(d) Not more than two of the events occur simultaneously.

$\overline{A \cap B \cap C}$

3) Show that $P\left[(A \cap B^c) \cup (B \cap A^c)\right] = P(A) + P(B) - 2P(A \cap B)$

Example

1) Suppose A and B are events for which $P(A) = x$, $P(B) = y$, $P(A \cap B) = z$.
Express each of the following probabilities in terms of x, y and z.

(a) $P(A \cup B) = P(A) + P(B) - P(A \cap B) = x + y - z.$

(b) $P(A^c \cap B^c) = P(\overline{A \cup B}) = 1 - P(A \cup B) = 1 - x - y + z.$

(c) $P(A^c \cap B) = P(B) - P(A \cap B) = y - z.$

(d) $P(A^c \cup B^c) = P(\overline{A \cap B}) = 1 - P(A \cap B) = 1 - z.$

2) Let A, B and C be three events. Express the following verbal statements in set notation.

(a) Atleast one of the events occurs.

$A \cup B \cup C$

(b) Exactly one of events occurs

$(A \cap \bar{B} \cap \bar{C}) \cup (A^c \cap B \cap C^c) \cup (A^c \cap B^c \cap C)$ $x \in A$, $x \notin B$, $x \notin C$

(c) Exactly two of the events occur

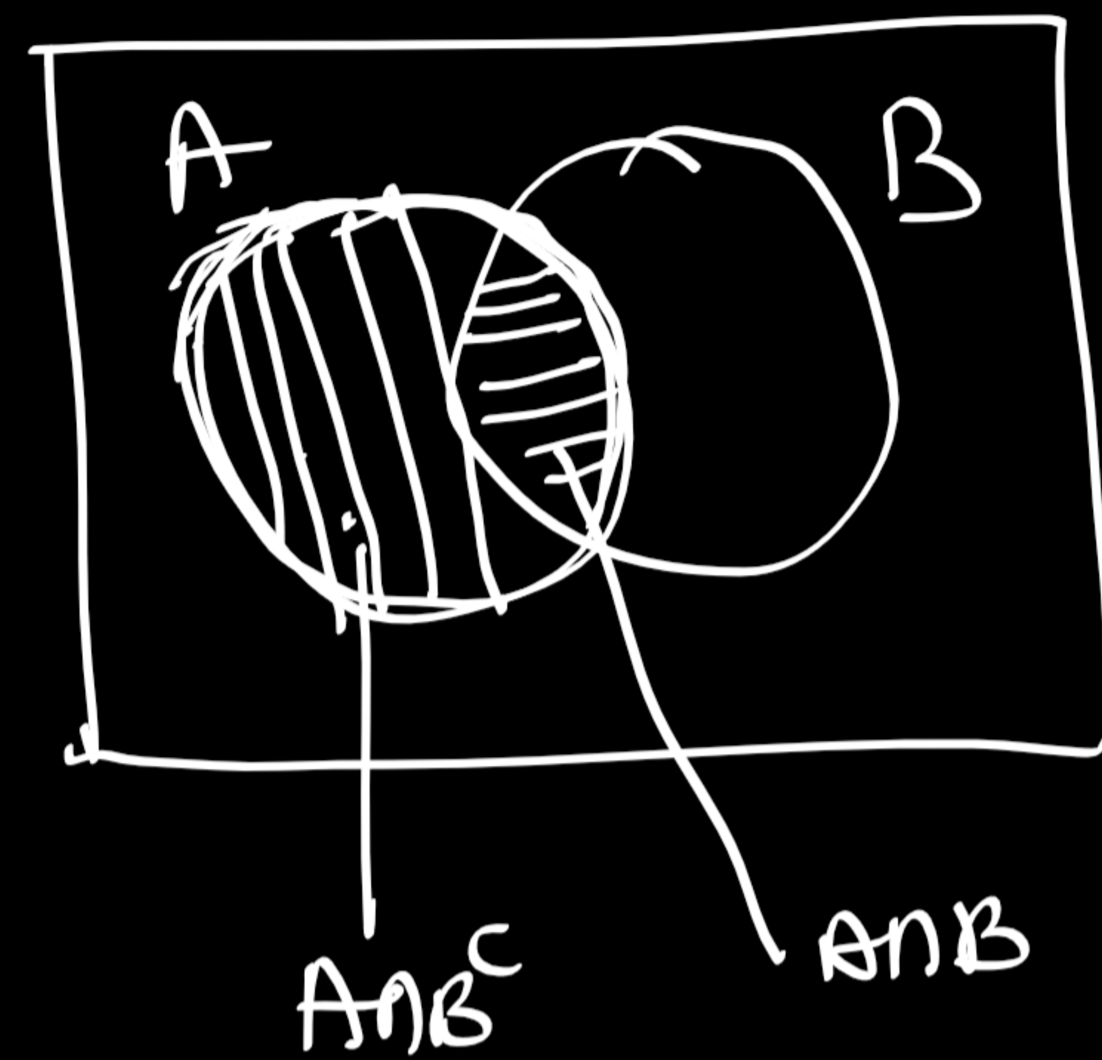
$(A \cap B \cap \bar{C}) \cup (A \cap \bar{B} \cap C) \cup (\bar{A} \cap B \cap C)$

