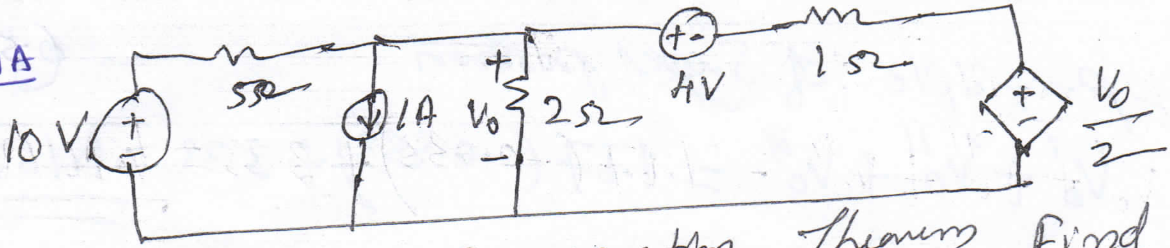
Q1A

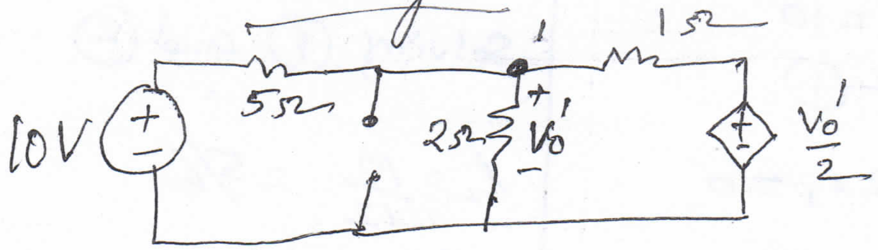
I CE 2154
Network Analysis and Signal



Superposition Theorem Find V_0

5

10V Source only acting.



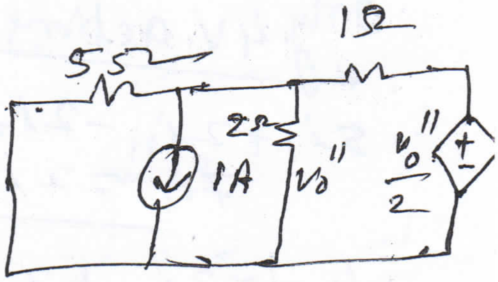
at node 1

$$\frac{V_0'}{2} + \frac{V_0' - \frac{V_0'}{2}}{1} + \frac{V_0' - 10}{5} = 0$$

(1.5 mark)

$$V_0' = 1.67\,V$$

Apply 1A Current Source



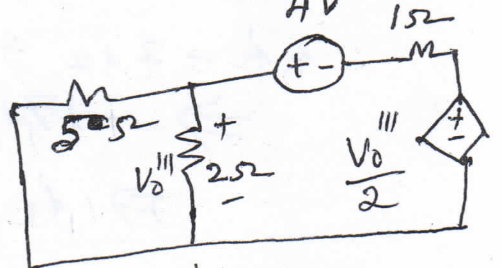
$$\frac{V_0''}{2} + 1 + \frac{V_0''}{5} + \frac{V_0'' - \frac{V_0''}{2}}{1} = 0$$

$$0.5V_0'' + 0.2V_0'' + V_0'' - 0.5V_0'' + 1 = 0$$

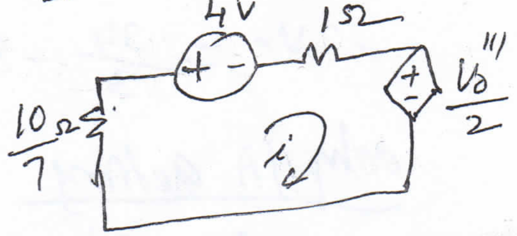
(1.5 mark)

$$V_0'' = -0.833\,V$$

Only 4V Source is acting



$$i \times \frac{10}{7} + 4 + 1 \times i + \frac{V_0'''}{2} = 0 \quad -(1)$$



from fig: $V_0''' = -i_{2\Omega} \times 2\Omega$

$$i_{2\Omega} = i\,\frac{5}{5+2} = \frac{5}{7}i$$

Sub in above Equ:

$$\therefore V_0''' = -\frac{5}{7}i \times 2 = -\frac{10}{7}i \quad -(2)$$

(1.5 mark)

