Basic Electrical Technology

[ELE 1051]

SINGLE PHASE AC CIRCUITS

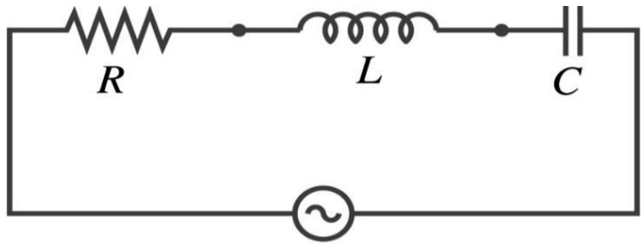
Recap

- Impedance, phasor & power triangles
- Concept of power factor and its significance
- Need for power factor improvement
- Tutorial 2a

Topics covered

- Resonance in series RLC circuit
- half power frequency, bandwidth
- Resonance in parallel circuits

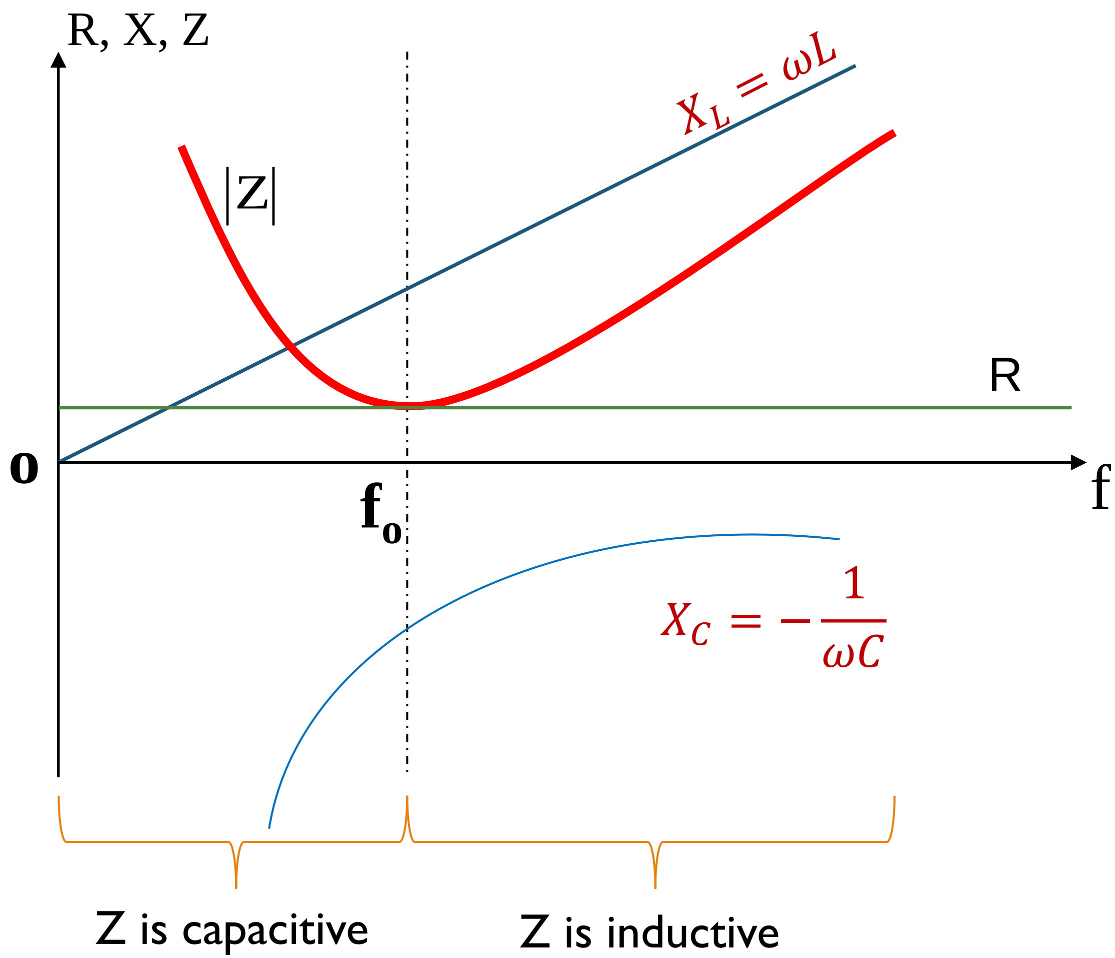
Series Resonance



v(t), variable frequency

$$Z = R + j(X_L \sim X_C)$$

$$|Z| = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$



'f_0 is called the resonant frequency'

