Basic Electrical Technology

[ELE 1051]

SINGLE PHASE AC CIRCUITS

Topics covered…

- Average value of an alternating waveform
- RMS value of an alternating waveform
- Representing AC
- R, L, C circuit response with AC supply
- Power associated with a pure R, L, C

Average value of Sinusoidal Alternating Current

Definition: *"It is that steady current which transfers the same amount of charge to any circuit during the given interval of time, as is transferred by the alternating current to the same circuit during the same time"*

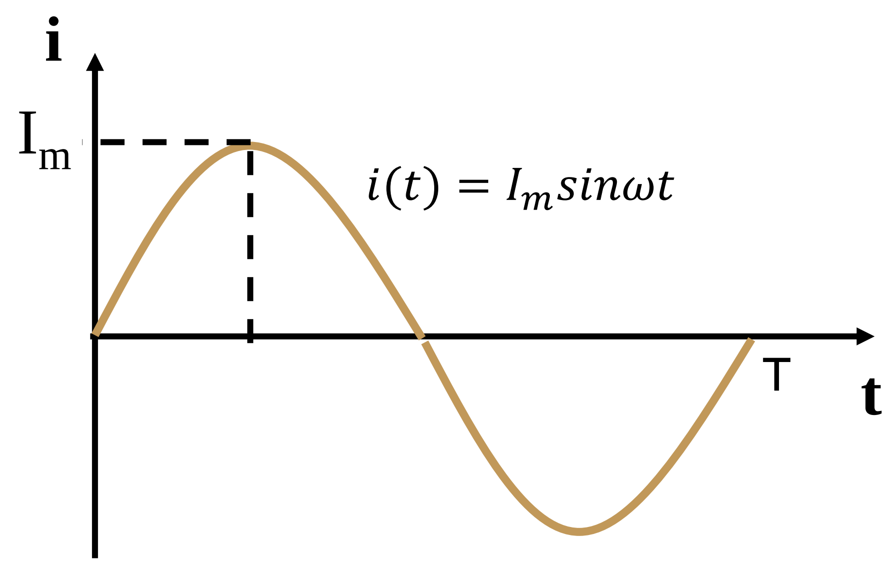
For a periodic function f(t) with period T,

$$\boldsymbol{F_{avg} = \frac{1}{T}\int_{0}^{T} f(t)dt}$$

For sinusoidal signal,

$$I_{avg} = \frac{1}{T/2}\int_{0}^{T/2} I_m \sin \omega t dt$$

$$\boxed{\boldsymbol{I_{avg} = \frac{2I_m}{\pi}}}$$



RMS value of Sinusoidal Alternating Current

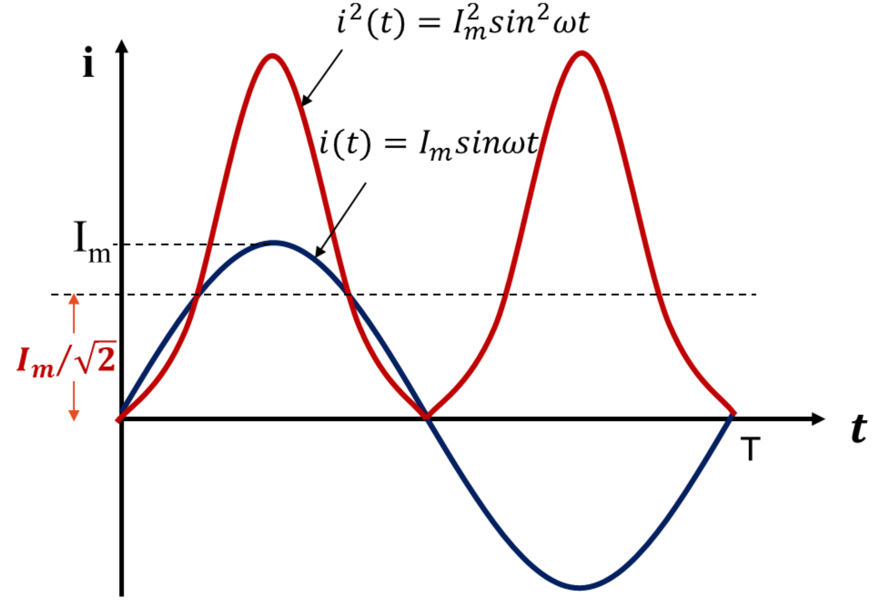
Definition: *"It is that value of direct current which when flowing through a circuit produces the same amount of heat for a given interval of time as that of the alternating current flowing through the same circuit during the same time"*

For a periodic function f(t) with period T,

$$\boldsymbol{F_{rms} = \sqrt{\frac{1}{T}\int_0^T f^2(t)dt}}$$

$$I_{rms} = \sqrt{\frac{1}{T}\int_0^T I_m^2 sin^2\omega t\ dt}$$

$$\boxed{\boldsymbol{I_{RMS} = \frac{I_m}{\sqrt{2}}}}$$



Form Factor & Peak Factor

$$\boldsymbol{Form\ Factor = \frac{RMS\ Value}{Average\ Value} = 1.11\ for\ sinusoidal}$$

$$\boldsymbol{Peak\ Factor = \frac{Maximum\ Value}{RMS\ Value} = \sqrt{2}\ for\ sinusoidal}$$

