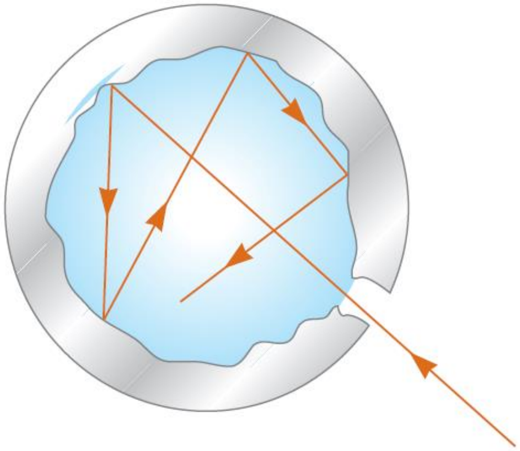
Chapter 3

QUANTUM PHYSICS

OBJECTIVES:

- To learn certain experimental results that can be understood only by particle theory of electromagnetic waves.
- To learn the particle properties of waves and the wave properties of the particles.
- To understand the uncertainty principle.

Blackbody Radiation and Planck's Hypothesis



- The electromagnetic radiation emitted by the black body is called **black-body radiation.**

Basic laws of radiation

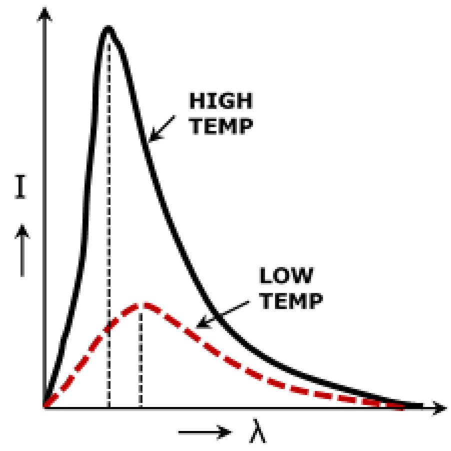
(1) All objects emit radiant energy.

(2) Hotter objects emit more energy (per unit area) than colder objects. (**Stefan's Law**)

$$P = \sigma A\, e\, T^4$$

(3) The peak of the wavelength distribution shifts to shorter wavelengths as the black body temperature increases. (**Wien's Displacement Law**)

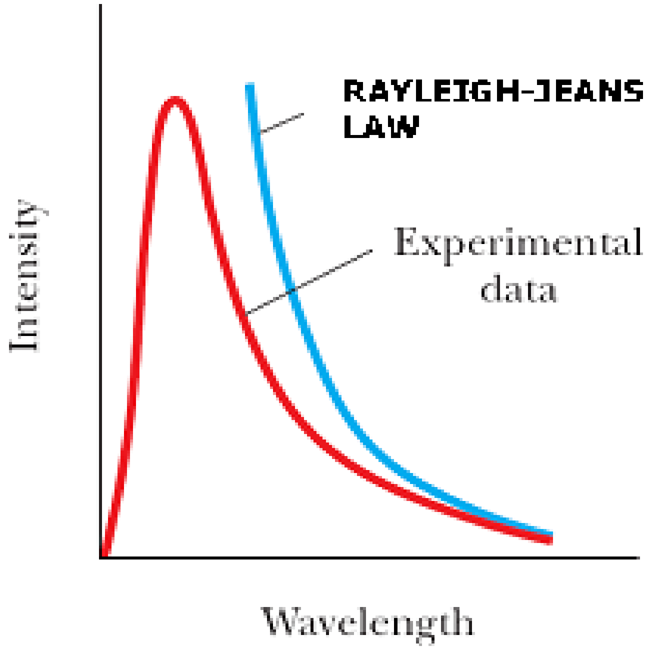
$$\lambda_m T = \text{constant}$$



(4) **Rayleigh-Jeans Law**: The intensity or power per unit area $I(\lambda,T)d\lambda$, emitted in the wavelength interval λ to $\lambda+d\lambda$ from a blackbody is given by

