Chapter 2

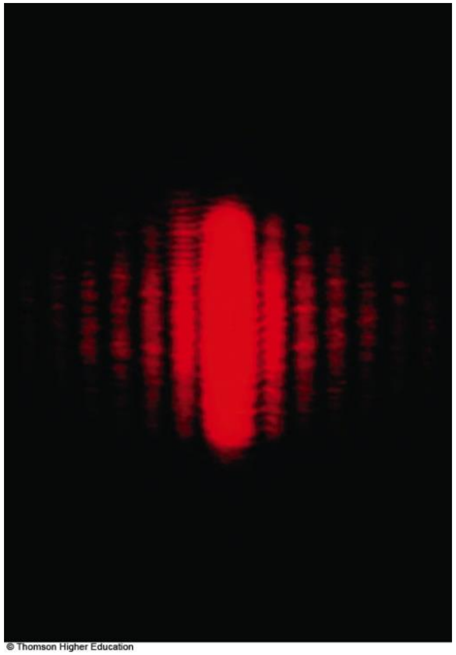
DIFFRACTION PATTERNS AND POLARIZATION

OBJECTIVES

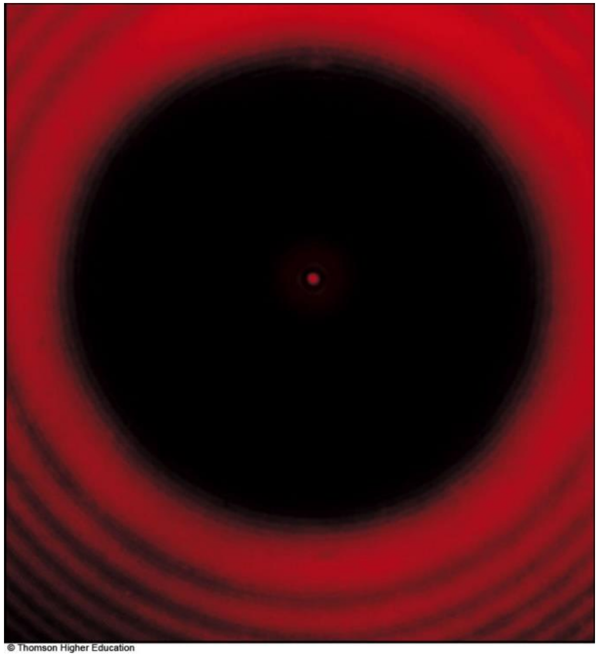
- To understand the principles of diffraction.
- To explain the intensity distribution in diffraction under various conditions.
- To explain the diffraction of light waves at single, multiple slits and circular apertures.
- To understand polarization phenomena and various techniques used to produce polarized light.

Introduction to Diffraction Patterns

Light of wavelength comparable to or larger than the width of a slit spreads out in all forward directions upon passing through the slit. This phenomenon is called *diffraction*



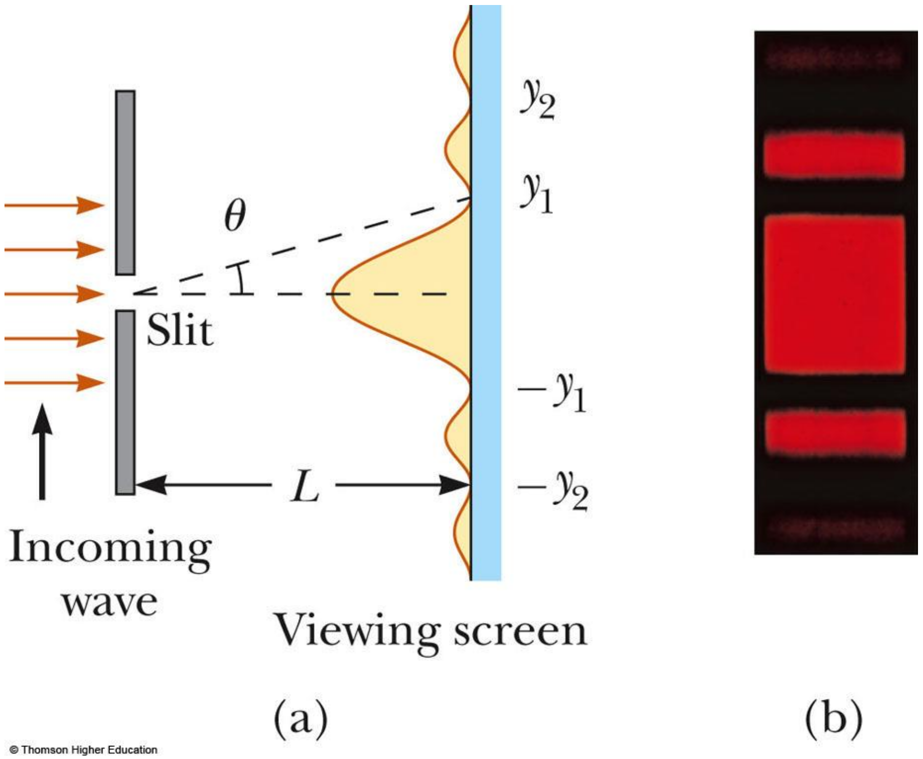
(a) The diffraction pattern that appears on a screen when light passes through a narrow vertical slit.