Series Resonance

- **When series RLC circuit is at resonance,**
  - Current is in phase with voltage
  - Circuit power factor is unity
  - $X_L = X_C$
  - $Z = R$

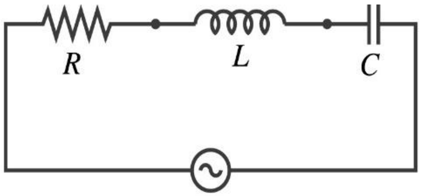
- **Resonant frequency for a series RLC circuit is obtained as follows:**

$$\text{Imaginary part of } Z_{eq} = 0$$

$$X_L = X_C$$

$$\omega_0 L = \frac{1}{\omega_0 C}$$

$$\boxed{f_0 = \frac{1}{2\pi\sqrt{LC}} \text{ hertz}}$$

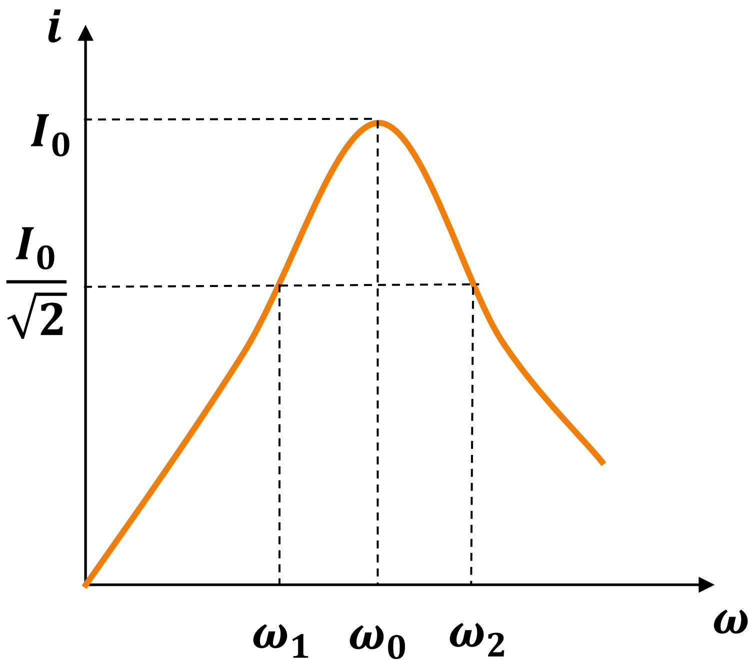


v(t), variable frequency

$$Z = R + j(X_L \sim X_C)$$

Current vs. Frequency in RLC Series Circuit

Variation of current with frequency



$$I_0 = I_{max} = \frac{V_{rms}}{R}$$

- **Half Power Frequency**

  'Frequency at which the power is half of the power at resonant frequency'

$$Power = \frac{1}{2} I_0^2 R = \left(\frac{I_0}{\sqrt{2}}\right)^2 R$$

$$At\ \omega_1\ and\ \omega_2, I = \frac{I_0}{\sqrt{2}}$$

$$\omega_1 = Lower\ half\ power\ frequency$$

$$\omega_2 = Upper\ half\ power\ frequency$$

$$\mathbf{Bandwidth = \omega_2 - \omega_1}$$

In practice the curve of $|I|$ against ω is not symmetrical about the resonant frequency

Half Power Frequency

$$Impedance\ at\ \omega_1 and\ \omega_2, |Z| = {V_0}\Big/{\frac{I_0}{\sqrt{2}}} = \sqrt{2}R$$

Below Resonant frequency $\omega_0, |X_C| > |X_L|$

At ω_1,

$$\sqrt{R^2 + (X_C - X_L)^2} = \sqrt{2}R$$

$$X_C - X_L = R$$

$$\frac{1}{\omega_1 C} - \omega_1 L = \mathrm{R}$$

$$\boxed{\boldsymbol{\omega_1 = \frac{-R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}}}$$