Theorem 5

If A and B are two events then

$$P(A\cap B^c) = P(A) - P(A\cap B)$$

$$A = (A\cap B^c) \cup (A\cap B)$$



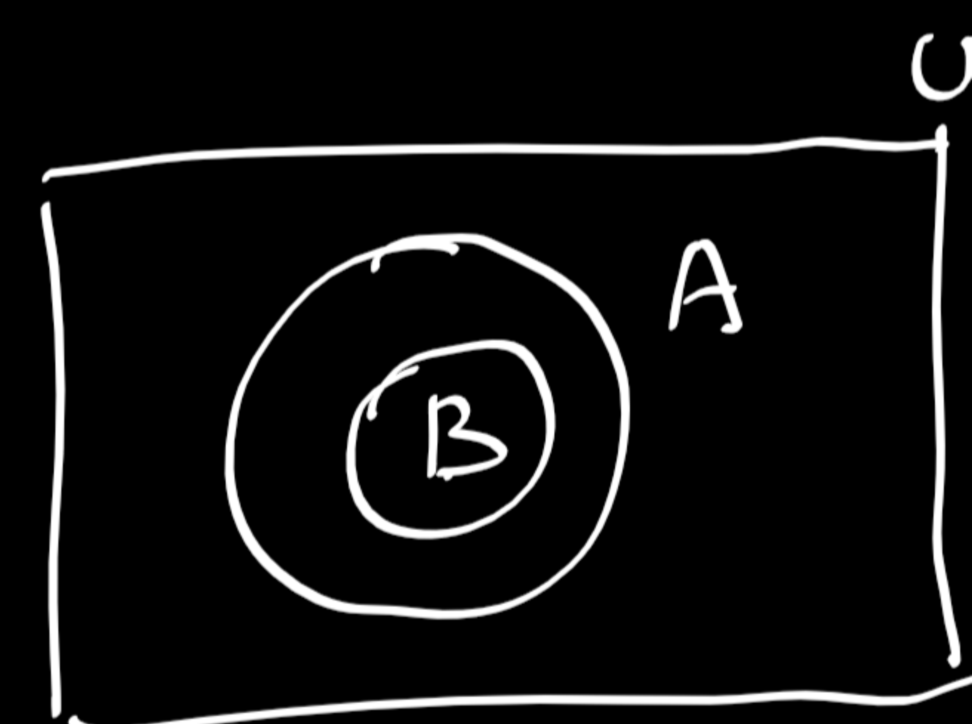
$$P(A) = P(A\cap B^c) + P(A\cap B)$$

$$\left\{ \because (A\cap B^c) \cap (A\cap B) = \phi \right\}$$

$$P(A\cap B^c) = P(A) - P(A\cap B)$$

Similarly, $P(A^c\cap B) = P(B) - P(A\cap B)$

Theorem 6 :- If $B \subseteq A$ then $P(A\cap B^c) = P(A) - P(B)$ and hence $P(B) \leq P(A)$.



