Title of My Thesis

SUBMITTED BY

Jan-Philipp Anton Konrad Christ

Titel meiner Arbeit

Bachelorarbeit

Fakultät für Physik
Quanten Vielteilchensysteme/ Theoretische Nanophysik
Ludwig-Maximilians-Universität
München

vorgelegt von

Jan-Philipp Anton Konrad Christ

München, 22.06.2023

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Bachelor Thesis

Faculty of Physics
Quantum Many-Body Systems/ Theoretical Nanophysics Group
Ludwig Maximilian University
Munich

submitted by

Jan-Philipp Anton Konrad Christ

Munich, 22.06.2023

First supervisor: Prof. Dr. Fabian Bohrdt, geb. Grusdt
Second supervisor: ...

NOTATION AND SYMBOLS

Here you can write which notation and conventions you want to use in your formulas.

For example, there are two different sign conventions for the metric tensor $\eta_{\mu\nu}$ in Minkowski space.

In (general) relativity, it is common to use the sign convention $\eta_{\mu\nu} = \mathrm{diag}(-,+,+,+)$, which is called the "mostly plus", "space dominant" or "east coast" sign convention.
Therefore, to lower the components of a contravariant vector $\boldsymbol{x} = (ct, x, y, z) =: (x^0, x^1, x^2, x^3)$, we get

$$
\begin{aligned}
x_0 &= \eta_{0\nu} x^\nu = -ct \neq x^0, \\
x_1 &= \eta_{1\nu} x^\nu = +x = x^1, \\
x_2 &= \eta_{2\nu} x^\nu = +y = x^2, \\
x_3 &= \eta_{3\nu} x^\nu = +z = x^3.
\end{aligned}
$$

On the other hand, in particle physics and quantum field theory, it is common to use the sign convention $\eta_{\mu\nu} = \mathrm{diag}(+,-,-,-)$, which is called the "mostly minus", "time dominant" or "west coast" sign convention.
Therefore, to lower the components of a contravariant vector $\boldsymbol{x} = (ct, x, y, z) =: (x^0, x^1, x^2, x^3)$, we get

$$
\begin{aligned}
x_0 &= \eta_{0\nu} x^\nu = +ct = x^0, \\
x_1 &= \eta_{1\nu} x^\nu = -x \neq x^1, \\
x_2 &= \eta_{2\nu} x^\nu = -y \neq x^2, \\
x_3 &= \eta_{3\nu} x^\nu = -z \neq x^3.
\end{aligned}
$$

Personally, I prefer the "mostly plus" sign convention $\eta_{\mu\nu} = \mathrm{diag}(-,+,+,+)$, since it is only necessary to flip the sign of one component by lowering the index instead of three – but that is of course up to you.

Another convention that is often declared at the beginning of a document are the units that are used.
For example, someone might prefer Planck units instead of SI units. Here, I want to show how to convert from SI units to Planck units.

$$[c]_{\mathrm{SI}} = 299\,792\,458\,\frac{\mathrm{m}}{\mathrm{s}} \implies \left[\sqrt{\frac{\hbar G}{c^3}}\right]_{\mathrm{SI}} \approx 1.616 \times 10^{-35}\,\mathrm{m} =: l_P$$
$$[\hbar]_{\mathrm{SI}} \approx 1.054 \times 10^{-34}\,\mathrm{kg}\frac{\mathrm{m}^2}{\mathrm{s}} \implies \left[\sqrt{\frac{\hbar G}{c^5}}\right]_{\mathrm{SI}} \approx 5.391 \times 10^{-44}\,\mathrm{s} =: t_p$$
$$[G]_{\mathrm{SI}} \approx 6.674 \times 10^{-11}\,\frac{\mathrm{m}^3}{\mathrm{kg} \cdot \mathrm{s}^2} \implies \left[\sqrt{\frac{\hbar c}{G}}\right]_{\mathrm{SI}} \approx 2.176 \times 10^{-8}\,\mathrm{kg} =: m_P$$
$$\left[\frac{1}{4\pi\varepsilon_0}\right]_{\mathrm{SI}} \approx 8.987 \times 10^{9}\,\mathrm{kg}\frac{\mathrm{m}^3}{\mathrm{s}^2 \cdot \mathrm{C}^2} \implies \left[\sqrt{4\pi\epsilon_0\hbar c}\right]_{\mathrm{SI}} \approx 1.875 \times 10^{-18}\,\mathrm{C} =: q_P$$
$$[k_B]_{\mathrm{SI}} \approx 1.381 \times 10^{-23}\,\mathrm{kg}\frac{\mathrm{m}^2}{\mathrm{s}^2 \cdot \mathrm{K}} \implies \left[\sqrt{\frac{\hbar c^5}{G k_B^2}}\right]_{\mathrm{SI}} \approx 1.416 \times 10^{32}\,\mathrm{K} =: T_P$$