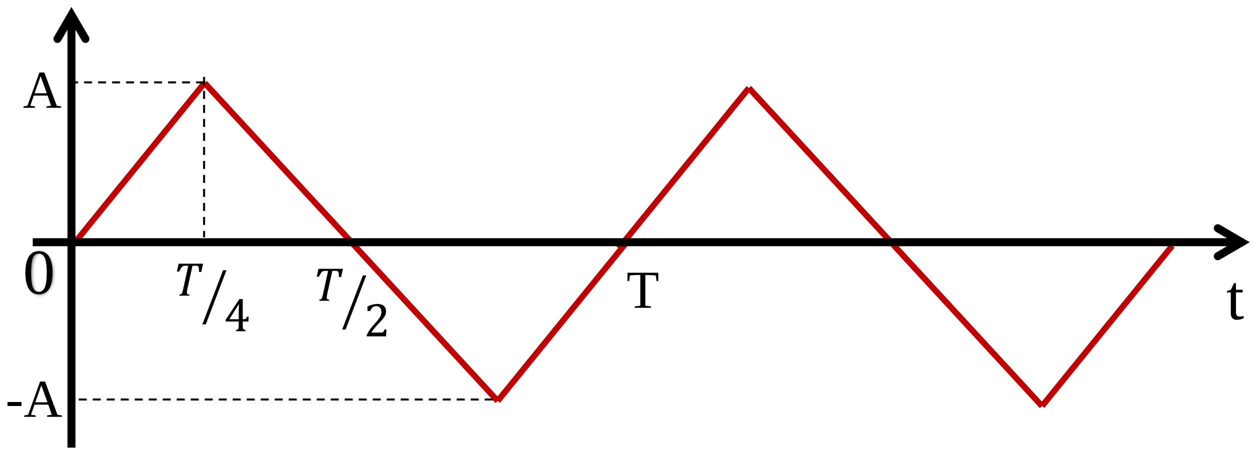
Exercise 1

If an alternating voltage has the equation **$v(t) = 141.4 \sin 377t$**, calculate

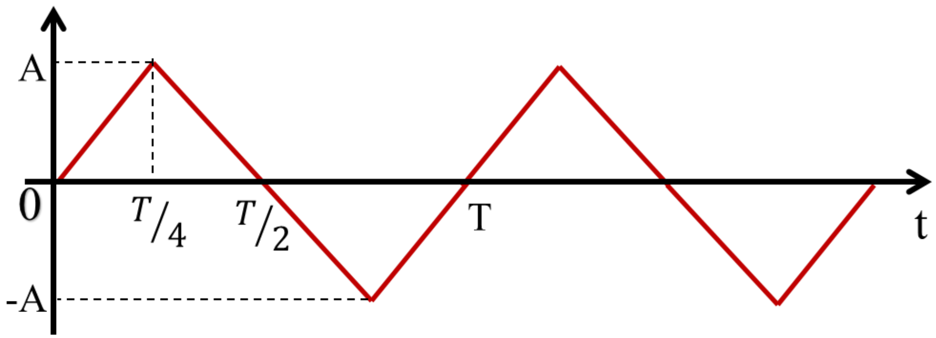
a. Maximum voltage value
b. RMS value of the voltage
c. Frequency
d. The instantaneous voltage when t = 3ms

Exercise 2

Find the Average value and RMS value of the given non-sinusoidal waveform



Solution:



Average Value

$$I_{avg} = \frac{1}{T/4}\int_0^{T/4} f(t).dt$$

$$I_{avg} = \frac{4}{T}\int_0^{T/4} \frac{4At}{T}.dt$$

$$I_{avg} = \frac{4}{T} \times \frac{4A}{T} \times \left[\frac{t^2}{2}\right]_0^{\frac{T}{4}}$$

$$I_{avg} = \frac{8A}{T^2} \times \left[\frac{T^2}{16}\right]$$

$$\boldsymbol{I_{avg} = \frac{A}{2}}$$

RMS Value

$$I_{rms}^2 = \frac{1}{T/4}\int_0^{T/4} f^2(t).dt$$

$$I_{rms}^2 = \frac{4}{T}\int_0^{T/4} \frac{16A^2t^2}{T^2}.dt$$

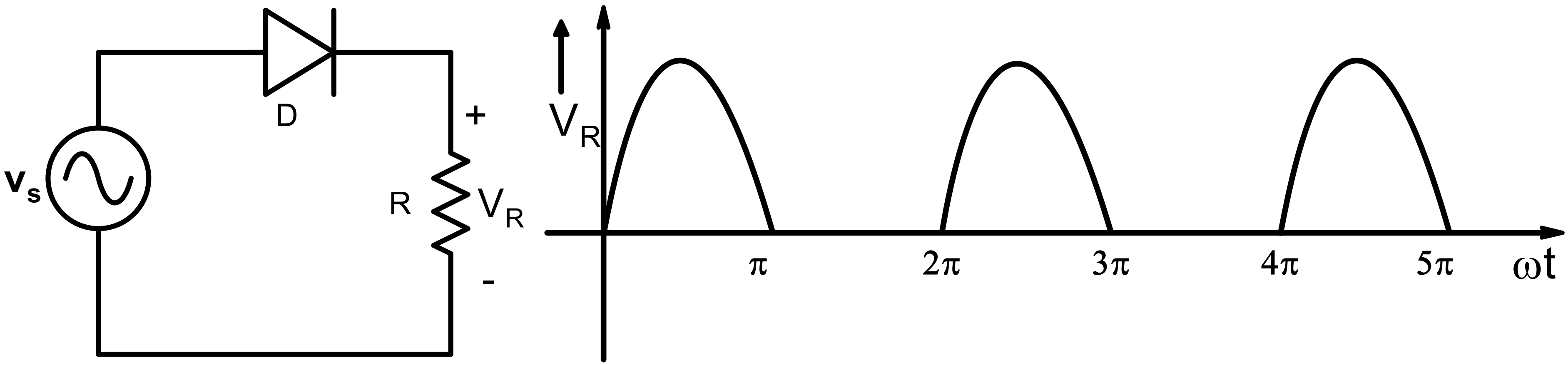
$$I_{rms}^2 = \frac{4}{T} \times \frac{16A^2}{T^2} \times \left[\frac{t^3}{3}\right]_0^{\frac{T}{4}}$$

