ICE 2154 NETWORK ANALYSIS AND SIGNALS

Motivation

Electrical circuits seem to be everywhere!

What is an Electric Circuit?

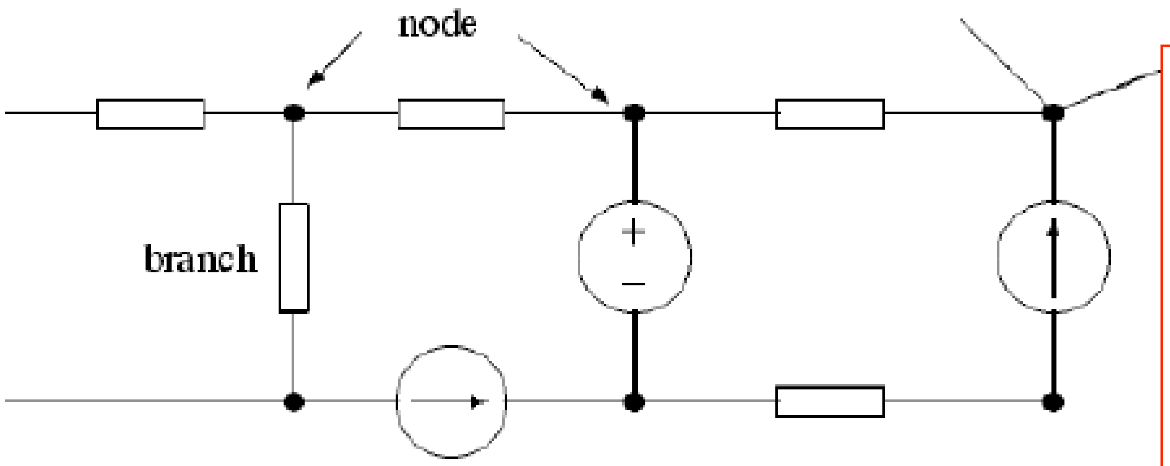
- In electrical engineering, we are usually interested in transferring energy or communicating signals from one point to another.

  To do this, we often require an interconnection of electrical components.

  "An electric circuit is an interconnection of electrical components."

Circuit

- Collection of devices such as sources and resistors in which terminals are connected together by conducting wires.
  - These wires converge in NODES
  - The devices are called BRANCHES of the circuit



<u>Circuit Analysis Problem:</u>
To find all currents and voltages in the branches of the circuit when the intensities of the sources are known.

Fundamental quantities

- Voltage — potential difference bet. 2 points
  - "across" quantity
  - analogous to 'pressure' between two points

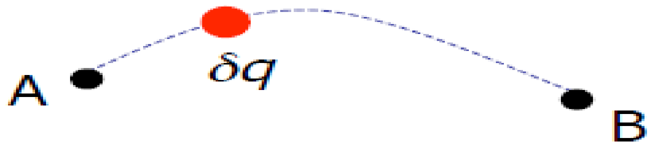
- Current — flow of charge through a material
  - "through" quantity
  - analogous to fluid flowing along a pipe

$$I = \lim_{\delta t \to 0} \frac{\delta q}{\delta t} = \frac{dq}{dt}$$

Power and energy

Work done in moving a charge δq from A to B having a potential difference of V is

$$W = V\,\delta q$$



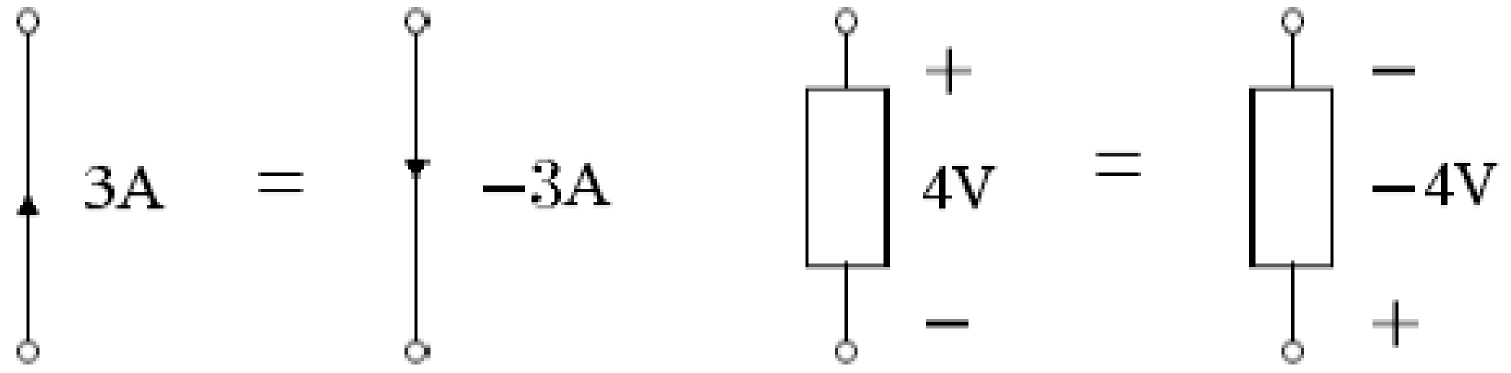
Power is work done per unit time, i.e.,

$$\boxed{P = \lim_{\delta t \to 0} V \frac{\delta q}{\delta t} = V \frac{dq}{dt} = VI}$$

Direction and polarity

- Current direction indicates the direction of flow of positive charge
- Voltage polarity indicates the relative potential between 2 points: + assigned to a higher potential point; and – assigned to a lower potential point.

- *NOTE: Direction and polarity are arbitrarily assigned on circuit diagrams. Actual direction and polarity will be governed by the sign of the value.*



Circuit Elements

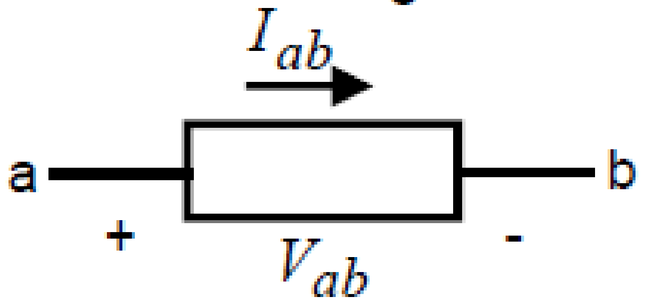
- Circuit components can be broadly classified as being either **active** or **passive**.

