机器人原理课程

05. 速度与静力b

课件来自于台湾大学 林沛群 教授 “机器人学导论”课程课件，在此表示感谢！

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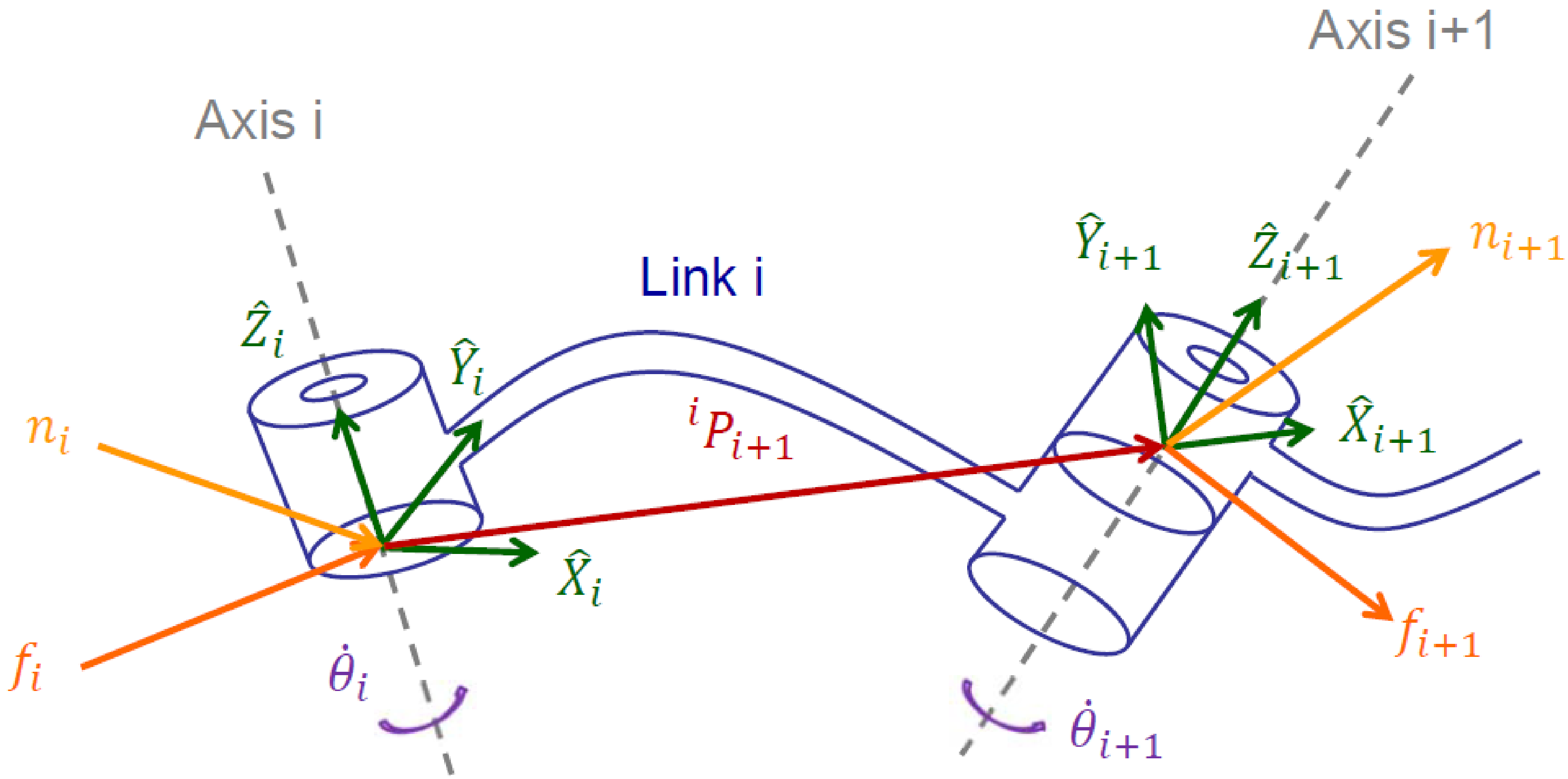
速度与静力

- When considering static forces
  - Lock all the joints
  - Write force-moment relationship
  - Compute static torque (ignore gravity)

速度与静力

- $${}^{i}f_i = {}^{i}f_{i+1} \xrightarrow{{}^{i}_{i+1}R} {}^{i}f_i = {}^{i}_{i+1}R\,{}^{i+1}f_{i+1}$$

$${}^{i}n_i = {}^{i}n_{i+1} + {}^{i}P_{i+1} \times {}^{i}f_{i+1} \xrightarrow{{}^{i}_{i+1}R} {}^{i}n_i = {}^{i}_{i+1}R\,{}^{i+1}n_{i+1} + {}^{i}P_{i+1} \times {}^{i}f_i$$