$$I_{rms}^2 = \frac{4}{T} \times \frac{16A^2}{T^2} \times \frac{1}{3} \times \left[\frac{T^3}{4^3}\right]$$

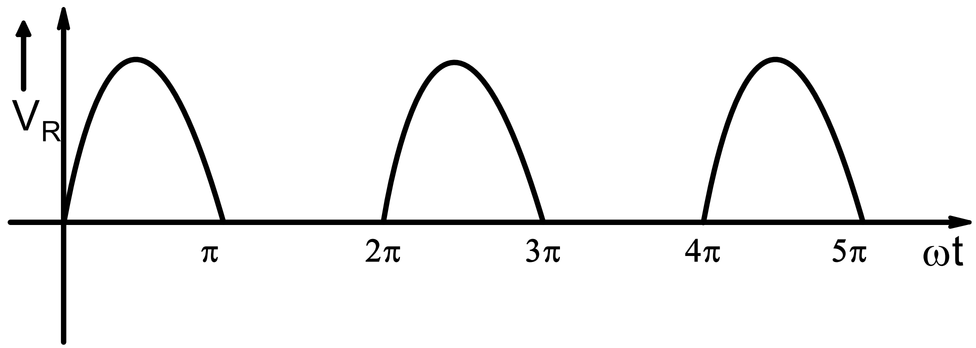
$$\boldsymbol{I_{rms} = \frac{A}{\sqrt{3}}}$$

Exercise 3

For the circuit shown below, sketch the voltage across the resistance, & then find the Average value and RMS value of the same.



Solution:



Average Value

$$V_{avg} = \frac{1}{2\pi}\left[\int_0^{\pi} V_m \sin\omega t\,.d\omega t \;+\; \int_{\pi}^{2\pi} 0.\,d\omega t\right]$$

$$V_{avg} = \frac{V_m}{2\pi}(-\cos\omega t)_0^{\pi}$$

$$V_{avg} = \frac{-V_m}{2\pi}(-1-1)$$

$$\boldsymbol{V_{avg} = \frac{V_m}{\pi}}$$

RMS Value

$$V_{rms}^2 = \frac{1}{2\pi}\left[\int_0^{\pi} V_m^2 sin^2\omega t.\,d\omega t \;+\; \int_{\pi}^{2\pi} 0.\,d\omega t\right]$$

$$V_{rms}^2 = \frac{V_m^2}{2\pi}\left[\int_0^{\pi} \frac{1-\cos 2\omega t}{2}.\,d\omega t\right]$$

$$V_{rms}^2 = \frac{V_m^2}{4\pi}\left[\omega t|_0^{\pi} - \sin 2\omega t|_0^{\pi}\right]$$

