Non-invasive measurement techniques

Optical measurements: Determination of the Damping of a Pendulum with Time of Flight

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Abstract

$\alpha_{alu} = (23.0 \pm 0.1) \cdot 10^{-6}\,\mathrm{K}^{-1}$
$\alpha_{sst} = (15.8 \pm 0.2) \cdot 10^{-6}\,\mathrm{K}^{-1}$, which is only 1 % off tabulated values [1, **?**].

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1 Introduction

A mechanical system cosisting only of a rigid body, with only one degree of freedom, rotation around a constant axis, from here on called a pendulum, is a system of great interest. Historically it has had a wide range of applications in science, mathemathics and in everyday life. Among the reasons for its continued importance as an educational tool in physics is its short, general equations of motion in the linearized, small amplitude case, and the ease and the great number of ways by which this case can be extended.

This experiment in particular will measure a physical pendulum's decreasing velocity in order to find and analyze its damping forces. A time of flight instrumentation is constructed and used to aquire the velocity data.

2 Theory

2.1 Time of Flight

Time of Flight (ToF) is a method that measures the time for an object to move a known distance. You can find the distance of an object or velocity or path length of a movement with this technique. We convert this technique to find velocity of pendulum. We have two parallel beams with optic setup. Pendulum make a harmonic oscillator between these beams. We know the distance between two beams. We take time data with photodiodes. When pendulum pass the beams, that cause picks at DAQ data. We can get time values with measure distance between these picks.

2.2 The pendulum

For a simple rigid body pendulum, using Newtons second law and balancing the forces gives us the following equation of motion:

$$\frac{d^2\theta}{dt^2} + \frac{mgl\sin\theta}{I_p} = 0, \tag{1}$$

where I_p is the bodys moment of inertia around the pivot point, θ is the angle between the line through the pivot and the bodys center of mass and the vertical line, m is the mass, g is the acceleration due to gravity and l is the distance from the pivot to the center of mass.

2.2.1 Introducing Friction

A physical pendulum will usually experience some friction between solid surfaces around its pivot point. This force may sometimes be modelled as being proportional to velocity[4, p. 30], and sometimes as independent of velocity [5].

To put both of these dependencies into 1, we introduce a term $F_l = -a\frac{d\theta}{dt} - c$ into it.

$$\frac{d^2\theta}{dt^2} - A\frac{d\theta}{dt} + \frac{mgl}{I_p}\sin\theta = \text{sgn}(\frac{d\theta}{dt})C, \tag{2}$$

where A and C are constants. If $C = 0$ this now discribes the motions of what is called a damped harmonic oscillator[3].

2.2.2 Introducing Drag

Air drag is the force between the air and the pendulum. This force is highly dependent on velocity. For low velocities we approximate the air to be small evenly distributed balls, bouncing off the pendulum as it travels through space. Increasing velocity by some factor will increase the number of bouncing by the same factor, so this introduces a linear term into Equation 2, and can be accounted for by adjusting the constant A.

For higher velocities, however, this model breaks down, and the air must be modelled as a fluid. Drag forces in fluids are proportional to the velocity squared[2], which for us means inserting a force $F_D = -B \cdot \left(\frac{d\theta}{dt}\right)^2$, where B is a constant, into Equation 2, giving us the equation of motion

$$\frac{d^2\theta}{dt^2} - B\frac{d\theta}{dt} + \left(1 + \frac{B}{A}\frac{d\theta}{dt}\right) + \frac{mgl}{I_p}\sin\theta = \text{sgn}(\frac{d\theta}{dt})C, \tag{3}$$

It looks like a damped harmonic oscillator, but with a damping ratio that depends on velocity.

At the pendulums lowest point we have, $\theta = \sin\theta = 0$

2.2.3 Energy Approach

The equation of energy concerning angular velocity of the pendulum is

$$E = \frac{(\frac{d\theta}{dt})^2 \cdot I_p}{2}.$$

As $\frac{d\theta}{dt} = const \cdot v$ and I_p is constant we can use the equation

$$E = d \cdot v_m^2,$$

where d is a constant and v_m is the maximum motion of the bottom part of the pendulum, when calculating energies.