速度与静力

- The joint toque required to maintain the static equilibrium
  - Revolute joint

$$\tau_i = {}^{i}n_i^T \, {}^{i}\hat{Z}_i$$

  - Prismatic joint

$$\tau_i = {}^{i}f_i^T \, {}^{i}\hat{Z}_i$$

速度与静力

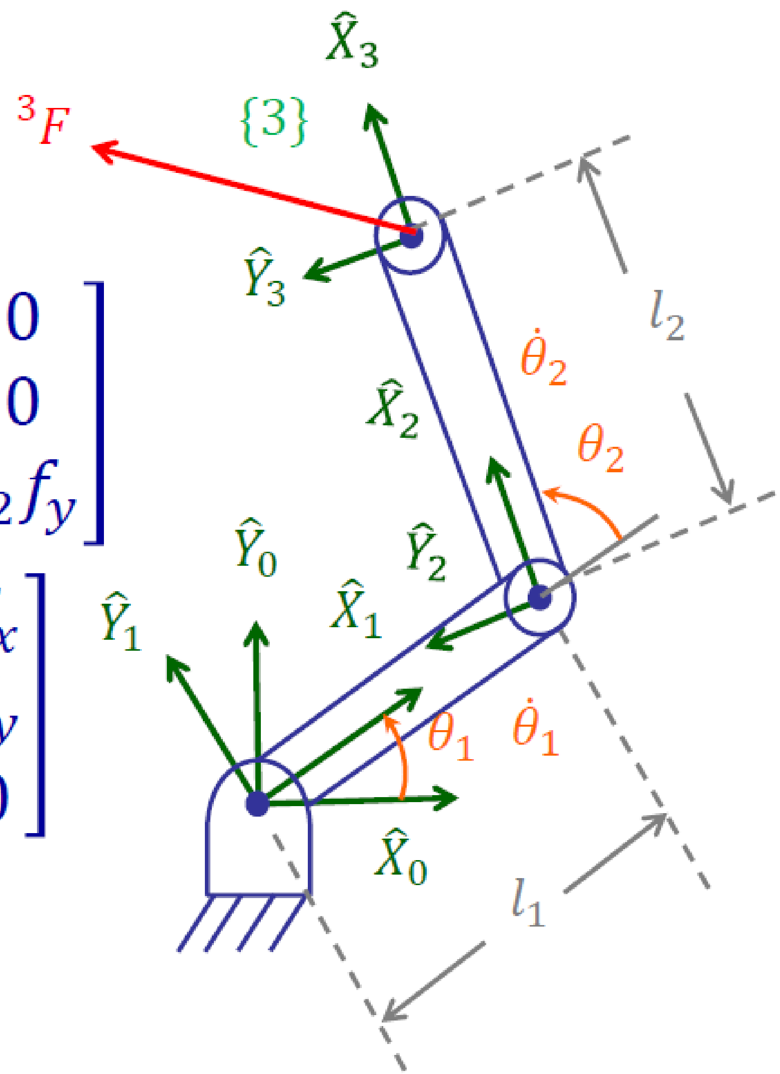
- Force "propagation" from link to link

$$^2f_2 = {}^2_3R\ {}^3f_3 = I\ {}^3F = \begin{bmatrix} f_x \\ f_y \\ 0 \end{bmatrix}$$

$$^2n_2 = {}^2_3R\ {}^3n_3 + {}^2P_3 \times {}^2f_2 = \begin{bmatrix} 0 \\ 0 \\ l_2 f_y \end{bmatrix}$$

$$^1f_1 = {}^1_2R\ {}^2f_2 = \begin{bmatrix} c_2 & -s_2 & 0 \\ s_2 & c_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} f_x \\ f_y \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} c_2 f_x - s_2 f_y \\ s_2 f_x + c_2 f_y \\ 0 \end{bmatrix}$$



