构造勒让德多项式，并存储在P中

```
0 1
1 x
2 3*x**2/2 - 1/2
3 x*(5*x**2 - 3)/2
4 35*x**4/8 - 15*x**2/4 + 3/8
5 x*(63*x**4 - 70*x**2 + 15)/8
6 231*x**6/16 - 315*x**4/16 + 105*x**2/16 - 5/16
7 x*(429*x**6 - 693*x**4 + 315*x**2 - 35)/16
8 6435*x**8/128 - 3003*x**6/32 + 3465*x**4/64 - 315*x**2/32 + 35/128
9 x*(12155*x**8 - 25740*x**6 + 18018*x**4 - 4620*x**2 + 315)/128
10 46189*x**10/256 - 109395*x**8/256 + 45045*x**6/128 - 15015*x**4/128 + 3465*x**2/256 - 63/2
11 x*(88179*x**10 - 230945*x**8 + 218790*x**6 - 90090*x**4 + 15015*x**2 - 693)/256
12 676039*x**12/1024 - 969969*x**10/512 + 2078505*x**8/1024 - 255255*x**6/256 + 225225*x**4/1
13 x*(1300075*x**12 - 4056234*x**10 + 4849845*x**8 - 2771340*x**6 + 765765*x**4 - 90090*x**2
14 5014575*x**14/2048 - 16900975*x**12/2048 + 22309287*x**10/2048 - 14549535*x**8/2048 + 4849
15 x*(9694845*x**14 - 35102025*x**12 + 50702925*x**10 - 37182145*x**8 + 14549535*x**6 - 29099
16 300540195*x**16/32768 - 145422675*x**14/4096 + 456326325*x**12/8192 - 185910725*x**10/4096
17 x*(583401555*x**16 - 2404321560*x**14 + 4071834900*x**12 - 3650610600*x**10 + 1859107250*x
18 2268783825*x**18/65536 - 9917826435*x**16/65536 + 4508102925*x**14/16384 - 4411154475*x**1
19 x*(4418157975*x**18 - 20419054425*x**16 + 39671305740*x**14 - 42075627300*x**12 + 26466926
20 34461632205*x**20/262144 - 83945001525*x**18/131072 + 347123925225*x**16/262144 - 49589132
21 x*(67282234305*x**20 - 344616322050*x**18 + 755505013725*x**16 - 925663800600*x**14 + 6942
22 263012370465*x**22/524288 - 1412926920405*x**20/524288 + 3273855059475*x**18/524288 - 4281
23 x*(514589420475*x**22 - 2893136075115*x**20 + 7064634602025*x**18 - 9821565178425*x**16 +
24 8061900920775*x**24/4194304 - 11835556670925*x**22/1048576 + 60755857577415*x**20/2097152
25 x*(15801325804719*x**24 - 96742811049300*x**22 + 260382246760350*x**20 - 405039050516100*x
26 61989816618513*x**26/8388608 - 395033145117975*x**24/8388608 + 556271163533475*x**22/41943
27 x*(121683714103007*x**26 - 805867616040669*x**24 + 2370198870707850*x**22 - 40793218659121
28 956086325095055*x**28/33554432 - 3285460280781189*x**26/16777216 + 20146690401016725*x**24
29 x*(1879204156221315*x**28 - 13385208551330770*x**26 + 42710983650155457*x**24 - 8058676160
30 7391536347803839*x**30/67108864 - 54496920530418135*x**28/67108864 + 180700315442965395*x*
31 x*(14544636039226909*x**30 - 110873045217057585*x**28 + 381478443712926945*x**26 - 7830347
32 916312070471295267*x**32/2147483648 - 4508837172160341 79*x**30/134217728 + 321531831129466
33 x*(1804857108504066435*x**32 - 14660993127540724272*x**30 + 54106046065924101480*x**28 - 1
34 7113260368810144185*x**34/4294967296 - 59560284580634192355*x**32/4294967296 + 28405674184
35 x*(14023284727082855679*x**34 - 120925426269772451145*x**32 + 476482276645073538840*x**30
36 1106281350692091948 01*x**36/17179869184 - 4908149654478999487 65*x**34/8589934592 + 3990539
37 x*(2182663205419532762 29*x**36 - 1991306431245765506418*x**34 + 8343854412614299129005*x**
38 8615775810866576693 25*x**38/34359738368 - 8075853860052271220473*x**36/34359738368 + 34847
39 x*(1701063429324939500975*x**38 - 16369974040646495717175*x**36 + 7268268474047044098425 7*
40 2687680218333404411 5405*x**40/274877906944 - 6634147374367264053 8025*x**38/68719476736 + 6
```

构造目标函数

计算勒让德多项式的模长

```
[2, 2/3, 2/5, 2/7, 2/9, 2/11, 2/13, 2/15, 2/17, 2/19, 2/21, 2/23, 2/25, 2/27, 2/29, 2/31, 2/3
```

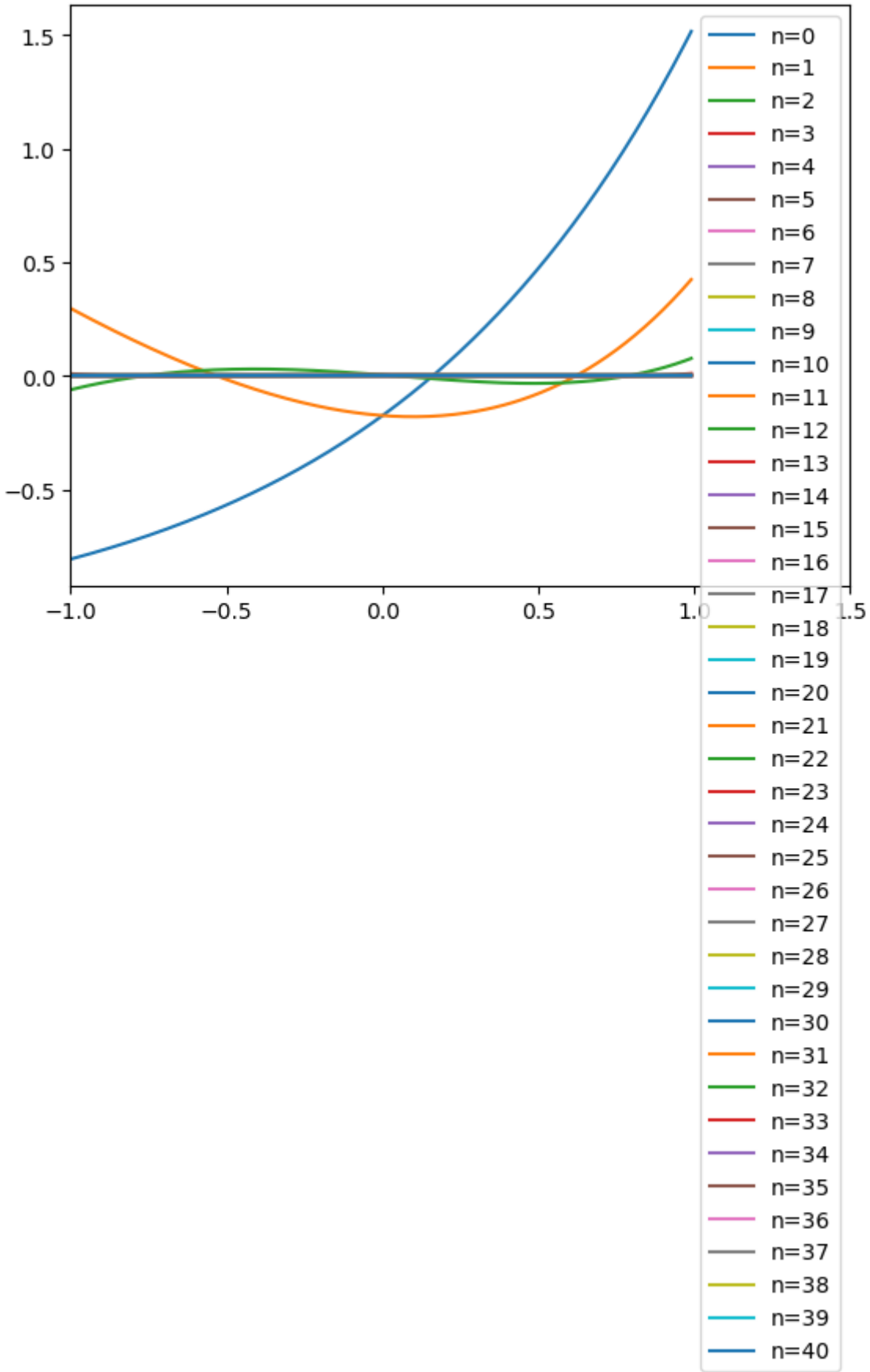
计算f在以勒让德多项式为基的空间中的分量

```
[-exp(-1)/2 + E/2, 3*exp(-1), -35*exp(-1)/2 + 5*E/2, -35*E/2 + 259*exp(-1)/2, -1197*exp(-1) +
```

由上面计算得出的系数构造最佳平方逼近多项式，以及其误差

```
0 sinh(1) exp(x) - sinh(1)
1 (6*x - 1 + exp(2))*exp(-1)/2 (-6*x + 2*exp(x + 1) - exp(2) + 1)*exp(-1)/2
2 (12*x + 5*(-7 + exp(2))*(3*x**2 - 1) - 2 + 2*exp(2))*exp(-1)/4 (-12*x + 5*(7 - exp(2))*(3*x
3 (-7*x*(-37 + 5*exp(2))*(5*x**2 - 3) + 12*x + 5*(-7 + exp(2))*(3*x**2 - 1) - 2 + 2*exp(2))*e
4 5*(-8379*x**4 + 1134*x**4*exp(2) - 70*x**3*exp(2) + 518*x**3 - 966*x**2*exp(2) + 7140*x**2
5 3*(-75999*x**5*exp(2) + 561561*x**5 - 27930*x**4 + 3780*x**4*exp(2) - 622230*x**3 + 84210*x
6 7*(-11586003*x**6 + 1567995*x**6*exp(2) - 65142*x**5*exp(2) + 481338*x**5 - 2134935*x**4*ex
7 (-307876140*x**7*exp(2) + 2274914070*x**7 - 81102021*x**6 + 10975965*x**6*exp(2) - 36714918
8 9*(-64784168735*x**8 + 8767583825*x**8*exp(2) - 273667680*x**7*exp(2) + 2022145840*x**7 - 1
9 5*(-568595781611*x**9*exp(2) + 4201386127939*x**9 - 116611503723*x**8 + 15781650885*x**8*ex
10 11*(-76432856441487*x**10 + 10344062275092*x**10*exp(2) - 258452628005*x**9*exp(2) + 19097
11 3*(-3339110227201385*x**11*exp(2) + 24672872789304103*x**11 - 560507613904238*x**10 + 7585
12 13*(-1093553931960443275*x**12 + 147996431116378075*x**12*exp(2) - 3082255594339740*x**11*
13 7*(-1429585255439097300 0*x**13*exp(2) + 1056328565064359341 50*x**13 - 2030885873640823225*
14 15*(-5522189897528575068195*x**14 + 747347133868390924245*x**14*exp(2) - 13342795717431574
15 (-6727218714441099398780 55*x**15*exp(2) + 49707796470781413799606 95*x**15 - 828328484629280
16 17*(-14972814056547383423513 3955*x**16 + 20263500311919759776786850*x**16*exp(2) - 3165749
17 9*(-52060434706575615255880 96425*x**17*exp(2) + 38467747258160344599395849775*x**17 - 5656
18 19*(-26241484805726514241908048 30825*x**18 + 35513987787322661847105930 8625*x**18*exp(2) -
19 5*(-10257244581450961115686796513 3820*x**19*exp(2) + 75791355632793588920311815008373 0*x**
20 21*(-57749665678688410512440101969942005*x**20 + 78155673614449869577342343340893 55*x**20*
21 11*(-12534051326612499245916195496689225 75*x**21*exp(2) + 92614808399215894951287229113929
22 23*(-38980704600952429738766094617847779947 5*x**22 + 52754646979326285957896912763121501800
23 3*(-74422471832959897165714520551762908425625*x**23*exp(2) + 54991181939482685586417001018
24 25*(-50681816152703714881057575555678929399150319*x**24 + 68590379439720894185472492756308
25 13*(-13190903917131287790977123664058424918229249 12*x**25*exp(2) + 97468329039287146446365
26 27*(-97615656512880872375842448984954833393791752048 9*x**26 + 13210842522498629266784287193
27 7*(-55034067491134338279437264916150569492959100995195*x**27*exp(2) + 40664981204432721061
28 29*(-21987661024986304069795723896931512626899095068157015*x**28 + 29757063325271511959846
29 15*(-13347321897786750210602805030452476162363046216630916 71*x**29*exp(2) + 98624110273231
30 31*(-11453344655787091703847950555993137881272067184871990445 79*x**30 + 155004164299749035
31 (-59584646713959100898821474161851006889513589306060450417494 0*x**31*exp(2) + 44027429720
32 33*(-54648250348960717888264861695944818583784715433565477885107269 5*x**32 + 7395836439361
33 17*(-18951018271289433382314200910433461295878111286214811211445196975*x**33*exp(2) + 1400
34 35*(-92501133583689011875989195909053624518956761389709287919004386802 27*x**34 + 125186671
35 9*(-13631602931854263574556242330273401035681079716092105753788395578492 79*x**35*exp(2) +
36 37*(-14112501375033707036911264167846043309552313193425038847672536475631385 33*x**36 + 190
37 19*(-55046450861910695990224942817451095158205401869438144920984062580456785800*x**37*exp(
38 39*(-60240000795636506420876984529477148377674275604083355878909766098210856093825*x**38 +
39 5*(-99201673704584192180949576475261364211928841810205451999058375950765984125587 45*x**39*
40 41*(-57210744272053449268803787789364842847740752268613440343273399621521133711734303 65*x*
```

画出图像



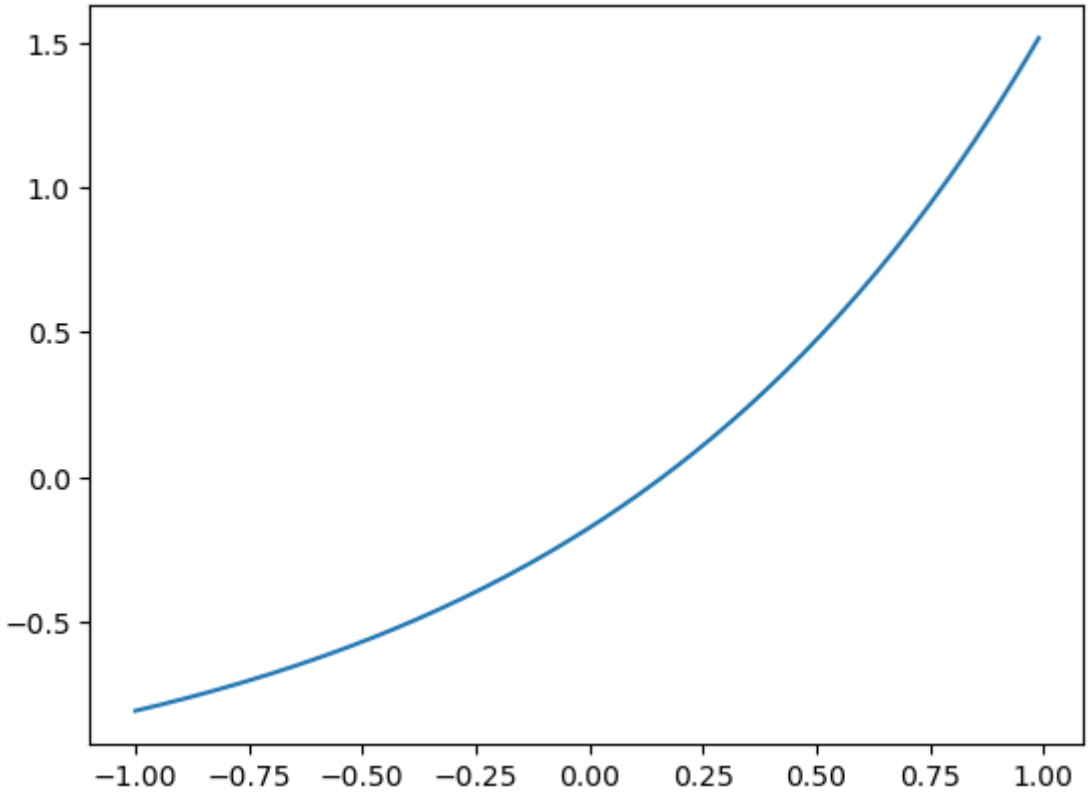
上面的计算得到的都是解析解，现在求数值解

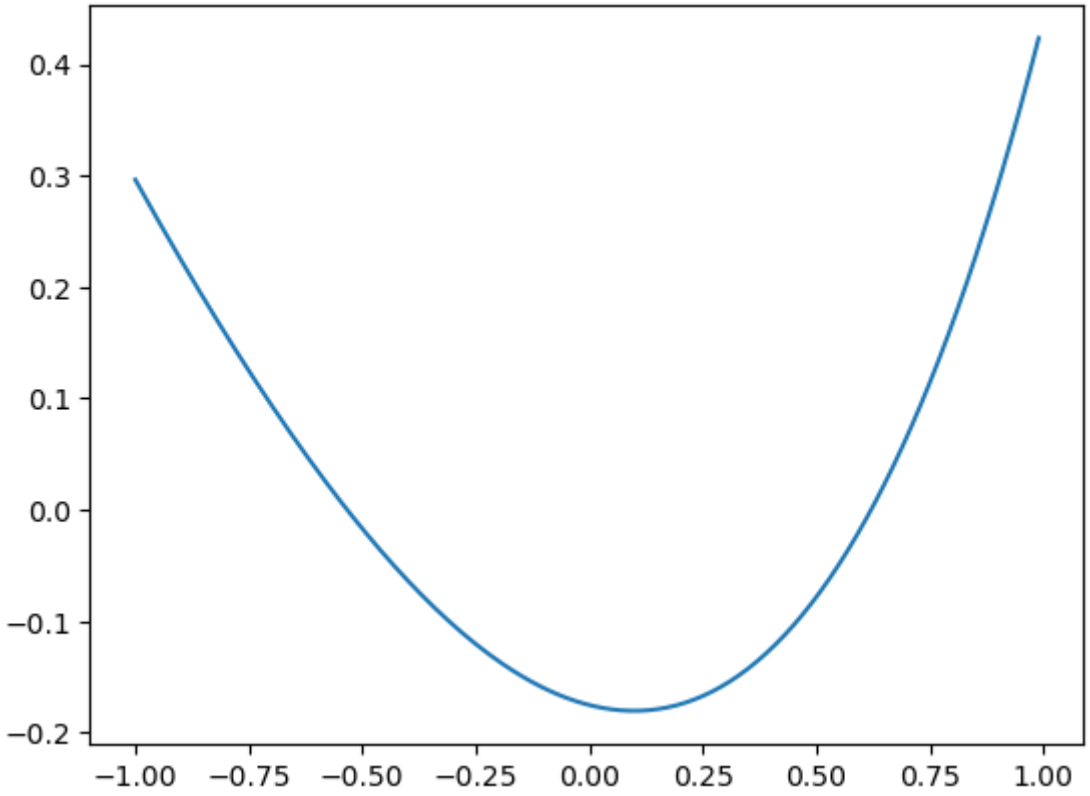
```
0 1.17520119364380 exp(x) - 1.1752011936438
1 1.10363832351433*x + 1.1752011936438 -1.10363832351433*x + 0.367879441171442*exp(x + 1) - 1
2 0.536721525971059*x**2 + 1.10363832351433*x + 0.996294018320115 -0.536721525971059*x**2 - 1
```

