Assignment-05

Review on Alphabets/Strings and Language.

1. Alphabets are finite set of symbols which is denoted by $\sum$. For example: $\sum = \{a,b,c,d\}$ or $\sum = \{0,1\}$.
2. Strings: Choosing the alphabets and creating a sequence of alphabets we make up.
3. Language : it is a set of strings of symbols over a finite alphabet.

Operations on Languages:

- Concatenation - merging of two languages together.
  $L_1L_2 = \{st: s \in L_1, t \in L_2\}$
- Nth power - set of strings made up from the language itself.
  $L^n = \{s_1s_2...s_n: s_1, s_2, ..., s_n \in L\}$
  where, s_n is a string of that language. For example: $L = \{ 0,1\}$
  $L^0 = \varepsilon$, $L^1 = \{ 0,1\} = L^1$, $L^2 = \{ 00,01,10,11\}$ and so on...
- Union - The new language will contain set of ALL strings from both the languages.
  $L_1 \cup L_2 = \{s: s \in L_1$ or $s \in L_2\}$
- Reversal-
  $L_1 \cup L_2 = \{s: s \in L_1$ or $s \in L_2\}$
- Complement- when in automata “does not exist” happens then it means complement. Suppose a language must not have 0’s at the end so the language will be all possible set of strings over alphabets $\{0,1\}$
  - **Example of the above operations are given below:**
    Given $L_1 = \{0,10\}$ and $L_2=\{\varepsilon, 1, 11, 111, \dots \}$
    $L_1 L_2 = \{01,011,0111,...\}$ U $\{101,1011,10111,...\}$ (combination of an element from L_1 and another element from L_2)

    $L_1^2 = \{00,100,010,1010\}$ || $L_2^2 = L_2$
    L_1 U $L_2 = \{0,10,\varepsilon, 1,11,111,1111,....\}$

- Star of a language-
  It means that any possible combination of strings made up from a set of strings of a language (no string/zero can also be present).
  It is the union of languages.
  $L^* = L^0 \cup L^1 \cup L^2 \cup \dots$

- **Example of "star" operation:**

Given, $L_1 = \{01, 0\}$, $L_2 = \{\varepsilon, 1, 11, 111, \ldots\}$.

If we need to find $L_1{}^*$ and $L_2{}^*$.

$L_1{}^* = L_0 \cup L_1 \cup L_2 \ldots..$

Here, $L_1^0 = \varepsilon$

$L_1^1 = L_1$,

$L_1^2 = \{010,0101,001,00\}$, so on..

Thus, $L_1{}^* = L_1^0 \cup L_1^1 \cup L_1^2 \cup \ldots$

$\{\varepsilon\} \cup \{01,0\} \cup \{010,0101,001,00\} \cup \ldots\ldots$

It is a language that does not have any consecutive 11s and starts with 0.

For $L_2{}^*$,

We repeat the same procedure and find that $L_2{}^* = L_2$.

Combining Languages:

It can be shown in another form too. For example,

$$(\{0\} \cup \{1\})^* \rightarrow 0(0+1)^*$$

This means that all possible combinations of strings containing 0 and 1 which begin from 0. Here, $(\{0\} \cup \{1\})^*$ we start from the first inner brackets(union of 0 and 1) till we reach the last bracket(outermost bracket).

Similarly ,

$$(\{0\}\{1\}^*) \cup (\{1\}\{0\}^*) \rightarrow 01^* + 10^*$$

The inner brackets of each side are solved then union of both strings. We find that the language of this example is the strings that contain either 0 at beginning followed by 1s or 1 at the beginning followed by 0s.

Regular Expression:

$\varnothing$, ε and a are called regular expression where a is the symbols of $\sum$.

Each regular expression are called language.

We can combine the regular expressions using these operations which are R+S, RS and R*. where R and S are just regular expressions. Using the regular expression and operations we can combine them to make a bigger regular expression.

A language will be called regular when it is expressed using regular expressions. However, not all languages can be expressed by regular expressions.