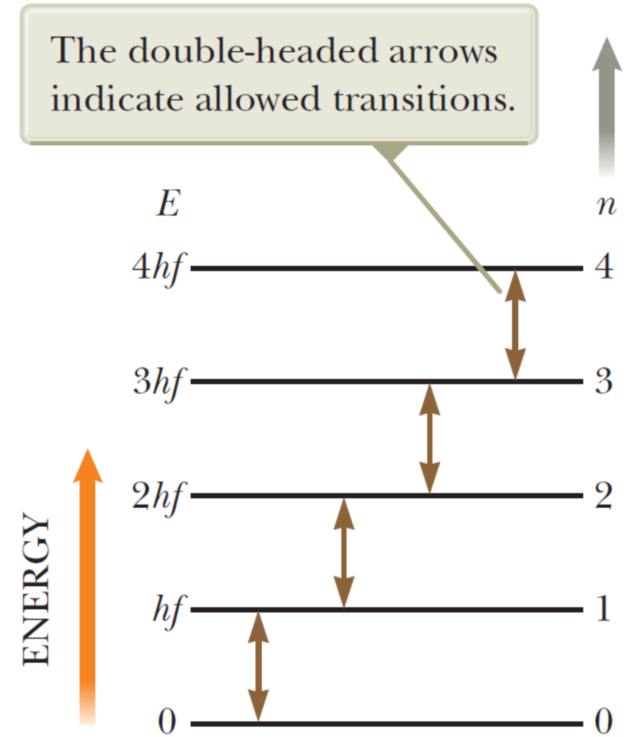
$$I(\lambda,T)=\frac{2\pi c k_B T}{\lambda^4}$$

- It agrees with experimental measurements only for long wavelengths.
- It predicts an energy output that diverges towards infinity as wavelengths become smaller and is known as the **ultraviolet catastrophe**.



(5) Planck's Law: The intensity or power per unit area $I(\lambda,T)d\lambda$, emitted in the wavelength interval λ to $\lambda+d\lambda$ from a blackbody is given by

$$I(\lambda,T)=\frac{2\pi hc^2}{\lambda^5}\frac{1}{e^{\frac{hc}{\lambda kT}}-1}$$



Assumptions of this law are:

- Energy of an oscillator in cavity walls:

  $E_n = n\,h\,f$

- Amount of emission / absorption of energy will be integral multiples of hf.

The results of Planck's law:

- The denominator [$exp(hc/\lambda kT)$] tends to infinity faster than the numerator (λ^{-5}), thus resolving the ultraviolet catastrophe and hence arriving at experimental observation:

$$I(\lambda, T) \to 0 \text{ as } \lambda \to 0.$$

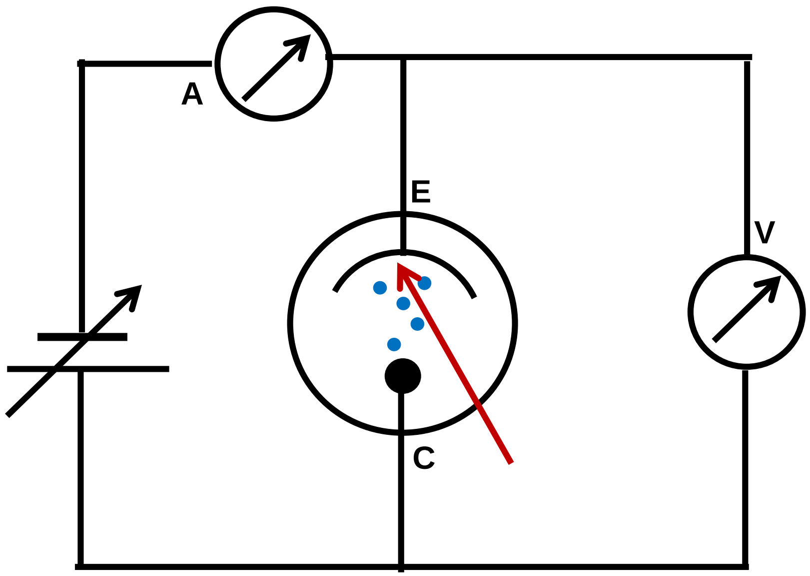
- For very large λ, $I(\lambda, T) \to 0$ as $\lambda \to \infty$.

$$exp\left(\frac{hc}{\lambda kT}\right) - 1 \cong \frac{hc}{\lambda kT} \Rightarrow \mathrm{I}(\lambda, T) \to 2\pi c \lambda^{-4} kT$$

- From a fit between Planck's law and experimental data, Planck’s constant was derived to be $\boldsymbol{h} = \mathbf{6.626 \times 10^{-34}}$ **J-s.**

Photoelectric Effect

Ejection of electrons from the surface of certain metals when it is irradiated by an electromagnetic radiation of suitable frequency is known as photoelectric effect.



