
Title of My Thesis



SUBMITTED BY

Jan-Philipp Anton Konrad Christ

Titel meiner Arbeit

Bachelorarbeit

FAKULTÄT FÜR PHYSIK
QUANTEN VIELTEILCHENSYSTEME/ THEORETISCHE NANOPHYSIK
LUDWIG-MAXIMILIANS-UNIVERSITÄT
MÜNCHEN

VORGELEGT VON

Jan-Philipp Anton Konrad Christ

MÜNCHEN, 22.06.2023

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FACULTY OF PHYSICS
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Jan-Philipp Anton Konrad Christ

MUNICH, 22.06.2023

Supervisor: Prof. Dr. Fabian Bohrdt, geb. Grusdt

NOTATION AND SYMBOLS

ABSTRACT

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CHAPTER 2

THEORETICAL BACKGROUND

A.1 Deriving the flow equations in the case of no n-dependence

First the canonical generator $\hat{\eta}$ has to be evaluated:

$$\hat{\eta} := \hat{\eta}(\lambda) := [\hat{\mathcal{H}}_0, \hat{\mathcal{H}}_{\text{int}}] = \left[\sum_k \omega_k \hat{a}_k^\dagger \hat{a}_k, \sum_{q \neq q'} V_{q,q'} \hat{a}_q^\dagger \hat{a}_{q'} + \sum_{p,p'} \left(W_{p,p'} \hat{a}_p^\dagger \hat{a}_{p'} + W_{p,p'}^* \hat{a}_p \hat{a}_{p'} \right) \right] \quad (\text{A.1})$$

$$\begin{aligned} &= \sum_k \sum_{q,q'} \omega_k V_{q,q'} [\hat{a}_k^\dagger \hat{a}_k, \hat{a}_q^\dagger \hat{a}_{q'}] + \sum_k \sum_{p,p'} \left(\omega_k W_{p,p'} [\hat{a}_k^\dagger \hat{a}_k, \hat{a}_p^\dagger \hat{a}_{p'}] + \omega_k W_{p,p'}^* [\hat{a}_k^\dagger \hat{a}_k, \hat{a}_p \hat{a}_{p'}] \right) \\ &= \sum_k \sum_{q,q'} \omega_k V_{q,q'} (\hat{a}_k^\dagger \hat{a}_{q'} \delta_{k,q} - \hat{a}_q^\dagger \hat{a}_k \delta_{k,q'}) \\ &+ \sum_k \sum_{p,p'} \left(\omega_k W_{p,p'} (\hat{a}_k^\dagger \hat{a}_p^\dagger \delta_{k,p'} + \hat{a}_k^\dagger \hat{a}_{p'}^\dagger \delta_{k,p}) - \omega_k W_{p,p'}^* (\hat{a}_p \hat{a}_k \delta_{k,p'} + \hat{a}_{p'} \hat{a}_k \delta_{k,p}) \right) \\ &= \sum_{q \neq q'} V_{q,q'} (\omega_q - \omega_{q'}) \hat{a}_q^\dagger \hat{a}_{q'} + \sum_{p,p'} \left(W_{p,p'} (\omega_p + \omega_{p'}) \hat{a}_p^\dagger \hat{a}_{p'}^\dagger - W_{p,p'}^* (\omega_p + \omega_{p'}) \hat{a}_p \hat{a}_{p'} \right) \end{aligned} \quad (\text{A.2})$$

Since $\hat{\eta}$ has the same form as $\hat{\mathcal{H}}_{\text{int}}$, $[\hat{\eta}, \hat{\mathcal{H}}_0]$ follows by inspection of A.2:

$$\begin{aligned} [\hat{\eta}, \hat{\mathcal{H}}_0] &= - \sum_{q \neq q'} V_{q,q'} (\omega_q - \omega_{q'})^2 \hat{a}_q^\dagger \hat{a}_{q'} \\ &- \sum_{p,p'} \left(W_{p,p'} (\omega_p + \omega_{p'})^2 \hat{a}_p^\dagger \hat{a}_{p'}^\dagger + W_{p,p'}^* (\omega_p + \omega_{p'})^2 \hat{a}_p \hat{a}_{p'} \right) \end{aligned} \quad (\text{A.3})$$

