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Ex 1. Obtain the response of unity feedback system whose open loop TF is $G(s) = \frac{4}{s(s+5)}$ and when input is unit step.

$G(s)$ - Forward path TF

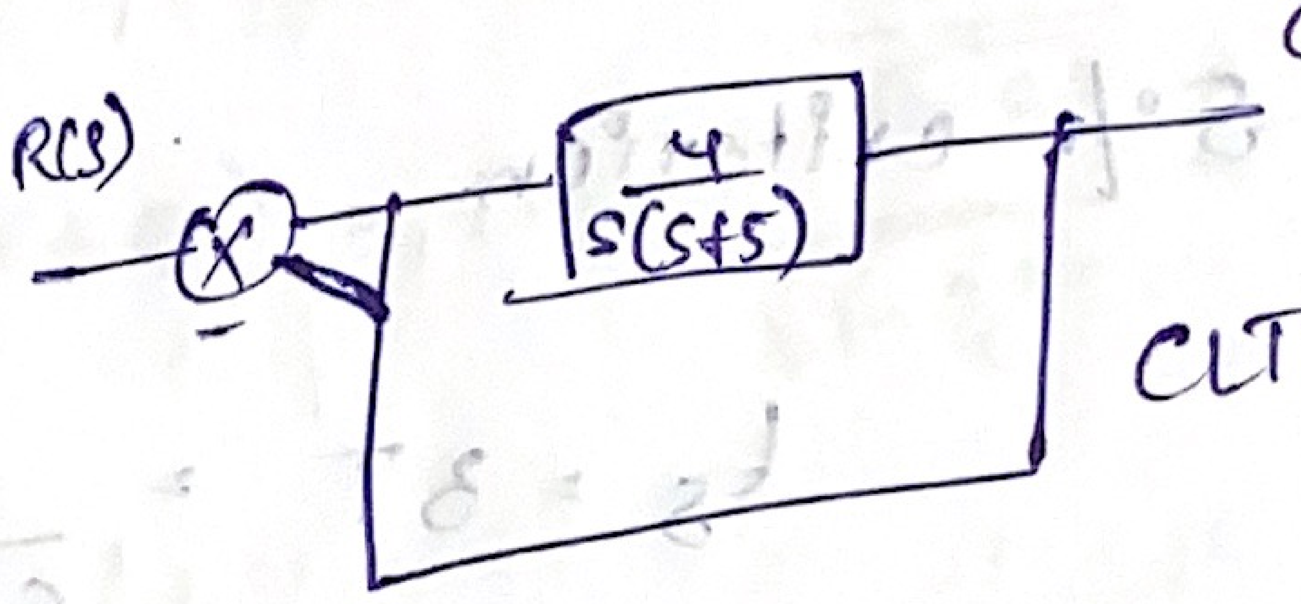
$H(s)$ - Feedback Path TF

$G(s)\cdot H(s)$ - Open loop TF

$H(s)$ is 1 & -ve

$\frac{G(s)}{1+G(s)H(s)}$ - closed loop TF or Overall TF

Sol. $G(s) = \frac{4}{s(s+5)}$



CLTF $= \frac{C(s)}{R(s)}$

Closed loop TF $\left(\frac{G}{1+GH}\right) = \frac{\frac{4}{s(s+5)}}{1+\frac{4}{s(s+5)}}$

$\frac{C(s)}{R(s)} = \frac{4}{s^2+5s+4} = \frac{4}{(s+4)(s+1)}$

Given $R(s) =$ unit step $= \frac{1}{s}$

$C(s) = R(s)\frac{4}{(s+4)(s+1)}$

$C(s) = \frac{1}{s}\frac{4}{(s+4)(s+1)}$

$C(s) = \frac{A}{s} + \frac{B}{s+4} + \frac{C}{s+1}$

$\frac{As+4A+Bs}{s(s+4)} + \frac{C}{s+1}$

$As^2+4As+Bs^2+As+4A+Bs+Cs^2+4C$

$s^2(A+B+C)+s(5A+B)+4A+4C$

compare

$A+B+C=0$

$5A+B=0$

$4A+4C=4.$

$A=1 \quad B=-1/3 \quad C=-4/3.$

$$C(s)=\frac{1}{s}-\frac{1}{3}\frac{1}{s+4}-\frac{4}{3}\frac{1}{(s+1)}$$

$$\mathcal{L}^{-1}(C(s))=1-\frac{1}{3}e^{-4t}-\frac{4}{3}e^{-t}.$$

$$G(s)=\frac{4}{s(s+8)}$$

$$CLTF=\frac{C(s)}{R(s)}=\frac{\frac{4}{s(s+8)}}{1+\frac{4}{s(s+8)}}=\frac{4}{s^2+8s+4}=\frac{4}{}$$