$$V_{rms}^2 = \frac{V_m^2}{4\pi}[\pi]$$

$$\boldsymbol{V_{rms} = \frac{V_m}{2}}$$

Representing AC

- Consider three sinusoidal signals $x(t), y(t)$ & $z(t)$ with same frequency

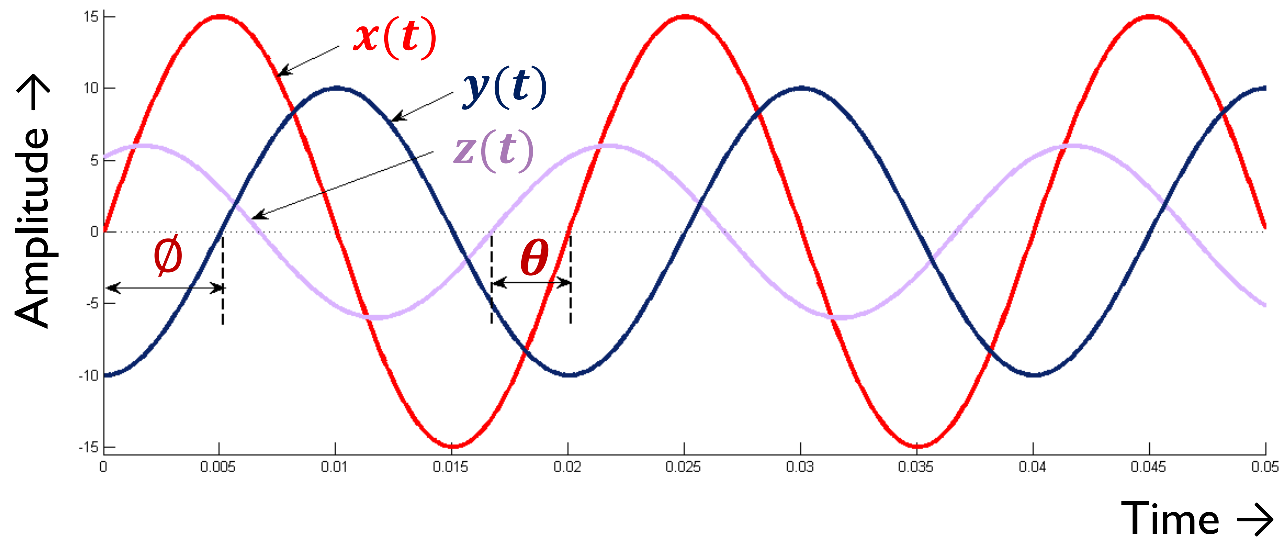
Mathematical Representation

$$x(t) = X_m sin(\omega t)$$

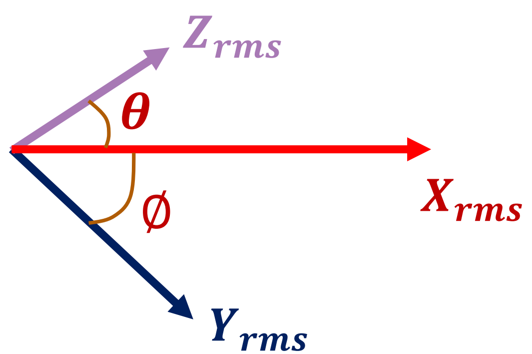
$$y(t) = Y_m \sin(\omega t - \emptyset)$$

$$z(t) = Z_m \sin(\omega t + \theta)$$

Graphical Representation

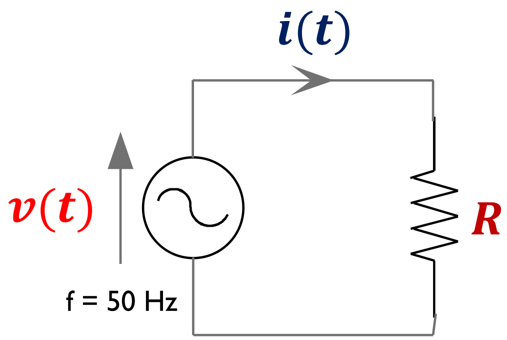


Phasor Representation



- Representing the relationship between sinusoidal signals with same frequency in graphical or mathematical form is tedious
- Phasor representation is often used

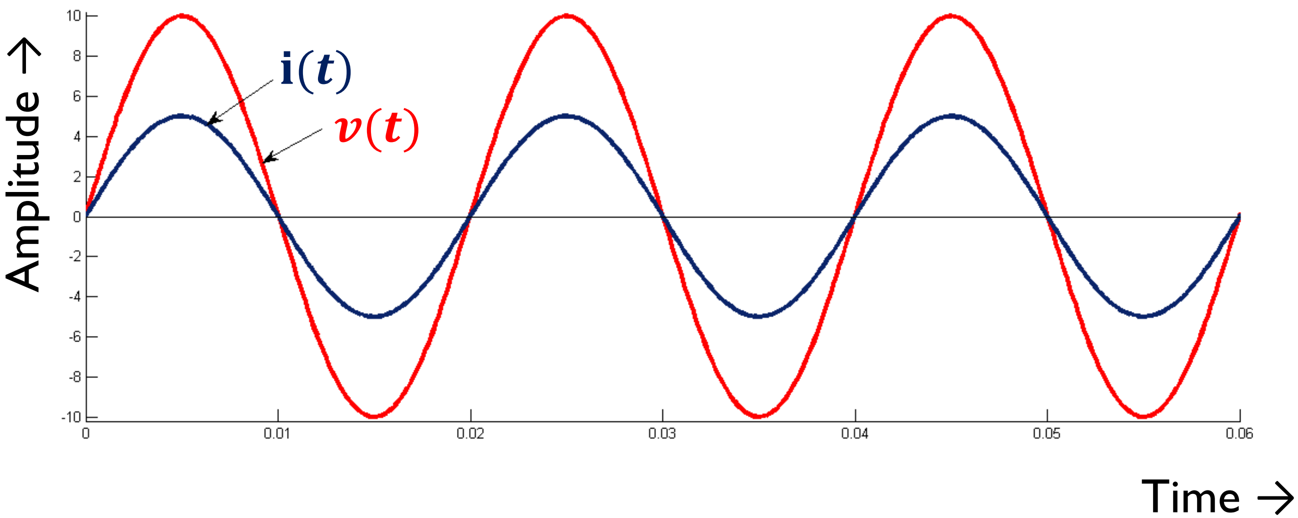
R circuit response with AC supply



$$i(t) = \frac{v(t)}{R}$$

'Current through the resistor is in phase with the voltage across it'