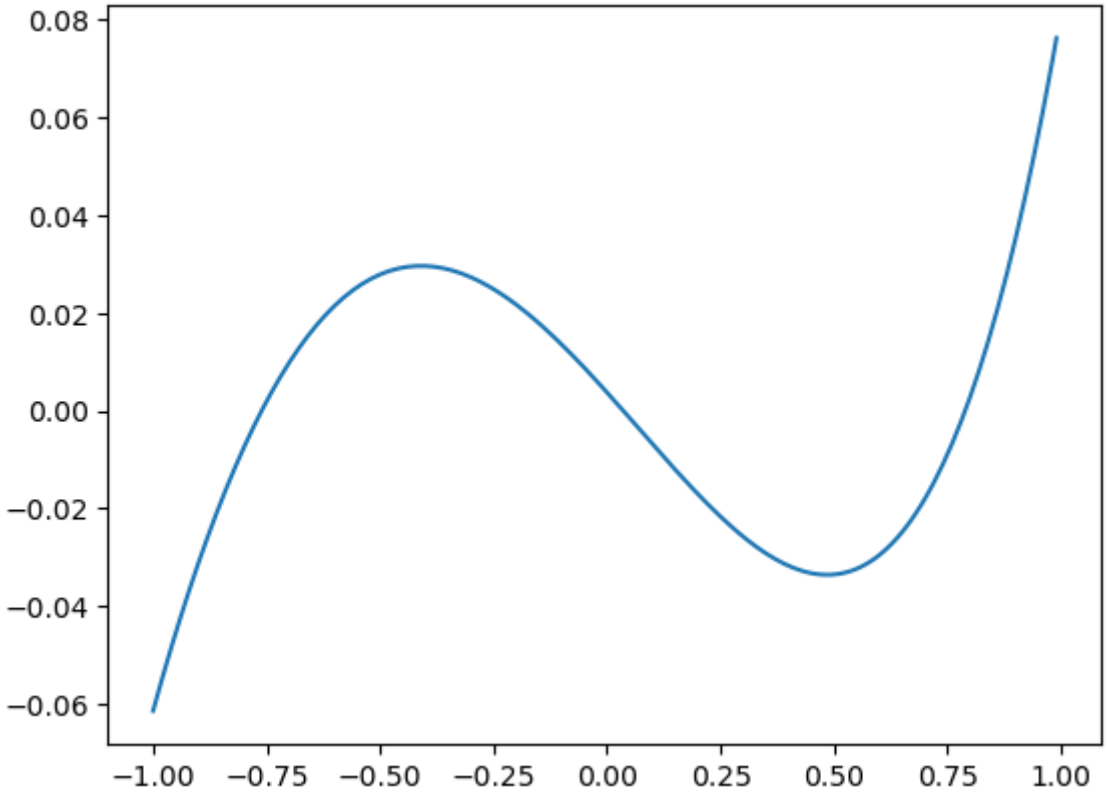
```
3 0.176139084171224*x**3 + 0.536721525971059*x**2 + 0.997954873011593*x + 0.996294018320115 -
4 0.0435974356513095*x**4 + 0.176139084171224*x**3 + 0.499352295412685*x**2 + 0.9979548730115
5 0.00865924075198658*x**5 + 0.0435974356513931*x**4 + 0.166517705557217*x**3 + 0.49935229541
6 0.00143587210097838*x**6 + 0.00865924074997907*x**5 + 0.0416394282894828*x**4 + 0.166517705
7 0.000204324281376349*x**7 + 0.00143587205815158*x**6 + 0.00832917602520447*x**5 + 0.0416394
8 2.54576224803951e-5*x**8 + 0.000204323938761921*x**7 + 0.00138832848177969*x**6 + 0.0083291
9 3.50837174579089e-6*x**9 + 2.54356951569839e-5*x**8 + 0.00019646881776429*x**7 + 0.00138821
10 3.85920892036997e-6*x**9 + 0.000200678863859239*x**7 + 0.00148193622542207*x**6 + 0.008335
11 0.0172443488049114*x**9 + 0.00134721475038 37*x**6 + 0.00862217440245568*x**5 + 0.041628935
12 0.298902045951797*x**12 + 0.0186813778719873*x**9 + 0.00934068893599365*x**5 + 0.166964814
13 -10.3006243528004*x**5 + 0.502960173476581*x**2 + 1.00592034695316*x + 0.999947694893128 2
14 -1412.65705409834*x**14 - 2825.31410819668*x**12 - 176.582131762292*x**4 + 1.3795479043929
15 192874.776452893*x**13 + 3013.66838207646*x**12 + 24109.3470566116*x**5 + 47.0885684699446
16 x**4*(3278871.19969918*x**12 - 13115484.7987967*x**10 + 204929.449981199*x**9 - 26230969.5
17 x**2*(-27773967.8092166*x**8 + 1735872.98807604*x**4 - 55547935.6184332*x**3 + 216984.1235
18 x**4*(-938142912.666872*x**11 - 1876285825.33374*x**9 + 1876285825.33374*x**7 + 234535728.
19 x*(16179508927509.5*x**12 - 126402413496.168*x**11 - 8089754463754.76*x**10 + 404487723187
20 x*(-543631499964320.0*x**17 + 4247121093471.25*x**8 - 1061780273367.81*x**6 + 132722534170
21 142379678562084.0*x**20 - 569518714248335.0*x**18 + 1.45796790847574e+17*x**15 + 284759357
22 x**5*(-4.87756536653701e+18*x**15 + 7.62119588521408e+16*x**14 - 1.9510261466148e+19*x**11
23 x**5*(6.51473078521813e+20*x**16 - 1.30294615704363e+21*x**12 + 6.51473078521813e+20*x**8 -
24 -1.35723558025378e+21*x**19 - 8.68630771362417e+22*x**14 + 6.78617790126889e+20*x**13 - 1.
25 x**6*(-7.22700801773531e+23*x**19 + 1.15632128283765e+25*x**15 + 4.51688001108457e+22*x**6
26 x**2*(-7.50496986457129e+23*x**23 - 7.685089141321e+26*x**22 + 6.00397589165703e+24*x**21 -
27 x*(2.04024885055514e+29*x**16 + 7.96972207248103e+26*x**13 + 5.10062212638786e+28*x**12 + 
28 x**4*(5.28278720233029e+28*x**21 + 2.70478704759311e+31*x**20 - 4.22622976186423e+29*x**15
29 x**2*(8.74392364523633e+29*x**26 + 8.95377781272201e+32*x**25 - 6.99513891618907e+30*x**24
30 x**3*(1.48035793170337e+34*x**27 - 1.1842863453627e+35*x**25 + 7.40178965851686e+33*x**20 -
31 -6.25914434916853e+37*x**27 + 9.77991304557582e+35*x**22 - 4.88995652278791e+35*x**18 - 6.
32 x**7*(-8.26207054090246e+39*x**21 - 6.45474261008004e+37*x**20 - 1.65241410818049e+40*x**1
33 x*(1.70248726297384e+40*x**25 - 4.35836739321303e+42*x**24 + 1.70248726297384e+40*x**23 +
34 x**8*(-1.121638667371e+42*x**21 + 5.74278997693952e+44*x**18 + 1.121638667371e+42*x**11 + 
35 2.95343484528318e+44*x**28 + 7.56079320392495e+46*x**25 + 3.78039660196247e+46*x**21 - 2.9
36 x**3*(-9.94664350383015e+48*x**25 - 7.77081523736731e+46*x**24 - 9.94664350383015e+48*x**1
37 x*(2.55386792665909e+48*x**31 - 1.02154717066364e+49*x**29 - 2.55386792665909e+48*x**17 + 
38 2.6839807768173e+51*x**38 - 8.38743992755407e+49*x**35 + 8.58873848581537e+52*x**34 - 3.43
39 x**6*(-1.12754720634294e+55*x**29 - 1.76179250991085e+53*x**26 - 3.52358501982169e+53*x**2
40 x**4*(-3.69835483680485e+56*x**34 - 2.95868386944388e+57*x**32 + 1.183473547777 55e+58*x**2
```

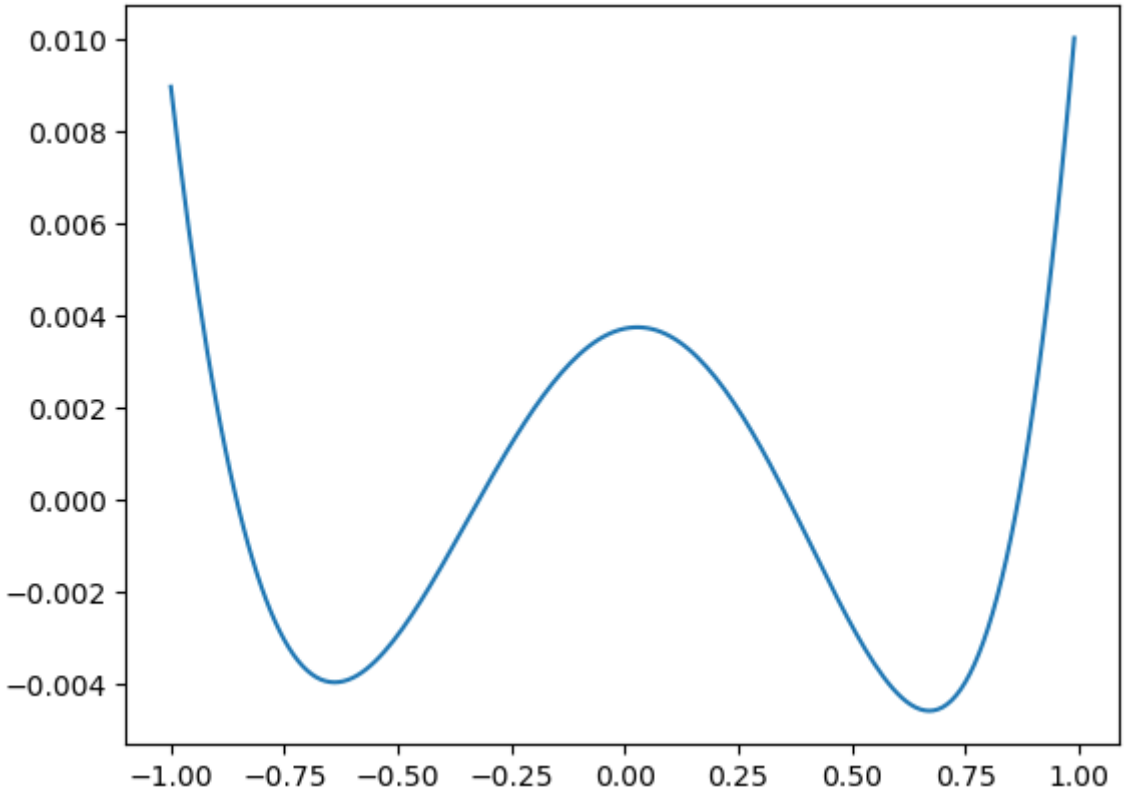
将数值解以latex公式形式给出

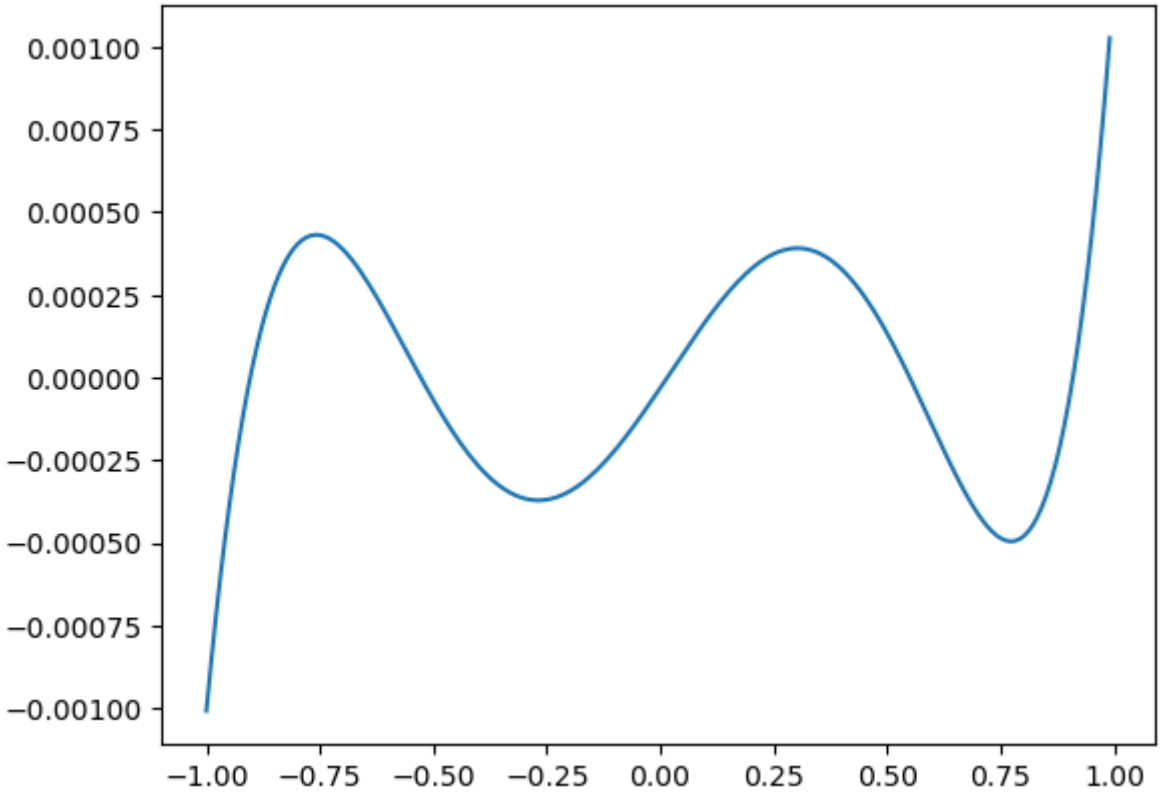
画出误差图像并计算误差极值

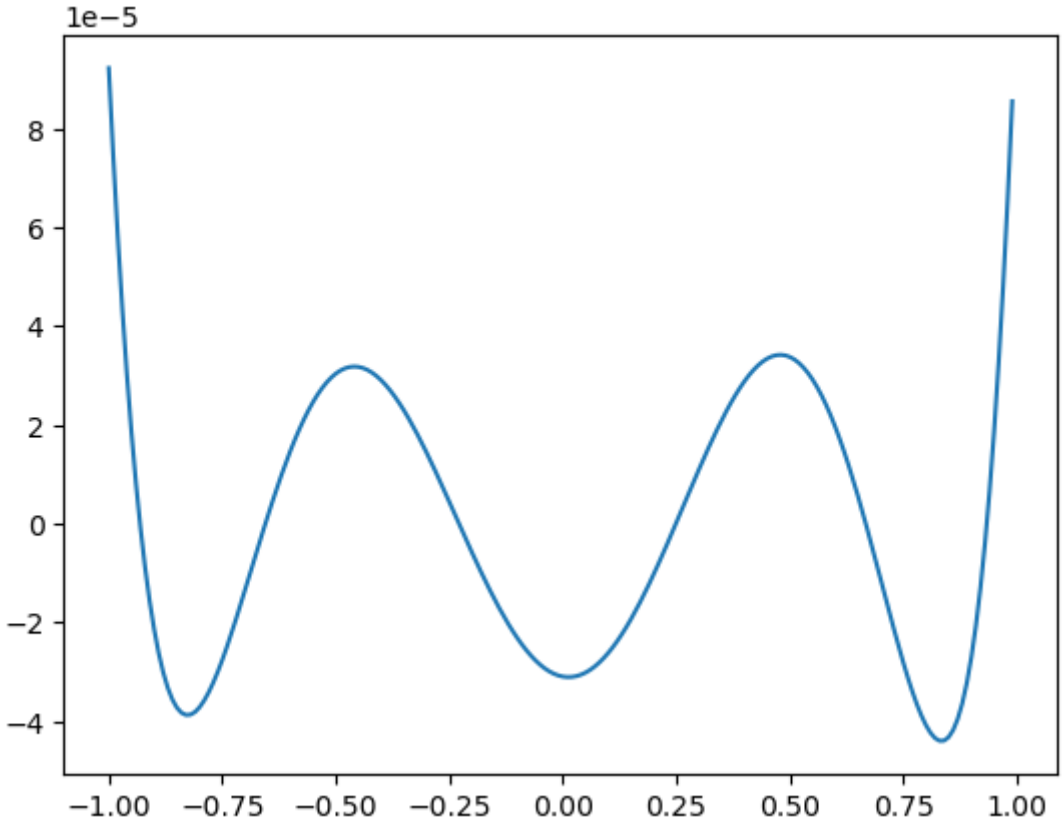

1.5
1.0
0.5
0.0
−0.5
−1.00
−0.75
−0.50
−0.25
0.00
0.25
0.50
0.75
1.00

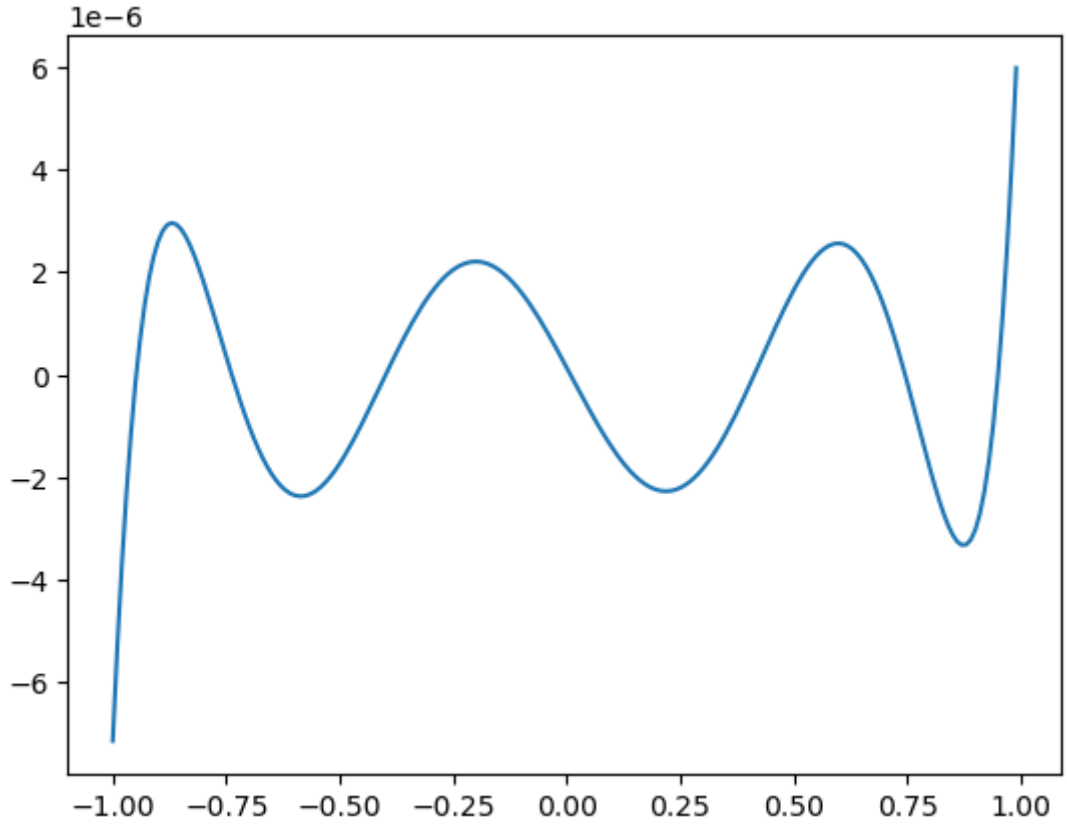

0.4
0.3
0.2
0.1
0.0
−0.1
−0.2
−1.00
−0.75
−0.50
−0.25
0.00
0.25
0.50
0.75
1.00

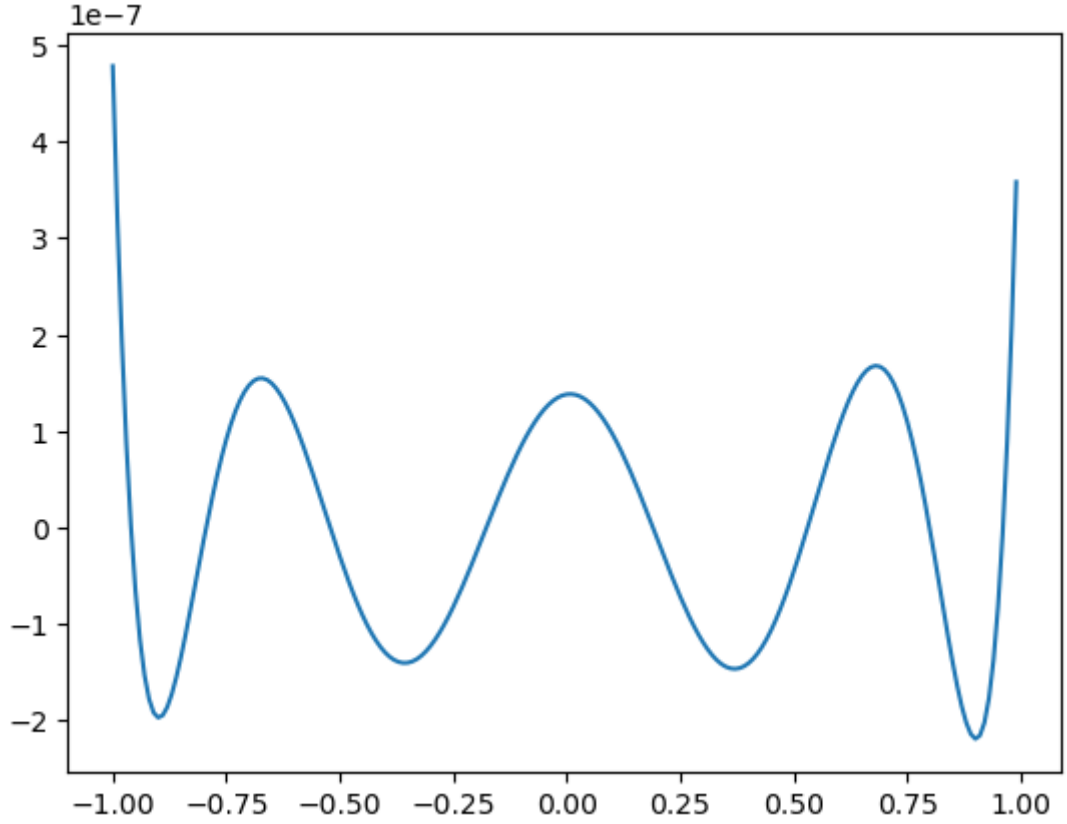

0.08
0.06
0.04
0.02
0.00
−0.02
−0.04
−0.06
−1.00
−0.75
−0.50
−0.25
0.00
0.25
0.50
0.75
1.00