$$P(A\cap B^c) = P(A) - P(A\cap B)$$

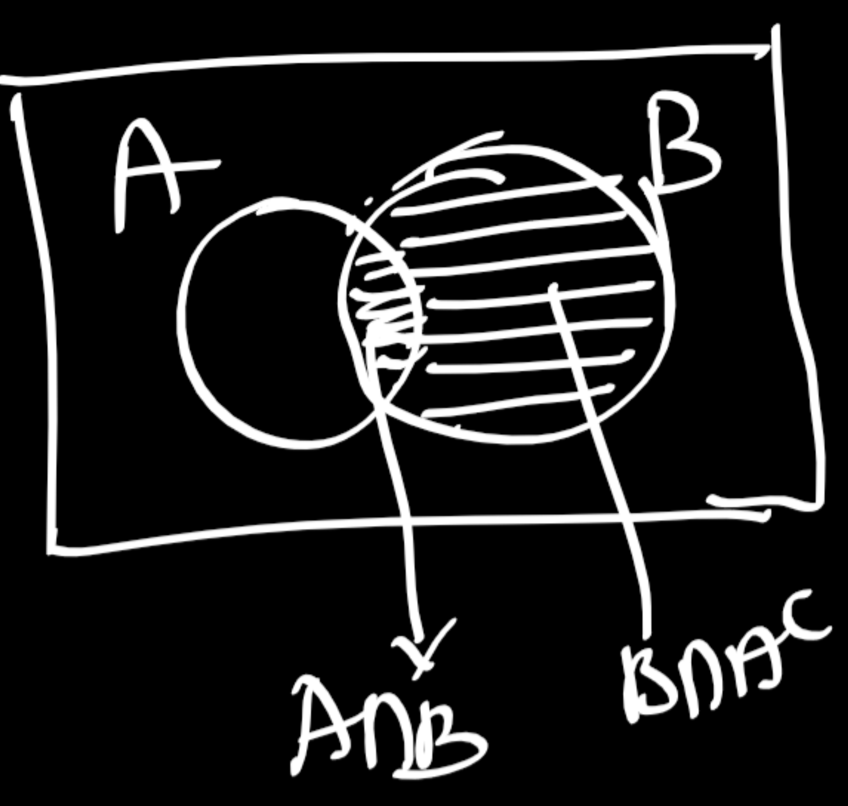
Since $B \subseteq A$, $(A\cap B) = B$

$$\therefore P(A\cap B^c) = P(A) - P(B)$$

$$P(A\cap B^c) \geq 0 \Rightarrow P(A) - P(B) \geq 0$$

$$\therefore P(B) \leq P(A) \quad //$$

$$B = (A \cap B) \cup (B \cap A^c)$$



$$P(B) = P\left[(A \cap B) \cup (B \cap A^c)\right]$$

$$= P(A \cap B) + P(B \cap A^c)$$

$$\left(\because (A \cap B) \cap (B \cap A^c) = \phi\right)$$

$$P(B \cap A^c) = P(B) - P(A \cap B)$$