The commutator of the generator and $\hat{\mathcal{H}}_{\text{int}}$ needs more work:

$$[\hat{\eta}, \hat{\mathcal{H}}_{\text{int}}] = \left[\sum_{q \neq q'} V_{q,q'} (\omega_q - \omega_{q'}) \hat{a}_q^\dagger \hat{a}_{q'} + \sum_{p,p'} \left(W_{p,p'} (\omega_p + \omega_{p'}) \hat{a}_p^\dagger \hat{a}_{p'}^\dagger - W_{p,p'}^* (\omega_p + \omega_{p'}) \hat{a}_p \hat{a}_{p'} \right), \right. \quad (\text{A.4})$$

$$\left. \sum_{\tilde{q} \neq \tilde{q}'} V_{\tilde{q},\tilde{q}'} \hat{a}_{\tilde{q}}^\dagger \hat{a}_{\tilde{q}'} + \sum_{\tilde{p},\tilde{p}'} \left(W_{\tilde{p},\tilde{p}'} \hat{a}_{\tilde{p}}^\dagger \hat{a}_{\tilde{p}'}^\dagger + W_{\tilde{p},\tilde{p}'}^* \hat{a}_{\tilde{p}} \hat{a}_{\tilde{p}'} \right) \right] \\ = \left[\sum_{q \neq q'} V_{q,q'} (\omega_q - \omega_{q'}) \hat{a}_q^\dagger \hat{a}_{q'}, \sum_{\tilde{q} \neq \tilde{q}'} V_{\tilde{q},\tilde{q}'} \hat{a}_{\tilde{q}}^\dagger \hat{a}_{\tilde{q}'} \right] \quad (\text{A.5})$$

$$+ \left[\sum_{q \neq q'} V_{q,q'} (\omega_q - \omega_{q'}) \hat{a}_q^\dagger \hat{a}_{q'}, \sum_{\tilde{p},\tilde{p}'} \left(W_{\tilde{p},\tilde{p}'} \hat{a}_{\tilde{p}}^\dagger \hat{a}_{\tilde{p}'}^\dagger + W_{\tilde{p},\tilde{p}'}^* \hat{a}_{\tilde{p}} \hat{a}_{\tilde{p}'} \right) \right] \quad (\text{A.6})$$

$$+ \left[\sum_{p,p'} \left(W_{p,p'} (\omega_p + \omega_{p'}) \hat{a}_p^\dagger \hat{a}_{p'}^\dagger - W_{p,p'}^* (\omega_p + \omega_{p'}) \hat{a}_p \hat{a}_{p'} \right), \sum_{\tilde{q} \neq \tilde{q}'} V_{\tilde{q},\tilde{q}'} \hat{a}_{\tilde{q}}^\dagger \hat{a}_{\tilde{q}'} \right] \quad (\text{A.7})$$

$$+ \left[\sum_{p,p'} \left(W_{p,p'} (\omega_p + \omega_{p'}) \hat{a}_p^\dagger \hat{a}_{p'}^\dagger - W_{p,p'}^* (\omega_p + \omega_{p'}) \hat{a}_p \hat{a}_{p'} \right), \sum_{\tilde{p},\tilde{p}'} \left(W_{\tilde{p},\tilde{p}'} \hat{a}_{\tilde{p}}^\dagger \hat{a}_{\tilde{p}'}^\dagger + W_{\tilde{p},\tilde{p}'}^* \hat{a}_{\tilde{p}} \hat{a}_{\tilde{p}'} \right) \right] \quad (\text{A.8})$$

In the following, A.5-A.8 will be evaluated separately:

A.5:

$$\begin{aligned}
& \left[\sum_{q \neq q'} V_{q,q'} (\omega_q - \omega_{q'}) \hat{a}_q^\dagger \hat{a}_{q'}, \sum_{\tilde{q} \neq \tilde{q}'} V_{\tilde{q},\tilde{q}'} \hat{a}_{\tilde{q}}^\dagger \hat{a}_{\tilde{q}'} \right] \\
&= \sum_{q \neq q'} \sum_{\tilde{q} \neq \tilde{q}'} V_{\tilde{q},\tilde{q}'} V_{q,q'} (\omega_q - \omega_{q'}) \left[\hat{a}_q^\dagger \hat{a}_{q'}, \hat{a}_{\tilde{q}}^\dagger \hat{a}_{\tilde{q}'} \right] \\
&= \sum_{q \neq q'} \sum_{\tilde{q} \neq \tilde{q}'} V_{\tilde{q},\tilde{q}'} V_{q,q'} (\omega_q - \omega_{q'}) \left(\hat{a}_q^\dagger \hat{a}_{\tilde{q}'} \delta_{q',\tilde{q}} - \hat{a}_{\tilde{q}}^\dagger \hat{a}_{q'} \delta_{q,\tilde{q}'} \right) \\
&= \sum_{q \neq q'} \sum_{\tilde{q}'} V_{q',\tilde{q}'} V_{q,q'} (\omega_q - \omega_{q'}) \hat{a}_q^\dagger \hat{a}_{\tilde{q}'} - \sum_{q \neq q'} \sum_{\tilde{q}} V_{\tilde{q},q} V_{q,q'} (\omega_q - \omega_{q'}) \hat{a}_{\tilde{q}}^\dagger \hat{a}_{q'} \\
&= \sum_{q,q'} \sum_{\tilde{q}'} V_{q',\tilde{q}'} V_{q,q'} (\omega_q - \omega_{q'}) \hat{a}_q^\dagger \hat{a}_{\tilde{q}'} - \sum_{q,q'} \sum_{\tilde{q}} V_{\tilde{q},q} V_{q,q'} (\omega_q - \omega_{q'}) \hat{a}_{\tilde{q}}^\dagger \hat{a}_{q'} \\
&= \sum_{q,q'} \sum_{\tilde{q}} V_{\tilde{q},q'} V_{q,\tilde{q}} (\omega_q - \omega_{\tilde{q}}) \hat{a}_q^\dagger \hat{a}_{q'} - \sum_{q,q'} \sum_{\tilde{q}} V_{q,\tilde{q}} V_{\tilde{q},q'} (\omega_{\tilde{q}} - \omega_{q'}) \hat{a}_q^\dagger \hat{a}_{q'} \\
&= \sum_{q \neq q'} \sum_{\tilde{q}} V_{\tilde{q},q'} V_{q,\tilde{q}} (\omega_q - \omega_{\tilde{q}}) \hat{a}_q^\dagger \hat{a}_{q'} - \sum_{q \neq q'} \sum_{\tilde{q}} V_{q,\tilde{q}} V_{\tilde{q},q'} (\omega_{\tilde{q}} - \omega_{q'}) \hat{a}_q^\dagger \hat{a}_{q'} \\
&+ \sum_k \sum_{\tilde{q}} V_{\tilde{q},k} V_{k,\tilde{q}} (\omega_k - \omega_{\tilde{q}}) \hat{a}_k^\dagger \hat{a}_{\tilde{q}} - \sum_k \sum_{\tilde{q}} V_{k,\tilde{q}} V_{\tilde{q},k} (\omega_{\tilde{q}} - \omega_k) \hat{a}_k^\dagger \hat{a}_{\tilde{q}} \\
&= \sum_{q \neq q'} \sum_{\tilde{q}} V_{\tilde{q},q'} V_{q,\tilde{q}} (\omega_q - \omega_{\tilde{q}}) \hat{a}_q^\dagger \hat{a}_{q'} - \sum_{q \neq q'} \sum_{\tilde{q}} V_{q,\tilde{q}} V_{\tilde{q},q'} (\omega_{\tilde{q}} - \omega_{q'}) \hat{a}_q^\dagger \hat{a}_{q'} \\
&+ \sum_k \sum_{\tilde{q}} 2V_{\tilde{q},k} V_{k,\tilde{q}} (\omega_k - \omega_{\tilde{q}}) \hat{a}_k^\dagger \hat{a}_{\tilde{q}} \tag{A.9}
\end{aligned}$$

