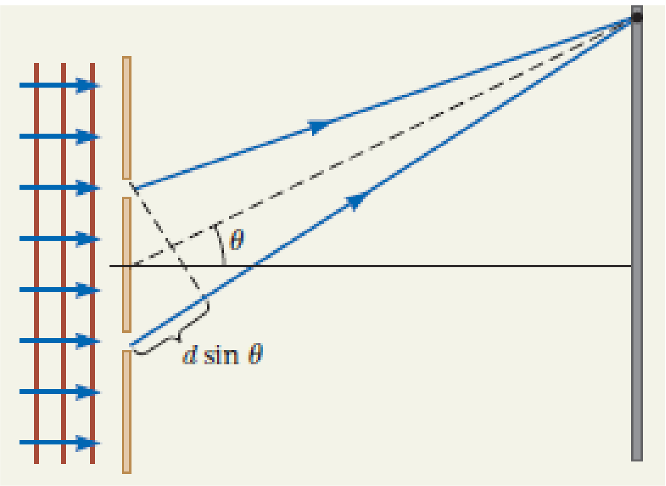
Chapter 1: Interference of Light Waves

P1: A viewing screen is separated from a double slit by 4.80 m. The distance between the two slits is 0.03 mm. Monochromatic light is directed toward the double slit and forms an interference pattern on the screen. The first dark fringe is 4.50 cm from the center line on the screen. (A) Determine the wavelength of the light. (B) Calculate the distance between adjacent bright fringes.

Ans:

$$y_{\text{dark}} = L\frac{\left(m+\frac{1}{2}\right)\lambda}{d} \qquad \text{(small angles)}$$



$$\lambda = \frac{y_{\text{dark}}d}{(m+\frac{1}{2})L} = \frac{(4.50\times10^{-2}\text{ m})(3.00\times10^{-5}\text{ m})}{(0+\frac{1}{2})(4.80\text{ m})}$$

λ = 562 nm

$$\Delta y = L\frac{\lambda}{d} = 4.80\text{ m}\left(\frac{5.62\times10^{-7}\text{ m}}{3.00\times10^{-5}\text{ m}}\right)$$

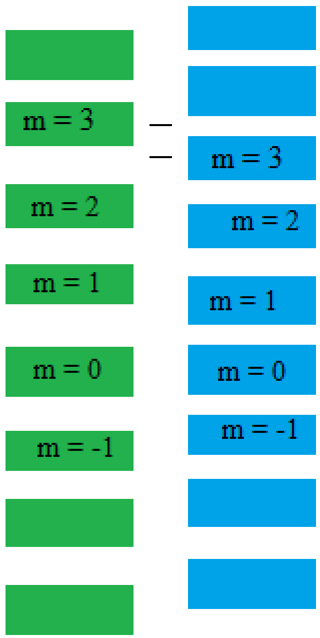
= 9.00 cm

P2 : A light source emits visible light of two wavelengths: λ = 430 nm and λ' = 510 nm. The source is used in a double-slit interference experiment in which L = 1.50m and d = 0.025 mm. Find the separation distance between the third-order bright fringes for the two wavelengths.

$$y'_{\text{bright}} - y_{\text{bright}} = L\frac{m\lambda'}{d} - L\frac{m\lambda}{d} = \frac{Lm}{d}(\lambda' - \lambda)$$

$$= \frac{(1.50\ \text{m})(3)}{0.025\,0 \times 10^{-3}\ \text{m}}(510 \times 10^{-9}\ \text{m} - 430 \times 10^{-9}\ \text{m})$$

= 1.44 cm



P4: A Young's interference experiment is performed with monochromatic light. The separation between the slits is 0.500 mm, and the interference pattern on a screen 3.30 m away shows the first side maximum 3.40 mm from the center of the pattern. What is the wavelength?

Ans:

The location of the bright fringes for small angles is given by,

$$y_{\text{bright}} = L\frac{m\lambda}{d}$$

For *m* = 1,

$$\lambda = \frac{y_{\text{bright}}}{L} = \frac{\left(3.40\times10^{-3}\ \text{m}\right)\left(0.500\times10^{-3}\ \text{m}\right)}{3.30\ \text{m}}$$

λ = 515 nm

P9 : In a double slit experiment, let L = 120 cm and d = 0.250 cm. The slits are illuminated with coherent 600-nm light. Calculate the distance y above the central maximum for which the average intensity on the screen is 75.0% of the maximum.