We know that the force of drag is

$$F_d = a + bv + cv^2.$$

The work done during one half period is found as

$$W = 2\int_0^s F_d ds,$$

where s is the distance travelled of the bottom part of the pendulum for one fourth period. Since $ds = rd\theta$ we get the equation

$$W = 2\int_0^{\theta_m} F_d d\theta,$$

where θ_m is the turning angle. We know that this work is much smaller than the total energy of the system. This is used to make the assumption

$$rg(1 - cos(\theta) + v^2 = v_m^2 = const,$$

wherer r is the length of the pendulum, and g is the acceleration of gravity. We can use this to find a relation between $d\theta$ and dv as:

$$d\theta = \frac{kvdv}{\sqrt{1-(v^2+d)^2}},$$

where k is a constant and $d = 2rg - v_m^2$. Using this we get an integration of finding the work.

$$W = 2k\int_{v_m}^0 F_d \frac{kvdv}{\sqrt{1-(v^2+d)^2}}$$

The velocities are not very high during our experiments. Thus we can Taylor expand around 0. This integral is solved using Taylor expansion around 0 and Wolfram Alpha which gives us the equation:

$$W = \frac{av_m^2}{2\sqrt{2rg-d^2}} + \frac{bv_m^3}{3\sqrt{1-d^2}} + \frac{v_m^4(ad - cd^2 + c)}{5(1-d^2)^{3/2}} + \mathcal{O}(5).$$

Using the definition of d, which is a function of v_m and Wolfram Alpha and Taylor expand the expression around $v_m = 0$ we get the final equation for the work of one half period:

$$W = Av + Bv^2 + Cv^3 + \mathcal{O}(4).$$

3 Experimental Setup

The experimental setup consists of one electric circuit described in section 3.1, one optical setup described in 3.2 and a PC with MATLAB and LabVIEW installed. The setup can be seen in Figure 2.