Graphical Representation

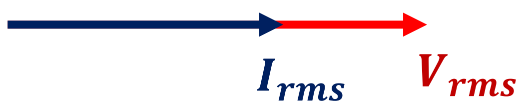


Mathematical Representation

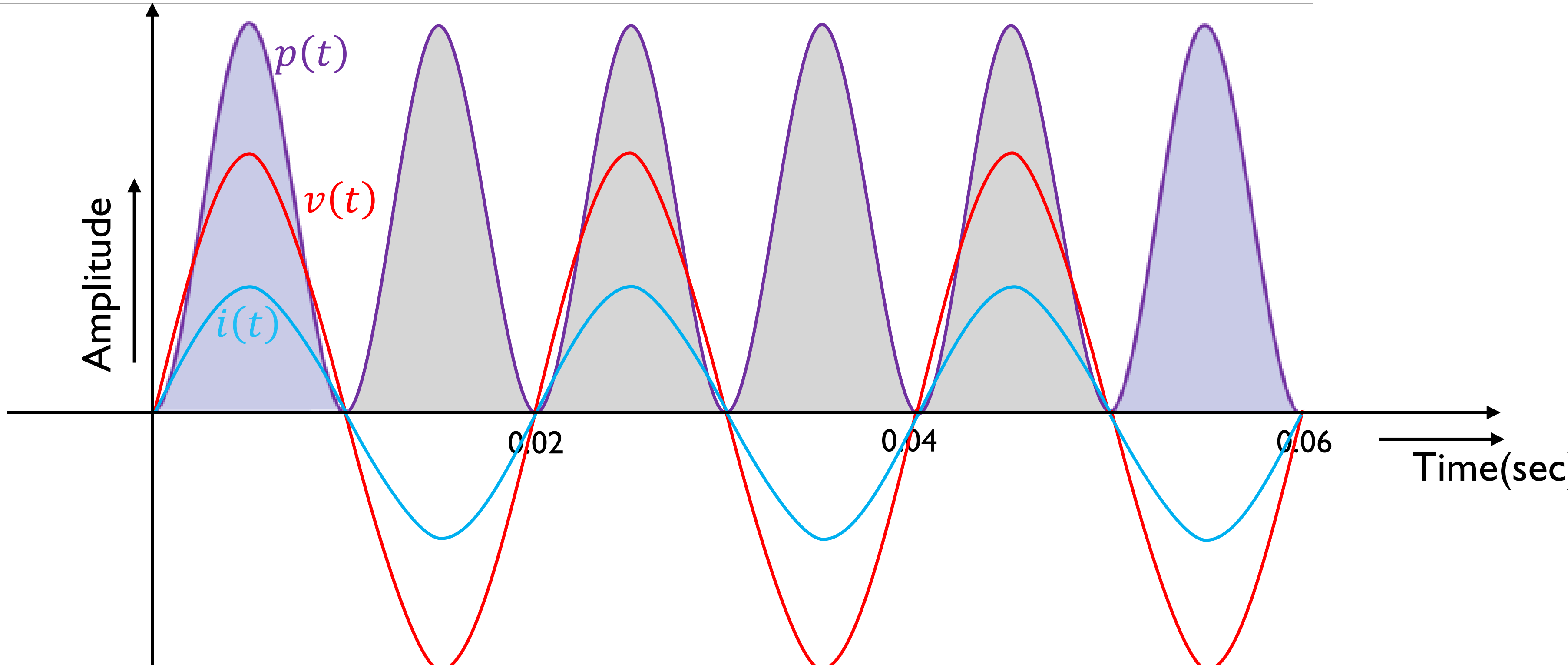
$$v(t) = V_m \sin(\omega t)$$

$$i(t) = I_m \sin(\omega t)$$

Phasor Representation



Power Associated - Pure Resistive Circuit



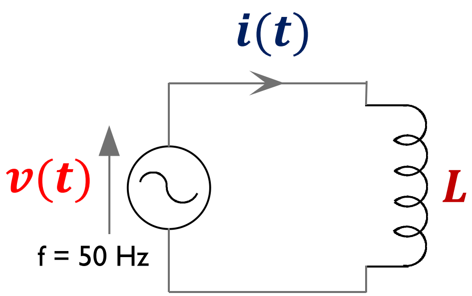
Instantaneous power,

$p(t) = v(t).\,i(t) = V_m I_m sin^2 \omega t$

$$\boldsymbol{Average\ Power, P = \frac{1}{T}\int_0^T p(t)dt}$$

$$\boxed{\boldsymbol{P_{avg} = \frac{V_m I_m}{2} = V_{rms} I_{rms} = \frac{V_{rms}^2}{R} = I_{rms}^2 R}}$$