$$\therefore P(A \cup B) = \underline{\underline{P(A) + P(B) - P(A \cap B)}}$$

4) <u>Theorem 4</u>

If A, B and C are any three events then

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$$

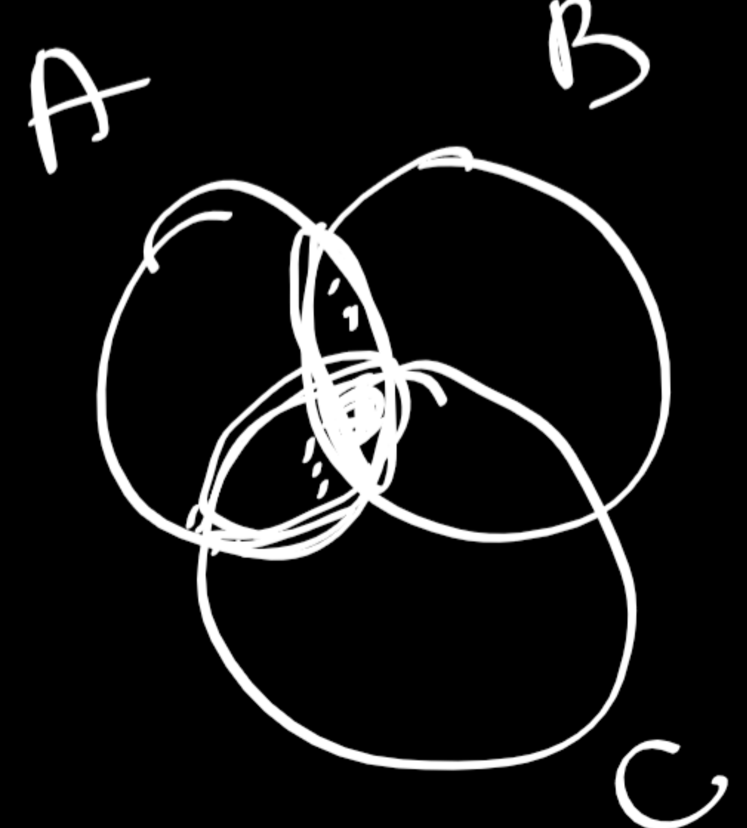
<u>Pf:-</u>

Let $B \cup C = D$

$$P(A \cup D) = P(A) + P(D) - P(A \cap D)$$

$$= P(A) + P(B \cup C) - P(A \cap (B \cup C))$$

$$= P(A) + P(B) + P(C) - P(B \cap C) - P\left[(A \cap B) \cup (A \cap C)\right]$$



$$= P(A) + P(B) + P(C) - P(B \cap C) - \left[P(A \cap B) + P(A \cap C) - P(A \cap B \cap C)\right]$$