Photoelectric Effect (T – Evacuated glass/ quartz tube, E – Emitter Plate / Photosensitive material / Cathode, C – Collector Plate / Anode, V – Voltmeter, A - Ammeter)

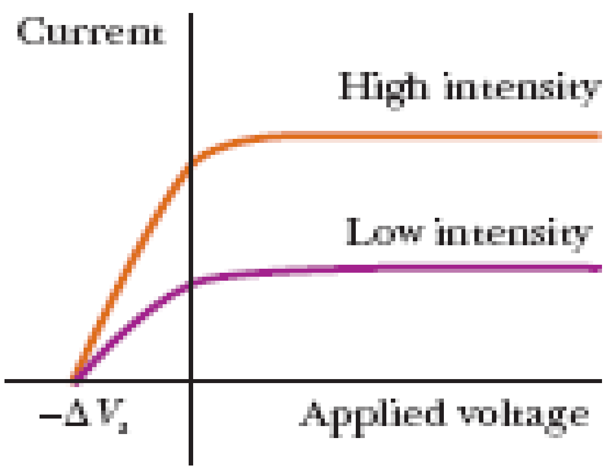
Classical Predictions

1. Electrons ejection should be frequency independent.
2. KE of the electrons should increase with intensity of light.
3. Measurable/ larger time interval between incidence of light and ejection of photoelectrons.
4. Ejection of photoelectron should not depend on light frequency.
5. K_{MAX} should not depend upon the frequency of the incident light.

Experimental results contradict classical predictions

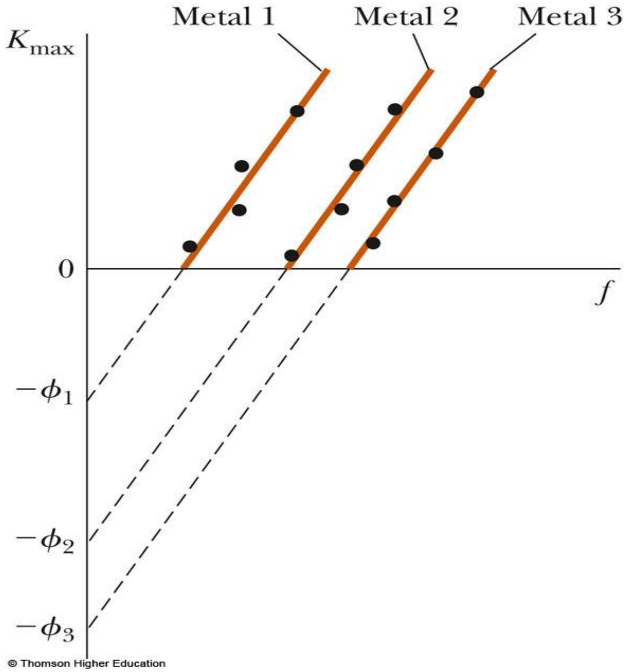
Experimental Observations

1. No photoemission for frequency below threshold frequency
2. K_{MAX} is independent of light intensity.
3. Instantaneous effect
4. KE of the most energetic photoelectrons is, $K_{MAX} = e\,\Delta V_s$
5. K_{MAX} increases with increasing f



Einstein's Interpretation of electromagnetic radiation:

1. Electromagnetic waves carry discrete energy packets (light quanta called photons now).
2. The energy E, per packet depends on frequency f: $E = hf$.
3. More intense light corresponds to more photons, not higher energy photons.
4. Each photon of energy E moves in vacuum at the speed of light: $c = 3 \times 10^8$ m/s and each photon carries a momentum, $p = E/c$.



Einstein's photoelectric equation

$$K_{max} = hf - \phi$$

Compton Effect

When X-rays are scattered by free/nearly free electrons, they suffer a change in their wavelength which depends on the scattering angle.