- An active element is capable of generating energy.
  - Example: current or voltage sources

- A passive element is an element that does not generate energy, however, they can either consume or store energy.
  - Example: resistors, capacitors, and inductors

Passive Sign Convention

- For calculating absorbed power: The power absorbed by any circuit element with terminals A and B is equal to the voltage drop from A to B multiplied by the current through the element from A to B, i.e., $P = V_{ab} \times I_{ab}$

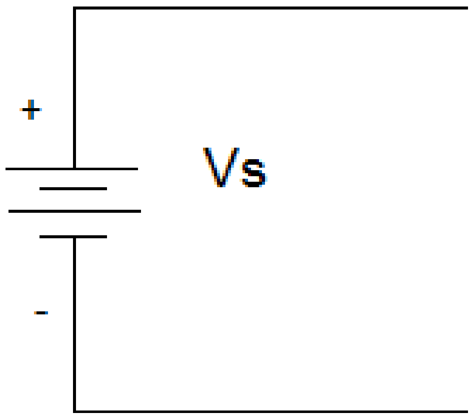


- With this convention if $P \geq 0$, then the element is absorbing (consuming) power. Otherwise (i.e., $P < 0$) is absorbing negative power or actually generating (delivering) power.

Voltage Sources

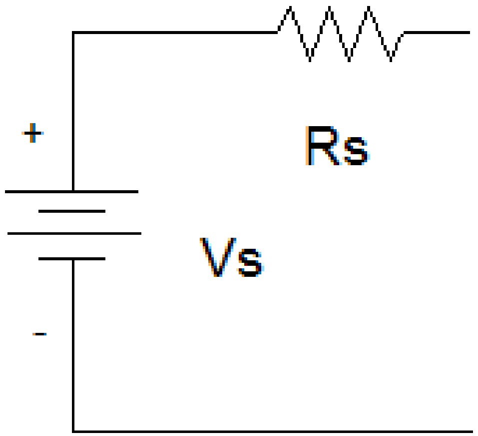
Ideal

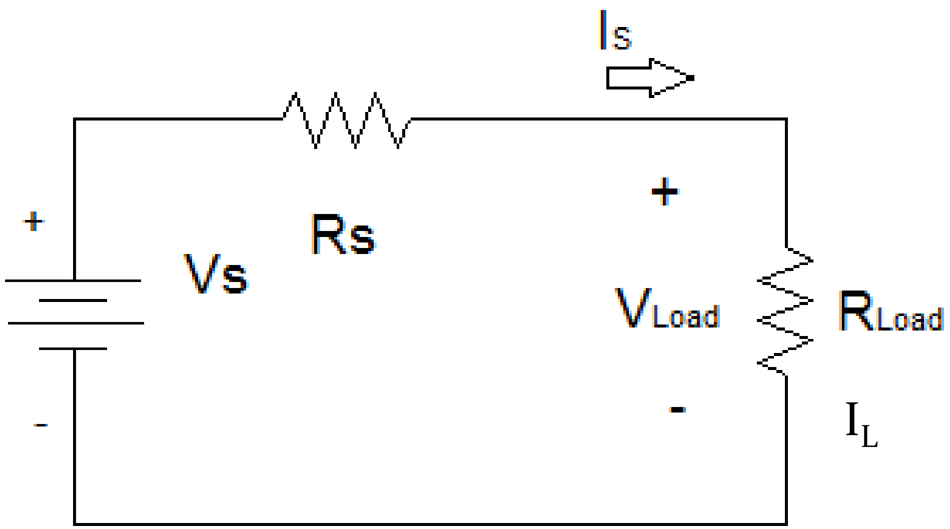
- An ideal voltage source has no internal resistance.
  - It can produce as much current as is needed to provide power to the rest of the circuit.



Real

- A real voltage sources is modeled as an ideal voltage source in series with a resistor.
  - There are limits to the current and output voltage from the source.





$$V_L = \frac{R_L}{R_L + R_S} V_S$$

$$I_L = V_L / R_L$$

$R_L = 0\Omega$

$$V_L = 0\text{V}$$

$$I_{L\max} = V_S / R_S$$

$$P_L = 0\text{W}$$

$R_L = \infty\Omega$

$$V_L = V_S$$

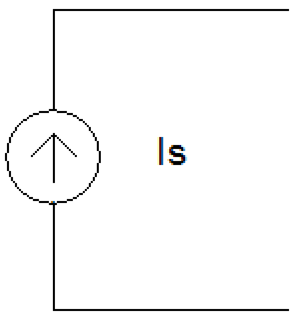
$$I_{L\min} = 0\text{A}$$

$$\text{P}_L = 0\text{W}$$

Current Sources

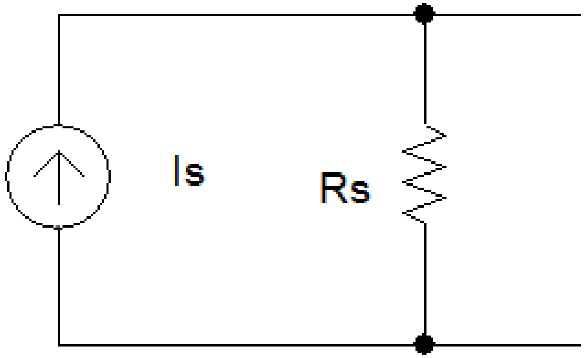
Ideal

- An ideal current source has no internal resistance.
  - It can produce as much voltage as is needed to provide power to the rest of the circuit.

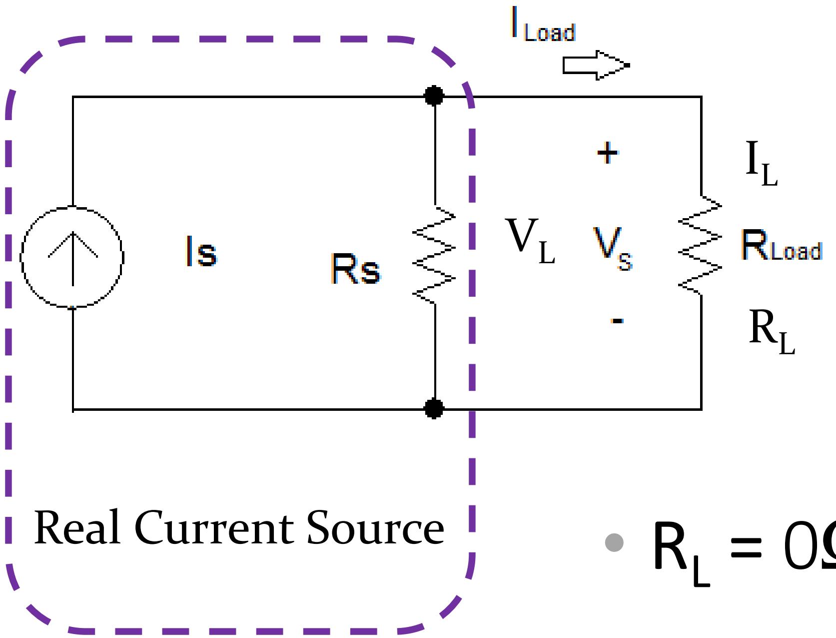


Real

- A real current sources is modeled as an ideal current source in parallel with a resistor.
  - Limitations on the maximum voltage and current.