$-8\pm\sqrt{64-16}$

$-8\pm\sqrt{48}$

$$C(s)=\frac{1}{s}\frac{4}{s^2+8s+4}.$$

$$C(s)=\frac{A}{s}+\frac{Bs+C}{s^2+8s+4}$$

$$C(s)=\frac{1}{s}+\frac{-(s+8)}{s^2+8s+4}$$

$$\mathcal{L}^{-1}(C(s))=\frac{1}{6}\Big(-3e^{(-4-2\sqrt{3})t}+2\sqrt{3}\,e^{(-4-2\sqrt{3})t}-3e^{(2\sqrt{3}-4)t}$$

$$-2\sqrt{3}\,e^{(2\sqrt{3}-4)t}+6\Big)$$

Q:3 The unity feedback system is characterised by an open loop TF $G(s) = \frac{K}{s(s+10)}$. Determine the gain K, so that the system will have a damping ratio of 0.5 for this value of K. Determine the settling time, peak overshoot & time at peak overshoot for a unit step input.

Sol. $CLTF \Rightarrow \frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)}$

$$= \frac{\frac{K}{s(s+10)}}{1+\frac{K}{s(s+10)}(1)} = \frac{K}{s(s+10)+K} = \frac{K}{s^2+10s+K}$$

The value of K can be obtained by comparing the above eq CLTF with its standard form.

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2} = \frac{K}{s^2+10s+K}$$

Given $\xi = 0.5$

$2\xi\omega_n = 10 \qquad \omega_n^2 = K$

$2(0.5)\omega_n = 10 \qquad \Rightarrow K = 10^2 = 100$

$\boxed{\omega_n = 10}$

settling time, t_s: 2% tolerance, $t_s = \frac{4}{\xi\omega_n} = \frac{4}{0.5(10)} = 0.8$ sec

5%, $t_s = \frac{3}{\xi\omega_n} = \frac{3}{5} = 0.6$ sec

Peak overshoot, $\% M_p = e^{-\xi\pi/\sqrt{1-\xi^2}} \times 100$

$\% M_p = \left(e^{-0.5\pi/\sqrt{1-0.5^2}}\right) \times 100$

$= 16.3\%$

Peak time, $t_p = \frac{\pi}{\omega_d}$

$t_p = \frac{\pi}{\omega_n\sqrt{1-\xi^2}} = 0.363$ sec.

Type number of control system

$T.F = \frac{A}{S^N(S+10)}$ if $N=0 \Rightarrow$ type '0' system.

$N=1 \Rightarrow$ type '1' system.

Type number - No. of poles at the origin.

Steady state error.

It is the value of error signal, $e(t)$, when $t \longrightarrow \infty$.

It is the measure of system accuracy.

These error arise from nature of input, ~~syst~~ type of a system & from non-linearity of system component.