Above Resonant frequency $\omega_0, |X_L| > |X_C|$

At ω_2,

$$\sqrt{R^2 + (X_L - X_C)^2} = \sqrt{2}R$$

$$X_L - X_C = R$$

$$\omega_2 L - \frac{1}{\omega_2 C} = \mathrm{R}$$

$$\boxed{\boldsymbol{\omega_2 = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}}}$$

$$\boldsymbol{\omega_2 \omega_1 = \frac{1}{LC} = \omega_0^2} \qquad \boldsymbol{\omega_2 - \omega_1 = \frac{R}{L}}$$

Quality Factor for series circuit

- At resonance, V_C and V_L can be very much greater than applied voltage

$$|V_C| = |I|X_C = \frac{V.X_C}{\sqrt{R^2 + (X_L - X_C)^2}}$$

At resonance, $X_L = X_C$

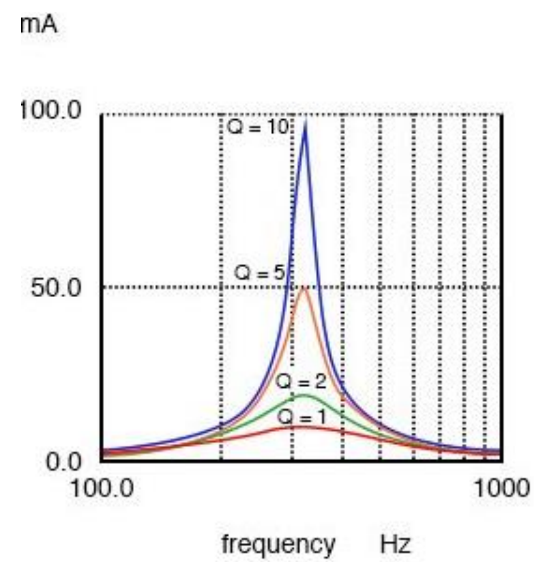
$$V_C = \frac{V}{R}X_C$$

$$V_C = \frac{V}{\omega_0 CR} = \boldsymbol{Q}V$$

Q is termed the Q factor or voltage magnification

$$\boxed{\boldsymbol{Q = \frac{1}{\omega_0 CR} = \frac{\omega_0 L}{R} = \frac{1}{R}\sqrt{\frac{L}{C}}}}$$

$$\boldsymbol{Q = \frac{Resonant\ frequency}{Bandwidth}}$$

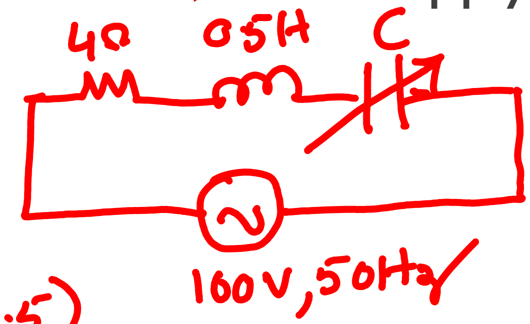


- High value of Q can lead to component damage
- Careful design necessary
- Larger the value of Q, more symmetrical the curve appears about the resonant frequency

Illustration 1

A circuit having a resistance of 4Ω and inductance of 0.5H and a variable capacitance in series, is connected across a 100V, 50Hz supply. Calculate:

a) The capacitance to give resonance
b) The voltages across the inductor and the capacitor
c) The Q factor of the circuit



Soln:- For resonance, $X_L = X_C = 2\pi(50)(0.5)$

(a) $\frac{1}{2\pi(50)C} = 50\pi = X_C$

$C = 20.26\mu F$

(b) $I = \frac{V}{R} = \frac{100}{4} = 25A$

$V_L = V_C = |I| X_C = 3926.99V$

(c) $V_C = QV$

$Q = \underline{39.26}$

Illustration 2

The bandwidth of a series resonant circuit is 500 Hz. If the resonant frequency is 6000 Hz, what is the value of Q? If R =10 Ω, what is the value of the inductive reactance at resonance? Calculate the inductance and capacitance of the circuit