(2) in (1) $\frac{10}{7}i + 4 + i + \frac{1}{2}\left(-\frac{10}{7}i\right) = 0$ $\Rightarrow i = -2.33\,A$

$V_0''' = -\frac{10}{7}(-2.33) = 3.33\,V$

By principle of Super position

$\therefore V_0' + V_0'' + V_0''' = 1.67 (-0.833) + 3.33 = 4.167$

(OR)

using 10 V

$5i_1 + 2i_1 - 2i_2 = 10$

$\Rightarrow 7i_1 - 2i_2 = 10$ — (1)

$2i_2 + i_2 + \frac{V_0}{2} - 2i_1 = 0$

$4i_1 - 6i_2 = V_0$

$2i_1 - 2i_2 = V_0$

$i_1 - i_2 = 2i_1 - 3i_2$

$i_1 = 2i_2$ — (2)

Solving (1) and (2)

$i_2 = \frac{10}{12} = 5/6$

$i_1 = 5/3$

$\therefore V_0 = \underline{5/3}$ V

only 4V acting

$5i_1 + 2i_1 - 2i_2 = 0$

$7i_1 = 2i_2$

$4 = -2i_2 + 2i_1 - i_2 - \frac{V_0}{2}$

$\frac{8 + V_0}{2} = 2i_1 - 3\left(\frac{7}{2}\right)i_1 = \frac{-17}{2} i_1$

$V_0 = -17i_1 - 8$

$2i_1 - 2i_2 = V_0$

$\Rightarrow -17i_1 - 8 = 2i_1 - 2i_2$

$\Rightarrow i_1 = \frac{-8}{12} = -2/3$

$\therefore V_0 = \frac{34}{3} - 8 = \frac{10}{3}$

only 1A acting

$-5 = 5i_1 + 2i_1 - 2i_2$

$7i_1 - 2i_2 = -5$

$2i_2 + i_2 - 2i_1 + \frac{V_0}{2} = 0$

I_1 I_2

$-2i_1 + 3i_2 = -V_0/$

$-2i_1 - 2i_2 = V_0$

$2i_1 - 2i_2 = 4i_1 -$

$\Rightarrow 2i_1 - 4i_2$

$i_1 = 2i_2$

$\Rightarrow 14i_2 - 2i_2 = -$

$i_2 = -5/12$

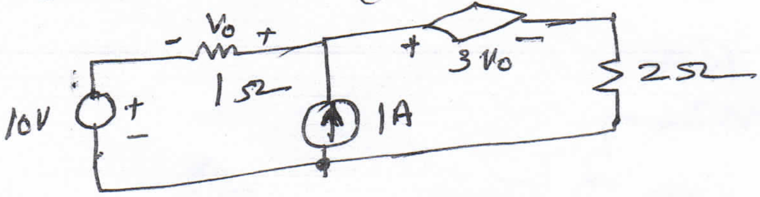
$i_1 = -5/6$

$\therefore V_0 = -5/6$

$V_0 = \frac{5}{3} + \frac{10}{3} - \frac{5}{6}$

$= \frac{25}{6} V = 4.16$

Q13 Obtain the current in 2Ω resistor By Thevenin's theorem



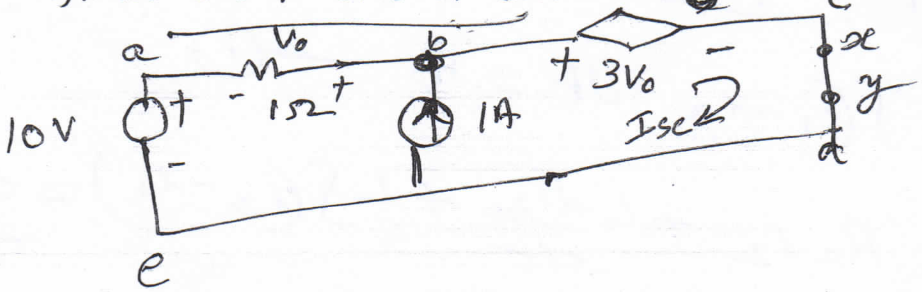
Soln Remove 2Ω

$\therefore V_{oc} = 10 + V_0 - 3V_0$

where V_0 is the drop across 1Ω resistor (V_0) $= 1A \times 1 = \underline{1V}$

$\therefore V_{oc} = 10 + 1 - 3 = \underline{8V}$

To find short circuit current



loop abcde, KVL

$$10 + V_0 - 3V_0 = 0 \quad (1)$$

Applying KCL at node b

$$\frac{V_0}{1} = 1 - I_{sc}$$

from (1)

$$10 + (1 - I_{sc}) - 3(1 - I_{sc}) = 0$$

$\therefore I_{sc} = -4A$

$$\therefore R_{th} = \frac{V_{th}}{I_{sc}} = \frac{8}{4} = 2\Omega$$

$\therefore$ Current flowing through 2Ω resistor

$$I_{2\Omega} = \frac{V_{oc}}{R_{th} + R_L} = \frac{8}{2+2} = \underline{2A} \text{ Ans}$$

