Title of My Thesis

SUBMITTED BY

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NOTATION AND SYMBOLS

λ	flow parameter; in the literature sometimes also denoted by B
$\hat{\cdot}$	denotes that $\cdot$ is an operator which does not commute with every other operator
1	indicates $1 \in \mathbb{N}$ or the identity operator $\hat{\mathbb{1}} =: \mathbb{1}$
$:\hat{A}:$	normal ordering of operator $\hat{A}$
$\hat{a}_k^\dagger$	k^{th} bosonic creation operator
$\hat{a}_k$	k^{th} bosonic annihilation operator
$[\hat{A}, \hat{B}]$	commutator of operators $\hat{A}, \hat{B}$
$\hat{A}^\dagger$	adjoint of $\hat{A}$
z^*	complex conjugate of $z \in \mathbb{C}$
$\delta_{\alpha,\beta}$	Kronecker-Delta of α, β
∂_x	partial derivative $\frac{\partial}{\partial x}$ w.r.t. x

ABSTRACT

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DETAILED CALCULATIONS

A.1 Deriving the flow equations in the case of no n-dependence

First the canonical generator $\hat{\eta}$ has to be evaluated:

$$\hat{\eta} := \hat{\eta}(\lambda) := \left[\hat{\mathcal{H}}_0, \hat{\mathcal{H}}_{\text{int}}\right] = \left[\sum_k \omega_k \hat{a}_k^\dagger \hat{a}_k, \sum_{q\neq q'} V_{q,q'} \hat{a}_q^\dagger \hat{a}_{q'} + \sum_{p,p'} \left(W_{p,p'} \hat{a}_p^\dagger \hat{a}_{p'}^\dagger + W_{p,p'}^* \hat{a}_p \hat{a}_{p'}\right)\right] \tag{A.1}$$

$$\begin{aligned}
&= \sum_k \sum_{q,q'} \omega_k V_{q,q'} \left[\hat{a}_k^\dagger \hat{a}_k, \hat{a}_q^\dagger \hat{a}_{q'}\right] + \sum_k \sum_{p,p'} \left(\omega_k W_{p,p'} \left[\hat{a}_k^\dagger \hat{a}_k, \hat{a}_p^\dagger \hat{a}_{p'}^\dagger\right] + \omega_k W_{p,p'}^* \left[\hat{a}_k^\dagger \hat{a}_k, \hat{a}_p \hat{a}_{p'}\right]\right) \\
&= \sum_k \sum_{q,q'} \omega_k V_{q,q'} \left(\hat{a}_k^\dagger \hat{a}_{q'} \delta_{k,q} - \hat{a}_q^\dagger \hat{a}_k \delta_{k,q'}\right) \\
&+ \sum_k \sum_{p,p'} \left(\omega_k W_{p,p'} \left(\hat{a}_k^\dagger \hat{a}_p^\dagger \delta_{k,p'} + \hat{a}_k^\dagger \hat{a}_{p'}^\dagger \delta_{k,p}\right) - \omega_k W_{p,p'}^* \left(\hat{a}_p \hat{a}_k \delta_{k,p'} + \hat{a}_{p'} \hat{a}_k \delta_{k,p}\right)\right) \\
&= \sum_{q\neq q'} V_{q,q'} (\omega_q - \omega_{q'}) \hat{a}_q^\dagger \hat{a}_{q'} + \sum_{p,p'} \left(W_{p,p'} (\omega_p + \omega_{p'}) \hat{a}_p^\dagger \hat{a}_{p'}^\dagger - W_{p,p'}^* (\omega_p + \omega_{p'}) \hat{a}_p \hat{a}_{p'}\right)
\end{aligned} \tag{A.2}$$

Since $\hat{\eta}$ has the same form as $\hat{\mathcal{H}}_{\text{int}}$, $\left[\hat{\eta}, \hat{\mathcal{H}}_0\right]$ follows by inspection of A.2:

$$\begin{aligned}
\left[\hat{\eta}, \hat{\mathcal{H}}_0\right] &= -\sum_{q\neq q'} V_{q,q'} (\omega_q - \omega_{q'})^2 \hat{a}_q^\dagger \hat{a}_{q'} \\
&- \sum_{p,p'} \left(W_{p,p'} (\omega_p + \omega_{p'})^2 \hat{a}_p^\dagger \hat{a}_{p'}^\dagger + W_{p,p'}^* (\omega_p + \omega_{p'})^2 \hat{a}_p \hat{a}_{p'}\right)
\end{aligned} \tag{A.3}$$

The commutator of the generator and $\hat{\mathcal{H}}_{\text{int}}$ needs more work:

$$\left[\hat{\eta}, \hat{\mathcal{H}}_{\text{int}}\right] = \left[\sum_{q\neq q'} V_{q,q'} (\omega_q - \omega_{q'}) \hat{a}_q^\dagger \hat{a}_{q'} + \sum_{p,p'} \left(W_{p,p'} (\omega_p + \omega_{p'}) \hat{a}_p^\dagger \hat{a}_{p'}^\dagger\right.\right. \tag{A.4}$$

$$\left.\left. - W_{p,p'}^* (\omega_p + \omega_{p'}) \hat{a}_p \hat{a}_{p'}\right), \sum_{\tilde{q}\neq \tilde{q}'} V_{\tilde{q},\tilde{q}'} \hat{a}_{\tilde{q}}^\dagger \hat{a}_{\tilde{q}'} + \sum_{\tilde{p},\tilde{p}'} \left(W_{\tilde{p},\tilde{p}'} \hat{a}_{\tilde{p}}^\dagger \hat{a}_{\tilde{p}'}^\dagger + W_{\tilde{p},\tilde{p}'}^* \hat{a}_{\tilde{p}} \hat{a}_{\tilde{p}'}\right)\right]$$