Soln :-

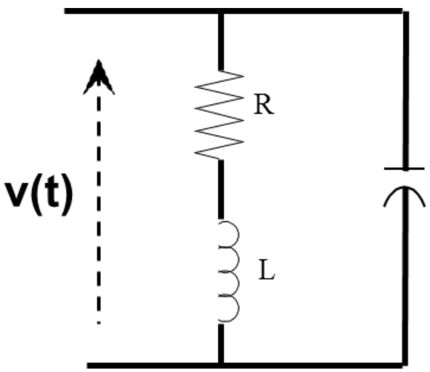
$$\omega_2 - \omega_1 = \frac{R}{L}$$

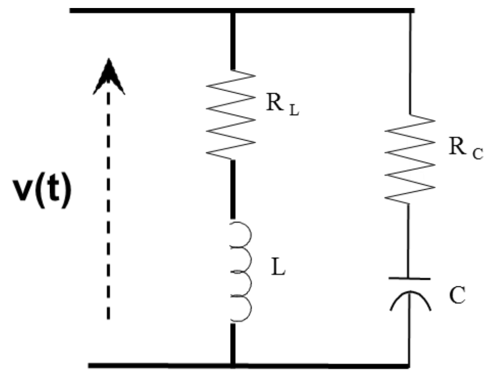
$$2\pi(500) = \frac{10}{L} \Rightarrow L = 3.18mH$$

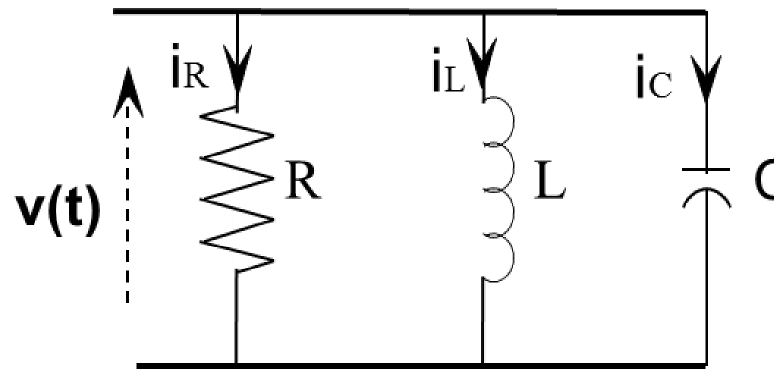
$$Q = \frac{f_0}{BW} = \frac{6000}{500} = 12$$

$$Q = \frac{1}{\omega_0 CR} = \frac{1}{2\pi(6000)C(10)} = 12 \Rightarrow C = 0.22\mu F$$

Resonance in parallel circuits







<u>Steps to obtain the expression of resonant frequency in parallel circuits</u>

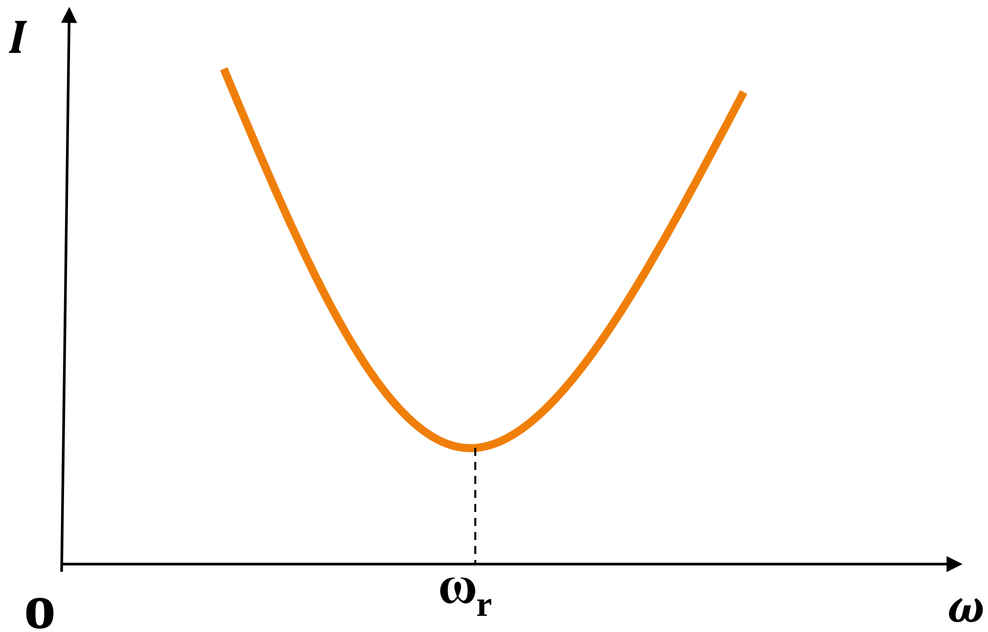
- Obtain the net admittance of the circuit ; $Y_{eq} = y_1 + y_2 + \cdots$

$$\mathbf{Y_{eq} = G_{eq} \pm jB_{eq}}$$

- Equate the imaginary part (susceptance) to zero; $B_{eq} = 0$ and obtain the expression of ω_r

