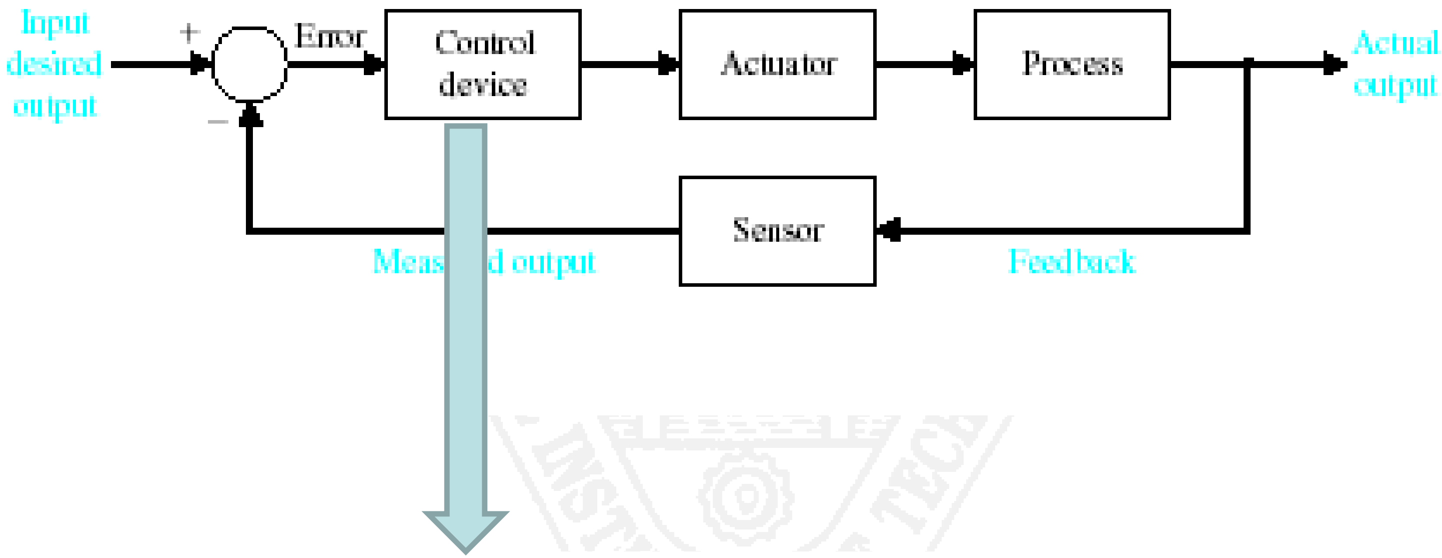
Controller principles

Examples of Control Systems



A negative feedback system block diagram depicting a basic closed-loop control system.
The control device is often called a "controller."

PROCESS CHARACTERISTICS

1.Process Equation

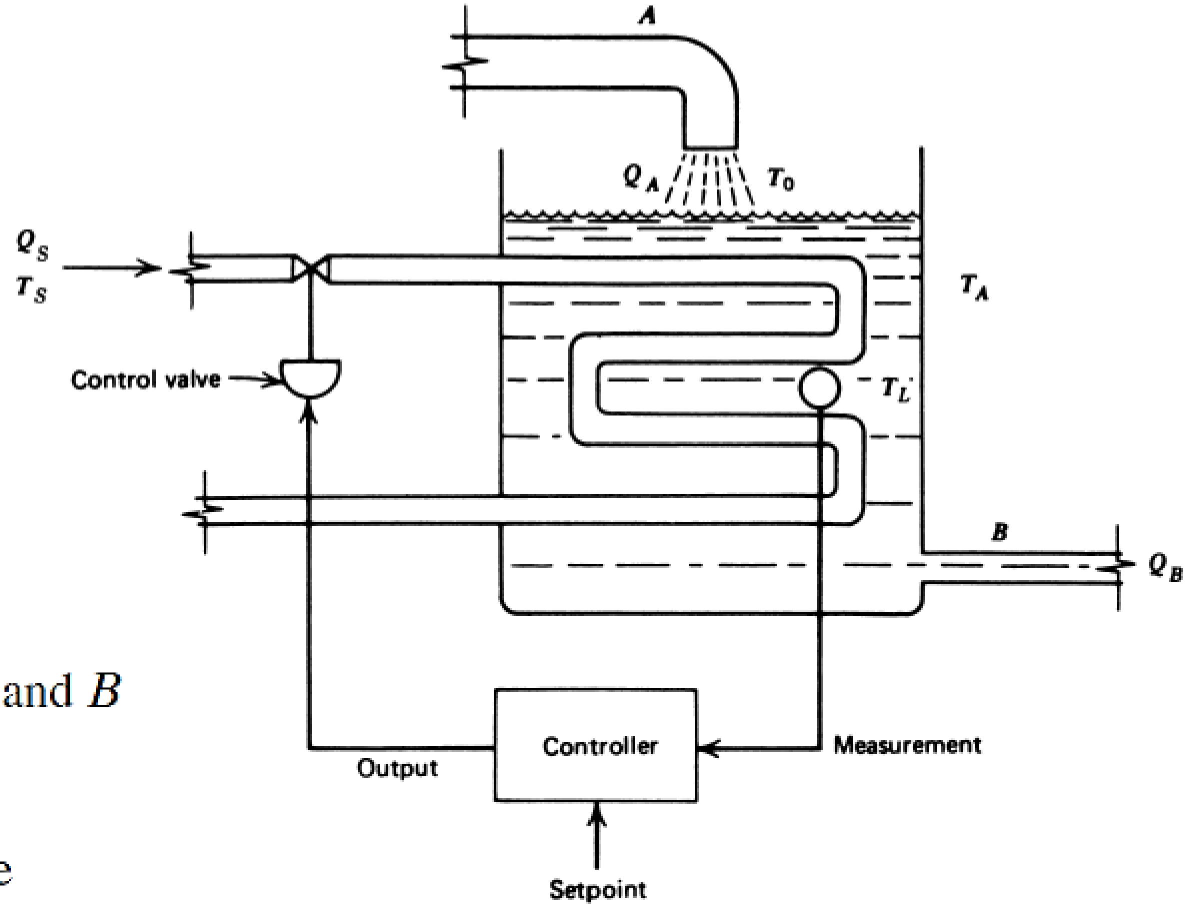
A process-control loop regulates some *dynamic variable* in a process.

This controlled variable, a process parameter, may depend on many other parameters

We have selected one of these other parameters to be our controlling parameter.

If a measurement of the **controlled variable** shows a deviation from the setpoint, then the **controlling parameter** is changed, which in turn changes the controlled variable.

PROCESS CHARACTERISTICS



$$T_L = F(Q_A, Q_B, Q_S, T_A, T_S, T_0)$$

Q_A, Q_B = flow rates in pipes A and B

Q_S = steam flow rate

T_A = ambient temperature

T_0 = inlet fluid temperature

T_S = steam temperature

PROCESS CHARACTERISTICS

2. Process Load

- From the process equation, or knowledge of and experience with the process, it is possible to identify a set of values for the process parameters that results in the controlled variable having the setpoint value.
- Another type of change involves a temporary variation of one of the load parameters. After the excursion, the parameter returns to its nominal value. This variation is called a transient. A transient is not a load change because it is not permanent.
- This set of parameters is called the *nominal set*. The term ***process load*** refers to this set of all parameters, **excluding the controlled variable.**

PROCESS CHARACTERISTICS

3. Process Lag

- At some point in time, a process-load change or transient causes a change in the controlled variable.
- The process-control loop responds to ensure that, some finite time later, the variable returns to the setpoint value. Part of this time is consumed by the process itself and is called the *process lag.*

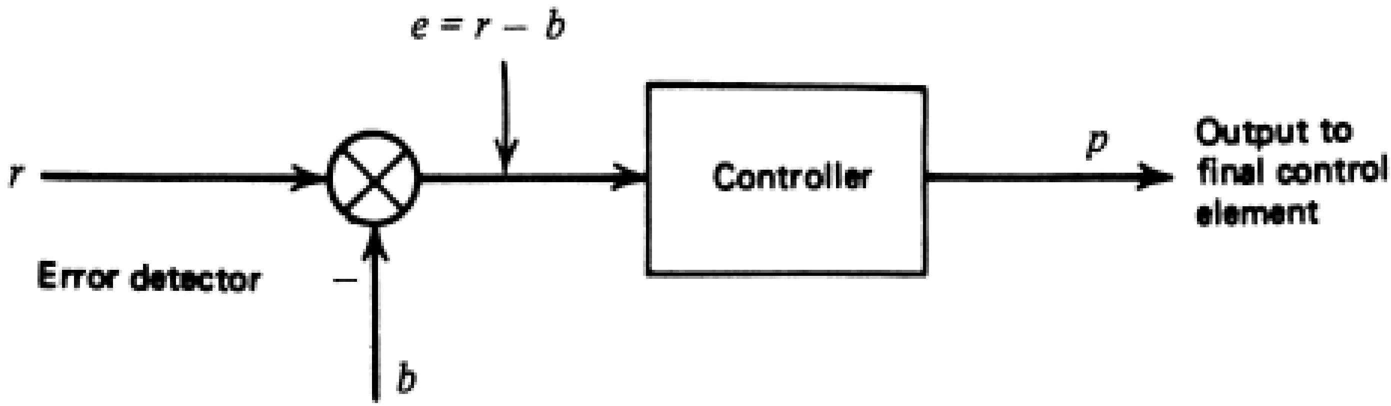
PROCESS CHARACTERISTICS

4. Self regulation

- Some processes adopt a specific value of the controlled variable for nominal load with no control operations.

- The control operations may be significantly affected by such ***self regulation.***

CONTROL SYSTEM PARAMETERS



$$e = r - b$$

- To describe controller operation in a general way, it is better to express the error as percent of the measured variable range (i.e., the span).

$$e_p = \frac{r - b}{b_{max} - b_{min}} \times 100$$

Measured variable range

- The measured value of a variable can be expressed as percent of span over a range of measurement by the equation

$$c_p = \frac{c - c_{\min}}{c_{\max} - c_{\min}} \times 100$$

c_p = measured value as percent of measurement range
c = actual measured value
$c_{\max}$ = maximum of measured value
$c_{\min}$ = minimum of measured value

Control Parameter Range

- Often, the output is expressed as a percentage where 0% is the minimum controller output and 100% the maximum.

- The controller output as a percent of full scale when the output varies between specified limits is given by

$$p = \frac{u - u_{\min}}{u_{\max} - u_{\min}} \times 100$$

p = controller output as percent of full scale
u = value of the output
$u_{\max}$ = maximum value of controlling parameter
$u_{\min}$ = minimum value of controlling parameter

Control system parameters

- Control Lag-When a process variable experiences a sudden change, then the time for the process control loop to make necessary adjustment to the final control element is called control lag.
- Dead time- is the elapsed time between the instant a deviation occurs and when the corrective action first occurs.
- Cycling-oscillation of the dynamic variable error about zero.

Controller Modes

- A controller generates a control signal to the final element, based on a measured deviation of the controlled variable from the set point.
- The choice is a complicated decision.
- Involves process characteristics, cost analysis, product rate, etc.
- P is the *percent of controller output* relative to its total range.

$$p = F(e_p)$$

Reverse and Direct Action

- *Direct action.*
  - when an increasing value of the controlled variable causes an increasing value of the controller output.
  - Eg: level-control.
- *Reverse action*
  - where an increase in a controlled variable causes a decrease in controller output.
  - Eg:a simple temperature control from a heater.

