Chapter 3: Quantum Physics

Heisenberg Uncertainty Principle

P 26: Use the uncertainty principle to show that if an electron was confined inside an atomic nucleus of diameter $2\text{x}10^{-15}$ m, it would have to be moving relativistically ($v \approx c$), while a proton confined to the same nucleus would be moving non-relativistically ($v < c$).

Given: Use the following relativistic relations for calculating the energy of a particle:

$E = \sqrt{p^2c^2 + m^2c^4}$ (E is the total energy and p is momentum)

$$E = \gamma mc^2 \text{ ; where } \gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \quad (v \text{ is velocity})$$

With $\Delta x = 2 \times 10^{-15}$ m, the uncertainty principle requires

$$\Delta p_x \geq (h/4\pi)\,\Delta x = 2.6 \times 10^{-20} \text{ kg. m/s} .$$

Assume $p = \Delta p_x = 2.6 \times 10^{-20}$ kg. m/s .

a) For an electron, $m_e = 9.11 \times 10^{-31}$ kg, hence its energy

$$E = \sqrt{p^2c^2 + m_e^2c^4}$$

Substituting the value of p and m_e we get

$$E = 48\ MeV \ldots\ldots..(1)$$

We also have the relation, $E = \gamma m_e c^2$

Substituting the value of m_e and c, we get

$$E = \gamma\ 0.511\ \mathrm{MeV} \ldots\ldots\ldots.(2)$$

Dividing eq.(2) by eq.(1) we get $\gamma = 94$

But $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \Rightarrow v = c\sqrt{1 - \frac{1}{\gamma^2}}$

Solving for v , we get

$$v = 0.99988\ c$$

Hence, if an electron exists inside a nucleus, its speed would be approximately equal the speed of light.

b) For a proton, we have to replace m by m_p = 1.6 x 10^{-27} kg in all equations in part a)

Solving we get

$$v = 1.8 \times 10^{7}\ m.s$$

i.e. inside the nucleus, proton has less than one-tenth the speed of light

When an atom makes a transition from an higher energy state to lower energy state, energy is emitted in the form of a photon of a given frequency f. The average time interval after excitation during which an atom radiates is called the lifetime t. If t = 10^{-8} s, use the uncertainty principle to compute the line width (or uncertainty in the frequency of the emitted photon) Δf produced by this finite lifetime.

Ans:

$(\Delta E) (\Delta t) \geq h/ 4\pi$

But we know, E = h f. So, $\Delta E = h \Delta f$

$(h \Delta f) (\Delta t) \geq h/ 4\pi$

$\Delta f \geq 1/4\pi \Delta t$

$\Delta f = 8 \times 10^6$ Hz