$$= \left[\sum_{q\neq q'} V_{q,q'} (\omega_q - \omega_{q'}) \hat{a}_q^\dagger \hat{a}_{q'}, \sum_{\tilde{q}\neq \tilde{q}'} V_{\tilde{q},\tilde{q}'} \hat{a}_{\tilde{q}}^\dagger \hat{a}_{\tilde{q}'}\right] \tag{A.5}$$

$$+ \left[\sum_{q\neq q'} V_{q,q'} (\omega_q - \omega_{q'}) \hat{a}_q^\dagger \hat{a}_{q'}, \sum_{\tilde{p},\tilde{p}'} \left(W_{\tilde{p},\tilde{p}'} \hat{a}_{\tilde{p}}^\dagger \hat{a}_{\tilde{p}'}^\dagger + W_{\tilde{p},\tilde{p}'}^* \hat{a}_{\tilde{p}} \hat{a}_{\tilde{p}'}\right)\right] \tag{A.6}$$

$$+ \left[\sum_{p,p'} \left(W_{p,p'} (\omega_p + \omega_{p'}) \hat{a}_p^\dagger \hat{a}_{p'}^\dagger - W_{p,p'}^* (\omega_p + \omega_{p'}) \hat{a}_p \hat{a}_{p'}\right), \sum_{\tilde{q}\neq \tilde{q}'} V_{\tilde{q},\tilde{q}'} \hat{a}_{\tilde{q}}^\dagger \hat{a}_{\tilde{q}'}\right] \tag{A.7}$$

$$+ \left[\sum_{p,p'} \left(W_{p,p'} (\omega_p + \omega_{p'}) \hat{a}_p^\dagger \hat{a}_{p'}^\dagger - W_{p,p'}^* (\omega_p + \omega_{p'}) \hat{a}_p \hat{a}_{p'}\right), \sum_{\tilde{p},\tilde{p}'} \left(W_{\tilde{p},\tilde{p}'} \hat{a}_{\tilde{p}}^\dagger \hat{a}_{\tilde{p}'}^\dagger + W_{\tilde{p},\tilde{p}'}^* \hat{a}_{\tilde{p}} \hat{a}_{\tilde{p}'}\right)\right] \tag{A.8}$$

In the following, A.5-A.8 will be evaluated separately:

A.5:

$$\begin{aligned}
&\left[\sum_{q\neq q'} V_{q,q'}(\omega_q-\omega_{q'})\hat{a}_q^\dagger\hat{a}_{q'}, \sum_{\tilde{q}\neq\tilde{q}'} V_{\tilde{q},\tilde{q}'}\hat{a}_{\tilde{q}}^\dagger\hat{a}_{\tilde{q}'}\right]\\
&=\sum_{q\neq q'}\sum_{\tilde{q}\neq\tilde{q}'} V_{\tilde{q},\tilde{q}'}V_{q,q'}(\omega_q-\omega_{q'})\left[\hat{a}_q^\dagger\hat{a}_{q'},\hat{a}_{\tilde{q}}^\dagger\hat{a}_{\tilde{q}'}\right]\\
&=\sum_{q\neq q'}\sum_{\tilde{q}\neq\tilde{q}'} V_{\tilde{q},\tilde{q}'}V_{q,q'}(\omega_q-\omega_{q'})\left(\hat{a}_q^\dagger\hat{a}_{\tilde{q}'}\delta_{q',\tilde{q}}-\hat{a}_{\tilde{q}}^\dagger\hat{a}_{q'}\delta_{q,\tilde{q}'}\right)\\
&=\sum_{q\neq q'}\sum_{\tilde{q}'} V_{q',\tilde{q}'}V_{q,q'}(\omega_q-\omega_{q'})\hat{a}_q^\dagger\hat{a}_{\tilde{q}'}-\sum_{q\neq q'}\sum_{\tilde{q}} V_{\tilde{q},q}V_{q,q'}(\omega_q-\omega_{q'})\hat{a}_{\tilde{q}}^\dagger\hat{a}_{q'}\\
&=\sum_{q,q'}\sum_{\tilde{q}'} V_{q',\tilde{q}'}V_{q,q'}(\omega_q-\omega_{q'})\hat{a}_q^\dagger\hat{a}_{\tilde{q}'}-\sum_{q,q'}\sum_{\tilde{q}} V_{\tilde{q},q}V_{q,q'}(\omega_q-\omega_{q'})\hat{a}_{\tilde{q}}^\dagger\hat{a}_{q'}\\
&=\sum_{q,q'}\sum_{\tilde{q}} V_{\tilde{q},q'}V_{q,\tilde{q}}(\omega_q-\omega_{\tilde{q}})\hat{a}_q^\dagger\hat{a}_{q'}-\sum_{q,q'}\sum_{\tilde{q}} V_{q,\tilde{q}}V_{\tilde{q},q'}(\omega_{\tilde{q}}-\omega_{q'})\hat{a}_q^\dagger\hat{a}_{q'}\\
&=\sum_{q\neq q'}\sum_{\tilde{q}} V_{\tilde{q},q'}V_{q,\tilde{q}}(\omega_q-\omega_{\tilde{q}})\hat{a}_q^\dagger\hat{a}_{q'}-\sum_{q\neq q'}\sum_{\tilde{q}} V_{q,\tilde{q}}V_{\tilde{q},q'}(\omega_{\tilde{q}}-\omega_{q'})\hat{a}_q^\dagger\hat{a}_{q'}\\
&+\sum_{k}\sum_{\tilde{q}} V_{\tilde{q},k}V_{k,\tilde{q}}(\omega_k-\omega_{\tilde{q}})\hat{a}_k^\dagger\hat{a}_k-\sum_{k}\sum_{\tilde{q}} V_{k,\tilde{q}}V_{\tilde{q},k}(\omega_{\tilde{q}}-\omega_k)\hat{a}_k^\dagger\hat{a}_k\\
&=\sum_{q\neq q'}\sum_{\tilde{q}} V_{\tilde{q},q'}V_{q,\tilde{q}}(\omega_q-\omega_{\tilde{q}})\hat{a}_q^\dagger\hat{a}_{q'}-\sum_{q\neq q'}\sum_{\tilde{q}} V_{q,\tilde{q}}V_{\tilde{q},q'}(\omega_{\tilde{q}}-\omega_{q'})\hat{a}_q^\dagger\hat{a}_{q'}\\
&+\sum_{k}\sum_{\tilde{q}} 2V_{\tilde{q},k}V_{k,\tilde{q}}(\omega_k-\omega_{\tilde{q}})\hat{a}_k^\dagger\hat{a}_k
\end{aligned}\tag{A.9}$$