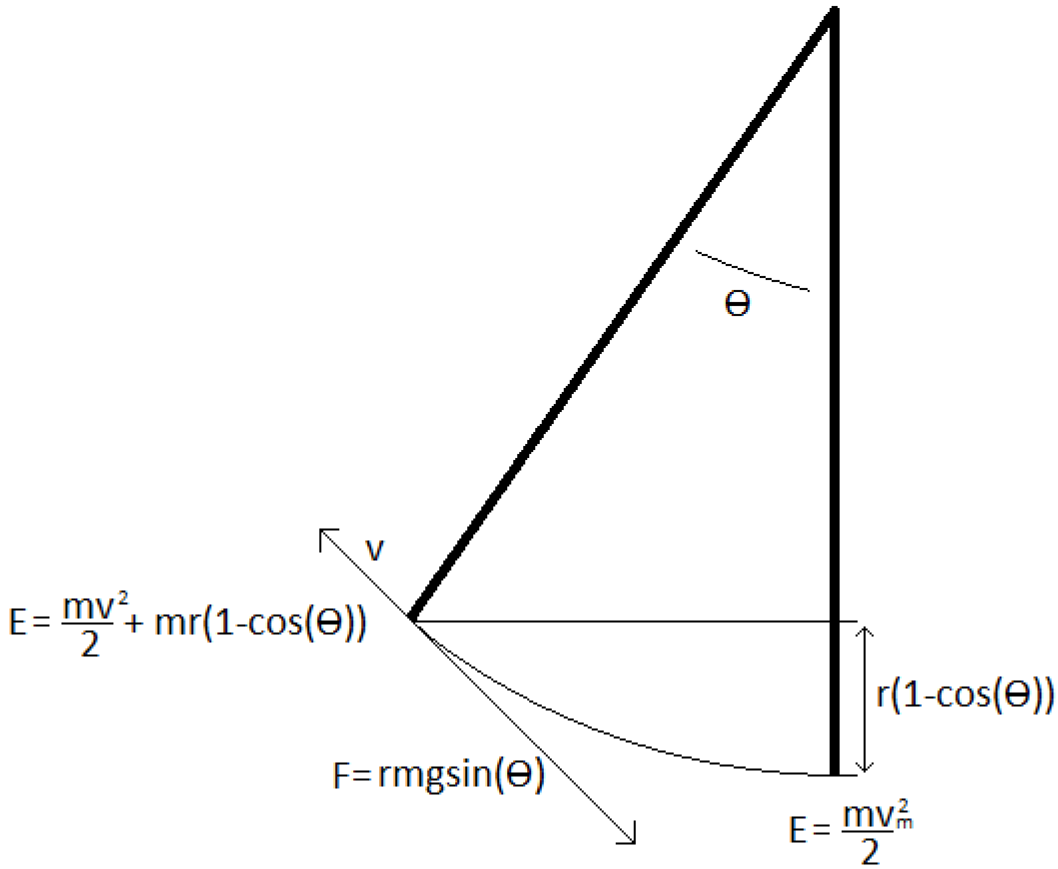

Figure 1: The energies of the moving pendulum are demonstrated

3.1 Electric circuit

The first mission is to convert light to voltage. This is done by using photodiodes that converts light into current, which is then converted using a current-to-voltage converter. Photodiodes are made from semiconductor materials and are based on p-n junction principle. We use Silicon photodiodes in the experiment. The electric circuit can be seen in Figure 3.

We connect a 100k resistor from input to output to create an inverting amplifier. We feed the operational amplifier with 15V and we use 470μF capacitor to reduce noise. The voltage is digitalized by a DAQ card that we attach to output of the amplifier. LabVIEW is used to collect the data.

3.2 Optical setup

The optical setup can be seen in Figure 4. We use a Helium-Neon laser(JDSU 1101) with a wavelength of 632,8nm. Firstly we set the laser, beam splitter,

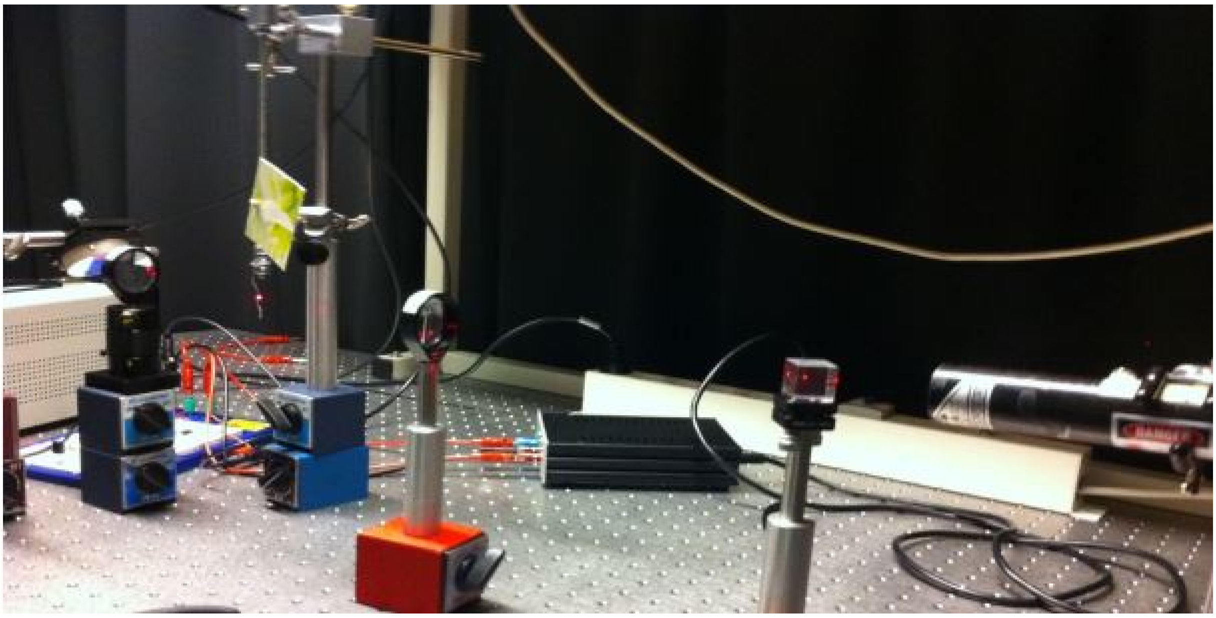

Figure 2: The electric circuit of the setup. D1 and D2 are the photodiodes.

lenses and mirror on the same height. We get two parallel beams with the beam splitter. We mount a wire to our pendulum. Because pendulum is too thick to get well data. Then we paint the wire with tipp-ex to prevent beam reflection from it. We use a straight mirror to angle the beam to photodiodes. We put two +100 lens to get more clear beams.