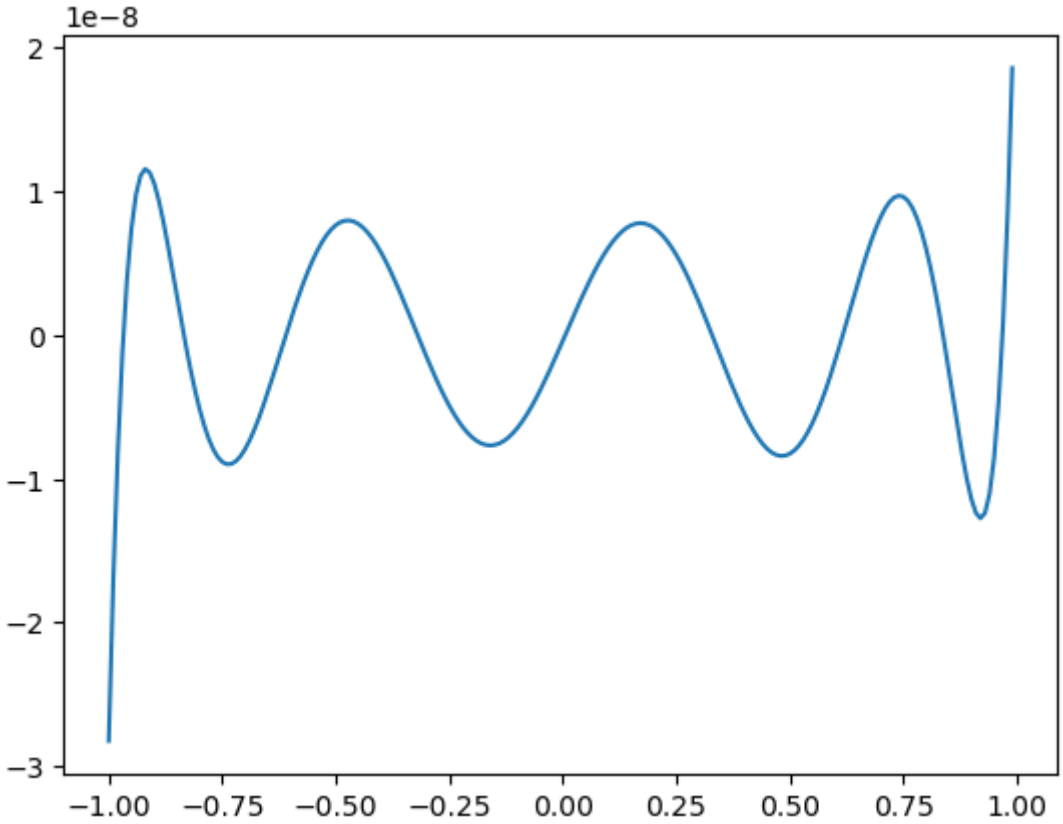

0.010
0.008
0.006
0.004
0.002
0.000
−0.002
−0.004
−1.00
−0.75
−0.50
−0.25
0.00
0.25
0.50
0.75
1.00

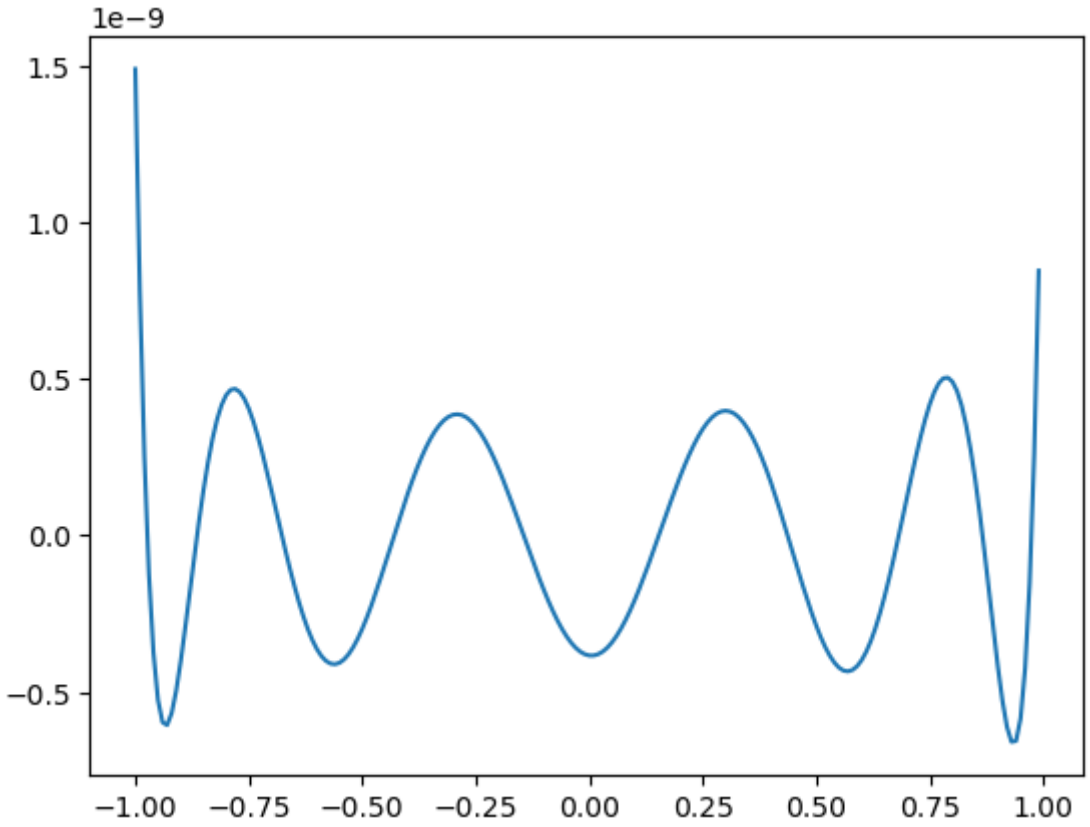

0.00100
0.00075
0.00050
0.00025
0.00000
−0.00025
−0.00050
−0.00075
−0.00100
−1.00
−0.75
−0.50
−0.25
0.00
0.25
0.50
0.75
1.00


1e−5
8
6
4
2
0
−2
−4
−1.00
−0.75
−0.50
−0.25
0.00
0.25
0.50
0.75
1.00


1e−6
6
4
2
0
−2
−4
−6
−1.00
−0.75
−0.50
−0.25
0.00
0.25
0.50
0.75
1.00


1e−7
5
4
3
2
1
0
−1
−2
−1.00
−0.75
−0.50
−0.25
0.00
0.25
0.50
0.75
1.00


1e−8
2
1
0
−1
−2
−3
−1.00
−0.75
−0.50
−0.25
0.00
0.25
0.50
0.75
1.00


1e−9
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1.0
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−0.25
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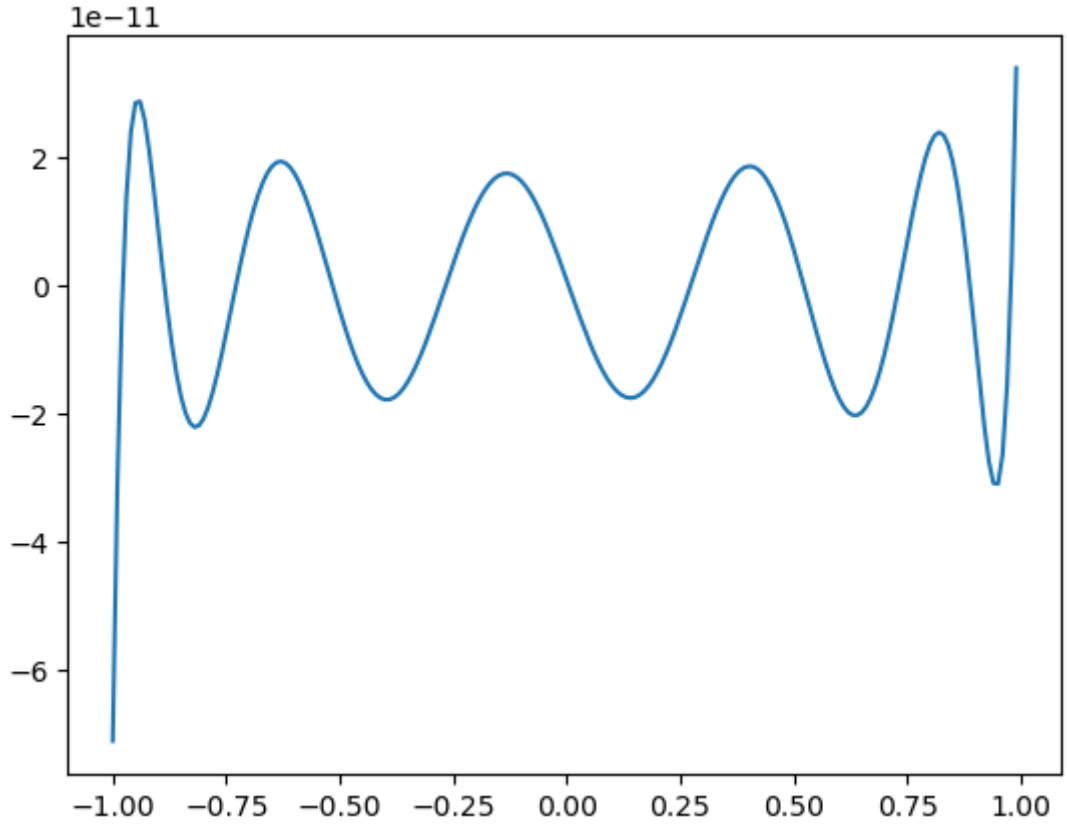
0.25
0.50
0.75
1.00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0
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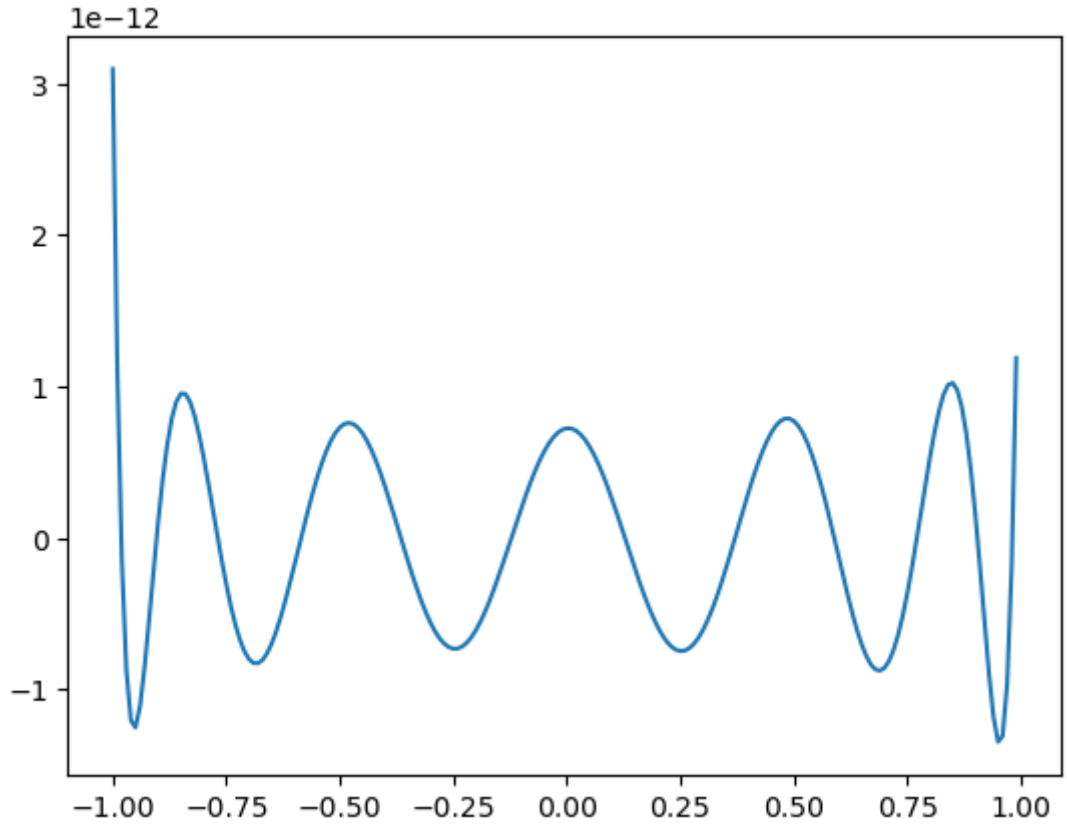
−4
−6
−1.00
−0.75
−0.50
−0.25
0.00
0.25
0.50
0.75
1.00


1e−12
3
2
1
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−1.00
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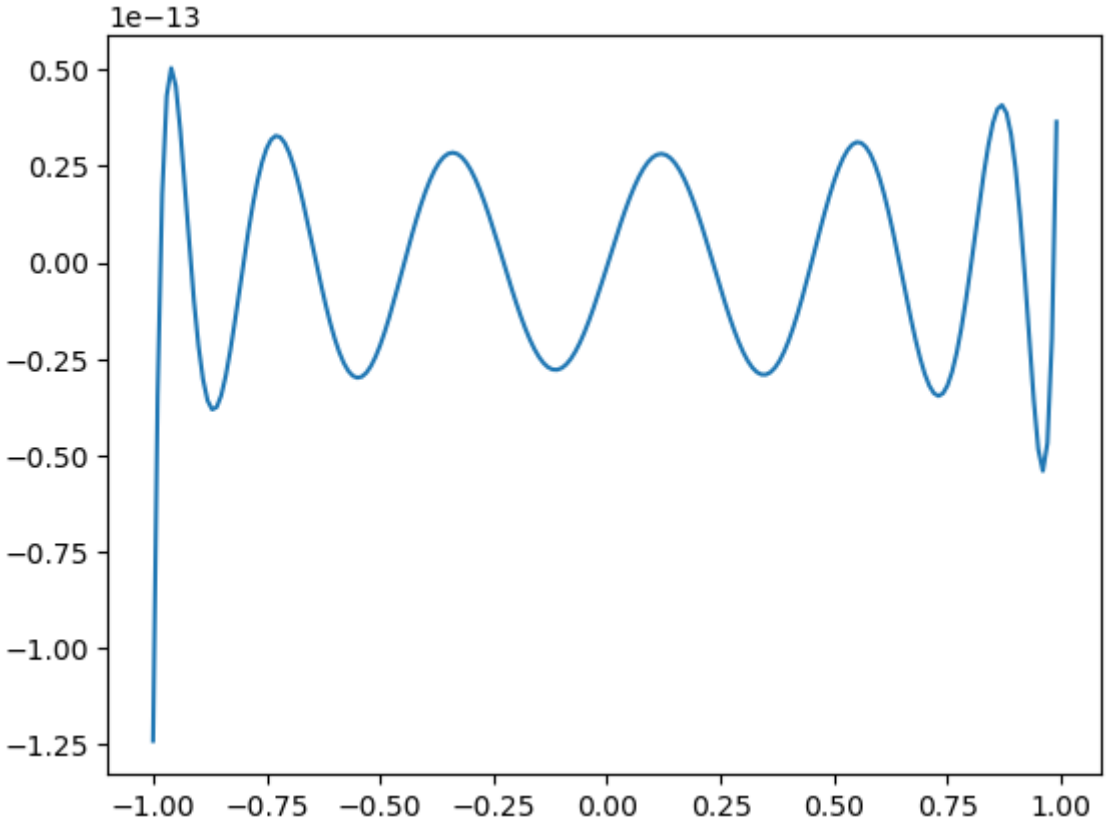
−0.50
−0.25
0.00
0.25
0.50
0.75
1.00


1e−13
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0.25
0.00
−0.25
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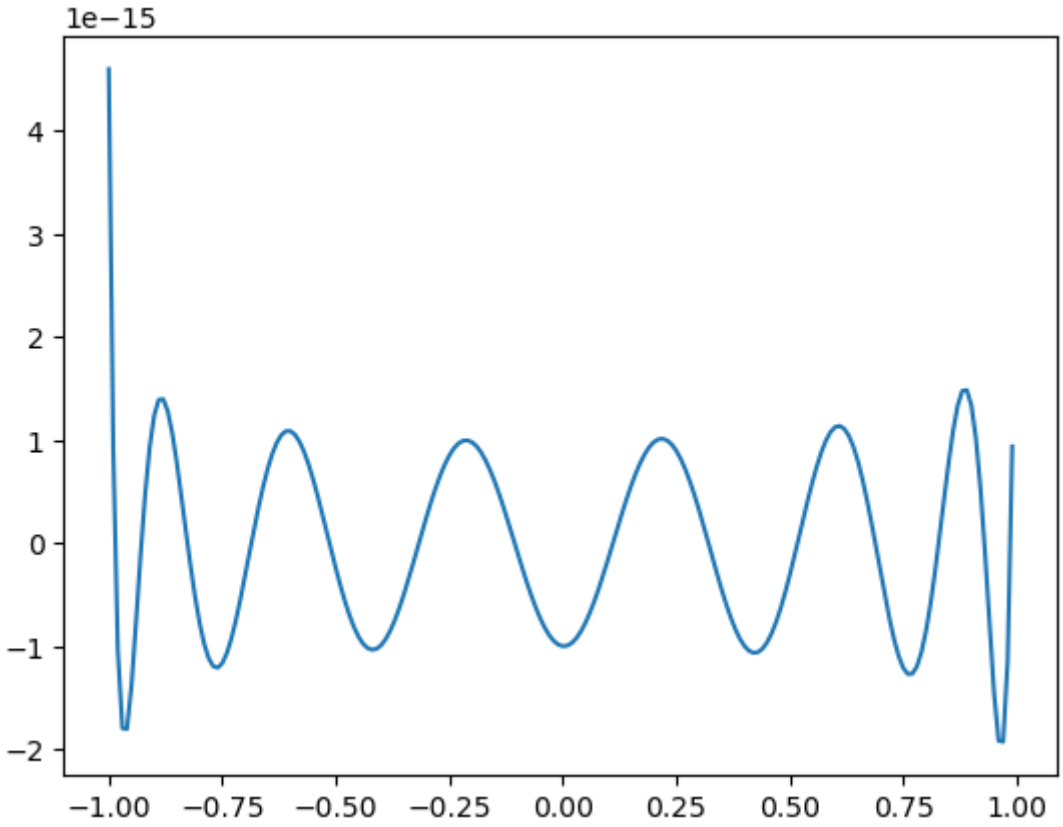
−0.75
−1.00
−1.25
−1.00
−0.75
−0.50
−0.25
0.00
0.25
0.50
0.75
1.00