A.6:

$$\begin{aligned}
&\left[\sum_{q\neq q'} V_{q,q'}(\omega_q-\omega_{q'})\hat{a}_q^\dagger\hat{a}_{q'}, \sum_{\tilde{p},\tilde{p}'}\left(W_{\tilde{p},\tilde{p}'}\hat{a}_{\tilde{p}}^\dagger\hat{a}_{\tilde{p}'}^\dagger+W_{\tilde{p},\tilde{p}'}^*\hat{a}_{\tilde{p}}\hat{a}_{\tilde{p}'}\right)\right]\\
&=\sum_{q\neq q'}\sum_{\tilde{p},\tilde{p}'} V_{q,q'}(\omega_q-\omega_{q'})\left(W_{\tilde{p},\tilde{p}'}\left[\hat{a}_q^\dagger\hat{a}_{q'},\hat{a}_{\tilde{p}}^\dagger\hat{a}_{\tilde{p}'}^\dagger\right]+W_{\tilde{p},\tilde{p}'}^*\left[\hat{a}_q^\dagger\hat{a}_{q'},\hat{a}_{\tilde{p}}\hat{a}_{\tilde{p}'}\right]\right)\\
&=\sum_{q,q'}\sum_{\tilde{p},\tilde{p}'} V_{q,q'}(\omega_q-\omega_{q'})\left(W_{\tilde{p},\tilde{p}'}\left(\hat{a}_q^\dagger\hat{a}_{\tilde{p}}^\dagger\delta_{q',\tilde{p}'}+\hat{a}_q^\dagger\hat{a}_{\tilde{p}'}^\dagger\delta_{q',\tilde{p}}\right)-W_{\tilde{p},\tilde{p}'}^*\hat{a}_{\tilde{p}}\left(\hat{a}_{\tilde{p}'}\hat{a}_{q'}\delta_{q,\tilde{p}}+\hat{a}_{\tilde{p}}\hat{a}_{q'}\delta_{\tilde{p}',q}\right)\right)\\
&=\sum_{p,p'}\sum_{q} V_{q,p'}(\omega_q-\omega_{p'})W_{p,p'}\hat{a}_q^\dagger\hat{a}_p^\dagger+\sum_{p,p'}\sum_{q} V_{q,p}(\omega_q-\omega_p)W_{p,p'}\hat{a}_q^\dagger\hat{a}_{p'}^\dagger\\
&-\sum_{p,p'}\sum_{q'} V_{p,q'}(\omega_p-\omega_{q'})W_{p,p'}^*\hat{a}_{p'}\hat{a}_{q'}-\sum_{p,p'}\sum_{q'} V_{p',q'}(\omega_{p'}-\omega_{q'})W_{p,p'}^*\hat{a}_p\hat{a}_{q'}\\
&=\sum_{p,p'}\sum_{q} V_{p',q}(\omega_{p'}-\omega_q)W_{p,q}\hat{a}_p^\dagger\hat{a}_{p'}^\dagger+\sum_{p,p'}\sum_{q} V_{p,q}(\omega_p-\omega_q)W_{q,p'}\hat{a}_p^\dagger\hat{a}_{p'}^\dagger\\
&-\sum_{p,p'}\sum_{q} V_{q,p}(\omega_q-\omega_p)W_{q,p'}^*\hat{a}_p\hat{a}_{p'}-\sum_{p,p'}\sum_{q} V_{q,p'}(\omega_q-\omega_{p'})W_{p,q}^*\hat{a}_p\hat{a}_{p'}\\
&=\sum_{p,p'}\sum_{q} V_{p',q}(\omega_{p'}-\omega_q)W_{p,q}\hat{a}_p^\dagger\hat{a}_{p'}^\dagger+\sum_{p,p'}\sum_{q} V_{p,q}(\omega_p-\omega_q)W_{q,p'}\hat{a}_p^\dagger\hat{a}_{p'}^\dagger\\
&-\sum_{p,p'}\sum_{q} V_{q,p}(\omega_q-\omega_p)W_{q,p'}^*\hat{a}_p\hat{a}_{p'}-\sum_{p,p'}\sum_{q} V_{q,p'}(\omega_q-\omega_{p'})W_{p,q}^*\hat{a}_p\hat{a}_{p'}\\
&=\sum_{p,p'}\sum_{q} V_{p,q}(\omega_p-\omega_q)(W_{q,p'}+W_{p',q})\hat{a}_p^\dagger\hat{a}_{p'}^\dagger\\
&+\sum_{p,p'}\sum_{q} V_{q,p}(\omega_p-\omega_q)(W_{q,p'}^*+W_{p',q}^*)\hat{a}_p\hat{a}_{p'}
\end{aligned}\tag{A.10}$$

