ICE 3153: MODERN CONTROL THEORY [3 1 0 4]

Introduction to Nonlinear Systems

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Introduction

- All physical systems exhibit non-linearities and time-varying parameters to some degree.
- The subject of nonlinear control deals with the analysis and the design of nonlinear control systems, *i.e.,* of control systems containing at least one nonlinear component.
- In the analysis, a nonlinear closed-loop system is assumed to have been designed, and we wish to determine the characteristics of the system's behavior.
- In the design, we are given a nonlinear plant to be controlled and some specifications of closed-loop system behavior, and our task is to construct a controller so that the closed loop system meets the desired characteristics.

Classification of Nonlinearity

- Nonlinearities can be classified as *inherent (natural)* and *intentional (artificial).*
- Inherent nonlinearities are those which naturally come with the system's hardware and motion.
- Examples of inherent nonlinearities include centripetal forces in rotational motion, and Coulomb friction between contacting surfaces.
- Intentional nonlinearities, on the other hand, are artificially introduced by the designer.
- Nonlinear control laws, such as adaptive control laws and bang-bang optimal control laws, are typical examples of intentional nonlinearities.

- Nonlinearities can also be classified in terms of their mathematical properties, as *continuous* and *discontinuous.*
- Because discontinuous nonlinearities cannot be locally approximated by linear functions, they are also called "hard" nonlinearities.
- Hard nonlinearities (such as, *e.g.,* backlash, hysteresis, or stiction)
- Whether a system in small range operation should be regarded as nonlinear or linear depends on the magnitude of the hard nonlinearities and on the extent of their effects on the system performance.

Why Nonlinear Control??

- **Improvement of existing control systems**

- **Analysis of hard nonlinearities**

- **Dealing with model uncertainties**

- **Design Simplicity**

Linear Systems

- Consider an LTI system of the form,

$$\dot{x} = Ax$$

- a linear system has a *unique equilibrium point* if A is nonsingular
- the equilibrium point is stable if all eigenvalues of A have negative real parts, *regardless of initial conditions*
- the transient response of a linear system is composed of the natural modes of the system, and the general solution can be solved analytically
- in the presence of an external input *u(t),*
- *First* it satisfies the *principle of superposition.*
- Second, the asymptotic stability of the system implies bounded-input bounded-output stability in the presence of u.
- Third, a sinusoidal input leads to a sinusoidal output of the same frequency.

Some properties of nonlinear dynamic systems are

- principle of superposition does not hold
- commutativity does not apply
- possible multiple isolated equilibrium points
- properties such as limit-cycle, bifurcation, chaos
- finite escape time: solutions of nonlinear systems may not exist for all times

Common Nonlinear System Behavior

- Multiple Equilibrium Points

- Nonlinear systems frequently have more than one equilibrium point

- an equilibrium point is a point where the system can stay forever without moving.

Consider the first order system

$$\dot{x} = -x + x^2 \tag{1.4}$$

with initial condition $x(0) = x_o$. Its linearization is

$$\dot{x} = -x \tag{1.5}$$

The solution of this linear equation is $x(t) = x_o\, e^{-t}$. It is plotted in Figure 1.3(a) for various initial conditions. The linearized system clearly has a unique equilibrium point at $x = 0$.

By contrast, integrating equation $dx/(-x + x^2) = dt$, the actual response of the nonlinear dynamics (1.4) can be found to be

