TABLE-2

ω rad/sec	0.5	1.0	3.0	6	10	20
∠G(jω) deg	−104	−118	−161 ≈ −160	−198	−222	−244.7 ≈ −244

On the same semilog sheet take another y-axis, choose appropriate scale and draw phase plot as shown in fig 6.9.2.

Step-3 : Find phase margin of uncompensated system.

Let, ϕ_{gc} = Phase of G(jω) at gain crossover frequency

γ = Phase margin of uncompensated system.

From the bode plot of uncompensated system we get, $\phi_{gc} = -226°$.

Now, $\gamma = 180° + \phi_{gc} = 180° - 226° = -46°$

Step-4 : Choose a new phase margin

The desired phase margin, $\gamma_d = 35°$

The phase margin of compensated system, $\gamma_n = \gamma_d + \epsilon$

Let initial choice of $\epsilon = 5°$

$\therefore \gamma_n = \gamma_d + \epsilon = 35° + 5° = 40°$.

Step-5 : Determine new gain crossover frequency

Let, ω_{gcn} = New gain crossover frequency and ϕ_{gcn} = Phase of G(jω) at ω_{gcn}

Now, $\gamma_n = 180° + \phi_{gcn}$, $\therefore \phi_{gcn} = \gamma_n - 180° = 40° - 180° = -140°$

From the bode plot we found that the frequency corresponding to a phase of −140° is 1.8 rad/sec.

Let, ω_{gcl} = Gain crossover frequency of lag compensator.

Choose ω_{gcl} such that, $\omega_{gcl} > \omega_{gcn}$. Let ω_{gcl} = 4 rad/sec.

Step-6 : Calculate β of lag compensator

From the bode plot we found that the db magnitude at ω_{gcl} is 23 db.

$\therefore |G(j\omega)|$ in db at $(\omega = \omega_{gcl}) = A_{gcl} = 23$ db.

Also, $A_{gcl} = 20\log\beta$; $\therefore \beta = 10^{A_{gcl}/20} = 10^{23/20} = 14$

Step-7 : Determine the transfer function of lag section.

The zero of the lag compensator is placed at a frequency one-tenth of ω_{gcl}.

$$\therefore \text{Zero of lag compensator, } z_{c1} = \frac{-1}{T_1} = \frac{\omega_{gcl}}{10}$$

$$\text{Now, } T_1 = \frac{10}{\omega_{gcl}} = \frac{10}{4} = 2.5$$

Pole of lag compensator, $p_{c1} = \frac{1}{\beta T_1} = \frac{1}{14 \times 2.5} = \frac{1}{35}$

Transfer function of lag section $\Big\}$ $G_1(s) = \beta\frac{(1+sT_1)}{(1+s\beta T_1)} = 14\frac{(1+2.5s)}{(1+35s)}$

Step-8 : Determine the transfer function of lead section.

Let $\alpha = 1/\beta$; $\therefore \alpha = 1/14 = 0.07$

The db gain (magnitude) corresponding to ω_m $\Big\}$ $= -20\log\frac{1}{\sqrt{\alpha}} = -20\log\frac{1}{\sqrt{0.07}} = -11.5 \text{ db} \approx -12 \text{ db}$

From the bode plot of uncompensated system the frequency ω_m corresponding to a db pair of -12 db is found to be 17 rad/sec.

$\therefore \omega_m = 17 \text{ rad / sec}$

$$\therefore \quad T_2 = \frac{1}{\omega_m\sqrt{\alpha}} = \frac{1}{17\sqrt{0.07}} = 0.22$$

Transfer function of lead section $\Big\}$ $G_2(s) = \alpha\frac{(1+sT_2)}{(1+s\alpha T_2)} = 0.07\frac{(1+0.22s)}{(1+0.0154s)}$

Step-10 : Determine the transfer function of lag-lead compensator.

Transfer function of lag - lead compensator $\Big\}$ $G_c(s) = G_1(s) \times G_2(s) = 14\frac{(1+2.5s)}{(1+35s)} \times 0.07\frac{(1+0.22s)}{(1+0.0154s)}$

$$= \frac{(1+2.5s)\,(1+0.22s)}{(1+35s)\,(1+0.0154s)}$$

Step-11 : Determine open loop transfer function of compensated system.

The lag-lead compensator is connected in series with G(s) as shown in fig 6.9.1.

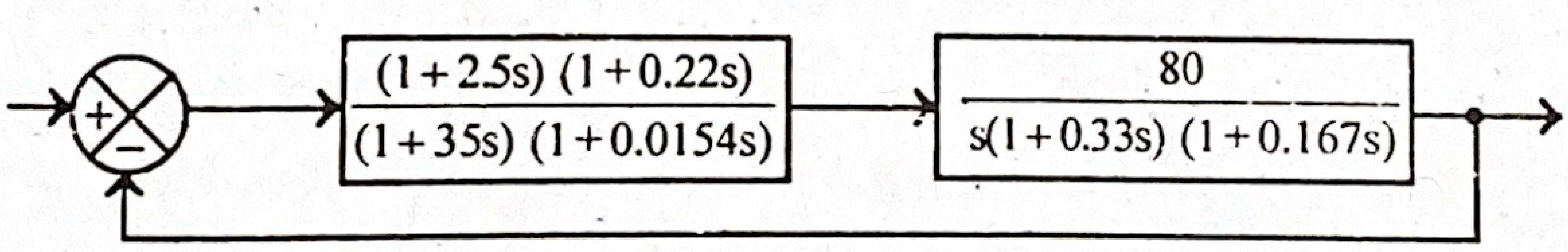


***Fig 6.9.1 :** Block diagram of lag-lead compensated system.*

Open loop transfer function of compensated system $\Big\}$ $G_0(s) = \frac{80(1+2.5s)\,(1+0.22s)}{s(1+35s)\,(1+0.0154s)\,(1+0.33s)\,(1+0.167s)}$

Step-12 : Bode plot of compensated system.

Put $s = j\omega$ in $G_0(s)$

$$\therefore G_0(j\omega) = \frac{80(1+j2.5\omega)\,(1+j0.22\omega)}{j\omega\,(1+j35\omega)\,(1+j0.0154\omega)\,(1+j0.33\omega)\,(1+j0.167\omega)}$$

MAGNITUDE PLOT

There are six corner frequencies, which are given below.

$$\omega_{c1} = \frac{1}{35} = 0.03\,\text{rad/sec}\;;\; \omega_{c2} = \frac{1}{2.5} = 0.4\,\text{rad/sec}\;;\; \omega_{c3} = \frac{1}{0.33} = 3\,\text{rad/sec}\;;$$

$$\omega_{c4} = \frac{1}{0.22} = 4.5\,\text{rad/sec};\; \omega_{c5} = \frac{1}{0.167} = 6\,\text{rad/sec}\;;\; \omega_{c6} = \frac{1}{0.0154} = 65\,\text{rad/sec}.$$

The various terms of $G_0(j\omega)$ are listed in table-3. Also the table shows the slope contributed by each term and the change in slope at the corner frequency.

TABLE-3

Term	Corner frequency rad/sec	Slope db/dec	Change in slope db/dec
$\frac{80}{j\omega}$	–	–20	–
$\frac{1}{1+j35\omega}$	$\omega_{c1} = \frac{1}{35} = 0.03$	–20	–20 –20 = –40
$1+j2.5\omega$	$\omega_{c2} = \frac{1}{2.5} = 0.4$	+20	–40 +20 = –20
$\frac{1}{1+j0.33\omega}$	$\omega_{c3} = \frac{1}{0.33} = 3$	–20	–20 –20 = –40
$1+j0.22\omega$	$\omega_{c4} = \frac{1}{0.22} = 4.5$	+20	–40 +20 = –20
$\frac{1}{1+j0.167\omega}$	$\omega_{c5} = \frac{1}{0.167} = 6$	–20	–20 –20 = –40
$\frac{1}{1+j0.0154\omega}$	$\omega_{c6} = \frac{1}{0.0154} = 65$	–20	–40 –20 = –60

Choose a low frequency ω_l such that $\omega_l < \omega_{c1}$ and choose a high frequency ω_h such that $\omega_h > \omega_{c6}$

Let $\omega_l = 0.01$ rad/sec and $\omega_h = 80$ rad/sec .

Let $A_0 = |G_0(j\omega)|$ in db.

$$\text{At } \omega = \omega_1,\quad A_0 = 20\log\frac{80}{0.01} = 78\text{ db}$$

$$\text{At } \omega = \omega_{c1},\quad A_0 = 20\log\frac{80}{0.03} = 68.5\text{ db} \approx 68\text{ db}$$

$$\text{At } \omega = \omega_{c2},\quad A_0 = -40\times\log\frac{0.4}{0.03} + 68 = 23\text{ db}$$

$$\text{At } \omega = \omega_{c3},\quad A_0 = -20\times\log\frac{3}{0.4} + 23 = 5\text{ db}$$

At $\omega = \omega_{c4}$, $A_0 = -40 \times \log\frac{4.5}{3} + 5 = -2$ db

At $\omega = \omega_{c5}$, $A_0 = -20 \times \log\frac{6}{4.5} + (-2) = -4$ db

At $\omega = \omega_{c6}$, $A_0 = -40 \times \log\frac{65}{6} + (-4) = -45$ db

At $\omega = \omega_h$, $A_0 = -60 \times \log\frac{80}{65} + (-45) = -50$ db

Using the values of A_0 at various frequencies the magnitude plot of compensated system is drawn as shown in fig 6.9.2.

PHASE PLOT

The phase angle of $G_0(j\omega)$ as a function of ω is given by

$$\phi_0 = \angle G_0(j\omega) = \tan^{-1}2.5\omega + \tan^{-1}0.22\omega - 90° - \tan^{-1}35\omega$$
$$-\tan^{-1} 0.0154\omega - \tan^{-1} 0.33\omega - \tan^{-1} 0.167\omega.$$

The phase angle of $G_0(j\omega)$ are calculated for various values of ω and listed in table-4.

TABLE-4

ω rad/sec	0.01	0.03	0.1	0.4	1	4	10	65	80
$\angle G_0(j\omega)$ deg	−108	−132	−152	−138	−126	−144	−168	−221 ≈ −220	−228

Using the values of ϕ_0 listed in table-4, the phase plot of compensated system is sketched as shown in fig 6.9.2.

Let ϕ_{gc0} = Phase of $G_0(j\omega)$ at the gain crossover frequency of compensated system.

and γ_0 = Phase margin of compensated system.

