Chapter 2: Diffraction Patterns and Polarization

P1 : Light of wavelength 540 nm passes through a slit of width 0.200 mm. (a) The width of the central maximum on a screen is 8.10 mm. How far is the screen from the slit? (b) Determine the width of the first bright fringe to the side of the central maximum.

Ans:

Given: $\lambda = 540 \times 10^{-9} m$ $\quad a = 0.2 \times 10^{-3} m$ $\quad W = 8.1 \times 10^{-3} m$

a)

$$W = \frac{2\lambda L}{a} \Rightarrow L = \frac{aW}{2\lambda} = \frac{0.2 \times 10^{-3} \times 8.1 \times 10^{-3}}{2 \times 540 \times 10^{-9}}$$

$$= 1.5\ m$$

b)

The first bright fringe is situated in between the first and second dark fringes.

If y_1 and y_2 are respectively the linear positions of first and second dark fringes, then the width of first bright fringe $= y_2 - y_1$

$$= \frac{2\lambda L}{a} - \frac{\lambda L}{a} = \frac{\lambda L}{a} = \frac{540 \times 10^{-9} \times 1.5}{0.2 \times 10^{-3}}$$

$$= 4.05\ mm$$

P 3: A screen is placed 50.0 cm from a single slit, which is illuminated with light of wavelength 690 nm. If the distance between the first and third minima in the diffraction pattern is 3.00 mm, what is the width of the slit?

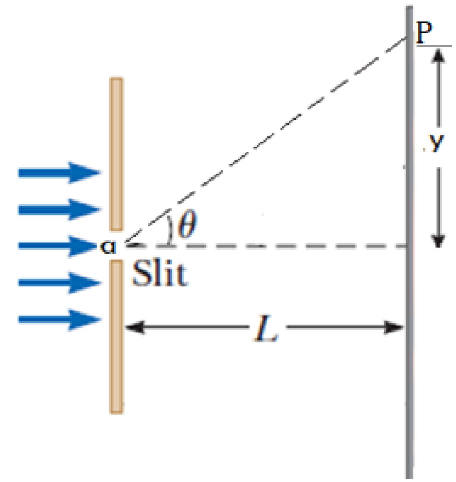
Ans:

Given: $\lambda = 690 \times 10^{-9} m \quad L = 50 \times 10^{-2} m \quad y_3 - y_1 = 3 \times 10^{-3} m$

$$y_3 - y_1 = \frac{3\lambda L}{a} - \frac{\lambda L}{a} = \frac{2\lambda L}{a}$$

$$a = \frac{2\lambda L}{y_3 - y_1} = \frac{2 \times 690 \times 10^{-9} \times 50 \times 10\text{^} - 2}{3 \times 10^{-3}} = 2.3 \times 10^{-4}\ m$$

P 5: A diffraction pattern is formed on a screen 120 cm away from a 0.4 mm wide slit. Monochromatic 546.1 nm light is used. Calculate the fractional intensity I/I_{max} at a point on the screen 4.10 mm from the center of the principal maximum. (Hint: Use small angle appproximation)



$$I = I_{max}\left[\frac{\sin(\pi a \sin\theta/\lambda)}{\pi a \sin\theta/\lambda}\right]^2$$

$$\sin\theta \approx \tan\theta = \frac{y}{L} = \frac{4.10 \times 10^{-3}\ \text{m}}{1.20\ \text{m}} = 3.417 \times 10^{-3}$$

$$\frac{\pi a \sin\theta}{\lambda} = \frac{\pi(4.00 \times 10^{-4}\ \text{m})(3.417 \times 10^{-3})}{546.1 \times 10^{-9}\ \text{m}} = 7.862\ \text{rad}$$

$$\frac{I}{I_{max}} = \left[\frac{\sin(\pi a \sin\theta/\lambda)}{\pi a \sin\theta/\lambda}\right]^2 = \left[\frac{\sin(7.862\ \text{rad})}{7.862\ \text{rad}}\right]^2 = 1.62 \times 10^{-2}$$

P 7: The angular resolution of a radio telescope is to be 0.100° when the incident waves have a wavelength of 3.00 mm. What is the minimum diameter required for the telescope's receiving dish (lens) to have this resolution?

Ans:

Given

$$\theta_{min} = 0.1° = \frac{0.1 \times 3.14}{180}\, rad = 0.00174\, rad$$

From Rayleigh's Criterion for circular aperture

$$\theta_{min} = 1.22\frac{\lambda}{D}$$

Hence

$$D = 1.22\frac{\lambda}{\theta_{min}} = 1.2\frac{3 \times 10^{-3}}{(0.00174)} = 2.07\, m$$

P 9 : Light of wavelength 500 nm is incident normally on a diffraction grating. If the third-order maximum of the diffraction pattern is observed at 32.0° (a) what is the number of rulings per centimeter for the grating? (b) Determine the total number of primary maxima that can be observed in this situation.

Ans:

a) Using grating equation, $d \sin\theta = m\lambda$

$$d = \frac{m\lambda}{\sin\theta} = \frac{3 \times 500 \times 10^{-9}}{\sin 32°} = 2.83 \times 10^{-6}\, m$$

$$\mathrm{N} = \frac{1}{d} = 3.53 \times 10^{5}\ (lines/m)$$

i.e., Number of ruling per cm = 3.53×10^{3}

b) To find the number of primary maxima that can be observed, we need to find the maximum order m that is allowed at wavelength λ.

From grating equation

$$m = \frac{d}{\lambda}\sin\theta$$

⇒ m is maximum when rhs is maximum. Maximum value of $\sin\theta$ is 1 ⇒

$$m_{max} \le \frac{d}{\lambda} = \frac{2.83 \times 10^{-6}}{500 \times 10^{-9}} = 5.66$$

$$m_{max} = 5$$

Total number of maxima that can be observed = $2\, m_{max} + 1 = 2 \times 5 + 1 = 11$

P 11 : The first-order diffraction maximum is observed at 12.6° for a crystal having a spacing between planes of atoms of 0.250 nm. (a) What wavelength x-ray is used to observe this first-order pattern? (b) How many orders can be observed for this crystal at this wavelength?

Ans:

a) From Bragg's law, $2d\sin\theta = m\lambda$, and $m = 1$

$$\lambda = 2d\sin\theta = 2(0.250\text{ nm})\sin 12.6^\circ$$

i.e., $\lambda = 0.109\text{ nm}$

b)
$$\frac{m\lambda}{2d} = \sin\theta \le 1$$

$$m \le \frac{2d}{\lambda} = \frac{2(0.250\text{ nm})}{0.109\text{ nm}} = 4.59$$

i.e., four orders can be observed.