Title of My Thesis

SUBMITTED BY

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Titel meiner Arbeit

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Faculty of Physics
Quantum Many-Body Systems/ Theoretical Nanophysics Group
Ludwig Maximilian University
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NOTATION AND SYMBOLS

λ	flow parameter; in the literature sometimes also denoted by B
$\hat{\cdot}$	denotes that $\cdot$ is an operator which does not commute with every other operator
1	indicates $1 \in \mathbb{N}$ or the identity operator $\hat{\mathbb{1}} =: \mathbb{1}$
$:\hat{A}:$	normal ordering of operator $\hat{A}$
$\hat{a}_k^\dagger$	k^{th} bosonic creation operator
$\hat{a}_k$	k^{th} bosonic annihilation operator
$[\hat{A}, \hat{B}]$	commutator of operators $\hat{A}, \hat{B}$
z^*	complex conjugate of $z \in \mathbb{C}$
$\delta_{\alpha,\beta}$	Kronecker-Delta of α, β
∂_x	partial derivative $\frac{\partial}{\partial x}$ w.r.t. x

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DETAILED CALCULATIONS

A.1 Deriving the flow equations in the case of no n-dependence

First the canonical generator $\hat{\eta}$ has to be evaluated:

$$\hat{\eta} := \hat{\eta}(\lambda) := \left[\hat{\mathcal{H}}_0, \hat{\mathcal{H}}_{\text{int}}\right] = \left[\sum_k \omega_k \hat{a}_k^\dagger \hat{a}_k, \sum_{q\neq q'} V_{q,q'} \hat{a}_q^\dagger \hat{a}_{q'} + \sum_{p,p'} \left(W_{p,p'} \hat{a}_p^\dagger \hat{a}_{p'}^\dagger + W_{p,p'}^* \hat{a}_p \hat{a}_{p'}\right)\right] \tag{A.1}$$

$$\begin{aligned}
&= \sum_k \sum_{q,q'} \omega_k V_{q,q'} \left[\hat{a}_k^\dagger \hat{a}_k, \hat{a}_q^\dagger \hat{a}_{q'}\right] + \sum_k \sum_{p,p'} \left(\omega_k W_{p,p'} \left[\hat{a}_k^\dagger \hat{a}_k, \hat{a}_p^\dagger \hat{a}_{p'}^\dagger\right] + \omega_k W_{p,p'}^* \left[\hat{a}_k^\dagger \hat{a}_k, \hat{a}_p \hat{a}_{p'}\right]\right) \\
&= \sum_k \sum_{q,q'} \omega_k V_{q,q'} \left(\hat{a}_k^\dagger \hat{a}_{q'} \delta_{k,q} - \hat{a}_q^\dagger \hat{a}_k \delta_{k,q'}\right) \\
&+ \sum_k \sum_{p,p'} \left(\omega_k W_{p,p'} \left(\hat{a}_k^\dagger \hat{a}_p^\dagger \delta_{k,p'} + \hat{a}_k^\dagger \hat{a}_{p'}^\dagger \delta_{k,p}\right) - \omega_k W_{p,p'}^* \left(\hat{a}_p \hat{a}_k \delta_{k,p'} + \hat{a}_{p'} + \hat{a}_{p'} \hat{a}_k \delta_{k,p}\right)\right) \\
&= \sum_{q\neq q'} V_{q,q'} (\omega_q - \omega_{q'}) \hat{a}_q^\dagger \hat{a}_{q'} + \sum_{p,p'} \left(W_{p,p'} (\omega_p + \omega_{p'}) \hat{a}_p^\dagger \hat{a}_{p'}^\dagger - W_{p,p'}^* (\omega_p + \omega_{p'}) \hat{a}_p \hat{a}_{p'}\right)
\end{aligned} \tag{A.2}$$