A.7:

$$
\begin{aligned}
&\left[\sum_{p,p'}\Big(W_{p,p'}(\omega_p+\omega_{p'})\hat a_p^\dagger\hat a_{p'}^\dagger - W_{p,p'}^*(\omega_p+\omega_{p'})\hat a_p\hat a_{p'}\Big), \sum_{\tilde q\neq\tilde q'} V_{\tilde q,\tilde q'}\hat a_{\tilde q}^\dagger\hat a_{\tilde q'}\right]\\
&=\sum_{p,p'}\sum_{\tilde q\neq\tilde q'} V_{\tilde q,\tilde q'}(\omega_p+\omega_{p'})\Big(W_{p,p'}\left[\hat a_p^\dagger\hat a_{p'}^\dagger,\hat a_{\tilde q}^\dagger\hat a_{\tilde q'}\right] - W_{p,p'}^*\left[\hat a_p\hat a_{p'},\hat a_{\tilde q}^\dagger\hat a_{\tilde q'}\right]\Big)\\
&=-\sum_{p,p'}\sum_{q\neq q'} V_{q,q'}(\omega_p+\omega_{p'})W_{p,p'}\left(\hat a_q^\dagger\hat a_p^\dagger\delta_{q',p'}+\hat a_q^\dagger\hat a_{p'}^\dagger\delta_{q',p}\right)\\
&-\sum_{p,p'}\sum_{q\neq q'} V_{q,q'}(\omega_p+\omega_{p'})W_{p,p'}^*\left(\hat a_p\hat a_{q'}\delta_{q,p'}+\hat a_{p'}\hat a_{q'}\delta_{q,p}\right)\\
&=-\sum_{p,p'}\sum_{q} V_{q,p'}(\omega_p+\omega_{p'})W_{p,p'}\hat a_q^\dagger\hat a_p^\dagger-\sum_{p,p'}\sum_{q} V_{q,p}(\omega_p+\omega_{p'})W_{p,p'}\hat a_q^\dagger\hat a_{p'}^\dagger\\
&-\sum_{p,p'}\sum_{q'} V_{p',q'}(\omega_p+\omega_{p'})W_{p,p'}^*\hat a_p\hat a_{q'}-\sum_{p,p'}\sum_{q'} V_{p,q'}(\omega_p+\omega_{p'})W_{p,p'}^*\hat a_{p'}\hat a_{q'}\\
&=-\sum_{p,p'}\sum_{q} V_{p',q}(\omega_p+\omega_{q})W_{p,q}\hat a_p^\dagger\hat a_{p'}^\dagger-\sum_{p,p'}\sum_{q} V_{p,q}(\omega_q+\omega_{p'})W_{q,p'}\hat a_p^\dagger\hat a_{p'}^\dagger\\
&-\sum_{p,p'}\sum_{q'} V_{q',p'}(\omega_p+\omega_{q'})W_{p,q'}^*\hat a_p\hat a_{p'}-\sum_{p,p'}\sum_{q'} V_{q',p}(\omega_{q'}+\omega_{p'})W_{q',p'}^*\hat a_p\hat a_{p'}\\
&=-\sum_{p,p'}\sum_{q} V_{p,q}(\omega_q+\omega_{p'})(W_{p',q}+W_{q,p'})\hat a_p^\dagger\hat a_{p'}^\dagger\\
&-\sum_{p,p'}\sum_{q} V_{q,p}(\omega_q+\omega_{p'})(W_{p',q}^*+W_{q,p'}^*)\hat a_p\hat a_{p'}
\end{aligned}
\tag{A.11}
$$

A.8:

$$
\begin{aligned}
&\left[\sum_{p,p'}\Big(W_{p,p'}(\omega_p+\omega_{p'})\hat a_p^\dagger\hat a_{p'}^\dagger - W_{p,p'}^*(\omega_p+\omega_{p'})\hat a_p\hat a_{p'}\Big), \sum_{\tilde p,\tilde p'}\Big(W_{\tilde p,\tilde p'}\hat a_{\tilde p}^\dagger\hat a_{\tilde p'}^\dagger+W_{\tilde p,\tilde p'}^*\hat a_{\tilde p}\hat a_{\tilde p'}\Big)\right]\\
&=\sum_{p,p'}\sum_{\tilde p,\tilde p'} W_{p,p'}(\omega_p+\omega_{p'})W_{\tilde p,\tilde p'}^*\left[\hat a_p^\dagger\hat a_{p'}^\dagger,\hat a_{\tilde p}\hat a_{\tilde p'}\right]-\sum_{p,p'}\sum_{\tilde p,\tilde p'} W_{p,p'}^*W_{\tilde p,\tilde p'}(\omega_p+\omega_{p'})\left[\hat a_p\hat a_{p'},\hat a_{\tilde p}^\dagger\hat a_{\tilde p'}^\dagger\right]\\
&=-\sum_{p,p'}\sum_{\tilde p,\tilde p'} W_{p,p'}(\omega_p+\omega_{p'}+\omega_{\tilde p}+\omega_{\tilde p'})W_{\tilde p,\tilde p'}^*\left[\hat a_{\tilde p}\hat a_{\tilde p'},\hat a_p^\dagger\hat a_{p'}^\dagger\right]\\
&=-\sum_{p,p'}\sum_{\tilde p,\tilde p'} W_{p,p'}(\omega_p+\omega_{p'}+\omega_{\tilde p}+\omega_{\tilde p'})W_{\tilde p,\tilde p'}^*\hat a_{\tilde p}\hat a_p^\dagger\delta_{\tilde p',p'}\\
&-\sum_{p,p'}\sum_{\tilde p,\tilde p'} W_{p,p'}(\omega_p+\omega_{p'}+\omega_{\tilde p}+\omega_{\tilde p'})W_{\tilde p,\tilde p'}^*\hat a_{\tilde p}\hat a_{p'}^\dagger\delta_{\tilde p',p}\\
&-\sum_{p,p'}\sum_{\tilde p,\tilde p'} W_{p,p'}(\omega_p+\omega_{p'}+\omega_{\tilde p}+\omega_{\tilde p'})W_{\tilde p,\tilde p'}^*\hat a_p^\dagger\hat a_{\tilde p'}\delta_{\tilde p,p'}\\
&-\sum_{p,p'}\sum_{\tilde p,\tilde p'} W_{p,p'}(\omega_p+\omega_{p'}+\omega_{\tilde p}+\omega_{\tilde p'})W_{\tilde p,\tilde p'}^*\hat a_{p'}^\dagger\hat a_{\tilde p'}\delta_{\tilde p,p}\\
&=-\sum_{p,p'}\sum_{\tilde p} W_{p,p'}(\omega_p+2\omega_{p'}+\omega_{\tilde p})W_{\tilde p,p'}^*\hat a_{\tilde p}\hat a_p^\dagger-\sum_{p,p'}\sum_{\tilde p} W_{p,p'}(2\omega_p+\omega_{p'}+\omega_{\tilde p})W_{\tilde p,p}^*\hat a_{\tilde p}\hat a_{p'}^\dagger\\
&-\sum_{p,p'}\sum_{\tilde p'} W_{p,p'}(\omega_p+2\omega_{p'}+\omega_{\tilde p'})W_{p',\tilde p'}^*\hat a_p^\dagger\hat a_{\tilde p'}-\sum_{p,p'}\sum_{\tilde p'} W_{p,p'}(2\omega_p+\omega_{p'}+\omega_{\tilde p'})W_{p,\tilde p'}^*\hat a_{p'}^\dagger\hat a_{\tilde p'}\\
&=-\sum_{p,p'}\sum_{\tilde p} W_{p,\tilde p}(\omega_p+2\omega_{\tilde p}+\omega_{p'})W_{p',\tilde p}^*\hat a_{p'}\hat a_p^\dagger-\sum_{p,p'}\sum_{\tilde p} W_{\tilde p,p}(2\omega_{\tilde p}+\omega_p+\omega_{p'})W_{p',\tilde p}^*\hat a_{p'}\hat a_p^\dagger\\
&-\sum_{p,p'}\sum_{\tilde p'} W_{p,\tilde p'}(\omega_p+2\omega_{\tilde p'}+\omega_{p'})W_{\tilde p',p'}^*\hat a_p^\dagger\hat a_{p'}-\sum_{p,p'}\sum_{\tilde p'} W_{\tilde p',p}(2\omega_{\tilde p'}+\omega_p+\omega_{p'})W_{\tilde p',p'}^*\hat a_p^\dagger\hat a_{p'}\\
&=-\sum_{p,p'}\sum_{\tilde p}(W_{p,\tilde p}+W_{\tilde p,p})(\omega_p+2\omega_{\tilde p}+\omega_{p'})W_{p',\tilde p}^*\hat a_{p'}\hat a_p^\dagger
\end{aligned}
$$

$$
\begin{aligned}
&- \sum_{p,p'} \sum_{\tilde{p}'} (W_{p,\tilde{p}'} + W_{\tilde{p}',p})(\omega_p + 2\omega_{\tilde{p}'} + \omega_{p'}) W^*_{\tilde{p}',p'} \hat{a}^\dagger_p \hat{a}_{p'} \\
&= - \sum_{p,p'} \sum_{\tilde{p}} (W_{p,\tilde{p}} + W_{\tilde{p},p})(\omega_p + 2\omega_{\tilde{p}} + \omega_{p'}) W^*_{p',\tilde{p}} (\delta_{p,p'} + \hat{a}^\dagger_p \hat{a}_{p'}) \\
&- \sum_{p,p'} \sum_{\tilde{p}} (W_{p,\tilde{p}} + W_{\tilde{p},p})(\omega_p + 2\omega_{\tilde{p}} + \omega_{p'}) W^*_{\tilde{p},p'} \hat{a}^\dagger_p \hat{a}_{p'} \\
&= - \sum_{p,p'} \sum_{\tilde{p}} (W_{p,\tilde{p}} + W_{\tilde{p},p})(\omega_p + 2\omega_{\tilde{p}} + \omega_{p'}) (W^*_{\tilde{p},p'} + W^*_{p',\tilde{p}}) \hat{a}^\dagger_p \hat{a}_{p'} \\
&- 2 \sum_{k} \sum_{\tilde{p}} (W_{k,\tilde{p}} + W_{\tilde{p},k})(\omega_k + \omega_{\tilde{p}}) W^*_{k,\tilde{p}} \\
&= - \sum_{q \neq q'} \sum_{\tilde{p}} (W_{q,\tilde{p}} + W_{\tilde{p},q})(\omega_q + 2\omega_{\tilde{p}} + \omega_{q'}) (W^*_{\tilde{p},q'} + W^*_{q',\tilde{p}}) \hat{a}^\dagger_q \hat{a}_{q'} \\
&- 2 \sum_{k} \sum_{\tilde{p}} (W_{k,\tilde{p}} + W_{\tilde{p},k})(\omega_k + \omega_{\tilde{p}}) (W^*_{\tilde{p},k} + W^*_{k,\tilde{p}}) \hat{a}^\dagger_k \hat{a}_k \\
&- 2 \sum_{k} \sum_{\tilde{p}} (W_{k,\tilde{p}} + W_{\tilde{p},k})(\omega_k + \omega_{\tilde{p}}) W^*_{k,\tilde{p}} \qquad \text{(A.12)}
\end{aligned}
$$