A.6:

$$\begin{aligned}
& \left[\sum_{q \neq q'} V_{q,q'} (\omega_q - \omega_{q'}) \hat{a}_q^\dagger \hat{a}_{q'}, \sum_{\tilde{p},\tilde{p}'} \left(W_{\tilde{p},\tilde{p}'} \hat{a}_{\tilde{p}}^\dagger \hat{a}_{\tilde{p}'}^\dagger + W_{\tilde{p},\tilde{p}'}^* \hat{a}_{\tilde{p}} \hat{a}_{\tilde{p}'} \right) \right] \\
&= \sum_{q \neq q'} \sum_{\tilde{p},\tilde{p}'} V_{q,q'} (\omega_q - \omega_{q'}) \left(W_{\tilde{p},\tilde{p}'} \left[\hat{a}_q^\dagger \hat{a}_{q'}, \hat{a}_{\tilde{p}}^\dagger \hat{a}_{\tilde{p}'}^\dagger \right] + W_{\tilde{p},\tilde{p}'}^* \left[\hat{a}_q^\dagger \hat{a}_{q'}, \hat{a}_{\tilde{p}} \hat{a}_{\tilde{p}'} \right] \right) \\
&= \sum_{q,q'} \sum_{\tilde{p},\tilde{p}'} V_{q,q'} (\omega_q - \omega_{q'}) \left(W_{\tilde{p},\tilde{p}'} \left(\hat{a}_q^\dagger \hat{a}_{\tilde{p}}^\dagger \delta_{q',\tilde{p}'} + \hat{a}_q^\dagger \hat{a}_{\tilde{p}'}^\dagger \delta_{q',\tilde{p}} \right) - W_{\tilde{p},\tilde{p}'}^* \left(\hat{a}_{\tilde{p}'} \hat{a}_{q'} \delta_{q,\tilde{p}} + \hat{a}_{\tilde{p}} \hat{a}_{q'} \delta_{\tilde{p}',q} \right) \right) \\
&= \sum_{p,p'} \sum_q V_{q,p'} (\omega_q - \omega_{p'}) W_{p,p'} \hat{a}_q^\dagger \hat{a}_p^\dagger + \sum_{p,p'} \sum_q V_{q,p} (\omega_q - \omega_p) W_{p,p'} \hat{a}_q^\dagger \hat{a}_p^\dagger \\
&- \sum_{p,p'} \sum_{q'} V_{p,q'} (\omega_p - \omega_{q'}) W_{p,p'}^* \hat{a}_{p'} \hat{a}_{q'} - \sum_{p,p'} \sum_{q'} V_{p',q'} (\omega_{p'} - \omega_{q'}) W_{p,p'}^* \hat{a}_{p'} \hat{a}_{q'} \\
&= \sum_{p,p'} \sum_q V_{p',q} (\omega_{p'} - \omega_q) W_{p,q} \hat{a}_p^\dagger \hat{a}_{p'}^\dagger + \sum_{p,p'} \sum_q V_{p,q} (\omega_p - \omega_q) W_{q,p'} \hat{a}_p^\dagger \hat{a}_{p'}^\dagger \\
&- \sum_{p,p'} \sum_q V_{q,p} (\omega_q - \omega_p) W_{q,p}^* \hat{a}_p \hat{a}_{p'} - \sum_{p,p'} \sum_q V_{q,p'} (\omega_q - \omega_{p'}) W_{p,q}^* \hat{a}_p \hat{a}_{p'} \\
&= \sum_{p,p'} \sum_q V_{p',q} (\omega_{p'} - \omega_q) W_{p,q} \hat{a}_p^\dagger \hat{a}_{p'}^\dagger + \sum_{p,p'} \sum_q V_{p,q} (\omega_p - \omega_q) W_{q,p'} \hat{a}_p^\dagger \hat{a}_{p'}^\dagger \\
&- \sum_{p,p'} \sum_q V_{q,p} (\omega_q - \omega_p) W_{q,p}^* \hat{a}_p \hat{a}_{p'} - \sum_{p,p'} \sum_q V_{q,p'} (\omega_q - \omega_{p'}) W_{p,q}^* \hat{a}_p \hat{a}_{p'} \\
&= \sum_{p,p'} \sum_q V_{p,q} (\omega_p - \omega_q) (W_{q,p'} + W_{p',q}) \hat{a}_p^\dagger \hat{a}_{p'}^\dagger \\
&+ \sum_{p,p'} \sum_q V_{q,p} (\omega_p - \omega_q) (W_{q,p'}^* + W_{p',q}^*) \hat{a}_p \hat{a}_{p'} \tag{A.10}
\end{aligned}$$