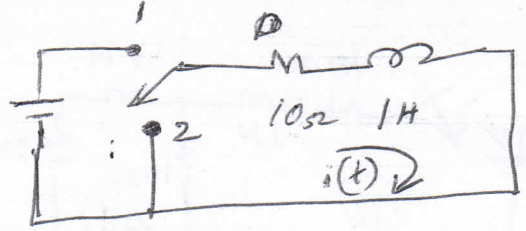
V_{th} = 1 mark

I_{sc} = 1 mark

R_{th} = ½ mark

I_L = ½ mark

Q1C



find $i(t)$

at $t=0$, $i_0 = \frac{10}{10} = 1A$

$10i + L\frac{di}{dt} = 0$

at $t=0$, $i_0 = 1$

$-10 = \frac{di}{dt}$; Integrating $i = -10t = -10 \times 5 \times 10^{-3} = -0.05A$

(1 mark)

Ch: eq

$s + 10 = 0$

$s = -10$

$i = K_1 e^{-10t}$

$1 = K_1$

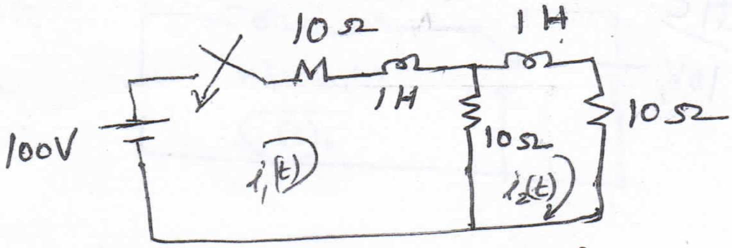
$t = 5msec$

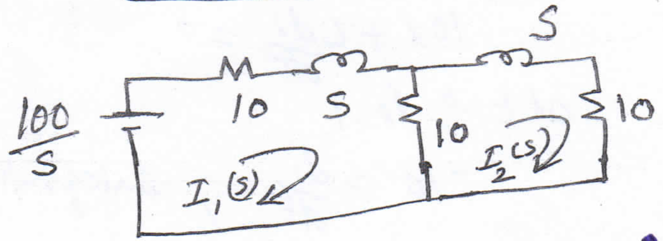
Soln: $i = e^{-10t} = e^{-5 \times 10^{-3} \times 10} = 0.9512A$

(1 mark)

Q.2A Switch closed

find $I_1(s)$, $I_2(s)$.





$$(S+20)I_1(s) - 10 I_2(s) = \frac{100}{S}$$

$$-10 I_1(s) + (S+20) I_2(s) = 0$$

$$Det = \begin{vmatrix} s+20 & -10 \\ -10 & s+20 \end{vmatrix}$$

$$= s^2+40s+300 = (s+10)(s+30)$$

$$\therefore I_1(s) = \frac{\begin{vmatrix} \frac{100}{s} & -10 \\ 0 & s+20 \end{vmatrix}}{(s+10)(s+30)} = \frac{100s+2000}{s(s+10)(s+30)}$$

Taking Laplace inverse

$$I_1(s) = \frac{6.66}{s} - \frac{5}{s+10} - \frac{1.67}{s+30}$$

(2.5 mark)

taking L^{-1}

$$I_1(t) = 6.66 - 5e^{-10t} - 1.67e^{-30t}$$

IIIly

$$I_2(s) = \frac{\begin{vmatrix} s+20 & \frac{100}{s} \\ -10 & 0 \end{vmatrix}}{(s+10)(s+30)} = \frac{1000}{s(s+10)(s+30)}$$

$$= \frac{3.33}{s} - \frac{5}{s+10} + \frac{1.67}{s+30}$$

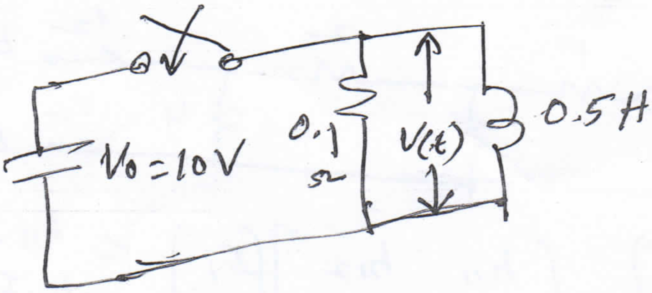
$$I_2(t) = 3.33 - 5e^{-10t} + 1.67e^{-30t}$$

