20201081 9 조정현 5주차 과제

[연습문제 3.1 part 2]

#6) (3)

소행렬식은 n차 행렬식 |A|에서 i행과 j열의

원소를 제외한 나머지로 만들어진 행렬식이다.

따라서 (1) ~ (4)중 나타낼 수 있는 행렬식을 표현하면

$$A = \begin{bmatrix} 2 & 3 & 0 \\ 4 & 2 & 1 \\ 6 & 5 & 2 \end{bmatrix}$$

(3) (1) (2) 이다. ∴ (3)만 소행렬식이 될 수 없다.

[연습문제 3.1 part 3]

#6) (1) -5 (2) 24 (3) 0

(1) $1\times3 - 2\times4 = -5$

(2) $4\times1\times6 + 6\times(-1)\times0 + 0\times0\times5 - 5\times1\times0 - 6\times0\times6 - 4\times(-1)\times0$

$= 24$

(3) $4\times2\times(-3) + 1\times2\times1 + 3\times5\times(-1) - (-1)\times2\times1 - 2\times5\times4 - 1\times3\times(-3)$

$= -24 + 14 - 15 + 2 - 40 + 63$

$= 0$

#11) (1) 11 (2) 4

(1) $2\begin{vmatrix} 1 & 2 \\ 4 & -1 \end{vmatrix} + 4\begin{vmatrix} 3 & 2 \\ 1 & -1 \end{vmatrix} + 3\begin{vmatrix} 3 & 1 \\ 1 & 4 \end{vmatrix}$

$= 2(-1-8) + 4(-3+2) + 3(12-1)$

$= -18 - 4 + 33$

$= 11$

(2) $4\begin{vmatrix} 5 & 2 \\ 1 & 3 \end{vmatrix} - 3\begin{vmatrix} 6 & 2 \\ 9 & 3 \end{vmatrix} + 0\begin{vmatrix} 6 & 5 \\ 9 & 1 \end{vmatrix}$

$= 4(15-14) - 3(18-18) + 0$

$= 4$

#12) (1) -11 (2) -29

(1) $3\begin{vmatrix} 3 & 2 \\ 5 & -1 \end{vmatrix} - 0\begin{vmatrix} 2 & 2 \\ 0 & -1 \end{vmatrix} + 4\begin{vmatrix} 2 & 3 \\ 0 & 5 \end{vmatrix}$

$= 3(-3-10) - 0 + 4(10-3)$

$= -39 + 28$

$= -11$

(2) $2\begin{vmatrix} 0 & 5 \\ 1 & 6 \end{vmatrix} - 3\begin{vmatrix} 4 & 5 \\ 5 & 6 \end{vmatrix} - 4\begin{vmatrix} 4 & 0 \\ 5 & 1 \end{vmatrix}$

$= 2(0-5) - 3(24-25) - 4(4-0)$

$= -10 - 3 - 16$

$= -29$

#15) (1) -83

2번째 열을 선택해서 풀이한다. 이유는 같은 행 안에 0이 가장 많이 들어있기 때문이다.

$(1 \quad 0 \quad -2 \quad 0)$ ← 2번째 열

$$1\times(-1)^{2+1}\begin{vmatrix} 2 & 2 & 3 \\ -1 & 1 & -2 \\ -3 & 0 & 2 \end{vmatrix} + (-2)\times(-1)^{2+3}\begin{vmatrix} 1 & 2 & 3 \\ 3 & -1 & -2 \\ 4 & -3 & 2 \end{vmatrix}$$

$$= -\begin{vmatrix} 2 & 2 & 3 \\ -1 & 1 & -2 \\ -3 & 0 & 2 \end{vmatrix} + 2\begin{vmatrix} 1 & 2 & 3 \\ 3 & -1 & -2 \\ 4 & -3 & 2 \end{vmatrix}$$

$= -\{2\times1\times2 + 2\times(-2)\times(-3) + (-1)\times0\times3$

$- 3\times1\times(-3) - (-2)\times0\times2 - 2\times(-1)\times2\}$

$= -(4 + 12 + 9 + 4)$

$= -29$

$= 2\{1\times(-1)\times2 + 2\times(-2)\times4 + 3\times(-3)\times3$

$- 3\times(-1)\times4 - (-2)\times(-3)\times1 - 2\times3\times2\}$

$= 2\{-2 - 16 - 27 + 12 - 6 + 12\}$

$= 2(-27) = -54$ ∴ $-29 - 54 = -83$

20201081 9 조정현 5주차 과제

[연습문제 3.2 part3]

#3) 26

$|A| = \det(A)$ 이고 $|B| = \det(B)$,

$|AB| = \det(AB) = \det(A) \times \det(B)$ 이다.