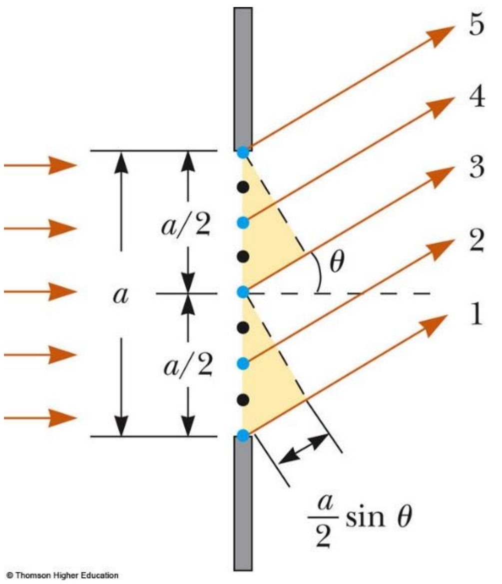
(b) Diffraction pattern created by the illumination of a penny, with the penny positioned midway between the screen and light source.

Diffraction Patterns from Narrow Slits

- Here we assume the observing screen is far from the slit and the rays reaching the screen are approximately parallel.
- In laboratory, this situation can also be achieved experimentally by using a converging lens to focus the parallel rays on a nearby screen.
- In this model, the pattern on the screen is called a **Fraunhofer diffraction pattern.**



- To analyze the diffraction pattern, let's divide the slit into two halves as shown in Figure.
- If this path difference is exactly half a wavelength (corresponding to a phase difference of 180°), the pairs of waves cancel each other and destructive interference results.
- Ray 1 and Ray 3 are out of phase by 180°
- Similarly Ray 2 and Ray 4 are out of phase by 180°



- Hence we can write $\frac{a}{2}\sin\theta = \pm\frac{\lambda}{2}$
- Dividing the slit into four equal parts and using similar reasoning, we find that the viewing screen is also dark when $\sin\theta = \pm 2\frac{\lambda}{a}$
- Therefore, the general condition for destructive interference is

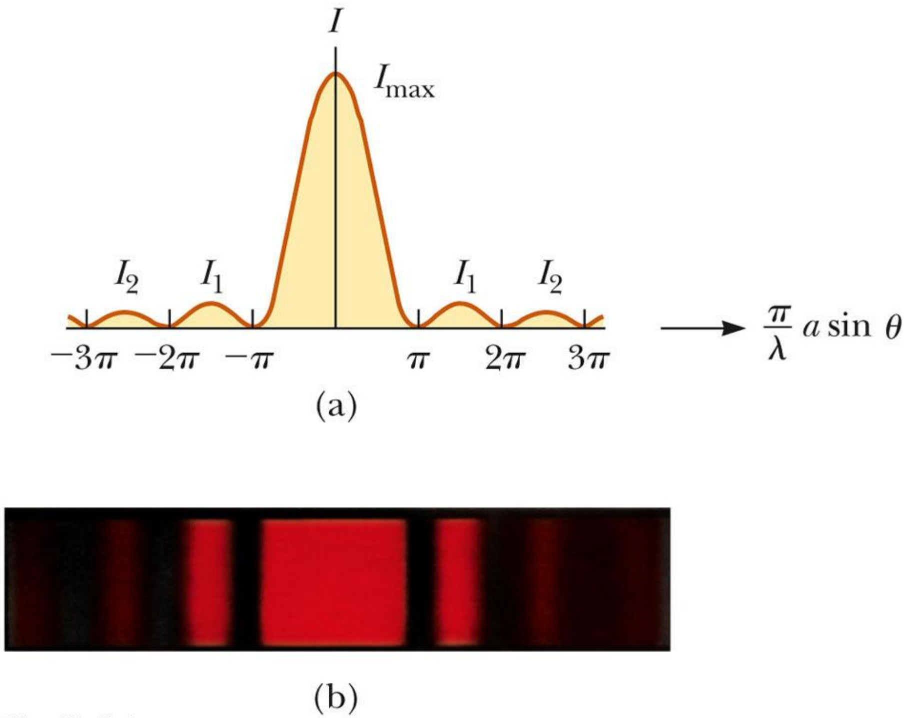
$$\sin\theta_{\text{dark}} = m\frac{\lambda}{a} \quad m = \pm 1, \pm 2, \pm 3, \ldots$$