- Appear as the resistance of the load on the source approaches Rs.



$$I_L = \frac{R_S}{R_L + R_S} I_S$$

$$V_L = I_L R_L$$

- $R_L = 0\Omega$

$$I_L = I_S$$

$$V_{L\min} = 0\text{V}$$

$$P_L = 0\text{W}$$

- $R_L = \infty\Omega$

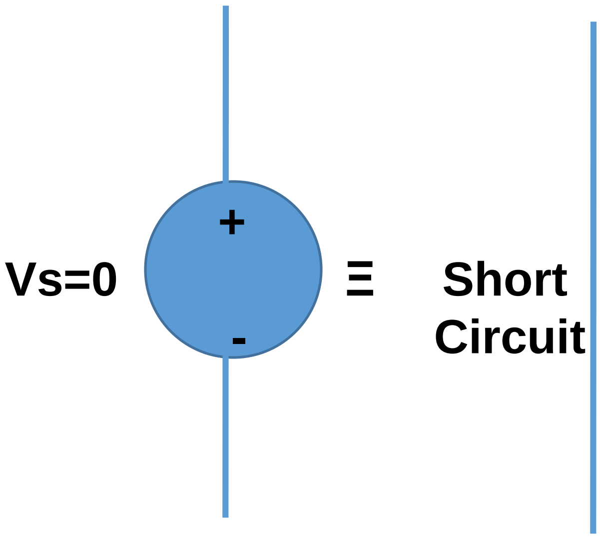
$$I_L = 0\text{A}$$

$$V_{L\max} = I_S R_S$$

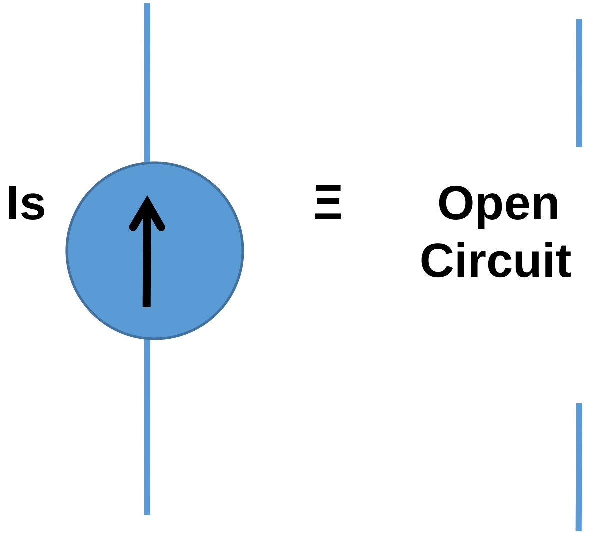
$$\text{P}_\text{L} = 0\text{W}$$

Sources when set to Zero

- Voltage Sources



- Current Sources

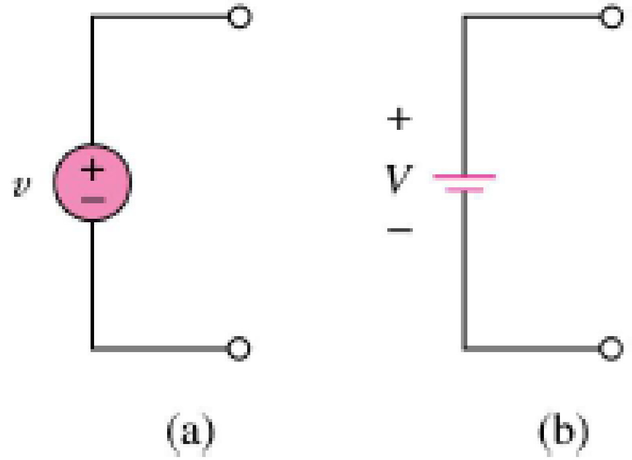


(Ideal) Voltage and Current Sources

- Independent sources: An (ideal) independent source is an active element that provides a specified voltage or current that is independent of other circuit elements and/or how the source is used in the circuit.
- Symbol for independent voltage source

  (a) Used for constant or time-varying voltage

  (b) Used for constant voltage (dc)

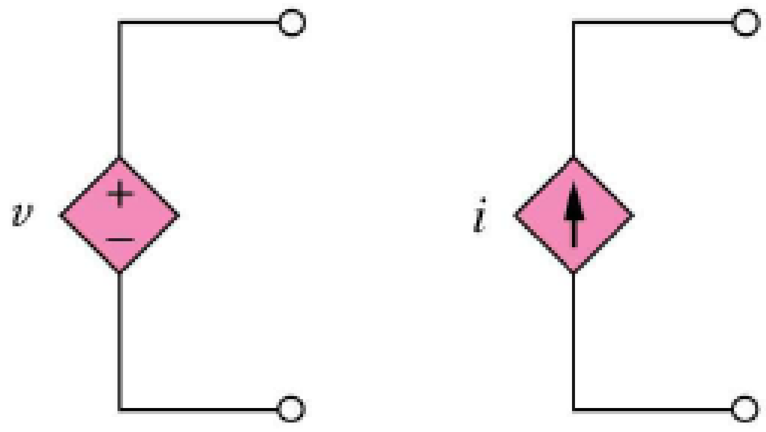


Question: Plot the v-i characteristic of the above dc source.

Ideal Dependent (Controlled) Source

- An ideal dependent (controlled) source is an active element whose quantity is controlled by a voltage or current of another circuit element.

- Dependent sources are usually presented by diamond-shaped symbols:

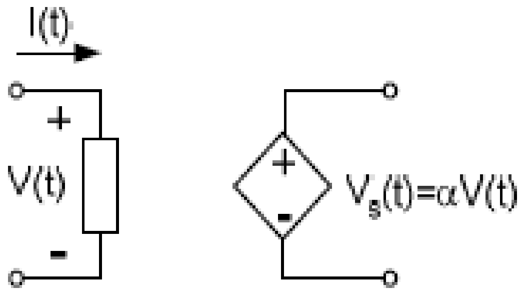


(a) (b)

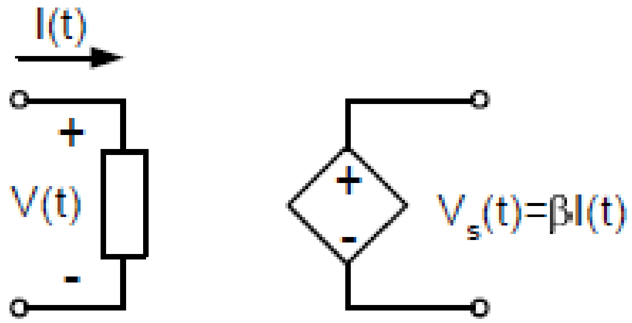
Dependent (Controlled) Source

- There are four types of dependent sources:

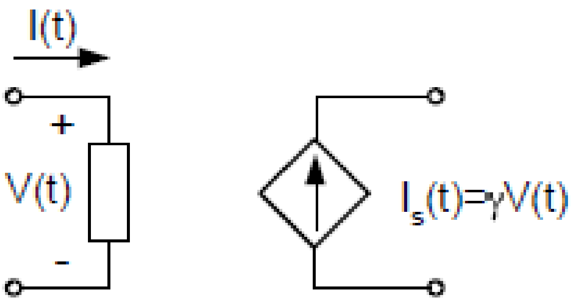
- Voltage-controlled voltage source (VCVS)



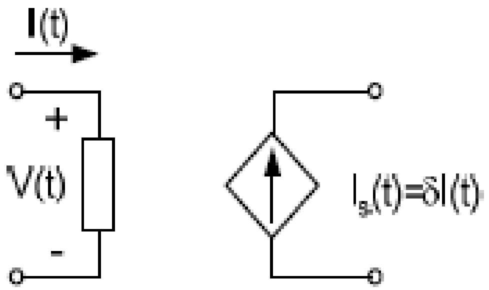
- Current-controlled voltage source (CCVS)



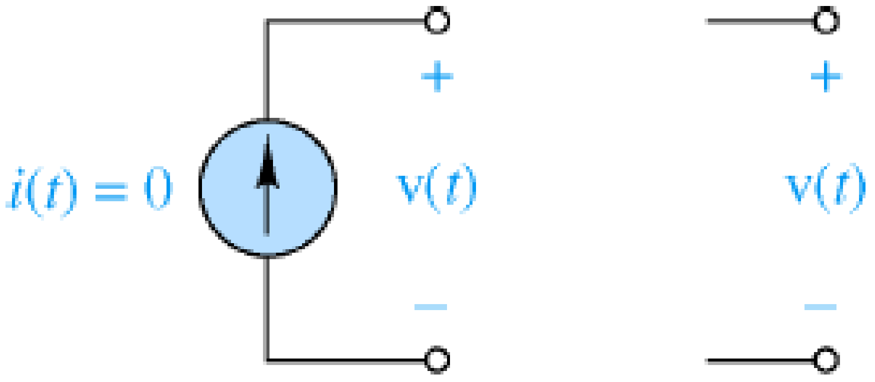
- Voltage-controlled current source (VCCS)



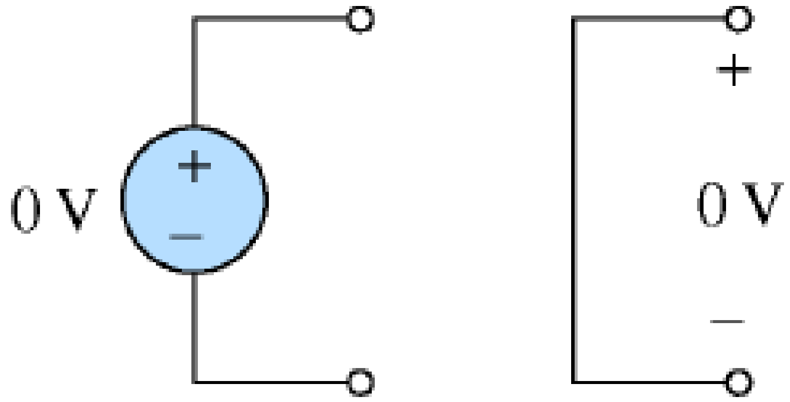
- Current-controlled current source (CCCS)



- A current source supplying zero current is equivalent to an open circuit:

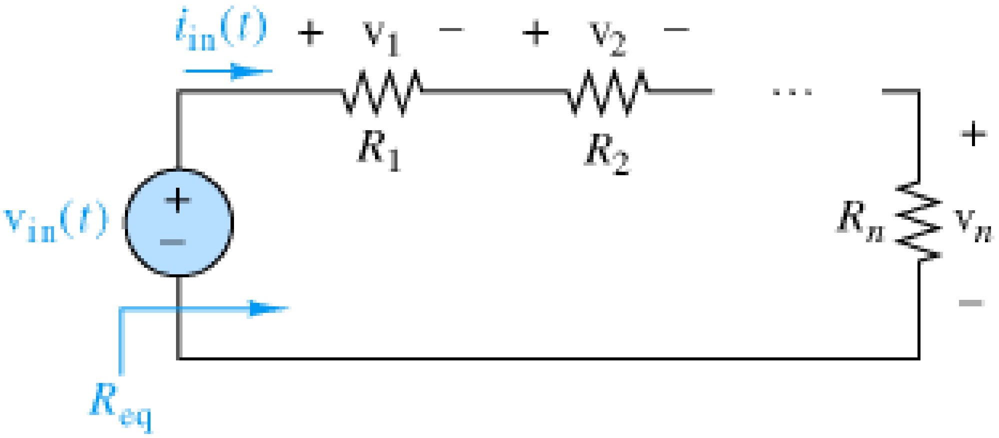


- A voltage source supplying 0V is equivalent to a short circuit:



Voltage Division

- In a series combination of n resistors, the voltage drop across the resistor R_j for $j=1,2, \ldots, n$ is:

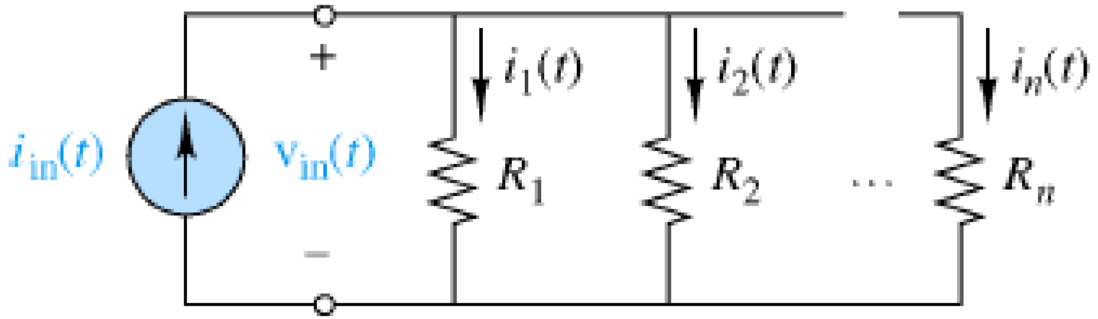


$$v_j(t) = \frac{R_j}{R_1 + R_2 + \cdots + R_n} v_{in}(t)$$

- What is the formula for two series resistors?!