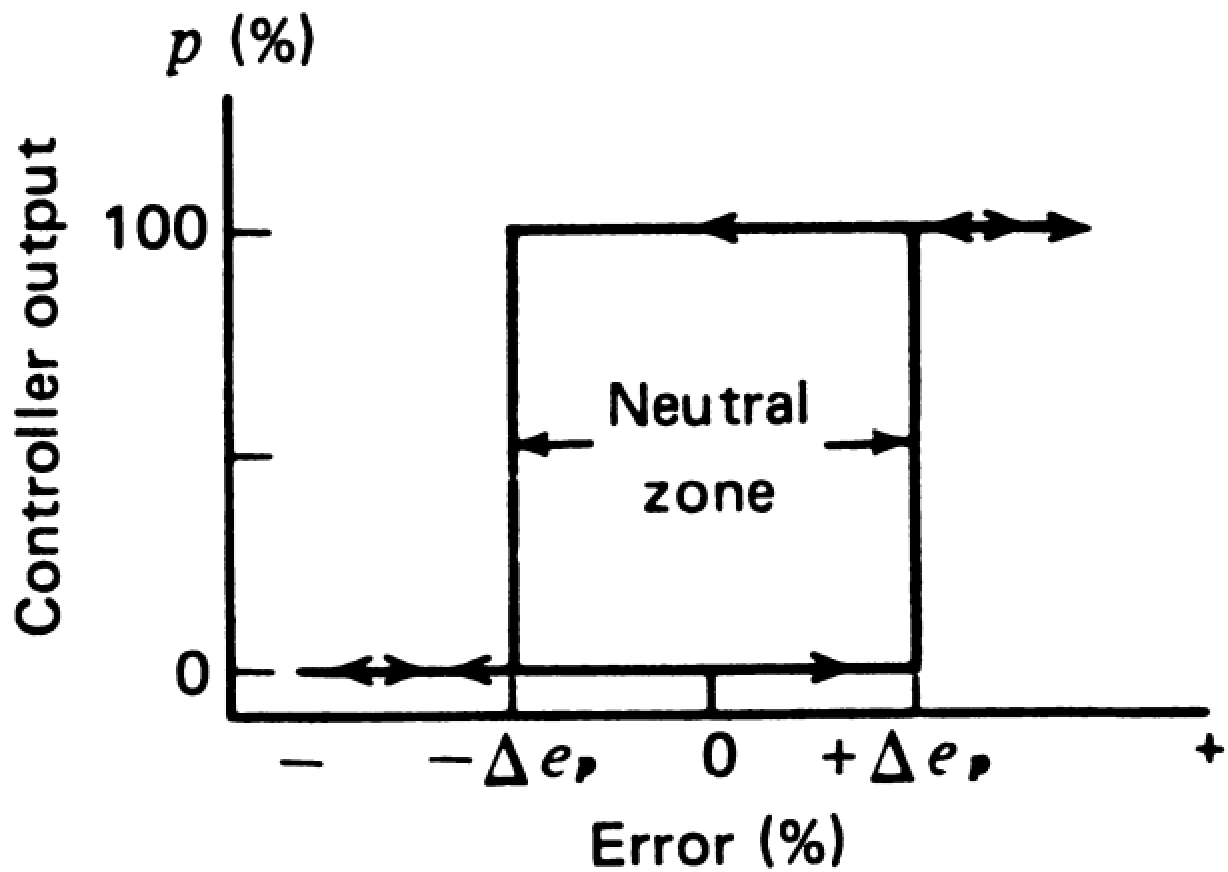
DISCONTINUOUS CONTROLLER MODES

- **Two-Position Mode (ON/OFF Mode)**

$$p = \begin{cases} 0\% & e_p < 0 \\ 100\% & e_p > 0 \end{cases}$$

- The measured value is less than the setpoint, full controller output results. When it is more than the setpoint, the controller output is zero.
  - Eg:A space heater

Neutral Zone/*differential gap,*



- A liquid-level control system linearly converts a displacement of 2 to 3 m into a 4- to 20-mA control signal. A relay serves as the two-position controller to open or close an inlet valve. The relay closes at 12 mA and opens at 10 mA. Find (a) the relation between displacement level and current, and (b) the neutral zone or displacement gap in meters.

Multiposition Mode

- provide several intermediate, rather than only two, settings of the controller output

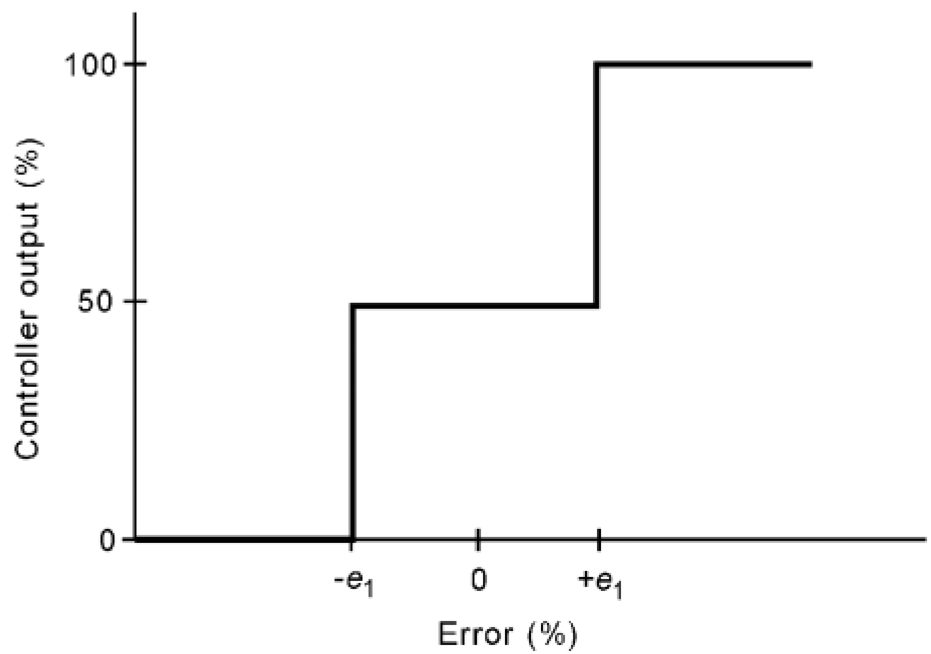
  reduce the cycling behavior and overshoot and undershoot inherent in the two-position mode

$$p = p_i \qquad e_p > |e_i| \; i = 1, 2, \ldots, n$$

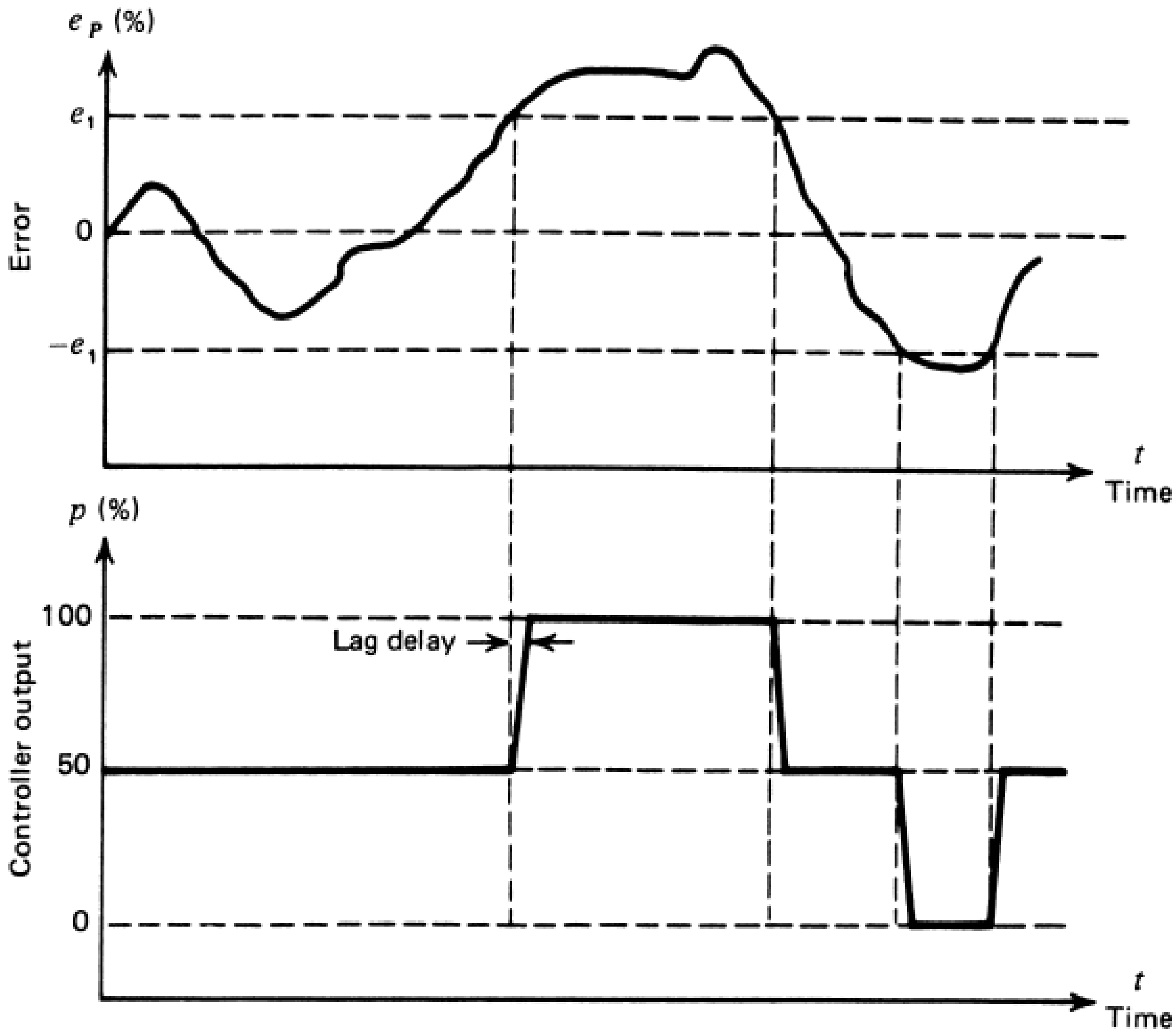
Three position controller

- As the error exceeds certain set limits , the controller output is adjusted to preset values

$$p = \begin{cases} 100 & e_p > e_2 \\ 50 & -e_1 < e_p < e_2 \\ 0 & e_p < \ -e_1 \end{cases}$$



Effect of lag



Floating-Control Mode

- Previously If the error exceeded some preset limit, the output was changed to a new setting **as quickly as possible**

- If the error is zero, the output does not change but remains (floats) at whatever setting it was when the error went to zero.

Single Speed

- In the single-speed floating-control mode, the output of the control element changes at a fixed rate when the error exceeds the neutral zone. *An* equation for this action is

$$\frac{dp}{dt} = \pm K_F \qquad |e_p| > \Delta e_p$$

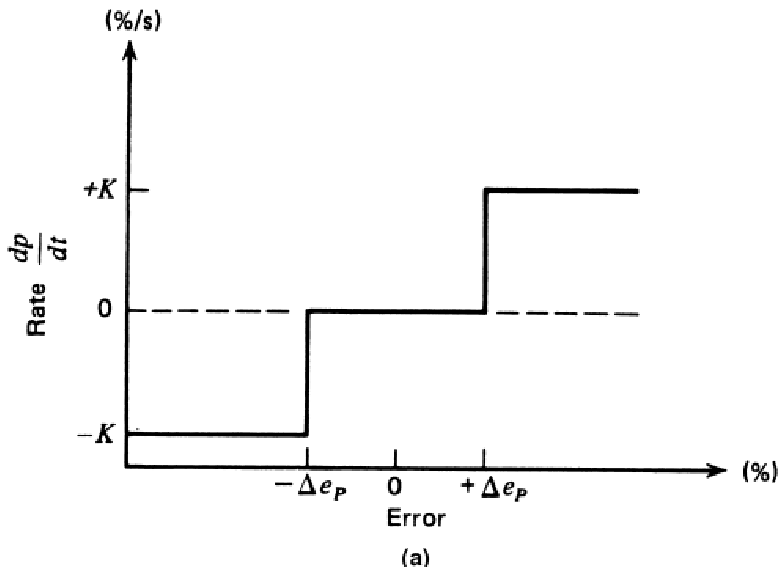
$\frac{dp}{dt}$ = rate of change of controller output with time