1e−15
4
3
2
1
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−2
−1.00
−0.75
−0.50
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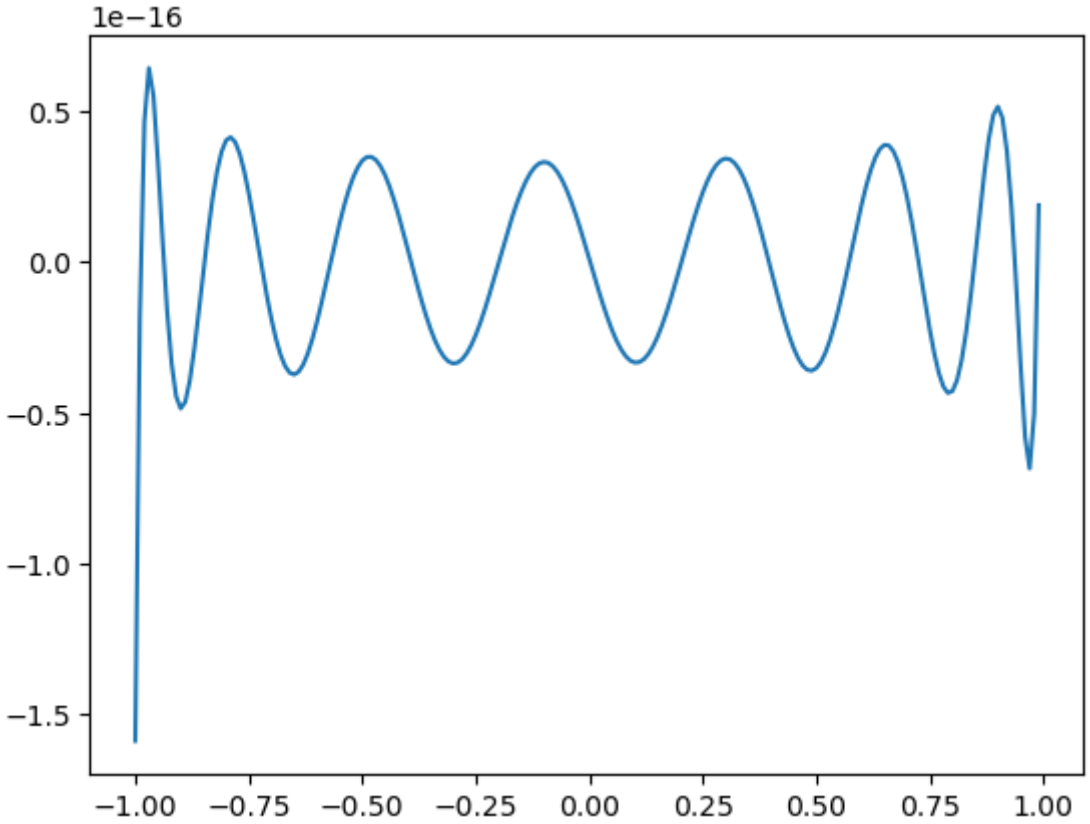
0.50
0.75
1.00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0.0
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−1.0
−1.5
−1.00
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−0.50
−0.25
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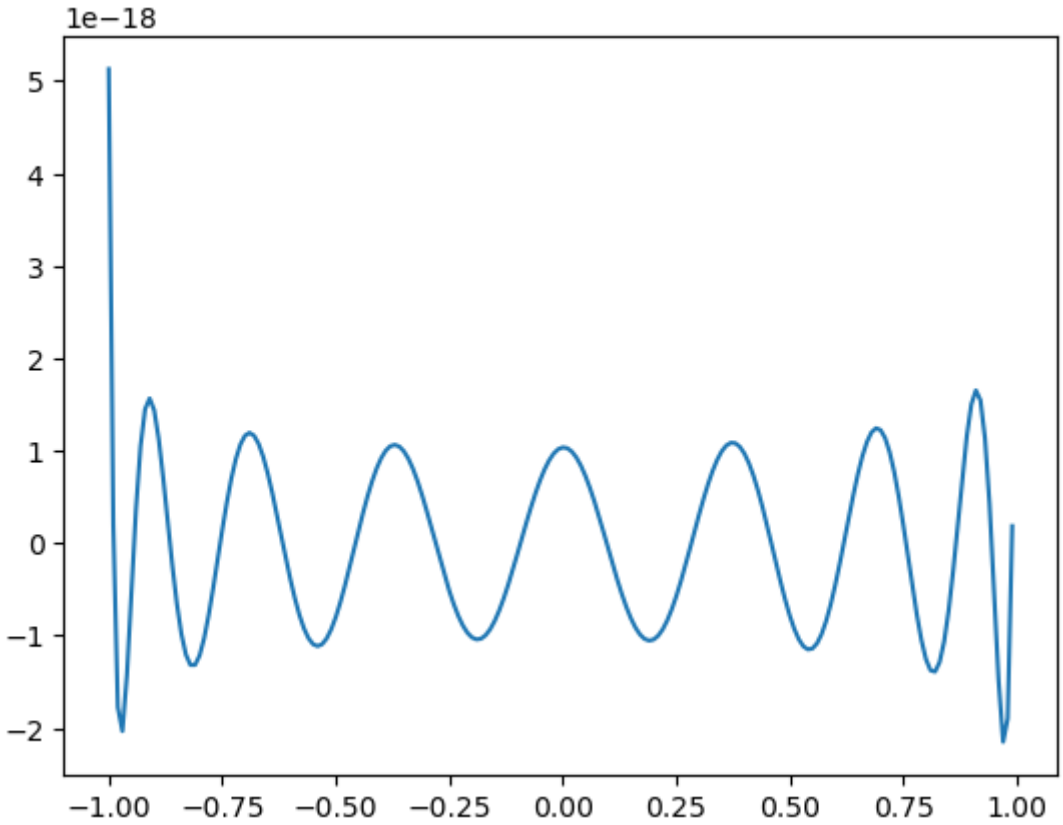
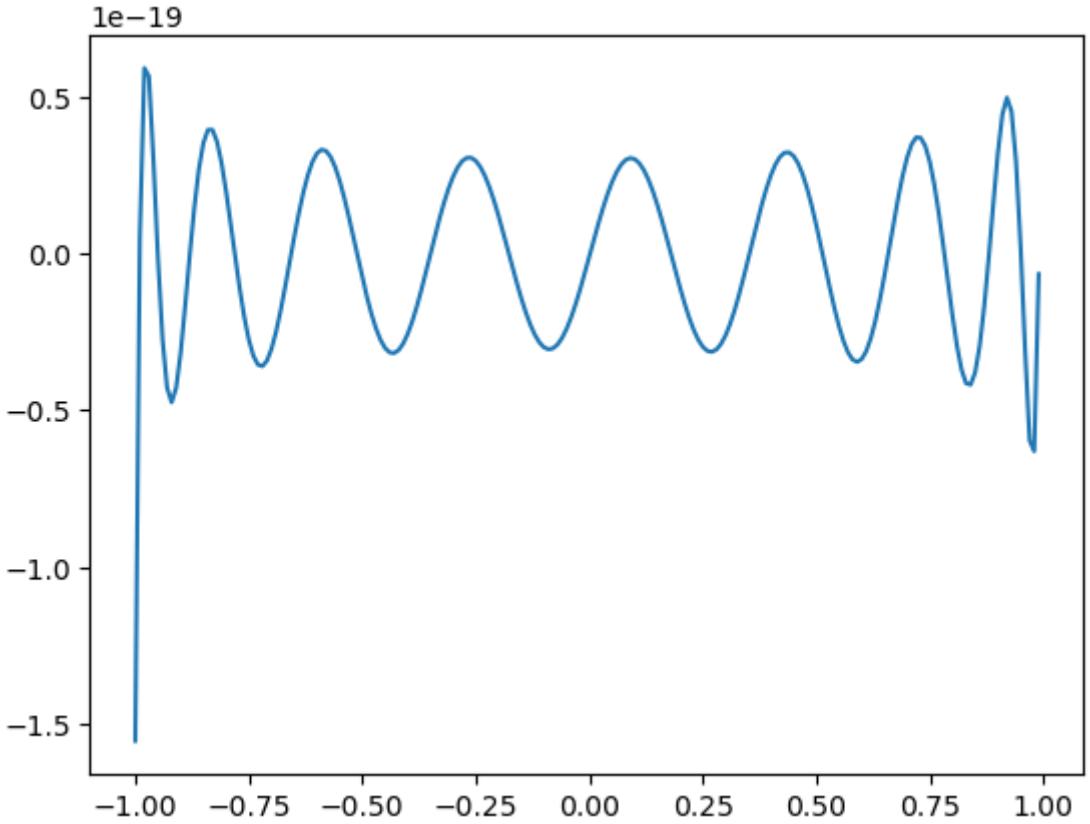
0.25
0.50
0.75
1.00


1e−18
5
4
3
2
1
0
−1
−2
−1.00
−0.75
−0.50
−0.25
0.00
0.25
0.50
0.75
1.00


1e−19
0.5
0.0
−0.5
−1.0
−1.5
−1.00
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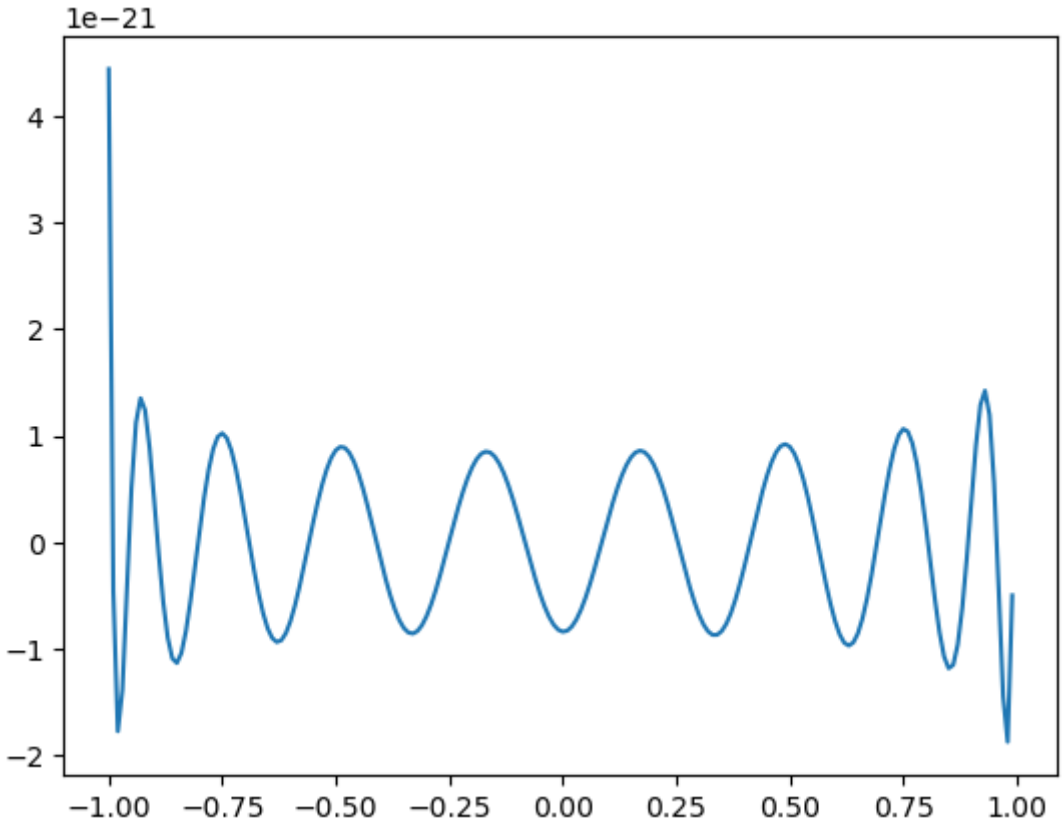
0.00
0.25
0.50
0.75
1.00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4
3
2
1
0
−1
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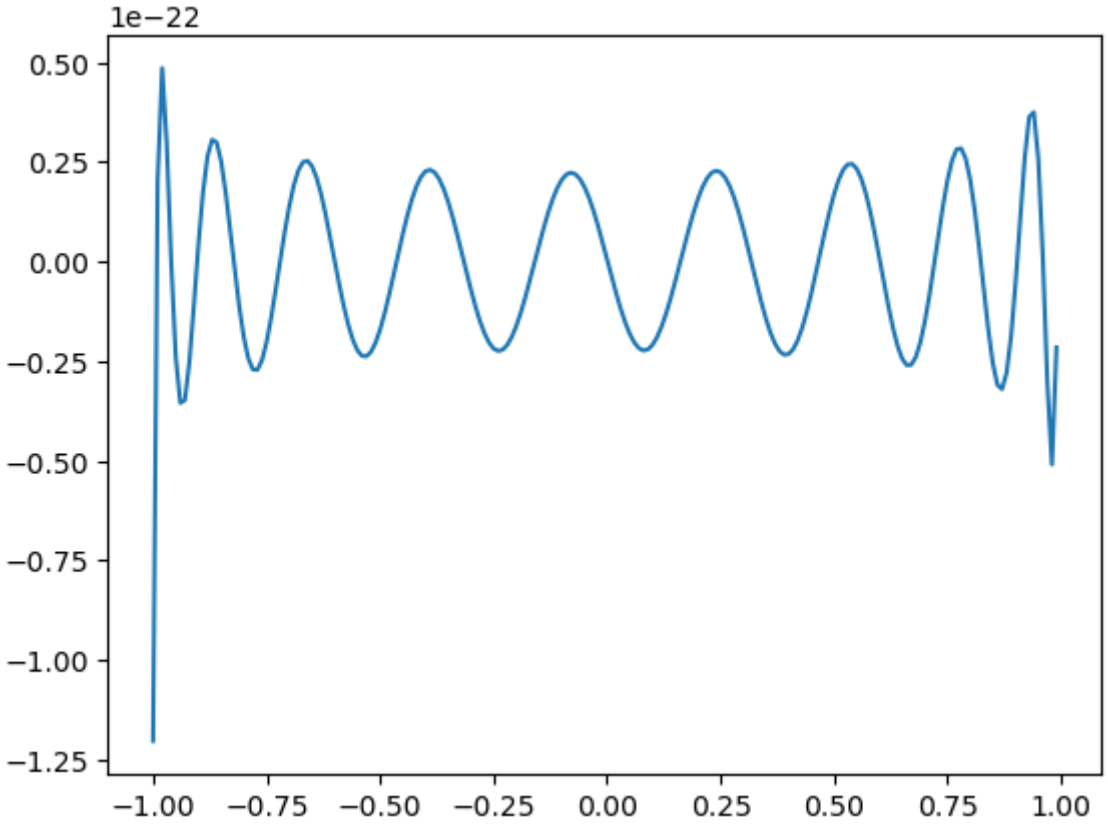
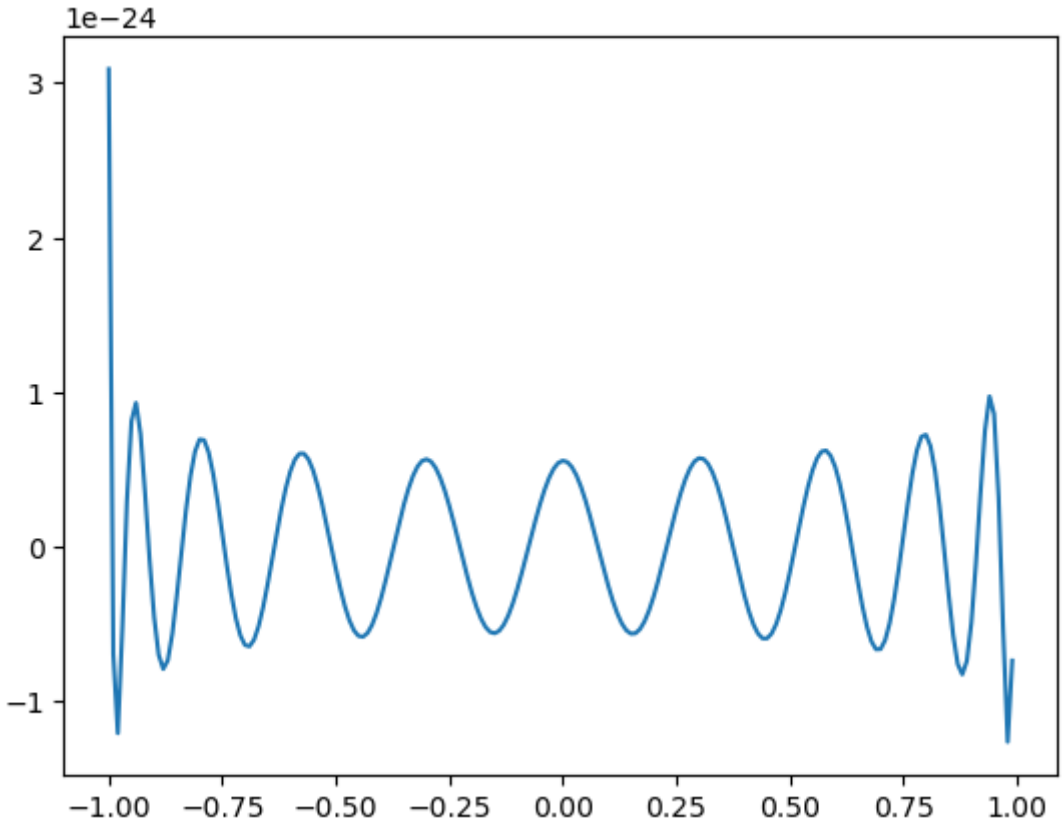
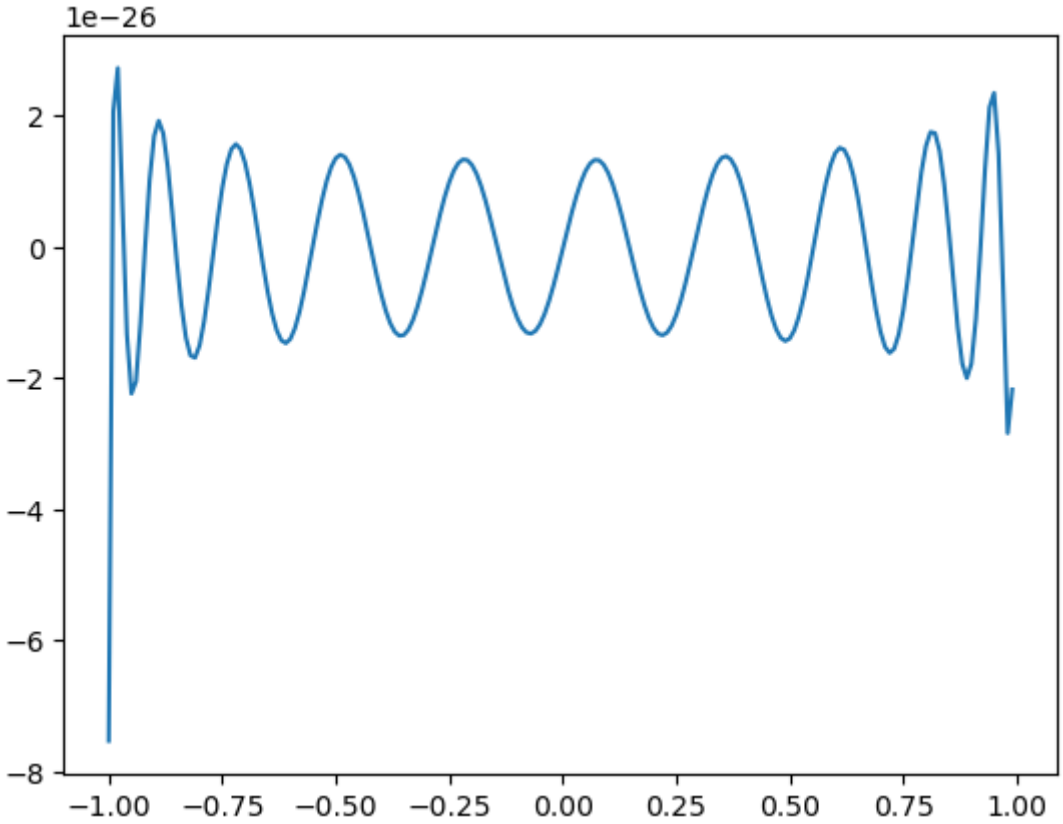
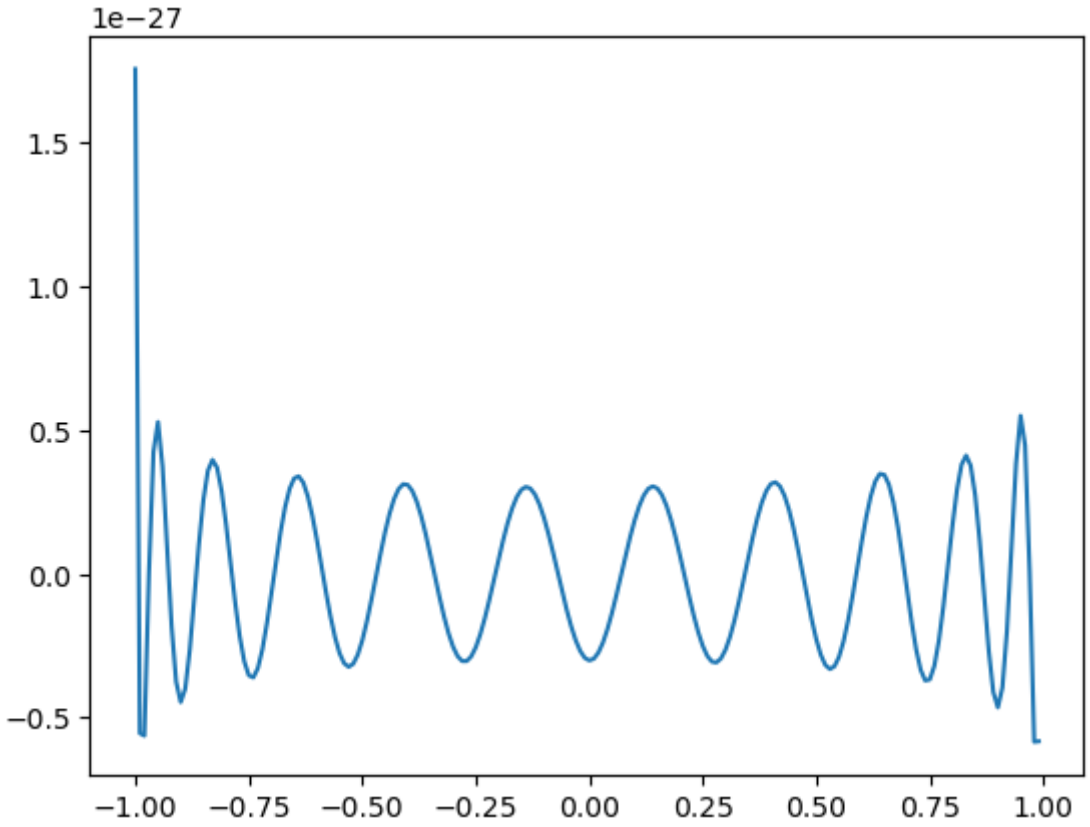
−1.00
−0.75
−0.50
−0.25
0.00
0.25
0.50
0.75
1.00