Intensity of Single-Slit Diffraction Patterns

Analysis of the intensity variation in a diffraction pattern from a single slit of width '*a*' shows that the intensity is given by

$$I = I_{max}\left[\frac{\sin\left(\pi a \sin\theta / \lambda\right)}{\pi a \sin\theta / \lambda}\right]^2$$

where I_{max} is the intensity at θ = 0 (the central maximum) and λ is the wavelength of light used to illuminate the slit.

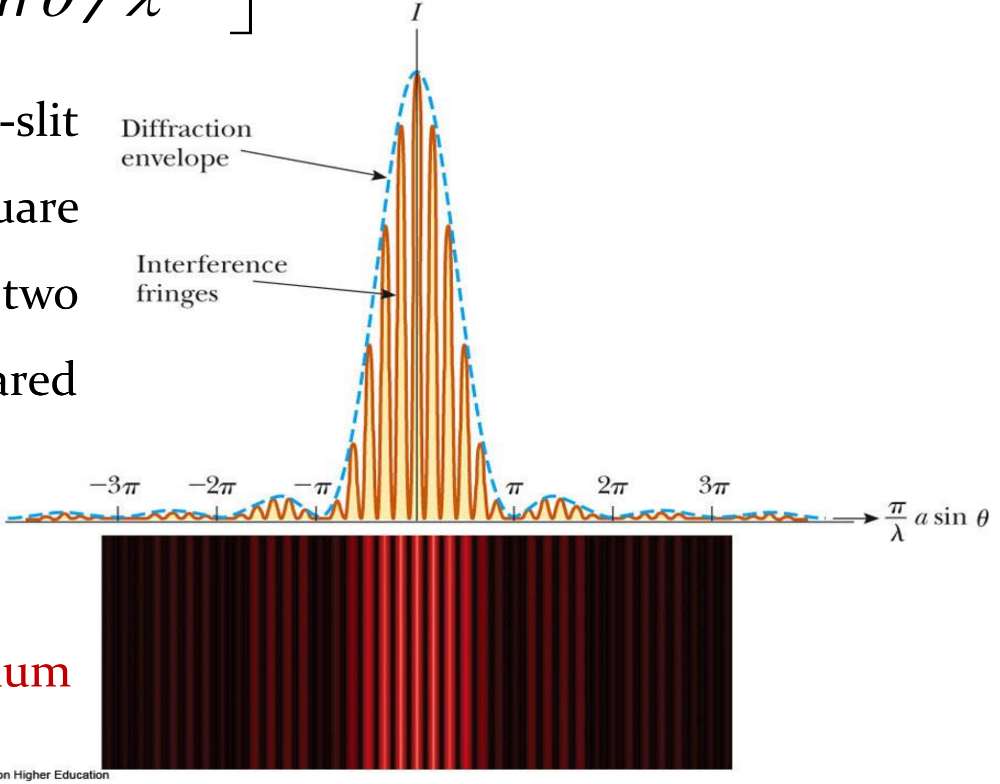


Intensity of Two-Slit Diffraction Patterns

When more than one slit is present, we must consider not only diffraction patterns due to the individual slits but also the interference patterns due to the waves coming from different slits. Intensity due to combined effect is given by

$$I = I_{max} \cos^2\left(\frac{\pi d \sin\theta}{\lambda}\right)\left[\frac{\sin(\pi a \sin\theta/\lambda)}{\pi a \sin\theta/\lambda}\right]^2$$

Above equation represents the single-slit diffraction pattern (the factor in square brackets) acting as an "envelope" for a two slit interference pattern (the cosine-squared factor).



If $\frac{d}{a} = m$ where m is an interger

In this case, m^{th} interference maximum coincides with first diffraction minimum.