From the bode plot of compensated system, we get, $\phi_{gc0} = -144°$

Now, $\gamma_0 = 180° + \phi_{gc0} = 180° - 144° = 36°$

CONCLUSION

The phase margin of the compensated system is satisfactory. Hence the design is acceptable.

RESULT

Transfer function of lag - lead compensator, $G_c(s) = \frac{(1+2.5s)(1+0.22s)}{(1+35s)(1+0.0154s)}$

$$\left.\begin{array}{l}\text{Open loop transfer}\\ \text{function of}\\ \text{compensated system}\end{array}\right\} G_0(s) = \frac{80(1+2.5s)(1+0.22s)}{s(1+35s)(1+0.0154s)(1+0.33s)(1+0.167s)}$$

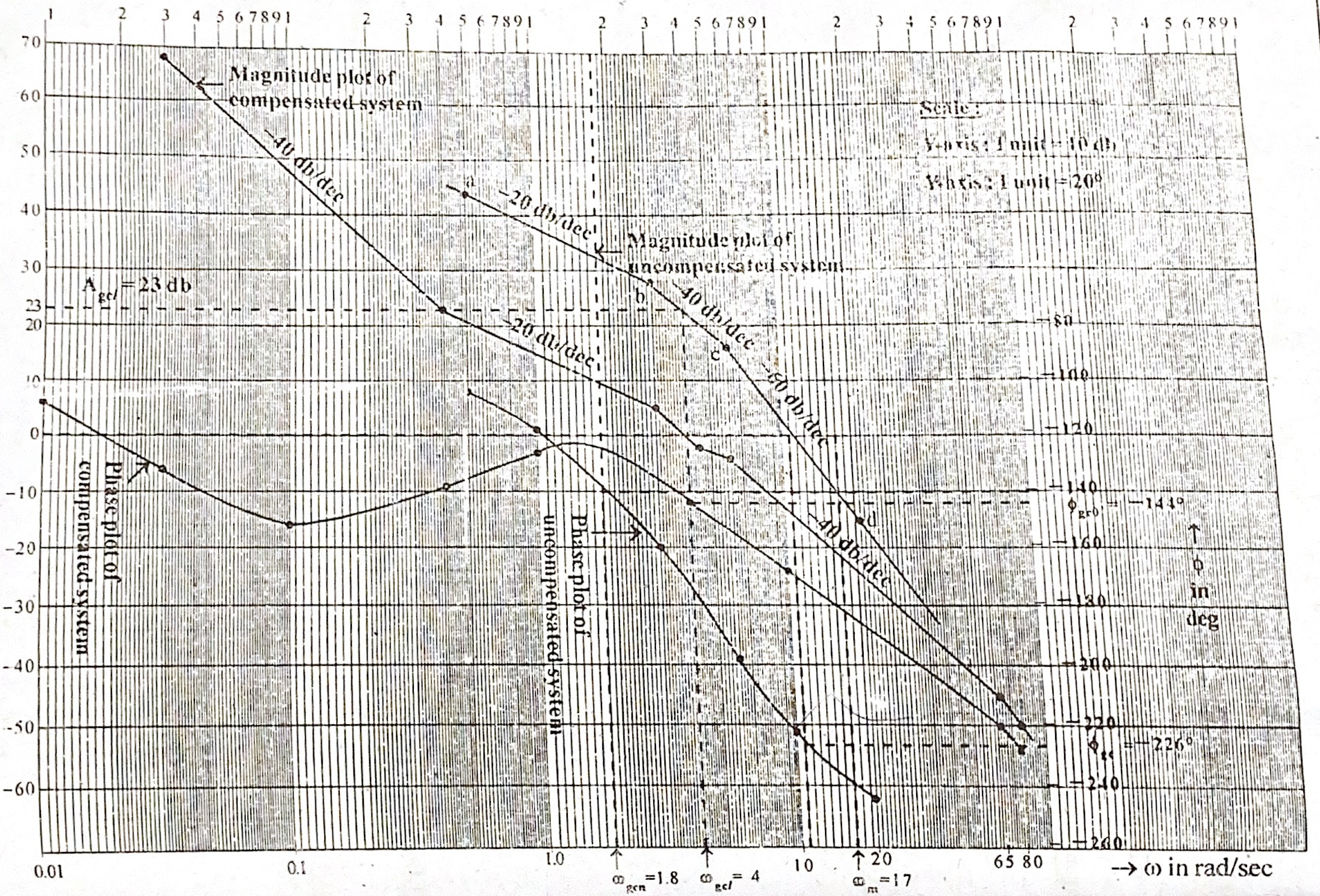


Fig 6.9.2 : Bode plot of $G(j\omega) = 80/j\omega(1 + j0.33\omega)(1 + j0.167\omega)$.

EXAMPLE 6.10

Design a lag-lead compensator for a system with open loop transfer function G(s) = K/s (s+0.5) to satisfy the following specifications. (i) Damping ratio of dominant closed-loop poles, ζ = 0.5. (ii) Undamped natural frequency of dominant closed loop poles, ω_n = 5 rad/sec. (iii) Velocity error constant, K_v = 80 sec^{-1}

SOLUTION

Step-1: Determine dominant pole, s_d.

$$\text{Dominant pole, } s_d = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2}$$

Given that, $\zeta = 0.5$ and $\omega_n = 5$ rad/sec.

$$\therefore s_d = -0.5 \times 5 \pm j5 \times \sqrt{1-0.5^2} = -2.5 \pm j4.3$$

Step-2 : Draw the pole-zero plot

The pole-zero plot of open loop transfer function is shown fig 6.10.1. Poles are represented by the symbol "x". The pole at point P is the dominant pole, s_d.

Step-3 : To find the angle to be contributed by lead network.

Let ϕ = Angle to be contributed by lead network to make point, P as a point on root locus.

$$\text{Now, } \phi = \begin{pmatrix} \text{sum of angles} \\ \text{contributed by poles} \\ \text{of uncompensated system} \end{pmatrix} - \begin{pmatrix} \text{sum of angles} \\ \text{contributed by zeros} \\ \text{of uncompensated system} \end{pmatrix} \pm n180°$$

From fig 6.10.1, we get,

$$\left.\begin{matrix} \text{Sum of angles contributed} \\ \text{by poles of uncompensated system} \end{matrix}\right\} = \theta_1 + \theta_2 = 120° + 115° = 235°$$

Since there is no finite zero in uncompensated system, there is no angle contribution by zeros

$$\therefore \phi = 235° \pm n180°$$

Let n = 1, $\quad \therefore \phi = 235° - 180° = 55°$.

Step-4 : To find the pole and zero of the lead section

Draw a line AP parallel to x-axis as shown in fig 6.10.1. The bisector PC is drawn to bisect the angle APO. The angles CPD and BPC are constructed as shown in fig 6.10.1. Here $\angle CPD = \angle BPC = \phi/2 = 55°/2 = 27.5° \approx 27°$.

From fig 6.10.1., Pole of the lead section, $p_{c2} = -10$

Zero of the lead section, $z_{c2} = -2.65$.

We know that, $z_{c2} = -1/T_2 \qquad \therefore T_2 = 1/2.65 = 0.377$

We know that, $p_{c2} = -1/\alpha T_2 \qquad \therefore \alpha T_2 = 1/10$ (or) $\alpha = 1/(T_2 \times 10) = 0.265$.

Step-5 : Transfer function of lead compensator

$$\text{Transfer function of lead section, } G_2(s) = \frac{(s+1/T_2)}{(s+1/\alpha T_2)} = \frac{(s+2.65)}{(s+10)}$$

Step-6 : To find gain, K

$$\left.\begin{array}{l}\text{Open loop transfer function}\\ \text{of lead compensated system}\end{array}\right\} G_{02}(s) = G_2(s) \times G(s) = \frac{(s+2.65)}{(s+10)} \times \frac{K}{s(s+0.5)} = \frac{K\,(s+2.65)}{s(s+0.5)\,(s+10)}$$

Here the value of K is given by the value of gain at the dominant pole s_d on the root locus. From magnitude condition K is given by,

$$K = \frac{\text{Product of vector lengths from all poles to } s = s_d}{\text{Product of vector lengths from all zeros to } s = s_d}$$

From fig 6.10.1, we get,

$$K = \frac{l_1 \times l_2 \times l_4}{l_3} = \frac{5 \times 4.75 \times 7.75}{4.3} = 42.8$$

$$G_{02}(s) = \frac{42.8\,(s+2.65)}{s\,(s+0.5)\,(s+10)}$$

Step-7 : To find velocity error constant of lead compensated system.

Let, K_{v2} = Velocity error constant of lead compensated system.

$$\therefore K_{v2} = \underset{s\to 0}{\mathrm{Lt}}\ s.\ G_{02}(s) = \underset{s\to 0}{\mathrm{Lt}}\ s\ \frac{42.8\,(s+2.65)}{s\,(s+0.5)\,(s+10)} = \frac{42.8 \times 2.65}{0.5 \times 10} = 22.684$$

Step-8 : To find the parameter, β

Let K_{vd} = Desired velocity error constant

A = The factor by which K_v is increased

Now, $A = K_{vd} / K_{v2} = 80 / 22.684 = 3.5267$

Select β, such that β > A. Let β = 4.

Step-9 : To find the transfer function of lag section.