Since $\hat{\eta}$ has the same form as $\hat{\mathcal{H}}_{\text{int}}$, $\left[\hat{\eta}, \hat{\mathcal{H}}_0\right]$ follows by inspection of A.2:

$$\begin{aligned}
\left[\hat{\eta}, \hat{\mathcal{H}}_0\right] &= -\sum_{q\neq q'} V_{q,q'} (\omega_q - \omega_{q'})^2 \hat{a}_q^\dagger \hat{a}_{q'} \\
&- \sum_{p,p'} \left(W_{p,p'} (\omega_p + \omega_{p'})^2 \hat{a}_p^\dagger \hat{a}_{p'}^\dagger + W_{p,p'}^* (\omega_p + \omega_{p'})^2 \hat{a}_p \hat{a}_{p'}\right)
\end{aligned} \tag{A.3}$$

The commutator of the generator and $\hat{\mathcal{H}}_{\text{int}}$ needs more work:

$$\left[\hat{\eta}, \hat{\mathcal{H}}_{\text{int}}\right] = \Bigg[\sum_{q\neq q'} V_{q,q'} (\omega_q - \omega_{q'}) \hat{a}_q^\dagger \hat{a}_{q'} + \sum_{p,p'} \Big(W_{p,p'} (\omega_p + \omega_{p'}) \hat{a}_p^\dagger \hat{a}_{p'}^\dagger \tag{A.4}$$

$$- W_{p,p'}^* (\omega_p + \omega_{p'}) \hat{a}_p \hat{a}_{p'}\Big), \sum_{\tilde{q}\neq\tilde{q}'} V_{\tilde{q},\tilde{q}'} \hat{a}_{\tilde{q}}^\dagger \hat{a}_{\tilde{q}'} + \sum_{\tilde{p},\tilde{p}'} \left(W_{\tilde{p},\tilde{p}'} \hat{a}_{\tilde{p}}^\dagger \hat{a}_{\tilde{p}'}^\dagger + W_{\tilde{p},\tilde{p}'}^* \hat{a}_{\tilde{p}} \hat{a}_{\tilde{p}'}\right)\Bigg]$$

$$= \left[\sum_{q\neq q'} V_{q,q'} (\omega_q - \omega_{q'}) \hat{a}_q^\dagger \hat{a}_{q'}, \sum_{\tilde{q}\neq\tilde{q}'} V_{\tilde{q},\tilde{q}'} \hat{a}_{\tilde{q}}^\dagger \hat{a}_{\tilde{q}'}\right] \tag{A.5}$$

$$+ \left[\sum_{q\neq q'} V_{q,q'} (\omega_q - \omega_{q'}) \hat{a}_q^\dagger \hat{a}_{q'}, \sum_{\tilde{p},\tilde{p}'} \left(W_{\tilde{p},\tilde{p}'} \hat{a}_{\tilde{p}}^\dagger \hat{a}_{\tilde{p}'}^\dagger + W_{\tilde{p},\tilde{p}'}^* \hat{a}_{\tilde{p}} \hat{a}_{\tilde{p}'}\right)\right] \tag{A.6}$$

$$+ \left[\sum_{p,p'} \left(W_{p,p'} (\omega_p + \omega_{p'}) \hat{a}_p^\dagger \hat{a}_{p'}^\dagger - W_{p,p'}^* (\omega_p + \omega_{p'}) \hat{a}_p \hat{a}_{p'}\right), \sum_{\tilde{q}\neq\tilde{q}'} V_{\tilde{q},\tilde{q}'} \hat{a}_{\tilde{q}}^\dagger \hat{a}_{\tilde{q}'}\right] \tag{A.7}$$

$$+ \left[\sum_{p,p'} \left(W_{p,p'} (\omega_p + \omega_{p'}) \hat{a}_p^\dagger \hat{a}_{p'}^\dagger - W_{p,p'}^* (\omega_p + \omega_{p'}) \hat{a}_p \hat{a}_{p'}\right), \sum_{\tilde{p},\tilde{p}'} \left(W_{\tilde{p},\tilde{p}'} \hat{a}_{\tilde{p}}^\dagger \hat{a}_{\tilde{p}'}^\dagger + W_{\tilde{p},\tilde{p}'}^* \hat{a}_{\tilde{p}} \hat{a}_{\tilde{p}'}\right)\right] \tag{A.8}$$

In the following, A.5-A.8 will be evaluated separately:

A.5:

$$
\begin{aligned}
&\left[\sum_{q\neq q'} V_{q,q'}(\omega_q-\omega_{q'})\hat{a}_q^\dagger \hat{a}_{q'}, \sum_{\tilde{q}\neq\tilde{q}'} V_{\tilde{q},\tilde{q}'}\hat{a}_{\tilde{q}}^\dagger \hat{a}_{\tilde{q}'}\right] \\
&=\sum_{q\neq q'}\sum_{\tilde{q}\neq\tilde{q}'} V_{\tilde{q},\tilde{q}'}V_{q,q'}(\omega_q-\omega_{q'})\left[\hat{a}_q^\dagger\hat{a}_{q'},\hat{a}_{\tilde{q}}^\dagger\hat{a}_{\tilde{q}'}\right] \\
&=\sum_{q\neq q'}\sum_{\tilde{q}\neq\tilde{q}'} V_{\tilde{q},\tilde{q}'}V_{q,q'}(\omega_q-\omega_{q'})\left(\hat{a}_q^\dagger\hat{a}_{\tilde{q}'}\delta_{q',\tilde{q}}-\hat{a}_{\tilde{q}}^\dagger\hat{a}_{q'}\delta_{q,\tilde{q}'}\right) \\
&=\sum_{q\neq q'}\sum_{\tilde{q}'} V_{q',\tilde{q}'}V_{q,q'}(\omega_q-\omega_{q'})\hat{a}_q^\dagger\hat{a}_{\tilde{q}'}-\sum_{q\neq q'}\sum_{\tilde{q}} V_{\tilde{q},q}V_{q,q'}(\omega_q-\omega_{q'})\hat{a}_{\tilde{q}}^\dagger\hat{a}_{q'} \\
&=\sum_{q,q'}\sum_{\tilde{q}'} V_{q',\tilde{q}'}V_{q,q'}(\omega_q-\omega_{q'})\hat{a}_q^\dagger\hat{a}_{\tilde{q}'}-\sum_{q,q'}\sum_{\tilde{q}} V_{\tilde{q},q}V_{q,q'}(\omega_q-\omega_{q'})\hat{a}_{\tilde{q}}^\dagger\hat{a}_{q'} \\
&=\sum_{q,q'}\sum_{\tilde{q}} V_{\tilde{q},q'}V_{q,\tilde{q}}(\omega_q-\omega_{\tilde{q}})\hat{a}_q^\dagger\hat{a}_{q'}-\sum_{q,q'}\sum_{\tilde{q}} V_{q,\tilde{q}}V_{\tilde{q},q'}(\omega_{\tilde{q}}-\omega_{q'})\hat{a}_q^\dagger\hat{a}_{q'} \\
&=\sum_{q\neq q'}\sum_{\tilde{q}} V_{\tilde{q},q'}V_{q,\tilde{q}}(\omega_q-\omega_{\tilde{q}})\hat{a}_q^\dagger\hat{a}_{q'}-\sum_{q\neq q'}\sum_{\tilde{q}} V_{q,\tilde{q}}V_{\tilde{q},q'}(\omega_{\tilde{q}}-\omega_{q'})\hat{a}_q^\dagger\hat{a}_{q'} \\
&+\sum_{k}\sum_{\tilde{q}} V_{\tilde{q},k}V_{k,\tilde{q}}(\omega_k-\omega_{\tilde{q}})\hat{a}_k^\dagger\hat{a}_k-\sum_{k}\sum_{\tilde{q}} V_{k,\tilde{q}}V_{\tilde{q},k}(\omega_{\tilde{q}}-\omega_k)\hat{a}_k^\dagger\hat{a}_k \\
&=\sum_{q\neq q'}\sum_{\tilde{q}} V_{\tilde{q},q'}V_{q,\tilde{q}}(\omega_q-\omega_{\tilde{q}})\hat{a}_q^\dagger\hat{a}_{q'}-\sum_{q\neq q'}\sum_{\tilde{q}} V_{q,\tilde{q}}V_{\tilde{q},q'}(\omega_{\tilde{q}}-\omega_{q'})\hat{a}_q^\dagger\hat{a}_{q'} \\
&+\sum_{k}\sum_{\tilde{q}} 2V_{\tilde{q},k}V_{k,\tilde{q}}(\omega_k-\omega_{\tilde{q}})\hat{a}_k^\dagger\hat{a}_k
\end{aligned}
\tag{A.9}
$$

