2) $y'' - 3y' + 2y = 1+2x, \quad y(1) = y(2) = 0, \quad h = 0.25$

$x_0 = 1 \quad x_1 = 1.25 \quad x_2 = 1.50 \quad x_3 = 1.75 \quad x_4 = 2.$

$y_0 = 0 \quad y_1 = \quad y_2 \quad y_3 \quad y_4 = 0$

$$y_i'' - 3y_i' + 2y_i = 1+2x_i$$

$$\frac{y_{i+1} - 2y_i + y_{i-1}}{h^2} - 3\left(\frac{y_{i+1} - y_{i-1}}{2h}\right) + 2y_i = 1+2x_i$$

$$16y_{i+1} - 32y_i + 16y_{i-1} - 6y_{i+1} + 6y_{i-1} + 2y_i = 1+2x_i$$

$$10y_{i+1} - 30y_i + 22y_{i-1} = 1+2x_i \longrightarrow (1)$$

Put $i = 1, 2, 3$ in (1)

$$10y_2 - 30y_1 = 3.5 \longrightarrow (2)$$

$$10y_3 - 30y_2 + 22y_1 = 4 \rightarrow (3)$$

$$-30y_3 + 22y_2 = 4.5 \rightarrow (4)$$

Solve (2), (3) and (4),

$$y_1 = y(1.25) = -0.292$$

$$y_2 = y(1.50) = -0.5261$$

$$y_3 = y(1.75) = -0.5358$$

Solve (2), (3) and (4), we get

$$y_1 = y(1.25) = 1.3513$$

$$y_2 = y(1.50) = 1.6349$$

$$y_3 = y(1.75) = 1.8508$$

1) Solve: $xy'' + y = 0, \quad y(1) = 1, \quad y(2) = 2, \quad h = 0.25$

$x_0 = 1 \qquad x_1 = 1.25 \qquad x_2 = 1.50 \qquad x_3 = 1.75 \qquad x_4 = 2$

$y_0 = 1 \qquad y_1 = \qquad y_2 = \qquad y_3 = \qquad y_4 = 2$

$$x_i y_i'' + y_i = 0$$

$$x_i \left[\frac{y_{i+1} - 2y_i + y_{i-1}}{h^2} \right] + y_i = 0$$

$$16x_i \left[y_{i+1} - 2y_i + y_{i-1} \right] + y_i = 0$$

$$16x_i y_{i+1} + (1 - 32x_i) y_i + 16 y_{i-1} = 0 \longrightarrow (1)$$

Put $i = 1$ in (1)

$$16x_1 y_2 + (1 - 32x_1) y_1 + 16y_0 = 0$$

$$20y_2 - 39y_1 + 16 = 0. \qquad \text{i.e } -39y_1 + 20y_2 = -16 \rightarrow (2)$$

Put $i=2$ in (1)

$$16x_2 y_3 + (1 - 32x_2) y_2 + 16y_1 = 0$$

$$24y_3 - 47y_2 + 16y_1 = 0 \qquad \text{i.e } 16y_1 - 47y_2 + 24y_3 = 0 \rightarrow (3)$$

Put $i=3$ in (1)

$$16x_3 y_4 + (1 - 32x_3) y_3 + 16y_2 = 0$$

$$56 - 55y_3 + 16y_2 = 0$$

$$\text{i.e, } 16y_2 - 55y_3 = -56 \rightarrow (3)$$

$(1) - (2) \implies \quad y_i' = \dfrac{y_{i+1} - y_{i-1}}{2h} \quad \rightarrow (3)$

$(1) + (2) \implies \quad y_i'' = \dfrac{y_{i+1} - 2y_i + y_{i-1}}{h^2} \quad \rightarrow (4)$

(3) and (4) are called the finite difference expressions for y_i' and y_i'' respectively.