Let zero of lag section, z_{c1} = 0.1 × second pole of G(s) = 0.1 × (−0.5) = −0.05

$$\text{Also, } z_{c1} = \frac{-1}{T_1}\ ;\quad \therefore T_1 = \frac{1}{0.05} = 20$$

$$\text{Pole of lag section, } p_{c1} = \frac{-1}{\beta T_1} = \frac{-1}{4 \times 20} = \frac{-1}{80} = -0.0125$$

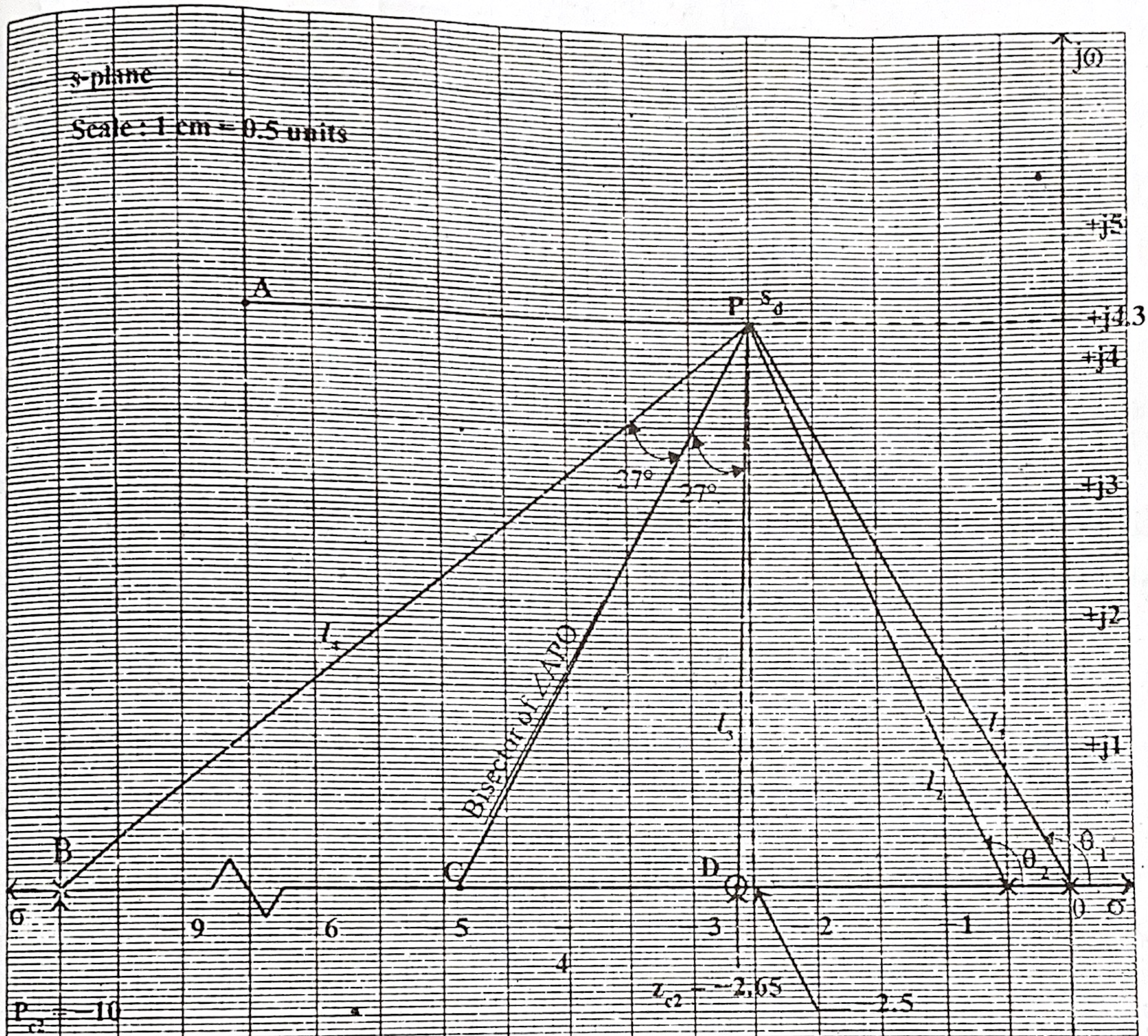
$$\text{Transfer function of lag section, } G_1(s) = \frac{(s+1/T_1)}{(s+1/\beta T_1)} = \frac{(s+0.05)}{(s+0.0125)}$$

Step-10 : Transfer function of compensated system

$$\left.\begin{array}{l}\text{Transfer function of}\\ \text{lag - lead compensator}\end{array}\right\} G_c(s) = G_1(s) \times G_2(s) = \frac{(s+0.05)\,(s+2.65)}{(s+0.0125)\,(s+10)}$$

The lag-lead compensator is connected in series with G(s) as shown in fig 6.10.2.

$$\left.\begin{array}{l}\text{Open loop transfer function}\\ \text{of compensated system}\end{array}\right\} G_0(s) = \frac{42.8\,(s+0.05)\,(s+2.65)}{s\,(s+0.0125)\,(s+0.5)\,(s+10)}$$



$$\theta_1 = 180° - \tan^{-1}\frac{4.3}{2.5} = 120°$$

$$\theta_2 = 180° - \tan^{-1}\frac{4.3}{2.0} = 115°$$

$$l_1 = 10 \times 0.5 = 5$$

$$l_2 = 9.5 \times 0.5 = 4.75$$

$$l_3 = 8.6 \times 0.5 = 4.3$$

$$l_4 = 15.5 \times 0.5 = 7.75$$

$$K_1 = \frac{l_1 \times l_2 \times l_4}{l_3} = \frac{5 \times 4.75 \times 7.75}{4.3} = 42.8$$

Fig 6.10.1 : Pole zero plot of lead compensated system.

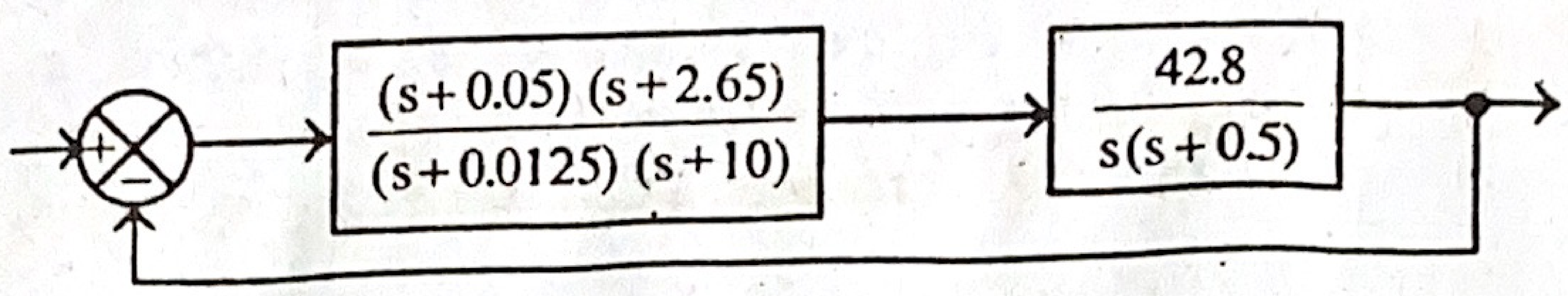


Fig 6.10.2 : Block diagram of lag-lead compensated system.

Step-11 : Check velocity error constant of compensated system.

Velocity error constant of compensated system $\Big\}$ $K_{vc} = \underset{s \to 0}{Lt}\ s\,G_0(s) = \underset{s \to 0}{Lt}\ s\ \dfrac{42.8\,(s+0.05)\,(s+2.65)}{s\,(s+0.0125)\,(s+0.5)\,(s+10)}$

$$= \frac{42.8 \times 0.05 \times 2.65}{0.0125 \times 0.5 \times 10} = 90.7$$

CONCLUSION

The velocity error constant of the compensated system satisfies the requirement. Hence the design is accepted.

RESULT

Transfer function of lag - lead compensator, $G_c(s) = \dfrac{(s+0.05)\,(s+2.65)}{(s+0.0125)\,(s+10)}$

Open loop transfer function of lag - lead compensated system $\Big\}$ $G_0(s) = \dfrac{42.8\,(s+0.05)\,(s+2.65)}{s\,(s+0.0125)\,(s+0.5)\,(s+10)}$

6.5 PI, PD AND PID CONTROLLERS

A controller with transfer function, $G_c(s)$ can be introduced in cascade with open loop transfer function, G(s) as shown in fig 6.21, to modify the transient and steady state response of the system.

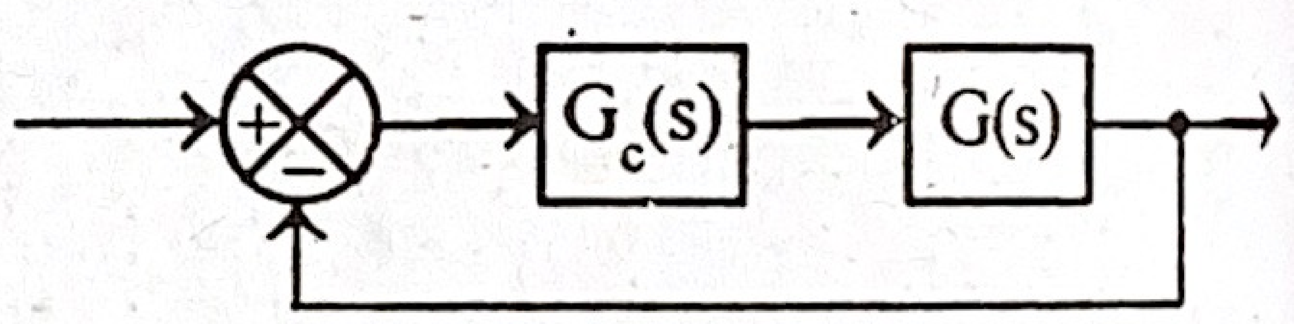


Fig 6.21 : Block diagram of a system with a controller in cascade.

The different types of controllers employed in control system are the following

1. *Proportional controller (P-controller)*
2. *Proportional-plus-integral controller (PI-controller)*
3. *Proportional-plus-derivative controller (PD-controller)*
4. *Proportional-plus-derivative-plus-integral controller (PID-controller)*

The proportional controller is a device that produces an output signal, u(t), which is proportional to the input signal, e(t).

In P-controller, $u(t) \propto e(t)$

$$\therefore u(t) = K_p\, e(t) \qquad .....(6.49)$$

where, K_p = Proportional gain or constant.

On taking laplace transform of equation (6.49) we get,

$$U(s) = K_p\, E(s)$$

Transfer function of P - controller, $G_c(s) = \dfrac{U(s)}{E(s)} = K_p$ .....(6.50)

6.7 SHORT QUESTIONS AND ANSWERS

Q6.1. *What are the time domain specifications needed to design a control system?*

The time domain specifications needed to design a control system are

1. Rise time, t_r
2. Peak overshoot, M_p
3. Settling time, t_s
4. Damping ratio, ζ
5. Natural frequency of oscillation, ω_n

Q6.2. *Write the necessary frequency domain specifications for design of a control system.*

The frequency domain specifications required to design a control system are,

1. Phase margin
2. Gain margin
3. Resonant peak
4. Bandwidth

Q6.3. *What are the two methods of designing a control system?*

Two methods of designing a control system are design using root locus and design using bode plot. In design using root locus, the system is designed to satisfy the specified time domain specifications. In design using bode plot, the system is designed to satisfy the specified frequency domain specifications.

Q6.4. *What is compensation?*

The compensation is the design procedure in which the system behaviour is altered to meet the desired specifications, by introducing additional device called compensator.

Q6.5. *What is compensator? What are the different types of compensator?*

A device inserted into the system for the purpose of satisfying the specifications is called compensator. The different types of compensators are lag compensator, lead compensator and lag-lead compensator.