A.6:

$$
\begin{aligned}
&\left[\sum_{q\neq q'} V_{q,q'}(\omega_q-\omega_{q'})\hat{a}_q^\dagger \hat{a}_{q'}, \sum_{\tilde{p},\tilde{p}'}\left(W_{\tilde{p},\tilde{p}'}\hat{a}_{\tilde{p}}^\dagger\hat{a}_{\tilde{p}'}^\dagger+W_{\tilde{p},\tilde{p}'}^*\hat{a}_{\tilde{p}}\hat{a}_{\tilde{p}'}\right)\right] \\
&=\sum_{q\neq q'}\sum_{\tilde{p},\tilde{p}'} V_{q,q'}(\omega_q-\omega_{q'})\left(W_{\tilde{p},\tilde{p}'}\left[\hat{a}_q^\dagger\hat{a}_{q'},\hat{a}_{\tilde{p}}^\dagger\hat{a}_{\tilde{p}'}^\dagger\right]+W_{\tilde{p},\tilde{p}'}^*\left[\hat{a}_q^\dagger\hat{a}_{q'},\hat{a}_{\tilde{p}}\hat{a}_{\tilde{p}'}\right]\right) \\
&=\sum_{q,q'}\sum_{\tilde{p},\tilde{p}'} V_{q,q'}(\omega_q-\omega_{q'})\left(W_{\tilde{p},\tilde{p}'}\left(\hat{a}_q^\dagger\hat{a}_{\tilde{p}}^\dagger\delta_{q',\tilde{p}'}+\hat{a}_q^\dagger\hat{a}_{\tilde{p}'}^\dagger\delta_{q',\tilde{p}}\right)-W_{\tilde{p},\tilde{p}'}^*\hat{a}_{\tilde{p}}\left(\hat{a}_{\tilde{p}'}\hat{a}_{q'}\delta_{q,\tilde{p}}+\hat{a}_{\tilde{p}}\hat{a}_{q'}\delta_{\tilde{p}',q}\right)\right) \\
&=\sum_{p,p'}\sum_{q} V_{q,p'}(\omega_q-\omega_{p'})W_{p,p'}\hat{a}_q^\dagger\hat{a}_p^\dagger+\sum_{p,p'}\sum_{q} V_{q,p}(\omega_q-\omega_p)W_{p,p'}\hat{a}_q^\dagger\hat{a}_{p'}^\dagger \\
&-\sum_{p,p'}\sum_{q'} V_{p,q'}(\omega_p-\omega_{q'})W_{p,p'}^*\hat{a}_{p'}\hat{a}_{q'}-\sum_{p,p'}\sum_{q'} V_{p',q'}(\omega_{p'}-\omega_{q'})W_{p,p'}^*\hat{a}_p\hat{a}_{q'} \\
&=\sum_{p,p'}\sum_{q} V_{p',q}(\omega_{p'}-\omega_q)W_{p,q}\hat{a}_p^\dagger\hat{a}_{p'}^\dagger+\sum_{p,p'}\sum_{q} V_{p,q}(\omega_p-\omega_q)W_{q,p'}\hat{a}_p^\dagger\hat{a}_{p'}^\dagger \\
&-\sum_{p,p'}\sum_{q} V_{q,p}(\omega_q-\omega_p)W_{q,p'}^*\hat{a}_p\hat{a}_{p'}-\sum_{p,p'}\sum_{q} V_{q,p'}(\omega_q-\omega_{p'})W_{p,q}^*\hat{a}_p\hat{a}_{p'} \\
&=\sum_{p,p'}\sum_{q} V_{p',q}(\omega_{p'}-\omega_q)W_{p,q}\hat{a}_p^\dagger\hat{a}_{p'}^\dagger+\sum_{p,p'}\sum_{q} V_{p,q}(\omega_p-\omega_q)W_{q,p'}\hat{a}_p^\dagger\hat{a}_{p'}^\dagger \\
&-\sum_{p,p'}\sum_{q} V_{q,p}(\omega_q-\omega_p)W_{q,p'}^*\hat{a}_p\hat{a}_{p'}-\sum_{p,p'}\sum_{q} V_{q,p'}(\omega_q-\omega_{p'})W_{p,q}^*\hat{a}_p\hat{a}_{p'} \\
&=\sum_{p,p'}\sum_{q} V_{p,q}(\omega_p-\omega_q)(W_{q,p'}+W_{p',q})\hat{a}_p^\dagger\hat{a}_{p'}^\dagger \\
&+\sum_{p,p'}\sum_{q} V_{q,p}(\omega_p-\omega_q)(W_{q,p'}^*+W_{p',q}^*)\hat{a}_p\hat{a}_{p'}
\end{aligned}
\tag{A.10}
$$

