Sol. Step 1.

$$e_{ss} = \frac{1}{15}$$

$$\frac{1}{K_v} = \frac{1}{15} \Rightarrow K_v = 15$$

$$K_v = \lim_{s \to 0} s \cdot G(s) \cdot H(s).$$

$$15 = \lim_{s \to 0} \cancel{s} \cdot \frac{K}{\cancel{s}(s+1)}$$

$$15 = \frac{K}{1} \Rightarrow K = 15$$

Step 2

$$G(s) = \frac{15}{s(s+1)}$$

It is in the time constant form.

$P_m = 13^\circ$

$$\omega_{c_1} = 1.$$

$$20 \log 15 = 23.52$$

ω	$\frac{1}{j\omega}$	$-\tan^{-1}\omega$	total.
0.1	-90°	-5.71	-95.71°
0.4	-90°	-21.80	-111.80°
1	-90°	-45	-135°
2	-90°	-63.43	-153.43°
3	-90°	-71.56	-161.56°
4	-90°	-75.96	-165.96°
10	-90°	-84.28	-174.28°
20	-90°	-87.13	-177.13°
40	-90°	-88.56	-178.56°

Step 3

The system requires a PM of 45°. But the available PM is 13°. So, lead compensation should be employed to impove PM.

Step 4

Find ϕ_m

$$\gamma_d \geq 45^\circ$$

Add. Phase lead, $\epsilon = 5^\circ$

Max. lead angle $\phi_m = \gamma_d - \gamma + \epsilon$

$$= 45 - 13 + 5 = 37^\circ$$

Step 5

Determ TF of lead compen

$$\alpha = \frac{1 - \sin\phi_m}{1 + \sin\phi_m} = \frac{1 - \sin 37^\circ}{1 + \sin 37^\circ} = 0.2486 \approx 0.25$$

The dB mag corresponding to.

$$\omega_m = -20 \log \frac{1}{\sqrt{\alpha}} = -6.02 \approx -6dB$$

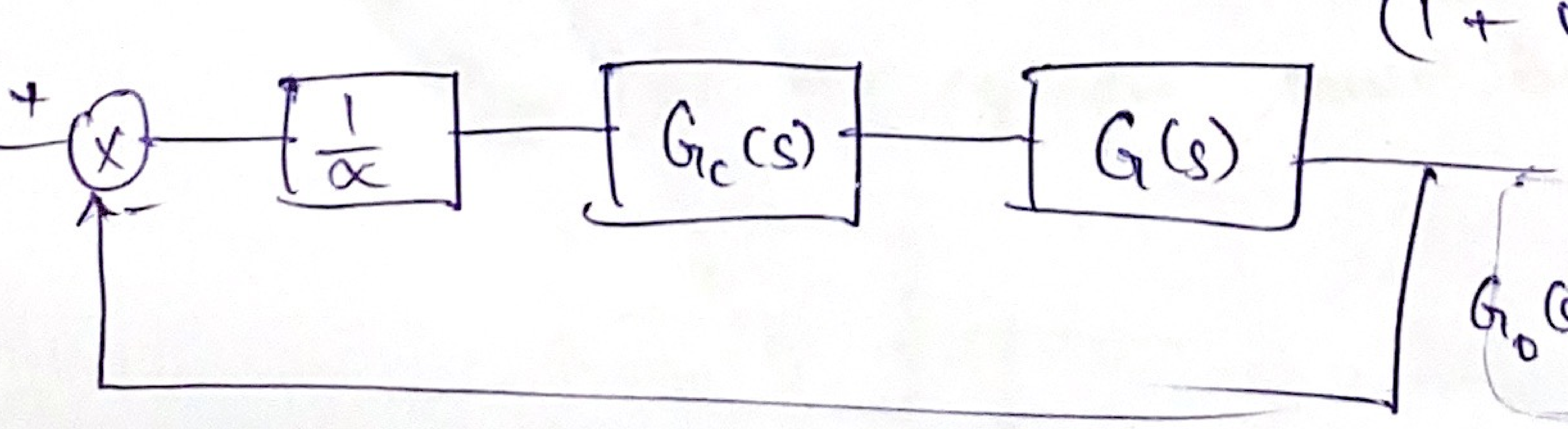
ω_m value corresponding to $-6dB$ is 5.6 rad/s.