Q6.6. *What are the two types of compensation schemes?*

The two types of compensation schemes employed in control system are series compensation and feedback or parallel compensation.

Q6.7. *What is series compensation?*

The series compensation is a design procedure in which a compensator is introduced in series with plant to alter the system behaviour and to provide satisfactory performance (i.e., to meet the desired specifications). The block diagram of series compensation scheme is shown in fig Q6.7.

$G_c(s)$ = Transfer function of series compensator
$G(s)$ = Open loop transfer function of the plant.
$H(s)$ = Feedback path transfer function.

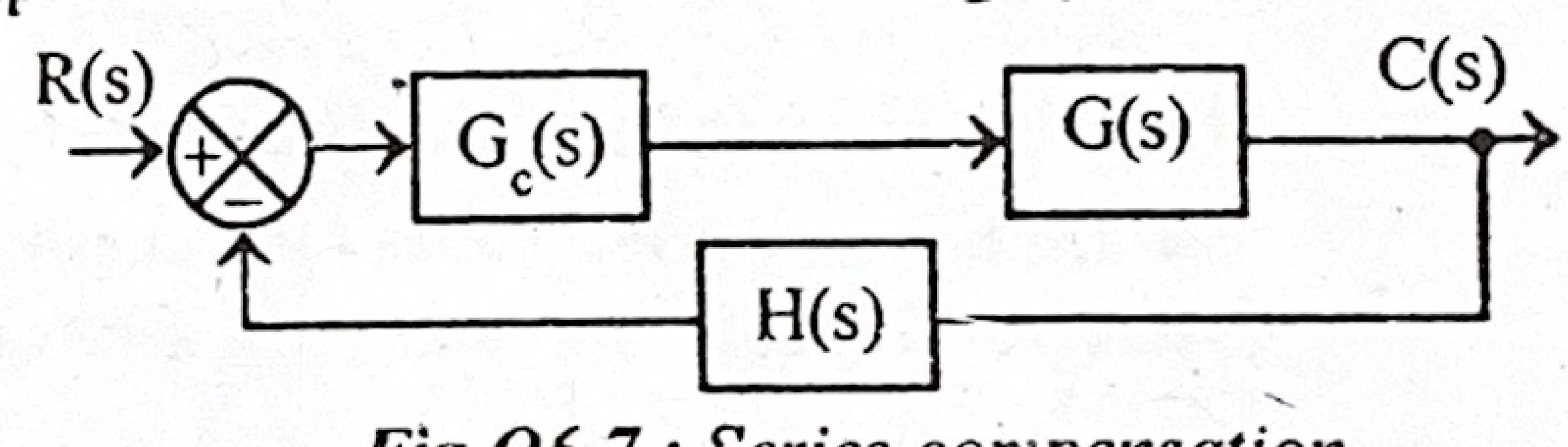


Fig Q6.7 : Series compensation.

Q6.8. *What is feedback compensation?*

The feedback compensation is a design procedure in which a compensator is introduced in the feedback path so as to meet the desired specifications. It is also called parallel compensation. The block diagram of feedback compensation scheme is shown in fig Q6.8.

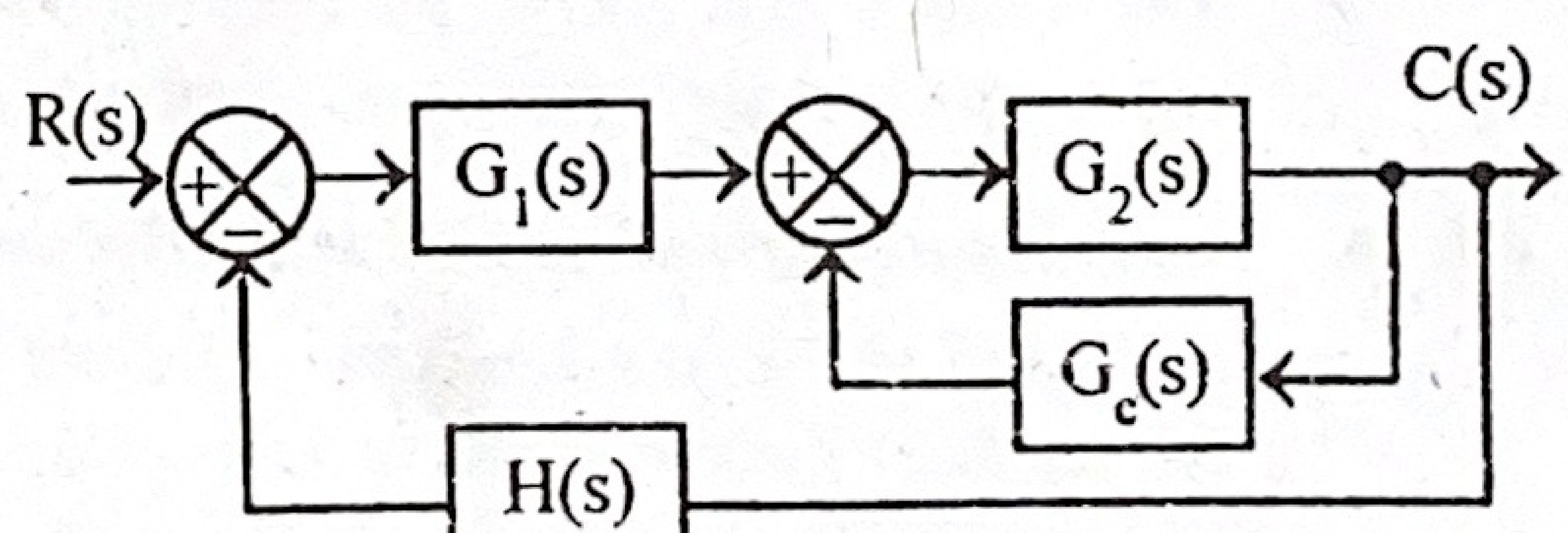


Fig Q6.8 : Feedback compensation

$G_c(s)$ = Transfer function of feedback compensator

$H(s)$ = Feedback path transfer function.

$G_1(s), G_2(s)$ = Open loop transfer function of the components of the plant.

Q6.9. ***What are the factors to be considered for choosing series or shunt/feedback compensation?***

The choice between series, shunt or feedback compensation depends on the following

1. Nature of signals in the system.
2. Power levels at various points.
3. Components available.
4. Designer's experience.
5. Economic considerations.

Q6.10. ***When lag/lead/lag-lead compensation is employed?***

Lag compensation is employed for a stable system for improvement in steady state performance.

Lead compensation is employed for stable/unstable system for improvement in transient-state performance.

Lag-lead compensation is employed for stable/unstable system for improvement in both steady-state and transient state performance.

Q6.11. ***Why compensation is necessary in feedback control system?***

In feedback control systems compensation is required in the following situations.

1. When the system is absolutely unstable, then compensation is required to stabilize the system and also to meet the desired performance.
2. When the system is stable, compensation is provided to obtain the desired performance.

Q6.12. ***Discuss the effect of adding a pole to open loop transfer function of a system.***

The addition of a pole to open loop transfer function of a system will reduce the steady-state error. The closer the pole to origin lesser will be the steady-state error. Thus the steady-state performance of the system is improved.

Also the addition of pole will increase the order of the system, which inturn makes the system less stable than the original system.

Q6.13. ***Discuss the effect of adding a zero to open loop transfer function of a system.***

The addition of a zero to open loop transfer function of a system will improve the transient response. The addition of zero reduces the rise time. If the zero is introduced close to origin then the peak overshoot will be larger. If the zero is introduced far away from the origin in the left half of s-plane then the effect of zero on the transient response will be negligible.

Q6.14. ***How root loci are modified when a zero is added to open loop transfer function?***

The addition of a zero to open loop transfer function will pull the root locus to the left which make the system more stable and reduce the settling time.

Q6.15. ***How the root loci are modified when a pole is added to open loop transfer function of the system?***

The addition of a pole to the open-loop transfer function has the effect of pulling the root locus to the right, which reduce the relative stability of the system and increase the settling time.

Q6.16. ***How control system design is carried using root locus?***

In design using root locus the transient response specifications are translated into desired locations for a pair of dominant closed loop poles.

In order to satisfy the performance specifications, the root loci should pass through these points. Hence a compensator is introduced in open loop transfer function which will reshape the root loci and force them to pass through the points where the dominant closed loop poles are located.

Q6.17. *What is the advantage in design using root locus.*

The advantage in design using root locus technique is that the information about closed loop transient response and frequency response are directly obtained from the pole-zero configuration of the system in the s-plane.

Q6.18. *What are the informations that can be obtained from frequency response plots?*

The low frequency region of frequency response plot provides information regarding the steady-state performance and high frequency region provides information regarding the transient performance of the system. The mid-frequency region provides information regarding relative stability.

Q6.19. *What are the advantages and disadvantages in frequency domain design.*

The advantages of frequency domain design are the following.

1. The effect of disturbances, sensor noise and plant uncertainities are easy to visualize and asses in frequency domain.
2. The experimental information can be used for design purposes.

The disadvantage of frequency response design is that it gives the information on closed loop system's transient response indirectly.

Q6.20. *What is lag compensation?*

The lag compensation is a design procedure in which a lag compensator is introduced in the system so as to meet the desired specifications.

Q6.21. *What is lag compensator? Give an example.*

A compensator having the characteristics of a lag network is called lag compensator. If a sinusoidal signal is applied to a lag compensator, then in steady state the output will have a phase lag with respect to input. An electrical lag compensator can be realised by a R-C network. The R-C network shown in fig Q6.21 is an example of electrical lag compensator.