A.7:

$$
\begin{aligned}
&\left[\sum_{p,p'}\Big(W_{p,p'}(\omega_p+\omega_{p'})\hat{a}_p^\dagger\hat{a}_{p'}^\dagger - W_{p,p'}^*(\omega_p+\omega_{p'})\hat{a}_p\hat{a}_{p'}\Big), \sum_{\tilde{q}\neq\tilde{q}'} V_{\tilde{q},\tilde{q}'}\hat{a}_{\tilde{q}}^\dagger\hat{a}_{\tilde{q}'}\right] \\
&= \sum_{p,p'}\sum_{\tilde{q}\neq\tilde{q}'} V_{\tilde{q},\tilde{q}'}(\omega_p+\omega_{p'})\left(W_{p,p'}\left[\hat{a}_p^\dagger\hat{a}_{p'}^\dagger, \hat{a}_{\tilde{q}}^\dagger\hat{a}_{\tilde{q}'}\right] - W_{p,p'}^*\left[\hat{a}_p\hat{a}_{p'}, \hat{a}_{\tilde{q}}^\dagger\hat{a}_{\tilde{q}'}\right]\right) \\
&= -\sum_{p,p'}\sum_{q\neq q'} V_{q,q'}(\omega_p+\omega_{p'})W_{p,p'}\left(\hat{a}_q^\dagger\hat{a}_p^\dagger\delta_{q',p'} + \hat{a}_q^\dagger\hat{a}_{p'}^\dagger\delta_{q',p}\right) \\
&- \sum_{p,p'}\sum_{q\neq q'} V_{q,q'}(\omega_p+\omega_{p'})W_{p,p'}^*\left(\hat{a}_p\hat{a}_{q'}\delta_{q,p'} + \hat{a}_{p'}\hat{a}_{q'}\delta_{q,p}\right) \\
&= -\sum_{p,p'}\sum_{q} V_{q,p'}(\omega_p+\omega_{p'})W_{p,p'}\hat{a}_q^\dagger\hat{a}_p^\dagger - \sum_{p,p'}\sum_{q} V_{q,p}(\omega_p+\omega_{p'})W_{p,p'}\hat{a}_q^\dagger\hat{a}_{p'}^\dagger \\
&- \sum_{p,p'}\sum_{q'} V_{p',q'}(\omega_p+\omega_{p'})W_{p,p'}^*\hat{a}_p\hat{a}_{q'} - \sum_{p,p'}\sum_{q'} V_{p,q'}(\omega_p+\omega_{p'})W_{p,p'}^*\hat{a}_{p'}\hat{a}_{q'} \\
&= -\sum_{p,p'}\sum_{q} V_{p',q}(\omega_p+\omega_{q})W_{p,q}\hat{a}_p^\dagger\hat{a}_{p'}^\dagger - \sum_{p,p'}\sum_{q} V_{p,q}(\omega_q+\omega_{p'})W_{q,p'}\hat{a}_p^\dagger\hat{a}_{p'}^\dagger \\
&- \sum_{p,p'}\sum_{q'} V_{q',p'}(\omega_p+\omega_{q'})W_{p,q'}^*\hat{a}_p\hat{a}_{p'} - \sum_{p,p'}\sum_{q'} V_{q',p}(\omega_{q'}+\omega_{p'})W_{q',p'}^*\hat{a}_p\hat{a}_{p'} \\
&= -\sum_{p,p'}\sum_{q} V_{p,q}(\omega_q+\omega_{p'})(W_{p',q}+W_{q,p'})\hat{a}_p^\dagger\hat{a}_{p'}^\dagger \\
&- \sum_{p,p'}\sum_{q} V_{q,p}(\omega_q+\omega_{p'})(W_{p',q}^*+W_{q,p'}^*)\hat{a}_p\hat{a}_{p'}
\end{aligned}
\tag{A.11}
$$