$$T = \frac{1}{\omega_m \sqrt{\alpha}} = \frac{1}{5.6\sqrt{0.25}} = 0.357 \approx 0.36$$

$$G_c(s) = \frac{s + Z_c}{s + P_c} = \frac{s + \frac{1}{T}}{s + \frac{1}{\alpha T}} = \alpha \frac{1 + sT}{1 + \alpha sT}$$

$$= \frac{0.25\ (1 + 0.36s)}{1 + (0.25)(0.36)s}$$

$$G_c(s) = \frac{0.25\ (1 + 0.36s)}{(1 + 0.09s)}$$



$$G_o(s) = \frac{(1 + 0.36s)\ 15}{s(1 + 0.09s)(1 + s)}$$

Q. Design a lead compensator with open loop tf

$\frac{k}{s(s+1)(s+5)}$ to satisfy following specs.

i) $K_v \geq 50$

ii) $PM \geq 20^\circ$

Sol.

$K_v = \lim_{s \to 0} s \cdot G(s) \cdot H(s)$

$50 = \lim_{s \to 0} \cancel{s} \cdot \frac{k}{\cancel{s}(s+1)(s+5)}$

$K = 250$

$G(s) = \frac{250}{s(s+1)(s+5)} = \frac{\overset{50}{\cancel{250}}}{s(1+s)(\cancel{5})(1+\frac{s}{5})}$

$= \frac{50}{s(1+s)(1+0.2s)}$

$\omega_{c_1} = 1$

$\omega_{c_2} = 5$

$20 \log 50 = 33.97 \approx 34.$

$-90 - \tan^{-1} 6.8 -$

		$-\tan^{-1}\omega$	$-\tan^{-1}(0.2\omega)$	total
0.1	-90°			-97.70
1	-90°			-149.03
2	-90°			-179.56
3	-90°			-198.42
4				-220.03
5	-90°			
6	-90°			-226.83
8	-90°			-236.20

$PM = -44^\circ$

$\phi_m = 20 - (-44^\circ) + 5^\circ$

$= 69^\circ$

Max lead value for a 1 lead comp is 60°.

$\alpha = \frac{1 - \sin 69^\circ}{1 + \sin 69^\circ} = 0.28$

So, here 2 lead com is used each providing $\frac{69}{2}$ lead $= 34.5$

$\omega_n = -20 \log \frac{1}{\sqrt{\alpha}} = -5.5\,dB$

$\omega_n = 7.8$ rad/sad.

$$T = \frac{1}{\omega_n \sqrt{\alpha}} = 0.24.$$

$$G_c(s) = \frac{\alpha^2 (1+sT)^2}{(1+\alpha sT)^2}$$

$$= \frac{(0.28)^2 (1+s(0.24))^2}{(1+0.28(s)(0.24)s)^2}$$

$$= 0.0784 \frac{(1+0.24s)^2}{(1+0.067s)^2}$$

$$G_o(s) = \frac{250(1+0.24s)^2}{s(1+0.067s)^2(s+1)(s+5)}$$

$$\frac{4(1+0.24s)^2}{s(s+1)(1+0.2s)(1+0.067s)^2}$$

3/6/22

Lag-Lead compensator

A compensator having the characteristics of lag lead network is called lag lead compensator.

In this when sinusoidal signal is applied, both phase lag & lead occurs in the o/p but in diff freq regions. Phase lag occurs in low freq region and plase lead occurs in high freq region. i.e phase angle varies from lag to lead, as the freq increased from 0 to ∞.

A lead compe basically increases bandwidth & speeds up the response and decreases & max overshoot in the step response.