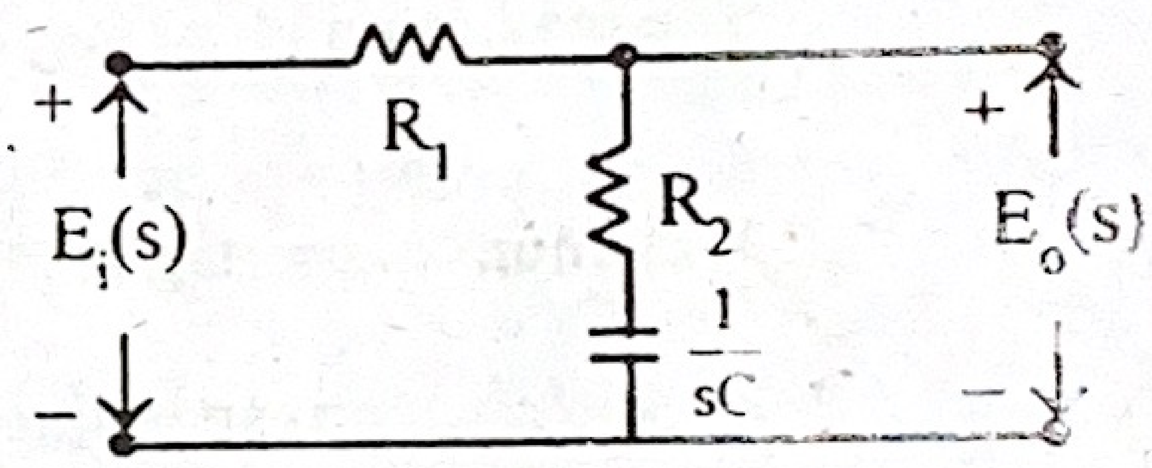


Fig Q6.21 : Lag compensator

Q6.22. *Write the transfer function of lag compensator and draw its pole-zero plot.*

$$\left.\begin{array}{l}\text{Transfer function of}\\ \text{lag compensator}\end{array}\right\} g_c(s) = \frac{s+\frac{1}{T}}{s+\frac{1}{\beta T}}$$

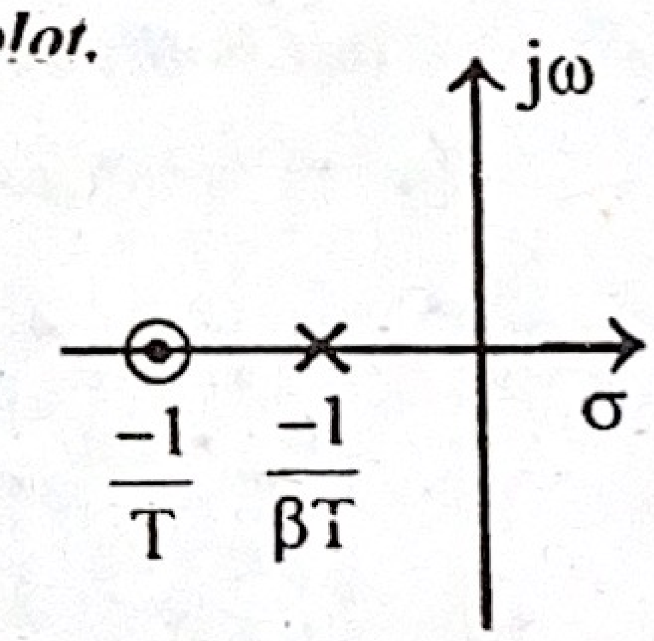


Fig Q 6.22 : Pole-zero plot of lag compensator

The lag compensator has a pole at $s = -1/\beta T$ and a zero at $s = -1/T$. Since $\beta > 1$ and $T > 0$, the pole of lag compensator is nearer to origin. The pole-zero plot of lag compensator is shown in fig Q6.22.

Q6.23. *What are the characteristics of lag compensation? When lag compensation is employed?*

The lag compensation improves the steady state performance, reduce the bandwidth and increases the rise time. (The increase in rise time results in slower transient response). If the pole introduced by the compensator is not cancelled by a zero in the system then the lag compensator increases the order of the system by one.

When a system is stable and does not satisfy the steady-state performance specifications then lag compensation can be employed so that the system s redesigned to satisfy the steady-state requirements.

Q6.24. ***Draw the bode plot of lag compensator.***

Let, $G_c(j\omega)$ = Sinusoidal transfer function of lag compensator.

The approximate magnitude plot and phase plot at $G_c(j\omega)$ are shown in fig Q6.24.

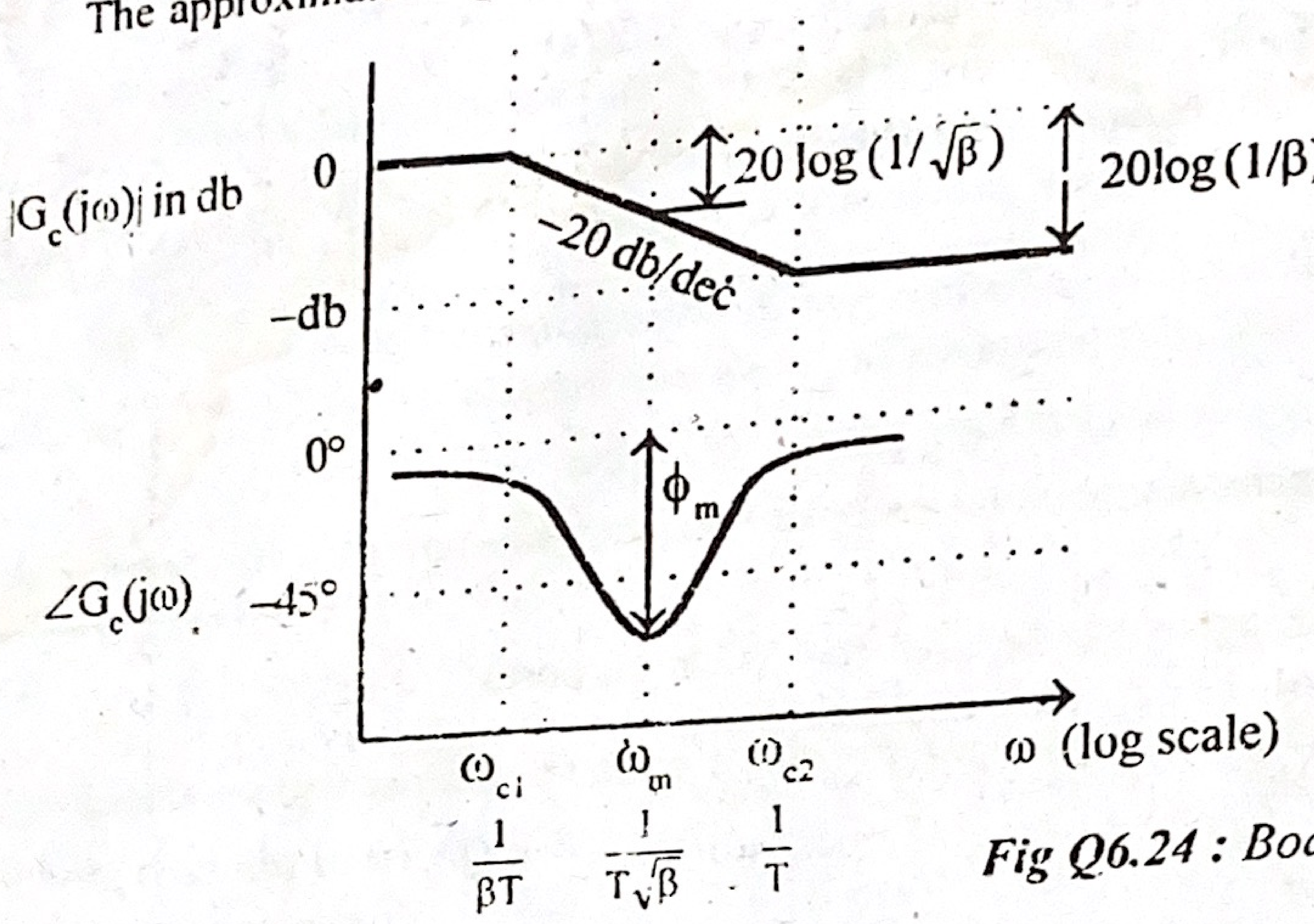


Fig Q6.24 : Bode plot of lag compensator.

Q6.25. ***When maximum phase lag occurs in lag compensator? Give the expressions for maximum lag angle and the corresponding frequency.***

The maximum phase lag occurs at the geometric mean of two corner frequencies of the lag compensator.

Maximum phase lag angle, $\phi_m = \tan^{-1}\dfrac{(1-\beta)}{2\sqrt{\beta}}$

Frequency corresponding to maximum phase lag angle $\Big\}$ $\omega_m = \sqrt{\omega_{c1}\,\omega_{c2}} = \sqrt{\dfrac{1}{T}\,\dfrac{1}{\beta T}} = \dfrac{1}{T\sqrt{\beta}}$.

Q6.26. ***What is the relation between ϕ_m and β in lag compensator?***

In lag compensator the ϕ_m and β are related by the expressions,

$$\phi_m = \tan^{-1}\left(\frac{1-\beta}{2\sqrt{\beta}}\right) \qquad \text{or} \qquad \beta = \frac{1-\sin\phi_m}{1+\sin\phi_m}$$

Since $\beta>1$, from the above expressions we can conclude that, larger the value of β the larger will be the value of ϕ_m.

Q6.27. ***Write the two equations that relates β and ϕ_m of lag compensator.***

The following two equations relates ϕ_m and β of lag compensator.

$$\phi_m = \tan^{-1}\frac{1-\beta}{2\sqrt{\beta}} \quad ; \quad \beta = \frac{1-\sin\phi_m}{1+\sin\phi_m}$$

Q6.28. ***What is lead compensation?***

The lead compensation is a design procedure in which a lead compensator is introduced in the system so as to meet the desired specifications.

Q6.29. What is lead compensator? Given an example.

A compensator having the characteristic of a lead network is called a lead compensator. If a sinusoidal signal is applied to a lead compensator, then in steady state the output will have a phase lead with respect to input. An electrical lead compensator can be realised by a R-C network. The R-C network shown in fig Q6.29. is an example of electrical lead compensator.

