2020108l9 조정현 6주차 과제

[연습문제 3.3 part 3]

#4) -2

$AA^{-1} = I$ 이므로

$$\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1\times1+2\times0 & 1\times a+2\times1 \\ 0\times1+1\times0 & 0\times a+1\times1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & a+2 \\ 0 & 1 \end{bmatrix}$$

$\therefore\ a+2=0$

$a=-2$

#6) (1) $\begin{bmatrix} 3 & -1 \\ -2 & 1 \end{bmatrix}$ (2) $\begin{bmatrix} \frac{-1}{3} & \frac{2}{3} \\ \frac{2}{3} & \frac{-1}{3} \end{bmatrix}$

(1) $[A \mid I] = \left[\begin{array}{cc|cc} 1 & 1 & 1 & 0 \\ 2 & 3 & 0 & 1 \end{array}\right]$ 2행에 $-2\ -2 \mid -2\ 0$ 더하기

$= \left[\begin{array}{cc|cc} 1 & 1 & 1 & 0 \\ 0 & 1 & -2 & 1 \end{array}\right]$ 1행에 $0\ -1 \mid 2\ -1$ 더하기

$= \left[\begin{array}{cc|cc} 1 & 0 & 3 & -1 \\ 0 & 1 & -2 & 1 \end{array}\right]$

$\therefore A^{-1} = \begin{bmatrix} 3 & -1 \\ -2 & 1 \end{bmatrix}$

(2) $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$

A의 여인수 행렬 $B = \begin{bmatrix} 1 & -2 \\ -2 & 1 \end{bmatrix}$

$\det(A) = 1\times1 - 2\times2 = -3$

$\mathrm{Adj}(A) = \begin{bmatrix} 1 & -2 \\ -2 & 1 \end{bmatrix}$

$\therefore A^{-1} = \frac{-1}{3}\begin{bmatrix} 1 & -2 \\ -2 & 1 \end{bmatrix}$

$= \begin{bmatrix} \frac{-1}{3} & \frac{2}{3} \\ \frac{2}{3} & \frac{-1}{3} \end{bmatrix}$

#8) 가역적이다.

A 행렬이 가역적이라는 것은 $AA^{-1} = I$ 가 성립한다는 뜻이다.

이것이 성립하기 위해서는 $\det(A) \neq 0$ 이어야한다.

따라서 $\det(A) = 0\times(0\times7+2\times9) - 3(1\times7+2\times4) - 5(-9\times1+0\times4)$

$\neq 0$ 이므로 다음 행렬은 가역적이다.

#10) ?

$A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$

#12) $\begin{bmatrix} 18 & 2 & 4 \\ -11 & 14 & 5 \\ -10 & -4 & -8 \end{bmatrix}$

$A = \begin{bmatrix} 2 & 3 & -4 \\ 0 & -4 & 2 \\ 1 & -1 & 5 \end{bmatrix}$ $\begin{bmatrix} + & - & + \\ - & + & - \\ + & - & + \end{bmatrix}$ 참고.

$\begin{vmatrix} -4 & 2 \\ -1 & 5 \end{vmatrix}$ $-\begin{vmatrix} 0 & 2 \\ 1 & 5 \end{vmatrix}$ $\begin{vmatrix} 0 & -4 \\ 1 & -1 \end{vmatrix}$

$-\begin{vmatrix} 3 & -4 \\ -1 & 5 \end{vmatrix}$ $\begin{vmatrix} 2 & -4 \\ 1 & 5 \end{vmatrix}$ $-\begin{vmatrix} 2 & 3 \\ 1 & -1 \end{vmatrix}$

$\begin{vmatrix} 3 & -4 \\ -4 & 2 \end{vmatrix}$ $-\begin{vmatrix} 2 & -4 \\ 0 & 2 \end{vmatrix}$ $\begin{vmatrix} 2 & 3 \\ 0 & -4 \end{vmatrix}$

$-20+2=18$ $-(0\times5-2)=2$ $0\times(-1)+4=4$

$-(3\times5-4)=-11$ $2\times5+4=14$ $-(2\times(-1)-3\times1)=5$

$3\times2-16=-10$ $-(2\times2+0)=-4$ $2\times(-4)=-8$

$\therefore \begin{bmatrix} 18 & 2 & 4 \\ -11 & 14 & 5 \\ -10 & -4 & -8 \end{bmatrix}$

[연습문제 3.4 part 3]

#1) $x_1 = 6$, $x_2 = -2$

$x_1 + x_2 = 4$

$2x_1 - x_2 = 14$

$\begin{bmatrix} 1 & 1 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 4 \\ 14 \end{bmatrix}$

$A \cdot z = b$ $\det(A) \neq 0$ 이기 때문에 가능함

$z = A^{-1}b$

$A^{-1} = \frac{1}{1\times(-1)-2\times1}\begin{bmatrix} -1 & -1 \\ -2 & 1 \end{bmatrix}$

$= \frac{-1}{3}\begin{bmatrix} -1 & -1 \\ -2 & 1 \end{bmatrix}$

$= \begin{bmatrix} \frac{1}{3} & \frac{1}{3} \\ \frac{2}{3} & \frac{-1}{3} \end{bmatrix}$

$\therefore \begin{bmatrix} \frac{1}{3} & \frac{1}{3} \\ \frac{2}{3} & \frac{-1}{3} \end{bmatrix}\begin{bmatrix} 4 \\ 14 \end{bmatrix} = \begin{bmatrix} 6 \\ -2 \end{bmatrix}$