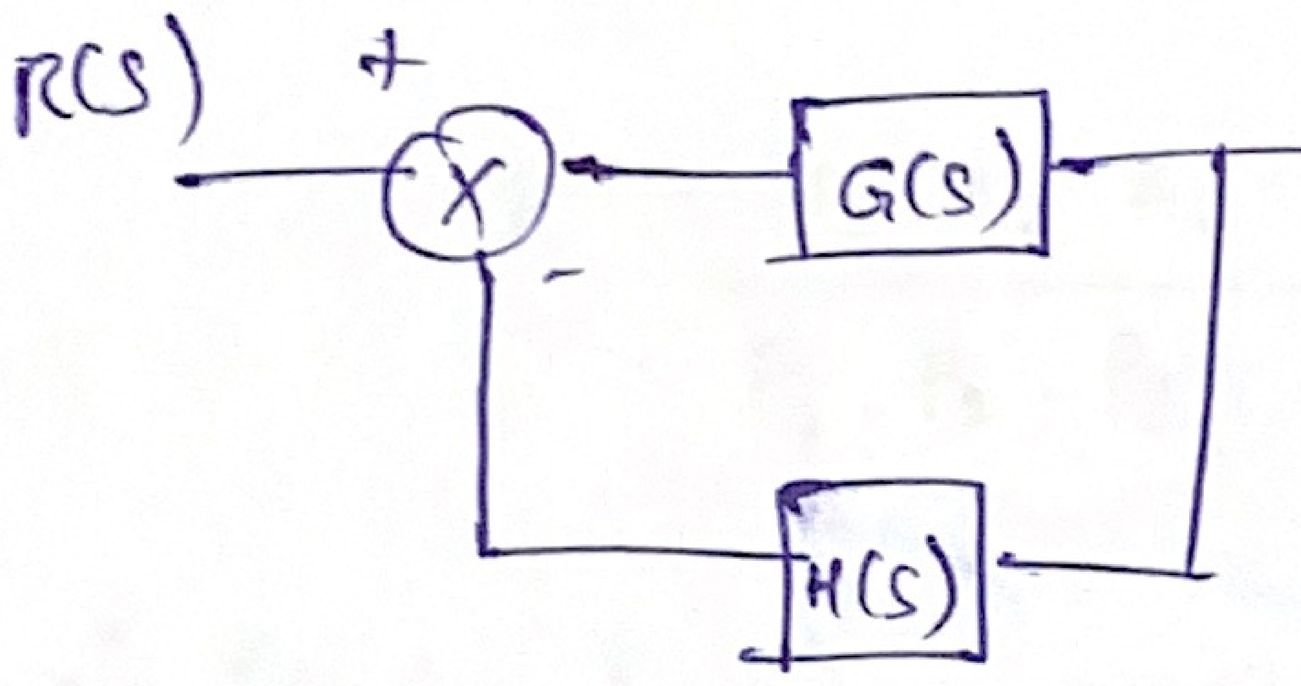
The steady state performance of a stable control system is generally decided by its steady state errors, to step, ramp and parabolic input.

$R(s)$ - Input signal.

$E(s)$ - error signal.

$C(s) \cdot H(s)$ - Feedback signal

$C(s)$ - output signal



$$E(s) = R(s) - C(s)\,H(s).$$

$$C(s) = E(s) \cdot G(s)$$

$$E(s) = R(s) - [E(s)\,G(s)]\,H(s)$$

$$E(s) + E(s)\,G(s)\,H(s) = R(s)$$

$$E(s)\,[1 + G(s)\,H(s)] = R(s)$$

$$\boxed{E(s) = \frac{R(s)}{1 + G(s)\,H(s)}}$$

let $e(t)$ = error signal in time domain.

then $e(t) = L^{-1}\{E(s)\} = L^{-1}\left\{\frac{R(s)}{1+G(s)\,H(s)}\right\}$

let e_{ss} be steady state error,

It is defined as the value of $e(t)$ when $t \to \infty$

$$\therefore e_{ss} = \boxed{\lim_{t\to\infty} e(t)}$$

$$F(s) = L\{f(t)\} \cdot \lim_{t\to\infty} f(t)$$

$$= \lim_{s\to 0} s\,F(s)$$

$$e_{ss} = \underset{t \to \infty}{Lt}\ e(t) = \underset{s \to 0}{Lt}\ s\,E(s)$$

$$= \underset{s \to 0}{Lt}\ \frac{s\,R(s)}{1+G(s)\,H(s)}$$

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Static error constants.

i) Positional error constant

$$K_p = \underset{s \to 0}{Lt}\ G(s)\,H(s)$$

ii) velocity error constant

$$K_v = \underset{s \to 0}{Lt}\ s\,G(s)\,H(s)$$

iii) acceleration error constant

$$K_a = \underset{s \to 0}{Lt}\ s^2\,G(s)\,H(s).$$

The K_p, K_v, K_a are in general called static error constants.

Steady state error when the input is unit step signal.

$$e_{ss} = \underset{s \to 0}{Lt}\ \frac{s\,R(s)}{1+G(s)\,H(s)}$$

when input is unit step $R(s) = 1/s$

$$\therefore e_{ss} = \underset{s \to 0}{Lt}\ \frac{s \times \frac{1}{s}}{1+G(s)H(s)} = \underset{s \to 0}{Lt}\ \frac{1}{1+G(s)H(s)} = \frac{1}{1+\underset{s \to 0}{Lt}\ G(s)\,H(s)}$$

$$\boxed{e_{ss} = \frac{1}{1+K_p}}$$ where $K_p = \underset{s \to 0}{Lt}\ G(s)\,H(s)$

constant K_p is called positional error constant.