Ans:

Intensity in a double-slit interference pattern is

$$I = I_{\max} \cos^2\left(\frac{\pi d}{\lambda L} y\right) \qquad \text{Or} \qquad y = \frac{\lambda L}{\pi d} \cos^{-1} \sqrt{\frac{I}{I_{\max}}}$$

For $I = 0.75\ I_{\max}$,

i.e.,

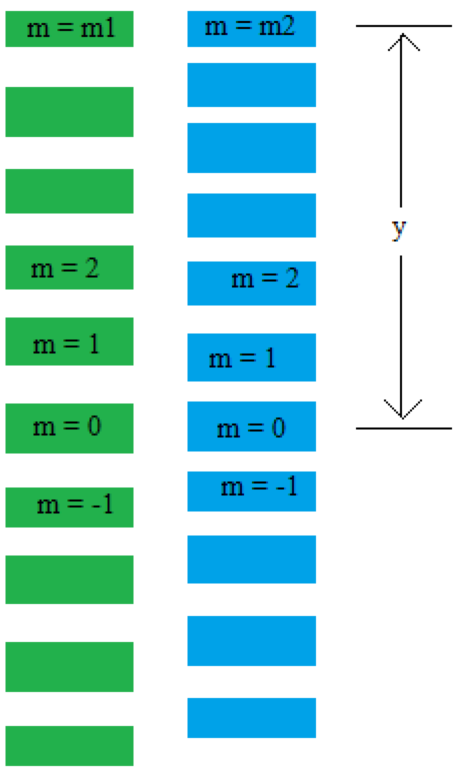
$$y = \frac{\left(6.00 \times 10^{-7}\ \text{m}\right)(1.20\ \text{m})}{\pi\left(2.50 \times 10^{-3}\ \text{m}\right)} \cos^{-1} \sqrt{0.750}$$

y = 48 μm

Put the calculator in radian mode.

P8: In a Young's interference experiment, the two slits are separated by 0.150 mm and the incident light includes two wavelengths: λ_1 = 540 nm (green) and λ_2 = 450 nm (blue). The overlapping interference patterns are observed on a screen 1.40 m from the slits. Calculate the minimum distance from the center of the screen to a point where a bright fringe of the green light coincides with a bright fringe of the blue light.

Let the m1-th order bright fringe of green light coincide with the m2-th order bright fringe of blue light.



Let the two fringes coincide at a distance y from the centre.

For green light,

$$y = \frac{m_1 \lambda_1 L}{d}$$

For blue light,

$$y = \frac{m_2 \lambda_2 L}{d}$$

Hence,

$$\frac{m_2}{m_1} = \frac{\lambda_1}{\lambda_2} = \frac{540\ nm}{450\ nm}$$

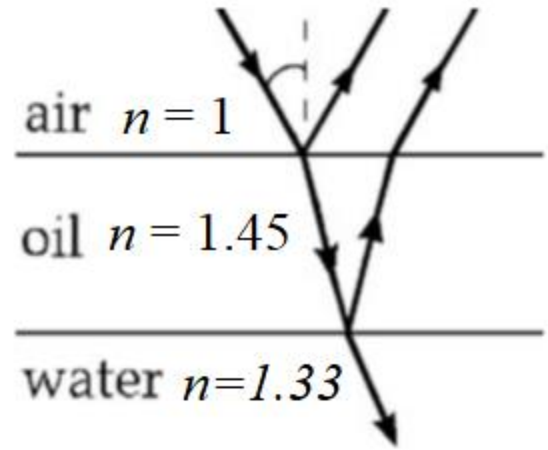
Thus smallest integers satisfying the equation are m_1 = 5 and m_2 = 6.

Using one of the previous equations

$$y = \frac{m_1 \lambda_1 L}{d} = \frac{5 \times 540 \times 10^{-9} \times 1.4}{0.150 \times 10^{-3}} = 2.52\ cm$$

P14 : An oil film (n = 1.45) floating on water is illuminated by white light at normal incidence. The film is 280 nm thick. Find (a) the wavelength and color of the light in the visible spectrum most strongly reflected and (b) the wavelength and color of the light in the spectrum most strongly transmitted. Explain your reasoning.

