Handwriting Assignment #3

Due to 10th Dec.

1. Given a list of n integers, v1,…, vn, the product-sum is the largest sum that can be formed by multiplying adjacent elements in the list. Each element can be matched with at most one of its neighbors. For example, given the list 1, 2, 3, 1 the product sum is 8 = 1 + (2 x 3) + 1, and given the list 2, 2, 1, 3, 2, 1, 2, 2, 1, 2 the product sum is 19 = (2 x 2) + 1 + (3 x 2) + 1 + (2 x 2) + 1 + 2

a) Compute the product-sum of 1, 4, 3, 2, 3, 4, 2.

b) Give the optimization formula for computing the product-sum of the first j elements.

c) Give a dynamic program for computing the value of the product sum of a list of integers.

2. Suppose we want to make change for n cents, using the least number of coins of 1, 10, and 25 cents. Describe a dynamic programming algorithm to find an optimal solution.

3.Suppose that the coin changing problem is solved by the standard greedy algorithm, where the coin values are 1, 4, 7 and 11.

Show that the greedy algorithm does not always find the fewest coins to yield a given amount of change.

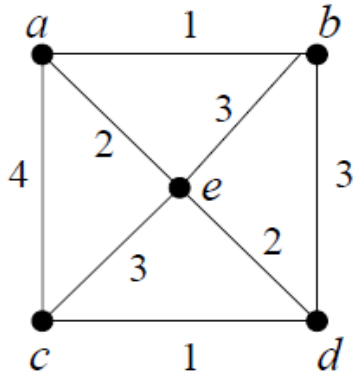
4. A table composed of N x M cells, each having a certain quantity of apples, is given. You start from the upper-left corner. At each step you can go down or right one cell. Write a function to find the maximum number of apples you can collect.

To understand the problem, refer to the below example.

2	0	5	3
1	3	4	2
1	0	3	8
8	**5**	3	1
1	**4**	**5**	**5**

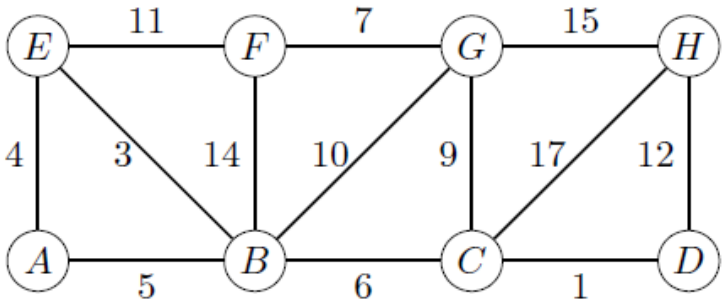
Max sum = 31

5. The following weighted graph is given.



a) Draw the progression of Prim's algorithm to find an MST.
b) Draw the progression of Kruskal's algorithm to find an MST.

6. Run Dijkstra's algorithm on the weighted graph below, using vertex A as the source. Write the vertices in the order which they are marked and compute all distances at each step.



7. Bellman-Ford and Dijkstra algorithms are competing algorithms for the single-source-shortest paths problem, although one of them can handle certain graph inputs that the other cannot. Suppose Gb is the set of graphs for which Bellman-Ford can find all shortest paths from a given source. Suppose Gd is the set of graphs for which Dijkstra can find all shortest paths from a given source. Which of Gb⊂Gd or Gd⊂Gb is true and why?