Non-invasive measurement techniques

Optical measurements: Determination of the Damping of a Pendulum with Time of Flight

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Abstract

$\alpha_{alu} = (23.0 \pm 0.1) \cdot 10^{-6}\,\mathrm{K}^{-1}$
$\alpha_{sst} = (15.8 \pm 0.2) \cdot 10^{-6}\,\mathrm{K}^{-1}$, which is only 1 % off tabulated values [1, **?**].

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1 Introduction

A mechanical system cosisting only of a rigid body, with only one degree of freedom, rotation around a constant axis, from here on called a pendulum, is a system of great interest. Historically it has had a wide range of applications in science, mathemathics and in everyday life. Among the reasons for its continued importance as an educational tool in physics is its short, general equations of motion in the linearized, small amplitude case, and the ease and the great number of ways by which this case can be extended.

This experiment in particular will measure a physical pendulum's decreasing velocity in order to find and analyze its damping forces. A time of flight instrumentation is constructed and used to aquire the velocity data.

2 Theory

2.1 Optics

2.2 Circuits

2.3 Friction

2.4 Drag

Air drag is a the force of friction from air. This force is described differently for different regions of velocity. For low velocities we approximate an object as though it was moving through a liquid with no turbulence. For this case the force is proportional to v as

$$F_d = -bv,$$

where F_d is force of drag, v is the velocity and b is a positive constant.

For higher velocities the force of drag can be described as

$$F_D = \frac{1}{2}\rho v^2 C_D A,$$

where F_D is the drag force, ρ is the mass density of the fluid and C_D is the drag coefficient. For our purpose all of the factors are constants, which allows us to use

$$F_D = -c \cdot v^2,$$

where c is a positive constant.

3 Experimental Setup

4 Procedure

5 Error calculations

5.1 Error of the time of flight instrumentation

One error of the measurement device is the error from the non-continous data collecting as demonstrated in Figure 1. Since our LabVIEW-setup collect data only at a rate of 50kHz and we measure at high velocities we will get an error from this. Since we know the maximum error in time as the rate of data collecting we know the maximum error as:

$$\Delta v = \Delta t \cdot s,$$

where Δv is the maximum error of velocity, Δt is the maximum error in time and s is the distance travelled during this time. This equation can be rewritten as:

$$\Delta v = \Delta t \cdot (\Delta t \cdot (v + \Delta v)) \Rightarrow \Delta v = \frac{v(\Delta t)^2}{1 - \Delta t}$$

5.2 Error of the slope of a linear fitting

To calculate the error of the slope of a linear fit we simply use MATLAB to calculate the following equation:

$$s = \mathrm{std}(f(x) - data(x)), \tag{1}$$

where s is the standard deviation of the residuals, x is the x-values of the region considered, $data$ is the data values of the region considered and f is the linear fit. This standard deviation will be a measure of how correct the linear approximation is.

6 Results

In Figure 2 we see the logarithm of work done by the friction and drag on the pendulum plotted versus the logarithm of velocity. The units of the work and velocity are a constant times their SI-unit. The slopes and the errors of the linear fits of the two different regions of the plot are calculated to be