4 Procedure

5 Error calculations

5.1 Error of the slope of a linear fitting

To calculate the error of the slope of a linear fit we simply use MATLAB to calculate the following equation:

$$s = \mathrm{std}(f(x) - data(x)), \tag{4}$$

where s is the standard deviation of the residuals, x is the x-values of the region considered, $data$ is the data values of the region considered and f is the linear fit. This standard deviation will be a measure of how correct the linear approximation is, but also gives an approximation of the total error from the time of flight intrumentation.

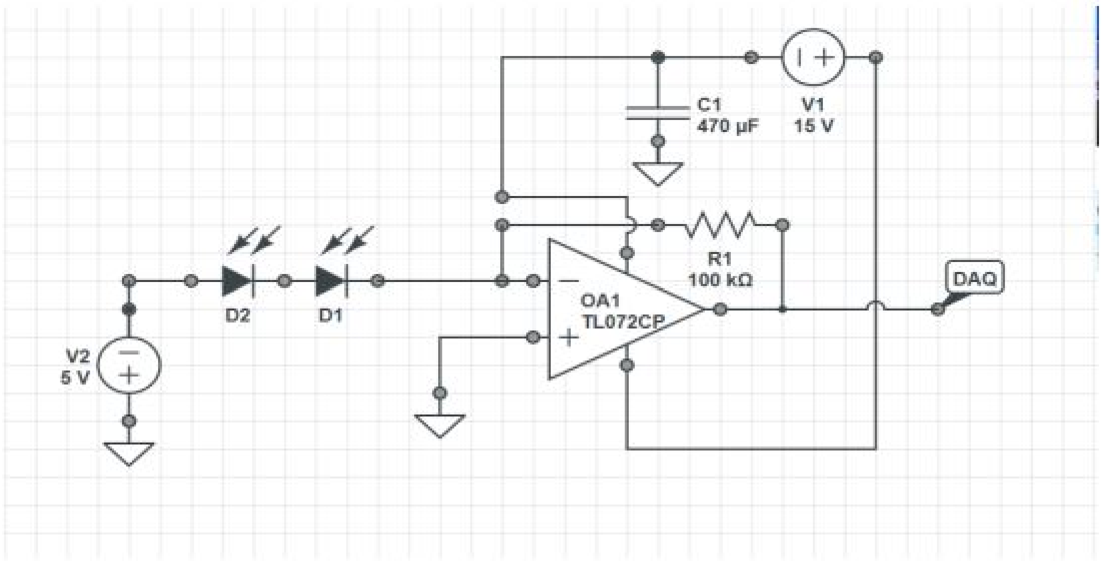

Figure 3: The electric circuit of the setup. D1 and D2 are the photodiodes.

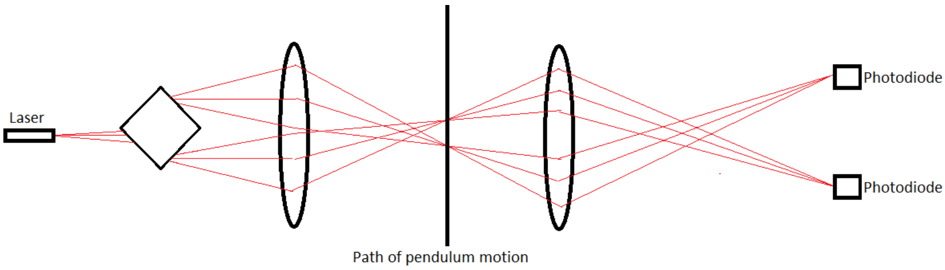

Figure 4: The optical setup seen from above. A mirror is also used to direct the lasers onto the photodiodes. The figure is not correct according to scale.

6 Results

In Figure 5 we see the logarithm of work done by the friction and drag on the pendulum plotted versus the logarithm of velocity. The units of the work and velocity are a constant times their SI-unit. The slopes and the errors of the linear fits of the two different regions of the plot are calculated to be

$$slope_{low} = \frac{\Delta \ln(W_{low})}{\Delta \ln(v_{low})} = 1.36(4),$$

$$slope_{high} = \frac{\Delta \ln(W_{high})}{\Delta \ln(v_{high})} = 2.60(3).$$

The values inside the parentheses are the standard deviation of the slope calculated using Equation 4.