Theorem 1

If ϕ is an empty set then $P(A)=0$

Pf:- Let A be any set

$$A = A \cup \phi \qquad A \cap \phi = \phi$$

$$P(A) = P(A \cup \phi) = P(A) + P(\phi),$$

$$\therefore \underline{P(\phi) = 0}$$

2) ## Theorem 2

For any event A,

$$P(A^c) = 1 - P(A)$$

Pf:-

$$A \cup A^c = S$$

$$P(A \cup A^c) = P(S) = 1$$

$$P(A) + P(A^c) = 1 \qquad \because A \cap A^c = \phi$$

$$\therefore \underline{P(A^c) = 1 - P(A)}$$

3) ## Theorem 3

If A and B are any two events then

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

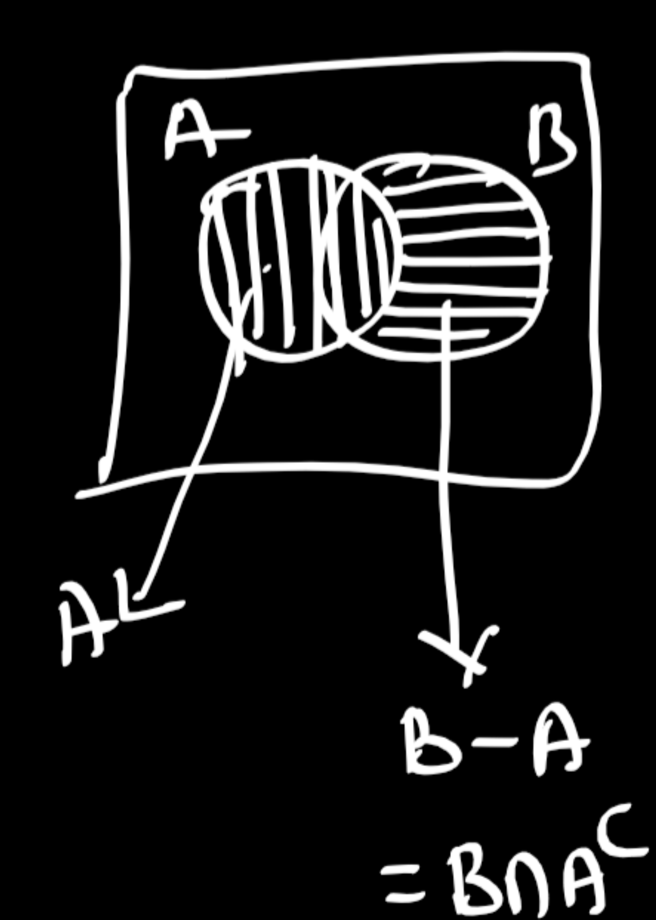
Pf:-

$$A \cup B = A \cup (B \cap A^c)$$

$$P(A \cup B) = P\left[A \cup (B \cap A^c)\right]$$

$$= P(A) + P(B \cap A^c)$$

$$(\because A \cap (B \cap A^c) = \phi)$$



(i) $0 \leq P(A) \leq 1$

(ii) $P(S) = 1$

(iii) $P(A \cup B) = P(A) + P(B)$ whenever A and B are disjoint, i.e., $A \cap B = \phi$

The above three conditions are called Kolmogorov's axioms of Probability.

Probability of an event

1) **Classical definition**

If the no: of elements in a sample space S is n and the no: of elements in an event A is m and n is finite then Probability of A is given by $P(A) = \frac{m}{n} = \frac{\text{Favourable Cases}}{\text{Exhaustive cases}}$

2) **Statistical definition**

If the no: of elements in a sample space S is n and the no: of elements in an event A is m and n is very large

Then $P(A) = \lim_{n \to \infty} \frac{m}{n}$.

3) **Axiomatic Definition**

Let S be the sample space associated with a random experiment E. Let A be an event. We consider a real no: P(A) called the probability of A satisfying the following conditions.

5) Elementary event → Event with only one element.

6) Mutually exclusive events (Disjoint events)

Two events A and B are said to be mutually exclusive if both of them cannot occur simultaneously.

If A and B are disjoint $A \cap B = \phi$.

7) Equally likely outcomes.

If all the outcomes of a random experiment have equal chances of occurence then the outcomes are said to be equally likely.

8) Favourable cases

Total no: of elements in an event

9) Exhaustive Cases

Total no: of elements in a Sample space.

Ex: If $S = \{HH, HT, TH, TT\}$, $A = \{HT, TT\}$

Then Exhaustive Cases = 4

Favourable cases = 2.

PROBABILITY

1. Random Experiment

If the repeatation of the experiment under identical conditions results in different outcomes then that experiment is called a random experiment.

Ex:- Tossing of a coin, Rolling of a die.

2) Sample Space (S)

The set of all outcomes of a random experiment

Ex:- In tossing two coins the sample space contains 4 outcomes

$$S = \{HH, HT, TH, TT\}$$

For a die, $S = \{1, 2, 3, 4, 5, 6\}$

3) Event :- Subset of a sample space.

If $S = \{HH, HT, TH, TT\}$

$A = \{HH\}$, $B = \{TH, HT\}$ → Events

4) Null event ϕ → Empty set.

Algebra of Sets

1) $A \cup \phi = A$ $\quad A \cap \phi = \phi$, $\quad A \cup A = A$, $\quad A \cap A = A$ (Idempotent)

2) $A \cup B = B \cup A$ $\quad A \cap B = B \cap A$ (Commutative)

3) $A \cup (B \cup C) = (A \cup B) \cup C$, $A \cap (B \cap C) = (A \cap B) \cap C$ } Associative law.

4) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$, $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ } Distributive law.