Current Division

- In a parallel combination of n resistors, the current through the resistor R_j for j=1,2, …, n is:



$$i_j(t) = \frac{G_j}{G_1 + G_2 + \cdots + G_n} i_{in}(t)$$

Delta-Wye Conversion

$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c}$$

$$R_2 = \frac{R_c R_a}{R_a + R_b + R_c}$$

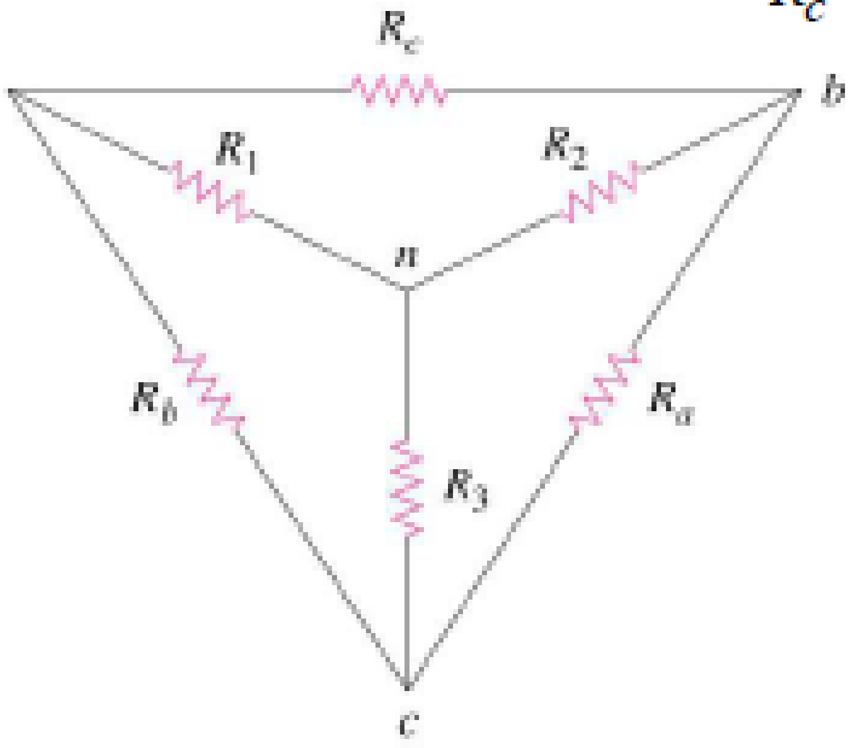
$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$$

Wye-Delta Conversion

$$R_a = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_1}$$

$$R_b = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_2}$$

$$R_c = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_3}$$



Kirchhoff's Current Law (KCL)

- **KCL:** The algebraic sum of the currents entering a node (or a **closed boundary**) is zero.

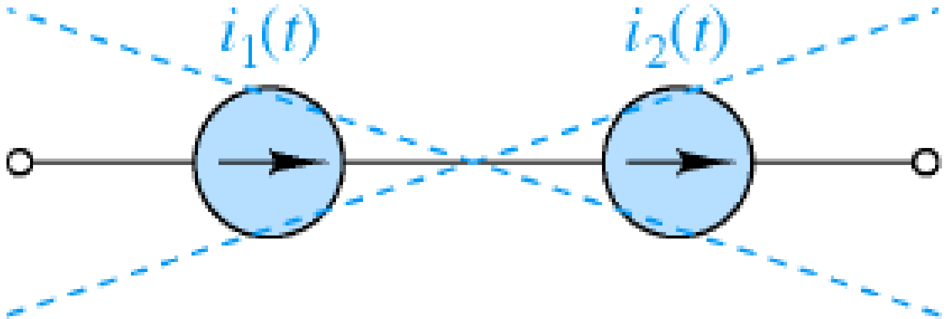
- The current entering a node may be regarded as positive while the currents leaving the node may be taken as negative or vice versa.

Kirchhoff's Voltage Law (KVL)

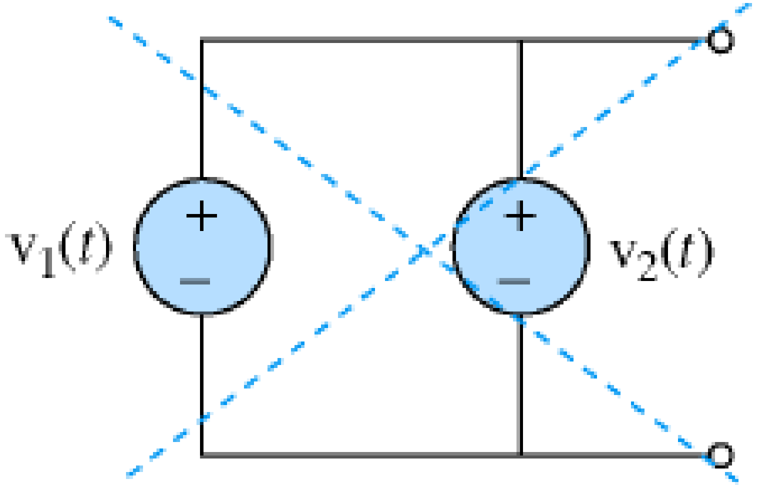
- **KVL**: The algebraic sum of the voltage drops around any closed path (or loop) is zero at any instance of time.

Sum of voltage drops=Sum of voltage rises

- A series connection of two different current sources is impossible. Why?



- A parallel connection of two different voltage sources is impossible. Why?



Terminology (Nodes and Branches)

- Please note that almost all components that we deal with in this course are two-terminal components (resistors, sources, …)