Using the expressions for the commutators of the generator and $\hat{\mathcal{H}}_0$ respectively $\hat{\mathcal{H}}_{\text{int}}$ derived above, the flow $\partial_\lambda \hat{\mathcal{H}}(\lambda) = [\hat{\eta}(\lambda), \hat{\mathcal{H}}(\lambda)]$ yields the following flow equations $\forall k, p, p', q, q'$ where $q \neq q'$:

$$
\begin{aligned}
\partial_\lambda \omega_k &= \sum_{\tilde{q}} 2 V_{\tilde{q},k} V_{k,\tilde{q}} (\omega_k - \omega_{\tilde{q}}) - 2 \sum_{\tilde{p}} (W_{k,\tilde{p}} + W_{\tilde{p},k})(\omega_k + \omega_{\tilde{p}})(W^*_{\tilde{p},k} + W^*_{k,\tilde{p}}) && \text{(A.13a)} \\
\partial_\lambda V_{q,q'} &= -V_{q,q'} (\omega_q - \omega_{q'})^2 - \sum_{\tilde{p}} (W_{q,\tilde{p}} + W_{\tilde{p},q})(\omega_q + \omega_{q'} + 2\omega_{\tilde{p}})(W^*_{\tilde{p},q'} + W^*_{q',\tilde{p}}) \\
&\quad + \sum_{\tilde{q}} V_{\tilde{q},q'} V_{q,\tilde{q}} (\omega_q + \omega_{q'} - 2\omega_{\tilde{q}}) && \text{(A.13b)} \\
\partial_\lambda W_{p,p'} &= -W_{p,p'} (\omega_p + \omega_{p'})^2 - \sum_{q} V_{p,q} (\omega_q + \omega_{p'})(W_{p',q} + W_{q,p'}) \\
&\quad + \sum_{q} V_{p,q} (\omega_p - \omega_q)(W_{q,p'} + W_{p',q}) && \text{(A.13c)} \\
\partial_\lambda W^*_{p,p'} &= -W^*_{p,p'} (\omega_p + \omega_{p'})^2 - \sum_{q} V_{q,p} (\omega_q + \omega_{p'})(W^*_{p',q} + W^*_{q,p'}) \\
&\quad + \sum_{q} V_{q,p} (\omega_p - \omega_q)(W^*_{q,p'} + W^*_{p',q}) && \text{(A.13d)} \\
\partial_\lambda \varepsilon &= -2 \sum_{p,p'} (W_{p,p'} + W_{p',p})(\omega_p + \omega_{p'}) W^*_{p,p'} && \text{(A.13e)}
\end{aligned}
$$

Obviously, equations A.13c and A.13d are not independent from each other, since they are related by complex conjugation. Seeing this is a good consistency check because complex conjugation was not explicitly used in the derivation of the these two equations.
Furthermore note that the flow equations A.13a-A.13e are exact in the sense that if the flow is completely traversed, the flow Hamiltonian will be exactly diagonal.

A.2 Deriving the flow equations with n-dependence

A.2.1 Useful preliminaries

Consider some operator $\hat{f}$ which depends on a number operator $\hat{n} = \hat{a}^\dagger\hat{a}$. The following relations will be used later:

$$\left[\hat{a}^\dagger, \hat{f}(\hat{n})\right] = \hat{a}^\dagger \left(\hat{f}(\hat{n}) - \hat{f}(\hat{n}+1)\right) \tag{A.14a}$$

$$\left[\hat{a}, \hat{f}(\hat{n})\right] = \hat{a} \left(\hat{f}(\hat{n}) - \hat{f}(\hat{n}-1)\right) \tag{A.14b}$$

$$\left[\hat{f}(\hat{n}), \hat{a}^\dagger\right] = \left(\hat{f}(\hat{n}) - \hat{f}(\hat{n}-1)\right)\hat{a}^\dagger \tag{A.14c}$$

$$\left[\hat{f}(\hat{n}), \hat{a}\right] = \left(\hat{f}(\hat{n}) - \hat{f}(\hat{n}+1)\right)\hat{a} \tag{A.14d}$$

