Neural network Notation

by Sue G.

training examples

$$X = \underbrace{\begin{bmatrix} x_{11} & \cdots & x_{1m} \\ x_2 & & \\ \vdots & & \\ x_{n_x} & & \end{bmatrix}}_{(n_x,\, m)} = \begin{bmatrix} | & | & & | \\ x^{(1)} & x^{(2)} & \cdots & x^{(m)} \\ | & | & & | \end{bmatrix}$$

different features

$x^{(1)}$

$$z_1^{[1](i)} = w_1^{[1]T} x^{(i)} + b_1^{[1]}$$

$$\underset{(l,\, n_x)}{W^{[1]}} = \begin{bmatrix} w_{11} & w_{12} & w_{13} & \cdots & w_{1n_x} \\ w_{21} & & & & \\ \vdots & & & & \\ w_{l1} & & & & \end{bmatrix} = \begin{bmatrix} w_1^T \\ w_2^T \\ \vdots \\ w_l^T \end{bmatrix}$$

$$z_1^{[1](1)} = w_1^{[1]} x^{(1)} + b^{[1]}$$

$$= w_1^{[1]} x^{(1)} + b^{[1]}$$

$$= w_{11} \cdot x_1^{(1)} + w_{12} \cdot x_2^{(1)} + \cdots + w_{1n_x} \cdot x_{n_x}^{(1)}$$

l: #layer neuron

if $n_x = 2$:

for loop

for $i = 1$ to m:

$$z^{[1](i)} = W^{[1]} x^{(i)} + b^{[1]}$$

$$a^{[1](i)} = \sigma(z^{[1](i)})$$

$$z^{[2](i)} = W^{[2]} a^{[1](i)} + b^{[2]}$$

$$a^{[2](i)} = \sigma(z^{[2](i)})$$

Vectorizing across multiple examples

$$Z^{[1]} = W^{[1]} X + b^{[1]}$$

$$A^{[1]} = \sigma(Z^{[1]})$$

$$Z^{[2]} = W^{[2]} A^{[1]} + b^{[2]}$$

$$A^{[2]} = \sigma(Z^{[2]})$$

$$Z^{[1]} = \begin{bmatrix} w_{11} \cdot x_1^{(1)} + w_{12} \cdot x_2^{(1)} + b & w_{11} \cdot x_1^{(2)} + w_{12} \cdot x_2^{(2)} + b & \cdots & w_{11} \cdot x_1^{(m)} + w_{12} \cdot x_2^{(m)} \\ w_{21} & & & \\ \vdots & & & \\ w_{l1} & & & \end{bmatrix}$$

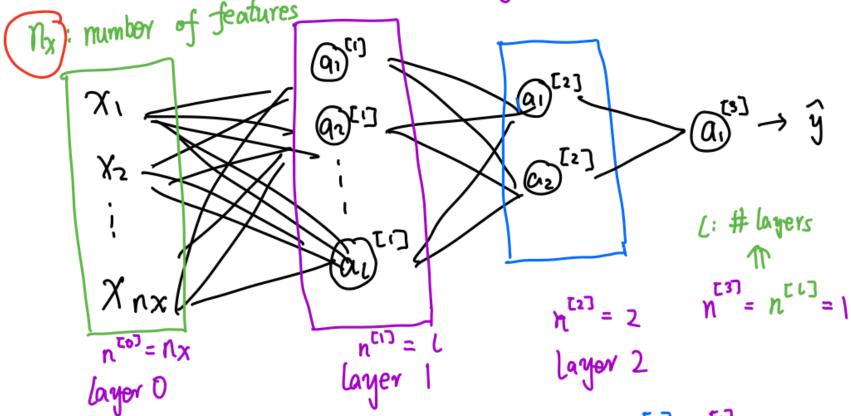
$$= \begin{bmatrix} z^{[1](1)} & z^{[1](2)} & \cdots & z^{[1](m)} \end{bmatrix}$$