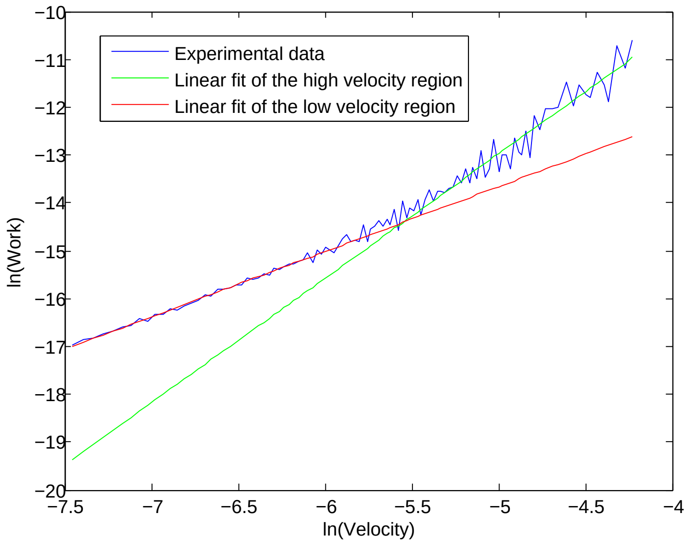

Figure 5: Work versus velocity data and linear fits for the pendulum.

In Figure 6 we see the logarithm of work done by the friction and drag on the pendulum with the added area plotted versus the logarithm of velocity. The units of the work and velocity are a constant times their SI-unit. The slopes and the errors of the linear fits of the two different regions of the plot are calculated to be

$$slope_{low} = \frac{\Delta \ln(W_{low})}{\Delta \ln(v_{low})} = 1.96(4),$$

$$slope_{high} = \frac{\Delta \ln(W_{high})}{\Delta \ln(v_{high})} = 2.63(7).$$

It is clear that the slopes of the high velocity region are similar and close to 3. The slope of the added-area pendulum for the low-velocity region is very close to 2. The slope of the ordinary pendulum for the low-velocity region is slightly above 1.

7 Discussion

The experimentally retrieved plots values are very precise and describing. They demonstrate the three types of frictional forces of

The behavior of the high-velocity region is not entirely as expected. We expected the slope of the added-area pendulum to be closer to 3 than the other pendulum. This is explained by the fact that the high-velocity regions are not exactly the same for the different types of pendula. We were not able to do a high enough velocity measurement due to the quick damping as well as the fluttering of the added area.

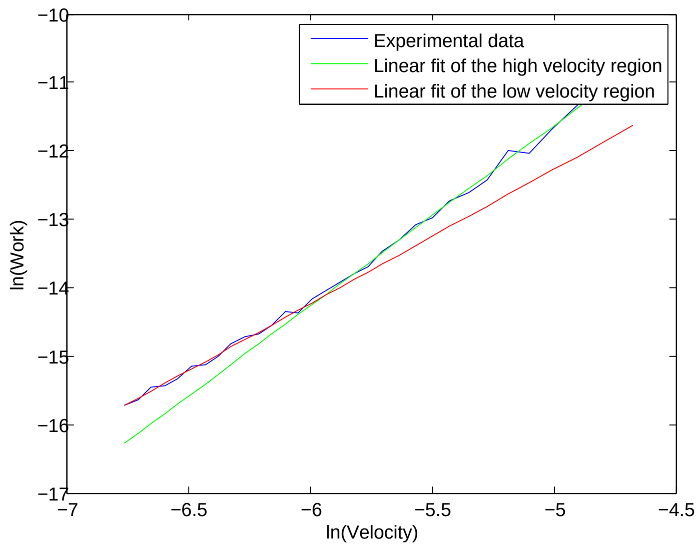

Figure 6: Work versus velocity data and linear fits for the pendulum with an added area.

8 Summary and Conclusions

References

[1] Nordling, C., Österman, J. (2006). *Physics Handbook* 8^{th}
Lund, Sweden, Studentlitteratur.

[2] Wikipedia. *Drag*
`http://en.wikipedia.org/wiki/Drag_(physics)` [2013-12-14]

[3] Wikipedia. *Harmonic oscillator*
`http://en.wikipedia.org/wiki/Harmonic_oscillator` [2013-12-14]

[4] Baker, Blackburn (2005). *The Pendulum, A Case Study in Physics*
Oxford, University Press.

[5] Wikipedia. *Friction*
`http://en.wikipedia.org/wiki/Friction` [2013-12-14]