II SEM - Engg. Mathematics II
MAT -1251 (I sessional)

Time: 1 Hr. **Date: 14.02.2020** **Time: 12.00 PM-1.00 PM** **Max. Marks: 15**

Q1. The equation of the right circular cone whose axis is $x = y = z$, vertex is the origin and the semi vertical angle is 45^0 is given by (0.5)

1. $x^2 + y^2 + z^2 = 0$
2. **$(x^2 + y^2 + z^2) = 2(x + y + z)^2$
3. $x^2 + y^2 + z^2 + xy + yz + zx = 0$
4. $2(x^2 + y^2 + z^2) = 3(x + y + z)^2$

Q2. The minimum value of $2x + 3y$, subject to the condition $x^2y^3 = 32,\ \ x > 0,\ y > 0$ is (0.5)

1. **10
2. 14
3. 0
4. 7

Q3. If $u = \log_e\left(\frac{x^2}{y}\right)$, then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} =$ (0.5)

1. **1
2. 0
3. u
4. $2u$

Q4. If $z = x^y$, then $\frac{\partial z}{\partial y}$ is (0.5)

1. yx^{y-1}
2. $x^y \log y$
3. ** $x^y \log x$
4. 0

Q5. The Maclaurin's series expansion of y = cos x up to two non-constant terms is (0.5)

1. $1 + \frac{x^2}{2!} + \frac{x^4}{4!}$
2. $1 - x + \frac{x^3}{3!}$

3. **$1-\frac{x^2}{2!}+\frac{x^4}{4!}$

4. $1+x+\frac{x^3}{3!}$

Q6. The value of $\lim_{x\to 1} x^{(1-x)^{-1}}$ is (0.5)

1. 1
2. -1
3. ** $\frac{1}{e}$
4. e

Q7. The equation of the right circular cylinder whose axis is Z- axis and radius 2 is (0.5)

1. $y^2+z^2=4$
2. ** $x^2+\ y^2=4$
3. $x^2+\ y^2+z^2=4$
4. $x^2+z^2=4$

Q8. The saddle point for the function x^3-y^2-3x is (0.5)

1. (0,0)
2. (1,1)
3. (0,1)
4. **(1,0)

Q9. The percentage error in the area of an ellipse when an error of 1% is made by measuring the major and minor axis is (0.5)

1. 0.02%
2. 2% **
3. 1%
4. 0.2%

Q10. If the functions $f(x)=\log_e x$ and $g(x)=\frac{1}{x}$ satisfies all the conditions of Cauchy's mean value theorem in $[1,e]$ then the value of c is (0.5)

1. e
2. $\frac{e^2}{e-1}$

3. $\frac{e}{e+1}$

4. ** $\frac{e}{e-1}$

Type: DES

Q11. By using Lagrange's method of undetermined multipliers, find the maximum and minimum distances of the point (3, 4, 12) from sphere $\boldsymbol{x^2+y^2+z^2=1}$. (2)

Let $f(x,y,z)=D^2=(x-3)^2+(y-4)^2+(z-12)^2$. $\emptyset(x,y,z): x^2+y^2+z^2-1$

$$f_x+\lambda\,\emptyset_x=0 \qquad 2(x-3)+\lambda\,(2x)=0$$

$$f_y+\lambda\,\emptyset_y=0 \quad => \quad 2(y-4)+\lambda\,(2y)=0 \quad => \quad -\lambda=\frac{x-3}{x}=\frac{y-4}{y}=\frac{z-12}{z}$$

$$f_z+\lambda\,\emptyset_z=0 \qquad 2(z-12)+\lambda\,(2z)=0 \qquad (1M)$$

$$x=\frac{3}{1+\lambda}\ ,\quad y=\frac{4}{1+\lambda}\ ,\quad z=\frac{12}{1+\lambda}$$

$$x^2+y^2+z^2=1 \quad => \quad \lambda=12 \quad \text{or} \quad \lambda=-14$$

$$(x,\ y,\ z)=\left(\frac{3}{13},\frac{4}{13},\frac{12}{13}\right) \text{ or } \left(\frac{-3}{13},\frac{-4}{13},\frac{-12}{13}\right) \qquad (0.5M)$$