$$slope_{low} = \frac{\Delta \ln(W_{low})}{\Delta \ln(v_{low})} = 1.36(4),$$

$$slope_{high} = \frac{\Delta \ln(W_{high})}{\Delta \ln(v_{high})} = 2.60(3).$$

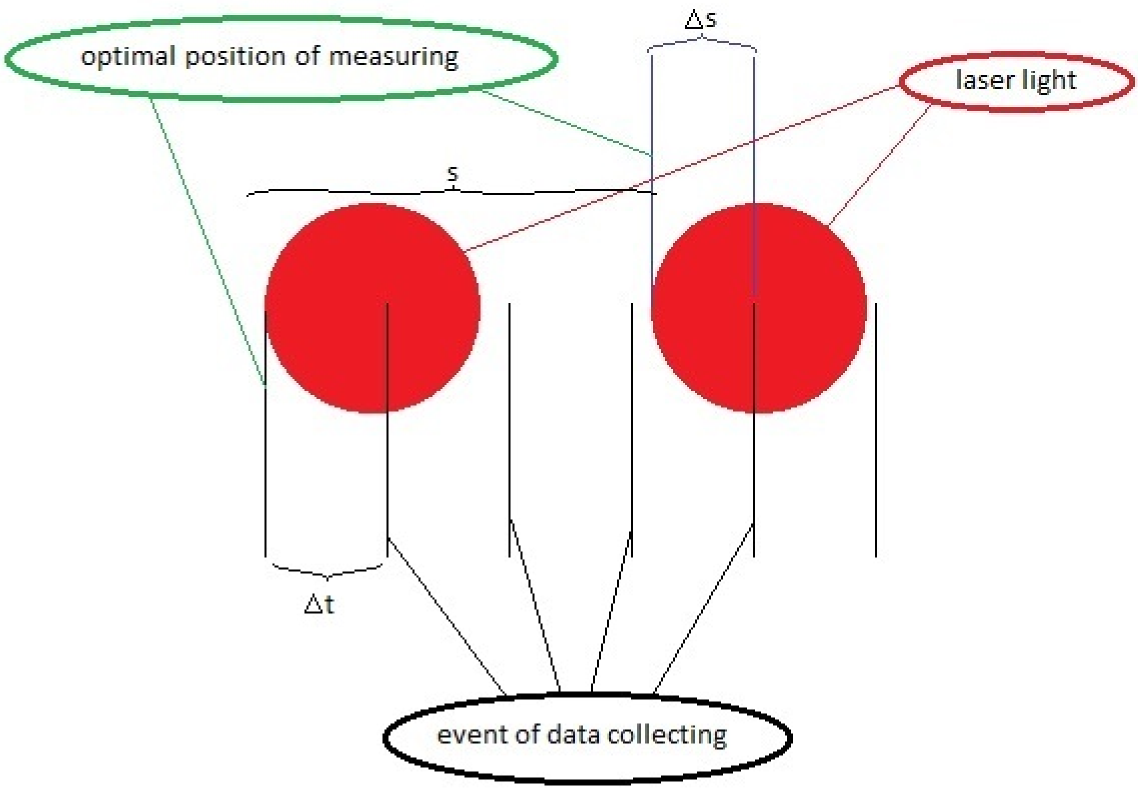

Figure 1: The source of the error of the time of flight instrumentation is demonstrated.

The values inside the parentheses are the standard deviation of the slope calculated using Equation 1.

In Figure 3 we see the logarithm of work done by the friction and drag on the pendulum with the added area plotted versus the logarithm of velocity. The units of the work and velocity are a constant times their SI-unit. The slopes and the errors of the linear fits of the two different regions of the plot are calculated to be

$$slope_{low} = \frac{\Delta \ln(W_{low})}{\Delta \ln(v_{low})} = 1.96(4),$$

$$slope_{high} = \frac{\Delta \ln(W_{high})}{\Delta \ln(v_{high})} = 2.63(7).$$

It is clear that the slopes of the high velocity region are similar and close to 3. The slope of the added-area pendulum for the low-velocity region is very close to 2. The slope of the ordinary pendulum for the low-velocity region is slightly above 1.

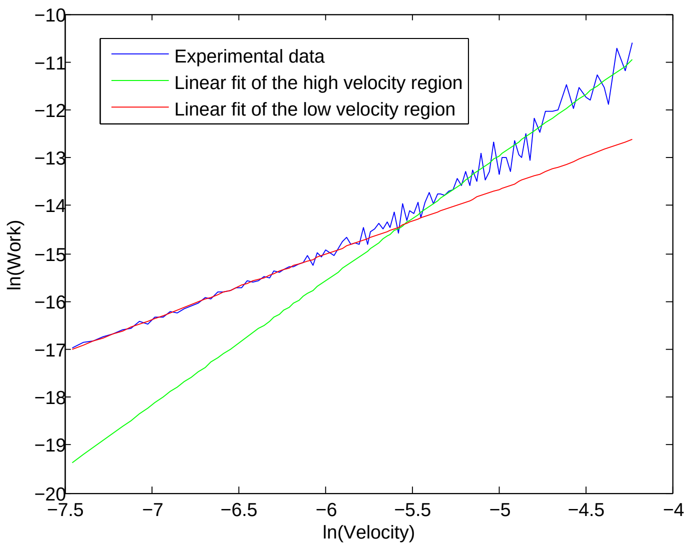

Figure 2: Work versus velocity data and linear fits for the pendulum.

7 Discussion

The behavior of the high-velocity region is not entirely as expected. We expected the slope of the added-area pendulum to be closer to 3 than the other pendulum. This is explained by the fact that the high-velocity regions are not exactly the same for the different types of pendula. We were not able to do a high enough velocity measurement due to the quick damping as well as the fluttering of the added area.

8 Summary and Conclusions

References

[1] Nordling, C., Österman, J. (2006). *Physics Handbook* 8^{th} Lund, Sweden, Studentlitteratur.

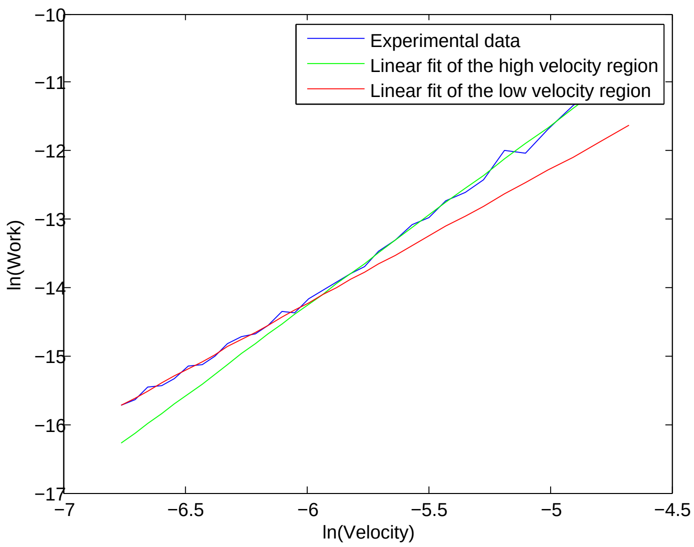

Figure 3: Work versus velocity data and linear fits for the pendulum with an added area.