Ans:



a) For strongly reflected light use constructive Interference condition,

$$2nt = \left(m + \frac{1}{2}\right)\lambda$$

$$\lambda_m = \frac{2nt}{m+1/2} = \frac{2(1.45)(280\text{ nm})}{m+1/2} = \frac{812\text{ nm}}{m+1/2}$$

When m = 0, λ = 1620 nm (infrared) ; m = 1, λ = 541 nm (green); m = 2, λ = 325 nm (ultraviolet)

Both infrared and ultraviolet light are invisible to the human eye, so the dominant color in reflected light is green.

b) Strongly transmitted light is minimally reflected. So use destructive interference condition,

$$2nt = m\lambda$$

$$\lambda_m = \frac{2nt}{m} = \frac{812\ \text{nm}}{m}$$

For $m = 1$, $\lambda = 812$ nm (near infrared) ;

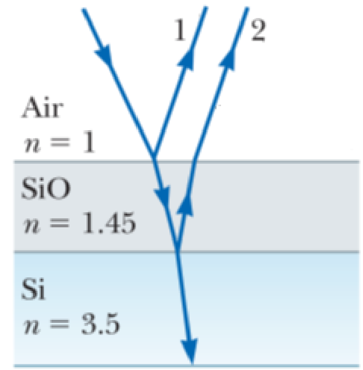
$m = 2$, $\lambda = 406$ nm (violet);

$m = 3$, $\lambda = 271$ nm (ultraviolet).

Of these, the only wavelength visible to the human eye is 406 nm. Thus, the dominant color in the transmitted light is violet.

P12 : Solar cells devices that generate electricity when exposed to sunlight are often coated with a transparent, thin film of silicon monoxide (SiO, *n* = 1.45) to minimize reflective losses from the surface. Suppose a silicon solar cell (*n* = 3.5) is coated with a thin film of silicon monoxide for this purpose. Determine the minimum film thickness that produces the least reflection at a wavelength of 550 nm, near the center of the visible spectrum.

Ans:



Condition for destructive interference is net path difference

$$\delta = \left(m + \frac{1}{2}\right)\lambda$$

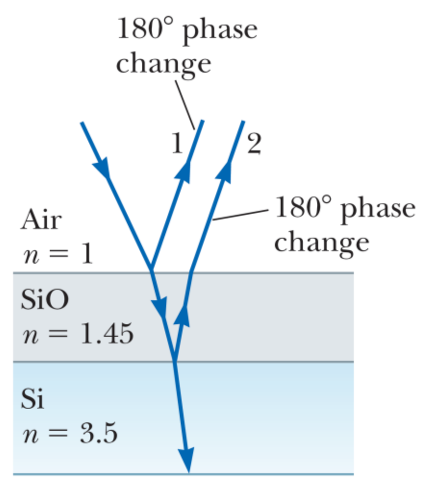
For Ray 1 there is a $\lambda/2$ path difference due to reflection.
Ray 2 travels an extra path $= 2nt$ inside the thin film (SiO) and also has a $\lambda/2$ path difference due to reflection.
Hence the net path difference between rays 1&2 is

$$\delta = 2nt + \frac{\lambda}{2} - \frac{\lambda}{2} = 2nt$$

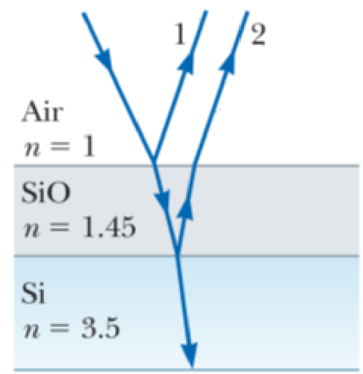
Hence for destructive interference of rays 1 & 2