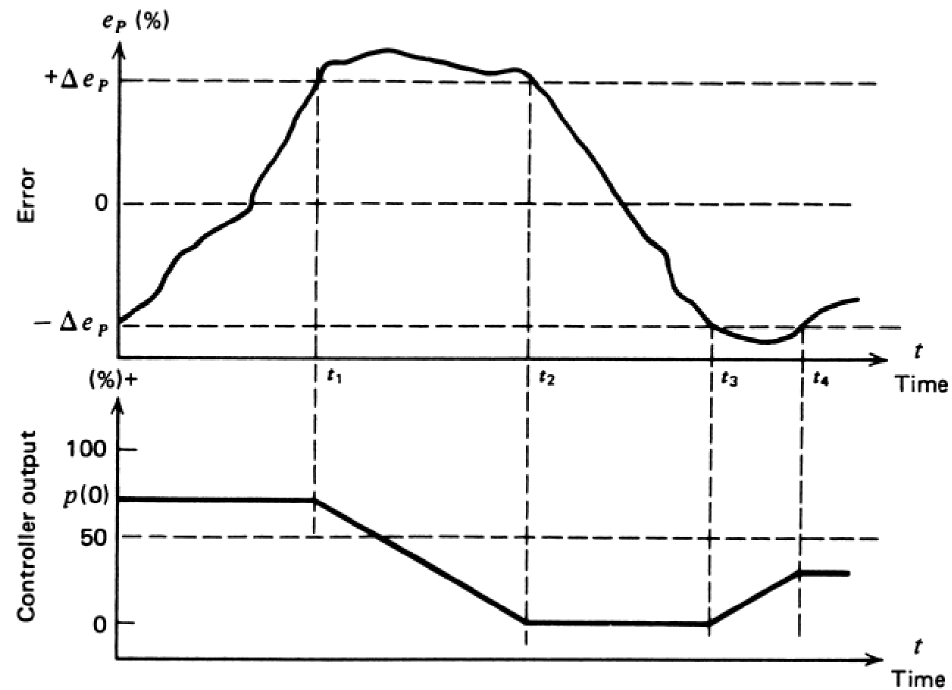
K_F = rate constant (%/s)

Δe_p = half the neutral zone

$$p = \pm K_F t + p(0) \qquad |e_p| > \Delta e_p$$

Single Speed





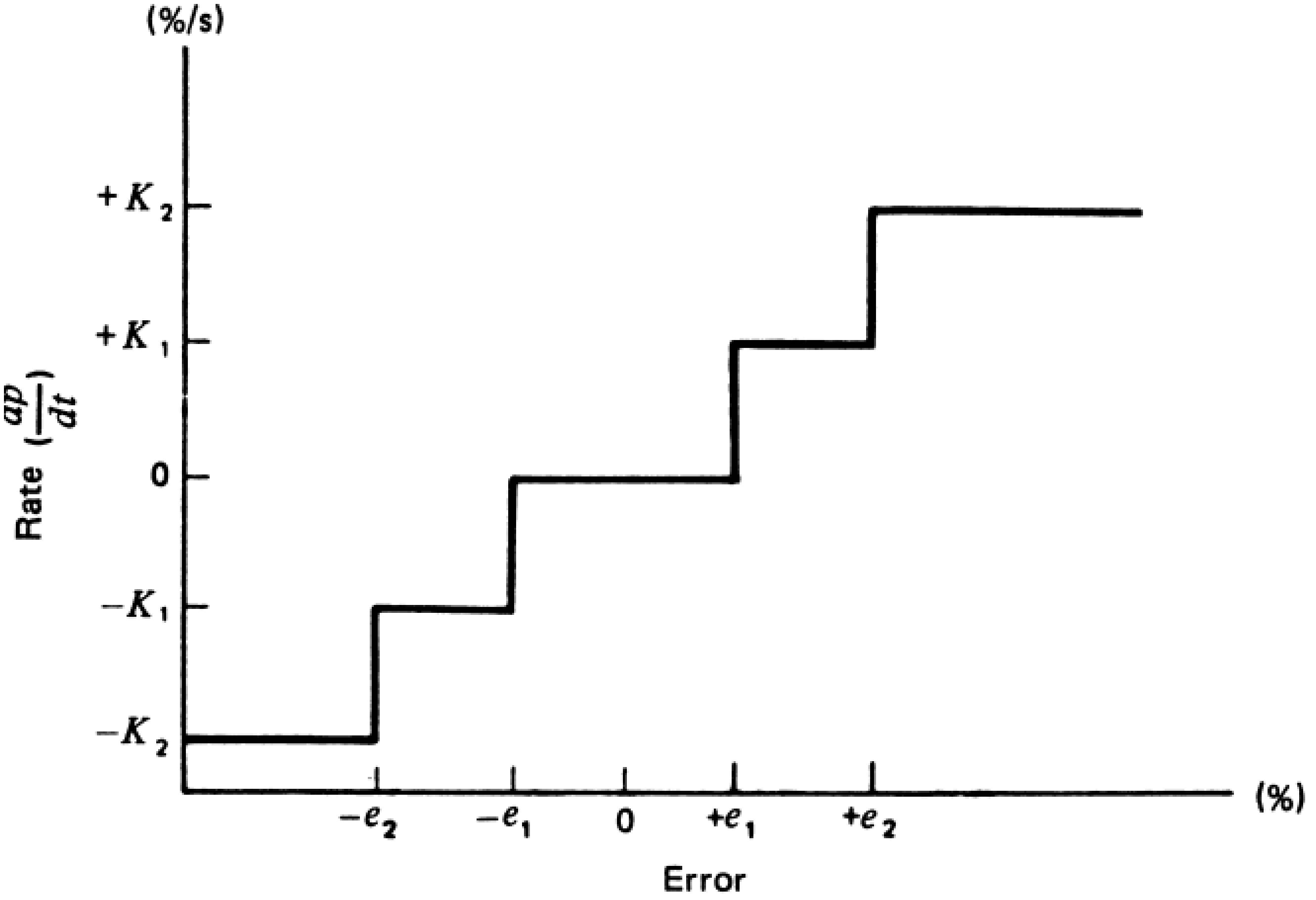
Multiple Speed

- In the floating multiple-speed control mode, not one but several possible speeds (rates) are changed by controller output.

$$\frac{dp}{dt} = \pm K_{Fi} \qquad |e_p| > e_{pi}$$

- If the error exceeds e_{pi}, then the speed is K_{fi}.If the error rises to exceed e_{p2}, the speed is increased to K_{f2} , and so on

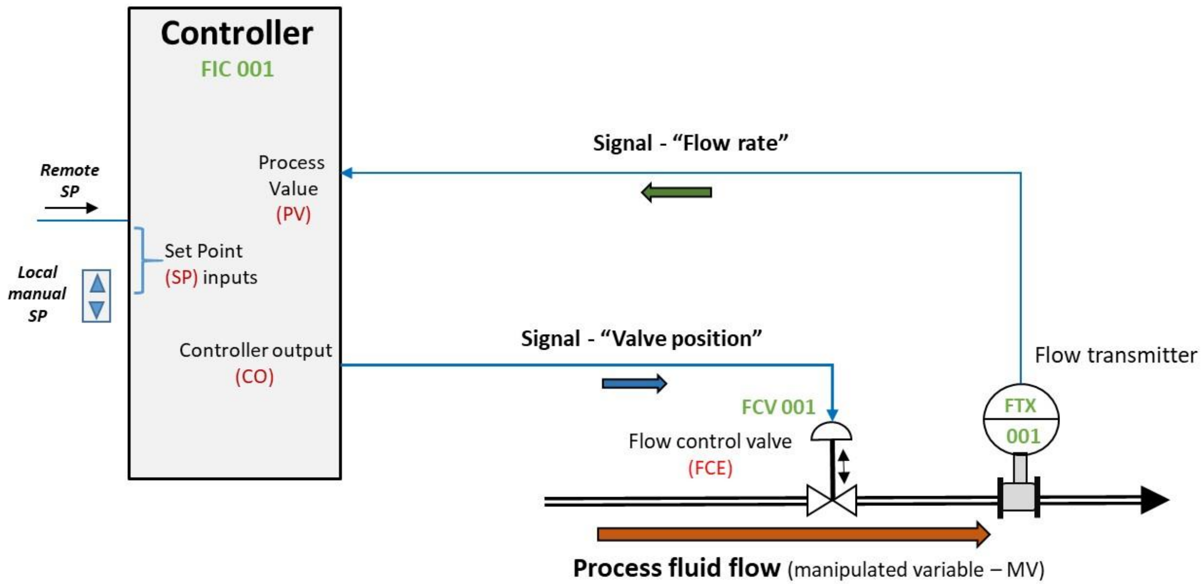
Multiple Speed



CONTINUOUS CONTROLLER MODES

The output of the controller changes smoothly in response to the error or rate of change of error.

Industrial process control loop



The basic building block of industrial process control systems is the "control loop" which contains all the elements to measure and control a process value at a desired setpoint. The controller may be a discrete piece of hardware, or a function within a large computerised DCS, SCADA or PLC system. Set points can be manually set locally or cascaded from another source.

An example is shown of a flow controller, with a flow transmitter and a control valve. The green text are "tags", which describe the function and identify the equipment. As each loop has a unique number the tags are unique within a plant to prevent confusion. In this case:
FIC = Flow indicating controller, FCV = Flow Control Valve, FTX = flow transmitter.

Standard practical control nomenclature is: SP = process set point, PV = process value, CO = controller output, FCE = final control element, MV = manipulated variable.

Proportional Control Mode

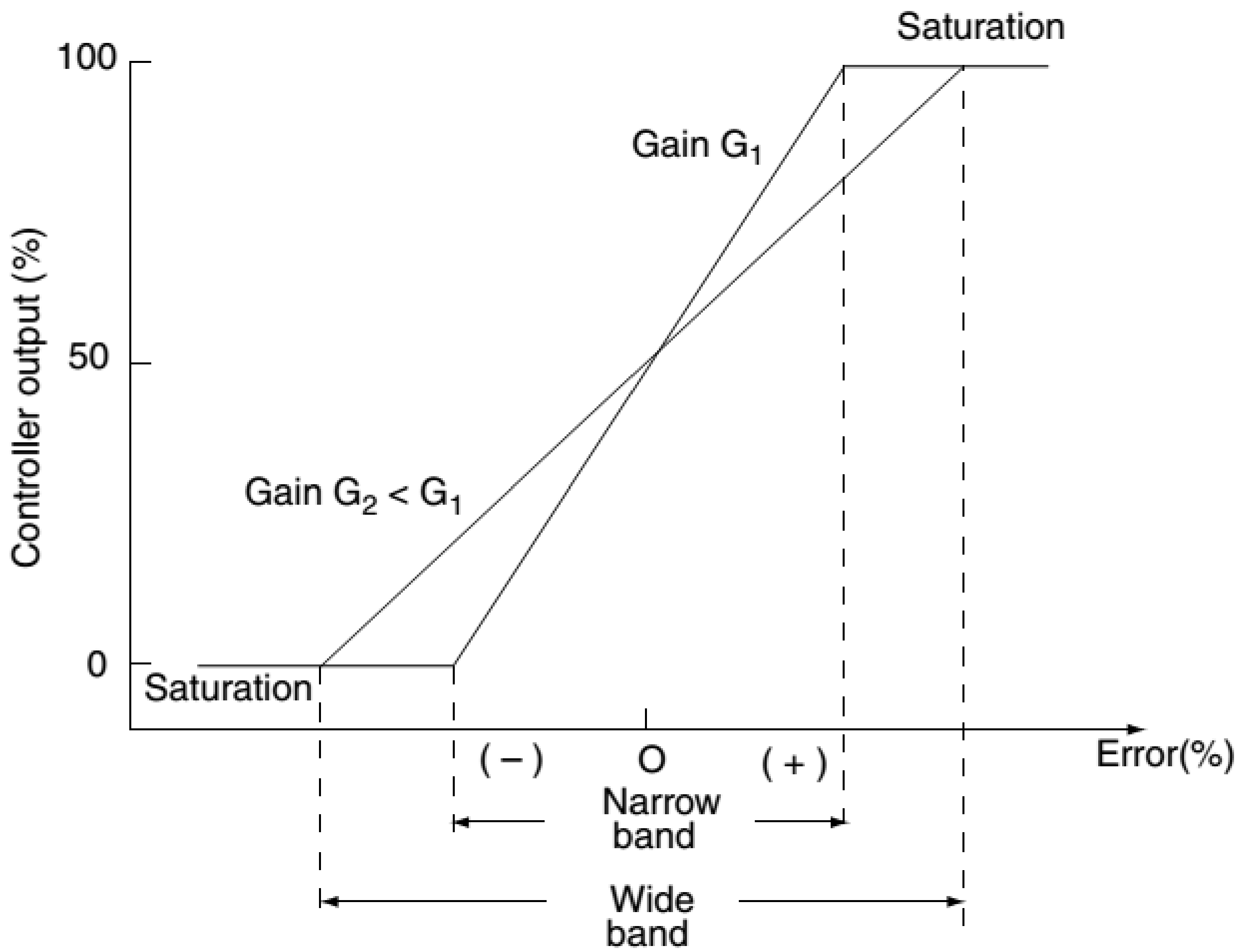
- The natural extension of two position and multiposition controllers.
- Smooth, linear relationship exists between the controller output and the error.
- The range of error to cover the 0% to 100% controller output is called the ***proportional band***

$$p = K_P e_p + p_0$$

Direct/reverse

K_P – Proportional gain between error and controller output(% per %)

p_0 - Controller output with no error (%)


Saturation
100
Gain G_1
Controller output (%)
50
Gain $G_2 < G_1$
0
Saturation
(−)
O
(+)
Error(%)
Narrow band
Wide band

- P_0 has been set to 50% and two different gains have been used.
- Note that the proportional band is dependent on the gain. A high gain means large response to an error, but also a narrow error band within which the output is not saturated

$$PB = \frac{100}{K_P}$$

Summary

1. If the error is zero, the output is a constant equal to p_o.

2. If there is error, for every 1% of error, a correction of K_p percent is added to or subtracted from p_o, depending on the sign of the error.

