不确定规划 Homework1

方言

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1. Let $\Lambda_1, \Lambda_2, ..., \Lambda_n$be independent events. Show that Λ_i and Λ_j are independent for any indexes i and j with $1 \leq i < j \leq n$.

由于 $\Lambda_1, \Lambda_2, ..., \Lambda_n$ 是独立事件，则:

$$\mathcal{M}\{\bigcap_{i=1}^{n} \Lambda_i^*\} = \bigwedge_{i=1}^{n} \mathcal{M}\{\Lambda_i^*\}$$

其中，Λ_i^* 可以从 $\{\Lambda_i, \Lambda_i^c, \Gamma\}, i = 1, 2, ..., n$ 中任取。

对任意给定的 i, j, 令 $\Lambda_t^* = \Gamma, t = 1, 2, ..., n, t \neq i, t \neq j$，则可以得到:

$$\mathcal{M}\{\bigcap_{i=1}^{n} \Lambda_i^*\} = \mathcal{M}\{\Lambda_i^* \cap \Lambda_j^* \cap \Gamma \cap ... \cap \Gamma\} = \mathcal{M}\{\Lambda_i^*\} \wedge \mathcal{M}\{\Lambda_j^*\} \wedge 1 \wedge ... \wedge 1$$

整理可得:

$$\mathcal{M}\{\Lambda_i^* \cap \Lambda_j^*\} = \mathcal{M}\{\Lambda_i^*\} \wedge \mathcal{M}\{\Lambda_j^*\}$$

因此，Λ_i 和 Λ_j 是独立的，$1 \leq i < j \leq n$。

2. Construct 100 independent events. (nontrival)

考虑 100 个不确定空间，分别记为 $(\Gamma_i, \mathcal{L}_i, \mathcal{M}_i), i = 1, 2, ..., 100$，在其中分别取非平凡的事件 $\Lambda_1, \Lambda_2, ..., \Lambda_{100}$，则:

$$\Lambda_1 = \Lambda_1 \times \Gamma_2 \times \Gamma_3 \times ... \times \Gamma_{100}$$

$$\Lambda_2 = \Gamma_1 \times \Lambda_2 \times \Gamma_3 \times ... \times \Gamma_{100}$$

$$...$$

$$\Lambda_{100} = \Gamma_1 \times \Gamma_2 \times \Gamma_3 \times ... \times \Lambda_{100}$$

此时，考虑独立性:

$$\bigcap_{i=1}^{100}(\Gamma_1 \times \Gamma_2 \times ... \times \Lambda_i^* \times ... \times \Gamma_{100}) = \bigcap_{i=1}^{100} \Lambda_i^* = \Lambda_1^* \times \Lambda_2^* \times ... \times \Lambda_{100}^*$$

其中 Λ_i^* 可以从 $\{\Lambda_i, \Lambda_i^c, \Gamma\}, i = 1, 2, ..., 100$ 中任取。由此可得:

$$\mathcal{M}\{\bigcap_{i=1}^{100} \Lambda_i^*\} = \mathcal{M}\{\Lambda_1^* \times \Lambda_2^* \times ... \times \Lambda_{100}^*\} = \bigwedge_{i=1}^{100} \mathcal{M}\{\Lambda_i^*\}$$

即，$\Lambda_1, \Lambda_2, ..., \Lambda_{100}$ 是独立事件。