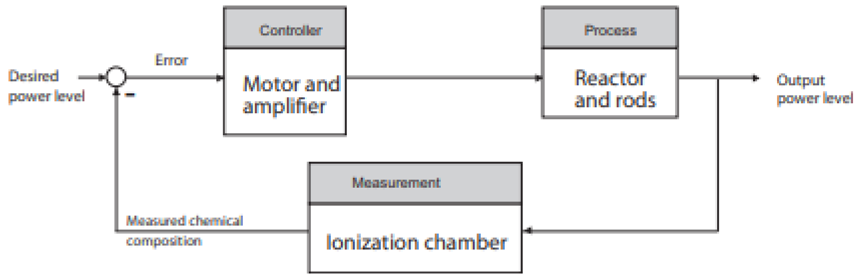
1、



2、

$$\frac{Y(s)}{R(s)} = T(s) = \frac{K_1K_2}{s^2 + (K_1 + K_2K_3 + K_1K_2)s + K_1K_2K_3} \, .$$

3、

(a) If

$$G(s) = \frac{1}{s^2 + 15s + 50} \quad \text{and} \quad H(s) = 2s + 15 \, ,$$

then the closed-loop transfer function of Figure E2.28(a) and (b) (in Dorf & Bishop) are equivalent.

(b) The closed-loop transfer function is

$$T(s) = \frac{1}{s^2 + 17s + 65} \, .$$

4、

The equations of motion for the two mass model of the robot are

$$M\ddot{x} + b(\dot{x} - \dot{y}) + k(x - y) = F(t)$$
$$m\ddot{y} + b(\dot{y} - \dot{x}) + k(y - x) = 0 \, .$$

Taking the Laplace transform and writing the result in matrix form yields

$$\begin{bmatrix} Ms^2 + bs + k & -(bs + k) \\ -(bs + k) & ms^2 + bs + k \end{bmatrix} \begin{bmatrix} X(s) \\ Y(s) \end{bmatrix} = \begin{bmatrix} F(s) \\ 0 \end{bmatrix} \, .$$

Solving for $Y(s)$ we find that

$$\frac{Y(s)}{F(s)} = \frac{\frac{1}{mM}(bs + k)}{s^2[s^2 + (1 + \frac{m}{M})\left(\frac{b}{m}s + \frac{k}{m}\right)]} \, .$$

5、

$$m_1\frac{d^2x}{dt^2} = -(k_1+k_2)x + k_2 y \quad \text{and} \quad m_2\frac{d^2y}{dt^2} = k_2(x-y)+u \ .$$

When $m_1 = m_2 = 1$ and $k_1 = k_2 = 1$, we have

$$\frac{d^2x}{dt^2} = -2x + y \quad \text{and} \quad \frac{d^2y}{dt^2} = x - y + u \ .$$

6 、

$$\begin{aligned} \dot{x}_1 &= -x_1 + \frac{1}{2}x_2 + r \\ \dot{x}_2 &= x_1 - \frac{3}{2}x_2 - r \\ y &= x_1 - \frac{3}{2}x_2 - r. \end{aligned}$$

In state-variable form we have

$$\dot{\mathbf{x}} = \begin{bmatrix} -1 & \frac{1}{2} \\ 1 & -\frac{3}{2} \end{bmatrix}\mathbf{x} + \begin{bmatrix} 1 \\ -1 \end{bmatrix} r \ , \quad y = \begin{bmatrix} 1 & -\frac{3}{2} \end{bmatrix}\mathbf{x} + \begin{bmatrix} -1 \end{bmatrix} r \ .$$

7、

$$\begin{aligned} Ri_1 + L_1\frac{di_1}{dt} + v &= v_a \\ L_2\frac{di_2}{dt} + v &= v_b \\ i_L = i_1 + i_2 &= C\frac{dv}{dt} \ . \end{aligned}$$

Let $x_1 = i_1, x_2 = i_2, x_3 = v, u_1 = v_a$ and $u_2 = v_b$. Then,

$$\dot{\mathbf{x}} = \begin{bmatrix} -\frac{R}{L_1} & 0 & -\frac{1}{L_1} \\ 0 & 0 & -\frac{1}{L_2} \\ \frac{1}{C} & \frac{1}{C} & 0 \end{bmatrix}\mathbf{x} + \begin{bmatrix} \frac{1}{L_1} & 0 \\ 0 & \frac{1}{L_2} \\ 0 & 0 \end{bmatrix}\mathbf{u}$$

$$y = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}\mathbf{x} + [0]\,\mathbf{u} \ .$$

8、

$$KG(s) \cdot \frac{1}{s} = \frac{(s+1)^2}{s(s^2+1)} \ .$$

We then compute the closed-loop transfer function as

$$T(s) = \frac{s^2+2s+1}{3s^3+5s^2+5s+1} = \frac{s^{-1}+2s^{-2}+s^{-3}}{3+5s^{-1}+5s^{-2}+s^{-3}} \ .$$

(a) The state variable model is

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1/3 & -5/3 & -5/3 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \\ 0 \\ 1/3 \end{bmatrix} r$$

$$y = \begin{bmatrix} 1 & 2 & 1 \end{bmatrix} \mathbf{x} \ .$$

9、

$$\dot{\mathbf{x}} = \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} d \ .$$

When $u_1 = 0$ and $u_2 = d = 1$, we have

$$\dot{x}_1 = 3x_1 + u_2$$
$$\dot{x}_2 = 2x_2 + 2u_2$$

So we see that we have two independent equations for x_1 and x_2. With $U_2(s) = 1/s$ and zero initial conditions, the solution for x_1 is found to be

$$x_1(t) = \mathcal{L}^{-1}\{X_1(s)\} = \mathcal{L}^{-1}\left\{\frac{1}{s(s-3)}\right\}$$
$$= \mathcal{L}^{-1}\left\{-\frac{1}{3s} + \frac{1}{3}\frac{1}{s-3}\right\} = -\frac{1}{3}\left(1-e^{3t}\right)$$

and the solution for x_2 is

$$x_2(t) = \mathcal{L}^{-1}\{X_2(s)\} = \mathcal{L}^{-1}\left\{\frac{2}{s(s-2)}\right\} = \mathcal{L}^{-1}\left\{-\frac{1}{s} + \frac{1}{s-2}\right\} = -1+e^{2t} \ .$$