Non-invasive measurement techniques

Optical measurements: Determination of the Damping of a Pendulum with Time of Flight

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Abstract

$\alpha_{alu} = (23.0 \pm 0.1) \cdot 10^{-6}\,\mathrm{K}^{-1}$
$\alpha_{sst} = (15.8 \pm 0.2) \cdot 10^{-6}\,\mathrm{K}^{-1}$, which is only 1 % off tabulated values [1, **?**].

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1 Introduction

A mechanical system cosisting only of a rigid body, with only one degree of freedom, rotation around a constant axis, from here on called a pendulum, is a system of great interest. Historically it has had a wide range of applications in science, mathemathics and in everyday life. Among the reasons for its continued importance as an educational tool in physics is its short, general equations of motion in the linearized, small amplitude case, and the ease and the great number of ways by which this case can be extended.

This experiment in particular will measure a physical pendulum's decreasing velocity in order to find and analyze its damping forces. A time of flight instrumentation is constructed and used to aquire the velocity data.

2 Theory

2.1 Optics

2.2 Circuits

2.3 maths

The forces considering air drag are described differently for different velocities. For very low velocities i.e. when the pendulum is at its turning point the drag force will be proportional to the velocity as

$$F_d = -bv,$$

where F_d is force of drag, v is the velocity and b is a positive constant. For higher velocities the force of

By beginning with the equation

$$v_{max} = Ce^{at} + De^{bt}.$$

By breaking out e^{at} and taking the logarithm we end up with the equation

$$\ln(v_{max}) = \ln(C + De^{\frac{b}{a}t}) + at$$

For small t the first term will be nearly constant. A linear fit can be made to find the slope a. In a similar fashion we can break out e^{bt}

3 Experimental Setup

4 Procedure

5 Error calculations

5.1 Error of the time of flight instrumentation

One error of the measurement device is the error from the non-continous data collecting as demonstrated in Figure 1. Since our LabVIEW-setup collect data only at a rate of some kHz and we measure fast velocities we will get an error from this. The error can be found as

$$v_{err} = \Delta t_{err} \cdot \Delta s,$$

where Δs is the length between the two laser lights, Δt is the rate of the data collecting and v is the velocity. The error in time is $< \Delta t$. Thus we set the maximum error in time as Δt. By the fact that $\Delta s = v \cdot (\Delta t + \Delta t_{err})$We

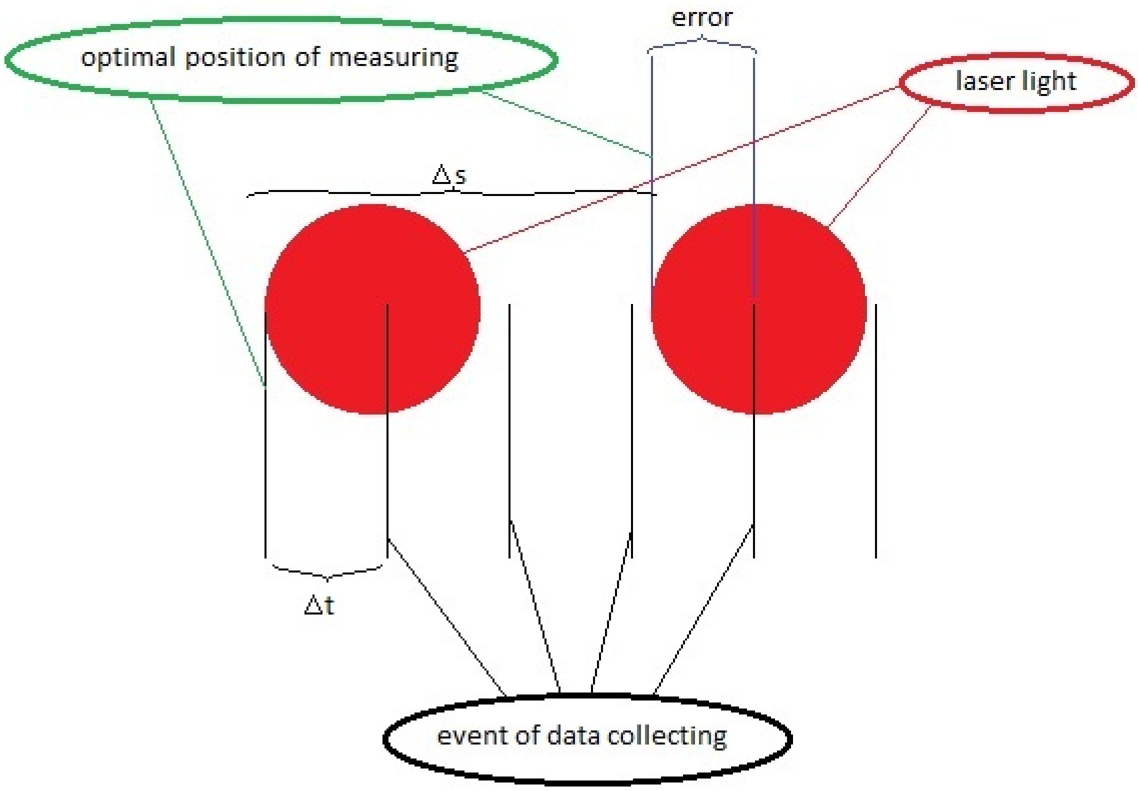

Figure 1: The source of the error of the time of flight instrumentation is demonstrated.

set $\Delta t = 50kHz$.