Resolution of Single-Slit and Circular Apertures

The ability of optical systems to distinguish between closely spaced objects is limited because of the wave nature of light.

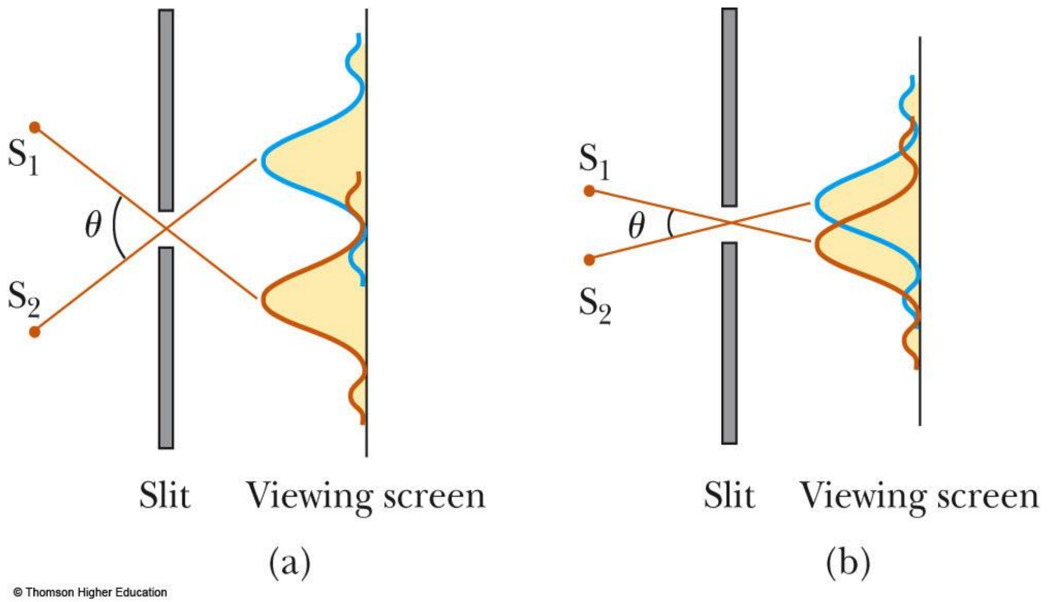
When the central maximum of one image falls on the first minimum of another image, the images are said to be just resolved. This limiting condition of resolution is known as **Rayleigh's criterion.**

In case of single-slit diffraction pattern, this occurs at the angle for which

$$\sin\theta = \frac{\lambda}{a}$$

Since $\lambda << a$

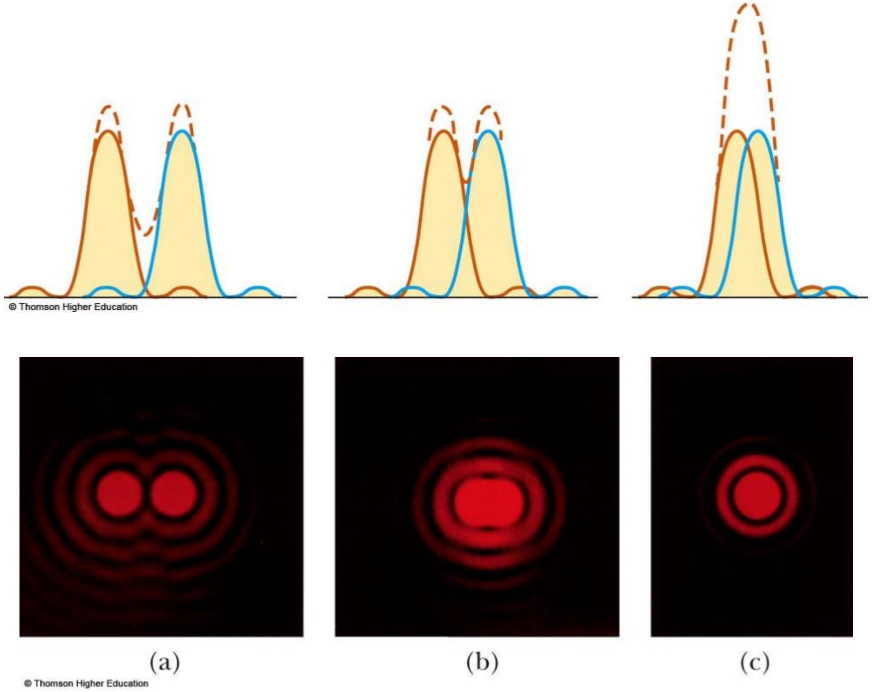
$$\theta_{min} = \frac{\lambda}{a}$$



Circular Aperture: Analysis shows that the limiting angle of resolution of the circular aperture is