A.8:

$$
\begin{aligned}
&\left[\sum_{p,p'}\Big(W_{p,p'}(\omega_p+\omega_{p'})\hat{a}_p^\dagger\hat{a}_{p'}^\dagger - W_{p,p'}^*(\omega_p+\omega_{p'})\hat{a}_p\hat{a}_{p'}\Big), \sum_{\tilde{p},\tilde{p}'}\Big(W_{\tilde{p},\tilde{p}'}\hat{a}_{\tilde{p}}^\dagger\hat{a}_{\tilde{p}'}^\dagger + W_{\tilde{p},\tilde{p}'}^*\hat{a}_{\tilde{p}}\hat{a}_{\tilde{p}'}\Big)\right] \\
&= \sum_{p,p'}\sum_{\tilde{p},\tilde{p}'} W_{p,p'}(\omega_p+\omega_{p'})W_{\tilde{p},\tilde{p}'}^*\left[\hat{a}_p^\dagger\hat{a}_{p'}^\dagger, \hat{a}_{\tilde{p}}\hat{a}_{\tilde{p}'}\right] - \sum_{p,p'}\sum_{\tilde{p},\tilde{p}'} W_{p,p'}^*W_{\tilde{p},\tilde{p}'}(\omega_p+\omega_{p'})\left[\hat{a}_p\hat{a}_{p'}, \hat{a}_{\tilde{p}}^\dagger\hat{a}_{\tilde{p}'}^\dagger\right] \\
&= -\sum_{p,p'}\sum_{\tilde{p},\tilde{p}'} W_{p,p'}(\omega_p+\omega_{p'}+\omega_{\tilde{p}}+\omega_{\tilde{p}'})W_{\tilde{p},\tilde{p}'}^*\left[\hat{a}_{\tilde{p}}\hat{a}_{\tilde{p}'}, \hat{a}_p^\dagger\hat{a}_{p'}^\dagger\right] \\
&= -\sum_{p,p'}\sum_{\tilde{p},\tilde{p}'} W_{p,p'}(\omega_p+\omega_{p'}+\omega_{\tilde{p}}+\omega_{\tilde{p}'})W_{\tilde{p},\tilde{p}'}^*\hat{a}_{\tilde{p}}\hat{a}_p^\dagger\delta_{\tilde{p}',p'} \\
&- \sum_{p,p'}\sum_{\tilde{p},\tilde{p}'} W_{p,p'}(\omega_p+\omega_{p'}+\omega_{\tilde{p}}+\omega_{\tilde{p}'})W_{\tilde{p},\tilde{p}'}^*\hat{a}_{\tilde{p}}\hat{a}_{p'}^\dagger\delta_{\tilde{p}',p} \\
&- \sum_{p,p'}\sum_{\tilde{p},\tilde{p}'} W_{p,p'}(\omega_p+\omega_{p'}+\omega_{\tilde{p}}+\omega_{\tilde{p}'})W_{\tilde{p},\tilde{p}'}^*\hat{a}_p^\dagger\hat{a}_{\tilde{p}'}\delta_{\tilde{p},p'} \\
&- \sum_{p,p'}\sum_{\tilde{p},\tilde{p}'} W_{p,p'}(\omega_p+\omega_{p'}+\omega_{\tilde{p}}+\omega_{\tilde{p}'})W_{\tilde{p},\tilde{p}'}^*\hat{a}_{p'}^\dagger\hat{a}_{\tilde{p}'}\delta_{\tilde{p},p} \\
&= -\sum_{p,p'}\sum_{\tilde{p}} W_{p,p'}(\omega_p+2\omega_{p'}+\omega_{\tilde{p}})W_{\tilde{p},p'}^*\hat{a}_{\tilde{p}}\hat{a}_p^\dagger - \sum_{p,p'}\sum_{\tilde{p}} W_{p,p'}(2\omega_p+\omega_{p'}+\omega_{\tilde{p}})W_{\tilde{p},p}^*\hat{a}_{\tilde{p}}\hat{a}_{p'}^\dagger \\
&- \sum_{p,p'}\sum_{\tilde{p}'} W_{p,p'}(\omega_p+2\omega_{p'}+\omega_{\tilde{p}'})W_{p',\tilde{p}'}^*\hat{a}_p^\dagger\hat{a}_{\tilde{p}'} - \sum_{p,p'}\sum_{\tilde{p}'} W_{p,p'}(2\omega_p+\omega_{p'}+\omega_{\tilde{p}'})W_{p,\tilde{p}'}^*\hat{a}_{p'}^\dagger\hat{a}_{\tilde{p}'} \\
&= -\sum_{p,p'}\sum_{\tilde{p}} W_{p,\tilde{p}}(\omega_p+2\omega_{\tilde{p}}+\omega_{p'})W_{p',\tilde{p}}^*\hat{a}_{p'}\hat{a}_p^\dagger - \sum_{p,p'}\sum_{\tilde{p}} W_{\tilde{p},p}(2\omega_{\tilde{p}}+\omega_p+\omega_{p'})W_{p',\tilde{p}}^*\hat{a}_{p'}\hat{a}_p^\dagger \\
&- \sum_{p,p'}\sum_{\tilde{p}'} W_{p,\tilde{p}'}(\omega_p+2\omega_{\tilde{p}'}+\omega_{p'})W_{\tilde{p}',p'}^*\hat{a}_p^\dagger\hat{a}_{p'} - \sum_{p,p'}\sum_{\tilde{p}'} W_{\tilde{p}',p}(2\omega_{\tilde{p}'}+\omega_p+\omega_{p'})W_{\tilde{p}',p'}^*\hat{a}_p^\dagger\hat{a}_{p'} \\
&= -\sum_{p,p'}\sum_{\tilde{p}}(W_{p,\tilde{p}}+W_{\tilde{p},p})(\omega_p+2\omega_{\tilde{p}}+\omega_{p'})W_{p',\tilde{p}}^*\hat{a}_{p'}\hat{a}_p^\dagger
\end{aligned}
$$