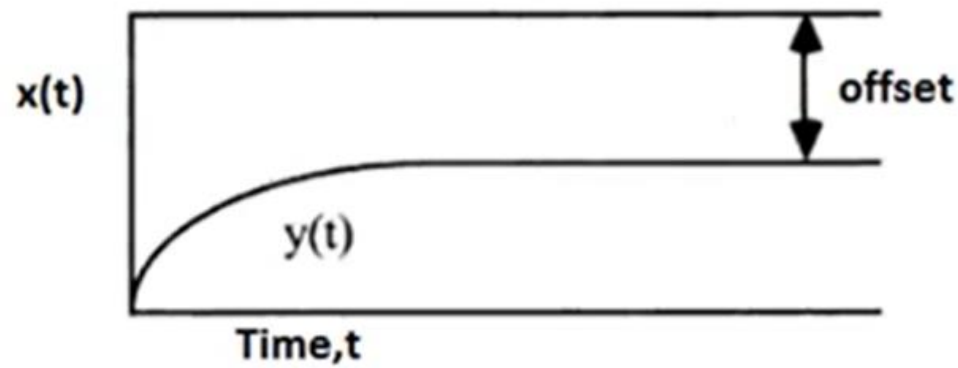
3. There is a band of error about zero of magnitude *PB* within which the output is not saturated at 0% or 100%.

Offset

- A permanent *residual error* in the operating point of the controlled variable when a change in load occurs.
- This error is referred to as *offset.*
- Can be minimized by a larger constant, K_p,

Limitation:

- The problem with PC is that there is offset when a set point change is made. PC can not keep the controlled variable on set point.
- Offset is that actual process output will not be able to achieve the desired SP change.
- The smaller the gain , the larger will be the offset.

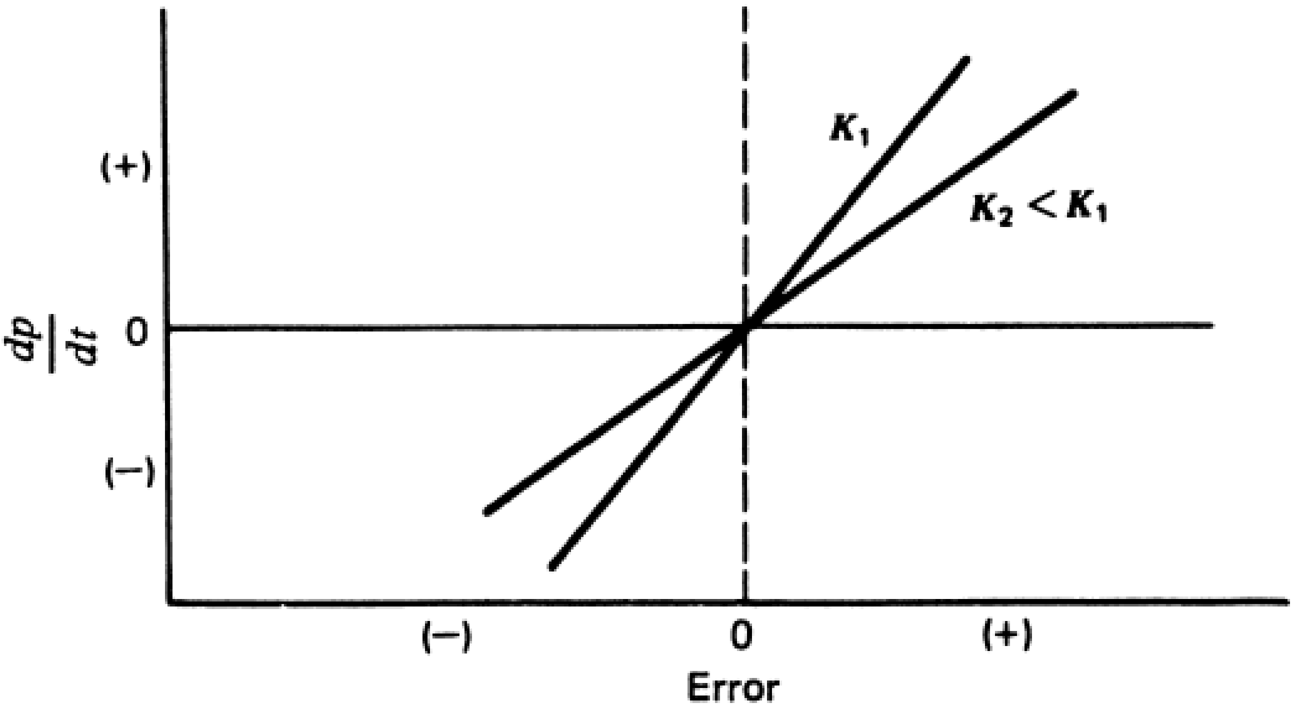


Integral controller

- The integral (I) control mode is also called reset mode.

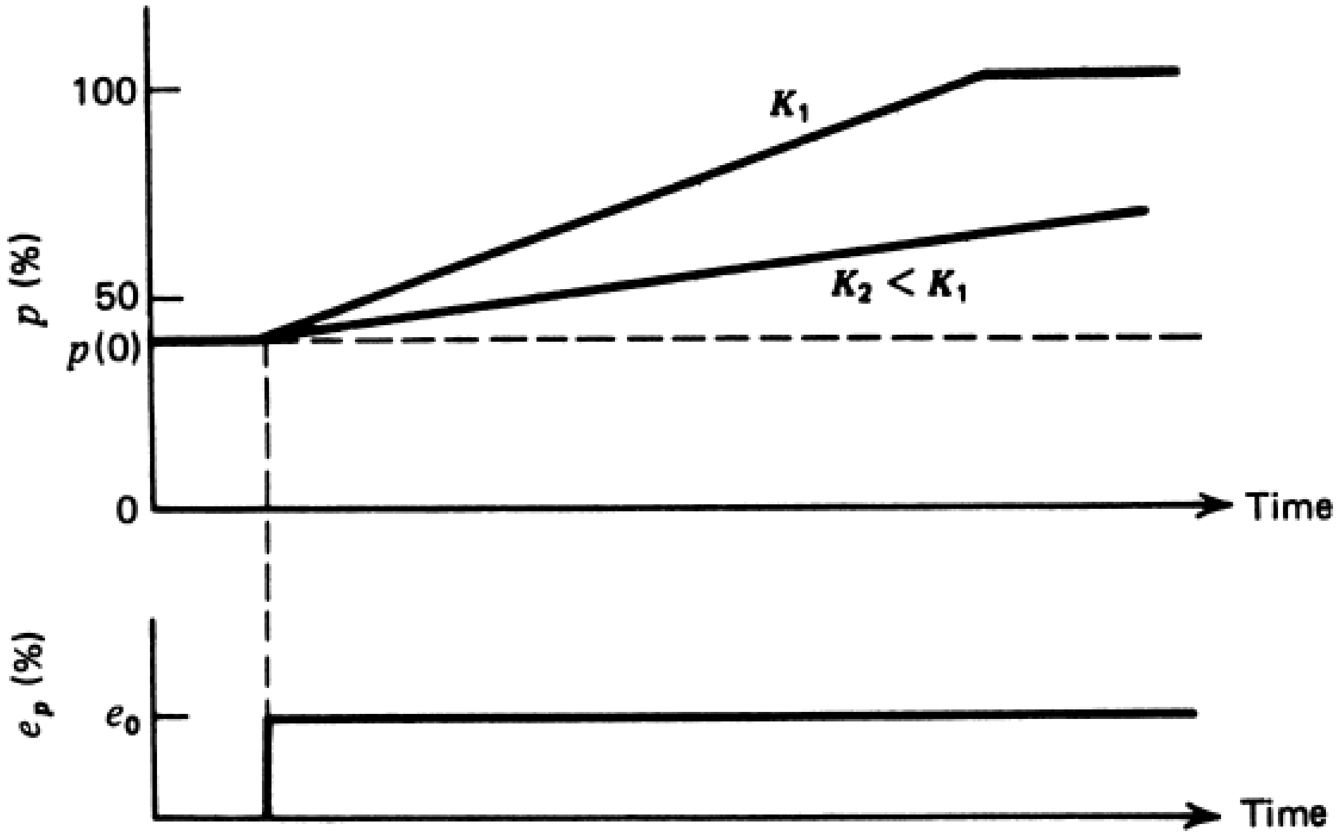
$$p(t) = K_I \int_0^t e_P \, dt + p(0) \qquad \frac{dp}{dt} = K_I e_p$$

- The integral mode, continuously looks at the total past history of the error by continuously integrating the area under the error curve.
- ***p*(0) is the controller output when the integral action starts.**



(a)

Integral controller



(b)

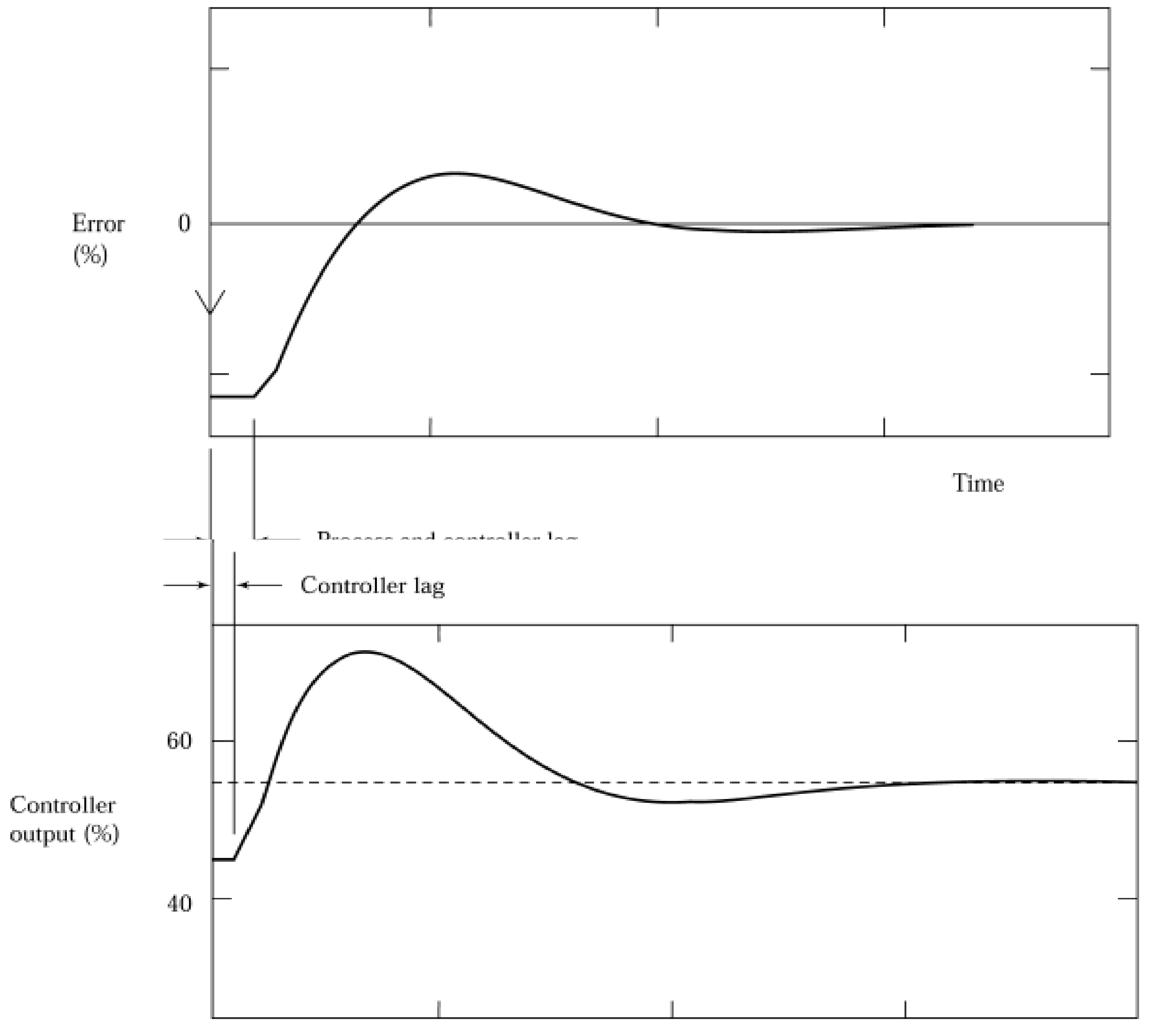
Integral controller

- 1. If the error is zero, the output stays fixed at a value equal to what it was when the error went to zero.
- 2. If the error is not zero, the output will begin to increase or decrease at a rate of KI percent/second for every 1% of error.
- The integral gain, , is often represented by the inverse, which is called the *integral time*,or the *reset action.*