$\therefore \det(AB) = (-2) \times (-13) = 26$

#5) (1) 1 (2) k (3) k (4) 1

(1) $1 \times (1-0) = 1$

(2) $k\,(1 \times 1 - 0) = k$

(3) $1\,(k \times 1 - 0) = k$

(4) $1 \times (1 \times 1 - 0) = 1$

#10) 6

$\det(BC) = \det(B) \times \det(C)$ 이므로

$\det(B) = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 4 & 1 \end{bmatrix}$

$\det(C) = \begin{bmatrix} 1 & 4 & 5 \\ 0 & 2 & 6 \\ 0 & 0 & 3 \end{bmatrix}$ 이라 하면,

$\det(B) = 1 \times (1 \times 1 - 0) = 1$

$\det(C) = 1 \times (2 \times 3 - 6 \times 0) - 4\,(0 \times 3 - 0 \times 6) + 5\,(0 \times 0 - 2 \times 0)$

$= 6$

$\therefore \det(B) \times \det(C) = 1 \times 6 = 6$

#12) (1) -20 (2) 5 (3) 4 (4) 16

(1) $2\,(3 \times 1 + 1 \times 1) - 1\,(0 \times 1 + 1 \times 4) + 2\,(0 \times 1 - 3 \times 4)$

$= 2 \times 4 - 4 - 24$

$= -20$

(2) $3\,(2 \times 3 - 1 \times 4) + 1\,(-1 \times 3 + 2 \times 1) + 5\,(-1 \times 4 + 4)$

$= 6 - 1 = 5$

(3) $2\,(3 \times 1 - 2 \times 0) - 4\,(-1 \times 1 - 0) + 3\,(-1 \times 2 - 3 \times 0)$

$= 2 \times 3 + 4 - 6 = 4$

(4) $1 \times (2 \times 7 + 1 \times 2) - 2\,(0 \times 7 + 0 \times 1) - 1\,(0 \times 2 - 0 \times 2)$

$= 16$

#15) 209

$$(-1)^{1+1} \times 3 \times \begin{vmatrix} -3 & 5 & -7 \\ 3 & -3 & 4 \\ 5 & 6 & 7 \end{vmatrix} + (-1)^{1+2} \times 7 \times \begin{vmatrix} 2 & 5 & -7 \\ -2 & -3 & 4 \\ 10 & 6 & 7 \end{vmatrix} + (-1)^{1+3} \times 4 \times \begin{vmatrix} 2 & -3 & -7 \\ -2 & 3 & 4 \\ 10 & 5 & 7 \end{vmatrix}$$

$$+ (-1)^{1+4} \times 6 \begin{vmatrix} 2 & -3 & 5 \\ -2 & 3 & -3 \\ 10 & 5 & 6 \end{vmatrix}$$

$$= -9 \begin{vmatrix} -3 & 4 \\ 6 & 7 \end{vmatrix} - 15 \begin{vmatrix} 3 & 4 \\ 5 & 7 \end{vmatrix} - 21 \begin{vmatrix} 3 & -3 \\ 5 & 6 \end{vmatrix} - 14 \begin{vmatrix} -3 & 4 \\ 6 & 7 \end{vmatrix} + 35 \begin{vmatrix} -2 & 4 \\ 10 & 7 \end{vmatrix}$$
$$+ 49 \begin{vmatrix} -2 & -3 \\ 10 & 6 \end{vmatrix} + 8 \begin{vmatrix} 3 & 4 \\ 5 & 7 \end{vmatrix} + 12 \begin{vmatrix} -2 & 4 \\ 10 & 7 \end{vmatrix} - 28 \begin{vmatrix} -2 & 3 \\ 10 & 5 \end{vmatrix} - 12 \begin{vmatrix} 3 & -3 \\ 5 & 6 \end{vmatrix}$$
$$- 18 \begin{vmatrix} -2 & -3 \\ 10 & 6 \end{vmatrix} - 30 \begin{vmatrix} -2 & 3 \\ 10 & 5 \end{vmatrix}$$

$$= -9\,(-21 - 24) - 15\,(21 - 20) - 21\,(18 + 15) - 14\,(-21 - 24) + 35\,(-14 - 40)$$
$$+ 49\,(-12 + 30) + 8\,(21 - 20) + 12\,(-14 - 40) - 28\,(-10 - 30) - 12\,(18 + 15)$$
$$- 18\,(-12 + 30) - 30\,(-10 - 30)$$

$= 209$