Classical Predictions:

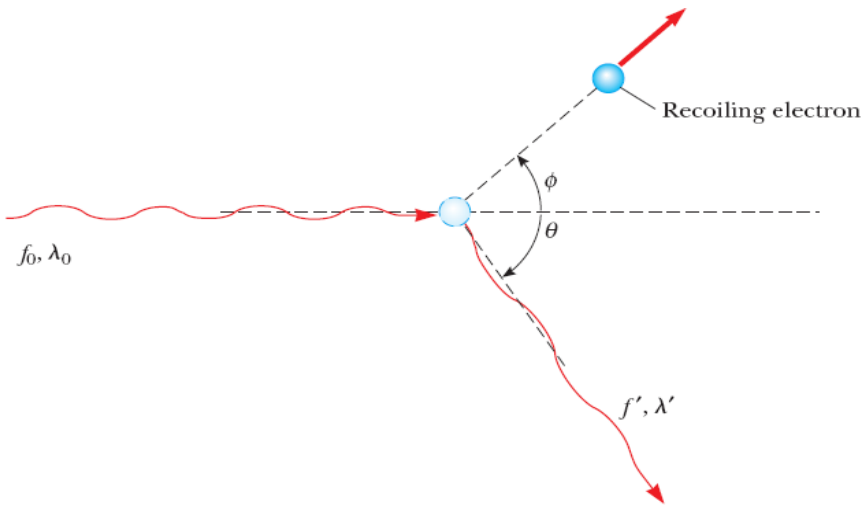
Effect of oscillating electromagnetic waves on electrons:

(a) oscillations in electrons, re-radiation in all directions

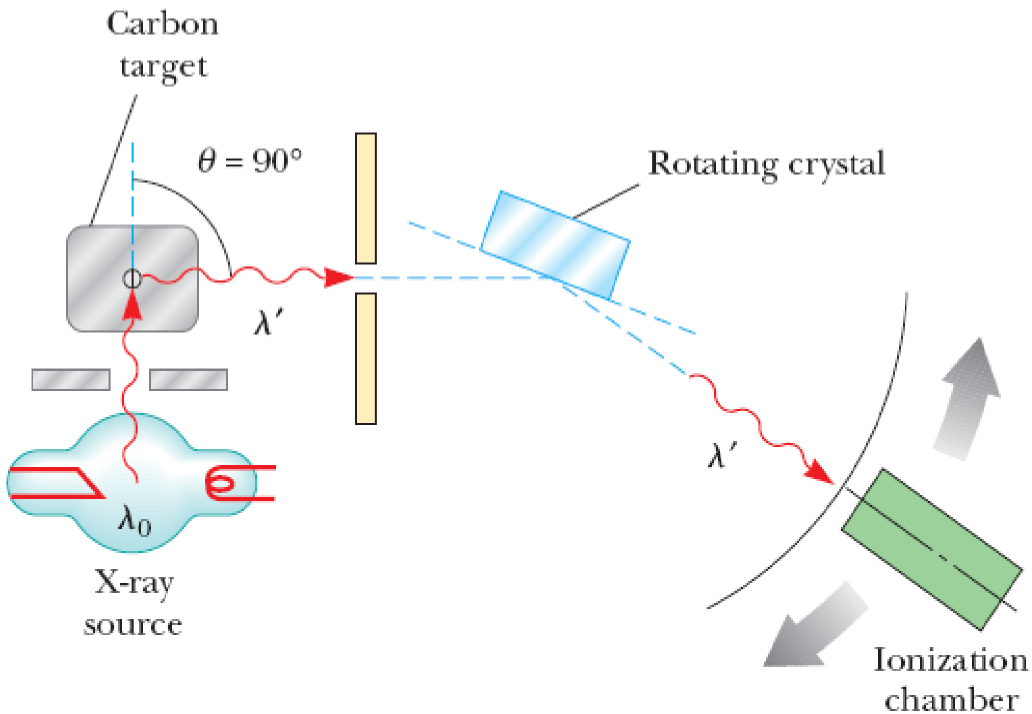
(b) radiation pressure - electrons accelerate in the direction of propagation of the waves

Different electrons will move at different speeds after the interaction.