1e−22
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
−1.25
−1.00
−0.75
−0.50
−0.25
0.00
0.25
0.50
0.75
1.00


1e−24
3
2
1
0
−1
−1.00
−0.75
−0.50
−0.25
0.00
0.25
0.50
0.75
1.00


1e−26
2
0
−2
−4
−6
−8
−1.00
−0.75
−0.50
−0.25
0.00
0.25
0.50
0.75
1.00


1e−27
1.5
1.0
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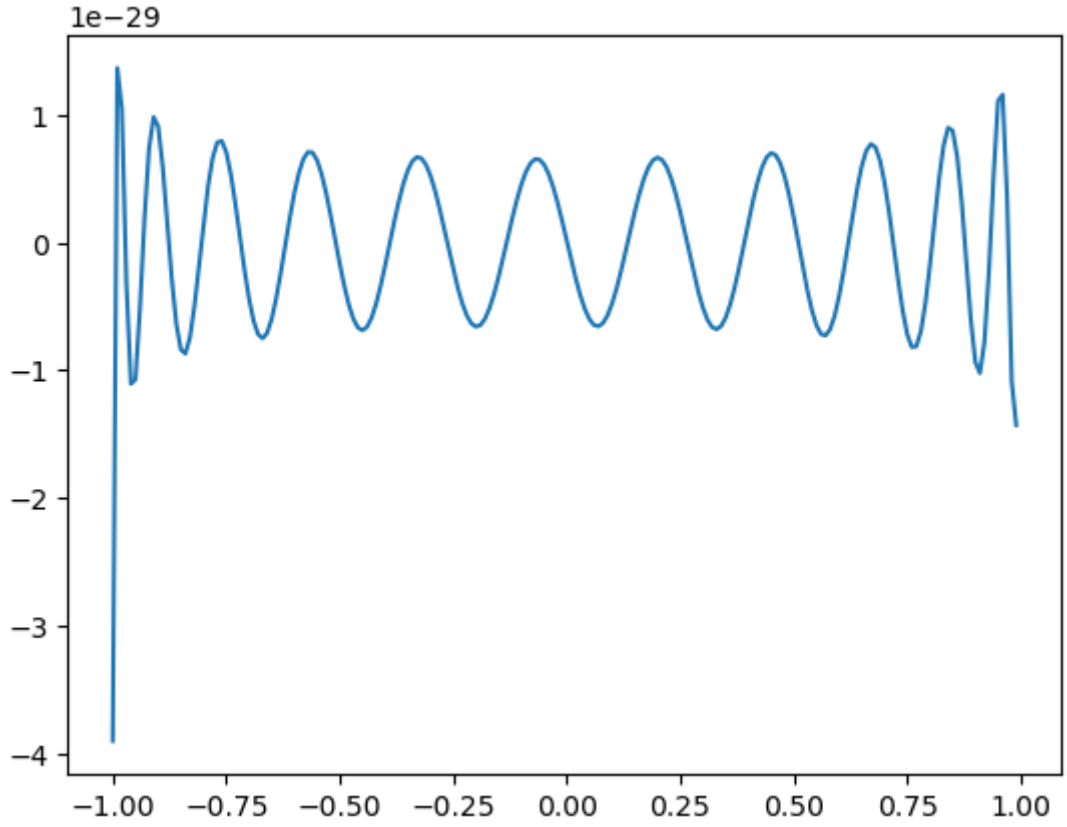
0.0
−0.5
−1.00
−0.75
−0.50
−0.25
0.00
0.25
0.50
0.75
1.00


1e−29
1
0
−1
−2
−3
−4
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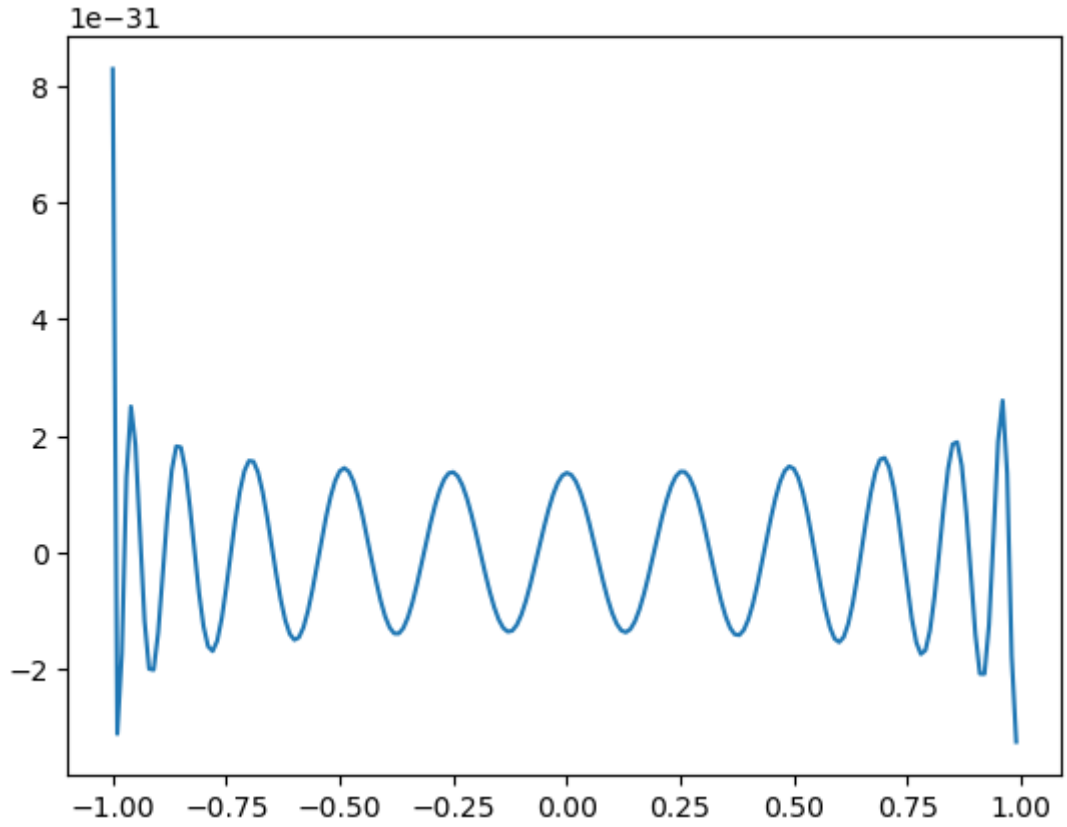
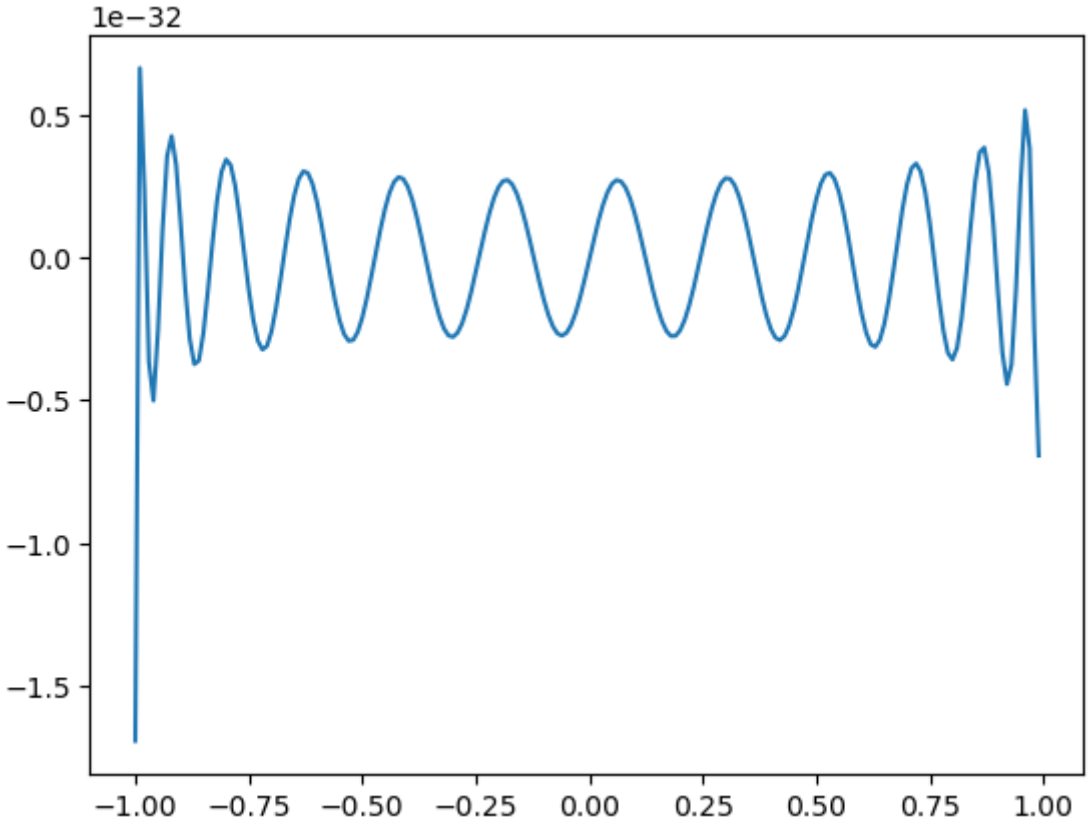
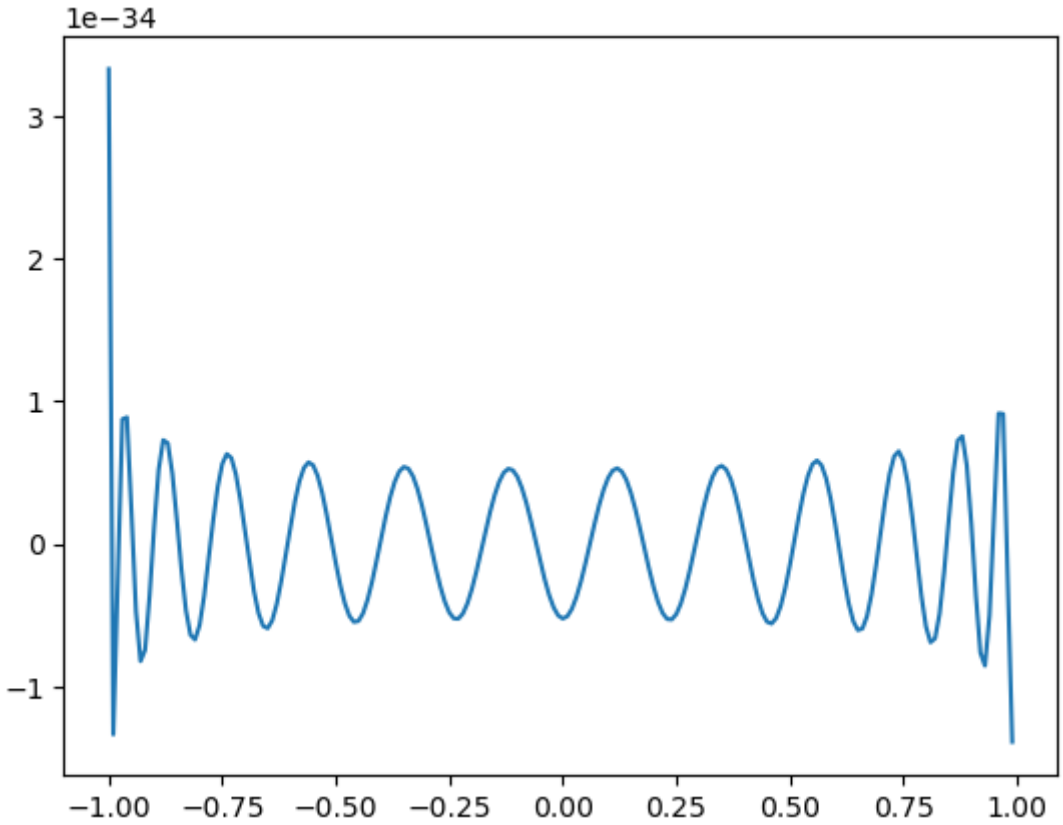
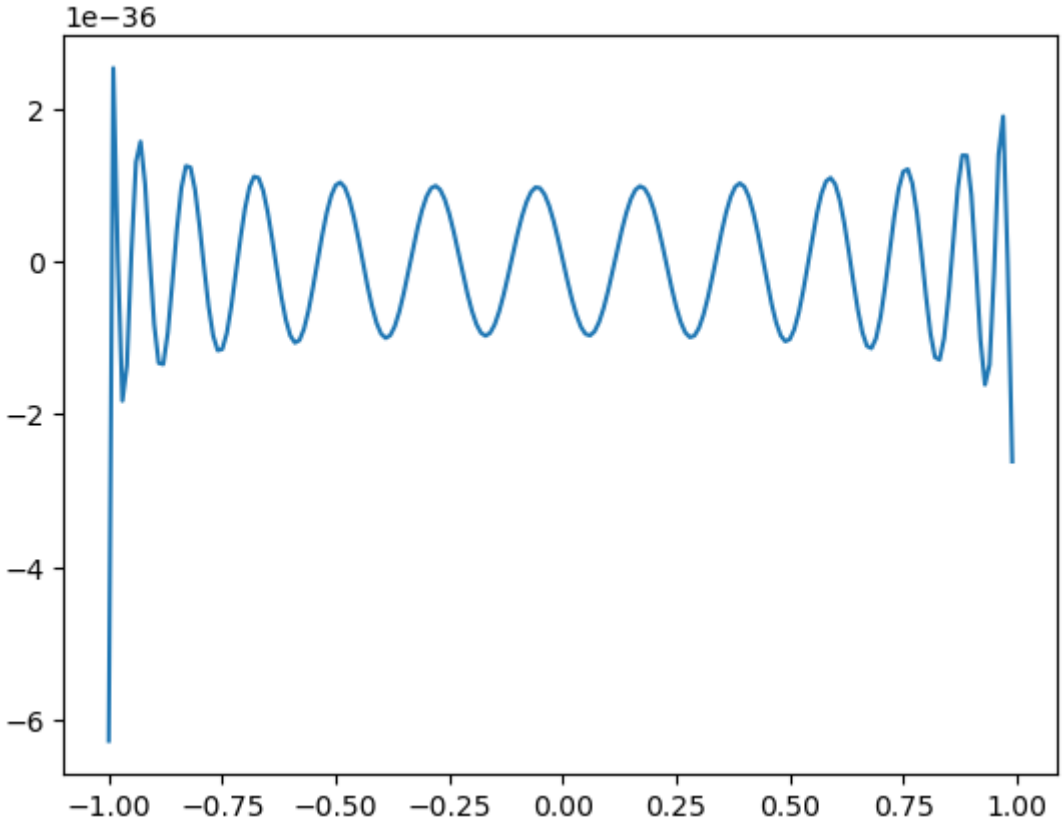
−0.50
−0.25
0.00
0.25
0.50
0.75
1.00