$$T_i = 1/K_i$$

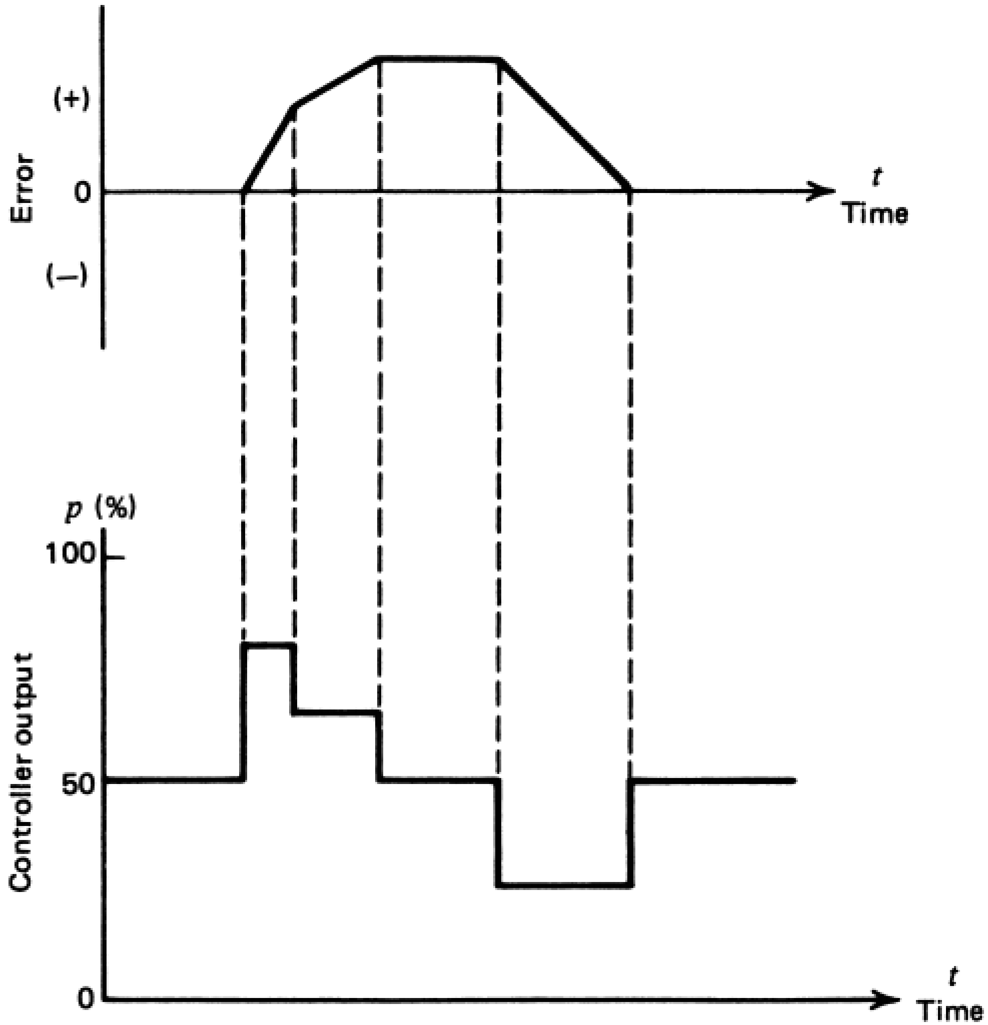
In minutes

Integral controller



An integral controller is used for speed control with a setpoint of 12 rpm within a range of 10 to 15 rpm. The controller output is 22% initially. The constant $K_I = -0.15\%$ controller output per second per percentage error. If the speed jumps to 13.5 rpm, calculate the controller output after 2 s for a constant e_p.

Derivative Control Mode



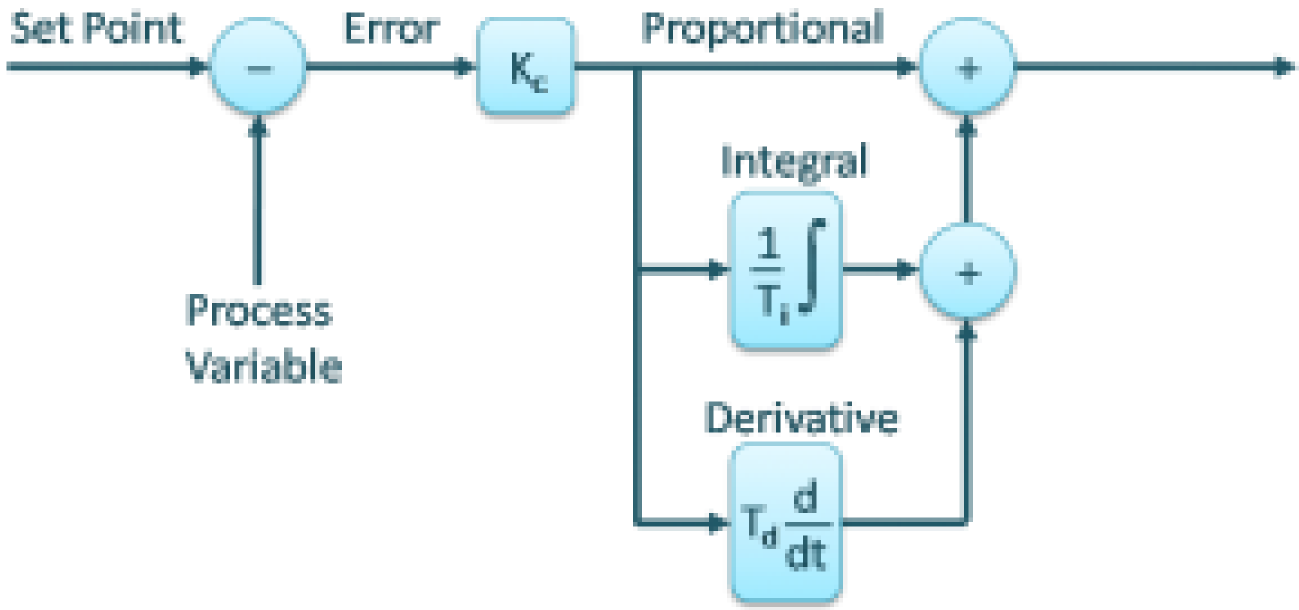
$$p(t) = K_D \frac{de_p}{dt}$$

Effect of controller parameters on the system response

CL RESPONSE	RISE TIME	OVERSHOOT	SETTLING TIME	S-S ERROR
Kp	Decrease	Increase	Small Change	Decrease
Ki	Decrease	Increase	Increase	Eliminate
Kd	Small Change	Decrease	Decrease	Small Change

COMPOSITE CONTROL MODES

Ideal configuration



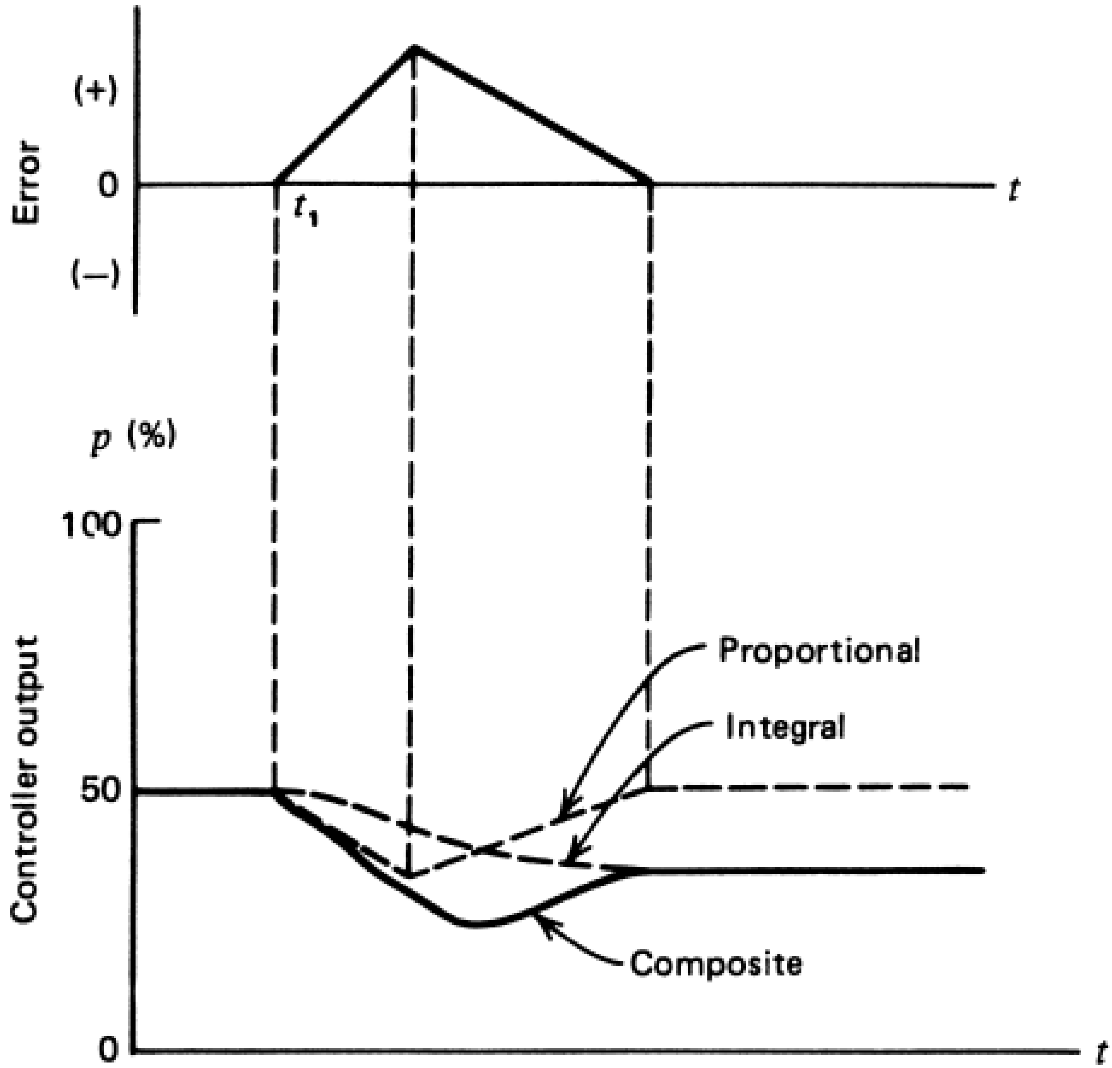
$$CO = K_c\left[E + \frac{1}{T_i}\int E \cdot dt + T_d\frac{dE}{dt}\right]$$

Proportional-Integral Control (PI)

$$p = K_P e_p + K_P K_I \int_0^t e_p \, dt + p_I(0)$$

$p_I(0)$ = integral term value at $t = 0$ (initial value)