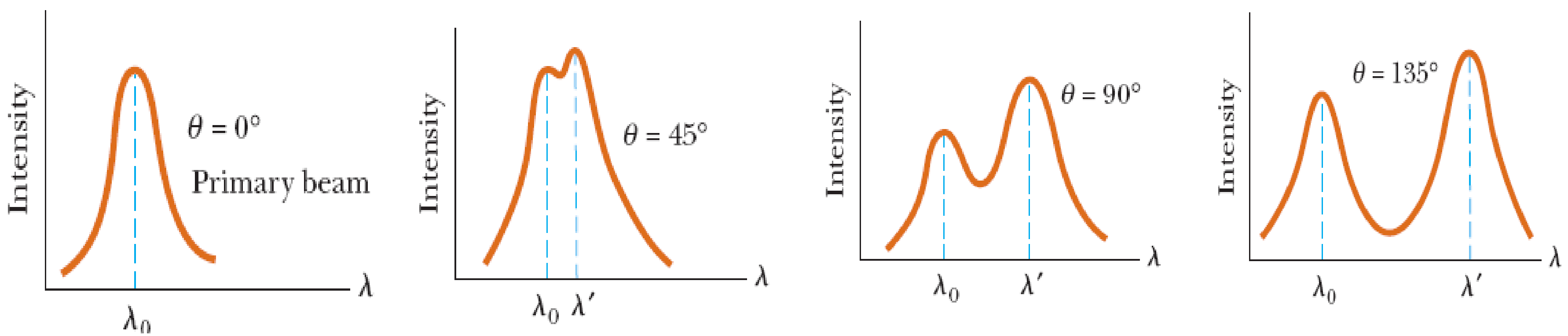
The scattered wave frequency should show a distribution of Doppler-shifted values



Schematic diagram of Compton's apparatus



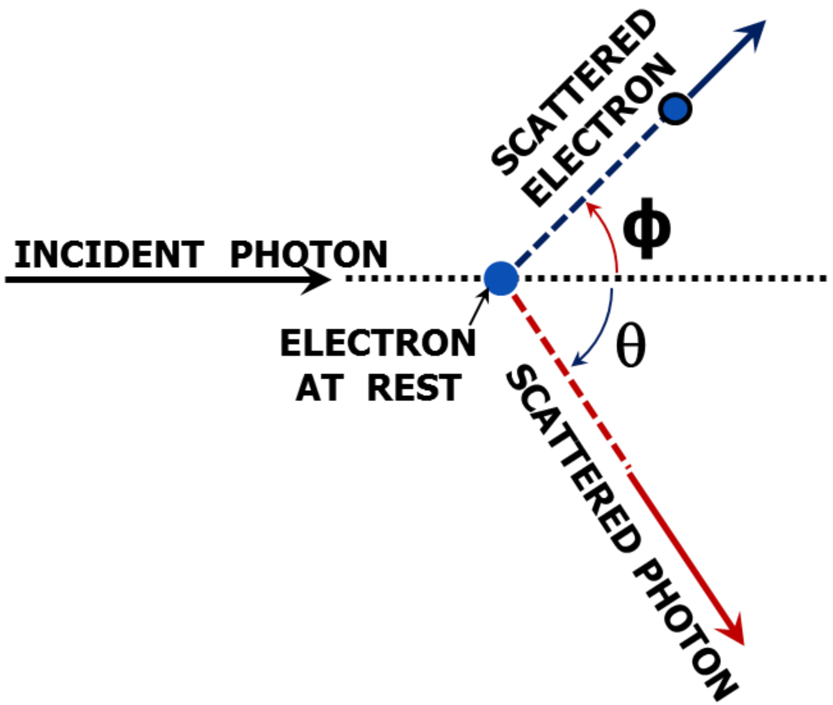
Graph of scattered x-ray intensity versus wavelength



Derivation of the Compton shift equation

Photon is treated as a particle having energy $E = hf_o = hc/\lambda_o$ and zero rest energy. Photons collide elastically with free electrons initially at rest as shown in figure.

In the scattering process, the total energy and total linear momentum of the system must be conserved.