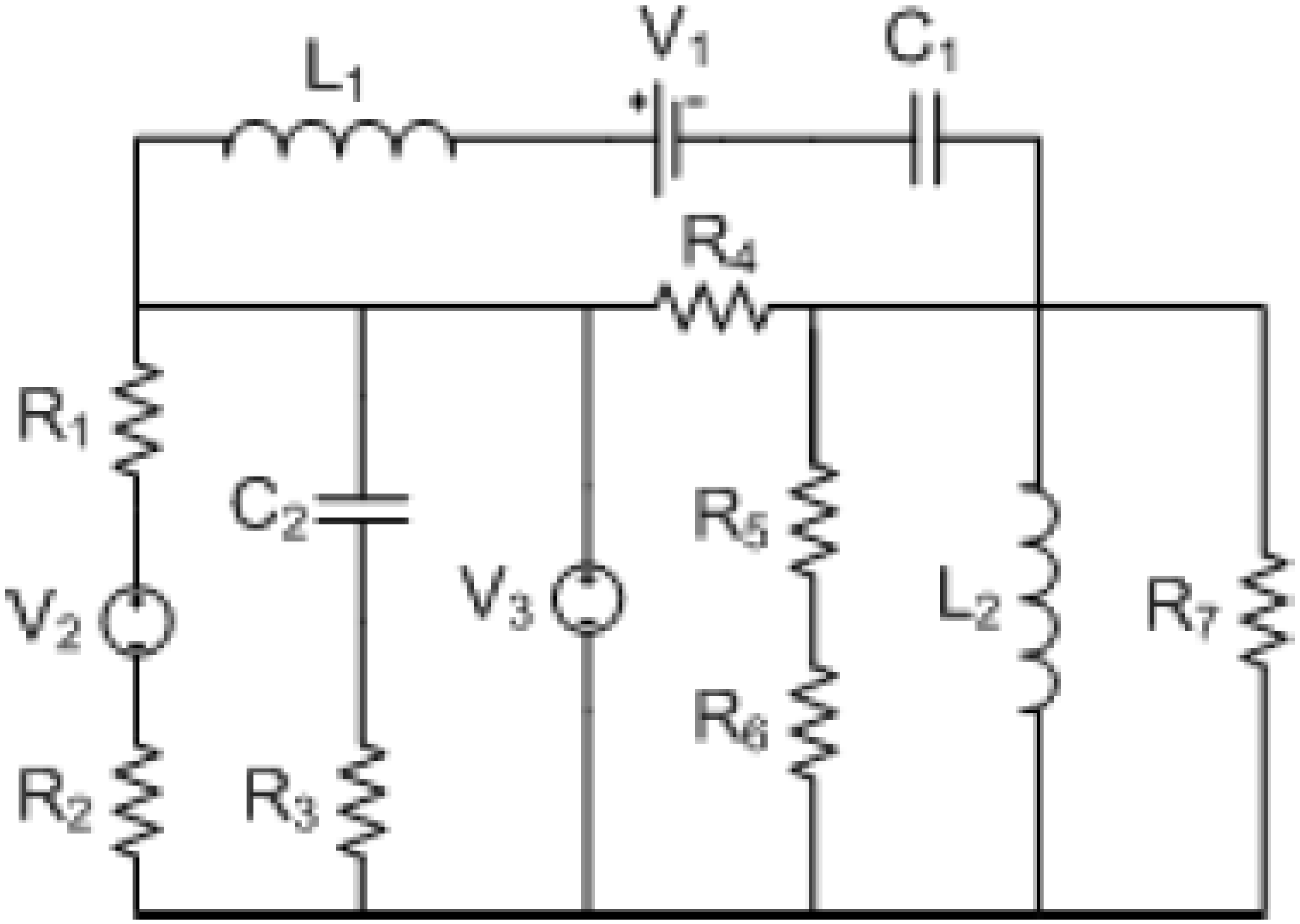
- A "**true node**" (or node for short) is the point of connection of three or more circuit elements. (The node includes the interconnection wires.)

- A "**binary node**" (or b-node for short) has only two components connected to it.

Loop

- A "**loop**" is any closed path in the circuit that does not cross any true node but once.

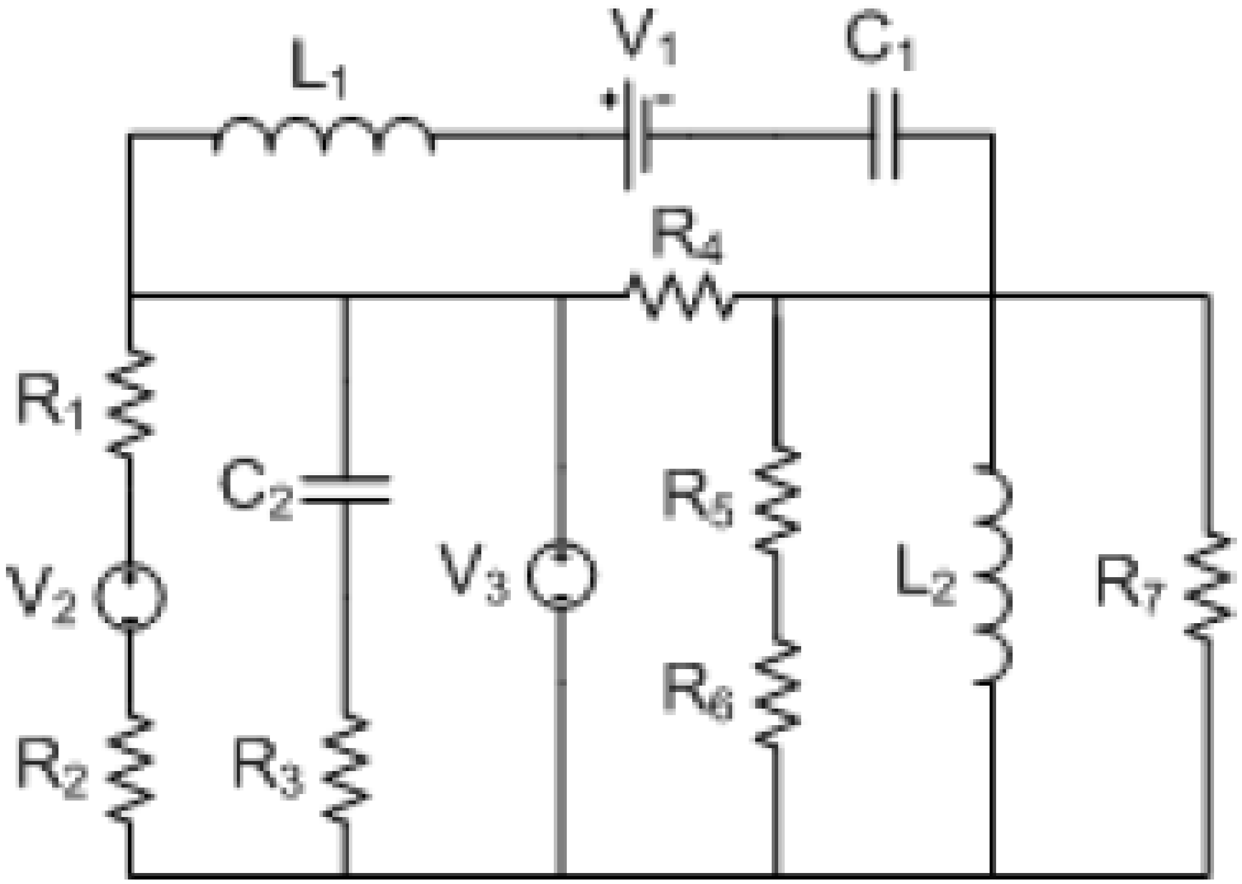
- In the following circuit identify the nodes (and their types).