$$\theta_{min} = 1.22\ \frac{\lambda}{D}$$

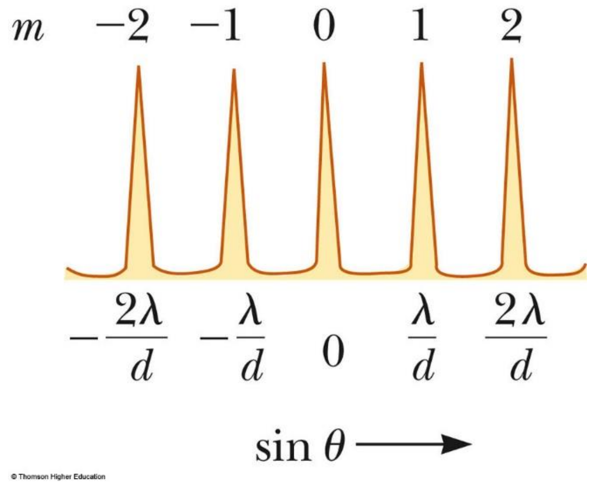
where D is the diameter of the aperture.

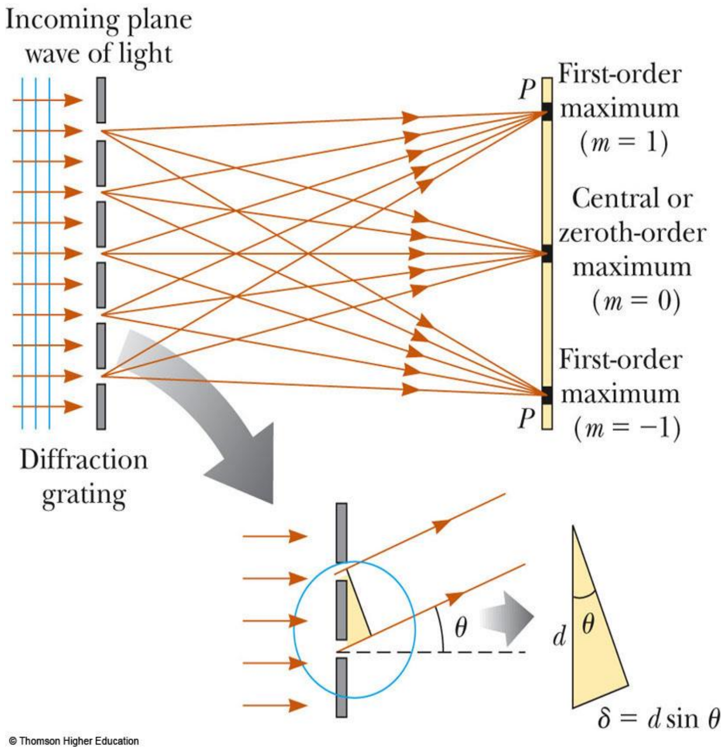


Diffraction Grating

- A *transmission grating* can be made by cutting parallel grooves on a glass plate with a precision ruling machine. The spaces between the grooves are transparent to the light and hence act as separate slits.
- A *reflection grating* can be made by cutting parallel grooves on the surface of a reflective material.

$$d \sin \theta_{\text{bright}} = m\lambda \quad m = 0, \pm 1, \pm 2, \pm 3, \ldots$$