L circuit response with AC supply



$$i(t) = \frac{1}{L}\int v(t)dt$$

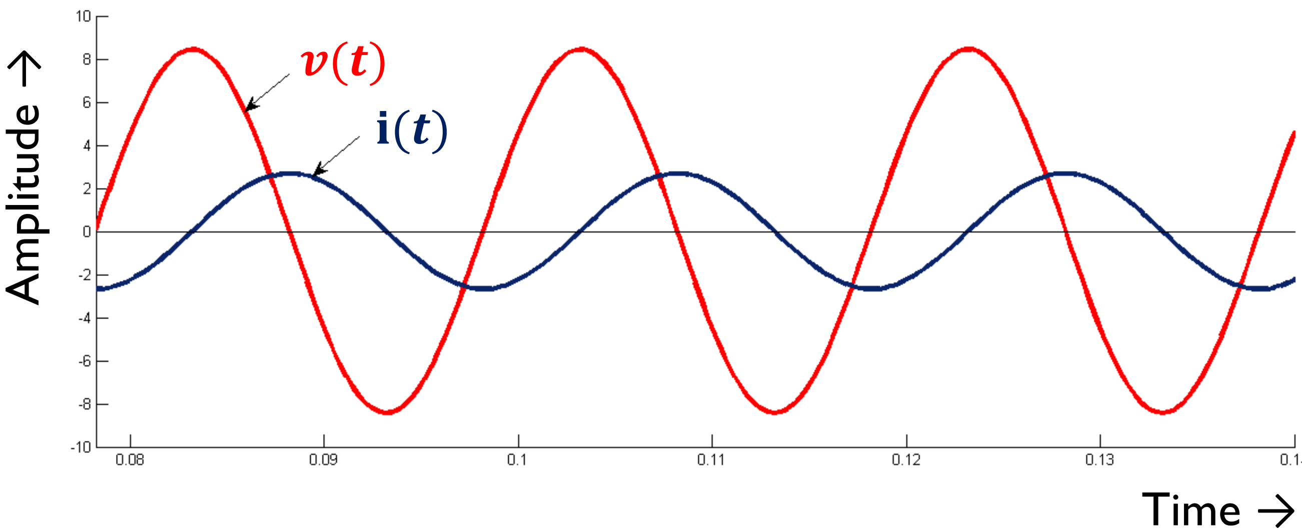
'Current through the inductor lags the voltage across it by 90°'

$$\bar{V} = V\angle 0° \qquad \bar{I} = I\angle -90°$$

$$\frac{\bar{V}}{\bar{I}} = \frac{V\angle 0°}{I\angle -90°} = jX_L \quad where \ \frac{V}{I} = X_L$$