λ_o = wavelength of the incident photon

$p_o = h/\lambda_o$ = momentum of the incident photon

$E_o = hc/\lambda_o$ = energy of the incident photon

λ' = wavelength of the scattered photon

$p' = h/\lambda'$ = momentum of the scattered photon

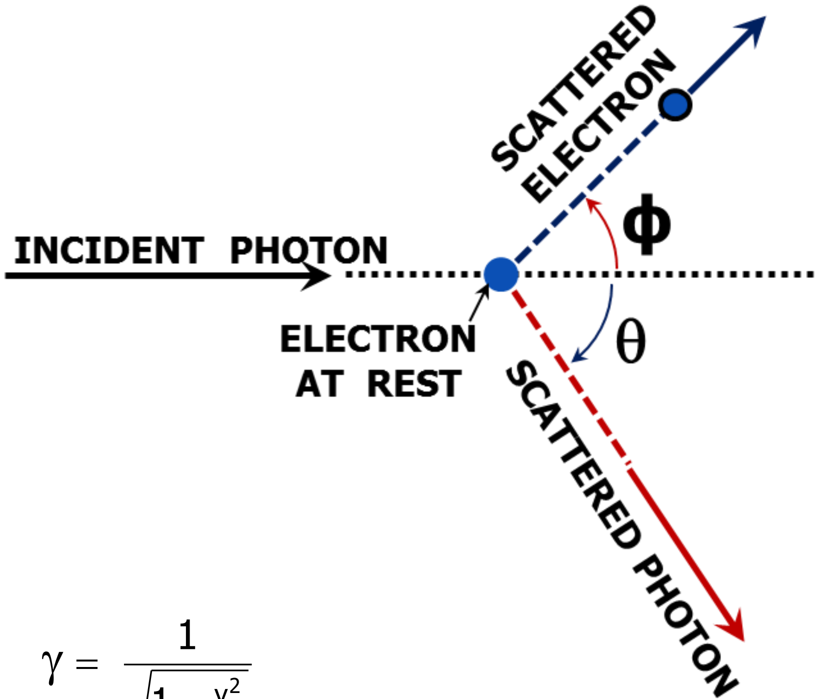
$E' = hc/\lambda'$ = energy of the scattered photon

Conservation of energy: $E_o = E' + K$

Conservation of momentum:

x-component: $p_o = p' \cos\theta + p\cos\phi$

y-component: $0 = p' \sin\theta - p\sin\phi$



Relativistic equations:

v = speed of the electron

m = mass of the electron

$p = \gamma\, m\, v$ = momentum of the electron where $\gamma = \dfrac{1}{\sqrt{1-\frac{v^2}{c^2}}}$

$E = \sqrt{p^2c^2 + m^2c^4}$ = total relativistic energy of the electron

$K = E - m\,c^2$ = kinetic energy of the electron

By using above relations and simplifying, we will get,

Compton shift $\boldsymbol{\lambda' - \lambda_o = \dfrac{h}{mc}(1-\cos\theta)}$

Photons and Electromagnetic Waves [Dual Nature of Light]

- Light exhibits diffraction and interference phenomena that are only explicable in terms of wave properties.
- Photoelectric effect and Compton Effect can only be explained taking light as photons / particle.
- This means true nature of light is not describable in terms of any single picture, instead both wave and particle nature have to be considered. In short, the particle model and the wave model of light complement each other.

de Broglie Hypothesis - Wave Properties of Particles

Wavelength associated with particle of mass m moving with velocity v is given by

de Broglie wavelength: $\boldsymbol{\lambda = \frac{h}{p} = \frac{h}{mv}}$

The momentum (p) of an electron accelerated through a potential difference of ΔV is