Proportional-integral (PI) action



- “Ideal” form of the PI Controller

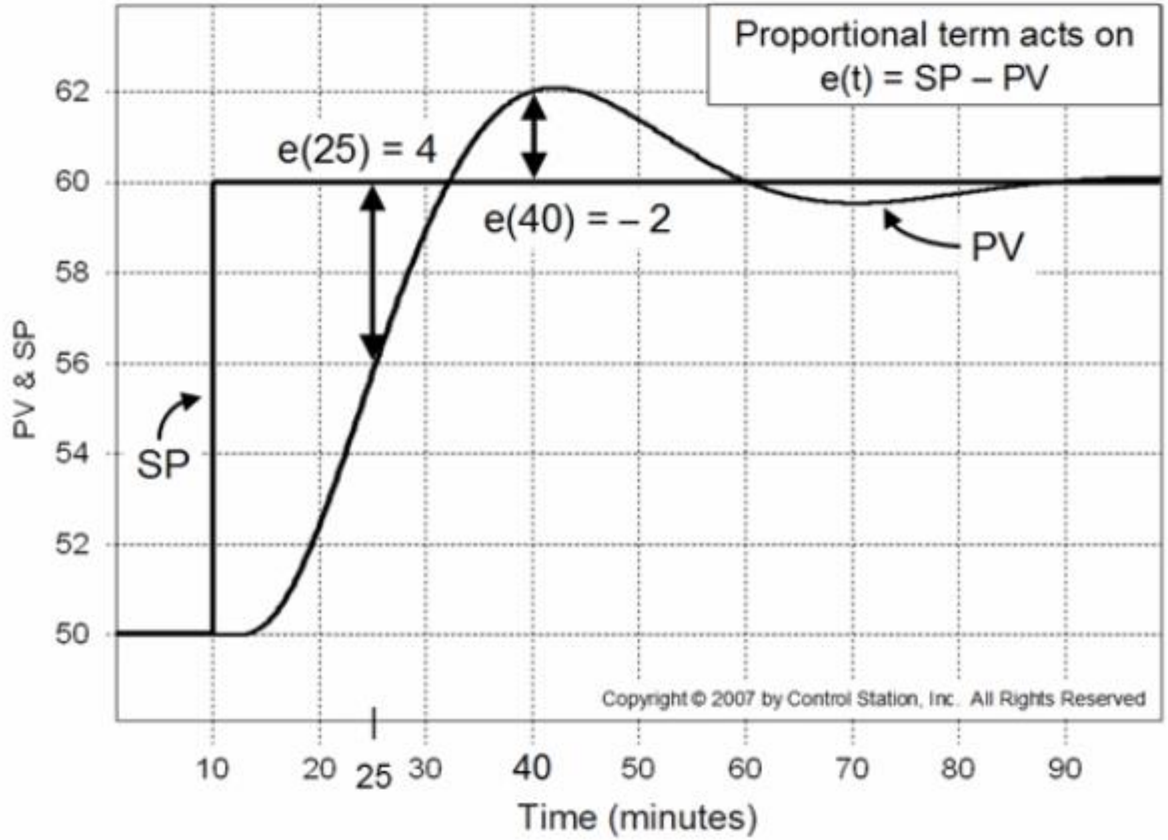
$$CO = CO_{bias} + Kc \cdot e(t) + \frac{Kc}{\tau_I} \int e(t)dt$$

where:

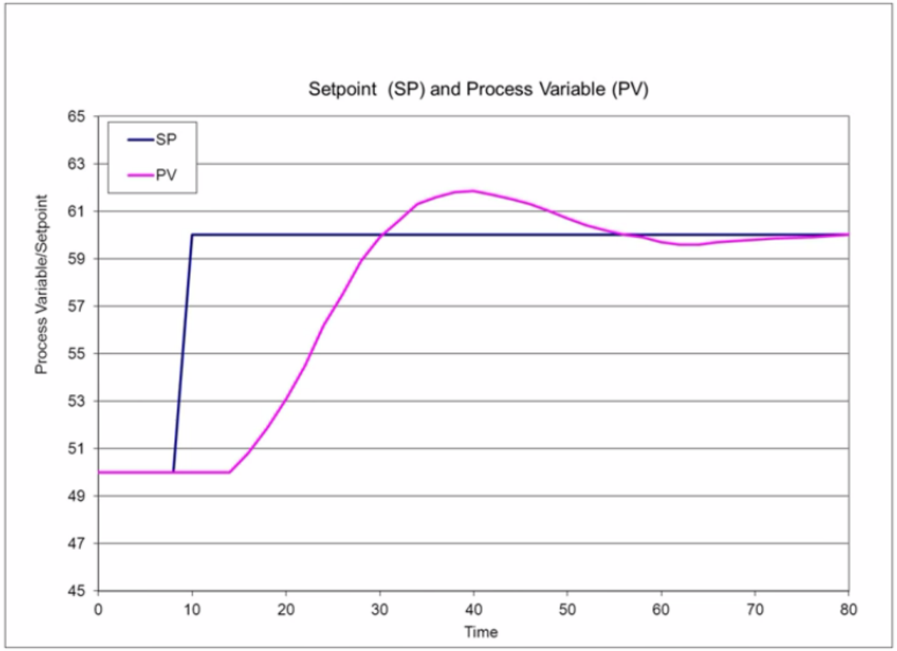
CO	= controller output signal
CO_{bias}	= controller bias or null value
PV	= measured process variable
SP	= set point
e(t)	= controller error = SP – PV
Kc	= controller gain (a tuning parameter)
τ_I	= controller **reset time** (a tuning parameter)

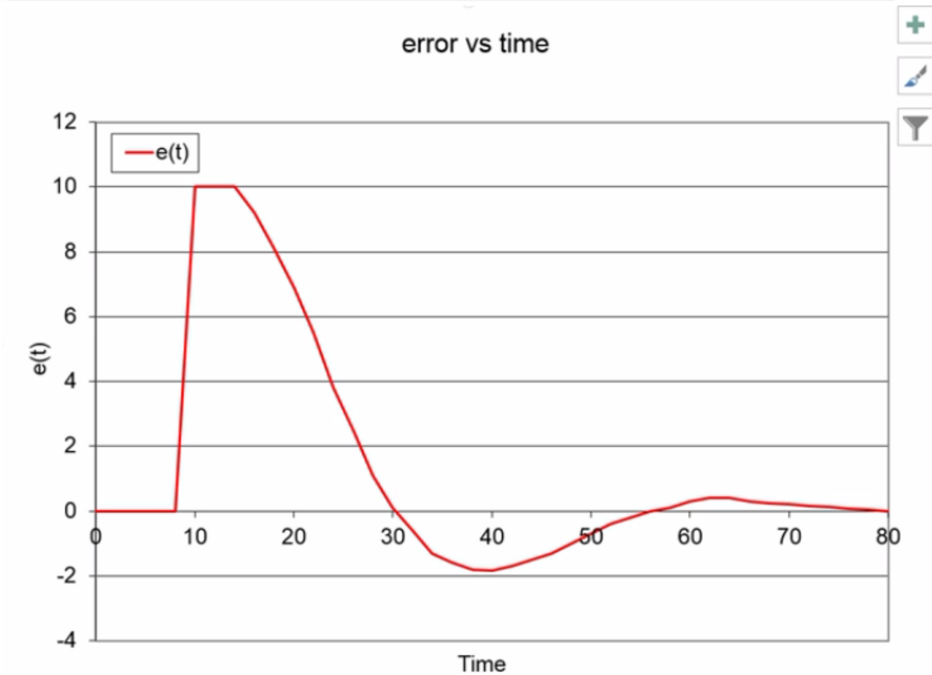
- τ_I is in denominator so smaller values provide a larger weighting to the integral term
- τ_I has units of time, and therefore is always positive

Function of the Proportional Term

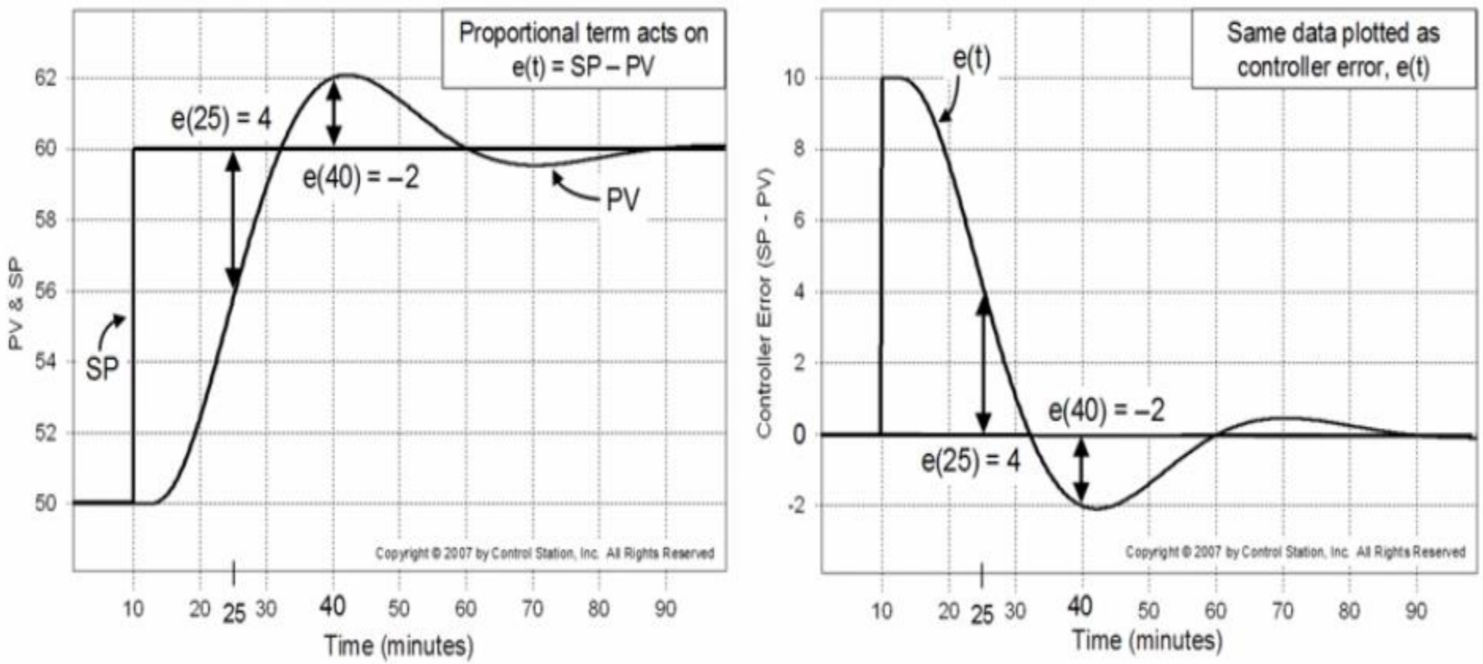


- The proportional term, $Kc \cdot e(t)$, immediately impacts CO based on the size of e(t) at a particular time t
- The past history and current trajectory of the controller error have no influence on the proportional term computation


Setpoint (SP) and Process Variable (PV)
SP
PV
Process Variable/Setpoint
Time
65
63
61
59
57
55
53
51
49
47
45
0
10
20
30
40
50
60
70
80

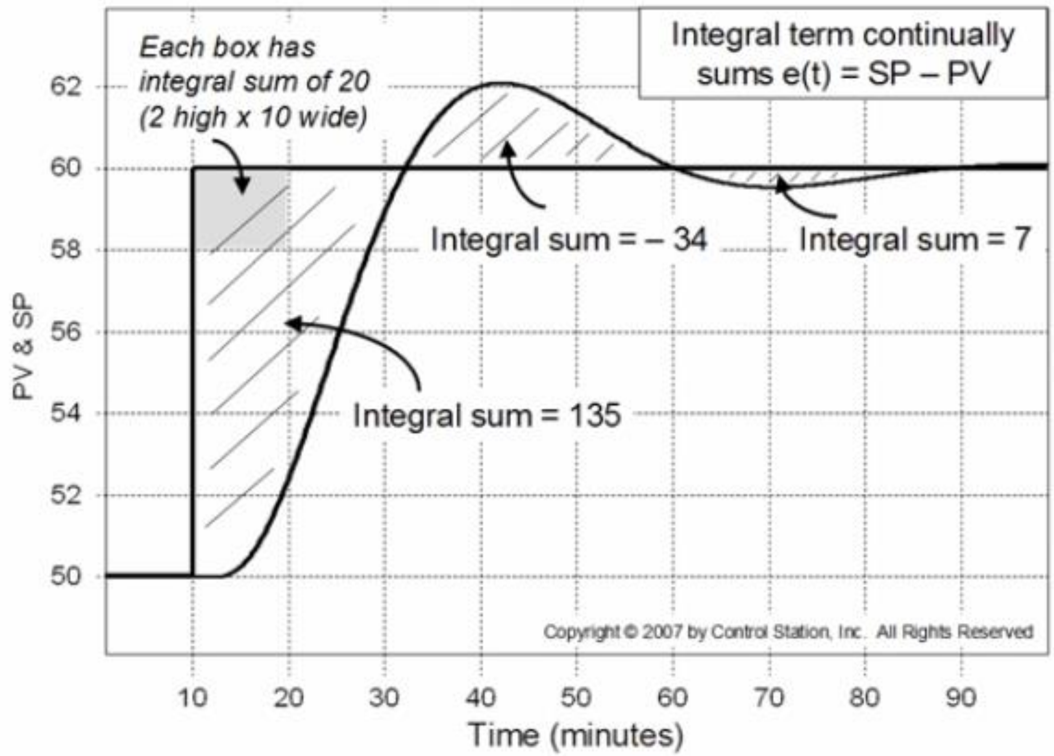

error vs time
e(t)
Time
12
10
8
6
4
2
0
-2
-4
0
10
20
30
40
50
60
70
80

Control Calculation is Based on Error, e(t)