X_L *is called* ***Inductive Reactance***

<u>*Graphical Representation*</u>



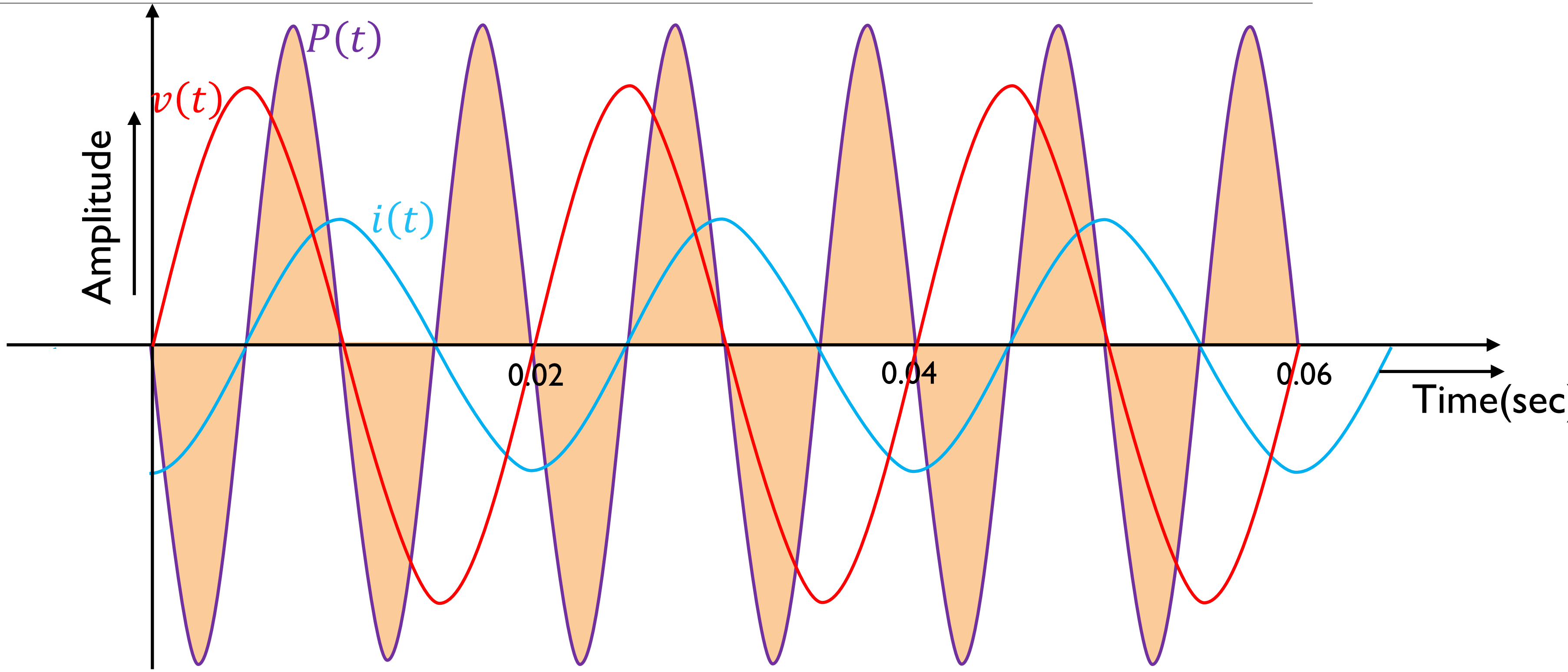
<u>*Mathematical Representation*</u>

$$v(t) = V_m \sin(\omega t)$$

$$i(t) = I_m \sin(\omega t - 90°)$$

<u>*Phasor Representation*</u>

Power Associated – Pure Inductive Circuit



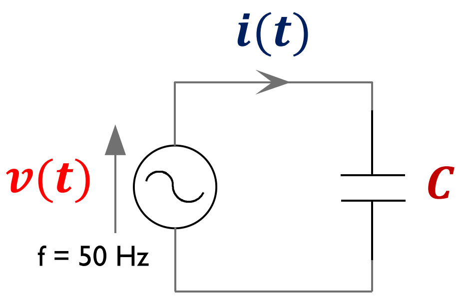
Instantaneous power,

$$p(t) = v(t).\,i(t)$$
$$= V_m I_m \sin \omega t \,.\, \sin(\omega t - 90\,°)$$
$$= -\frac{V_m I_m}{2} \sin 2\omega t$$

$$\boldsymbol{Average\ Power, P = \frac{1}{T}\int_0^T p(t)dt}$$

$$\boxed{\boldsymbol{P_{avg} = 0}}$$

C circuit response with AC supply



$$i(t) = C\frac{dv(t)}{dt}$$

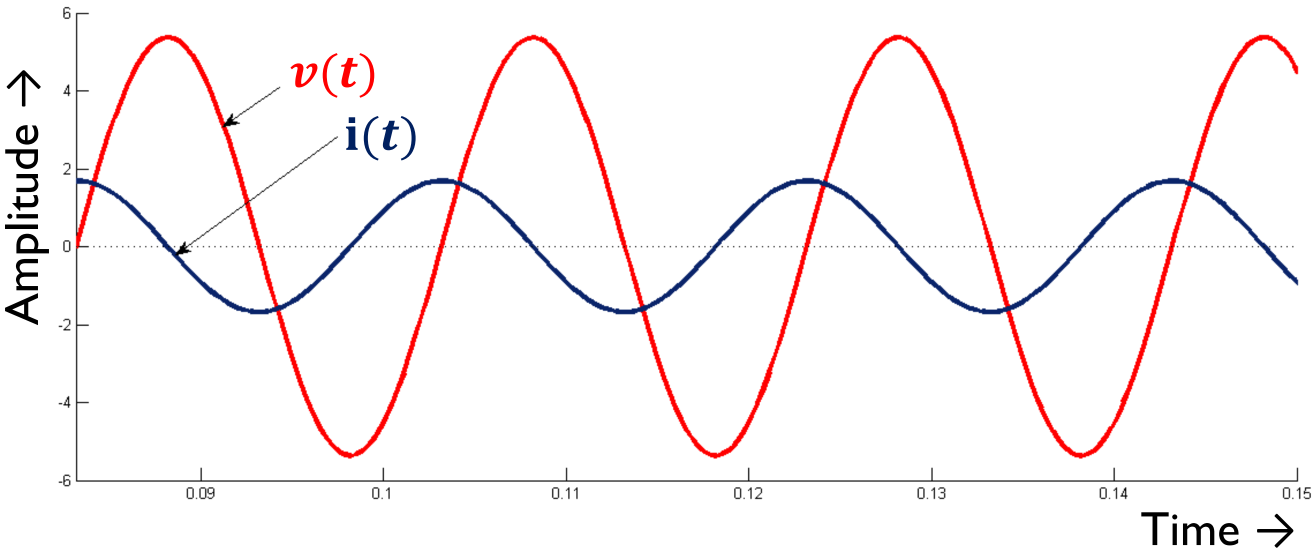
'Current through the capacitor leads the voltage across it by 90°'

$\bar{V} = V\angle 0°$ $\quad$ $\bar{I} = I\angle 90°$

$$\frac{\bar{\boldsymbol{V}}}{\bar{\boldsymbol{I}}} = \frac{\boldsymbol{V\angle 0°}}{\boldsymbol{I\angle 90°}} = \boldsymbol{-jX_C} \quad where\ \frac{V}{I} = X_C$$