scan through training examples

$$A^{[1]} = \sigma(Z^{[1]}) = \begin{bmatrix} a^{[1](1)} & \cdots & a^{[1](m)} \\ & & \\ & & \end{bmatrix}$$

different nodes / hidden units

$n^{[l]}$: number of hidden units of the l^{th} layer



$a^{[0]} = X$

$a^{[L]} = \hat{y}$

$z^{[l]} = W^{[l]} a^{[l-1]} + b^{[l]}$

$a^{[l]} = g^{[l]}(z^{[l]}) = \hat{y}$

$z^{[1]} = W^{[1]} \underbrace{A^{[0]}}_{= X} + b^{[1]}$

$A^{[1]} = g^{[1]}(z^{[1]})$

$\vdots$

$z^{[L]} = W^{[L]} A^{[L-1]} + b^{[L]}$

$A^{[L]} = g^{[L]}(z^{[L]}) = \hat{Y}$

After Vectorized

$W^{[1]} : \underbrace{n^{[1]} \cdot n_x}_{\text{shape}}$, $n_x = n^{[0]}$ $\quad W^{[2]} : n^{[2]} \cdot n^{[1]}$

$b^{[1]} : n^{[1]} \cdot 1 \quad b^{[2]} : n^{[2]} \cdot 1$

$A^{[1]} : n^{[1]} \cdot m \quad A^{[2]} : n^{[2]} \cdot m$

$X^{[1]} : n_x \cdot m \quad X^{[2]} : n_x \cdot m$

$A^{[1]} = \sigma(WX + b)$

$(n^{[1]} \cdot \cancel{n_x}) \cdot (\cancel{n_x} \cdot m) + (n^{[1]} \cdot 1)$

$= n^{[1]} \cdot m \quad$ broadcasting $\Rightarrow n^{[1]} \cdot m$

$\Rightarrow$

$W^{[l]} : (n^{[l]}, n^{[l-1]})$

$b^{[l]} : (n^{[l]}, 1)$

$dW^{[l]} : (n^{[l]}, n^{[l-1]})$

$db^{[l]} : (n^{[l]}, 1)$

$z^{[l]}, a^{[l]} : (n^{[l]}, 1)$

W, b, dW, db same

$Z^{[l]}, A^{[l]} : (n^{[l]}, m)$

Dimentions are Different After Vectorized

Justification for vectorized implementation

$z^{[1](1)} = W^{[1]} x^{(1)} + \cancel{b^{[1]}} \rightarrow 0$, $\quad z^{[1](2)} = W^{[1]} x^{(2)} + \cancel{b^{[1]}} \rightarrow 0$, $\quad z^{[1](3)} = W^{[1]} x^{(3)} + \cancel{b^{[1]}} \rightarrow 0$

$W^{[1]} = \begin{bmatrix} \text{—} \\ \text{—} \\ \text{—} \end{bmatrix} \quad W^{[1]} x^{(1)} = \begin{bmatrix} \cdot \\ \cdot \\ \cdot \\ \cdot \end{bmatrix} \quad W^{[1]} x^{(2)} = \begin{bmatrix} \cdot \\ \cdot \\ \cdot \\ \cdot \end{bmatrix} \quad W^{[1]} x^{(3)} = \begin{bmatrix} \cdot \\ \cdot \\ \cdot \\ \cdot \end{bmatrix}$

$$W^{[1]} \underbrace{\begin{bmatrix} | & | & | & \\ x^{(1)} & x^{(2)} & x^{(3)} & \cdots \\ | & | & | & \end{bmatrix}}_{X} = \begin{bmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{bmatrix} = \begin{bmatrix} | & | & | & \\ z^{[1](1)} & z^{[1](2)} & z^{[1](3)} & \cdots \\ | & | & | & \end{bmatrix} = Z^{[1]}$$

$+b^{[1]} \quad +b^{[1]} \quad +b^{[1]}$

$Z^{[1]} = W^{[1]} X + b^{[1]}$

$W^{[1]} x^{(i)} = z^{[1](i)}$

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