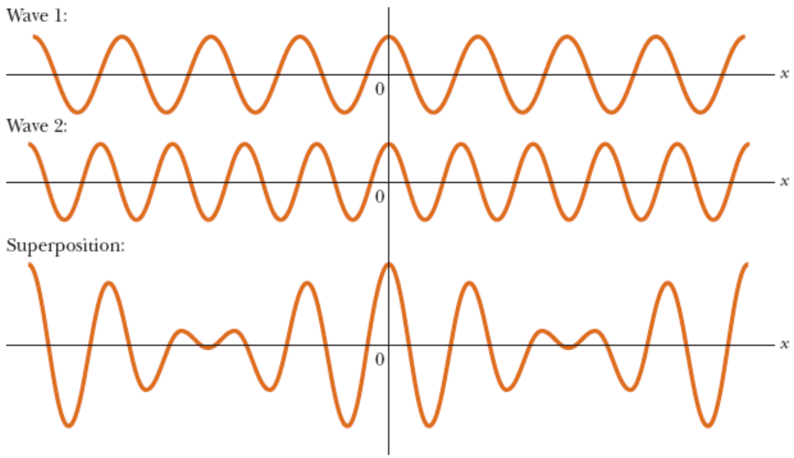
$$p = m\,v = \sqrt{2\,m\,e\,\Delta V}$$

Frequency of the matter wave associated with the particle is $\frac{E}{h}$, where E is total relativistic energy of the particle.

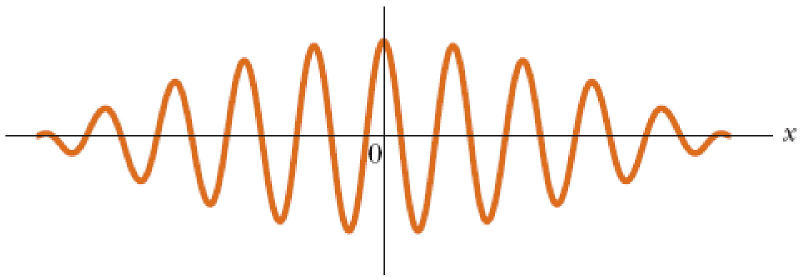
$$f = \frac{E}{h}$$

The Quantum Particle

If we add up large number of waves such that constructive interference takes place in small localized region of space a **wavepacket**, which represents a quantum particle can be formed.



Superposition of two waves



Wave packet

Mathematical representation of a wave packet:

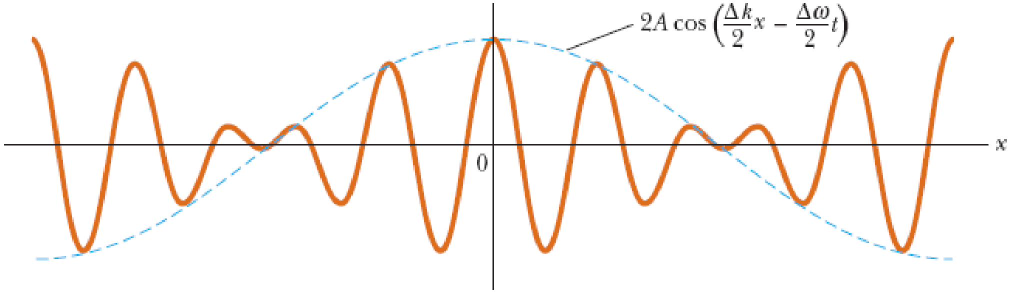
$$y_1 = A\,cos(k_1x - \omega_1 t) \quad \text{and} \quad y_2 = A\,cos(k_2x - \omega_2 t)$$

$$\text{where } k = 2\pi/\lambda\,, \qquad \omega = 2\pi f$$

The resultant wave $\quad y = y_1 + y_2$

$$y = 2A\left[cos\left(\frac{\Delta k}{2}x - \frac{\Delta\omega}{2}t\right)\ cos\left(\frac{k_1+k_2}{2}x - \frac{\omega_1+\omega_2}{2}t\right)\right]$$

where $\Delta k = k_1 - k_2$ and $\Delta\omega = \omega_1 - \omega_2$.



Phase speed, the speed with which wave crest of individual wave moves, is given by

$$v_p = f\,\lambda \qquad \text{or} \qquad v_p = \frac{\omega}{k}$$

Group speed, the speed of the wave packet, is given by

$$v_g = \frac{\left(\frac{\Delta\omega}{2}\right)}{\left(\frac{\Delta k}{2}\right)} = \frac{\Delta\omega}{\Delta k}$$

Relation between group speed (v_g) and phase speed (v_p):

$$v_P = \frac{\omega}{k} = f\,\lambda \qquad \therefore \qquad \omega = k\,v_P$$

$$\text{But } v_g = \frac{d\omega}{dk} = \frac{d(k v_P)}{dk} = k\,\frac{dv_P}{dk} + v_P$$