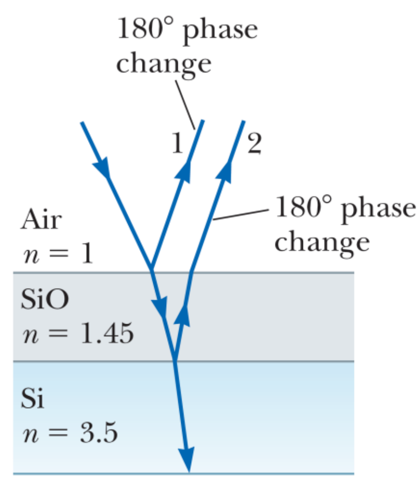
$$2nt = \left(m + \frac{1}{2}\right)\lambda$$

Smallest thickness corresponds to $m = 0$. Putting $m = 0$ in the above equation and rearranging we get



$$t = \frac{\lambda}{4n} = \frac{550 \times 10^{-9}}{4 \times 1.45} = 98.2\ nm$$





P18 : In a Newton's rings experiment, a plano-convex glass (n = 1.52) lens having radius r = 5.00 cm is placed on a flat plate as shown in Figure. When light of wavelength 650 nm is incident normally, 55 bright rings are observed, with the last one precisely on the edge of the lens. (a) What is the radius of curvature (R) of the convex surface of the lens? (b) What is the focal length of the lens?

Ans:

a)

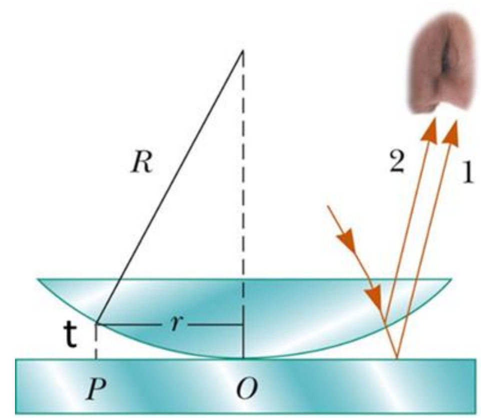
For bright rings $r = \sqrt{\left(m + \frac{1}{2}\right) R\lambda}$

For the 55th bright ring, m = 54

As per the statement given above, the radius of the 55th bright fringe (or fringe of order m = 54) is 5 cm.

From the above equation $R = \frac{r^2}{\left(m+\frac{1}{2}\right)\lambda} = \frac{\left(5\times10^{-2}\right)^2}{\left(54+\frac{1}{2}\right)\times650\times10^{-9}}$

R = 70.6 m



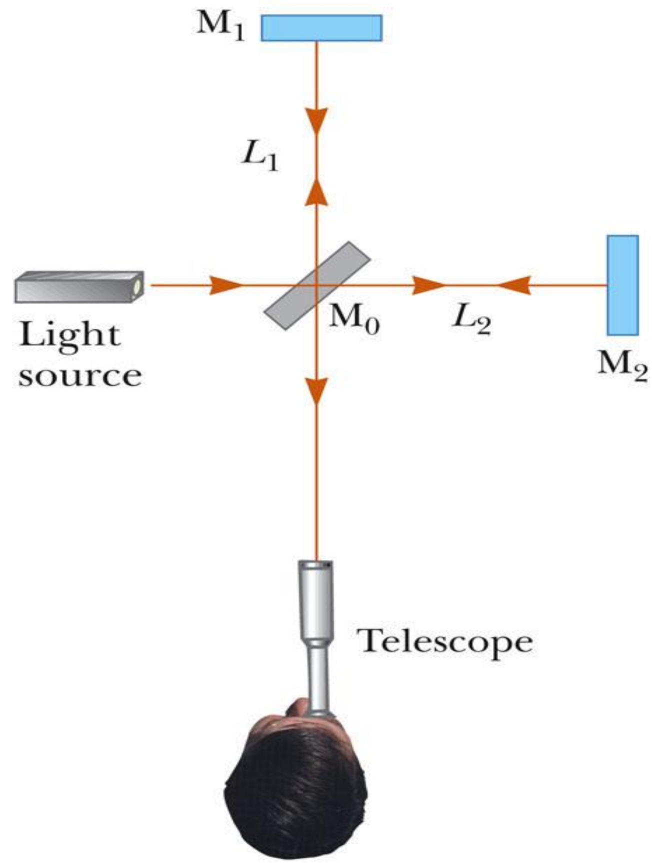
b) Using lens makers formula for plano-convex lens we get,

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = 0.520\left(\frac{1}{\infty} - \frac{1}{-70.6\text{ m}}\right)$$

So, f = 136 m

P 20 : Mirror M_1 in Figure is moved through a displacement ΔL. During this displacement, 250 fringe reversals (formation of successive dark or bright bands) are counted. The light being used has a wavelength of 632.8 nm. Calculate the displacement ΔL.