$\boldsymbol{X_C}$ *is called* ***Capacitive Reactance***

Graphical Representation

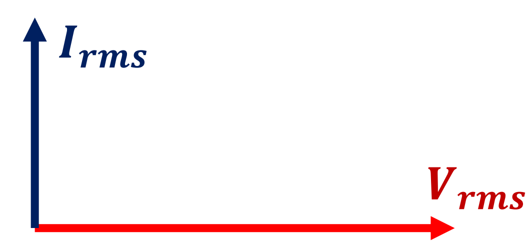


Mathematical Representation

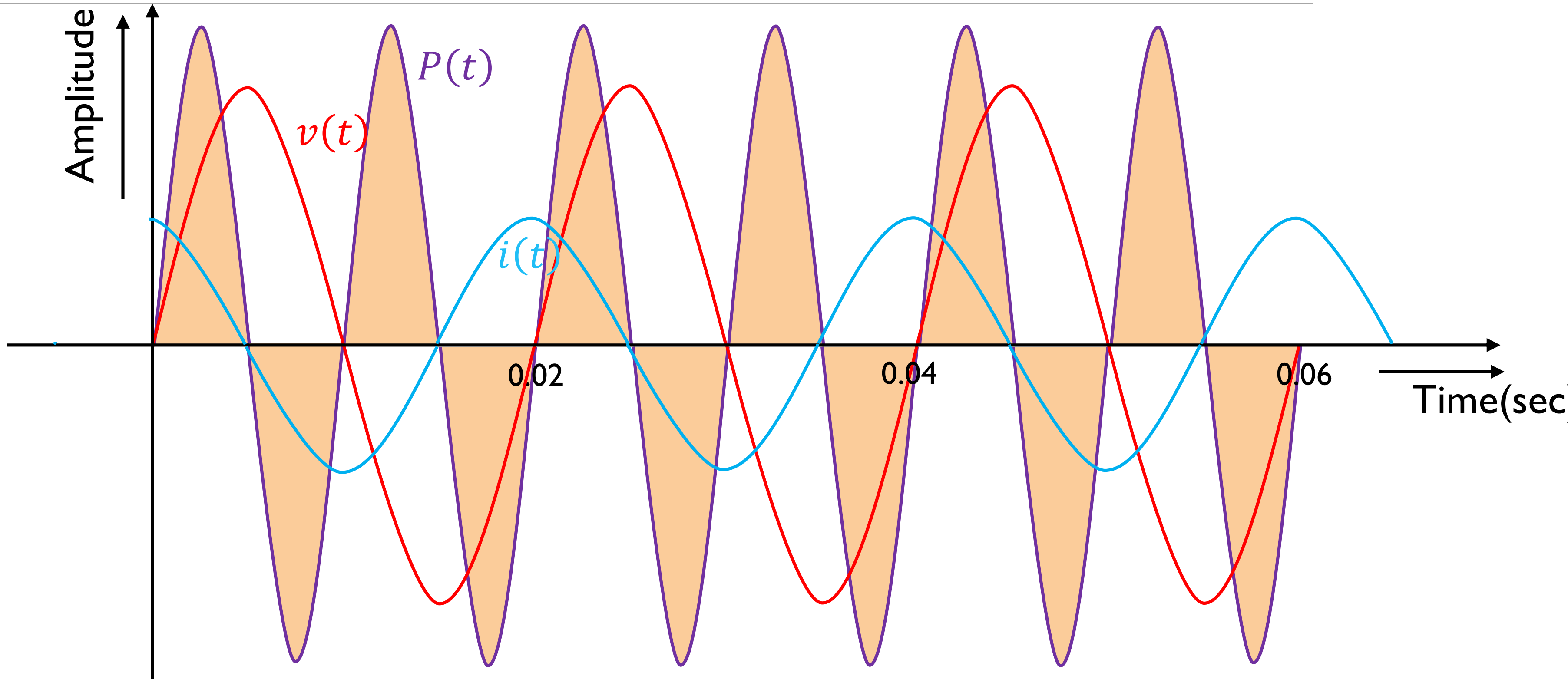
$$v(t) = V_m \sin(\omega t)$$

$$i(t) = I_m \sin(\omega t + 90°)$$

Phasor Representation



Power Associated – Pure capacitive Circuit



Instantaneous power,

$$p(t) = v(t).i(t)$$
$$= V_m I_m \sin\omega t \,.\, \sin(\omega t + 90\,°)$$
$$= \frac{V_m I_m}{2} \sin 2\omega t$$

$$\boldsymbol{Average\ Power, P = \frac{1}{T}\int_0^T p(t)dt}$$

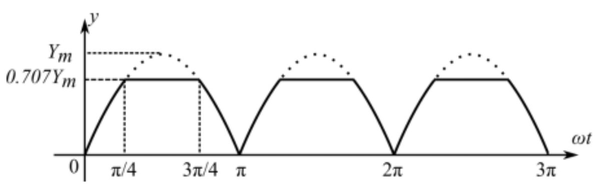
$$\boxed{\boldsymbol{P_{avg} = 0}}$$

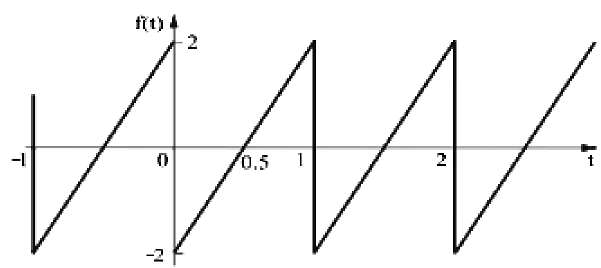
Practice Quiz Questions

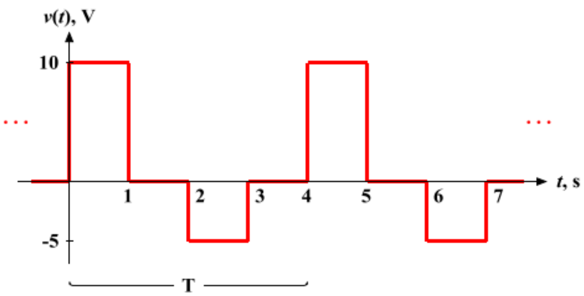
Link:

https://forms.microsoft.com/Pages/ResponsePage.aspx?id=Qr2-Kf_xPUyWiAZ-NGDcHxdWYFLgt7pFmZiZBmJlji1UMDA0OU84WlVHRTINV1NHOVVPOUI3ODBITyQlQCN0PWcu

Q1	Q2	Q3
Find the average and RMS value of the waveform	Find the average and RMS value of the waveform	Find the average and RMS value of the waveform







Deadline: 17 December 2020, 9AM