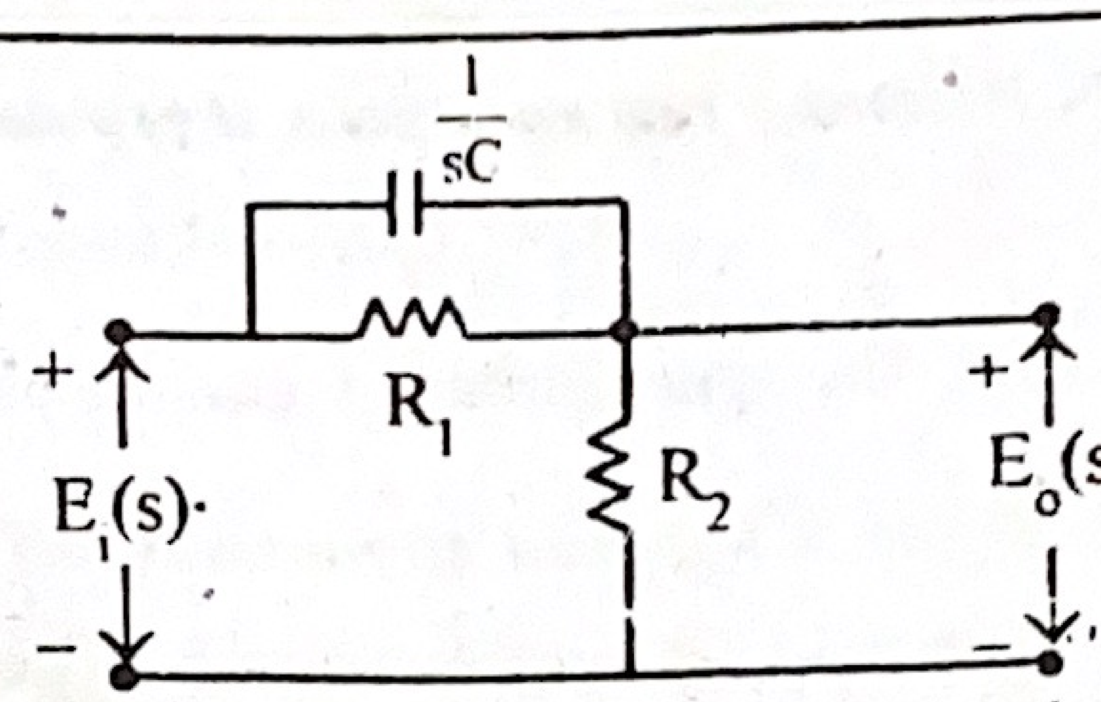
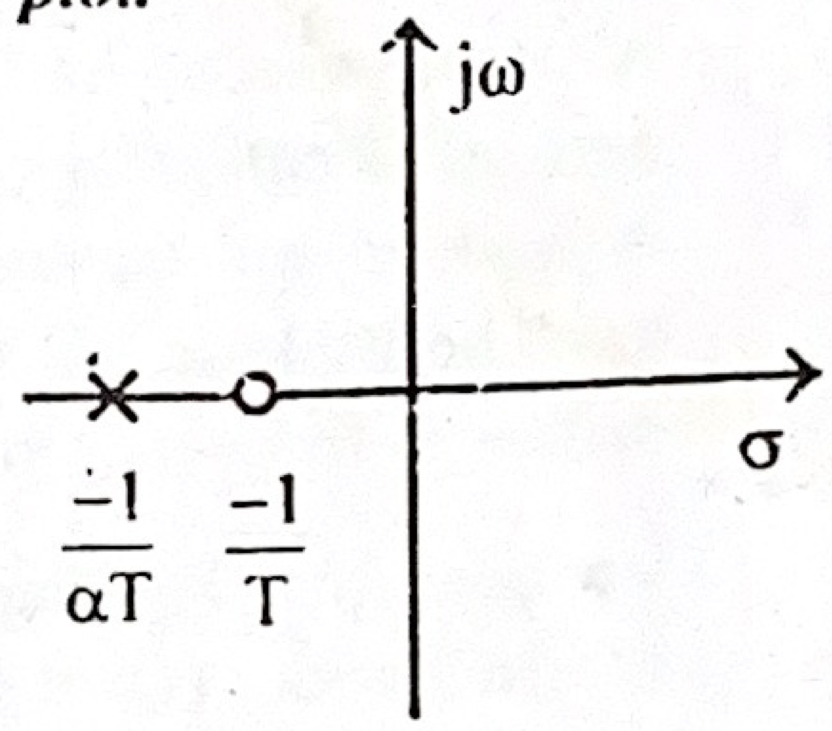


Fig Q6.29 : Electrical lead compensator

Q6.30. Write the transfer function of lead compensator and draw its pole-zero plot.

$$\text{Transfer function of lead compensator, } g_c(s) = \frac{s+\frac{1}{T}}{s+\frac{1}{\alpha T}}$$



The lead compensator has a pole at $s = -1/\alpha T$ and a zero at $s = -1/T$. Since $\alpha < 1$ and $T > 0$, the zero of lead compensator is nearer to origin. The pole-zero plot of lead compensator is shown in fig Q6.30.

Fig Q6.30 : Pole-zero plot of lead compensator

Q6.31. What are the characteristics of lead compensation?. When lead compensation is employed?

The lead compensation increases the bandwidth and improves the speed of response. It also reduces the peak overshoot. If the pole introduced by the compensator is not cancelled by a zero in the system, then lead compensation increases the order of the system by one. When the given system is stable/unstable and requires improvement in transient state response then lead compensation is employed.

Q6.32. Draw the bode plot of lead compensator.

Let, $G_c(j\omega)$ = Sinusoidal transfer function of lead compensator. The approximate magnitude plot and phase plot of $G_c(j\omega)$ are shown in fig Q6.32.

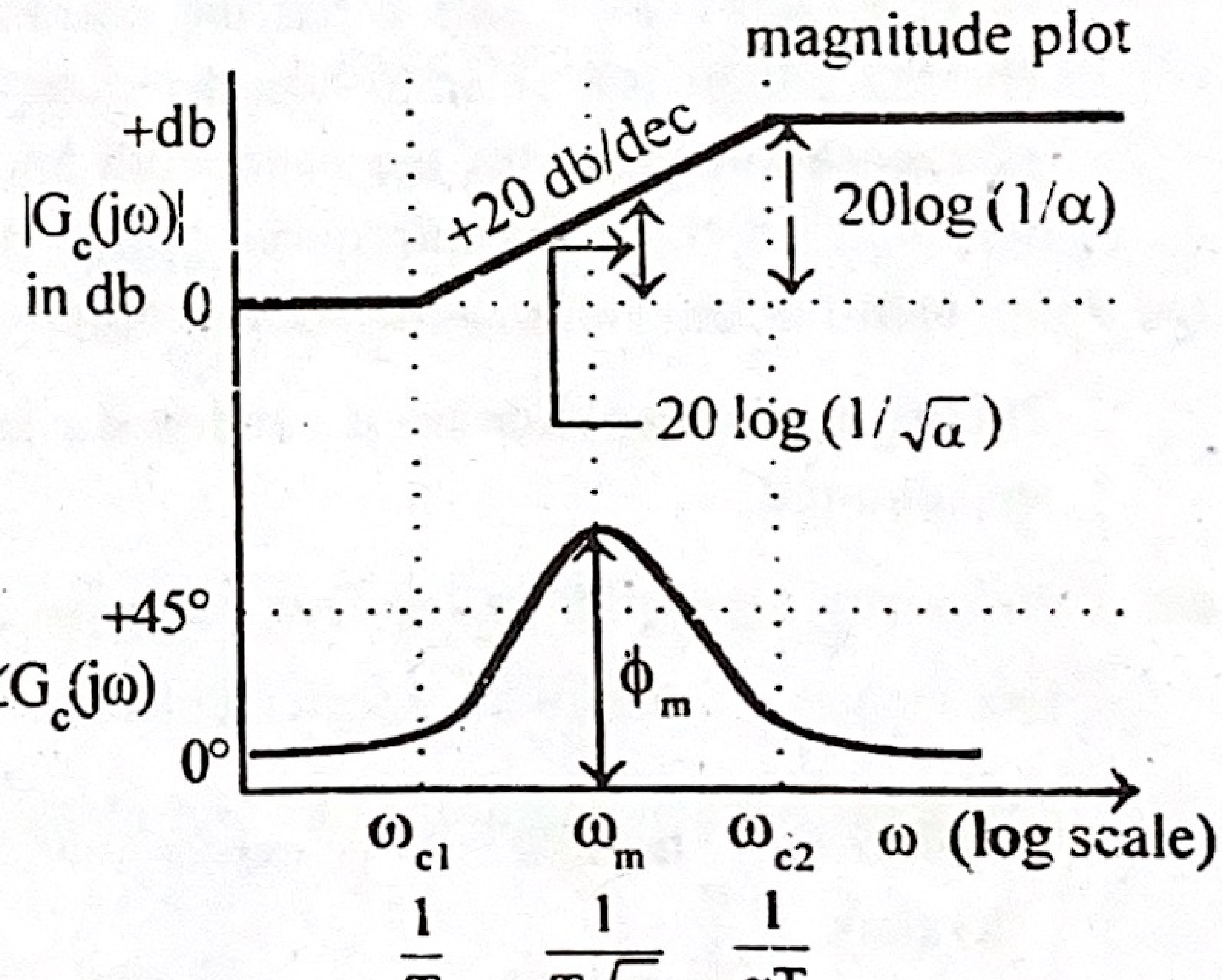


Fig Q6.32 : Bode plot of lead compensator

Q6.33. What is the relation between ϕ_m and α in lead compensator?

In lead compensator the ϕ_m and α are related by the expression,

$$\phi_m = \tan^{-1}\left(\frac{1-\alpha}{2\sqrt{\alpha}}\right) \quad \text{or} \quad \alpha = \frac{1-\sin\phi_m}{1+\sin\phi_m}$$

Since $\alpha < 1$ from the above expressions we can conclude that, smaller the value of α the larger will be the value of ϕ_m.

Q6.34. When maximum phase lead occurs in lead compensator? Give the expressions for maximum lead angle and the corresponding frequency.

The maximum phase lead occurs at the geometric mean of two corner frequencies of the lead compensator.

$$\text{Maximum phase lead angle, } \phi_m = \tan^{-1}\frac{(1-\alpha)}{2\sqrt{\alpha}}$$

$$\left.\begin{array}{l}\text{Frequency corresponding to}\\ \text{maximum phase lead angle}\end{array}\right\} \omega_m = \sqrt{\omega_{c1}\,\omega_{c2}} = \sqrt{\frac{1}{T}\,\frac{1}{\alpha T}} = \frac{1}{T\sqrt{\alpha}}$$

Q6.35. ***Write the two equations that relates α and ϕ_m of lead compensator***

$$\phi_m = \tan^{-1}\left(\frac{1-\alpha}{2\sqrt{\alpha}}\right) \quad ; \quad \alpha = \frac{1-\sin\phi_m}{1+\sin\phi_m}$$

Q6.36. ***What is lag-lead compensation?***

The lag-lead compensation is a design procedure in which a lag-lead compensator is introduced in the system so as to meet the desired specifications.

Q6.37. ***What is lag-lead compensator? Given an example.***

A compensator having the characteristics of lag-lead network is called lag-lead compensator. If a sinusoidal signal is applied to a lag-lead compensator then the output will have both phase lag and lead with respect to input, but in different frequency regions.