Substituting for k in terms of λ, we get

$$\boldsymbol{v_g = v_P - \lambda\left(\frac{dv_P}{d\lambda}\right)}$$

Relation between group speed (v_g) and particle speed (u):

$$\omega = 2\,\pi\,f = 2\,\pi\,\frac{E}{h} \quad \text{and} \quad k = \frac{2\,\pi}{\lambda} = \frac{2\,\pi}{h/p} = \frac{2\,\pi\,p}{h}$$

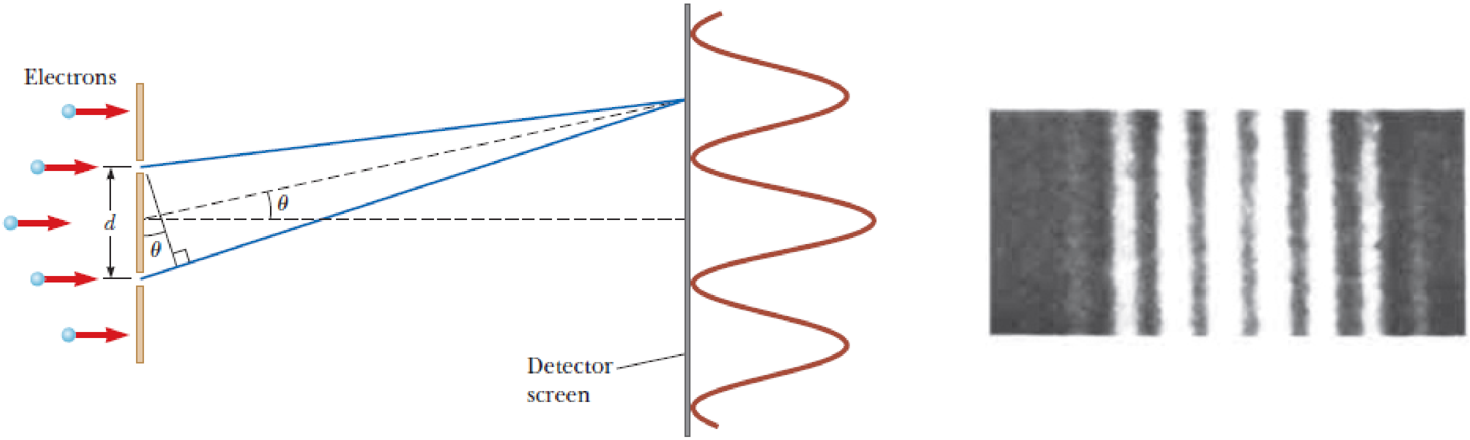
$$v_g = \frac{d\omega}{dk} = \frac{\frac{2\,\pi}{h}\,dE}{\frac{2\,\pi}{h}\,dp} = \frac{d\,E}{d\,p}$$

For a classical particle moving with speed u, the kinetic energy E is given by

$$E = \tfrac{1}{2} m\,u^2 = \frac{p^2}{2\,m} \quad \text{and} \quad dE = \frac{2\,p\,dp}{2\,m} \quad \text{or} \quad \frac{d\,E}{d\,p} = \frac{p}{m} = u$$

$$\boldsymbol{v_g = \frac{d\omega}{dk} = \frac{d\,E}{d\,p} = u}$$

Double–Slit Experiment Revisited



(a) (b)

(a) Schematic of electron beam interference experiment, (b) Photograph of a double-slit interference pattern produced by electrons

$\boldsymbol{d \sin\theta = m\lambda}$, where m is the order number and λ is the electron wavelength.

The electrons are detected as particles at a localized spot on the detector screen at some instant of time, but the probability of arrival at the spot is determined by finding the intensity of two interfering waves.

Uncertainty Principle

Heisenberg uncertainty principle: It is fundamentally impossible to make simultaneous measurements of a particle's position and momentum with infinite accuracy.

$$(\Delta x)(\Delta p_x) \geq h / 4\pi$$

One more relation expressing uncertainty principle is related to energy and time which is given by

$$(\Delta E)(\Delta t) \geq h / 4\pi$$