Ans:

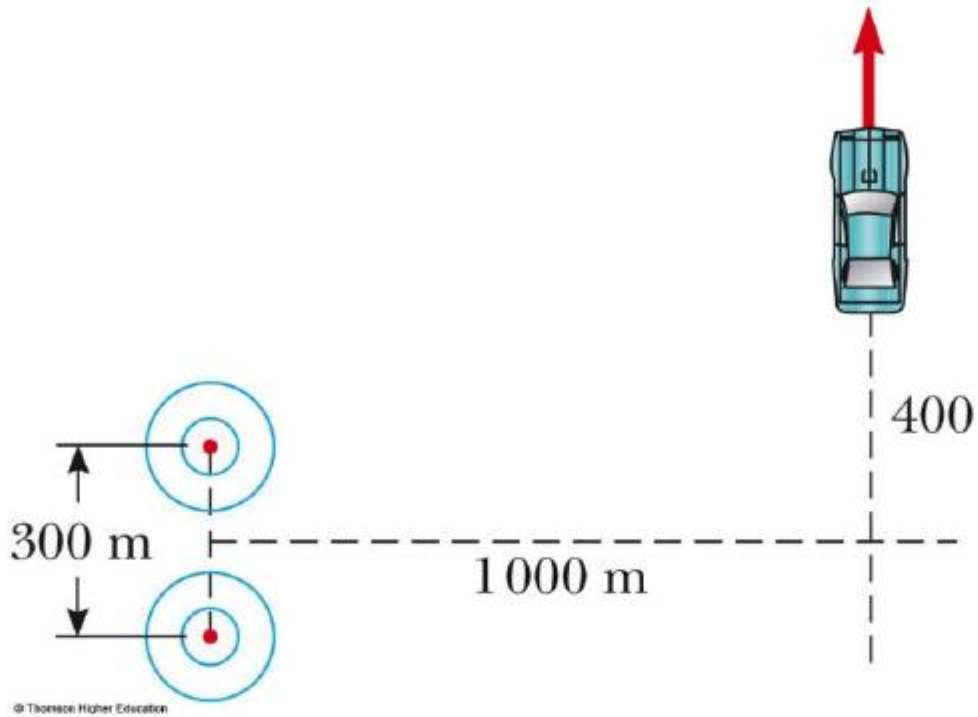


$$\mathbf{m} = \frac{\boldsymbol{\Delta L}}{(\boldsymbol{\lambda}/\mathbf{4})}$$

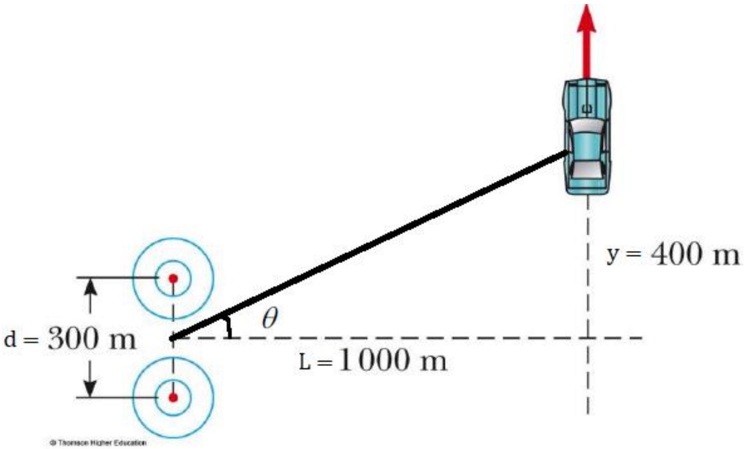
m = 250, $\lambda = 632.8\ nm$

$\Delta L = m(\lambda/4) = 39.6\ \mu m$

Two radio antennas separated by $d = 300$ m as shown in figure simultaneously broadcast identical signals at the same wavelength. A car travels due north along a straight line at position $x = 1000$ m from the center point between the antennas, and its radio receives the signals. (a) If the car is at the position of the second maximum after that at point O when it has traveled a distance $y = 400$ m northward, what is the wavelength of the signals? (b) How much farther must the car travel from this position to encounter the next minimum in reception? *Note:* Do not use the small-angle approximation in this problem.