These can be proved by induction for $\hat{f}(\hat{n}) = \hat{n}^k, k \in \mathbb{N}$ and from there simply extended to well-behaved $\hat{f}$ via power series. Equations A.14 are still valid for functions depending on $\{\hat{n}_k\}_k$, because all $\hat{n}_k$ pairwise commute.
We will write $\hat{f}\left(\hat{n}_1, \hat{n}_2, \ldots\right) =: \hat{f}$ and $\hat{f}\left(\hat{n}_1, \hat{n}_2, \ldots, \hat{n}_k \pm 1, \hat{n}_{k+1}, \ldots\right) =: \hat{f}(\hat{n}_k \pm 1)$ where it is understood that $\hat{f}(\hat{n}_k \pm 1, \hat{n}_k \pm 1) =: \hat{f}(\hat{n}_k \pm 2)$.
Using this notation, it is evident that a simple induction for $n_1, n_2 \in \mathbb{N}_0$ yields the following relation:

$$\begin{aligned}&\left[\hat{f}(\hat{n}), \hat{a}^\dagger_{k_1}\hat{a}^\dagger_{k_2}\cdots\hat{a}^\dagger_{k_{n_1}}\hat{a}_{k_1}\hat{a}_{k_2}\cdots\hat{a}_{k_{n_2}}\right]\\ &= \left(\hat{f} - \hat{f}\left(\hat{n}_{k_1}-1, \hat{n}_{k_2}-1, \ldots, \hat{n}_{k_{n_1}}, \hat{n}_{k_1}+1, \hat{n}_{k_2}+1 \ldots \hat{n}_{k_{n_2}}+1\right)\right)\hat{a}^\dagger_{k_1}\hat{a}^\dagger_{k_2}\cdots\hat{a}^\dagger_{k_{n_1}}\hat{a}_{k_1}\hat{a}_{k_2}\cdots\hat{a}_{k_{n_2}}\end{aligned} \tag{A.15}$$

Furthermore, applying the recurrence relation introduced to define the normal ordering procedure can be used to successively normal order operators. Let $\hat{O} := \hat{a}^\dagger_{k_1}\hat{a}^\dagger_{k_2}\cdots\hat{a}^\dagger_{k_{n_1}}\hat{a}_{k_1}\hat{a}_{k_2}\cdots\hat{a}_{k_{n_2}}$. Then:

$$\begin{aligned}\hat{a}_q : \hat{O} : &=: \hat{O}\hat{a}_q : + \sum_k : \frac{\partial \hat{O}}{\partial \hat{a}^\dagger_k} :\\ &=: \hat{O}\hat{a}_q : + \sum_{i=1}^{n_1} \delta_{k_i,q} : \hat{a}^\dagger_{k_1}\hat{a}^\dagger_{k_2}\cdots\hat{a}^\dagger_{k_{i-1}}\hat{a}^\dagger_{k_{i+1}}\cdots\hat{a}^\dagger_{k_{n_1}}\hat{a}_{k_1}\hat{a}_{k_2}\cdots\hat{a}_{k_{n_2}} :\end{aligned} \tag{A.16a}$$

$$\hat{a}^\dagger_q : \hat{O} : =: \hat{a}^\dagger_q\hat{O} : \tag{A.16b}$$

A.2.2 The canonical generator

The first step in the calculating the flow equations is again to calculate the canonical commutator $\hat{\eta} := \left[\hat{\mathcal{H}}_0, \hat{\mathcal{H}}_{\text{int}}\right]$:

$$\begin{aligned}\hat{\eta} &= \left[\sum_k \hat{\omega}_k : \hat{a}^\dagger_k\hat{a}_k :, \sum_{q\neq q'} \hat{V}_{q,q'} : \hat{a}^\dagger_q\hat{a}_{q'} : + \sum_{p,p'}\left(\hat{W}_{p,p'} : \hat{a}^\dagger_p\hat{a}^\dagger_{p'} : + \hat{W}^\dagger_{p,p'} : \hat{a}_p\hat{a}_{p'} :\right)\right]\\ &= \sum_k \sum_{q\neq q'} \left[\hat{\omega}_k : \hat{a}^\dagger_k\hat{a}_k :, \hat{V}_{q,q'} : \hat{a}^\dagger_q\hat{a}_{q'} :\right] && \text{(A.17)}\\ &+ \sum_k \sum_{p,p'} \left[\hat{\omega}_k : \hat{a}^\dagger_k\hat{a}_k :, \hat{W}_{p,p'} : \hat{a}^\dagger_p\hat{a}^\dagger_{p'} :\right] && \text{(A.18)}\end{aligned}$$

$$+\sum_k \sum_{p,p'} \left[\hat{\omega}_k : \hat{a}_k^\dagger \hat{a}_k :, \hat{W}_{p,p'}^\dagger : \hat{a}_p \hat{a}_{p'} :\right] \tag{A.19}$$

In the following, the terms A.17-A.19 will be evaluated separately:

A.17

$$\begin{aligned}
&\left[\hat{\omega}_k : \hat{a}_k^\dagger \hat{a}_k :, \hat{V}_{q,q'} : \hat{a}_q^\dagger \hat{a}_{q'} :\right] \\
&= \hat{\omega}_k \hat{V}_{q,q'} \left[: \hat{a}_k^\dagger \hat{a}_k :, : \hat{a}_q^\dagger \hat{a}_{q'} :\right] + \hat{\omega}_k \underbrace{\left[: \hat{a}_k^\dagger \hat{a}_k :, \hat{V}_{q,q'}\right]}_{=0} : \hat{a}_q^\dagger \hat{a}_{q'} : \\
&+ \hat{V}_{q,q'} \left[\hat{\omega}_k, : \hat{a}_q^\dagger \hat{a}_{q'} :\right] : \hat{a}_k^\dagger \hat{a}_k : + \underbrace{\left[\hat{\omega}_k, \hat{V}_{q,q'}\right]}_{=0} : \hat{a}_q^\dagger \hat{a}_{q'} :: \hat{a}_k^\dagger \hat{a}_k : \\
&= \hat{\omega}_k \hat{V}_{q,q'} \left(: \hat{a}_k^\dagger \hat{a}_{q'} : \delta_{k,q} - : \hat{a}_q^\dagger \hat{a}_k : \delta_{k,q'}\right) + \hat{V}_{q,q'} \left(\hat{\omega}_k - \hat{\omega}_k(\hat{n}_q - 1, \hat{n}_{q'} + 1)\right) : \hat{a}_q^\dagger \hat{a}_{q'} :: \hat{a}_k^\dagger \hat{a}_k : \\
&= \hat{\omega}_k \hat{V}_{q,q'} \left(: \hat{a}_k^\dagger \hat{a}_{q'} : \delta_{k,q} - : \hat{a}_q^\dagger \hat{a}_k : \delta_{k,q'}\right) \\
&+ \hat{V}_{q,q'} \left(\hat{\omega}_k - \hat{\omega}_k(\hat{n}_q - 1, \hat{n}_{q'} + 1)\right) \left(: \hat{a}_q^\dagger \hat{a}_k^\dagger \hat{a}_{q'} \hat{a}_k :\right) + \delta_{q',k} : \hat{a}_q^\dagger \hat{a}_k :
\end{aligned} \tag{A.20}$$

A.18

$$\begin{aligned}
&\left[\hat{\omega}_k : \hat{a}_k^\dagger \hat{a}_k :, \hat{W}_{p,p'} : \hat{a}_p^\dagger \hat{a}_{p'}^\dagger :\right] \\
&= \hat{W}_{p,p'} \left[\hat{\omega}_k, : \hat{a}_p^\dagger \hat{a}_{p'}^\dagger :\right] : \hat{a}_k^\dagger \hat{a}_k : + \hat{W}_{p,p'} \hat{\omega}_k \left[: \hat{a}_k^\dagger \hat{a}_k :, : \hat{a}_p^\dagger \hat{a}_{p'}^\dagger :\right] \\
&= \hat{W}_{p,p'} \left(\omega_k - \hat{\omega}(\hat{n}_{p'} - 1, \hat{n}_p - 1)\right) : \hat{a}_p^\dagger \hat{a}_{p'}^\dagger :: \hat{a}_k^\dagger \hat{a}_k : + \hat{W}_{p,p'} \hat{\omega}_k \left(: \hat{a}_k^\dagger \hat{a}_p^\dagger : \delta_{k,p'} + : \hat{a}_k^\dagger \hat{a}_{p'}^\dagger : \delta_{k,p}\right) \\
&= \hat{W}_{p,p'} \left(\omega_k - \hat{\omega}(\hat{n}_{p'} - 1, \hat{n}_p - 1)\right) : \hat{a}_p^\dagger \hat{a}_{p'}^\dagger \hat{a}_k^\dagger \hat{a}_k : + \hat{W}_{p,p'} \hat{\omega}_k \left(: \hat{a}_k^\dagger \hat{a}_p^\dagger : \delta_{k,p'} + : \hat{a}_k^\dagger \hat{a}_{p'}^\dagger : \delta_{k,p}\right)
\end{aligned} \tag{A.21}$$

A.19

$$\begin{aligned}
&\left[\hat{\omega}_k : \hat{a}_k^\dagger \hat{a}_k :, \hat{W}_{p,p'}^\dagger : \hat{a}_p \hat{a}_{p'} :\right] \\
&= \hat{W}_{p,p'}^\dagger \left[\hat{\omega}_k, : \hat{a}_p \hat{a}_{p'} :\right] : \hat{a}_k^\dagger \hat{a}_k : + \hat{W}_{p,p'}^\dagger \hat{\omega}_k \left[: \hat{a}_k^\dagger \hat{a}_k :, : \hat{a}_p \hat{a}_{p'} :\right] \\
&= \hat{W}_{p,p'}^\dagger \left(\omega_k - \hat{\omega}(\hat{n}_{p'} + 1, \hat{n}_p + 1)\right) : \hat{a}_p \hat{a}_{p'} :: \hat{a}_k^\dagger \hat{a}_k : + \hat{W}_{p,p'}^\dagger \hat{\omega}_k \left(: \hat{a}_p \hat{a}_k : \delta_{k,p'} + : \hat{a}_{p'} \hat{a}_k : \delta_{k,p}\right) \\
&= \hat{W}_{p,p'}^\dagger \left(\omega_k - \hat{\omega}(\hat{n}_{p'} + 1, \hat{n}_p + 1)\right) \left(: \hat{a}_k^\dagger \hat{a}_k \hat{a}_p \hat{a}_{p'} : + \delta_{k,p} : \hat{a}_{p'} \hat{a}_k : + \delta_{k,p'} : \hat{a}_p \hat{a}_k :\right) \\
&+ \hat{W}_{p,p'}^\dagger \hat{\omega}_k \left(: \hat{a}_p \hat{a}_k : \delta_{k,p'} + : \hat{a}_{p'} \hat{a}_k : \delta_{k,p}\right)
\end{aligned} \tag{A.22}$$

APPENDIX B

THE SECOND APPENDIX

Here comes the second appendix.

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DECLARATION OF AUTHORSHIP

I hereby declare that I have written this thesis independently and by myself and that I have not used any sources or auxiliary materials other than those indicated in the thesis.

Munich, 22.06.2023 . . . .