(2.5 mark)

find V (t)

Q 2 B

KCL



$$C\frac{dv}{dt} + GV + \frac{1}{L}\int V\,dt = 0 \quad \text{— (1)}$$

~~diff~~

$$C\frac{d^2v}{dt^2} + G\frac{dv}{dt} + \frac{V}{L} = 0$$

ch: equ.

$$s^2 + 10s + 2 = 0$$

roots are $-9.7958, -0.2042$

$s_1 = 4.3, \quad s_2 = -5.3$

$$\therefore V(t) = K_1 e^{-9.7t} + K_2 e^{-.204t} \quad \text{— (2)}$$

at $t=0$, $V(0) = 10V$

$$10 = K_1 + K_2 \quad \text{— (3)}$$

diff: $\frac{dv}{dt} = -9.7K_1 e^{-9.7t} - 0.204 K_2 e^{-0.204t}$ — (4)

from (1) at $t = 0$, $V_0 = 10$

$$C\frac{dv}{dt} = -10 \times 10 + 0$$

$$\boxed{\frac{1}{L}\int V\,dt = 0 \quad \text{Inductor current} = 0}$$

$$\therefore \left.\frac{dV}{dt}\right|_{t=0} = -100$$

(4) gives $-100 = -9.7K_1 - 0.204K_2$ — (5)

where $K_1 = -4.896$, $K_2 = 14.896$

(2) gives $\therefore V(t) = -0.0209\, e^{-0.204t} + 10.0209\, e^{-9.79t}$

$K_1 = -0.2128$

$K_2 = 10.218$

$s_1, s_2 = -5 \pm \sqrt{23}$

$-0.204, \; -9.79$

$V(t) = K_1 e^{s_1 t} + K_2 e^{s_2 t}$

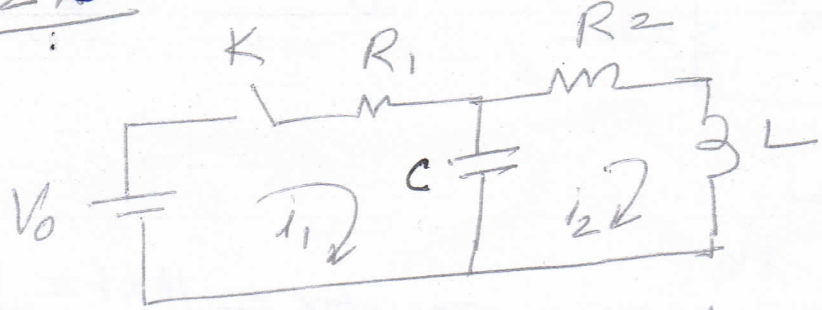
(2 mark)

$K_1 = -0.02$

$K_2 = 10.0209$

(1 mark)

Q2 E



at $t=0$, switch Closed

$i_1(0) = \frac{V}{R_1}$, $i_2(0) = 0$ Ans (1 mark)

first loop

$R_1 i_1 + \frac{1}{C}\int (i_1 - i_2)\,dt = V_0$ — (1)

Second loop $R_2 i_2 + L\frac{di_2}{dt} + \frac{1}{C}\int (i_2 - i_1) = 0$ — (2)

at $t=0$, $\frac{1}{C}\int (i_2 - i_1) = 0$ $\because$ Short ckt

(2) Gives $R_2 i_2 + L_2 \frac{di_2}{dt} = 0$

$\because$ at $t=0$, $i_2(0) = 0$ $\therefore \frac{di_2}{dt} = 0$ Ans

diff (1)