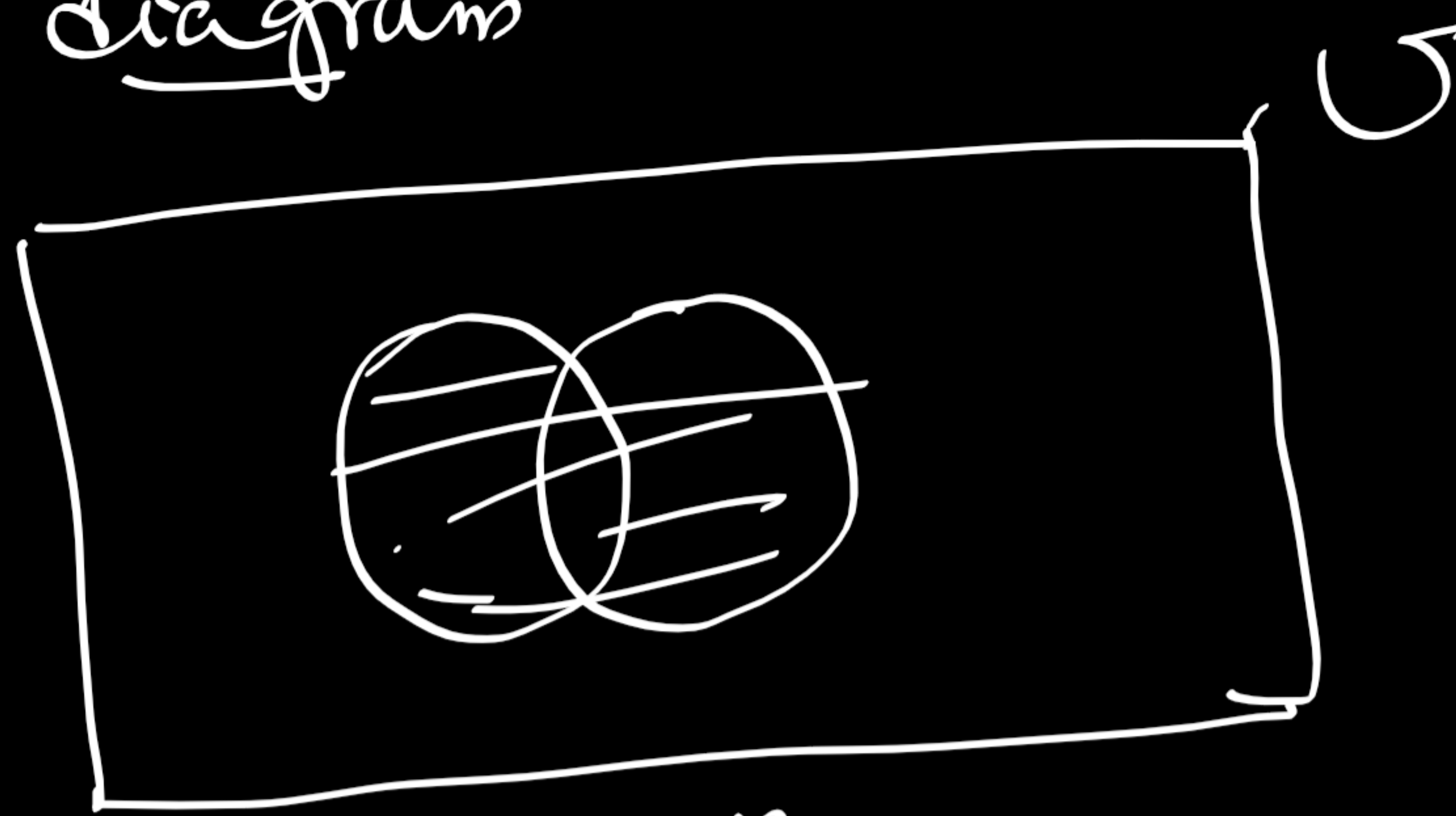
5) $(A \cup B)^C = A^C \cap B^C$, $(A \cap B)^C = A^C \cup B^C$ } De Morgan's law.