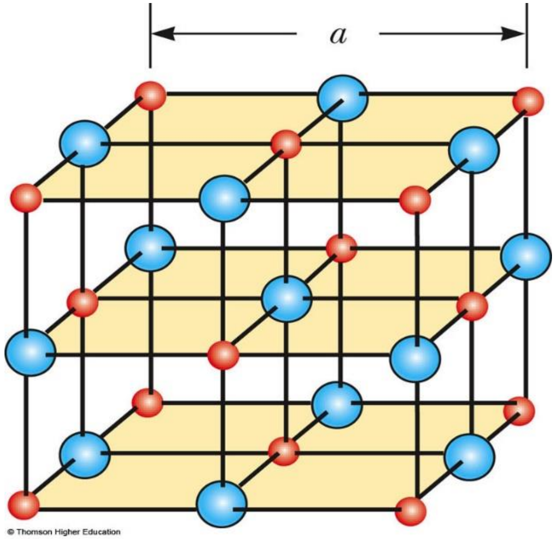


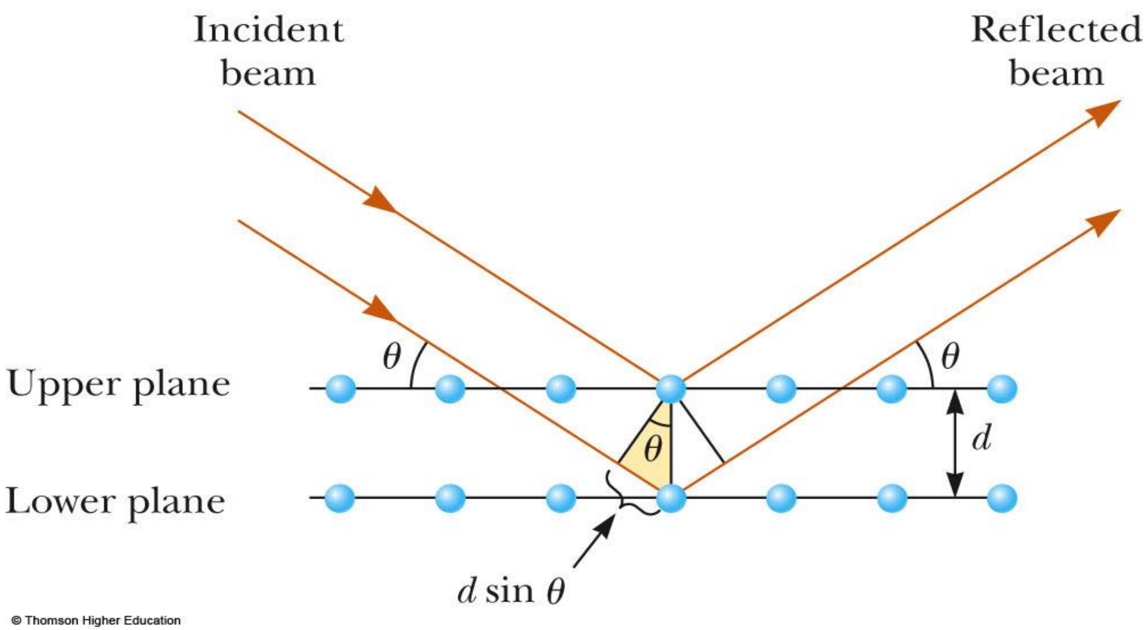
Diffraction of X-Rays by Crystals

- Crystal acts as 3D grating for X-rays.
- Condition for constructive interference (maxima in the reflected beam) is

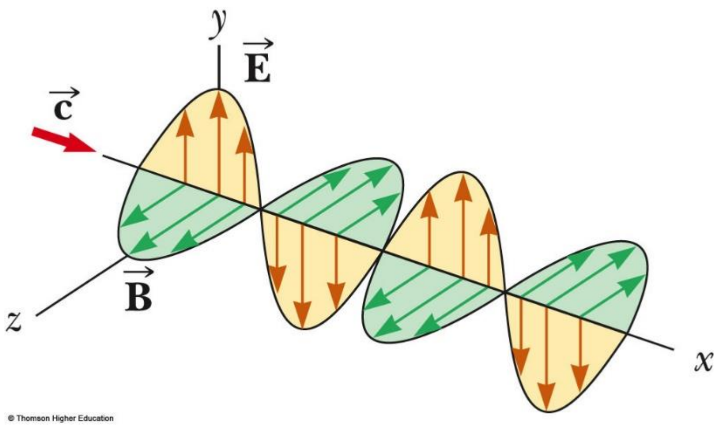
$$2d \sin\theta = m\lambda \quad m = 1, 2, 3, \ldots$$

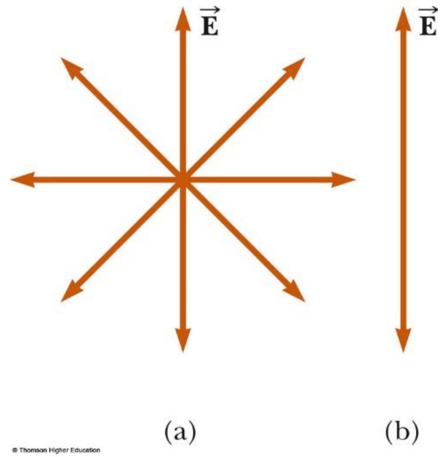
This condition is known as **Bragg's law**





Polarization of Light Waves





Schematic diagram of an electromagnetic wave propagating at velocity c in the x direction. The electric field vibrates in the xy plane, and the magnetic field vibrates in the xz plane.