A.7:

$$\begin{aligned}
& \left[\sum_{p,p'} \left(W_{p,p'}(\omega_p + \omega_{p'}) \hat{a}_p^\dagger \hat{a}_{p'}^\dagger - W_{p,p'}^*(\omega_p + \omega_{p'}) \hat{a}_p \hat{a}_{p'} \right), \sum_{\tilde{q} \neq \tilde{q}'} V_{\tilde{q},\tilde{q}'} \hat{a}_{\tilde{q}}^\dagger \hat{a}_{\tilde{q}'}^\dagger \right] \\
&= \sum_{p,p'} \sum_{\tilde{q} \neq \tilde{q}'} V_{\tilde{q},\tilde{q}'}(\omega_p + \omega_{p'}) \left(W_{p,p'} \left[\hat{a}_p^\dagger \hat{a}_{p'}^\dagger, \hat{a}_{\tilde{q}}^\dagger \hat{a}_{\tilde{q}'}^\dagger \right] - W_{p,p'}^* \left[\hat{a}_p \hat{a}_{p'}, \hat{a}_{\tilde{q}}^\dagger \hat{a}_{\tilde{q}'}^\dagger \right] \right) \\
&= - \sum_{p,p'} \sum_{q \neq q'} V_{q,q'}(\omega_p + \omega_{p'}) W_{p,p'} \left(\hat{a}_q^\dagger \hat{a}_p^\dagger \delta_{q',p'} + \hat{a}_q^\dagger \hat{a}_{p'}^\dagger \delta_{q',p} \right) \\
&\quad - \sum_{p,p'} \sum_{q \neq q'} V_{q,q'}(\omega_p + \omega_{p'}) W_{p,p'}^* \left(\hat{a}_p \hat{a}_{q'} \delta_{q,p'} + \hat{a}_{p'} \hat{a}_{q'} \delta_{q,p} \right) \\
&= - \sum_{p,p'} \sum_q V_{q,p'}(\omega_p + \omega_{p'}) W_{p,p'} \hat{a}_q^\dagger \hat{a}_p^\dagger - \sum_{p,p'} \sum_q V_{q,p}(\omega_p + \omega_{p'}) W_{p,p'} \hat{a}_q^\dagger \hat{a}_{p'}^\dagger \\
&\quad - \sum_{p,p'} \sum_{q'} V_{p',q'}(\omega_p + \omega_{p'}) W_{p,p'}^* \hat{a}_p \hat{a}_{q'} - \sum_{p,p'} \sum_{q'} V_{p,q'}(\omega_p + \omega_{p'}) W_{p,p'}^* \hat{a}_{p'} \hat{a}_{q'} \\
&= - \sum_{p,p'} \sum_q V_{p',q}(\omega_p + \omega_q) W_{p,q} \hat{a}_p^\dagger \hat{a}_{p'}^\dagger - \sum_{p,p'} \sum_q V_{p,q}(\omega_q + \omega_{p'}) W_{q,p} \hat{a}_p^\dagger \hat{a}_{p'}^\dagger \\
&\quad - \sum_{p,p'} \sum_{q'} V_{q',p'}(\omega_p + \omega_{q'}) W_{p,q'}^* \hat{a}_p \hat{a}_{p'} - \sum_{p,p'} \sum_{q'} V_{q',p}(\omega_{q'} + \omega_{p'}) W_{q',p}^* \hat{a}_{p'} \hat{a}_{p'} \\
&= - \sum_{p,p'} \sum_q V_{p,q}(\omega_q + \omega_{p'}) (W_{p',q} + W_{q,p'}) \hat{a}_p^\dagger \hat{a}_{p'}^\dagger \\
&\quad - \sum_{p,p'} \sum_q V_{q,p}(\omega_q + \omega_{p'}) (W_{p',q}^* + W_{q,p'}^*) \hat{a}_p \hat{a}_{p'} \tag{A.11}
\end{aligned}$$