1e−31
8
6
4
2
0
−2
−1.00
−0.75
−0.50
−0.25
0.00
0.25
0.50
0.75
1.00


1e−32
0.5
0.0
−0.5
−1.0
−1.5
−1.00
−0.75
−0.50
−0.25
0.00
0.25
0.50
0.75
1.00


1e−34
3
2
1
0
−1
−1.00
−0.75
−0.50
−0.25
0.00
0.25
0.50
0.75
1.00


1e−36
2
0
−2
−4
−6
−1.00
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−0.50
−0.25
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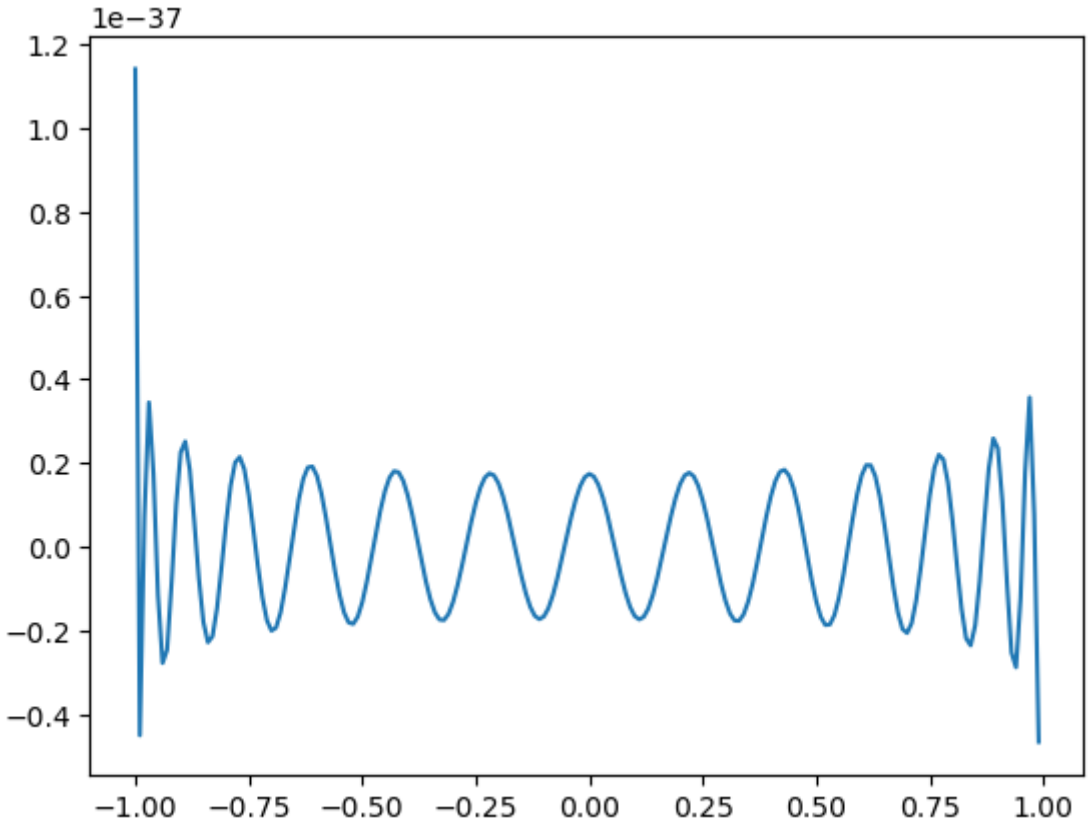
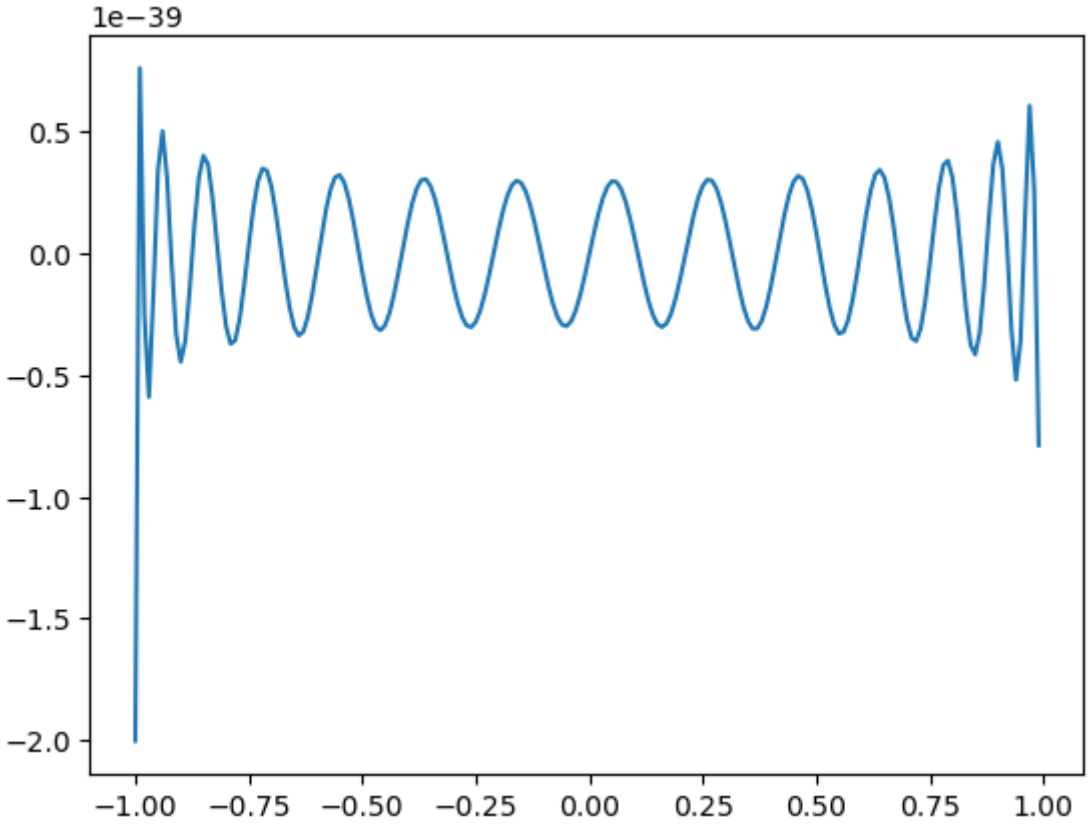
0.25
0.50
0.75
1.00


1e−37
1.2
1.0
0.8
0.6
0.4
0.2
0.0
−0.2
−0.4
−1.00
−0.75
−0.50
−0.25
0.00
0.25
0.50
0.75
1.00


1e−39
0.5
0.0
−0.5
−1.0
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−1.00
−0.75
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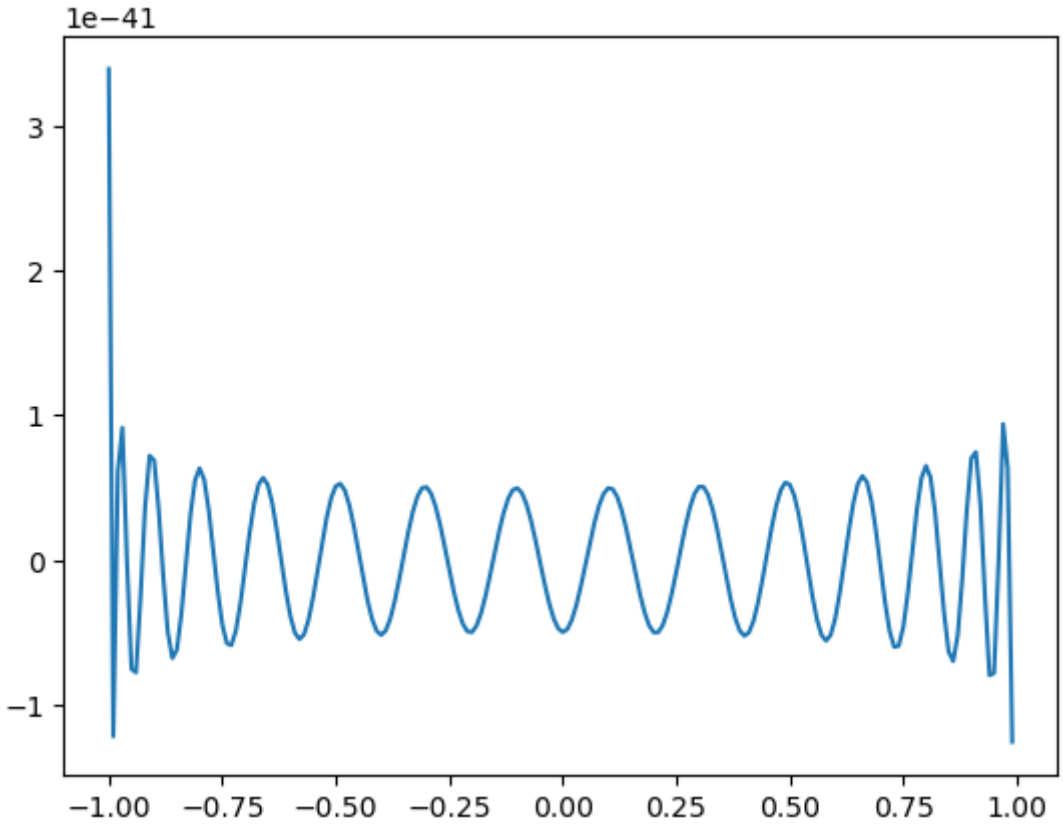
−0.25
0.00
0.25
0.50
0.75
1.00


1e−41
3
2
1
0
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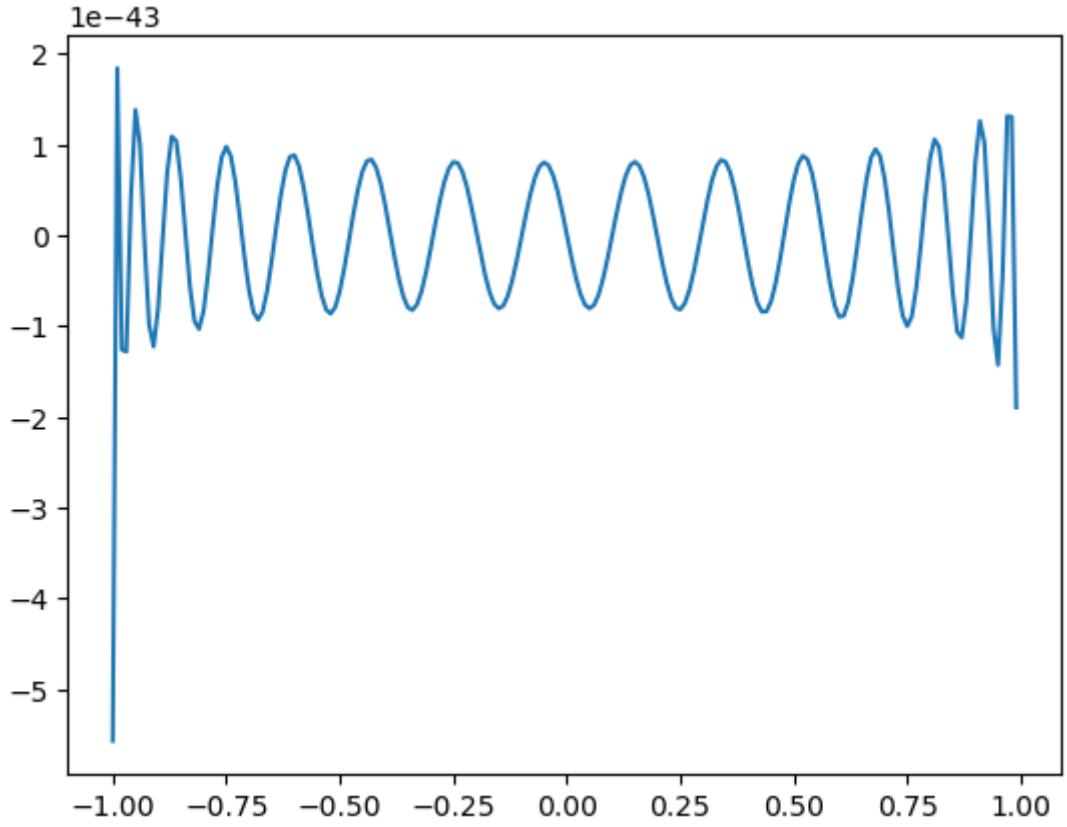
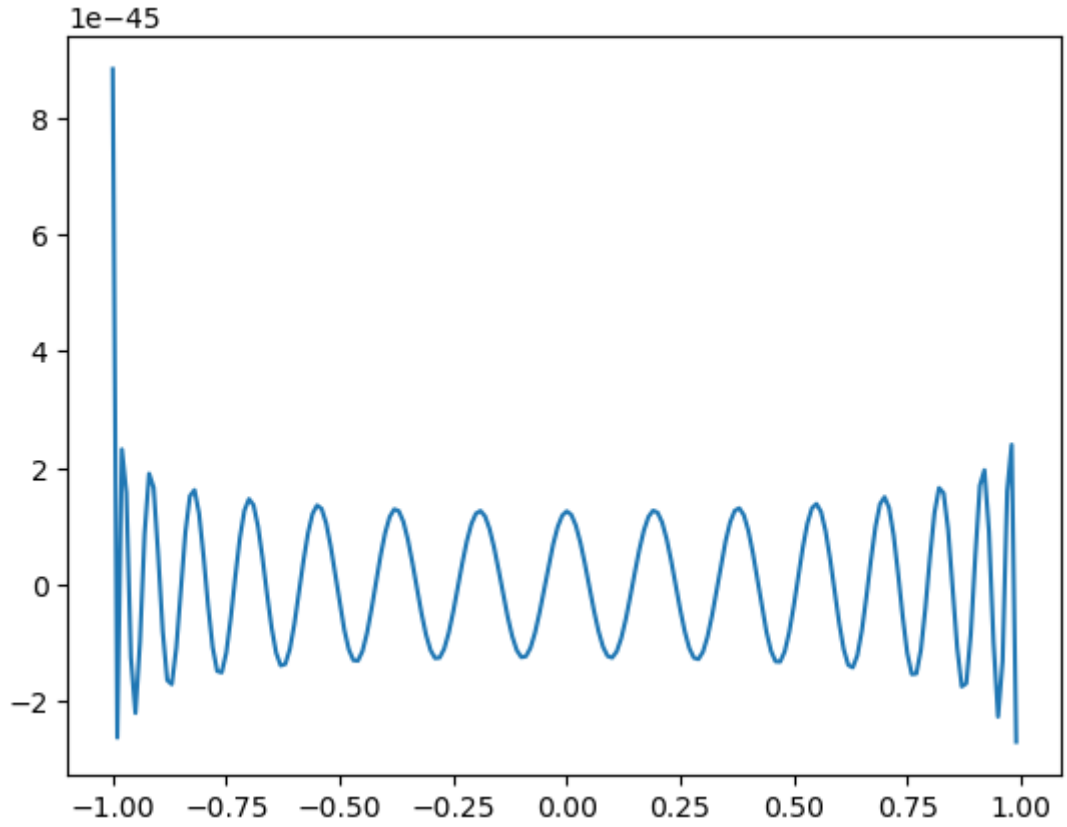
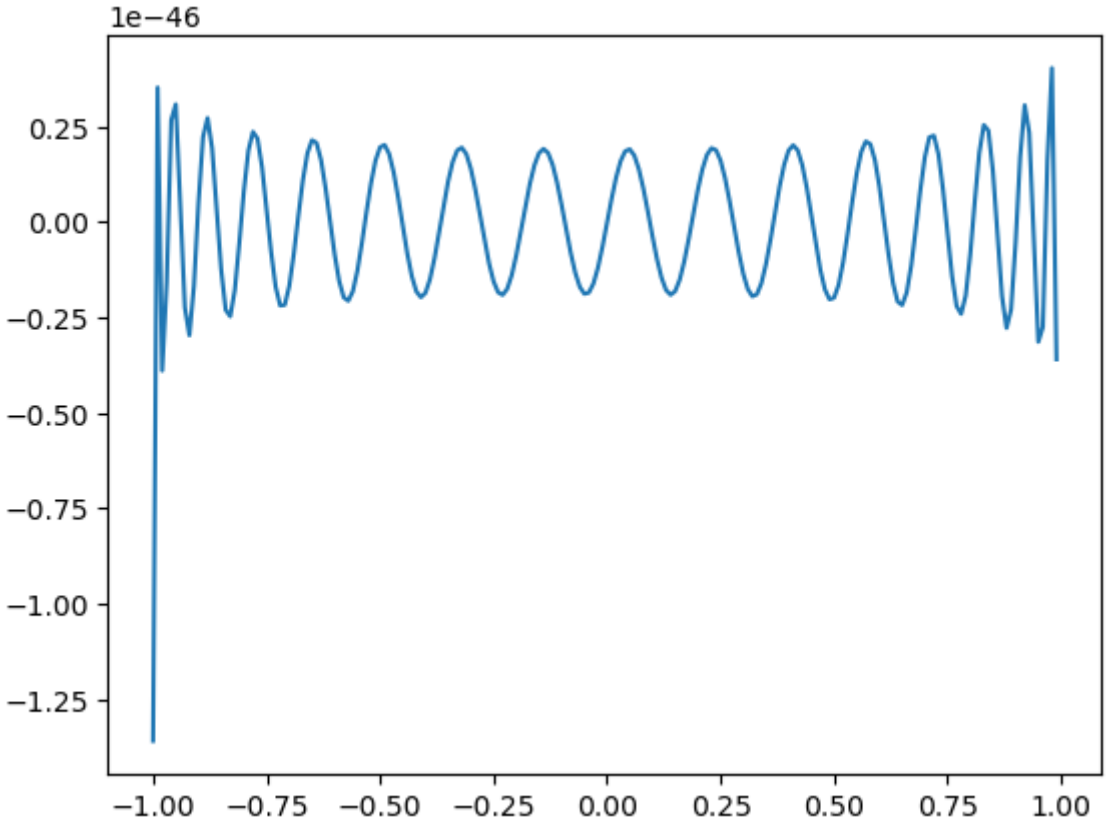
−1.00
−0.75
−0.50
−0.25
0.00
0.25
0.50
0.75
1.00


1e−43
2
1
0
−1
−2
−3
−4
−5
−1.00
−0.75
−0.50
−0.25
0.00
0.25
0.50
0.75
1.00


1e−45
8
6
4
2
0
−2
−1.00
−0.75
−0.50
−0.25
0.00
0.25
0.50
0.75
1.00


1e−46
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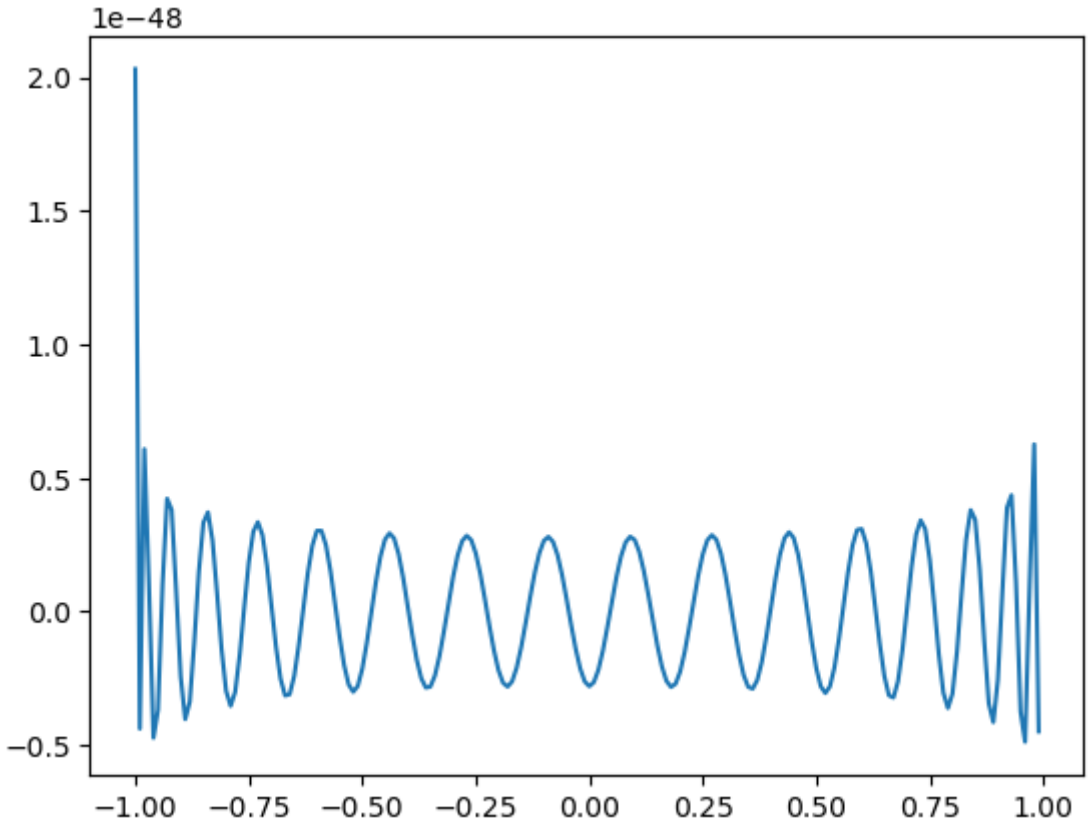
0.00
−0.25
−0.50
−0.75
−1.00
−1.25
−1.00
−0.75
−0.50
−0.25
0.00
0.25
0.50
0.75
1.00