Branch

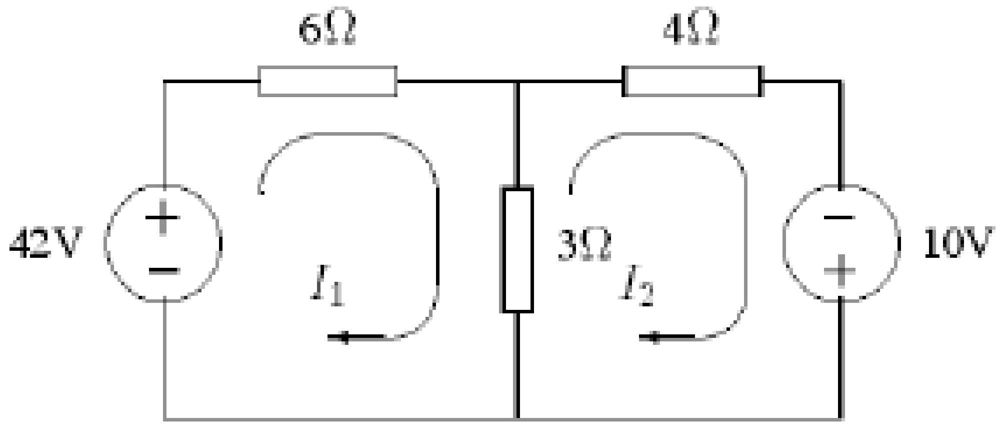
- A branch is a collection of elements that are connected between two "**true nodes**" that includes only those two true nodes (and does not include any other true nodes).

- In our example:



Mesh analysis

Step 1: Define meshes and unknowns
Each window is a mesh. Here, we have two meshes. For each one, we "imagine" a current circulating around it. So, we have two such currents, I_1 and I_2 — unknowns to be found.



Step 2: Set up KVL equations

Mesh 1: $-42 + 6I_1 + 3(I_1 - I_2) = 0$

Mesh 2: $3(I_2 - I_1) + 4I_2 - 10 = 0$

Step 3: Simplify and solve

$$\begin{aligned} 9I_1 - 3I_2 &= 42 \\ -3I_1 + 7I_2 &= 10 \end{aligned}$$

which gives $\boxed{I_1 = 6\text{ A and } I_2 = 4\text{ A.}}$

Once we know the mesh currents, we can find anything in the circuit!

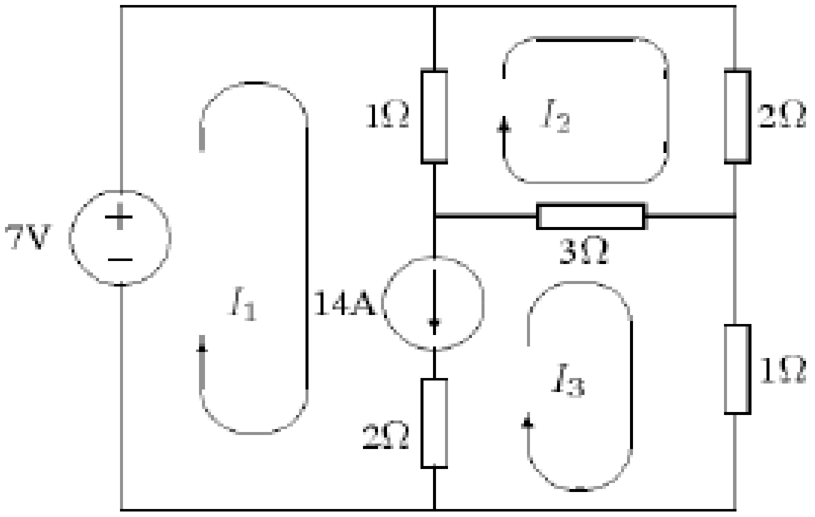
e.g., current flowing down the 3Ω resistor in the middle is equal to $I_1 - I_2$;
current flowing up the 42V source is I_1 ;
current flowing down the 10V source is I_2 ;
and voltages can be found via Ohm's law.

In general, we formulate the solution in terms of unknown mesh currents:

[R] [I] = [V]   — mesh equation

where [R] is the resistance matrix
[I] is the unknown mesh current vector
[V] is the source vector

Problem with current sources

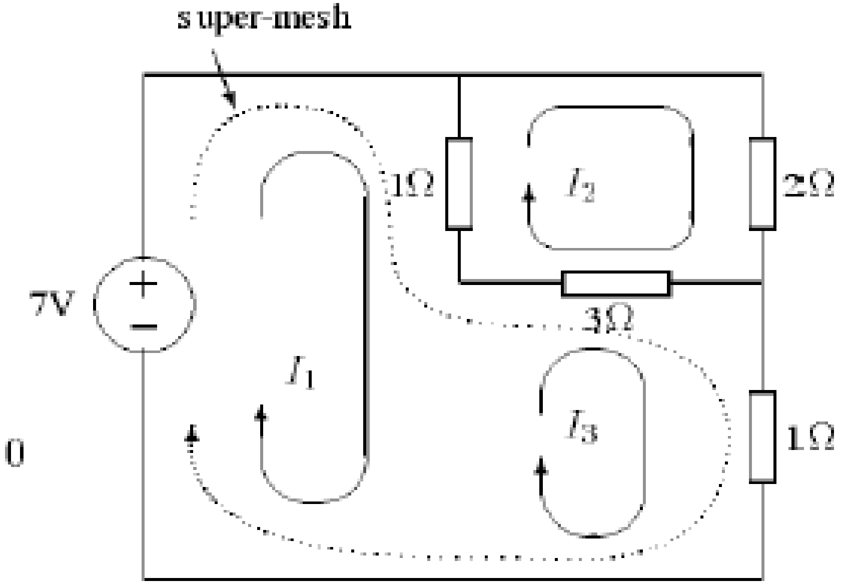


The mesh method may run into trouble if the circuit has current source(s).

Suppose we define the unknowns in the same way, i.e., I_1, I_2 and I_3 .

The trouble is that we don't know what voltage is dropped across the 14A source! How can we set up the KVL equation for meshes 1 and 3?

One solution is to ignore meshes 1 and 3. Instead we look at the **supermesh** containing 1 and 3.