Minimum distance =12 and Maximum distance =14. (0.5M)

Q12. Find the equation of the sphere having the circle

$x^2+\ y^2+z^2+10y-4z-8=0\,,\ \ x+y+z=3$, as a great circle. (2)

Equation of sphere passing through the given circle is

$(x^2+y^2+z^2+10y-4z-8)+k(x+y+z-3)=0.$ (0.5M)

The given circle is a great circle. Therefore centre of the sphere lies on the plane $x+y+z-3$. Centre of the sphere is $\left(\frac{-k}{2},-5-\frac{k}{2},\ 2-\frac{k}{2}\right)$.

Centre lies on the plane $\frac{-k}{2}-5-\frac{k}{2}+\ 2-\frac{k}{2}=3\ ,\quad k=-4.$ (1M)

Equation of the sphere is $x^2+y^2+z^2-4x+6y-8z+4=0.$ (0.5M)

Q13. Expand $f(x,y)=log_e(x+e^y)$ by Taylor's series in powers of $x-1$ and y such that it includes all terms up to second degree. (2)

$$f_x=\frac{1}{x+e^y} \qquad (f_x)_{(1,0)}=\frac{1}{2}$$

$$f_y=\frac{e^y}{x+e^y} \qquad (f_y)_{(1,0)}=\frac{1}{2}$$

$$f_{xx} = \frac{-1}{(x+e^y)^2} \qquad (f_{xx})_{(1,0)} = \frac{-1}{4}$$
$$f_{yy} = \frac{-xe^y}{(x+e^y)^2} \qquad (f_{yy})_{(1,0)} = \frac{1}{4}$$
$$f_{xy} = \frac{-e^y}{(x+e^y)^2} \qquad (f_{xy})_{(1,0)} = \frac{-1}{4} \qquad (1.5M)$$

$$f(x,y) = log_e 2 + \frac{1}{2}[(x-1)+y] + \frac{1}{8}[-(x-1)^2 + y^2 - 2(x-1)y] + \cdots \qquad (0.5M)$$

Q14. If $z = f(x,y)$ where $x = e^u \cos v$ and $y = e^u \sin v$, show that

$y\frac{\partial z}{\partial u} + x\frac{\partial z}{\partial v} = e^{2u}\frac{\partial z}{\partial y}$ (2).

$$\frac{\partial z}{\partial u} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial u} = \frac{\partial z}{\partial x}e^u \cos v + \frac{\partial z}{\partial y}e^u \sin v = x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} \qquad (0.5M)$$
$$\frac{\partial z}{\partial v} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial v} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial v} = \frac{\partial z}{\partial x}(-e^u \sin v) + \frac{\partial z}{\partial y}e^u \cos v = -y\frac{\partial z}{\partial x} + x\frac{\partial z}{\partial y} \qquad (0.5M)$$
$$y\frac{\partial z}{\partial u} + x\frac{\partial z}{\partial v} = (x^2+y^2)\frac{\partial z}{\partial y} = e^{2u}(cos^2 v + sin^2 v)\frac{\partial z}{\partial y} = e^{2u}\frac{\partial z}{\partial y}. \qquad (1M)$$

Q15. In the formula $w = \left(\frac{x^3}{y}\right)^{0.5}$; x is subjected to an increase of 2%. Calculate approximately the percentage change needed in y to ensure that w remains unchanged. (2)

log w = 0.5(3log x-log y) (0.5M)

$\frac{\delta w}{w} = 0.5(\frac{3\delta x}{x} - \frac{\delta y}{y})$

$0 = 0.5(3 * 0.02 - \frac{\delta y}{y})$ (1M)

$\frac{\delta y}{y} = 0.06$

The percentage change needed in y is 6%. (0.5M)