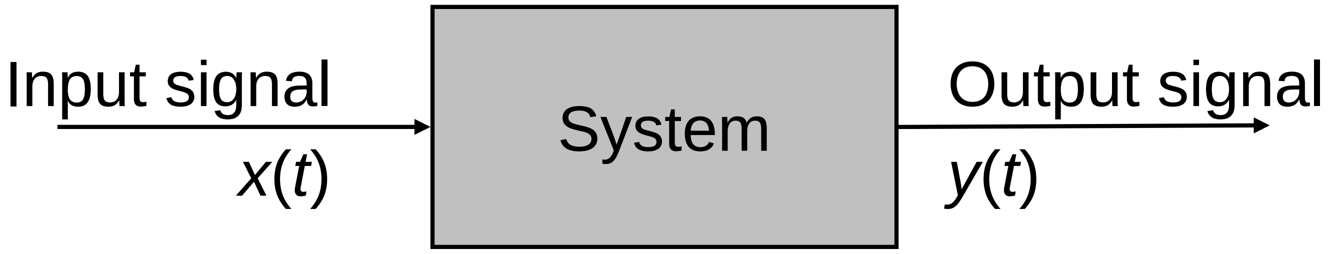
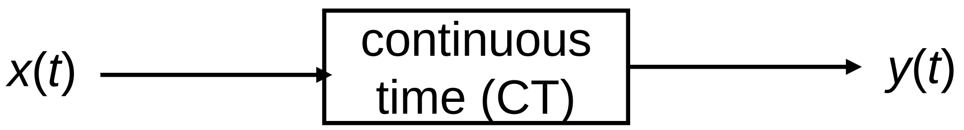
System Properties

What is a System?

- Systems process input signals to produce output signals
- A system takes a signal as an input and transforms it into another signal
- It is an operator operating on signals to produce output signal
- Examples:
  - A circuit involving a capacitor can be viewed as a system that transforms the source voltage (signal) to the voltage (signal) across the capacitor
  - A CD player takes the signal on the CD and transforms it into a signal sent to the loud speaker
  - A communication system is generally composed of three sub-systems, the transmitter, the channel and the receiver. The channel typically attenuates and adds noise to the transmitted signal which must be processed by the receiver

How is a System Represented?





How Are Signal & Systems Related ?

- How to design a system to process a signal in particular ways?

- Design a system to restore or enhance a particular signal
  - Remove **high frequency** background communication noise
  - Enhance **noisy** images from spacecraft

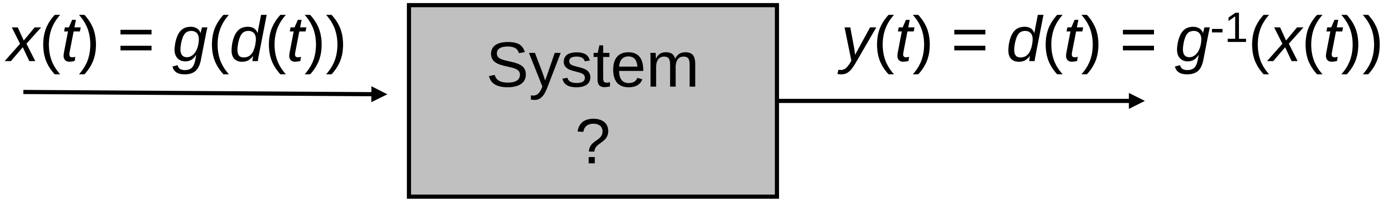
- Assume a signal is represented as
  - $x(t) = d(t) + n(t)$
- Design a system to remove the unknown “noise” component $n(t)$, so that $y(t) \approx d(t)$

$$x(t) = d(t) + n(t) \longrightarrow \boxed{\begin{array}{c}\text{System}\\ ?\end{array}} \longrightarrow y(t) \approx d(t)$$

How Are Signal & Systems Related ?

- How to design a system to extract specific pieces of information from signals
  - Estimate the heart rate from an electrocardiogram
  - Estimate economic indicators (bear, bull) from stock market values

- Assume a signal is represented as
  - $x(t) = g(d(t))$
- Design a system to "invert" the transformation $g()$, so that $y(t) = d(t)$



How Are Signal & Systems Related ?

- How to design a (dynamic) system to modify or control the output of another (dynamic) system
  - – Control an aircraft's altitude, velocity, heading by adjusting throttle, rudder, ailerons
  - – Control the temperature of a building by adjusting the heating/cooling energy flow.

- Assume a signal is represented as
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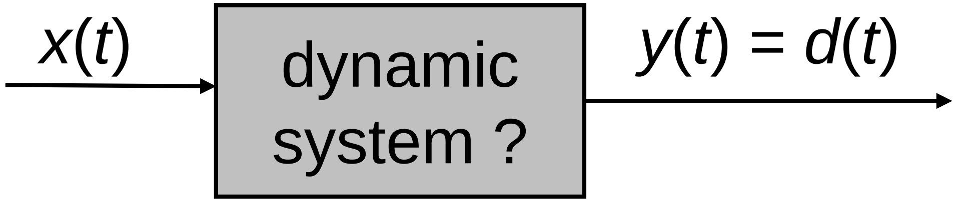


Examples of Simple Systems

- To get some idea of typical systems (and their properties), consider the electrical circuit example:

$$\frac{dv_c(t)}{dt}+\frac{1}{RC}v_c(t)=\frac{1}{RC}v_s(t)$$

which is a **first order**, **CT differential** equation.