#3) $x=2,\ y=-1,\ z=1$

$$2x+2y-z=1$$
$$x+y-z=0$$
$$3x+2y-3z=1$$

$$\underbrace{\begin{bmatrix} 2 & 2 & -1 \\ 1 & 1 & -1 \\ 3 & 2 & -3 \end{bmatrix}}_{A} \cdot \underbrace{\begin{bmatrix} x \\ y \\ z \end{bmatrix}}_{z} = \underbrace{\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}}_{b}$$

$$z = A^{-1}b$$

$$\det(A) = 2(-3+2) - 2(-3+3) - 1(2-3)$$
$$= -2+1 = -1$$

$$A^{-1} = -\begin{bmatrix} \begin{vmatrix} 1 & -1 \\ 2 & -3 \end{vmatrix} & -\begin{vmatrix} 1 & -1 \\ 3 & -3 \end{vmatrix} & \begin{vmatrix} 1 & 1 \\ 3 & 2 \end{vmatrix} \\ -\begin{vmatrix} 2 & -1 \\ 2 & -3 \end{vmatrix} & \begin{vmatrix} 2 & -1 \\ 3 & -3 \end{vmatrix} & -\begin{vmatrix} 2 & 2 \\ 3 & 2 \end{vmatrix} \\ \begin{vmatrix} 2 & -1 \\ 1 & -1 \end{vmatrix} & -\begin{vmatrix} 2 & -1 \\ 1 & -1 \end{vmatrix} & \begin{vmatrix} 2 & 2 \\ 1 & 1 \end{vmatrix} \end{bmatrix} = \begin{bmatrix} 1 & -4 & 1 \\ 0 & 3 & -1 \\ 1 & -2 & 0 \end{bmatrix}$$

$$\therefore \begin{bmatrix} 1 & -4 & 1 \\ 0 & 3 & -1 \\ 1 & -2 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1\times1+1\times1 \\ -1\times1 \\ 1\times1 \end{bmatrix}$$
$$= \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$$

$\therefore x=2,\ y=-1,\ z=1$

#7) 유일한 해가 존재하지 않아 해를 구할 수 없다.

$$3x+y-z=3$$
$$2x+2y-3z=1$$
$$-x+y-2z=-2$$

$$\underbrace{\begin{bmatrix} 3 & 1 & -1 \\ 2 & 2 & -3 \\ -1 & 1 & -2 \end{bmatrix}}_{A} \cdot \underbrace{\begin{bmatrix} x \\ y \\ z \end{bmatrix}}_{z} = \underbrace{\begin{bmatrix} 3 \\ 1 \\ -2 \end{bmatrix}}_{b}$$

$$\det(A) = 3(-4+3) - (-4-3) - (2\times1+2)$$

$= -3+7-4 = 0$ 이므로 다음의 선형 시스템은 크래머 공식으로 계산할 수 없다. 즉, 유일한 해가 존재하지 않아 해를 구할 수 없다.

#8) $x_1=4,\ x_2=-2,\ x_3=3$

$$\underbrace{\begin{bmatrix} 2 & 4 & 6 \\ 4 & 5 & 6 \\ 3 & 1 & -2 \end{bmatrix}}_{\text{행렬 } A}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 18 \\ 24 \\ 4 \end{bmatrix}$$

$$\det(A) = 2(-10-6) - 4(-8-18) + 6(4-15)$$
$$= -32 + 104 - 66$$

$= 6$ 이므로 A^{-1}는 존재 가능하다.

$$\begin{vmatrix} 18 & 4 & 6 \\ 24 & 5 & 6 \\ 4 & 1 & -2 \end{vmatrix} = 24$$

$$\begin{vmatrix} 2 & 18 & 6 \\ 4 & 24 & 6 \\ 3 & 4 & -2 \end{vmatrix} = -12$$

$$\begin{vmatrix} 2 & 4 & 18 \\ 4 & 5 & 24 \\ 3 & 1 & 4 \end{vmatrix} = 18$$

$$\therefore x_1 = \frac{24}{6} = 4$$
$$x_2 = \frac{-12}{6} = -2$$
$$x_3 = \frac{18}{6} = 3$$

#9) $x_1=1,\ x_2=-3,\ x_3=2$

$$\underbrace{\begin{bmatrix} 1 & -3 & -2 \\ 2 & -4 & -3 \\ -3 & 6 & 8 \end{bmatrix}}_{\text{행렬 } A}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 8 \\ -5 \end{bmatrix}$$

$$\det(A) = 1(-32+18) + 3(16-9) - 2(12-12)$$

$= -14+21 = 7$ 이므로 A^{-1}는 존재 가능하다.

$$\begin{vmatrix} 6 & -3 & -2 \\ 8 & -4 & -3 \\ -5 & 6 & 8 \end{vmatrix} = 7$$

$$\begin{vmatrix} 1 & 6 & -2 \\ 2 & 8 & -3 \\ -3 & -5 & 8 \end{vmatrix} = -21$$

$$\begin{vmatrix} 1 & -3 & 6 \\ 2 & -4 & 8 \\ -3 & 6 & -5 \end{vmatrix} = 14$$

$$\therefore x_1 = \frac{7}{7} = 1$$
$$x_2 = \frac{-21}{7} = -3$$
$$x_3 = \frac{14}{7} = 2$$