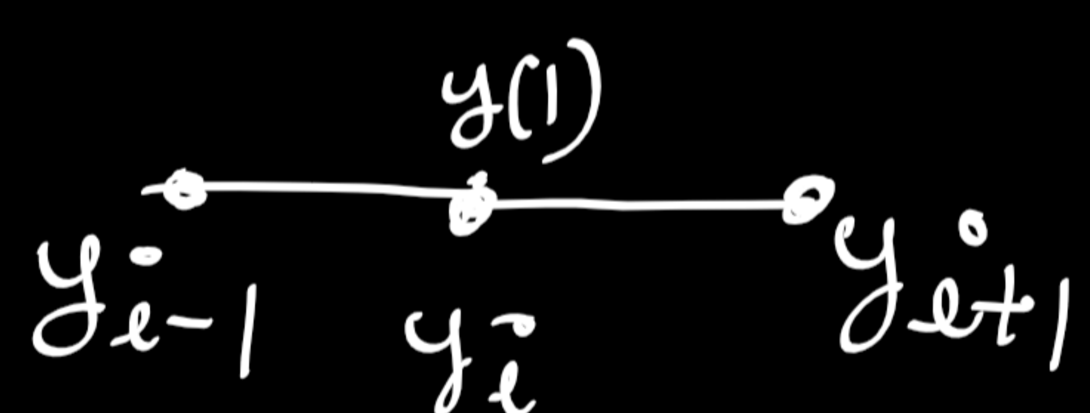
Ex:- $\dfrac{d^2y}{dx^2} = 2+3x^2, \quad 0<x<2, \quad h=0.5, \quad y(0)=0, \quad y(2)=0$

what is the value of d^2y/dx^2 at $y(1)$

$x_0 = 0 \quad x_1 = 0.5 \quad x_2 = 1 \quad x_3 = 1.5 \quad x_4 = 2$

$y_0 = 0 \quad y_1 = y(0.5) \quad y_2 = y(1) \quad y_3 = y(1.5) \quad y_4 = 0$

$$y_i'' = \frac{y(1.5) - 2y(1) + y(0.5)}{(0.5)^2}$$



Consider a second order B.V.P (Boundary value problem)

$$p(x)y''(x) + q(x)\,y'(x) + r(x)\,y(x) = f(x),$$

given $y(x_0) = a$, $y(x_n) = b$.

Divide $[x_0, x_n]$ into n subintervals of length h.

$$h = \frac{x_n - x_0}{n}$$

x_0

$x_1 = x_0 + h$

$x_2 = x_1 + h$

$\vdots$

x_n

Let $x_i = x_0 + ih$, $i = 1, 2, \dots, n$

We use the following notations.

$y(x_i) = y_i$, $y'(x_i) = y_i'$, $y''(x_i) = y_i''$

$p(x_i) = p_i$, $q(x_i) = q_i$, $r(x_i) = r_i$, $f(x_i) = f_i$

Let $y(x_i+h) = y_{i+1}$

By Taylor's Series expansion

$$y_{i+1} = y(x_i+h) = y(x_i) + h\,y'(x_i) + \frac{h^2}{2!}\,y''(x_i) + \dots$$

$$\therefore \quad y_{i+1} = y_i + h y_i' + \frac{h^2}{2!} y_i'' + \dots \longrightarrow (1)$$

$$y_{i-1} = y(x_i - h) = y(x_i) - h y'(x_i) + \frac{h^2}{2!} y''(x_i) - \dots$$

$$y_{i-1} = y_i - h y_i' + \frac{h^2}{2!} y_i'' - \dots \longrightarrow (2)$$

Finite difference method.

Consider a second order D.E $F(x, y, y', y'') = 0$.
Its general solution contains two arbitrary constants. To determine these constants we need to prescribe two conditions. The conditions are called initial conditions if y and y' are specified at a certain value of x. The D.E together with the initial condition is called the the initial value problem.

If y or y' or their combination is prescribed at two different values of x, then the conditions are called boundary conditions and the D.E together with the boundary conditions is called a boundary value problem.

Engg. Mathematics IV (MAT-2258)

1. Finite difference method for solving D.Es (ODE + PDE)
2. Probability
3. Difference Equations
4. Z-Transforms.

Laplace $\int_0^\infty$

Fourier $\int_{-\infty}^\infty$

$\sum$