A.8:

$$\begin{aligned}
& \left[\sum_{p,p'} \left(W_{p,p'}(\omega_p + \omega_{p'}) \hat{a}_p^\dagger \hat{a}_{p'}^\dagger - W_{p,p'}^*(\omega_p + \omega_{p'}) \hat{a}_p \hat{a}_{p'} \right), \sum_{\tilde{p},\tilde{p}'} \left(W_{\tilde{p},\tilde{p}'} \hat{a}_{\tilde{p}}^\dagger \hat{a}_{\tilde{p}'}^\dagger + W_{\tilde{p},\tilde{p}'}^* \hat{a}_{\tilde{p}} \hat{a}_{\tilde{p}'} \right) \right] \\
&= \sum_{p,p'} \sum_{\tilde{p},\tilde{p}'} W_{p,p'}(\omega_p + \omega_{p'}) W_{\tilde{p},\tilde{p}'}^* \left[\hat{a}_p^\dagger \hat{a}_{p'}^\dagger, \hat{a}_{\tilde{p}} \hat{a}_{\tilde{p}'} \right] - \sum_{p,p'} \sum_{\tilde{p},\tilde{p}'} W_{p,p'}^* W_{\tilde{p},\tilde{p}'}(\omega_p + \omega_{p'}) \left[\hat{a}_p \hat{a}_{p'}, \hat{a}_{\tilde{p}}^\dagger \hat{a}_{\tilde{p}'}^\dagger \right] \\
&= - \sum_{p,p'} \sum_{\tilde{p},\tilde{p}'} W_{p,p'}(\omega_p + \omega_{p'} + \omega_{\tilde{p}} + \omega_{\tilde{p}'}) W_{\tilde{p},\tilde{p}'}^* \left[\hat{a}_{\tilde{p}} \hat{a}_{\tilde{p}'}^\dagger, \hat{a}_p^\dagger \hat{a}_{p'}^\dagger \right] \\
&= - \sum_{p,p'} \sum_{\tilde{p},\tilde{p}'} W_{p,p'}(\omega_p + \omega_{p'} + \omega_{\tilde{p}} + \omega_{\tilde{p}'}) W_{\tilde{p},\tilde{p}'}^* \hat{a}_{\tilde{p}} \hat{a}_p^\dagger \delta_{\tilde{p}',p'} \\
&\quad - \sum_{p,p'} \sum_{\tilde{p},\tilde{p}'} W_{p,p'}(\omega_p + \omega_{p'} + \omega_{\tilde{p}} + \omega_{\tilde{p}'}) W_{\tilde{p},\tilde{p}'}^* \hat{a}_{\tilde{p}} \hat{a}_{p'}^\dagger \delta_{\tilde{p},p} \\
&\quad - \sum_{p,p'} \sum_{\tilde{p},\tilde{p}'} W_{p,p'}(\omega_p + \omega_{p'} + \omega_{\tilde{p}} + \omega_{\tilde{p}'}) W_{\tilde{p},\tilde{p}'}^* \hat{a}_p^\dagger \hat{a}_{\tilde{p}'} \delta_{\tilde{p},p'} \\
&\quad - \sum_{p,p'} \sum_{\tilde{p},\tilde{p}'} W_{p,p'}(\omega_p + \omega_{p'} + \omega_{\tilde{p}} + \omega_{\tilde{p}'}) W_{\tilde{p},\tilde{p}'}^* \hat{a}_{p'}^\dagger \hat{a}_{\tilde{p}} \delta_{\tilde{p},p} \\
&= - \sum_{p,p'} \sum_{\tilde{p}} W_{p,p'}(\omega_p + 2\omega_{p'} + \omega_{\tilde{p}}) W_{\tilde{p},p}^* \hat{a}_{\tilde{p}} \hat{a}_p^\dagger - \sum_{p,p'} \sum_{\tilde{p}} W_{p,p'}(2\omega_p + \omega_{p'} + \omega_{\tilde{p}}) W_{\tilde{p},p}^* \hat{a}_{\tilde{p}} \hat{a}_{p'}^\dagger \\
&\quad - \sum_{p,p'} \sum_{\tilde{p}'} W_{p,p'}(\omega_p + 2\omega_{p'} + \omega_{\tilde{p}'}) W_{\tilde{p}',p}^* \hat{a}_p^\dagger \hat{a}_{\tilde{p}'} - \sum_{p,p'} \sum_{\tilde{p}'} W_{p,p'}(2\omega_p + \omega_{p'} + \omega_{\tilde{p}'}) W_{\tilde{p}',p}^* \hat{a}_{p'}^\dagger \hat{a}_{\tilde{p}'} \\
&= - \sum_{p,p'} \sum_{\tilde{p}} W_{p,\tilde{p}}(\omega_p + 2\omega_{\tilde{p}} + \omega_{p'}) W_{p',\tilde{p}}^* \hat{a}_{p'}^\dagger \hat{a}_{\tilde{p}} - \sum_{p,p'} \sum_{\tilde{p}} W_{\tilde{p},p}(2\omega_{\tilde{p}} + \omega_p + \omega_{p'}) W_{p',\tilde{p}}^* \hat{a}_{p'}^\dagger \hat{a}_{\tilde{p}} \\
&\quad - \sum_{p,p'} \sum_{\tilde{p}'} W_{p,\tilde{p}'}(\omega_p + 2\omega_{\tilde{p}'} + \omega_{p'}) W_{\tilde{p}',p}^* \hat{a}_p^\dagger \hat{a}_{\tilde{p}'} - \sum_{p,p'} \sum_{\tilde{p}'} W_{\tilde{p}',p}(2\omega_{\tilde{p}'} + \omega_p + \omega_{p'}) W_{\tilde{p}',p}^* \hat{a}_{p'}^\dagger \hat{a}_{\tilde{p}'} \\
&= - \sum_{p,p'} \sum_{\tilde{p}} (W_{p,\tilde{p}} + W_{\tilde{p},p})(\omega_p + 2\omega_{\tilde{p}} + \omega_{p'}) W_{p',\tilde{p}}^* \hat{a}_{p'}^\dagger \hat{a}_{\tilde{p}}
\end{aligned}$$