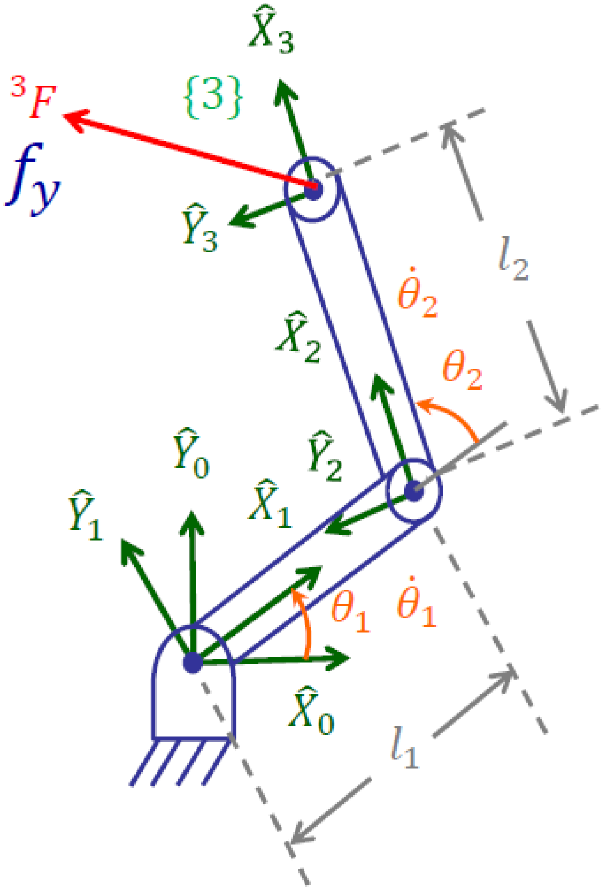
速度与静力

$$ {}^{1}n_{1} = {}^{1}_{2}R\,{}^{2}n_{2} + {}^{1}P_{2} \times {}^{1}f_{1} = \begin{bmatrix} 0 \\ 0 \\ l_1 s_2 f_x + l_1 c_2 f_y + l_2 f_y \end{bmatrix} $$

- Therefore,

$$ \tau_1 = {}^{1}n_1^T\,{}^{1}\widehat{Z_1} = l_1 s_2 f_x + (l_1 c_2 + l_2) f_y $$

$$ \tau_2 = {}^{2}n_2^T\,{}^{2}\widehat{Z_2} = l_2 f_y $$



$$ \Rightarrow \tau = \begin{bmatrix} l_1 s_2 & l_1 c_2 + l_2 \\ 0 & l_2 \end{bmatrix} \begin{bmatrix} f_x \\ f_y \end{bmatrix} $$

速度与静力

- The principal of virtual work

$$F \cdot \delta \mathcal{X} = \Gamma \cdot \delta \Theta$$

$$F^T \delta \mathcal{X} = F^T J \delta \Theta = \Gamma^T \delta \Theta$$

$$\Gamma = J^T F$$

Respect to frame $\{0\}$

$$\Rightarrow \quad \Gamma = {}^0 J^T \, {}^0 F$$

"inverse" Cartesian torque to joint torque without using IK technique

速度与静力

- General velocity and force representations

$$\boldsymbol{\nu} = \begin{bmatrix} v \\ \omega \end{bmatrix} \qquad \boldsymbol{\mathcal{F}} = \begin{bmatrix} F \\ N \end{bmatrix}$$

- Frame transformation

$${}^{i+1}\omega_{i+1} = {}^{i+1}_{i}R\ {}^{i}\omega_i + \dot{\theta}_{i+1}\,{}^{i+1}\hat{Z}_{i+1}$$

$${}^{i+1}v_{i+1} = {}^{i+1}_{i}R(\ {}^{i}v_i + \ {}^{i}\omega_i \times \ {}^{i}P_{i+1})$$

$$\Big\downarrow\ i = A, i+1 = B, \dot{\theta} = 0$$

$$\begin{bmatrix} {}^{A}v_A \\ {}^{A}\omega_A \end{bmatrix} = \begin{bmatrix} {}^{A}_{B}R & {}^{A}P_{B\,ORG} \times {}^{A}_{B}R \\ 0 & {}^{A}_{B}R \end{bmatrix} \begin{bmatrix} {}^{B}v_B \\ {}^{B}\omega_B \end{bmatrix}$$

$${}^{A}\boldsymbol{\nu}_A = {}^{A}_{B}T_v\ {}^{B}\boldsymbol{\nu}_B \qquad P\times = \begin{bmatrix} 0 & -p_z & p_y \\ p_z & 0 & -p_x \\ -p_y & p_x & 0 \end{bmatrix}$$

速度与静力

$$\Big\downarrow \text{"inverse"}$$

$$\begin{bmatrix} {}^{B}v_{B} \\ {}^{B}\omega_{B} \end{bmatrix} = \begin{bmatrix} {}^{B}_{A}R & -{}^{B}_{A}R\, {}^{A}P_{BORG}\times \\ 0 & {}^{B}_{A}R \end{bmatrix} \begin{bmatrix} {}^{A}v_{A} \\ {}^{A}\omega_{A} \end{bmatrix}$$

$${}^{B}\boldsymbol{v}_{B} = {}^{B}_{A}T_{v}\, {}^{A}\boldsymbol{v}_{A}$$

- Similarly,

$$\begin{bmatrix} {}^{A}F_{A} \\ {}^{A}N_{A} \end{bmatrix} = \begin{bmatrix} {}^{A}_{B}R & 0 \\ {}^{A}P_{BORG}\times {}^{A}_{B}R & {}^{A}_{B}R \end{bmatrix} \begin{bmatrix} {}^{B}F_{B} \\ {}^{B}N_{B} \end{bmatrix}$$

$${}^{A}\boldsymbol{\mathcal{F}}_{A} = {}^{A}_{B}T_{f}\, {}^{B}\boldsymbol{\mathcal{F}}_{B}$$

$$\Rightarrow \quad {}^{A}_{B}T_{f} = {}^{A}_{B}T_{v}{}^{T}$$

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