$R_1 \frac{di_1}{dt} + \frac{i_1 - i_2}{C} = 0$

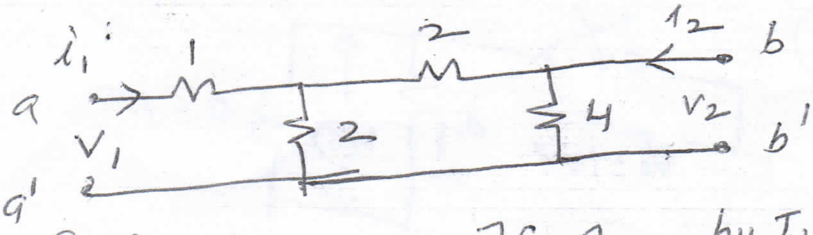
$\frac{di_1}{dt} = \frac{-(i_1 - i_2)}{R_1 C}$

at $t=0$, $i_1 = \frac{V}{R_1}$ and $i_2 = 0$

$\therefore \frac{di_1}{dt} = \frac{-V}{R_1^2 C}$ Ans (1 mark)

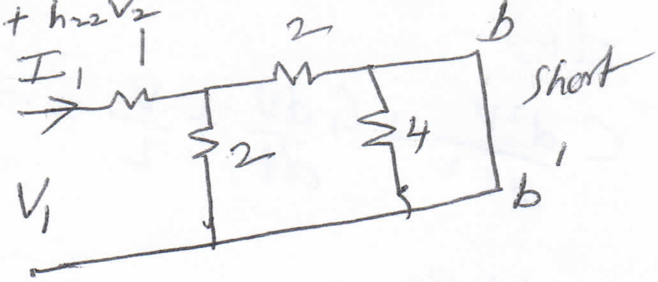
Q3A

h Parameters



$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix} \quad \begin{matrix} = h_{11} I_1 + h_{12} V_2 \\ = h_{21} I_1 + h_{22} V_2 \end{matrix}$$

$\boxed{\text{1 mark}}$

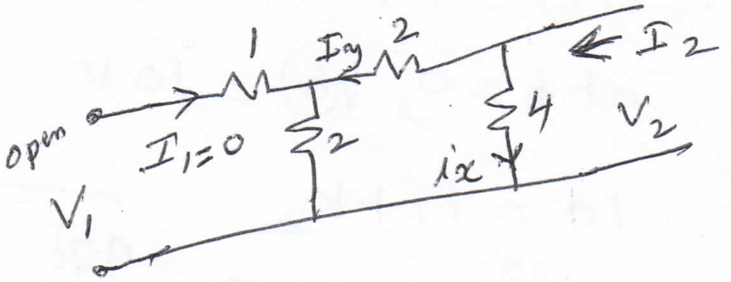


$b b'$ Short CKted $V_2 = 0$

$$h_{11} = = \left.\frac{V_1}{I_1}\right|_{V_2=0} = 2\,\Omega$$

$$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0} = -\frac{1}{2}$$

If aa' open $\Rightarrow I_1 = 0$



$V_1 = I_y \cdot 2$

$\therefore I_y = \frac{I_2}{2}$

$V_2 = I_x \cdot 4 \cdot 0$

$I_x = \frac{I_2}{4}$

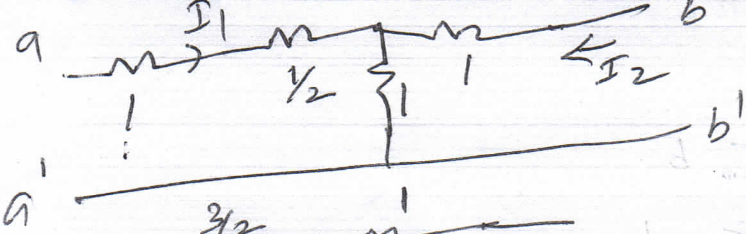
$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0} = \frac{1}{2}$$

$$h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0} = \frac{1}{2}\,\mho$$

$$\begin{bmatrix} 2 & 1/2 \\ -1/2 & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}$$

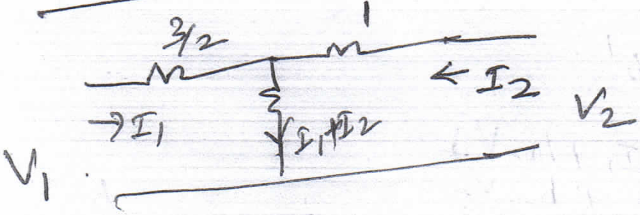
$\boxed{\text{Each parameter 1 mark each}}$

(OR)



