Assignment 3

AI1110: Probability and Random Variables Indian Institute of Technology Hyderabad

Sudarshan Shivashankar AI22BTECH11027

Question 10.15.1.14: One card is drawn from a well-shuffled deck of 52 cards. Find the probability of getting

- 1) a king of red colour
- 2) a face card
- 3) a red face card
- 4) the jack of hearts
- 5) a spade
- 6) the queen of diamonds

Solution:

VARIABLE	RANGE	DESCRIPTION
X	{0,1}	The Random variable denoting the Color of the card.
Y	{0,1,2,3}	The Random variable denoting the Type of the card.
Z	{0,1,2,3,4,5,6,7,8,9,10,11,12}	The Random variable denoting the Value of the card.

TABLE 1

Total number of cards = 52

$$n(S) = 52 \tag{1}$$

$$\Pr\left(E\right) = \frac{n(E)}{n(S)}\tag{2}$$

For X={0,1},
$$\Pr(X) = \frac{n(X)}{n(S)} = \frac{26}{52} = \frac{1}{2}$$
 (3)

For Y={0,1,2,3},
$$Pr(Y) = \frac{n(Y)}{n(S)} = \frac{13}{52} = \frac{1}{4}$$
 (4)

For Z={0,1},
$$\Pr(X) = \frac{n(Z)}{n(S)} = \frac{4}{52} = \frac{1}{13}$$
 (5)

a) Total number of kings of red colour = 2

PROBABILITY	RANGE OF RANDOM VARIABLE	VALUE OF PROBABILITY
Pr(X)	{0,1}	$\frac{1}{2}$
Pr (<i>Y</i>)	{0,1,2,3}	$\frac{1}{4}$
Pr(Z)	{0,1,2,3,4,5,6,7,8,9,10,11,12}	$\frac{1}{13}$

TABLE 2

$$Pr(A \text{ King of Red colour}) = Pr((X = 1), (Z = 12))$$
(6)

$$= (\Pr(X = 1))(\Pr(Z = 12)) \tag{7}$$

$$\Pr((X=1), (Z=12)) = \frac{1}{2} \times \frac{1}{13}$$
 (8)

$$=\frac{1}{26}=0.038\tag{9}$$

$$\therefore \Pr((X=1), (Z=12)) = 0.038 \tag{10}$$

b) Number of cards that are face cards = 12

$$Pr(A Face card) = Pr((Z=12)+(Z=11)+(Z=10)+(Z=1))$$
 (11)

$$=\frac{4}{13}=0.23\tag{12}$$

$$\therefore \Pr((Z=12)+(Z=11)+(Z=10)+(Z=1)) = 0.23$$
(13)

c) Number of cards that are red face cards = 6

$$Pr(A \text{ Red Face card}) = Pr((X=1),((Z=12)+(Z=11)+(Z=10))$$
(14)

$$Pr(A \text{ Red Face card}) = (Pr(X=1))(Pr((Z=12)+(Z=11)+(Z=10)))$$
 (15)

$$=\frac{1}{2}\times\frac{3}{13}\tag{16}$$

$$=\frac{3}{26}=0.11\tag{17}$$

$$\therefore \Pr((X=1),((Z=12)+(Z=11)+(Z=10))) = 0.11$$
(18)

d) Number of cards that are jack of hearts = 1

$$Pr (The Jack of Hearts) = Pr ((Z=10), (Y=2))$$
(19)

$$Pr((Z=10),(Y=2)) = (Pr(Z=10))(Pr(Y=2))$$
(20)

$$=\frac{1}{13}\times\frac{1}{4}\tag{21}$$

$$=\frac{1}{52}=0.019\tag{22}$$

$$\therefore \Pr((Z=10),(Y=2)) = 0.019 \tag{23}$$

e) Number of cards that are spade = 13

$$Pr(A Spade) = Pr(Y=1)$$
 (24)

$$=\frac{1}{4}=0.25\tag{25}$$

$$\therefore \Pr(Y=1) = 0.25$$
 (26)

f) Number of cards that are queens of diamonds = 1

$$Pr (The Queen of Diamonds) = Pr ((Z=11),(Y=3))$$
 (27)

$$Pr((Z=11),(Y=3)) = (Pr(Z=11))(Pr(Y=3))$$
(28)

$$= \frac{1}{13} \times \frac{1}{4}$$
 (29)
= $\frac{1}{52} = 0.019$ (30)

$$=\frac{1}{52}=0.019\tag{30}$$

$$\therefore \Pr((Z=11),(Y=3)) = 0.019$$
 (31)