$$
\begin{aligned}
&- \sum_{p,p'} \sum_{\tilde{p}'} (W_{p,\tilde{p}'} + W_{\tilde{p}',p})(\omega_p + 2\omega_{\tilde{p}'} + \omega_{p'}) W^*_{\tilde{p}',p'} \hat{a}^\dagger_p \hat{a}_{p'} \\
&= - \sum_{p,p'} \sum_{\tilde{p}} (W_{p,\tilde{p}} + W_{\tilde{p},p})(\omega_p + 2\omega_{\tilde{p}} + \omega_{p'}) W^*_{p',\tilde{p}} (\delta_{p,p'} + \hat{a}^\dagger_p \hat{a}_{p'}) \\
&- \sum_{p,p'} \sum_{\tilde{p}} (W_{p,\tilde{p}} + W_{\tilde{p},p})(\omega_p + 2\omega_{\tilde{p}} + \omega_{p'}) W^*_{\tilde{p},p'} \hat{a}^\dagger_p \hat{a}_{p'} \\
&= - \sum_{p,p'} \sum_{\tilde{p}} (W_{p,\tilde{p}} + W_{\tilde{p},p})(\omega_p + 2\omega_{\tilde{p}} + \omega_{p'}) (W^*_{\tilde{p},p'} + W^*_{p',\tilde{p}}) \hat{a}^\dagger_p \hat{a}_{p'} \\
&- 2 \sum_{k} \sum_{\tilde{p}} (W_{k,\tilde{p}} + W_{\tilde{p},k})(\omega_k + \omega_{\tilde{p}}) W^*_{k,\tilde{p}} \\
&= - \sum_{q \neq q'} \sum_{\tilde{p}} (W_{q,\tilde{p}} + W_{\tilde{p},q})(\omega_q + 2\omega_{\tilde{p}} + \omega_{q'}) (W^*_{\tilde{p},q'} + W^*_{q',\tilde{p}}) \hat{a}^\dagger_q \hat{a}_{q'} \\
&- 2 \sum_{k} \sum_{\tilde{p}} (W_{k,\tilde{p}} + W_{\tilde{p},k})(\omega_k + \omega_{\tilde{p}}) (W^*_{\tilde{p},k} + W^*_{k,\tilde{p}}) \hat{a}^\dagger_k \hat{a}_k \\
&- 2 \sum_{k} \sum_{\tilde{p}} (W_{k,\tilde{p}} + W_{\tilde{p},k})(\omega_k + \omega_{\tilde{p}}) W^*_{k,\tilde{p}}
\end{aligned}
\tag{A.12}
$$