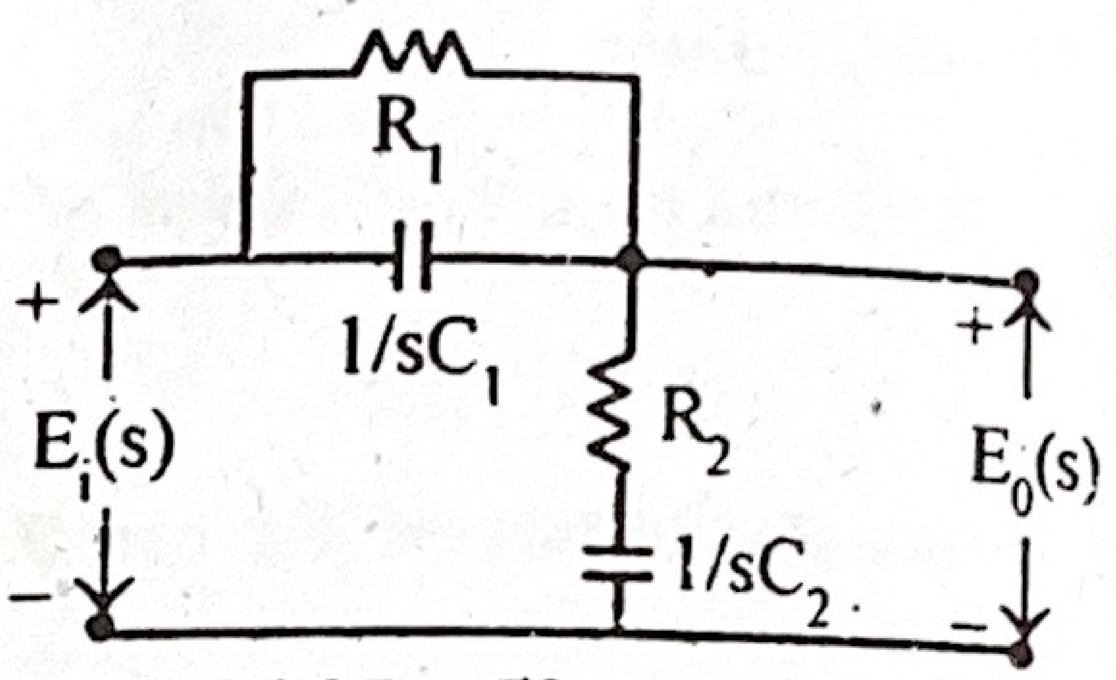


Fig Q6.37 : Electrical lag-lead compensator

An electrical lag-lead compensator can be realised by a R-C network. The R-C network shown in fig Q6.37 is an example of electrical lag-lead compensator.

Q6.38. ***Write the transfer function of lag-lead compensator and draw its pole-sero plot.***

$$\left.\begin{array}{l}\text{Transfer function of}\\ \text{lag - lead compensator}\end{array}\right\} G_c(s) = \underbrace{\frac{(s+1/T_1)}{(s+1/\beta T_1)}}_{\text{lag section}} \; \underbrace{\frac{(s+1/T_2)}{(s+1/\alpha T_2)}}_{\text{lead section}}$$

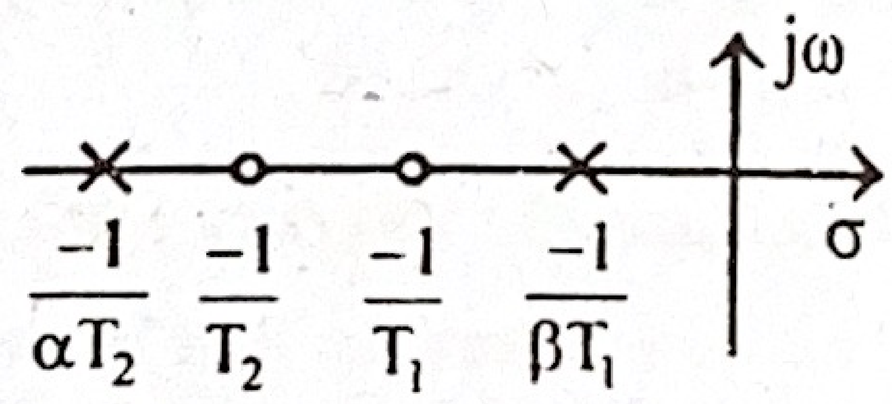


Fig Q 6.38 : Pole-zero plot of lag-lead compensator

Q6.39. ***What are the characteristics of lag-lead compensation? When lag-lead compensation is employed?***

The lag-lead compensation has the characteristics of both lag compensation and lead compensation. The lag compensation improves the steady state performance and decreases the bandwidth. The lead compensation increases the bandwidth and improves the speed of response. It also reduces the peak overshoot. If the poles introduced by the compensator is not cancelled by zeros in the system then the lag-lead compensator increases the order of the system by two.

The lag-lead compensation is employed when improvements in both steady-state and transient response are required.

Q6.40. ***Draw the bode plot of lag-lead compensator.***

Let, $G_c(j\omega)$ = Sinusoidal transfer function of lag-lead compensator.

The approximate magnitude plot and phase plot of $G_c(j\omega)$ are shown in fig Q6.40.

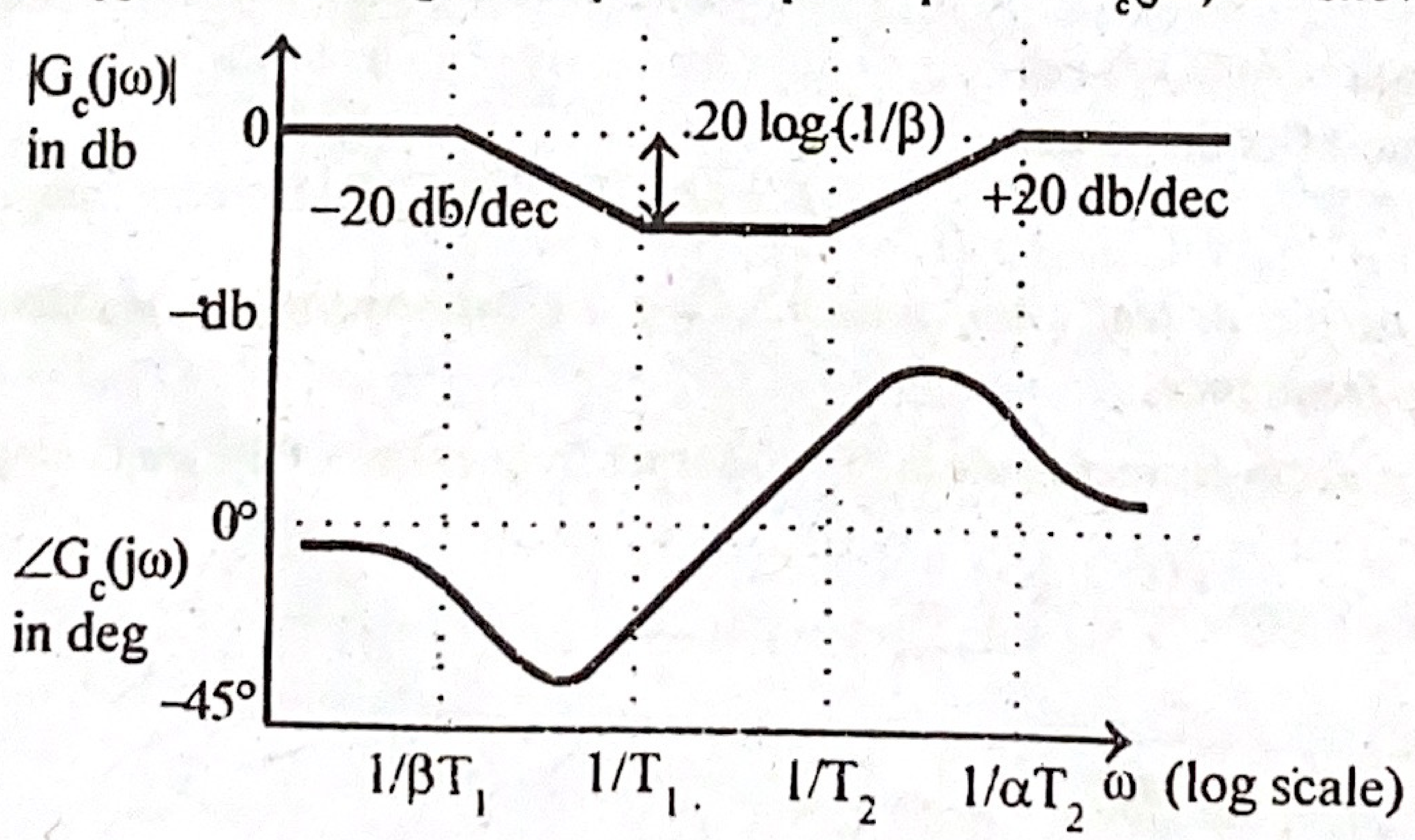


Fig Q6.40 : Bode plot of lag-lead compensator

Q6.41. What is P-controller and what are its characteristics?

The proportional controller is a device that produces an output signal which is proportional to the input signal.

The proportional controller improves the steady state tracking accuracy, disturbance signal rejection and relative stability. It also decreases the sensitivity of the system to parameter variations.

Q6.42. What is PI-controller and what are its effect on system performance?

The PI-controller is a device that produces an output signal consisting of two terms-one proportional to input signal and the other proportional to the integral of input signal.

The introduction of PI-controller in the system reduces the steady state error and increases the order and type number of the system by one.

Q6.43. Write the transfer function of PI-controller.

$$\left.\begin{array}{l}\text{Transfer function}\\ \text{of PI - controller}\end{array}\right\} G_c(s) = K_p + \frac{K_i}{s} = \frac{K_p\,(s + K_i / K_p)}{s}$$

Q6.44. What is PD-controller and what are its effect on system performance?

The PD-controller is a device that produces an output signal consisting of two terms-one proportional to input signal and the other proportional to the derivative of input signal.

The PD-controller increases the damping of the system which results in reducing the peak overshoot.

Q6.45. Write the transfer function of PD-controller.

Transfer function of PD - controller, $G_c(s) = K_p + K_d s = K_d\,(s + K_p / K_d)$

Q6.46. What is PID controller and what are its effect on system performance?

The PID controller is a device which produces an output signal consisting of three terms-one proportional to input signal, another one proportional to integral of input signal and the third one proportional to derivative of input signal.

The PID controller stabilises the gain, reduces the steady state error and peak overshoot of the system.