6) $A \cup A^C = U$, $\quad A \cap A^C = \phi$

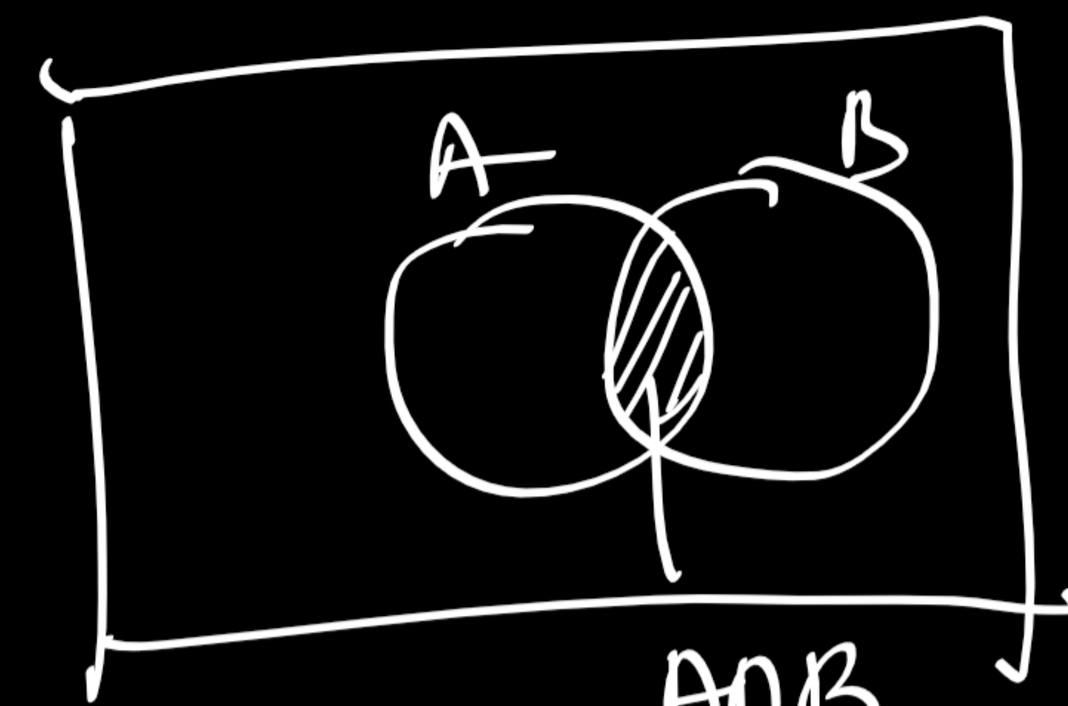
7) $(A^C)^C = A$, $\quad U^C = \phi$

8) $A \cup U = U$, $\quad A \cap U = A$

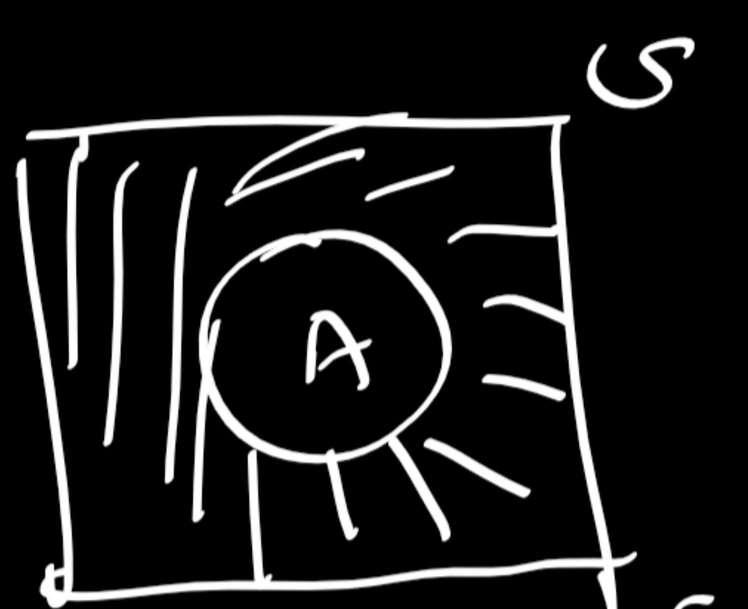
Venn diagrams



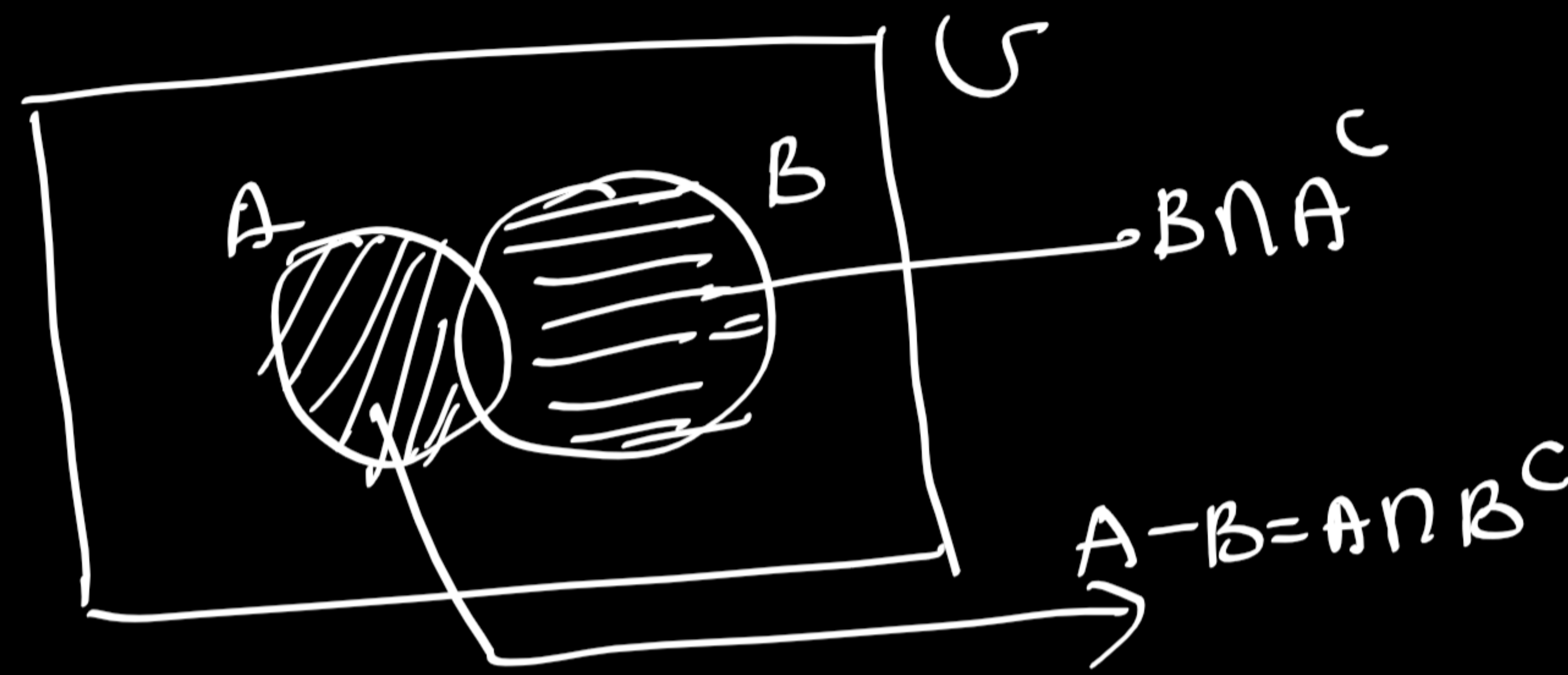
$A \cup B$



$A \cap B$



A^C



Set theory

1) Universal Set

2) Subset
If A and B are two sets
then $x \in A \Rightarrow x \in B$ then $A \subseteq B$

3) Null net (empty set)

4) Complement of a set
A^c, A' or $\bar{A} = \{x : x \in U, x \notin A\}$

5) Union of two set
$A \cup B = \{x : x \in A$ or $x \in B$ or $x \in$ both$\}$

6) Intersection of two sets
$A \cap B = \{x : x \in A$ and $x \in B\}$

7) Difference between two sets (Relative complement)
$A - B = A \cap B^c = \{x : x \in A$ but $x \notin B\}$
OR
$\{x : x \in A$ and $x \in B^c\}$

$B - A = B \cap A^c = \{x : x \in A^c$ and $x \in B\}$