Δ Star Conversion

$$\frac{R_1 R_2}{R_1+R_2+R_3}$$



Loop 1

$$\frac{3}{2}I_1 + I_1 + I_2 = V_1$$

$$\frac{5}{2}I_1 + I_2 = V_1 \quad \text{— (1)}$$

Loop 2

$$I_2 + I_1 + I_2 = V_2$$

$$I_1 + 2I_2 = V_2 \quad \text{— (2)}$$

∴ Z parameter

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} 5/2 & 1 \\ 1 & 2 \end{bmatrix}$$

$$\Delta = \frac{5}{2} \times 2 - 1 = \underline{\underline{4}}$$

$$\therefore \text{h parameter} = \begin{bmatrix} \frac{|Z|}{Z_{22}} & \frac{Z_{12}}{Z_{22}} \\ -\frac{Z_{21}}{Z_{22}} & \frac{1}{Z_{22}} \end{bmatrix} = \underline{\underline{\begin{bmatrix} 2 & \frac{1}{2} \\ -1/2 & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}}}$$

Q3B $y(n) = n\,x(n)$

system is M, NC, L, TV, US, NIV with illustration — 3M

3C $x(t) = e^{-2t}u(t)$

$$E = \int_0^{\infty} (e^{-2t})^2\,dt = \int_0^{\infty} e^{-4t}\,dt = \frac{1}{4}$$ (1.5M) — 2M

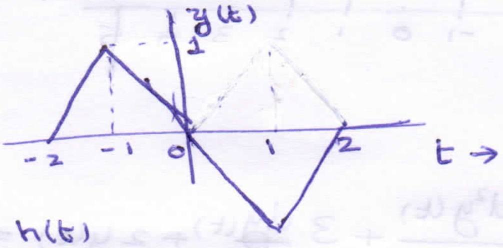
$P = 0$ (0.5M)

4A (i) $y(t) = x(t) * h(t)$

$$= [\delta(t+1) - \delta(t-1)] * [r(t+1) - 2r(t) + r(t-1)]$$

$$= r(t+2) - 2r(t+1) + r(t) - \{r(t) - 2r(t-1) + r(t-2)\}$$

$$= r(t+2) - 2r(t+1) + 2r(t-1) - r(t-2)$$ — 2M

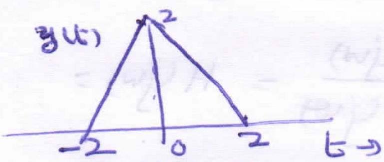


— 0.5M

(ii) $y(t) = u(t) * h(t)$

$$= u(t) * \{u(t+2) - 2u(t) + u(t-2)\}$$

$$= r(t+2) - 2r(t) + r(t-2)$$ — 2M



— 0.5M

5M

4B

$$e^{-2t}u(t) \xleftrightarrow{FT} \frac{1}{2+j\omega}$$

$$t\,e^{-2t}u(t) \xleftrightarrow{FT} \frac{1}{(2+j\omega)^2}$$ — 1M

$$t\,e^{-2t}\sin t\,u(t) = t\,e^{-2t}\frac{(e^{jt} - e^{-jt})}{2j} \xleftrightarrow{FT} \frac{1}{2j}\left[\frac{1}{(2+j(\omega-1))^2} - \frac{1}{(2+j(\omega+1))^2}\right]$$ — 1M

$$\therefore x(t) = \frac{d}{dt}\{t\,e^{-2t}\sin t\,u(t)\} \xleftrightarrow{FT} j\omega \cdot \frac{1}{2j}\{\quad\}$$

$$= \frac{\omega}{2}\left\{\frac{1}{[2+j(\omega-1)]^2} - \frac{1}{[2+j(\omega+1)]^2}\right\}$$ — 1M

3M

4C

$$x(t) = \cos(\pi t) + \cos(4\pi t) + \sin(5\pi t)$$

$$= \frac{e^{j\pi t} + e^{-j\pi t}}{2} + \frac{e^{j4\pi t} + e^{-j4\pi t}}{2} + \frac{e^{j5\pi t} - e^{-j5\pi t}}{2j} \quad - \tfrac{1}{2}M$$