$$\begin{aligned}
& - \sum_{p,p'} \sum_{\tilde{p}'} (W_{p,\tilde{p}'} + W_{\tilde{p}',p}) (\omega_p + 2\omega_{\tilde{p}'} + \omega_{p'}) W_{\tilde{p}',p'}^* \hat{a}_p^\dagger \hat{a}_{p'} \\
& = - \sum_{p,p'} \sum_{\tilde{p}} (W_{p,\tilde{p}} + W_{\tilde{p},p}) (\omega_p + 2\omega_{\tilde{p}} + \omega_{p'}) W_{p',\tilde{p}}^* (\delta_{p,p'} + \hat{a}_p^\dagger \hat{a}_{p'}) \\
& - \sum_{p,p'} \sum_{\tilde{p}} (W_{p,\tilde{p}} + W_{\tilde{p},p}) (\omega_p + 2\omega_{\tilde{p}} + \omega_{p'}) W_{\tilde{p},p'}^* \hat{a}_p^\dagger \hat{a}_{p'} \\
& = - \sum_{p,p'} \sum_{\tilde{p}} (W_{p,\tilde{p}} + W_{\tilde{p},p}) (\omega_p + 2\omega_{\tilde{p}} + \omega_{p'}) (W_{\tilde{p},p'}^* + W_{p',\tilde{p}}^*) \hat{a}_p^\dagger \hat{a}_{p'} \\
& - 2 \sum_k \sum_{\tilde{p}} (W_{k,\tilde{p}} + W_{\tilde{p},k}) (\omega_k + \omega_{\tilde{p}}) W_{k,\tilde{p}}^* \\
& = - \sum_{q \neq q'} \sum_{\tilde{p}} (W_{q,\tilde{p}} + W_{\tilde{p},q}) (\omega_q + 2\omega_{\tilde{p}} + \omega_{q'}) (W_{\tilde{p},q'}^* + W_{q',\tilde{p}}^*) \hat{a}_q^\dagger \hat{a}_{q'} \\
& - 2 \sum_k \sum_{\tilde{p}} (W_{k,\tilde{p}} + W_{\tilde{p},k}) (\omega_k + \omega_{\tilde{p}}) (W_{\tilde{p},k}^* + W_{k,\tilde{p}}^*) \hat{a}_k^\dagger \hat{a}_k \\
& - 2 \sum_k \sum_{\tilde{p}} (W_{k,\tilde{p}} + W_{\tilde{p},k}) (\omega_k + \omega_{\tilde{p}}) W_{k,\tilde{p}}^* \tag{A.12}
\end{aligned}$$