(a) A representation of an unpolarized light beam viewed along the direction of propagation. The transverse electric field can vibrate in any direction in the plane of the page with equal probability. (b) A linearly polarized light beam with the electric field vibrating in the vertical direction.

Polarization by Selective Absorption

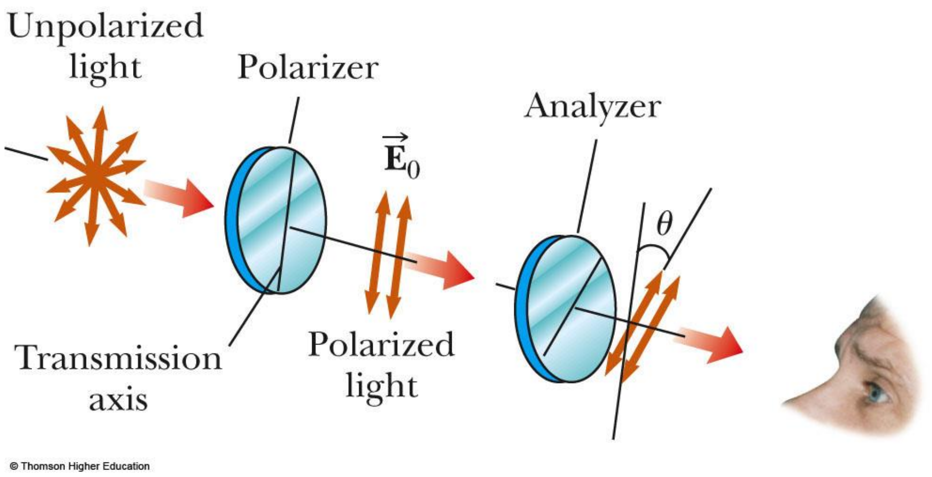
Polaroid is a device which polarizes unpolarized light when light passes through it.

Malus's law

The component of $\vec{E}_0$ parallel to the analyzer axis, which is transmitted through the analyzer, is $E_0 \cos\theta$.

Intensity is square of magnitude of field vector. Hence,

$$I = I_{max} \cos^2 \theta$$



Two polarizing sheets whose transmission axes make an angle u with each other. Only a fraction of the polarized light incident on the analyzer is transmitted through it.

Polarization by Reflection

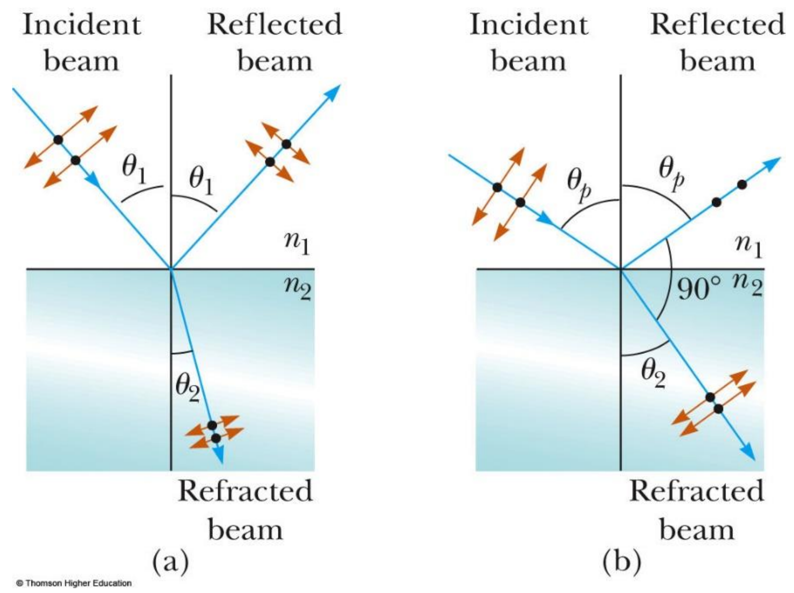
For one particular angle of incidence (θ_p), the reflected light is completely polarized.