Comparing with

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}, \quad \omega_0 = \pi$$

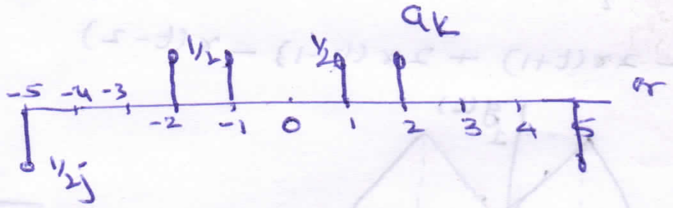
$$a_1 = a_{-1} = 1/2$$

$$a_2 = a_{-2} = 1/2$$

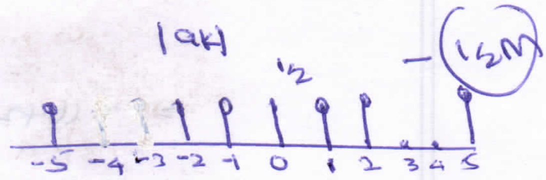
$$a_3 = \frac{1}{2j}$$

$$a_4 = -1/2j$$

$a_k = 0$ otherwise — 1M



or



— 1/2M

2M

5A

$$\frac{d^2y(t)}{dt^2} + 3\frac{dy(t)}{dt} + 2y(t) = \frac{d\,x(t)}{dt} + 3x(t)$$

(i) FT ⇒ $\left[(j\omega)^2 + 3j\omega + 2\right] Y(j\omega) = (j\omega + 3) X(j\omega)$

$$\frac{Y(j\omega)}{X(j\omega)} = H(j\omega) = \frac{j\omega + 3}{(j\omega)^2 + 3j\omega + 2} \quad - 1.5M$$

(ii) Take IFT $\quad H(j\omega) = \frac{A}{(j\omega + 1)} + \frac{B}{(j\omega + 2)}$

$\Rightarrow A = 2, B = -1$

$$\therefore h(t) = (2e^{-t} - e^{-2t})u(t) \quad - 1.5M$$

(iii) $Y(j\omega) = X(j\omega) \cdot H(j\omega)$, $x(t) = e^{-3t}u(t)$ — 0.5M

$$= \frac{1}{3 + j\omega} \cdot \frac{j\omega + 3}{(j\omega + 1)(j\omega + 2)}$$

$$= \frac{1}{(j\omega + 1)(j\omega + 2)} = \frac{A}{j\omega + 1} + \frac{B}{j\omega + 2}$$

A = 1
B = −1

$$y(t) = (e^{-t} - e^{-2t})u(t)$$

1.5M

5M

5B $X(j\omega) = \left[\frac{2}{\omega}\sin\omega\right]\left[\frac{1}{(j\omega+1)}\right]$

$= X_1(j\omega)\cdot X_2(j\omega)$

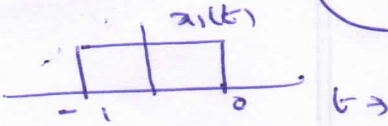
$\therefore x(t) = x_1(t) * x_2(t)$

$= rect(t) * e^{-t}u(t)$

$\frac{2}{\omega}\sin\omega \xleftrightarrow{FT} rect(t)$

$= \begin{cases} 1, & |t|<1 \\ 0 & |t|>1 \end{cases}$

$\frac{1}{(j\omega+1)} \longleftrightarrow e^{-t}u(t)$ — 1.5M



$t<-1, \quad x(t) = 0$

$-1<t<1 \quad x(t) = \int_0^{t+1} e^{-\tau}d\tau$

$= 1 - e^{-(t+1)}$

$t>1 \quad x(t) = \int_{t-1}^{t+1} e^{-\tau}d\tau$

$= \underline{e^{-t}(e^{2}-e^{-2})}$ — 1.5M

5C $X = \int_{-\infty}^{\infty}\left(\frac{\sin\pi t}{\pi t}\right)^2 dt$

using Parsevals theorem

$\int_{-\infty}^{\infty}|x(t)|^2 dt = \frac{1}{2\pi}\int_{-\infty}^{\infty}|X(j\omega)|^2 d\omega$ — 1/2 m

$\therefore x(t) = \frac{\sin\pi t}{\pi t} \xleftrightarrow{FT} X(j\omega) = \begin{cases} 1 & |\omega|<\pi \\ 0 & |\omega|>\pi \end{cases}$ 1/2 m

$\int_{-\infty}^{\infty}\left(\frac{\sin\pi t}{\pi t}\right)^2 dt = \frac{1}{2\pi}\int_{-\pi}^{\pi} d\omega$

$= \underline{1}$ — 1M

2M