

SECOND SESSIONAL EXAMINATION, EVEN SEMESTER, (2022-2023)

Common To All

Time -2hrs

Maximum Marks - 45

1. Attempt any FIVE questions.

Q N	QUESTION	Marks	CO	BL
a.	Examine the convergence of the sequence $\left\{\frac{2n+1}{n}\right\}$.	2	CO3	L2
b.	State necessary condition for convergent series.	2	CO3	L1
c.	Examine the convergence of the series $\sum \frac{n+1}{n}$.	2	CO3	L2
d.	Find the constant term if the function $f(x) = \pi - x$ is expanded in Fourier series defined in $(0, 2\pi)$.	2	CO3	L3
e.	Find the constant term if the function $f(x) = x^2 - 2$ is expanded in Fourier series defined in $(-2, 2)$.	2	CO3	L3
f.	Examine the convergence of the series $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \dots$	2	CO3	L2

2. Attempt any ONE of the following.

Q N	QUESTION	Marks	CO	BL
a.	Find half range Fourier sine series of $f(x)$ defined over the range $0 < x < 4$ as $f(x) = \begin{cases} x & 0 < x < 2 \\ 4 - x & 2 < x < 4 \end{cases}$	5	CO3	L3
b.	Obtain a Fourier series to represent the function $f(x) = x $ and hence deduce $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots$	5	CO3	L4
c.	Test the convergence of the series $\frac{x}{1.2} + \frac{x^2}{3.4} + \frac{x^3}{5.6} + \dots$	5	CO3	L3

3. Attempt any FIVE questions.

Q N	QUESTION	Marks	CO	BL
a.	State necessary condition for a function to be analytic.	2	CO4	L1
b.	Show that $u = \frac{1}{2} \log(x^2 + y^2)$ is harmonic.	2	CO4	L3
c.	Find the image of $x = 2$ under the transformation of $w = \frac{1}{z}$.	2	CO4	L2
d.	Find the fixed point under the transformation $w = \frac{2z+6}{z+7}$.	2	CO4	L1
e.	Determine a, b, c, d so that the function $f(z) = (x^2 + axy + by^2) + i(cx^2 + dxy + y^2)$ is analytic.	2	CO4	L2
f.	Show that z^2 is analytic everywhere.	2	CO4	L3

4. Attempt any ONE of the following.

Q N	QUESTION	Marks	CO	BL
a.	Prove that $w = \frac{z}{1-z}$ maps the upper half of the z -plane into upper half of the w -plane. What is the image of the circle $ z = 1$ under this transformation?	5	CO4	L3
b.	Determine the analytic function whose real part is $e^{2x}(x \cos 2y - y \sin 2y)$.	5	CO4	L4
c.	Show that the function $f(z) = \begin{cases} \frac{x^3 y^5 (x+iy)}{x^6 + y^{10}} & z \neq 0 \\ 0 & z = 0 \end{cases}$ is not analytic at the origin even though it satisfies Cauchy Riemann equations at origin.	5	CO4	L4

5. Attempt any FIVE questions.

Q N	QUESTION	Marks	CO	BL
a.	Evaluate $\int_0^{1+i} (z - \bar{z}) dz$ along the line $y = 2x$	2	CO5	L2
b.	Evaluate $\oint_C \frac{e^{-z}}{z+1} dz$, where C is the circle $ z = \frac{1}{2}$	2	CO5	L2
c.	Expand $\cos z$ in a Taylor's series about $z = \frac{\pi}{4}$	2	CO5	L3
d.	Find the residue of $\frac{z^2}{(z-1)(z-2)^2}$ at the pole $z = 2$	2	CO5	L2
e.	Find the residue of $z \cos \frac{1}{z}$ at $z = 0$	2	CO5	L3
f.	What is conformal transformation?	2	CO5	L1

6. Attempt any ONE of the following.

Q N	QUESTION	Marks	CO	BL
a.	Expand $\frac{1}{(z+1)(z+3)}$ in Taylor's or Laurent's series in the regions I. $1 < z < 3$ II. $1 < z+1 < 2$	5	CO5	L3
b.	Evaluate $\oint_C \frac{z-3}{z^2+2z+5} dz$, where c is the circle $ z+1-i = 2$	5	CO5	L3
c.	Evaluate $\int_0^{3+i} \bar{z}^2 dz$ along the real axis from $z = 0$ to $z = 3$ and then along a line parallel to imaginary axis from $z = 3$ to $z = 3+i$.	5	CO5	L3

Bloom's Taxonomy Level (BL) :-

Remember (L1),

Understanding (L2),

Apply (L3),

Analyze (L4), Evaluating (L5),

Creating (L6)

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