For maximum: $d\sin\theta = m\lambda$
Given m = 2
From the figure,



At the $m = 2$ maximum,

$$\tan\theta = \frac{400\text{ m}}{1\,000\text{ m}} = 0.400 \rightarrow \theta = 21.8°$$

So $\lambda = \dfrac{d\sin\theta}{m} = \dfrac{(300\text{ m})\sin 21.8°}{2} = \boxed{55.7\text{ m}}$.

The next minimum encountered is the $m = 2$ minimum, and at that point,

$$d\sin\theta = \left(m + \frac{1}{2}\right)\lambda$$

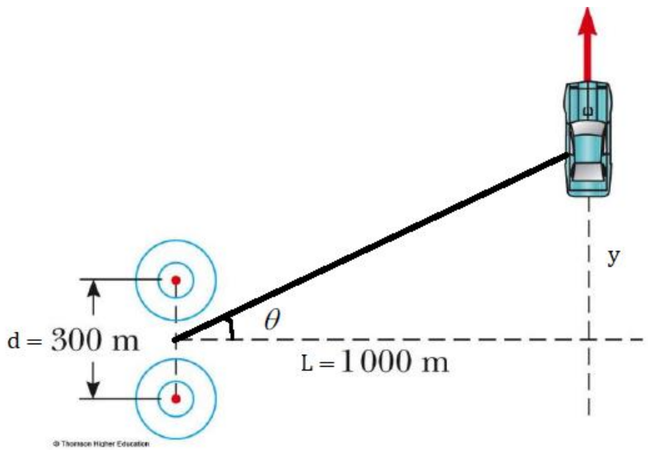
which becomes $d\sin\theta = \dfrac{5}{2}\lambda$,

or $\quad \sin\theta = \dfrac{5}{2}\dfrac{\lambda}{d} = \dfrac{5}{2}\left(\dfrac{55.7\text{ m}}{300\text{ m}}\right) = 0.464 \rightarrow \theta = 27.7°$,

so $\quad y = (1\,000\text{ m})\tan 27.7° = 524\text{ m}.$

Therefore, the car must travel an additional

$$524\text{ m} - 400\text{ m} = \boxed{124\text{ m}}$$



Show that the two waves with wave functions $E_1 = 6.00\sin(100\pi t)$ and $E_2 = 8.00\sin(100\pi t + \pi/2)$ add to give a wave with the wave function $E_R \sin(100\pi t + \phi)$. Find the required values for E_R and ϕ.

$$E_{\text{res}} = E_1 + E_2$$

$$E_1 + E_2 = 6.00 \sin(100\pi t) + 8.00 \sin(100\pi t + \pi/2)$$

Sin (A+B) = Sin A Cos B + Cos A Sin B

$$= 6.00 \sin(100\pi t) + \left[8.00 \sin(100\pi t)\cos(\pi/2) + 8.00 \cos(100\pi t)\sin(\pi/2)\right]$$

$$E_1 + E_2 = 6.00 \sin(100\pi t) + 8.00 \cos(100\pi t)$$

$$E_{\text{res}} = E_R \sin(100\pi t + \phi) = E_R \sin(100\pi t)\cos\phi + E_R \cos(100\pi t)\sin\phi$$

The equation $E_1 + E_2 = E_R \sin(100\pi t + \phi)$ is satisfied if we require

$$600 = E_R^{-} \cos\phi \quad \text{and} \quad 8.00 = E_R \sin\phi$$

or $$(6.00)^2 + (8.00)^2 = E_R{}^2(\cos^2\phi + \sin^2\phi) \rightarrow \boxed{E_R = 10.0}$$

and $$\tan\phi = \sin\phi/\cos\phi = 8.00/6.00 = 1.33 \rightarrow \boxed{\phi = 53.1^\circ}$$