不确定规划小测 2

方言
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1 Problem Definition

Assume I have filled a urn with 100 balls that are either red or black. The distribution function is completely unknown to you. The problem is the number of red balls in the urn.

2 Emergency

Samples	#(Red Balls)	Samples	#(Red Balls)
1	27	11	60
2	54	12	37
3	72	13	47
4	13	14	38
5	55	15	53
6	19	16	58
7	48	17	20
8	62	18	51
9	37	19	36
10	73	20	51

3 Uncertain Variable

We denote #(Red Balls) as η, which is the uncertain variable.

4 Belief Degree

Since the distribution function is completely unknown, we have:

$$\eta = k \text{ with belief degree } \frac{1}{101}, \quad k = 0, 1, 2, ..., 100$$

5 Uncertain Distribution

According to the emergency, we assume that η has a normal uncertainty distribution $N(50, 20)$:

$$\Phi(\eta) = (1 + \exp\left(\frac{\pi(50-\eta)}{20\sqrt{3}}\right))^{-1}$$

6 Uncertain Measure

$$\mathcal{M}\{\eta \leq x\} = (1 + \exp\left(\frac{\pi(50-x)}{20\sqrt{3}}\right))^{-1}$$

7 Inverse Uncertain Distribution

η has a inverse normal uncertainty distribution:

$$\Phi^{-1}(\alpha) = 50 + \frac{20\sqrt{3}}{\pi} \ln \frac{\alpha}{1-\alpha}$$

8 Expectation

$$\begin{aligned} \mathbb{E}(\eta) &= \int_0^1 \Phi^{-1}(\alpha)\, d\alpha \\ &= 50 \end{aligned}$$

9 Test

Given significance level $\alpha = 0.05$, we have:

$$\begin{cases} \Phi^{-1}(\alpha/2) = 9.60 \\ \Phi^{-1}(1-\alpha/2) = 90.40 \end{cases}$$

It follows $\alpha \times 10 = 1$, then we have:

$$W = \{(x_1, x_2, ..., x_{20}) : \text{there are at least 1 of indexes } (1 \leq \text{i} \leq 20) \text{ such that } x_i \leq 9.60 \text{ or } x_i \geq 90.40\}$$

According to the emergency, $9.60 \leq x_i \leq 90.40,\ i = 1, 2, ..., 20$. Therefore, it is reasonable to use noraml uncertainty distribution $N(50, 20)$