1e−48
2.0
1.5
1.0
0.5
0.0
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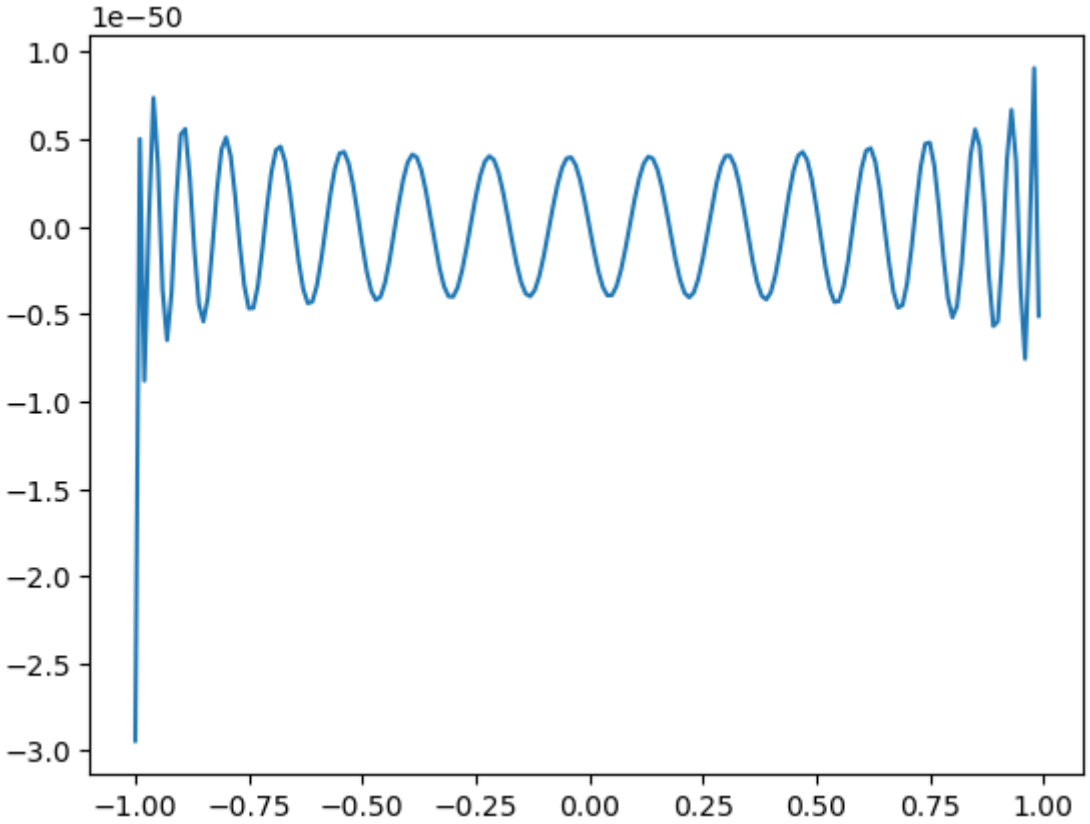
−1.00
−0.75
−0.50
−0.25
0.00
0.25
0.50
0.75
1.00


1e−50
1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
−2.5
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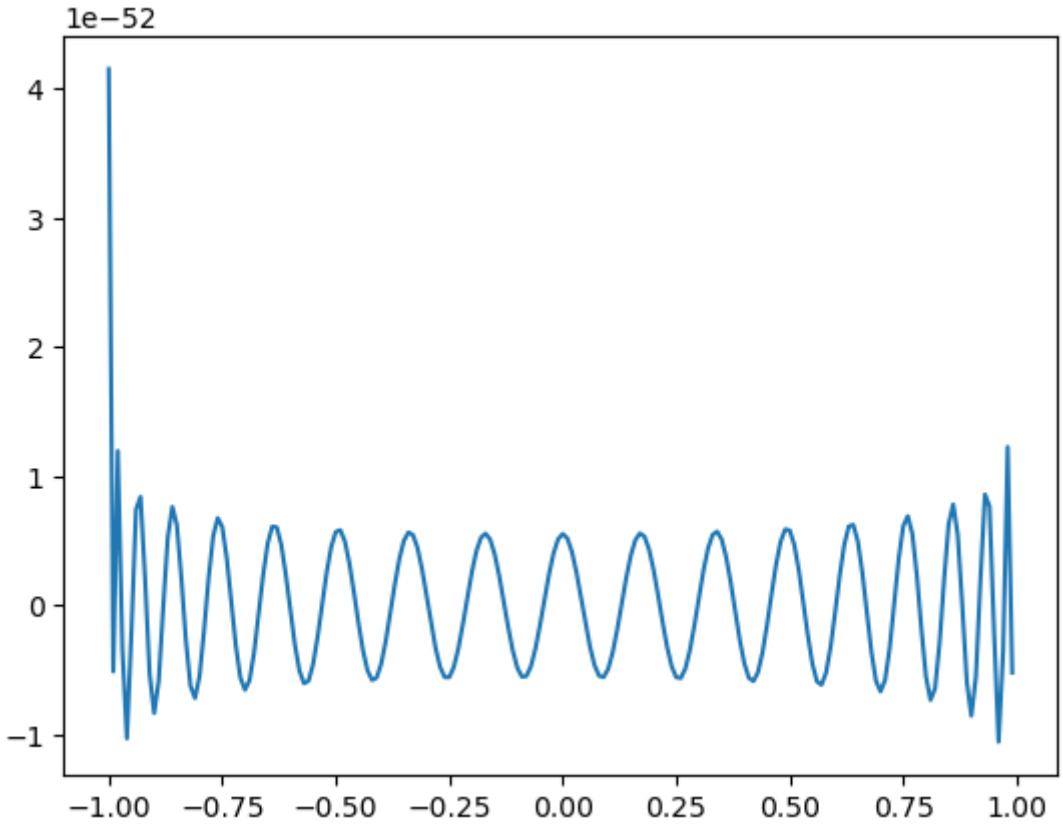
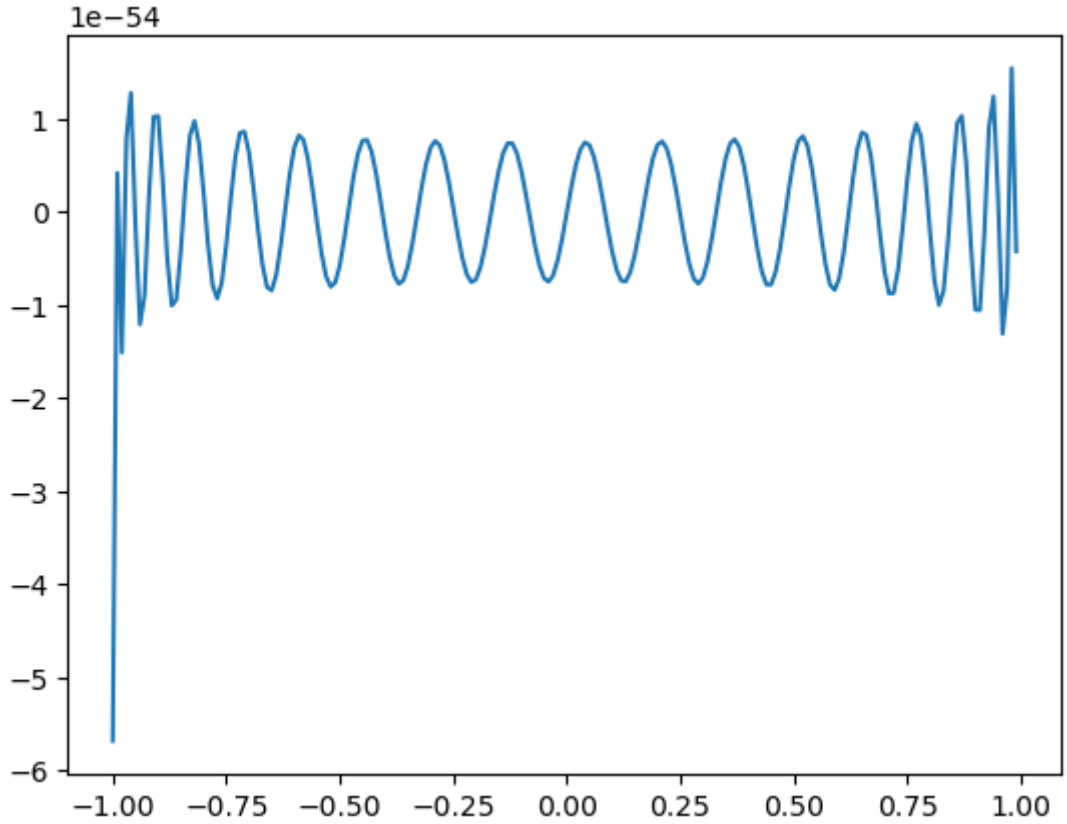
−0.50
−0.25
0.00
0.25
0.50
0.75
1.00


1e−52
4
3
2
1
0
−1
−1.00
−0.75
−0.50
−0.25
0.00
0.25
0.50
0.75
1.00


1e−54
1
0
−1
−2
−3
−4
−5
−6
−1.00
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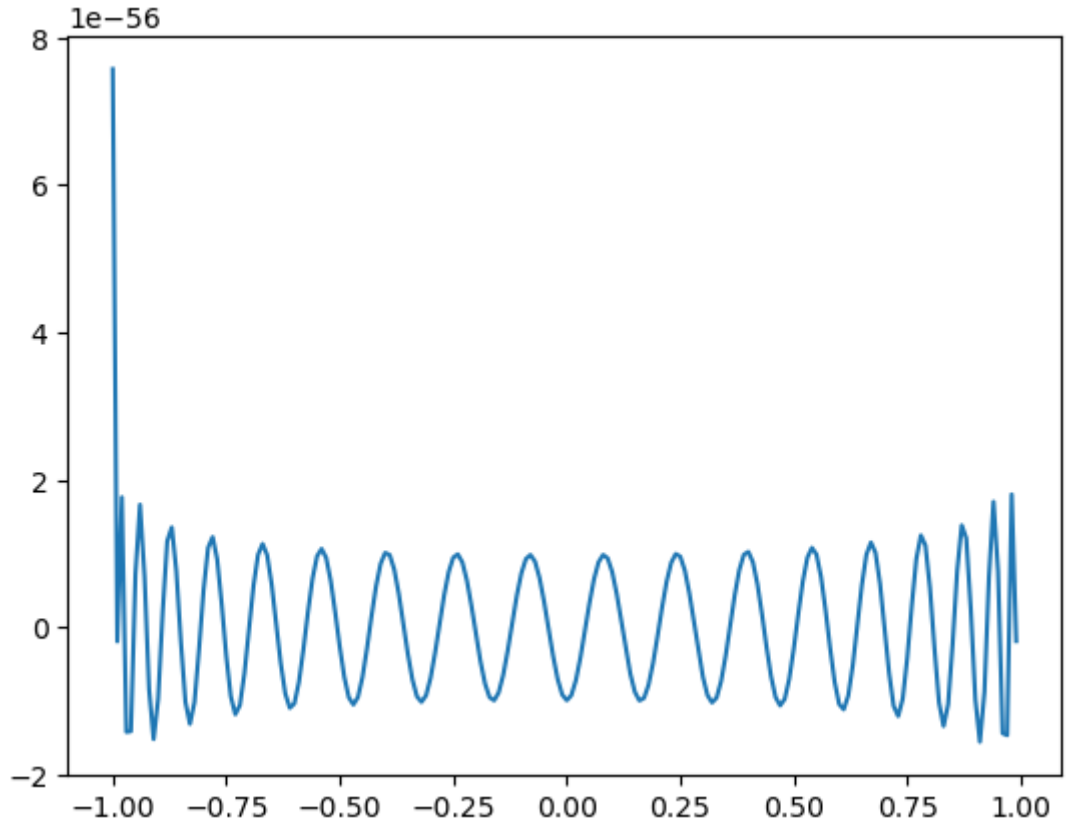
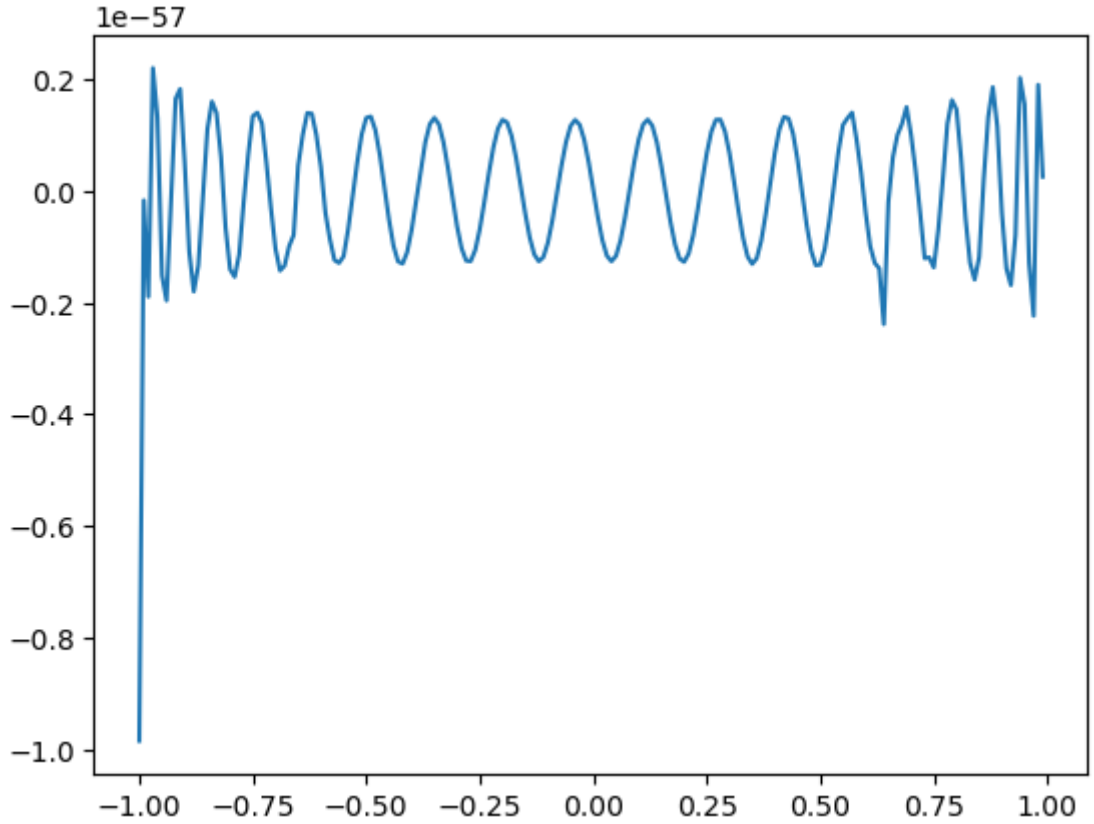
−0.50
−0.25
0.00
0.25
0.50
0.75
1.00


1e−56
8
6
4
2
0
−2
−1.00
−0.75
−0.50
−0.25
0.00
0.25
0.50
0.75
1.00


1e−57
0.2
0.0
−0.2
−0.4
−0.6
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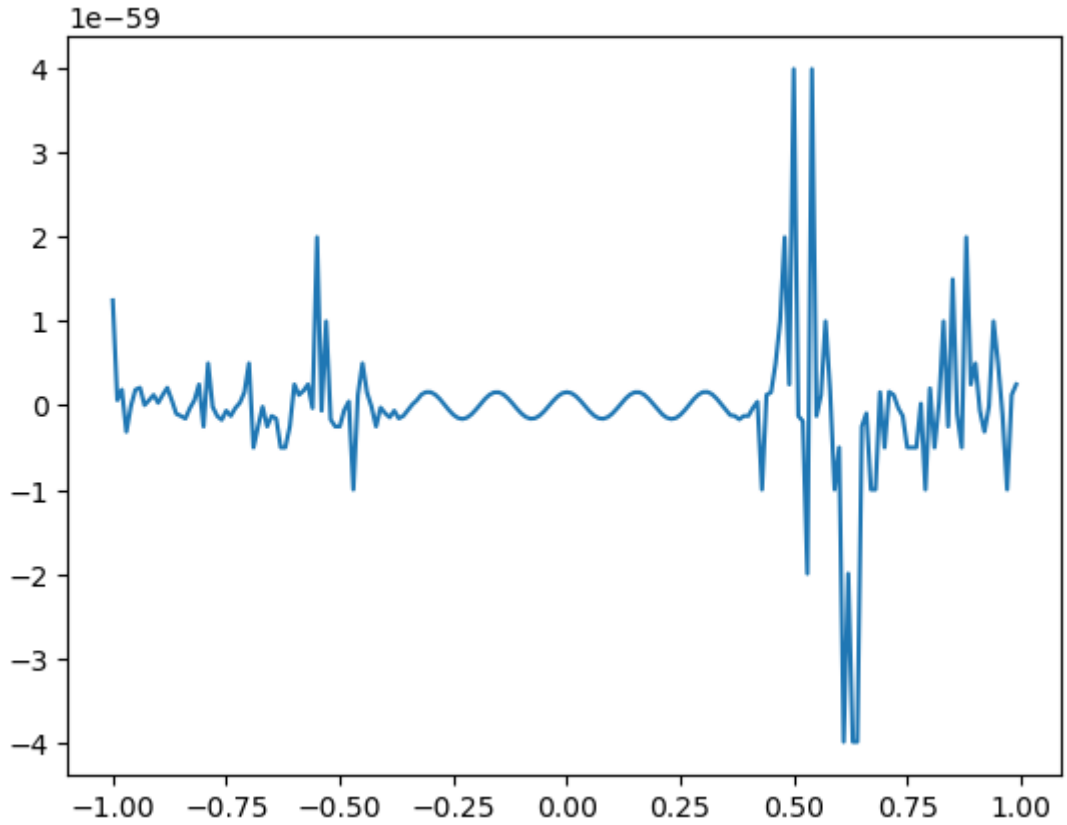
0.00
0.25
0.50
0.75
1.00