5.2 Error of the slope of a linear fitting

To calculate the error of the slope of a linear fit we simply use MATLAB to calculate the following equation:

$$s = \mathrm{std}(f(x) - data(x)), \tag{1}$$

where s is the standard deviation of the residuals, x is the x-values of the region considered, $data$ is the data values of the region considered and f is the linear fit. This standard deviation will be a measure of how correct the linear approximation is.

6 Results

In Figure 2 we see the logarithm of work done by the friction and drag on the pendulum with the added area plotted versus the logarithm of velocity. The units of the work and velocity are an unknown constant times their SI-unit. The slopes and the errors of the linear fits of the two different regions of the plot are calculated to be

$$slope_{low} = \frac{\Delta \ln(W_{low})}{\Delta \ln(v_{low})} = 1.36(4),$$

$$slope_{high} = \frac{\Delta \ln(W_{high})}{\Delta \ln(v_{high})} = 2.60(3).$$

The values inside the parentheses are the standard deviation of the slope calculated using Equation 1.

In Figure 3 we see the logarithm of work done by the friction and drag on the pendulum with the added area plotted versus the logarithm of velocity. The units of the work and velocity are an unknown constant times their SI-unit. The slopes and the errors of the linear fits of the two different regions of the plot are calculated to be

$$slope_{low} = \frac{\Delta \ln(W_{low})}{\Delta \ln(v_{low})} = 1.96(4),$$

$$slope_{high} = \frac{\Delta \ln(W_{high})}{\Delta \ln(v_{high})} = 2.63(7).$$

It is clear that the slopes of the high velocity region are similar and close to 3. The slope of the added-area pendulum for the low-velocity region is very close to 2. The slope of the ordinary pendulum for the low-velocity region is slightly above 1.

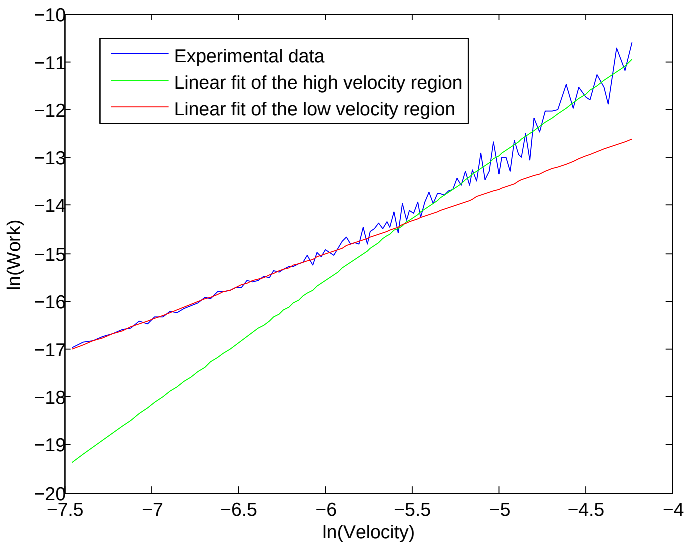

Figure 2: Work versus velocity data and linear fits for the pendulum.

7 Discussion

The behavior of the high-velocity region is not entirely as expected. We expected the slope of the added-area pendulum to be closer to 3 than the other pendulum. This is explained by the fact that the high-velocity regions are not exactly the same for the different types of pendula. We were not able to do a high enough velocity measurement due to the quick damping as well as the fluttering of the added area.

8 Summary and Conclusions

References

[1] Nordling, C., Österman, J. (2006). *Physics Handbook* 8^{th}
Lund, Sweden, Studentlitteratur.

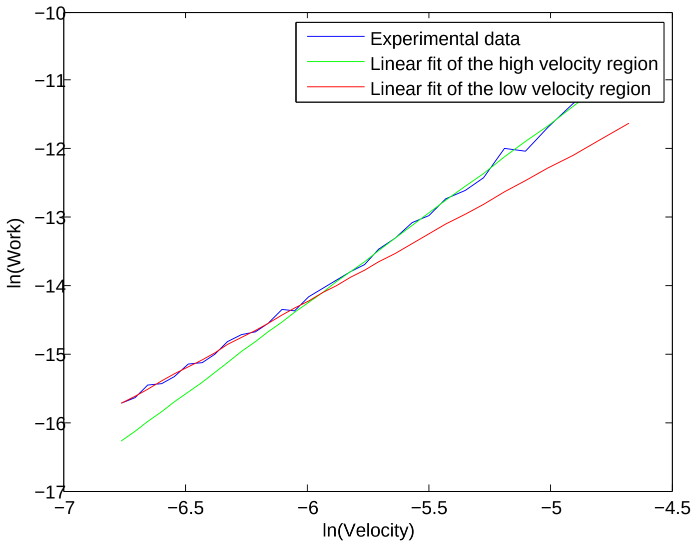

Figure 3: Work versus velocity data and linear fits for the pendulum with an added area.