- Examples of **first order**, **DT difference** equations:

$$y[n]=x[n]+1.01y[n-1]$$

where *y* is the monthly bank balance, and *x* is monthly net deposit

- Example of second order system includes: $$a\frac{d^2y(t)}{dt^2}+b\frac{dy(t)}{dt}+cy(t)=x(t)$$

System described by **order** and **parameters** (*a*, *b*, *c*)

System with and without Memory

- A system is said to be memoryless if its output for each value of the independent variable at a given time is dependent on the input at only that same time (Static system)

$$y[n] = (2x[n] - x^2[n])^2$$

- e.g. a resistor is a memoryless CT system where $x(t)$ is current and $y(t)$ is the voltage
- A DT system with memory is an accumulator (integrator)

$$y[n] = \sum_{k=-\infty}^{n} x[k]$$

- and a delay

$$y[n] = x[n-1]$$

- Roughly speaking, a memory corresponds to a mechanism in the system that retains information about input values other than the current time.

$$\begin{aligned} y[n] &= \sum_{k=-\infty}^{n-1} x[k] + x[n] \\ &= y[n-1] + x[n] \end{aligned}$$

System Causality

- A system is causal if the output at any time depends on values of the output at only the present and past times. Referred to as non-anticipative, as the system output does not anticipate future values of the input
- OR A system with input x and output y is said to be causal if, for every real to, y(to) does not depend on x(t) for some t > to.
- If the independent variable t represents time, a system must be causal in order to be physically realizable.
- Noncausal systems can sometimes be useful in practice, however, since the independent variable need not always represent time. For example, in some situations, the independent variable might represent position.
- Most physical systems are causal

E.g. The accumulator system is causal: $y[n] = \sum_{k=-\infty}^{n} x[k]$

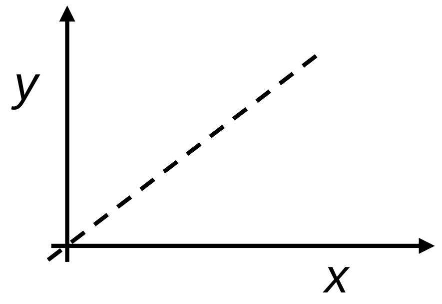
because $y[n]$ only depends on $x[n]$, $x[n\text{-}1]$, …

E.g. The averaging/filtering system is non-causal $y[n] = \frac{1}{2M+1} \sum_{k=-M}^{M} x[n-k]$

because $y[n]$ depends on $x[n+1]$, $x[n+2]$, …

Linear Systems

The most important property that a system possesses is **linearity**
It means allows any system response to be analysed as the sum of simpler responses (convolution)
Simplistically, it can be imagined as a line



- Specifically, a linear system must satisfy the two properties:
- **1 Additive**: the response to $x_1(t)+x_2(t)$ is $y_1(t) + y_2(t)$
- **2 Scaling**: the response to $ax_1(t)$ is $ay_1(t)$
- **Combined**: $ax_1(t)+bx_2(t) \rightarrow ay_1(t) + by_2(t)$

- E.g. Linear $y(t) = 3*x(t)$ why?
  Non-linear $y(t) = 3*x(t)+2$, $y(t) = 3*x^2(t)$ why?
- (equivalent definition for DT systems)

- The linearity property is also referred to as the superposition property.

- Practically speaking, linear systems are much easier to design and analyze than nonlinear systems.

Suppose an input signal $x[n]$ is made of a linear sum of other (basis/simpler) signals $x_k[n]$:

$$x[n] = \sum_k a_k x_k[n] = a_1 x_1[n] + a_2 x_2[n] + a_3 x_3[n] + \cdots$$

then the (linear) system response is:

$$y[n] = \sum_k a_k y_k[n] = a_1 y_1[n] + a_2 y_2[n] + a_3 y_3[n] + \cdots$$

The basic idea is that if we understand how simple signals get affected by the system, we can work out how complex signals are affected, by expanding them as a linear sum

Linear Systems

- Linear systems play a crucial role in most areas of science
  - Closed form solutions often exist
  - Theoretical analysis is considerably simplified
  - Non-linear systems can often be regarded as linear, for small perturbations, so-called linearization

Time Invariance

- A system is time invariant if its behaviour and characteristics are fixed over time
- We would expect to get the same results from an input-output experiment, if the same input signal was fed in at a different time

- A system H is said to be time invariant (TI) if, for every function x and every real number t0, the following condition holds:

$$y(t - t_0) = H\, x'(t) \text{ where } y = H\, x \text{ and } x'(t) = x(t - t_0)$$

- In other words, a system is time invariant if a time shift (i.e., advance or delay) in the input always results only in an identical time shift in the output.