Using Snell's law of refraction

$$\frac{n_2}{n_1} = \frac{\sin\theta_p}{\sin\theta_2}$$

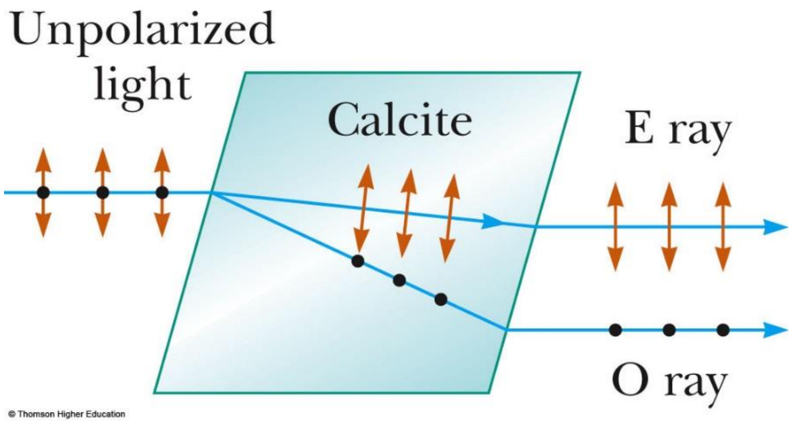
But, $\theta_2 = 90 - \theta_p$. So, we can write,

$$\tan\theta_p = \frac{n_2}{n_1}$$

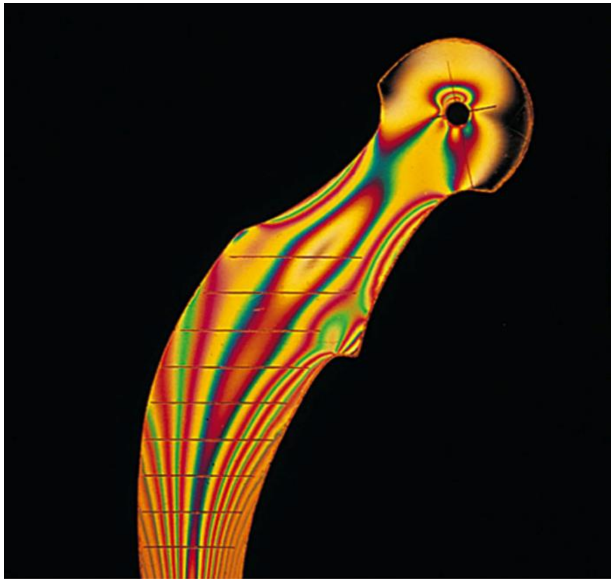


This expression is called **Brewster's law**, and the polarizing angle $\boldsymbol{\theta_p}$ is sometimes called **Brewster's angle**

Polarization by Double Refraction



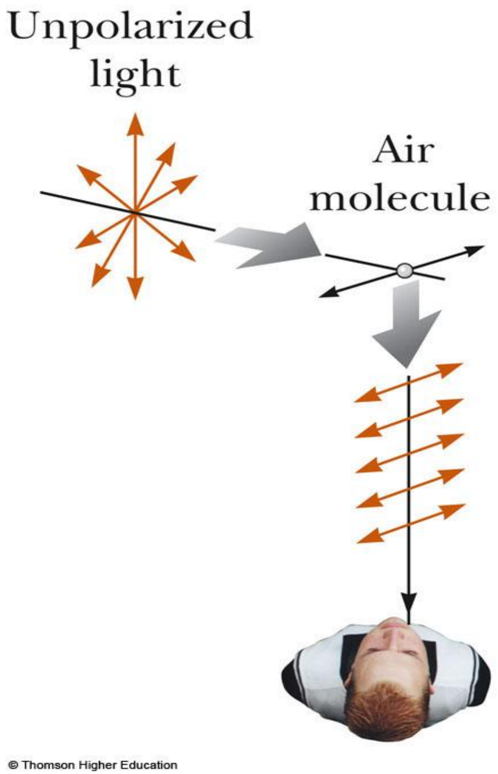
Unpolarized light incident at an angle to the optic axis in a calcite crystal splits into an ordinary (O) ray and an extraordinary (E) ray



The pattern is produced when the plastic model is viewed between a polarizer and analyzer oriented perpendicular to each other. Such patterns are useful in the optimal design of architectural components

Polarization by Scattering

The scattering of unpolarized sunlight by air molecules.



Optical Activity

- A material is said to be optically active if it rotates the plane of polarization of any light transmitted through the material.
- The angle through which the light is rotated by a specific material depends on the length of the path through the material and on concentration if the material is in solution.