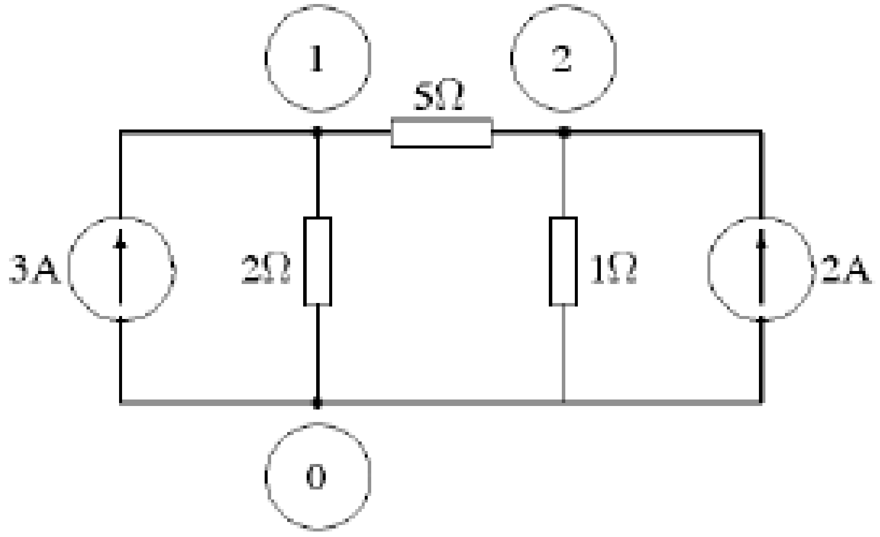
So, we set up KVL equations for mesh 2 and the supermesh:

Mesh 2: $(I_2 - I_1) \times 1 + I_2 \times 2 + (I_2 - I_3) \times 3 = 0$

Supermesh: $-7 + (I_1 - I_2) \times 1 + (I_3 - I_2) \times 3 + I_3 \times 1 = 0$

One more equation: $I_1 - I_3 = 14$

Nodal analysis



Step 1: Define unknowns
Each node is assigned a number. Choose a reference node which has zero potential. Then, each node has a voltage w.r.t. the reference node. Here, we have V_1 and V_2 — unknowns to be found.

Step 2: Set up KCL equation for each node

Node 1: $$-3 + \frac{V_1}{2} + \frac{V_1 - V_2}{5} = 0$$

Node 2: $$\frac{V_2 - V_1}{5} + \frac{V_2}{1} - 2 = 0$$

Step 3: Simplify and solve

$$\begin{bmatrix} \frac{1}{2} + \frac{1}{5} & -\frac{1}{5} \\ -\frac{1}{5} & \frac{1}{5} + 1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

which gives $V_1 = 5$ V and $V_2 = 2.5$ V.

Once we know the nodal voltages, we can find anything in the circuit!

e.g., voltage across the 5Ω resistor in the middle is equal to $V_1 - V_2$;
voltage across the 3A source is V_1 ;
voltage across the 2A source is V_2 ;
and currents can be found via Ohm's law.

Nodal analysis

In general, we formulate the solution in terms of unknown nodal voltages:

[G] [V] = [I]   — nodal equation

where [G] is the conductance matrix
[V] is the unknown node voltage vector
[I] is the source vector

Problem with voltage sources

The nodal method may run into trouble if the circuit has voltage source(s).

Suppose we define the unknowns in the same way, i.e., V_1, V_2 and V_3 .

The trouble is that we don't know what current is flowing through the 2V source! How can we set up the KCL equation for nodes 2 and 3?

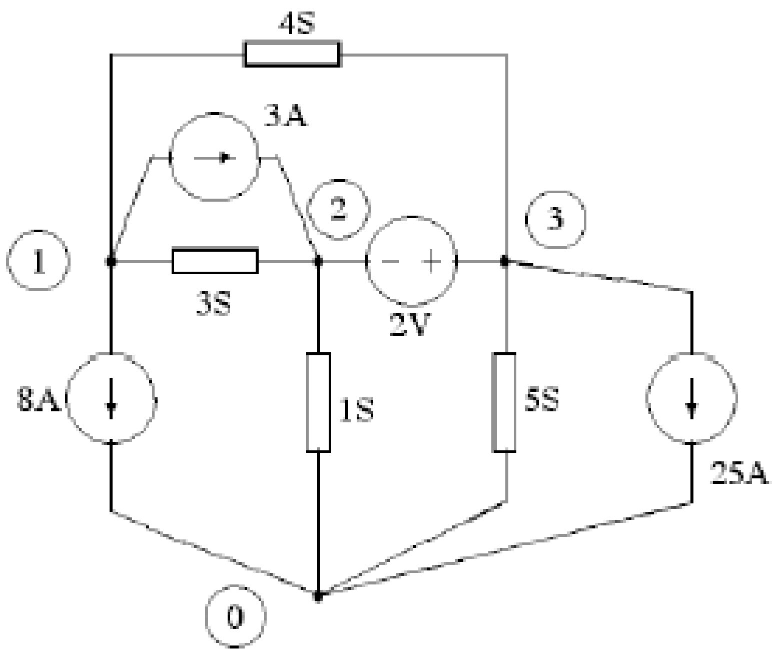
One solution is to ignore nodes 1 and 3. Instead we look at the **supernode** merging 2 and 3.

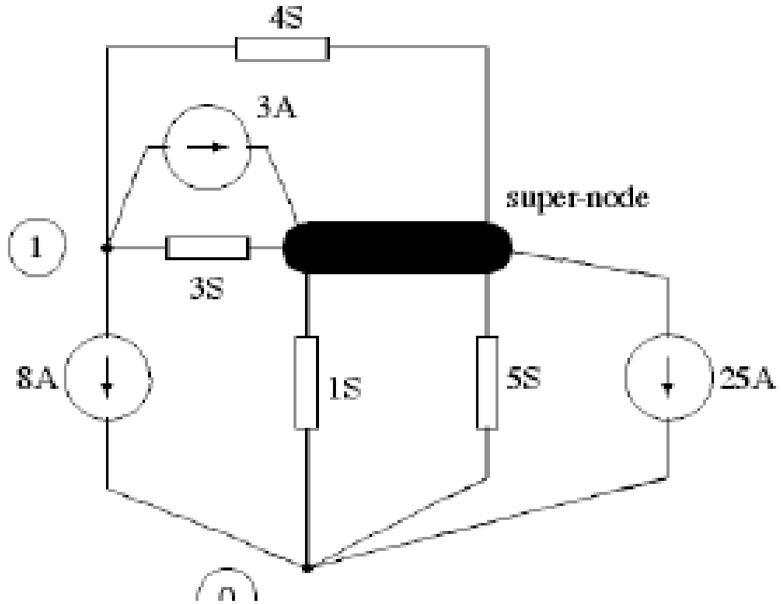
So, we set up KCL equations for node 1 and the supernode:

$$8 + (V_1 - V_2) \times 3 + 3 + (V_1 - V_3) \times 4 = 0$$

$$(V_2 - V_1) \times 3 - 3 + V_2 \times 1 + (V_3 - V_1) \times 4 + V_3 \times 5 + 25 = 0$$

One more equation: $V_3 - V_2 = 2$





References

- http://pongsak.ee.engr.tu.ac.th/le325/NetworkTheorem.pdf
- http://bapirajueca1.blogspot.com/2017/03/unit-6-network-theorems-ppt.html?m=1
-
- https://www.iare.ac.in/sites/default/files/PPT/IARE_EC_PPT_1.pdf