- Here is identical data plotted two ways
- To the right is a plot of error, where: e(t) = SP – PV
- Error e(t) continually changes size and sign with time

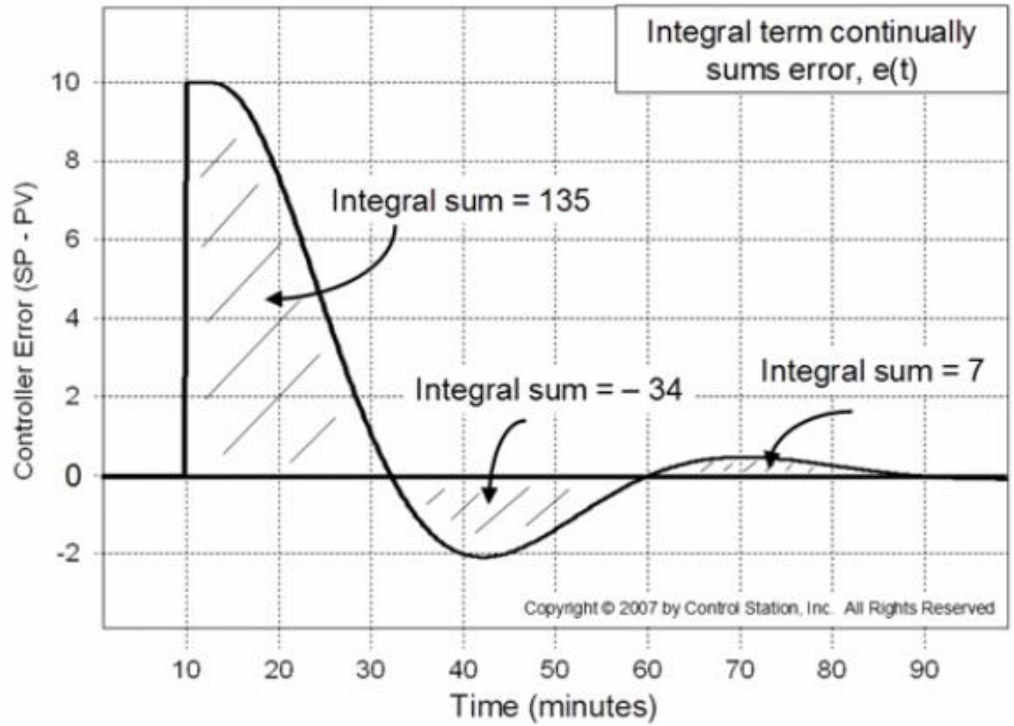
Integral Term Continually Sums the Value: SP – PV



- The integral is the sum of the area between SP and PV
- At t=32 min, when the PV first reaches the SP, the integral is:

$$\int_{0\,\text{min}}^{32\,\text{min}} e(t)dt = 135$$

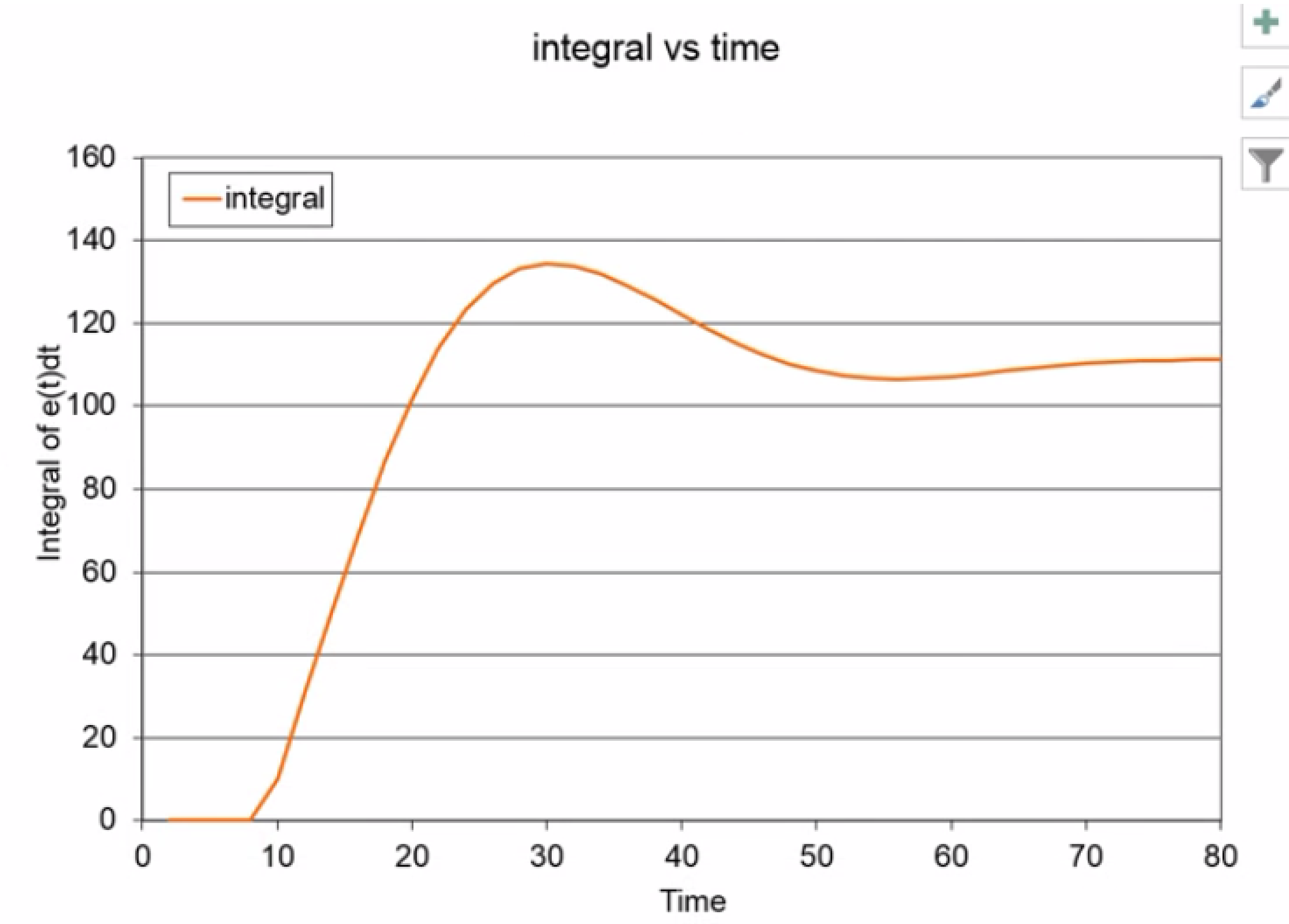
Integral of Error is the Same as Integral of: SP – PV



- At t = 60 min, the total integral is: 135 – 34 = 101
- When the dynamics have ended, e(t) is constant at zero and the total integral has a final residual value: 135 – 34 + 7 = 108

Function of the Integral Term

- The integral term continually sums up error, e(t)
- Through constant summing, integral action accumulates influence based on how long and how far the measured PV has been from SP over time.
- Even a small error, if it persists, will have a sum total that grows over time and the amount added to CO_{bias} will similarly grow.
- The continual summing of integration starts from the moment the controller is put in automatic


integral vs time
integral
Integral of e(t)dt
160
140
120
100
80
60
40
20
0
0
10
20
30
40
50
60
70
80
Time

Advantage of PI Control – No Offset

- The PI controller stops computing changes in CO when e(t) equals zero for a sustained period

$$CO = CO_{bias} + Kc \cdot e(t) + \frac{Kc}{\tau_I} \int e(t)dt$$

- At that point, the proportional term equals zero, and the integral term may have a residual value

$$CO = CO_{bias} + 0 + \underbrace{\frac{Kc}{\tau_I}(108)}_{\text{Integral acts as "moving bias" term}}$$

- This residual value, when added to CO_{bias}, essentially creates an overall "moving bias" that tracks changes in operating level
- This moving bias eliminates offset, making PI control the **most widely used industry algorithm**

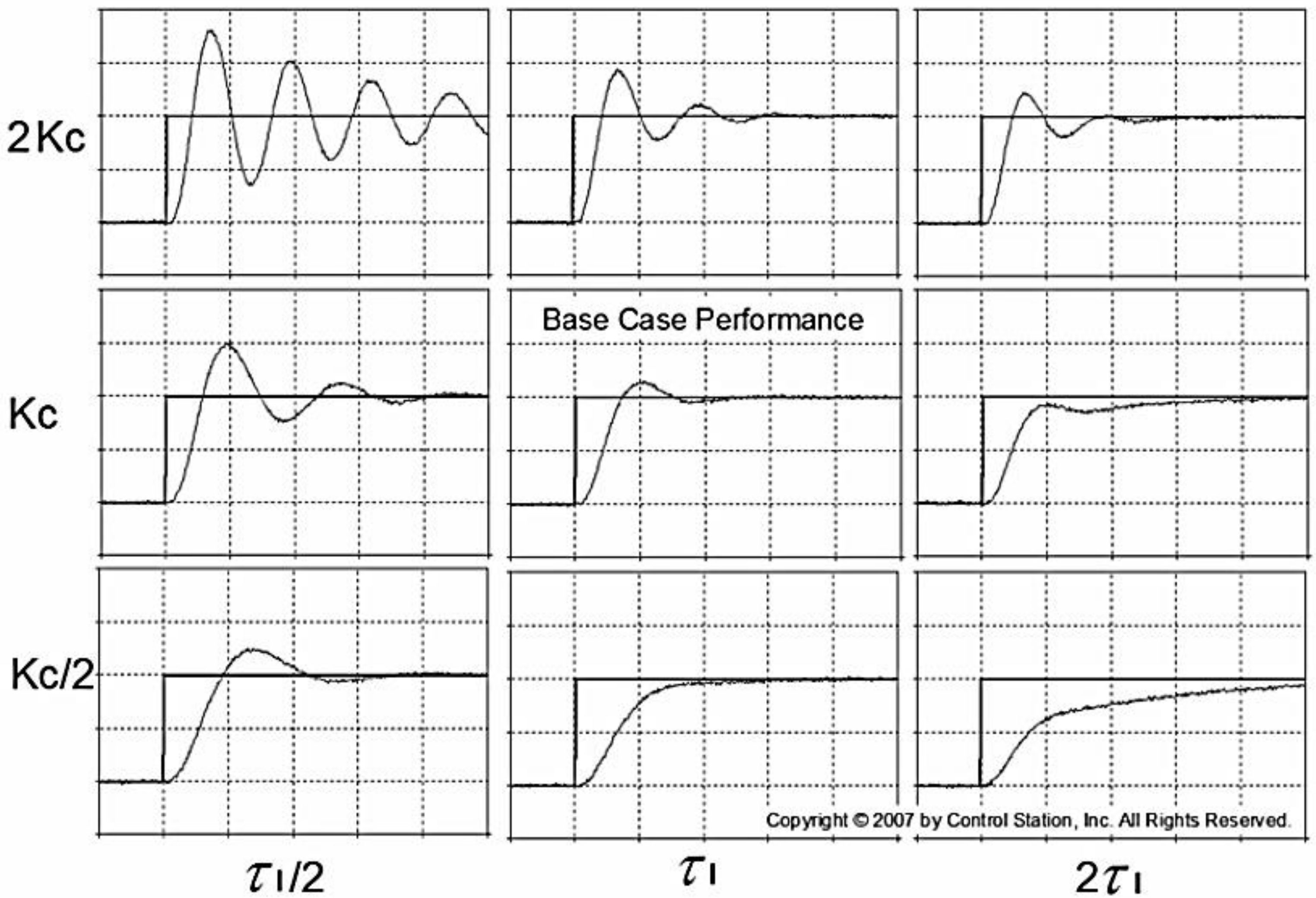
Disadvantage of PI control-interaction

- Integral action tends to oscillatory or rolling behavior of PV
- There are two tuning parameters (Kc and τ_I) and the interact with each other

$$CO = CO_{bias} + Kc \cdot e(t) + \frac{Kc}{\tau_I} \int e(t)dt$$

- This interaction can make it challenging to arrive at best tuning values.