Using the expressions for the commutators of the generator and $\hat{\mathcal{H}}_0$ respectively $\hat{\mathcal{H}}_{\text{int}}$ derived above, the flow $\partial_\lambda \hat{\mathcal{H}}(\lambda) = [\hat{\eta}(\lambda), \hat{\mathcal{H}}(\lambda)]$ yields the following flow equations $\forall k, p, p', q, q'$ where $q \neq q'$:

$$\partial_\lambda \omega_k = \sum_{\tilde{q}} 2V_{\tilde{q},k} V_{k,\tilde{q}} (\omega_k - \omega_{\tilde{q}}) - 2 \sum_{\tilde{p}} (W_{k,\tilde{p}} + W_{\tilde{p},k}) (\omega_k + \omega_{\tilde{p}}) (W_{\tilde{p},k}^* + W_{k,\tilde{p}}^*) \tag{A.13a}$$

$$\begin{aligned}
\partial_\lambda V_{q,q'} &= -V_{q,q'} (\omega_q - \omega_{q'})^2 - \sum_{\tilde{p}} (W_{q,\tilde{p}} + W_{\tilde{p},q}) (\omega_q + 2\omega_{\tilde{p}} + \omega_{q'}) (W_{\tilde{p},q'}^* + W_{q',\tilde{p}}^*) \\
&+ \sum_{\tilde{q}} V_{\tilde{q},q'} V_{q,\tilde{q}} (\omega_q - \omega_{\tilde{q}}) - \sum_{\tilde{q}} V_{q,\tilde{q}} V_{\tilde{q},q'} (\omega_{\tilde{q}} - \omega_{q'}) \tag{A.13b}
\end{aligned}$$

$$\begin{aligned}
\partial_\lambda W_{p,p'} &= -W_{p,p'} (\omega_p + \omega_{p'})^2 - \sum_q V_{p,q} (\omega_q + \omega_{p'}) (W_{p',q} + W_{q,p'}) \\
&+ \sum_q V_{p,q} (\omega_p - \omega_q) (W_{q,p'} + W_{p',q}) \tag{A.13c}
\end{aligned}$$

$$\begin{aligned}
\partial_\lambda W_{p,p'}^* &= -W_{p,p'}^* (\omega_p + \omega_{p'})^2 - \sum_q V_{q,p} (\omega_q + \omega_{p'}) (W_{p',q}^* + W_{q,p'}^*) \\
&+ \sum_q V_{q,p} (\omega_p - \omega_q) (W_{q,p'}^* + W_{p',q}^*) \tag{A.13d}
\end{aligned}$$

$$\partial_\lambda \varepsilon = -2 \sum_{p,p'} (W_{p,p'} + W_{p',p}) (\omega_p + \omega_{p'}) W_{p,p'}^* \tag{A.13e}$$

Obviously, equations A.13c and A.13d are not independent from each other, since they are related by complex conjugation. It is nevertheless a good consistency check to see that the two independently derived equations are equivalent.

APPENDIX B

THE SECOND APPENDIX

Here comes the second appendix.

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DECLARATION OF AUTHORSHIP

I hereby declare that I have written this thesis independently and by myself and that I have not used any sources or auxiliary materials other than those indicated in the thesis.

Munich, 22.06.2023

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