1e−59
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3
2
1
0
−1
−2
−3
−4
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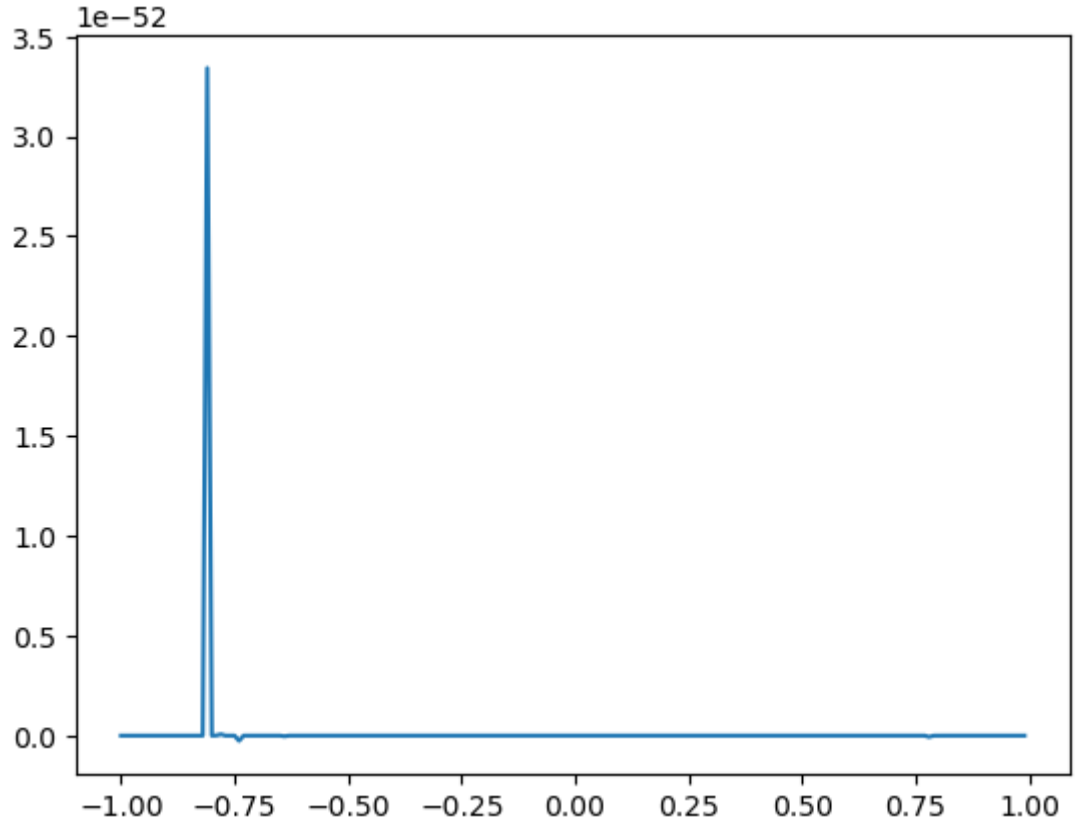
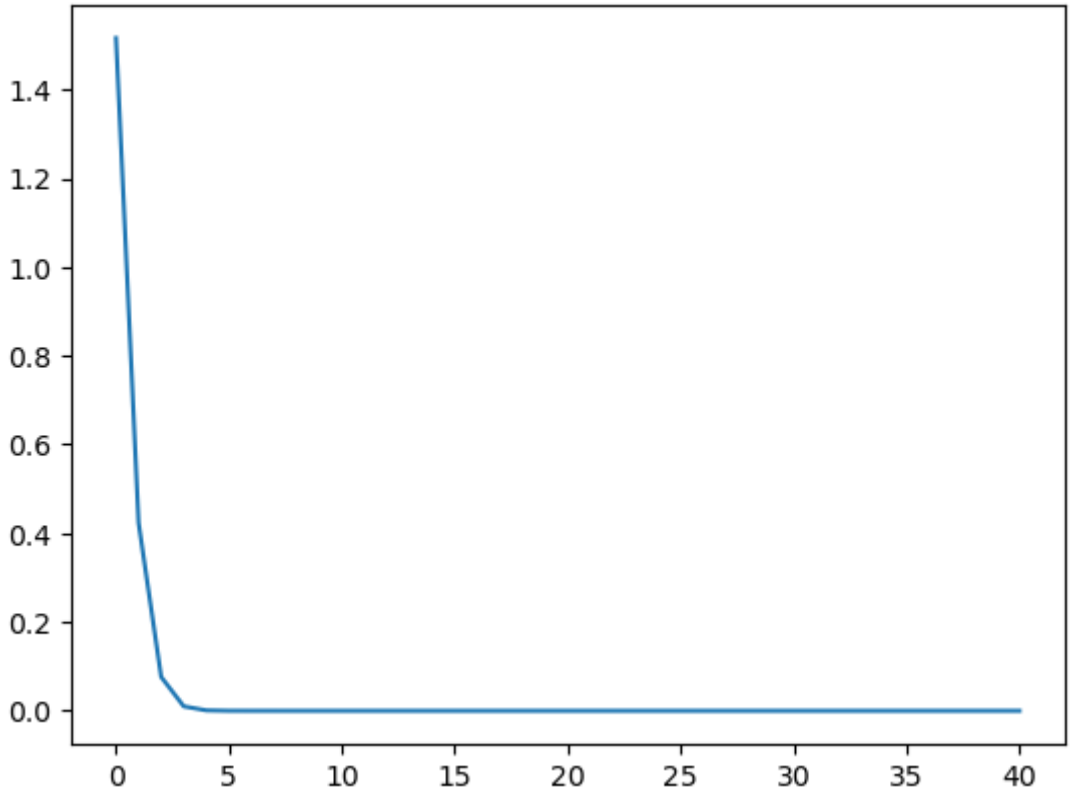
−0.75
−0.50
−0.25
0.00
0.25
0.50
0.75
1.00

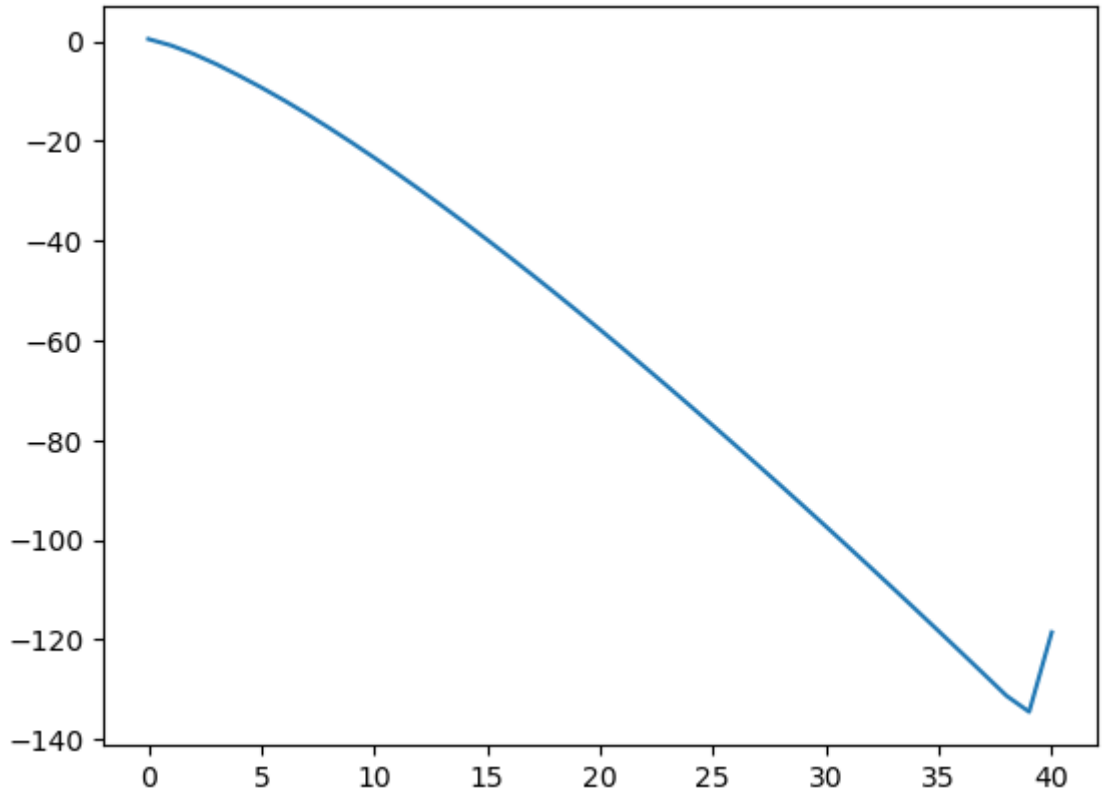



```
[199 199 199 199 199   0   0   0   0   0   0   0   0   0   0   0   0   0
   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
   0   0   0 150  19]
[1.51603327870547 0.423431338426280 0.0762977461457297 0.0100167849111821
 0.00102592199263293 9.23120414227268e-5 7.14229769069819e-6
 4.78243618165514e-7 2.82283563996151e-8 1.48978475696039e-9
 7.11018883495221e-11 3.09719590327914e-12 1.24085050395897e-13
 4.60201888391880e-15 1.58878264460653e-16 5.13042000954950e-18
 1.55609595210140e-19 4.44960778609840e-21 1.20347295712696e-22
 3.08785232006107e-24 7.53581816158400e-26 1.75346188126356e-27
 3.89850286123413e-29 8.29843589115682e-31 1.69426265859726e-32
 3.32335912780489e-34 6.27271836167756e-36 1.14087082267490e-37
 2.00214311955731e-39 3.39443675398567e-41 5.56614442906541e-43
 8.83737388435451e-45 1.35991750193116e-46 2.03017940185315e-48
 2.94290650979934e-50 4.1457604e-52 5.68019e-54 7.57493e-56 9.84e-58
 0.e-59 0.e-52]
```

画出误差极值图像和极值对数图像




```
/var/folders/22/vljd7pts6hx7gvsntnkblyvm0000gn/T/ipykernel_68092/1593124892.py:4: Deprecation
Deprecated in NumPy 1.20; for more details and guidance: https://numpy.org/devdocs/release/1.
  log_error = np.log(np.array(max_error, dtype=np.float))
```



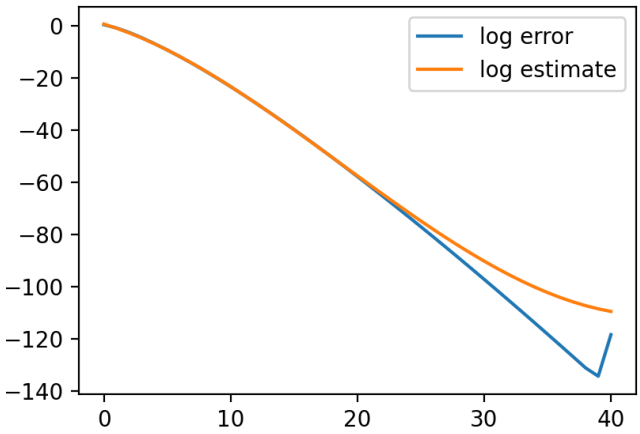
对对数误差进行最小二乘法

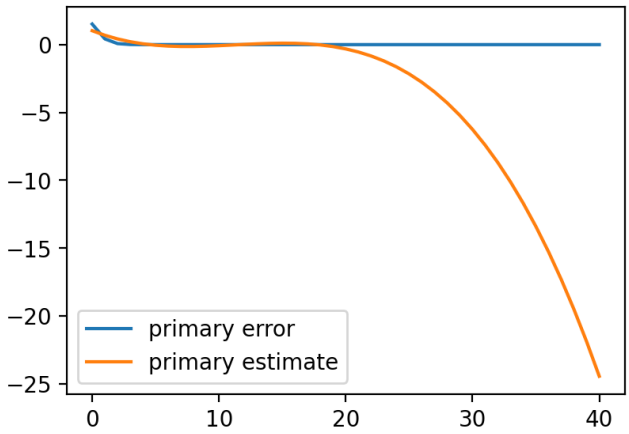
```
[   0.41609724   -0.85936391   -2.57311188   -4.6034931    -6.88216357
   -9.29033597  -11.84947603  -14.55314557  -17.38293882  -20.32463419
```

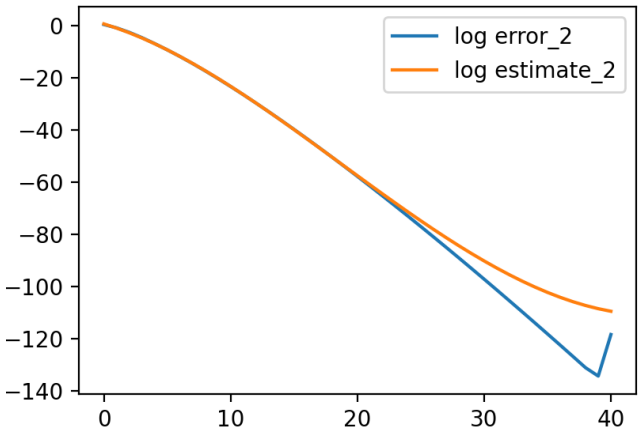
```
  -23.36690722  -26.50052396  -29.71780917  -33.0122813   -36.3783934
  -39.81134414  -43.30693668  -46.861471    -50.47166054  -54.13456642
  -57.84754501  -61.60820546  -65.4143751   -69.26407083  -73.15547534
  -77.0869171   -81.05685354  -85.06385659  -89.10660046  -93.18385097
  -97.29445639 -101.43733942 -105.61149024 -109.8159603  -114.04985695
 -118.31233862 -122.60261049 -126.9199208  -131.26330707 -134.47055303
 -118.52816788]
[ 0.65107    -1.48276383 -0.11101957  0.00197944]
[1.02590682793058 -0.372383279356719 0.0371489358711674
 -0.00109407977978011]
```

画图

```
[   0.65107      -0.94073398   -2.74270048   -4.74295292   -6.92961466
   -9.29080911  -11.81465964  -14.48928964  -17.30282251  -20.24338163
  -23.29909038  -26.45807216  -29.70845036  -33.03834836  -36.43588954
  -39.8891973   -43.38639503  -46.91560611  -50.46495393  -54.02256188
  -57.57655334  -61.11505171  -64.62618037  -68.09806271  -71.51882211
  -74.87658197  -78.15946567  -81.3555966   -84.45309815  -87.44009371
  -90.30470666  -93.03506039  -95.61927829  -98.04548374 -100.30180014
 -102.37635087 -104.25725933 -105.93264889 -107.39064294 -108.61936488
 -109.60693809]
```



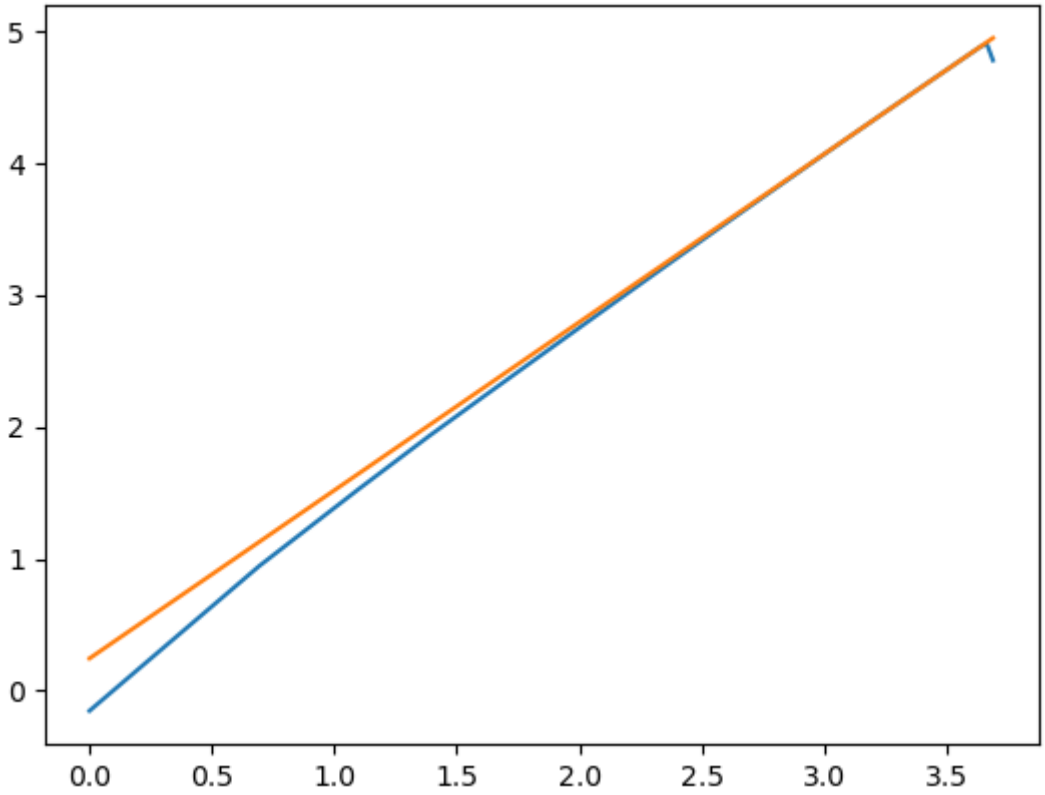




考虑log error和n的次数关系

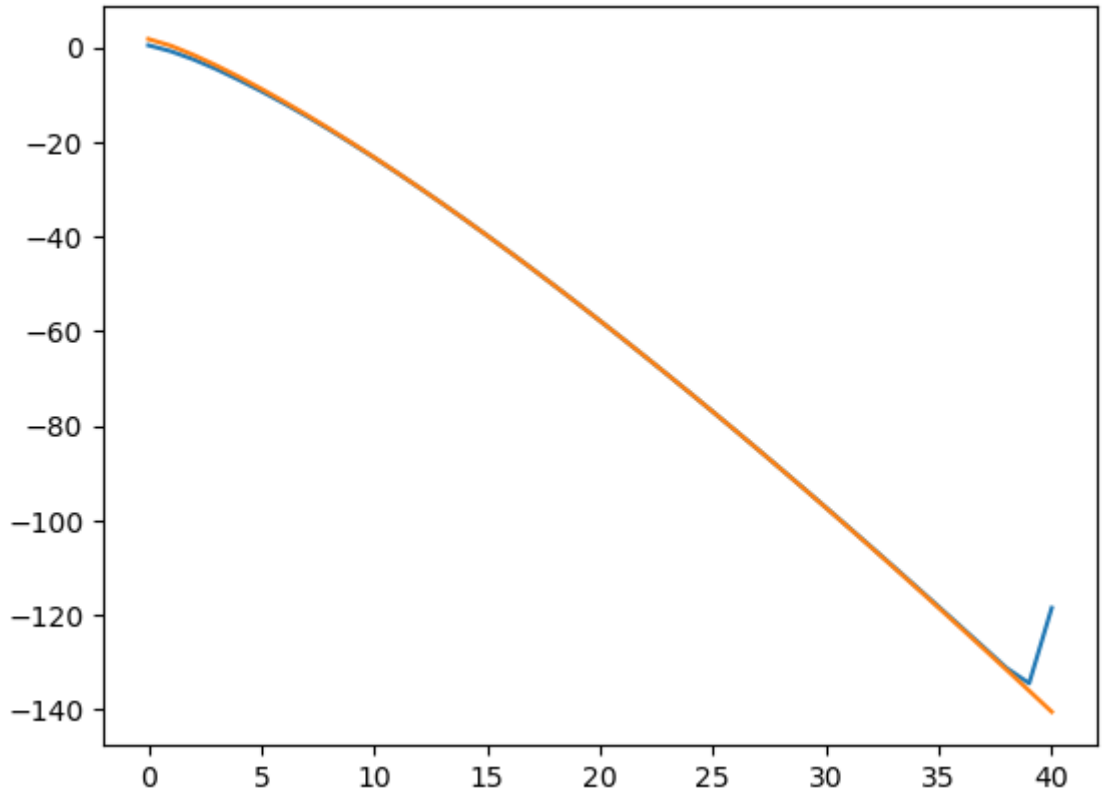
```
[0.24500778 1.27382186]
/var/folders/22/vljd7pts6hx7gvsntnkblyvm0000gn/T/ipykernel_68092/3235719765.py:3: RuntimeWarn
  deg_A = np.transpose([np.ones_like(n), np.log(n)])[25:35]
/var/folders/22/vljd7pts6hx7gvsntnkblyvm0000gn/T/ipykernel_68092/3235719765.py:10: RuntimeWar
  deg_est_loglogerr = np.matmul(np.transpose([np.ones_like(n), np.log(n)]), deg_coef)
```

```
/var/folders/22/vljd7pts6hx7gvsntnkblyvm0000gn/T/ipykernel_68092/3235719765.py:12: RuntimeWar
  plt.plot(np.log(n)[log_error < 0], np.log(-log_error[log_error < 0]))
/var/folders/22/vljd7pts6hx7gvsntnkblyvm0000gn/T/ipykernel_68092/3235719765.py:13: RuntimeWar
  plt.plot(np.log(n), deg_est_loglogerr)
```



构造1.27次回归

```
[-1.24869328 -0.17524121  1.73244439]
```



关于最佳平方逼近多项式的收敛性的猜想：

1.intuition：在赋范线性函数空间里的一组有限正交基下,随着基的数量增多,其张成的线性子空间越大，原函数在该子空间内的投影在均方意义下逐渐收敛到函数本身。