$$x(t) = \frac{x_o\, e^{-t}}{1 - x_o + x_o e^{-t}}$$

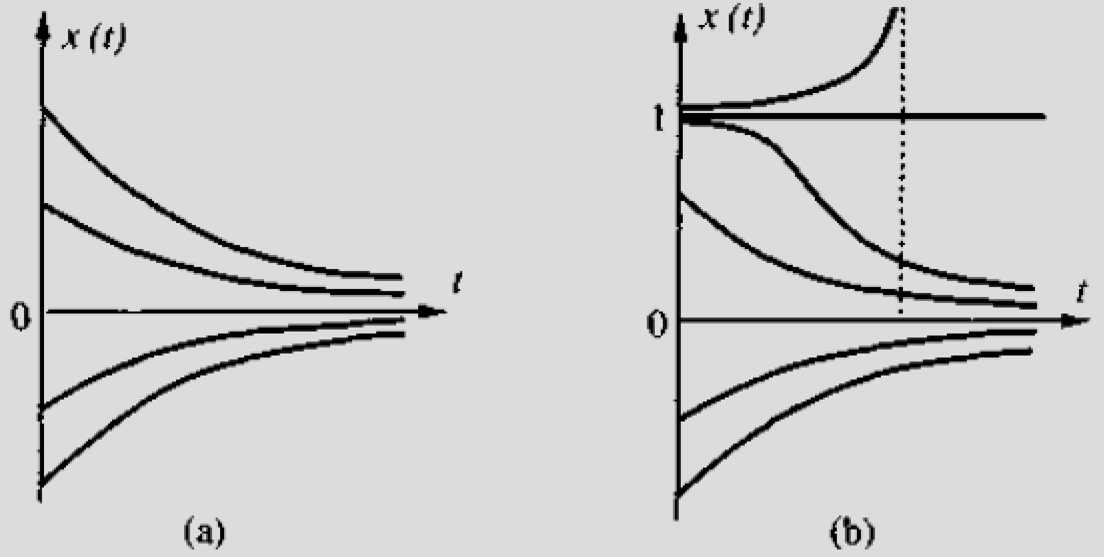


Figure 1.3 : Responses of the linearized system (a) and the nonlinear system (b)

- **Limit Cycles**
- Nonlinear systems can display oscillations of fixed amplitude and fixed period without external excitation.
- These oscillations are called limit cycles, or self-excited oscillations.

The second-order nonlinear differential equation

$$m\ddot{x} + 2c(x^2 - 1)\dot{x} + kx = 0$$

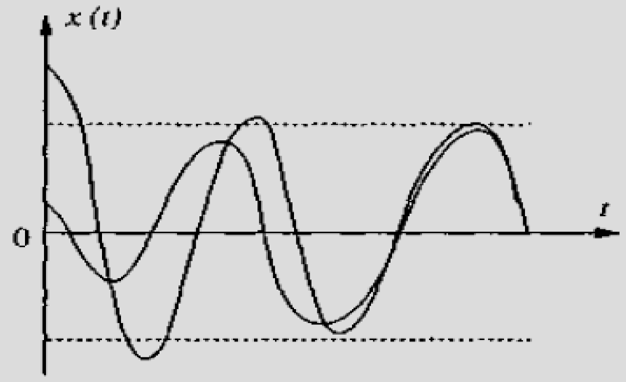


Figure 1.4 : Responses of the Van der Pol oscillator

- **Bifurcations**
- As the parameters of nonlinear dynamic systems are changed, the stability of the equilibrium point can change (as it does in linear systems) and so can the number of equilibrium points.
- Values of these parameters at which the qualitative nature of the system's motion changes are known as *critical* or *bifurcation* values.
- The phenomenon of bifurcation, *i.e.,* quantitative change of parameters leading to qualitative change of system properties, is the topic of bifurcation theory.