Using the expressions for the commutators of the generator and $\hat{\mathcal{H}}_0$ respectively $\hat{\mathcal{H}}_{\text{int}}$ derived above, the flow $\partial_\lambda \hat{\mathcal{H}}(\lambda) = [\hat{\eta}(\lambda), \hat{\mathcal{H}}(\lambda)]$ yields the following flow equations $\forall k, p, p', q, q'$ where $q \neq q'$:

$$
\partial_\lambda \omega_k = \sum_{\tilde{q}} 2 V_{\tilde{q},k} V_{k,\tilde{q}} (\omega_k - \omega_{\tilde{q}}) - 2 \sum_{\tilde{p}} (W_{k,\tilde{p}} + W_{\tilde{p},k})(\omega_k + \omega_{\tilde{p}})(W^*_{\tilde{p},k} + W^*_{k,\tilde{p}}) \tag{A.13a}
$$

$$
\begin{aligned}
\partial_\lambda V_{q,q'} = &- V_{q,q'} (\omega_q - \omega_{q'})^2 - \sum_{\tilde{p}} (W_{q,\tilde{p}} + W_{\tilde{p},q})(\omega_q + \omega_{q'} + 2\omega_{\tilde{p}})(W^*_{\tilde{p},q'} + W^*_{q',\tilde{p}}) \\
&+ \sum_{\tilde{q}} V_{\tilde{q},q'} V_{q,\tilde{q}} (\omega_q + \omega_{q'} - 2\omega_{\tilde{q}})
\end{aligned}
\tag{A.13b}
$$

$$
\begin{aligned}
\partial_\lambda W_{p,p'} = &- W_{p,p'} (\omega_p + \omega_{p'})^2 - \sum_{q} V_{p,q} (\omega_q + \omega_{p'})(W_{p',q} + W_{q,p'}) \\
&+ \sum_{q} V_{p,q} (\omega_p - \omega_q)(W_{q,p'} + W_{p',q})
\end{aligned}
\tag{A.13c}
$$

$$
\begin{aligned}
\partial_\lambda W^*_{p,p'} = &- W^*_{p,p'} (\omega_p + \omega_{p'})^2 - \sum_{q} V_{q,p} (\omega_q + \omega_{p'})(W^*_{p',q} + W^*_{q,p'}) \\
&+ \sum_{q} V_{q,p} (\omega_p - \omega_q)(W^*_{q,p'} + W^*_{p',q})
\end{aligned}
\tag{A.13d}
$$

$$
\partial_\lambda \varepsilon = -2 \sum_{p,p'} (W_{p,p'} + W_{p',p})(\omega_p + \omega_{p'}) W^*_{p,p'} \tag{A.13e}
$$

Obviously, equations A.13c and A.13d are not independent from each other, since they are related by complex conjugation. Seeing this is a good consistency check because complex conjugation was not explicitly used in the derivation of the these two equations.
Further note that the flow equations A.13a-A.13e are exact in the sense that if the flow is completely traversed, the flow Hamiltonian will be exactly diagonal.

APPENDIX B

THE SECOND APPENDIX

Here comes the second appendix.

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DECLARATION OF AUTHORSHIP

I hereby declare that I have written this thesis independently and by myself and that I have not used any sources or auxiliary materials other than those indicated in the thesis.

Munich, 22.06.2023 . . . .