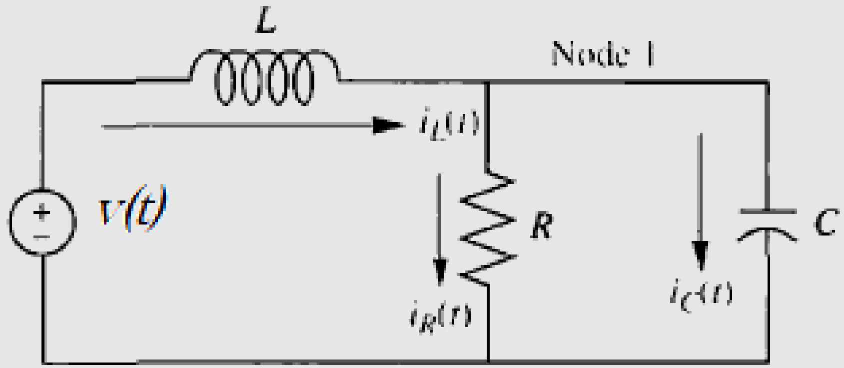
$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}-R/L & -1/L\\ 1/C & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}1\backslash \mathrm{L}\\ 0\end{bmatrix}[V_i]$$

$$\begin{bmatrix}Y_1\\ Y_2\end{bmatrix} = \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}$$

Question 3:

Ex 3.1, CSE – 6th Edition, Norman. S. Nise

For the electrical network shown in figure find a state space representation if the output is the current through the resistor.



Hint:

No. of states required = 2; States I_L and V_c

Write two first order differential equation

Write KCL at node 1 and KVL for outer loop

$$C\frac{dv_C}{dt} = i_C$$

$$L\frac{di_L}{dt} = v_L$$

Apply KCL at Node 1:

$$\begin{aligned} i_C &= -i_R + i_L \\ &= -\frac{1}{R}v_C + i_L \end{aligned}$$

Apply KVL in outer-loop:

$$v_L = -v_C + v(t)$$

Differential Equation:

$$\begin{aligned} C\frac{dv_C}{dt} &= -\frac{1}{R}v_C + i_L \\ L\frac{di_L}{dt} &= -v_C + v(t) \end{aligned}$$

$$\begin{aligned} \frac{dv_C}{dt} &= -\frac{1}{RC}v_C + \frac{1}{C}i_L \\ \frac{di_L}{dt} &= -\frac{1}{L}v_C + \frac{1}{L}v(t) \end{aligned}$$

Output Equation:

$$i_R = \frac{1}{R}v_C$$

State-space Model:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -1/(RC) & 1/C \\ -1/L & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1\backslash \mathrm{L} \end{bmatrix} [V_i]$$

$$\begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} = [1/R\ 0] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$