- In simple terms, a time invariant system is a system whose behavior does not change with respect to time.

- Practically speaking, compared to time-varying systems, time-invariant systems are much easier to design and analyze, since their behavior does not change with respect to time.

E.g. The following CT system is **time-invariant** $y(t) = \sin(x(t))$

because it is invariant to a time shift, i.e. $x_2(t) = x_1(t\text{-}t_0)$

$$y_2(t) = \sin(x_2(t)) = \sin(x_1(t - t_0)) = y_1(x_1(t - t_0))$$

E.g. The following DT system is **time-varying** $y[n] = nx[n]$

Because the **system parameter** that multiplies the input signal is time varying, this can be verified by substitution

$$x_1[n] = \delta[n] \Rightarrow y_1[n] = 0$$
$$x_2[n] = \delta[n-1] \Rightarrow y_2[n] = \delta[n-1]$$

System Stability

- Informally, a stable system is one in which small input signals lead to responses that do not diverge
- If an input signal is bounded, then the output signal must also be bounded, if the system is stable

$$\forall x : |x| < U \rightarrow |y| < V$$

- **E.g.** Consider the DT system of the bank account

$$y[n] = x[n] + 1.01y[n-1]$$

This grows without bound, due to 1.01 multiplier. This system is unstable.

- **E.g.** Consider the CT electrical circuit, is stable if *RC*>0, because it dissipates energy

$$\frac{dv_c(t)}{dt} + \frac{1}{RC}v_c(t) = \frac{1}{RC}v_s(t)$$

Invertible and Inverse Systems

- A system is said to be **invertible** if distinct inputs lead to distinct outputs (similar to matrix invertibility)
- If a system is invertible, an inverse system exists which, when **cascaded** with the original system, yields an output equal to the input of the first signal
- E.g. the CT system is invertible: $y(t) = 2x(t)$

  because $w(t) = 0.5*y(t)$ recovers the original signal $x(t)$

- E.g. the CT system is not-invertible $y(t) = x^2(t)$

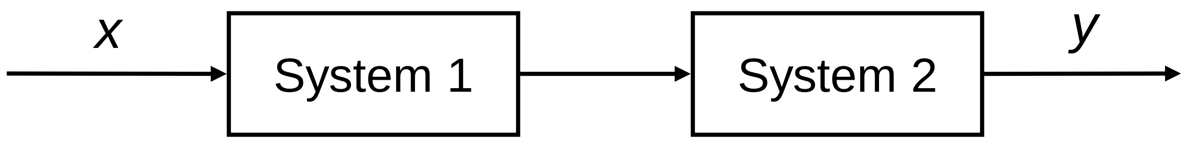
  because distinct input signals lead to the same output signal

- Widely used as a design principle:
  - Encryption, decryption
  - System control, where the reference signal is input

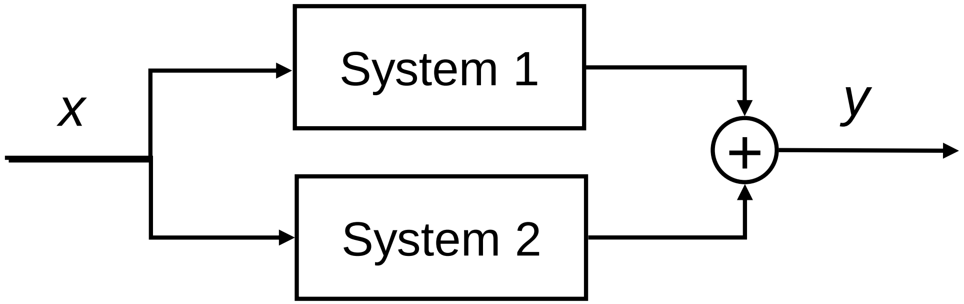
System Structures

- Systems are generally composed of components (sub-systems).
- We can use our understanding of the components and their interconnection to understand the operation and behaviour of the overall system

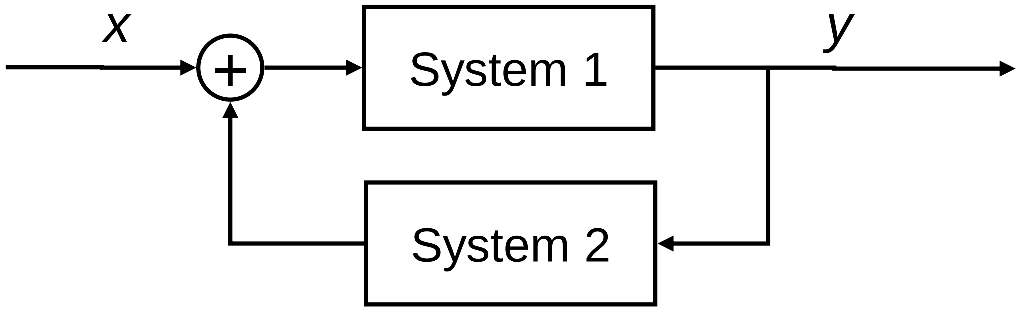
- **Series/cascade**



- **Parallel**



- **Feedback**



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