Q6.47. Write the transfer function of PID controller.

$$\left.\begin{array}{l}\text{Transfer function}\\ \text{of PID - controller}\end{array}\right\} G_c(s) = K_p + \frac{K_i}{s} + K_d s = \frac{K_d s^2 + K_p s + K_i}{s}$$

Q6.48. What is feedback compensation?

The feedback compensation is a design procedure in which a compensator is placed in an internal feedback path around one or more components of the forward path so as to meet the desired specifications.

Q6.49. Draw the block diagram of a feedback compensation scheme.

The block diagram of a popular feedback scheme employed in control systems is shown in fig Q6.49.

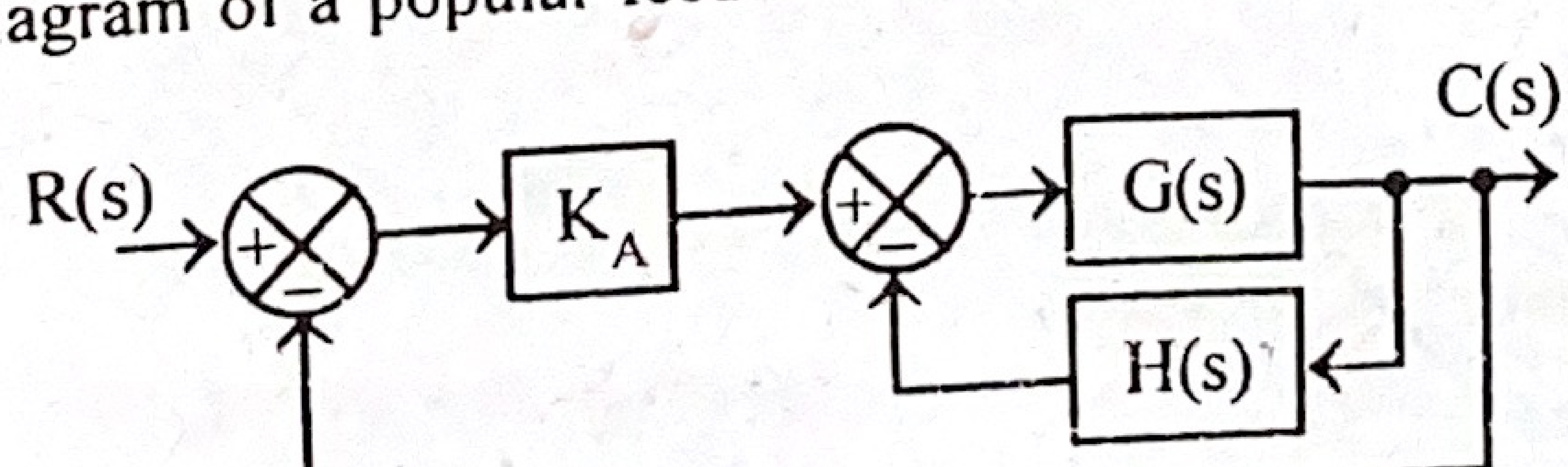


Fig Q6.49 : Feedback compensation scheme

Here. $H(s)$ = Transfer function of feedback compensator

K_A = A parameter to adjust the velocity error constant of the system.

Q6.50. ***What is the disadvantage in rate feedback and how it is eliminated?***

The disadvantage in the rate feedback is that the system velocity error constant, K_v is reduced. This undesirable effect can be eliminated by reducing the feedback signal in the low frequencies by introducing a high pass filter in cascade with rate device as shown in fig Q6.50.

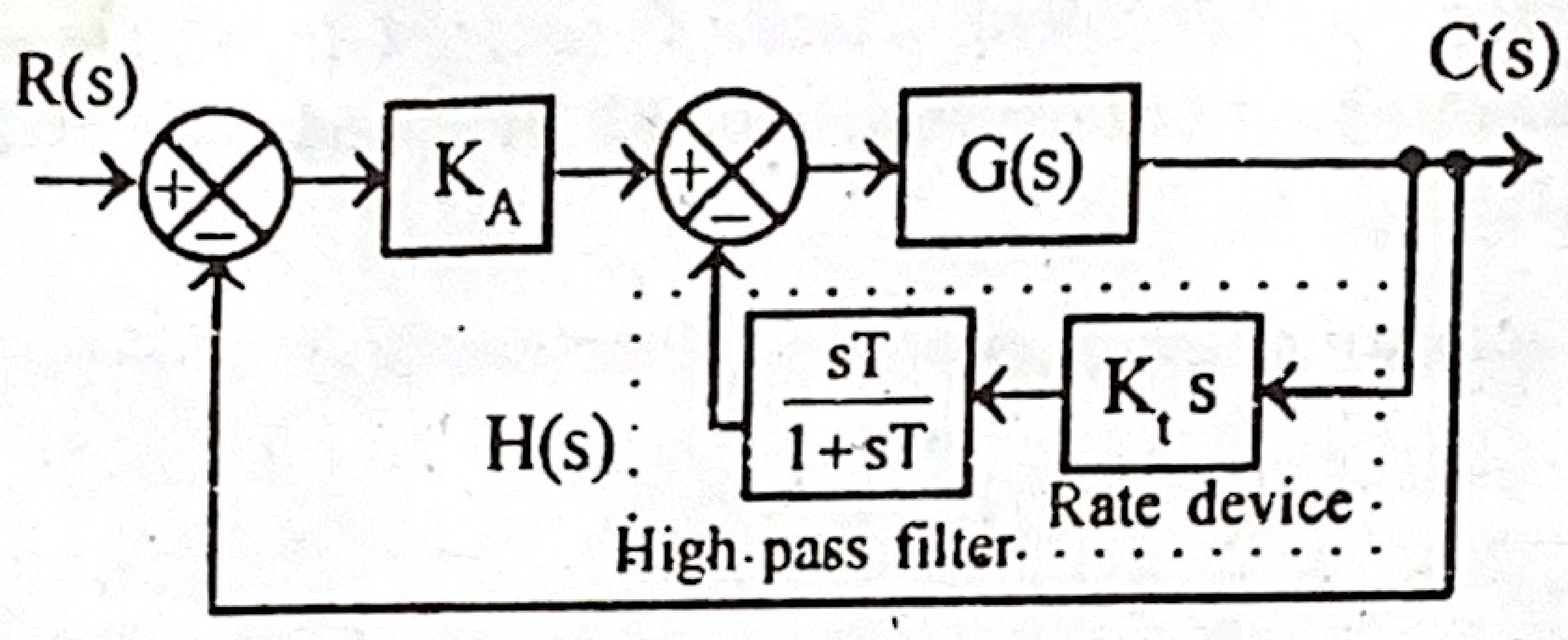


Fig Q.6.50 : Feedback compensation with high-pass filter in cascade with rate device

6.8 EXERCISES

1. Design a phase lag network for a system having $G(s) = K/s(1+0.2s)^2$ to have a phase margin of 30°.

2. The open loop transfer function of a certain unity feedback control system is given by $G(s) = K/s(s+1)$. It is desired to have the velocity error constant, $K_v = 10$ and the phase margin to be atleast 60°. Design a phase lag series compensator.

3. The controlled plant of a unity feedback system is $G(s) = K/s(s+5)$. It is specified that velocity error constant of the system be equal to 15, while the damping ratio is 0.6 and velocity error is less than 0.25 rad per unit ramp input. Design a suitable lag compensator.

4. Consider the unity feedback system with an open loop transfer function $G(s) = K/s(s+2)^2$. Design a suitable lead compensator so that phase margin is atleast 50° and velocity error constant is 20 s^{-6}

5. The open loop transfer function of certain unity feedback control system is given by $G(s) = K/s(0.1s+1)(0.2s+1)$. It is desired to have the phase margin to be atleast 30°. Design a suitable phase lead series compensator.

6. A unity feedback system with open loop transfer function $G(s) = K/s^2(s+1.5)$ is to be lead compensated to satisfy the following specifications.

   (i) Damping ratio = 0.45

   (ii) Undamped natural frequency, ω_n = 2.2 rad/sec

   (ii) Velocity error constant, K_v = 30.

7. Consider a unity feedback control system whose forward transfer function is $G(s) = K/s(s+2)(s+8)$. Design a lag-lead compensator so that $K_v = 80\ s^{-1}$ and dominant closed loop poles are located at $-2 \pm j2\sqrt{3}$

8. The open loop transfer function of uncompensated system is $G(s) = K/s\ (s+1)(s+4)$. Design a lag-lead compensator to meet the following specifications.

   (i) Velocity error constant ≥ 5

   (ii) Damping ratio = 0.4.

9. Consider a unity feedback system with open loop transfer function, $G(s) = K/s(2s+1)(0.5s+1)$. Design a suitable lag-lead compensator to meet the following specifications.

(i) $K_v = 30$ (ii) Phase margin ≥ 50

10. Consider a unity feedback system with open loop transfer function. $G(s)=1/s(s+1)$. Design a PD controller so that the phase margin of the system is 30° at a frequency of 2 rad/sec.

11. A unity feedback system has an open loop transfer function as $G(s) = 50/(s+3)(s+1)$. Design a PI controller so that phase margin of the system is 35° at a frequency of 1.2 rad/sec.

12. Consider a unity feedback system with open loop transfer function, $G(s)=20/(s+0.5)(s+2)(s+4)$. Design a PID controller so that the phase margin of the system is 30° at a frequency of 2 rad/sec and steady state error for unit ramp input is 0.1.

13. Consider a unity feedback system with open loop transfer function $G(s) = 10/s(s+4)$. The dominant poles are $-2 \pm j\sqrt{6}$. Design a suitable PD controller.

14. Consider a unity feedback system with open loop transfer function $G(s) = 10/(s+1)(s+2)$. Design a PI controller so that the closed loop has damping ratio of 0.707 and natural frequency of oscillation as 1.2 rad/sec.

15. Consider a unity feedback system with $G(s) = 50/(s+2)(s+10)$. Design a PID controller to satisfy the following specifications.

(i) $K_v \geq 2$

(ii) Damping ratio = 0.6

(iii) Natural frequency of oscillations = 2 rad/sec.

16. Design a feedback compensation scheme for a unity feedback system with open loop transfer function $G(s) = 1/s^2(s+5)$ to satisfy the following specifications. (i) phase margin of the system should be atleast 50°. (ii) Velocity error constant, $K_v \geq 20$.

17. Consider a unity feedback system with open loop transfer function, $G(s) = K/s(s+1)(s+4)$. Design a feedback compensator to satisfy the following specifications.

(a) Maximum overshoot, $M_p \leq 12\%$

(b) Settling time, $t_s = 10s$

(c) Velocity error constant, $K_v = 10$.