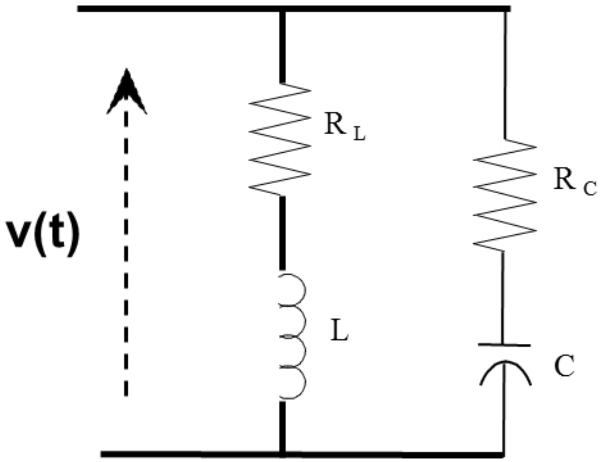
The expression for resonant frequency depends on circuit configuration

Current vs. Frequency in parallel Circuits



- At resonance
  - Impedance is maximum
  - Resultant current minimum

Parallel resonance circuits



$$y_{eq} = \frac{1}{R_L + jX_L} + \frac{1}{R_C - jX_c}$$

$$y_{eq} = \frac{R_C - jX_c + R_L + jX_L}{(R_L R_C + X_L X_c) - j(R_L X_C - X_L R_C)}$$

Rationalizing ;

$$y_{eq} = \frac{\left((R_L R_C + X_L X_c) + j(R_L X_C - X_L R_C)\right)(R_C - jX_c + R_L + jX_L)}{(R_L R_C + X_L X_c)^2 + (R_L X_C - X_L R_C)^2}$$

Separating the real & imaginary terms ;

$$y_{eq} = \frac{1}{(R_L R_C + X_L X_c)^2 + (R_L X_C - X_L R_C)^2} (R_L^2 R_C + R_L R_C^2 - R_L X_C^2 - X_L^2 R_C) + \boldsymbol{j}(R_L^2 X_C + X_C X_L^2 - X_L R_C^2 - X_C^2 X_L)$$

Parallel resonance circuits

Equating the imaginary part to zero;

$B_{eq} = 0$;

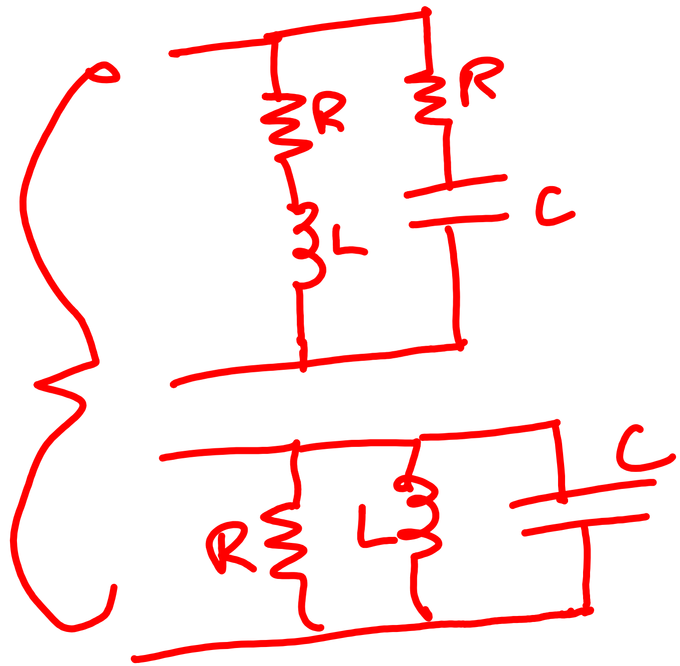
$R_L^2 X_C + X_C X_L^2 - X_L R_C^2 - X_C^2 X_L = 0$

Solving for ω_0;

$$\boldsymbol{\omega_0 = \frac{1}{\sqrt{LC}}\sqrt{\frac{R_L^2 C - L}{R_C^2 C - L}}}$$

If $R_L = R_c$:

$$\boxed{\boldsymbol{f_0 = \frac{1}{2\pi\sqrt{LC}}}}$$



Exercise 1

A parallel circuit with an RL series branch (R = 20 Ω and L = 50 mH) and an RC series branch (R = 10 Ω and C = 100 μF) are connected to a variable frequency voltage source. Find at what frequency the circuit will resonate?