In Planck units, we have $[c]_{\mathrm{P}} = 1, [\hbar]_{\mathrm{P}} = 1, [G]_{\mathrm{P}} = 1, \left[\frac{1}{4\pi\epsilon_0}\right]_{\mathrm{P}} = 1, [k_B]_{\mathrm{P}} = 1$ and therefore $l_P = 1, t_P = 1, m_P = 1, q_P = 1, T_P = 1$, which is very convienent to express and manipulate equations.
Of course, this “trick” demands to remind ourself, which physical quantity we deal with while manipulating equations.

ABSTRACT

Here you can write a summary, what your thesis is about or short introduction to your thesis.

Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

CONTENTS

CHAPTER 1 INTRODUCTION

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1.1 Section 01

1.1.1 Subsection 1-01

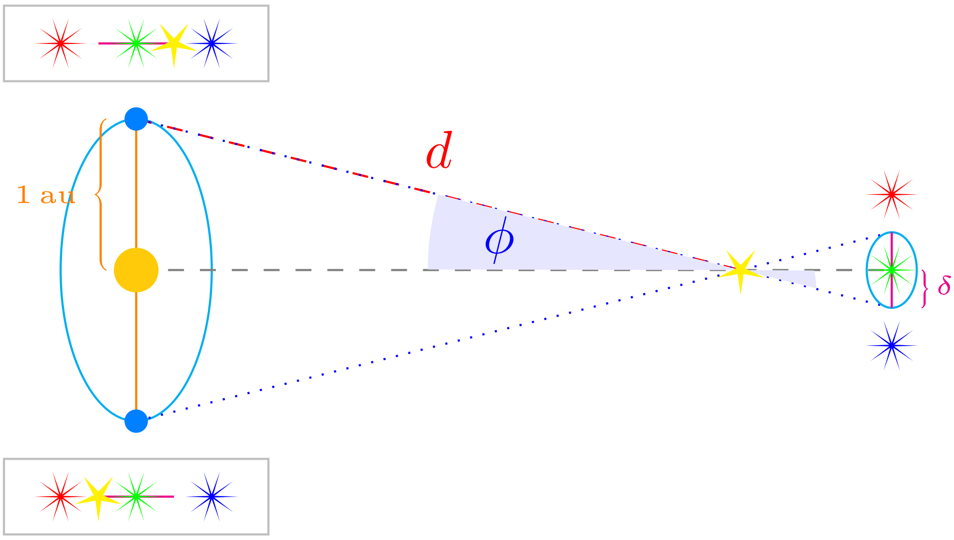

Figure 1.1: Some graphics made with Tikz. Have a look into the directory “`figures/tikz/parallax`”.

1.2 Section 02

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CHAPTER 2

THEORETICAL BACKGROUND

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2.1 Second Quantization

2.1.1 Occupation number representation and the Fock Space

I can cite stuff [**Leavitt**].

2.1.2 Bosonic creation and annihilation operators

Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

2.1.3 One-Body-Operators

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2.1.5 Change of basis

2.2 Section 02

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CHAPTER 3

CHAPTER 02

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CHAPTER 4

CONCLUSION

APPENDIX A

DETAILED CALCULATIONS

A.1 Deriving the flow equations in the case of no n-dependence

First the canonical generator $\hat{\eta}$ has to be evaluated:

$$\hat{\eta} := \hat{\eta}(\lambda) := \left[\hat{\mathcal{H}}_0, \hat{\mathcal{H}}_{\text{int}}\right] = \left[\sum_k \omega_k \hat{a}_k^\dagger \hat{a}_k, \sum_{q\neq q'} V_{q,q'} \hat{a}_q^\dagger \hat{a}_{q'} + \sum_{p,p'} \left(W_{p,p'} \hat{a}_p^\dagger \hat{a}_{p'}^\dagger + W_{p,p'}^* \hat{a}_p \hat{a}_{p'}\right)\right] \tag{A.1}$$

$$\begin{aligned}
&= \sum_k \sum_{q,q'} \omega_k V_{q,q'} \left[\hat{a}_k^\dagger \hat{a}_k, \hat{a}_q^\dagger \hat{a}_{q'}\right] + \sum_k \sum_{p,p'} \left(\omega_k W_{p,p'} \left[\hat{a}_k^\dagger \hat{a}_k, \hat{a}_p^\dagger \hat{a}_{p'}^\dagger\right] + \omega_k W_{p,p'}^* \left[\hat{a}_k^\dagger \hat{a}_k, \hat{a}_p \hat{a}_{p'}\right]\right) \\
&= \sum_k \sum_{q,q'} \omega_k V_{q,q'} \left(\hat{a}_k^\dagger \hat{a}_{q'} \delta_{k,q} - \hat{a}_q^\dagger \hat{a}_k \delta_{k,q'}\right) \\
&+ \sum_k \sum_{p,p'} \left(\omega_k W_{p,p'} \left(\hat{a}_k^\dagger \hat{a}_p^\dagger \delta_{k,p'} + \hat{a}_k^\dagger \hat{a}_{p'}^\dagger \delta_{k,p}\right) - \omega_k W_{p,p'}^* \left(\hat{a}_p \hat{a}_k \delta_{k,p'} + \hat{a}_{p'} + \hat{a}_{p'} \hat{a}_k \delta_{k,p}\right)\right) \\
&= \sum_{q\neq q'} V_{q,q'} (\omega_q - \omega_{q'}) \hat{a}_q^\dagger \hat{a}_{q'} + \sum_{p,p'} \left(W_{p,p'} (\omega_p + \omega_{p'}) \hat{a}_p^\dagger \hat{a}_{p'}^\dagger - W_{p,p'}^* (\omega_p + \omega_{p'}) \hat{a}_p \hat{a}_{p'}\right)
\end{aligned} \tag{A.2}$$

Since $\hat{\eta}$ has the same form as $\hat{\mathcal{H}}_{\text{int}}$, $\left[\hat{\eta}, \hat{\mathcal{H}}_0\right]$ follows by inspection of A.2:

$$\begin{aligned}
\left[\hat{\eta}, \hat{\mathcal{H}}_0\right] &= -\sum_{q\neq q'} V_{q,q'} (\omega_q - \omega_{q'})^2 \hat{a}_q^\dagger \hat{a}_{q'} \\
&\quad - \sum_{p,p'} \left(W_{p,p'} (\omega_p + \omega_{p'})^2 \hat{a}_p^\dagger \hat{a}_{p'}^\dagger + W_{p,p'}^* (\omega_p + \omega_{p'})^2 \hat{a}_p \hat{a}_{p'}\right)
\end{aligned} \tag{A.3}$$

The commutator of the generator and $\hat{\mathcal{H}}_{\text{int}}$ needs more work:

$$\begin{aligned}
\left[\hat{\eta}, \hat{\mathcal{H}}_{\text{int}}\right] = \Bigg[&\sum_{q\neq q'} V_{q,q'}(\omega_q - \omega_{q'})\hat{a}_q^\dagger \hat{a}_{q'} + \sum_{p,p'} \Big(W_{p,p'}(\omega_p + \omega_{p'})\hat{a}_p^\dagger \hat{a}_{p'}^\dagger \qquad \text{(A.4)}\\
&- W^*_{p,p'}(\omega_p + \omega_{p'})\hat{a}_p \hat{a}_{p'}\Big), \sum_{\tilde{q}\neq\tilde{q}'} V_{\tilde{q},\tilde{q}'}\hat{a}_{\tilde{q}}^\dagger \hat{a}_{\tilde{q}'} + \sum_{\tilde{p},\tilde{p}'} \Big(W_{\tilde{p},\tilde{p}'}\hat{a}_{\tilde{p}}^\dagger \hat{a}_{\tilde{p}'}^\dagger + W^*_{\tilde{p},\tilde{p}'}\hat{a}_{\tilde{p}} \hat{a}_{\tilde{p}'}\Big) \Bigg]\\
= &\left[\sum_{q\neq q'} V_{q,q'}(\omega_q - \omega_{q'})\hat{a}_q^\dagger \hat{a}_{q'}, \sum_{\tilde{q}\neq\tilde{q}'} V_{\tilde{q},\tilde{q}'}\hat{a}_{\tilde{q}}^\dagger \hat{a}_{\tilde{q}'} \right] \qquad \text{(A.5)}\\
+ &\left[\sum_{q\neq q'} V_{q,q'}(\omega_q - \omega_{q'})\hat{a}_q^\dagger \hat{a}_{q'}, \sum_{\tilde{p},\tilde{p}'} \Big(W_{\tilde{p},\tilde{p}'}\hat{a}_{\tilde{p}}^\dagger \hat{a}_{\tilde{p}'}^\dagger + W^*_{\tilde{p},\tilde{p}'}\hat{a}_{\tilde{p}} \hat{a}_{\tilde{p}'}\Big) \right] \qquad \text{(A.6)}\\
+ &\left[\sum_{p,p'} \Big(W_{p,p'}(\omega_p + \omega_{p'})\hat{a}_p^\dagger \hat{a}_{p'}^\dagger - W^*_{p,p'}(\omega_p + \omega_{p'})\hat{a}_p \hat{a}_{p'}\Big), \sum_{\tilde{q}\neq\tilde{q}'} V_{\tilde{q},\tilde{q}'}\hat{a}_{\tilde{q}}^\dagger \hat{a}_{\tilde{q}'} \right] \qquad \text{(A.7)}\\
+ &\left[\sum_{p,p'} \Big(W_{p,p'}(\omega_p + \omega_{p'})\hat{a}_p^\dagger \hat{a}_{p'}^\dagger - W^*_{p,p'}(\omega_p + \omega_{p'})\hat{a}_p \hat{a}_{p'}\Big), \sum_{\tilde{p},\tilde{p}'} \Big(W_{\tilde{p},\tilde{p}'}\hat{a}_{\tilde{p}}^\dagger \hat{a}_{\tilde{p}'}^\dagger + W^*_{\tilde{p},\tilde{p}'}\hat{a}_{\tilde{p}} \hat{a}_{\tilde{p}'}\Big) \right] \\
&\qquad \text{(A.8)}
\end{aligned}$$

In the following, A.5-A.8 will be evaluated separately:

A.5:

$$\begin{aligned}
&\left[\sum_{q\neq q'} V_{q,q'}(\omega_q - \omega_{q'})\hat{a}_q^\dagger \hat{a}_{q'}, \sum_{\tilde{q}\neq\tilde{q}'} V_{\tilde{q},\tilde{q}'}\hat{a}_{\tilde{q}}^\dagger \hat{a}_{\tilde{q}'} \right]\\
&= \sum_{q\neq q'} \sum_{\tilde{q}\neq\tilde{q}'} V_{\tilde{q},\tilde{q}'} V_{q,q'}(\omega_q - \omega_{q'}) \left[\hat{a}_q^\dagger \hat{a}_{q'}, \hat{a}_{\tilde{q}}^\dagger \hat{a}_{\tilde{q}'}\right]\\
&= \sum_{q\neq q'} \sum_{\tilde{q}\neq\tilde{q}'} V_{\tilde{q},\tilde{q}'} V_{q,q'}(\omega_q - \omega_{q'}) \left(\hat{a}_q^\dagger \hat{a}_{\tilde{q}'}\delta_{q',\tilde{q}} - \hat{a}_{\tilde{q}}^\dagger \hat{a}_{q'}\delta_{q,\tilde{q}'}\right)\\
&= \sum_{q\neq q'} \sum_{\tilde{q}'} V_{q',\tilde{q}'} V_{q,q'}(\omega_q - \omega_{q'})\hat{a}_q^\dagger \hat{a}_{\tilde{q}'} - \sum_{q\neq q'} \sum_{\tilde{q}} V_{\tilde{q},q} V_{q,q'}(\omega_q - \omega_{q'})\hat{a}_{\tilde{q}}^\dagger \hat{a}_{q'}\\
&= \sum_{q,q'} \sum_{\tilde{q}'} V_{q',\tilde{q}'} V_{q,q'}(\omega_q - \omega_{q'})\hat{a}_q^\dagger \hat{a}_{\tilde{q}'} - \sum_{q,q'} \sum_{\tilde{q}} V_{\tilde{q},q} V_{q,q'}(\omega_q - \omega_{q'})\hat{a}_{\tilde{q}}^\dagger \hat{a}_{q'}\\
&= \sum_{q,q'} \sum_{\tilde{q}} V_{\tilde{q},q'} V_{q,\tilde{q}}(\omega_q - \omega_{\tilde{q}})\hat{a}_q^\dagger \hat{a}_{q'} - \sum_{q,q'} \sum_{\tilde{q}} V_{q,\tilde{q}} V_{\tilde{q},q'}(\omega_{\tilde{q}} - \omega_{q'})\hat{a}_q^\dagger \hat{a}_{q'}\\
&= \sum_{q\neq q'} \sum_{\tilde{q}} V_{\tilde{q},q'} V_{q,\tilde{q}}(\omega_q - \omega_{\tilde{q}})\hat{a}_q^\dagger \hat{a}_{q'} - \sum_{q\neq q'} \sum_{\tilde{q}} V_{q,\tilde{q}} V_{\tilde{q},q'}(\omega_{\tilde{q}} - \omega_{q'})\hat{a}_q^\dagger \hat{a}_{q'}\\
&+ \sum_{k} \sum_{\tilde{q}} V_{\tilde{q},k} V_{k,\tilde{q}}(\omega_k - \omega_{\tilde{q}})\hat{a}_k^\dagger \hat{a}_k - \sum_{k} \sum_{\tilde{q}} V_{k,\tilde{q}} V_{\tilde{q},k}(\omega_{\tilde{q}} - \omega_k)\hat{a}_k^\dagger \hat{a}_k\\
&= \sum_{q\neq q'} \sum_{\tilde{q}} V_{\tilde{q},q'} V_{q,\tilde{q}}(\omega_q - \omega_{\tilde{q}})\hat{a}_q^\dagger \hat{a}_{q'} - \sum_{q\neq q'} \sum_{\tilde{q}} V_{q,\tilde{q}} V_{\tilde{q},q'}(\omega_{\tilde{q}} - \omega_{q'})\hat{a}_q^\dagger \hat{a}_{q'}\\
&+ \sum_{k} \sum_{\tilde{q}} 2V_{\tilde{q},k} V_{k,\tilde{q}}(\omega_k - \omega_{\tilde{q}})\hat{a}_k^\dagger \hat{a}_k \qquad \text{(A.9)}
\end{aligned}$$

A.6:

$$\left[\sum_{q\neq q'} V_{q,q'}(\omega_q-\omega_{q'})\hat{a}_q^\dagger\hat{a}_{q'}, \sum_{\tilde{p},\tilde{p}'}\left(W_{\tilde{p},\tilde{p}'}\hat{a}_{\tilde{p}}^\dagger\hat{a}_{\tilde{p}'}^\dagger + W_{\tilde{p},\tilde{p}'}^*\hat{a}_{\tilde{p}}\hat{a}_{\tilde{p}'}\right)\right] \tag{A.10}$$
$$= \sum_{q\neq q'}\sum_{\tilde{p},\tilde{p}'} V_{q,q'}(\omega_q-\omega_{q'})\left(W_{\tilde{p},\tilde{p}'}\left[\hat{a}_q^\dagger\hat{a}_{q'}, \hat{a}_{\tilde{p}}^\dagger\hat{a}_{\tilde{p}'}^\dagger\right] + W_{\tilde{p},\tilde{p}'}^*\hat{a}_{\tilde{p}}\left[\hat{a}_q^\dagger\hat{a}_{q'}, \hat{a}_{\tilde{p}'}\right]\right)$$

Appendix B

The Second Appendix

Here comes the second appendix.

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DECLARATION OF AUTHORSHIP

I hereby declare that I have written this thesis independently and by myself and that I have not used any sources or auxiliary materials other than those indicated in the thesis.

Munich, 22.06.2023 . . . .