day/date

Assignment 1

Q1 P1) Both John and Bill can be correct as Big-Oh gives an upper bound function hence $O(n^3 + 200n)$ or $O(n^3)$, ~~hence~~ this means running time will not go beyond this.
$\Omega(n^3)$ gives lower bound, meaning it is greater or equal to n^3. ~~The algorithim run time can be theta (n^3).~~
Hence both statements hold if the run time is between n^3 and $n^3 + 200n$

Q1 P2) Ω tells the lower bound.
John states it is $n^2 + 200n \leq f(n)$ while Bill states $n^3 \leq f(n)$
Bills algorithim will more likely give the ~~correct answer~~ better runtime as it will give the closer answer, since the algorithim is greater than or equal to n^3.

Q2) $n \leq \log_2 n \leq 1 n\log_2 n \leq n^2 + \log_2 n \leq n^3 \leq 2^n$ ~~$\leq \log 2(n)$~~

Q3) $2^n < n < \log 2(n) < n^2 < n\log_2(n) < n^2 + \log_2(n) < n^3$

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Q4) P1 $\frac{n^3+2n}{2n+1} = O(n^2)$ $\frac{n^3+2n}{n^2(2n+1)} \le C$

$\frac{n^3+2n}{2n+1} \le Cn^2$ $\frac{n^3+2n}{2n^3+n^2}$ $\frac{\cancel{n}(n^2+2)}{n^2(2n+1)} = \frac{n^2+2}{n(2n+1)} \le C$

$\frac{n^2+2n}{n^2(2n+1)} \le C$ $\frac{n^2+2n^2}{2n^2+n^2} \le C$

$\frac{\cancel{n^2+2n}}{n}$ $\frac{n(2+n)}{n^2(2n+1)} \le C$

$\frac{2+n}{2n^2+n} \le C$ $\frac{n^2+n^2}{2n^2+n^2} > \frac{2+n}{2n^2+n}$ $n>1$

$\frac{1+1}{2(1)+1} = \frac{2}{3}$

$\frac{n^2+2}{2n^2+n} \le \frac{n^2+2n^2}{2n^2+n^2}$

$\le \frac{3n^2}{3n^2} 1$

$\frac{n^3+2n}{n^2(2n+1)} \le C$ $C=3$

$\frac{n^3+2n}{n^2(2n+1)} \le 3$

$n^3+2n \le 3(2n^3+n^2)$

$n^3+2n \le 6n^3+3n^2$

$2n \le 5n^3+3n^2$

$\frac{-5n^3}{n^2}+\frac{2n}{n^2} \le \frac{3n^2}{n^2}$

$-5n+\frac{2}{n} \le 3$ for $n \geq 2$

$C=3$ $n_0=2$

means ...

Q4P2) $c_1 n^3 \leq (n+3)^3 \leq c_2 n^3$ $n \geq n_0$

$n_0 = 3$ $c_1 = 1/16$ $c_2 = 16$

lower bound

$(n+3)^3 \geq 3^3 = 27 \geq 27/16 * n^3$ $c_1 = 1/16$

upper bound

$\cancel{(n+3)^3 \leq (n+3)}$

$(n+3)^3 <= 16n^3$

$n^3 + 9n^2 + 27n + 27 \leq 16n^3$

$n^3 + 9n^2 + 27n^2 + 27n \leq 16n^3$

$n^3 + 36n^2 \leq 16n^3$

$(n+3)^3 = \Theta(n^3)$ with $C_1 = 1/16$, $C_2 = 16$, $n0 = 3$

Finally BingO!

$c = 8 \quad n_0 = 2$

Bonus Question: This is because the statement means ~~only provides~~ a lower bound for running time, while $O(n^2)$ is an upper bound meaning ~~num~~ values lower the n^2 and not above it

Finally BingO!