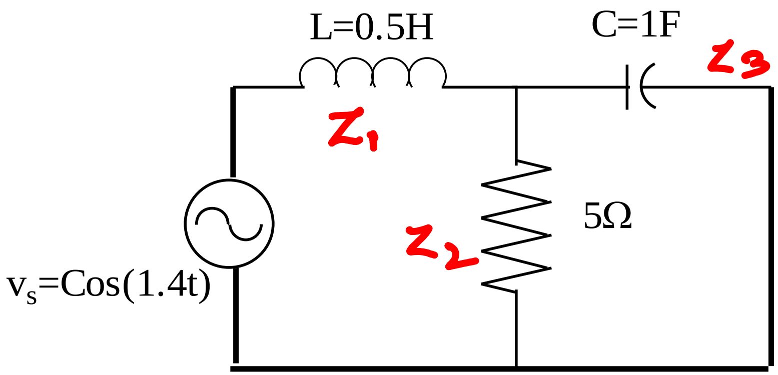
Ans:

$$\omega_c = \frac{1}{\sqrt{LC}}\sqrt{\frac{R_L^2 C - L}{R_C^2 C - L}} = 223.6067\, rad/sec = 35.58\, Hz$$

Exercise 2

Show that circuit given in figure will be at resonance at supply frequency

$\omega_0 = 1.4 \text{ rad/s}$



$Z_{eq} = Z_1 + (Z_2 \| Z_3)$

Soln:-

$Z_1 = j(1.4)(0.5) = 0.7j$

$Z_2 = 5$

$Z_3 = \frac{1}{1.4 \times 1} = -\frac{5j}{7}$

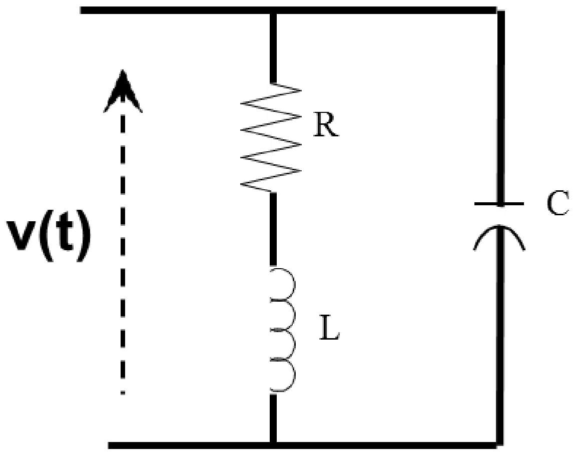
$Z_2 \| Z_3 = \frac{(5)\left(-\frac{5j}{7}\right)}{5 - \frac{5j}{7}}$

$Z_2 \| Z_3 = 0.1 - 0.7j$

$Z_{eq} = \cancel{0.7j} + 0.1 - \cancel{0.7j} = 0.1\Omega$

Exercise 3

Obtain the expression for resonant frequency for the given parallel circuit



$$Z_1 = R + jX_L$$

$$Z_2 = -jX_C$$

$$Y_{eq} = \frac{1}{R+jX_L} - \frac{1}{jX_C}$$

$$= \frac{R - jX_L}{R^2 + X_L^2} + \frac{j}{X_C}$$

$$= \frac{(R - jX_L)X_C + j(R^2 + X_L^2)}{X_C(R^2 + X_L^2)}$$

$$B_{eq} = 0$$

$$-X_L X_C + (R^2 + X_L^2) = 0$$

$$X_L X_C = R^2 + X_L^2$$

$$\frac{\cancel{\omega_r} L}{\cancel{\omega_r} C} = R^2 + \omega_r^2 L^2$$

$$\omega_r^2 = \frac{\frac{L}{C} - R^2}{L^2}$$

$$\omega_r^2 = \frac{1}{CL} - \frac{R^2}{L^2}$$

$$\omega_r = \sqrt{\frac{1}{LC} - \left(\frac{R}{L}\right)^2}$$