PI controller tuning guide



Integral Action and Reset Windup

- The math makes it possible for the error sum (the integral) to grow very large.

$$CO = CO_{bias} + Kc \cdot e(t) + \underbrace{\frac{Kc}{\tau_I} \int e(t)dt}_{integral}$$

- The integral term can grow so large that the total CO signal stops making sense (it can be signaling for a valve to be open 120% or negative 15%)
- "Windup" is when the CO grows to exceed the valve limits because the integral has reached a huge positive/negative value
- It is associated with the integral term, so it is called *reset windup*
- The controller can't regulate the process until the error changes sign and the integral term shrinks sufficiently so that the CO value again makes sense (moves between 0 – 100%).

- **Is the process of accumulating the integral component beyond the saturation limits of final control element.**

Summary of PI Controller

- When the error is zero, the controller output is fixed at the value that the integral term had when the error went to zero.

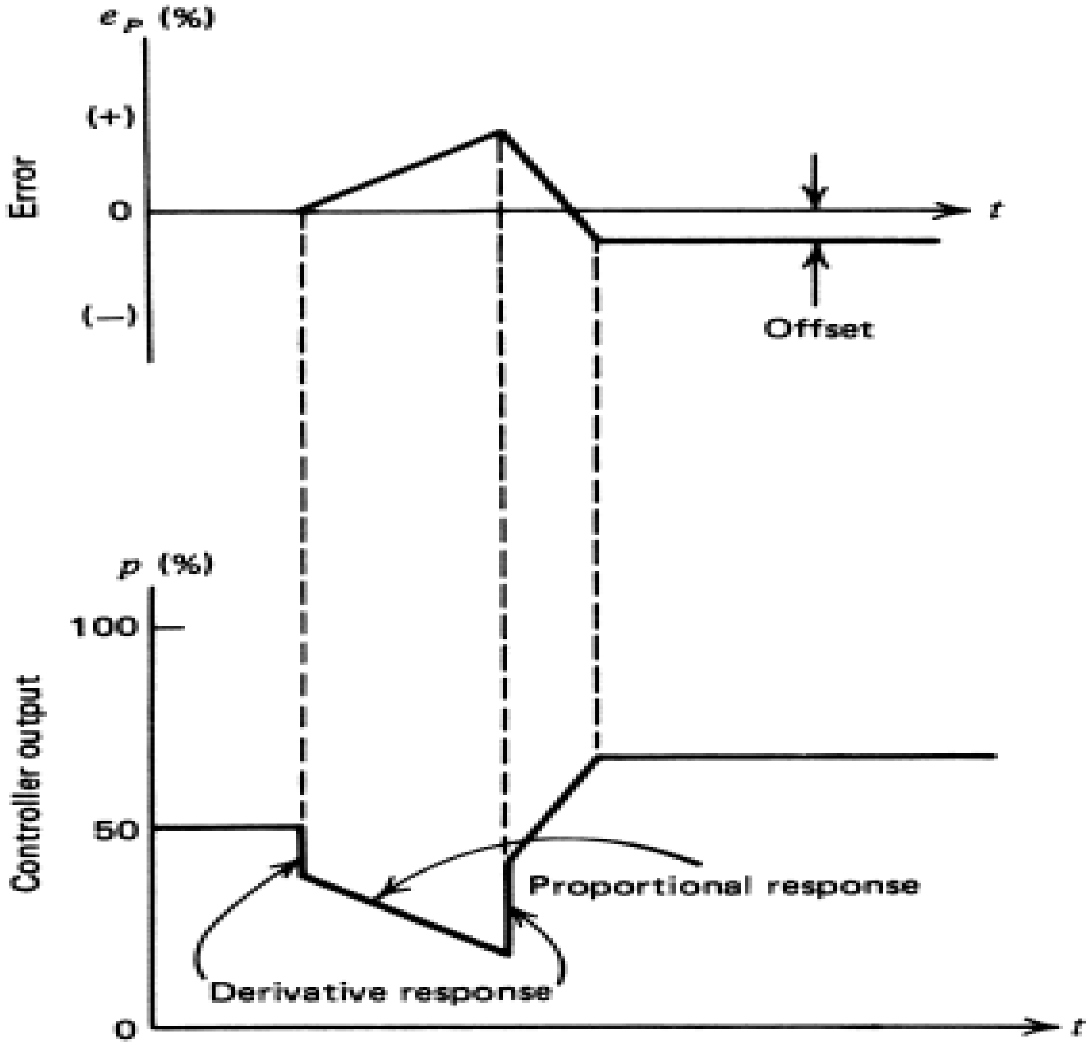
- If the error is not zero, the proportional term contributes a correction, and the integral term begins to increase or decrease the accumulated value [initially, $P_I(0)$].

Proportional-Derivative Control Mode (PD)

$$p = K_P e_p + K_P K_D \frac{de_p}{dt} + p_0$$

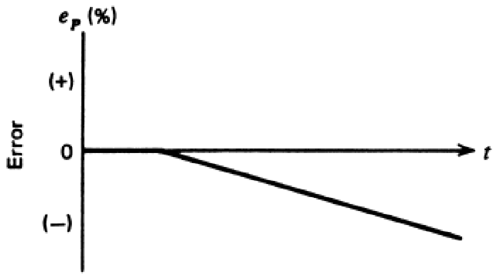
- Cannot eliminate the offset of proportional controllers.
- It can, however, handle fast process load changes as long as the load change offset error is acceptable.

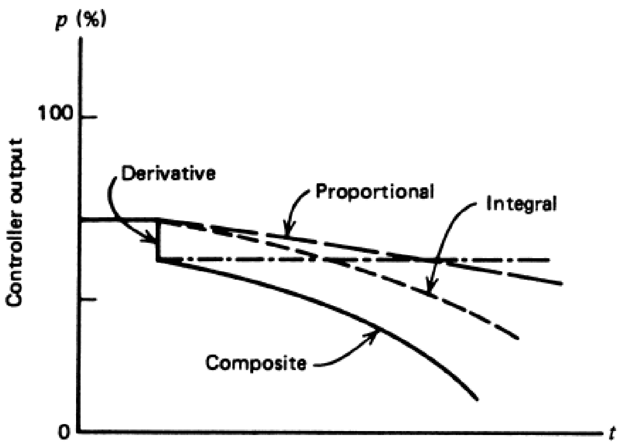
Proportional-derivative (PD) action



Three-Mode Controller (PID)

$$p = K_P e_p + K_P K_I \int_0^t e_p \, dt + K_P K_D \frac{de_p}{dt} + p_I(0)$$





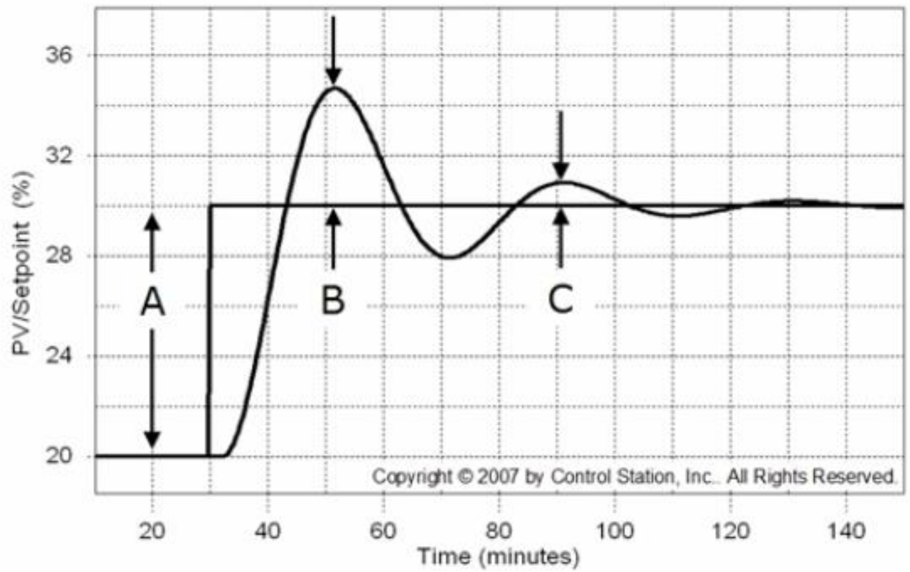
Evaluating Controller Performance

- Bioreactors can't tolerate sudden operating changes because the fragile living cell cultures could die.
  - » "good" control means PV moves ***slowly***

- Packaging/filling stations can be unreliable. Upstream process must ramp back quickly if a container filling station goes down.
  - » "good" control means PV moves ***quickly***

- The operator or engineer defines what is good or best control performance based on their knowledge of:
  - goals of production
  - capabilities of the process
  - impact on down stream units
  - desires of management

Performance Analysis

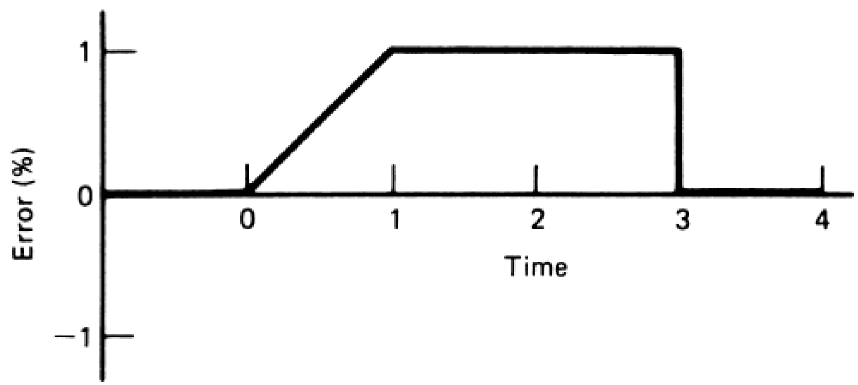


- Rise Time = When PV first reaches SP
- Peak Time = Time of first peak
- Overshoot Ratio = B/A
- Decay Ratio = C/B
- Settling Time = Time when PV remains < 5% of SP

Problems

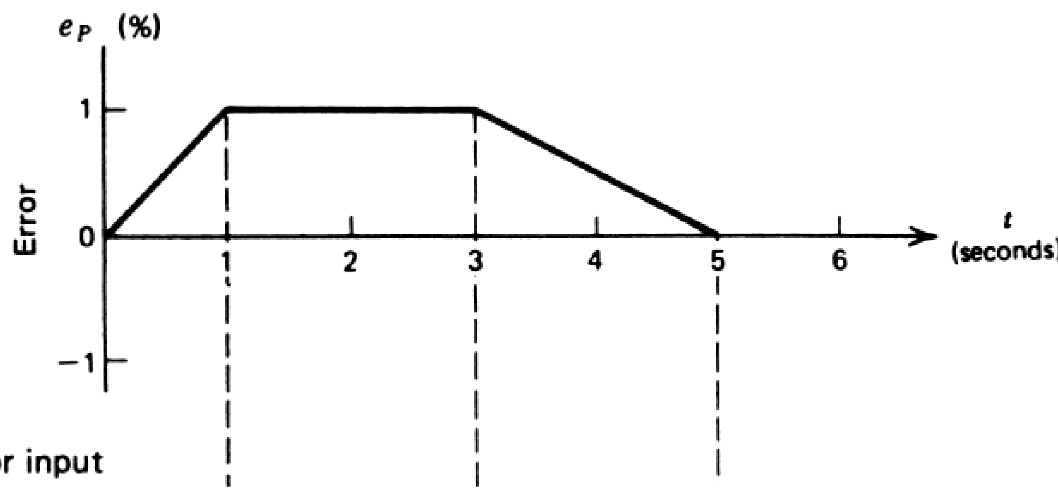
Given the error of Figure 20 (top), plot a graph of a proportional-integral controller output as a function of time.

$$K_P = 5,\ K_I = 1.0\ \text{s}^{-1},\ \text{and}\ p_I(0) = 20\%$$



Problems

Suppose the error, Figure 22a, is applied to a proportional-derivative controller with $K_P = 5$, $K_D = 0.5$ s, and $p_0 = 20\%$. Draw a graph of the resulting controller output.



a) Error input

Let us combine everything and see how the error of Figure 22a produces an output in the three-mode controller with $K_P = 5$, $K_I = 0.7\ \mathrm{s}^{-1}$, $K_D = 0.5$ s, and $p_I(0) = 20\%$. Draw a plot of the controller output.