Assignment 3

AI1110: Probability and Random Variables
Indian Institute of Techonology Hyderabad

Sudarshan Shivashankar
AI22BTECH11027

Question 10.15.1.14 : One card is drawn from a well-shuffled deck of 52 cards. Find the probability of getting

1) a king of red colour
2) a face card
3) a red face card
4) the jack of hearts
5) a spade
6) the queen of diamonds

Solution:

VARIABLE	RANGE	DESCRIPTION
X	{0,1}	The Random variable denoting the Color of the card.
Y	{0,1,2,3}	The Random variable denoting the Type of the card.
Z	{0,1,2,3,4,5,6,7,8,9,10,11,12}	The Random variable denoting the Value of the card.

TABLE 1

Total number of cards $= 52$

$$n(S) = 52 \tag{1}$$

$$\Pr(E) = \frac{n(E)}{n(S)} \tag{2}$$

$$\text{For X=\{0,1\}}, \Pr(X) = \frac{\text{n(X)}}{n(S)} = \frac{26}{52} = \frac{1}{2} \tag{3}$$

$$\text{For Y=\{0,1,2,3\}}, \Pr(Y) = \frac{\text{n(Y)}}{n(S)} = \frac{13}{52} = \frac{1}{4} \tag{4}$$

$$\text{For Z=\{0,1\}}, \Pr(X) = \frac{\text{n(Z)}}{n(S)} = \frac{4}{52} = \frac{1}{13} \tag{5}$$

PMF OF XYZ:

PROBABILITY	RANGE OF RANDOM VARIABLE	VALUE OF PROBABILITY
$\Pr(X = i)$	$i \in \{0, 1\}$	$\frac{1}{2}$
$\Pr(Y = i)$	$i \in \{0, 1, 2, 3\}$	$\frac{1}{4}$
$\Pr(Z = i)$	$i \in \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$	$\frac{1}{13}$

TABLE 2

The Above Table represents the Probability Mass Function of the Random Variables X,Y,Z.

a) Total number of kings of red colour = 2

$$\Pr(\text{A King of Red colour}) = \Pr((X = 1), (Z = 12)) \tag{6}$$
$$= (\Pr(X = 1))(\Pr(Z = 12)) \tag{7}$$
$$\Pr((X = 1), (Z = 12)) = \frac{1}{2} \times \frac{1}{13} \tag{8}$$
$$= \frac{1}{26} = 0.038 \tag{9}$$
$$\therefore \Pr((X = 1), (Z = 12)) = 0.038 \tag{10}$$

b) Number of cards that are face cards = 12

$$\Pr(\text{A Face card}) = \Pr((\text{Z=12})+(\text{Z=11})+(\text{Z=10})+(\text{Z=1})) \tag{11}$$
$$= \frac{4}{13} = 0.23 \tag{12}$$
$$\therefore \Pr((\text{Z=12})+(\text{Z=11})+(\text{Z=10})+(\text{Z=1})) = 0.23 \tag{13}$$

c) Number of cards that are red face cards = 6

$$\Pr(\text{A Red Face card}) = \Pr((\text{X=1}),((\text{Z=12})+(\text{Z=11})+(\text{Z=10})) \tag{14}$$

$$\Pr(\text{A Red Face card}) = (\Pr(\text{X=1}))(\Pr((\text{Z=12})+(\text{Z=11})+(\text{Z=10}))) \tag{15}$$
$$= \frac{1}{2} \times \frac{3}{13} \tag{16}$$
$$= \frac{3}{26} = 0.11 \tag{17}$$
$$\therefore \Pr((\text{X=1}),((\text{Z=12})+(\text{Z=11})+(\text{Z=10}))) = 0.11 \tag{18}$$

d) Number of cards that are jack of hearts = 1

$$\Pr(\text{The Jack of Hearts}) = \Pr((\text{Z=10}),(\text{Y=2})) \tag{19}$$

$$\Pr((\text{Z=10}),(\text{Y=2})) = (\Pr(Z = 10))(\Pr(Y = 2)) \tag{20}$$
$$= \frac{1}{13} \times \frac{1}{4} \tag{21}$$
$$= \frac{1}{52} = 0.019 \tag{22}$$
$$\therefore \Pr((\text{Z=10}),(\text{Y=2})) = 0.019 \tag{23}$$

e) Number of cards that are spade = 13

$$\Pr(\text{A Spade}) = \Pr(\text{Y}=1) \tag{24}$$
$$= \frac{1}{4} = 0.25 \tag{25}$$
$$\therefore \Pr(\text{Y}=1) = 0.25 \tag{26}$$

f) Number of cards that are queens of diamonds = 1

$$\Pr(\text{The Queen of Diamonds}) = \Pr((\text{Z}=11),(\text{Y}=3)) \tag{27}$$
$$\Pr((\text{Z}=11),(\text{Y}=3)) = (\Pr(Z = 11))(\Pr(Y = 3)) \tag{28}$$
$$= \frac{1}{13} \times \frac{1}{4} \tag{29}$$
$$= \frac{1}{52} = 0.019 \tag{30}$$
$$\therefore \Pr((\text{Z}=11),(\text{Y}=3)) = 0.019 \tag{31}$$