$$\ddot{x} + \alpha x + x^3 = 0$$

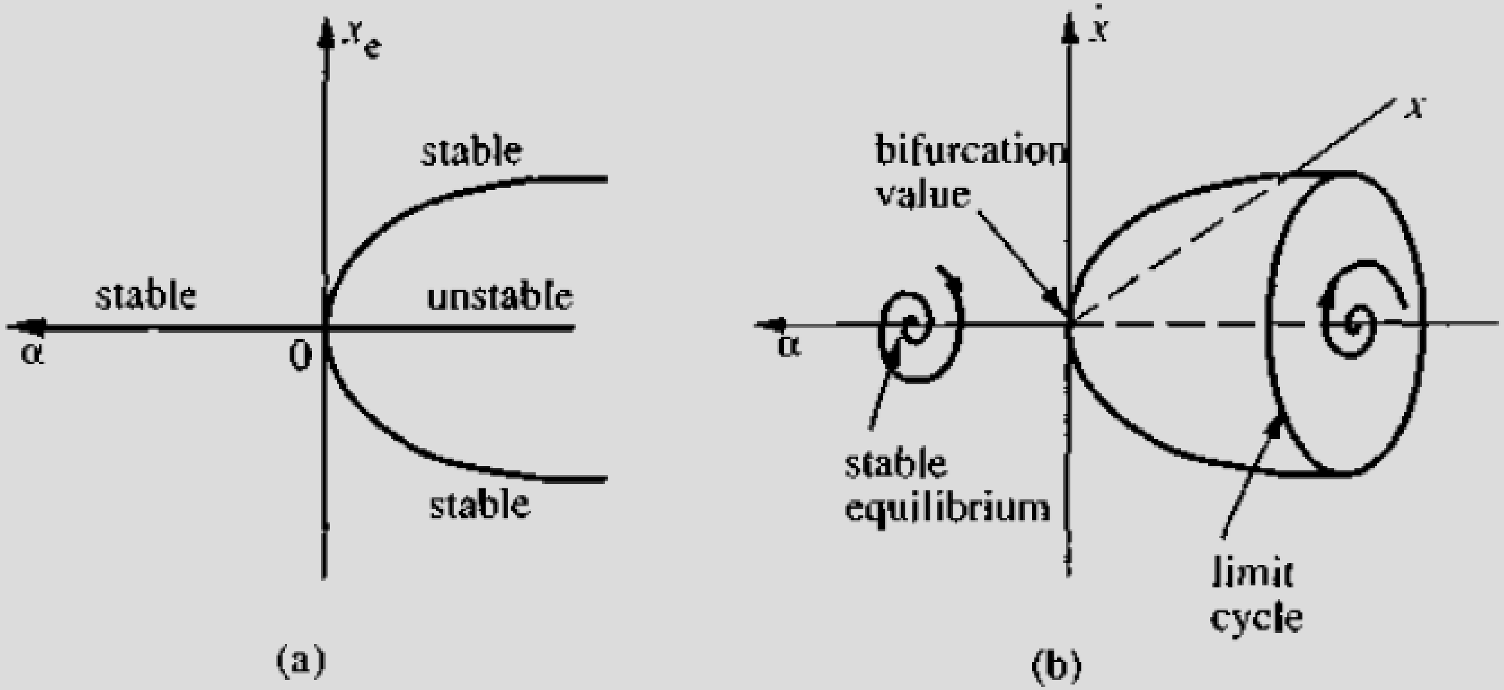


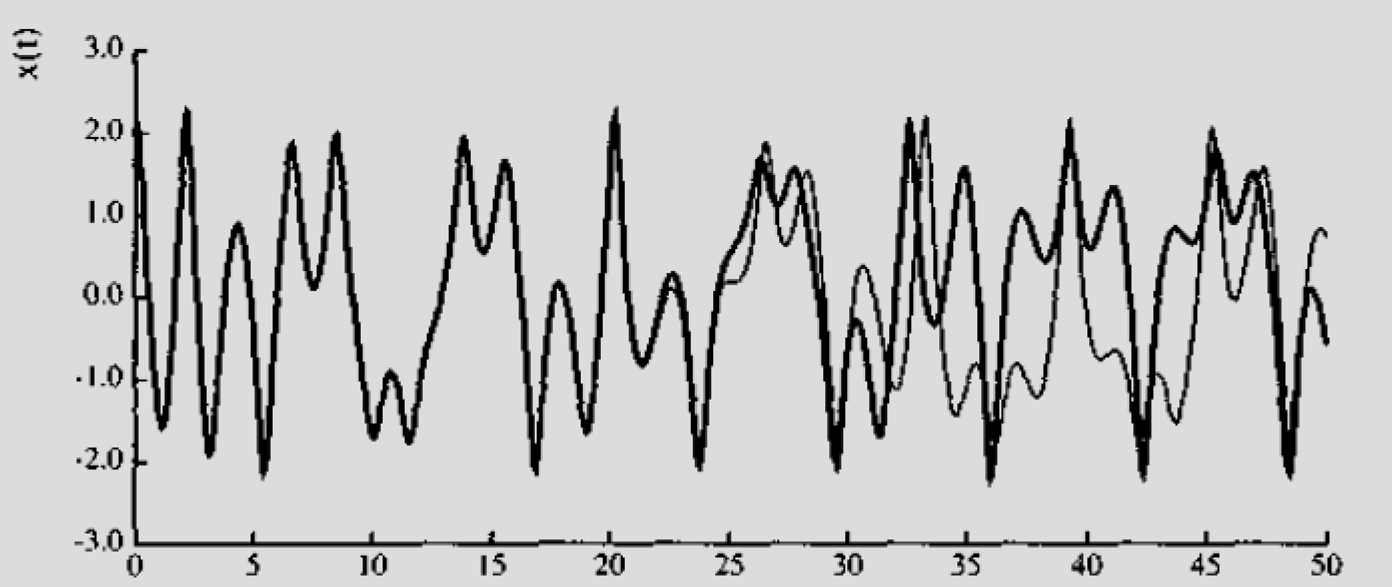
Figure 1.5 : (a) a pitchfork bifurcation; (b) a Hopf bifurcation

- **Chaos**
- For stable linear systems, small differences in initial conditions can only cause small differences in output.
- Nonlinear systems, however, can display a phenomenon called *chaos,* by which we mean that the system output is extremely sensitive to initial conditions.
- The essential feature of chaos is the unpredictability of the system output.
- Even if we have an exact model of a nonlinear system and an extremely accurate computer, the system's response in the long-run still cannot be well predicted.

- Chaotic phenomena can be observed in many physical systems. The most commonly seen physical problem is turbulence in fluid mechanics

As an example of chaotic behavior, let us consider the simple nonlinear system

$$\ddot{x} + 0.1\dot{x} + x^5 = 6\sin t$$



- **Jump Response**
- In the frequency response of a nonlinear systems, the amplitude of the response may jump from one point to another for increasing or decreasing values of frequency.