不确定规划课程作业 3

方言
2021210929

2022 年 3 月 24 日

1 Let ξ be an uncertain variable with regular uncertainty distribution Φ, and let f be a continuous and strictly decreasing function. Show that $f(\xi)$ has an inverse uncertainty distribution

$$\Psi^{-1}(\alpha) = f(\Phi^{-1}(1-\alpha))$$

证明:

1）由于 f 和 Φ^{-1} 都是连续的，显然，$\Psi^{-1}(\alpha) = f(\Phi^{-1}(1-\alpha))$ 是一个关于 α 的连续函数。并且由于 f 严格递减，则 $\Psi^{-1}(\alpha)$ 关于 α 严格递增。

2）推导可得:

$$\mathcal{M}\{f(\xi) \le \Psi^{-1}(\alpha)\} = \mathcal{M}\{f(\xi) \le f(\Phi^{-1}(1-\alpha))\} = \mathcal{M}\{\xi \ge \Phi^{-1}(1-\alpha)\} = 1-(1-\alpha) = \alpha$$

因此:

$$\mathcal{M}\{f(\xi) \le \Psi^{-1}(\alpha)\} = \alpha$$

由定理可知，$f(\xi)$ 的逆不确定分布为 $\Psi^{-1}(\alpha)$

2 Let ξ be an uncertain variable with regular uncertainty distribution Φ, Show that $\Phi(\xi)$ is always a linear uncertain variable $\mathcal{L}(0,1)$ whose inverse uncertainty distribution is

$$\Psi^{-1}(\alpha) = \alpha$$

证明:

由定理可知，对一个连续且严格递增的函数 f，我们有:

$$\Psi^{-1}(\alpha) = f(\Phi^{-1}(\alpha))$$

由于 Φ 满足连续和严格递增的条件，即不确定变量 $\Phi(\xi)$ 的逆分布函数:

$$\Psi^{-1}(\alpha) = \Phi(\Phi^{-1}(\alpha)) = \alpha = (1-\alpha)\cdot 0 + \alpha \cdot 1$$

因此，不确定变量 $\Phi(\xi)$ 的逆分布函数为线性不确定变量 $\mathcal{L}(0,1)$ 的逆分布函数，则 $\Phi(\xi)$ 为线性不确定变量 $\mathcal{L}(0,1)$

3 Let $\xi_1, \xi_2, ...\xi_n$ be independent uncertain variables. Show that ξ_i and ξ_j are independent for any indexes i and j with $1 \leq i < j \leq n$.

证明:

由于 $\xi_1, \xi_2, ...\xi_n$ 是独立的不确定变量，则:

$$\mathcal{M}\{\bigcap_{i=1}^{n}(\xi_i \in B_i)\} = \bigwedge_{i=1}^{n} \mathcal{M}\{\xi \in B_i\}$$

其中，$B_1, B_2, ..., B_n$ 为 $\mathbb{R}$ 上的布雷尔集。

对任意满足 $1 \leq i < j \leq n$ 的 i 和 j，令 $\xi_t(\gamma) \equiv c_t$，$c_t \in B_t$，$t \neq i, j$，此时 $\{\xi_t \in B_t\} = \Gamma$，则 $\mathcal{M}\{\xi_t \in B_t\} = 1$，因此:

$$\mathcal{M}\{\bigcap_{i=1}^{n}(\xi_i \in B_i)\} = \mathcal{M}\{(\xi_i \in B_i) \cap (\xi_j \in B_j) \cap \Gamma \cap ... \cap \Gamma\} = \mathcal{M}\{\xi \in B_i\} \wedge \mathcal{M}\{\xi \in B_j\} \wedge 1 \wedge ... \wedge 1$$

整理可得:

$$\mathcal{M}\{(\xi_i \in B_i) \cap (\xi_j \in B_j)\} = \mathcal{M}\{\xi \in B_i\} \wedge \mathcal{M}\{\xi \in B_j\}$$

因此，ξ_i 和 ξ_j 是独立的，$1 \leq i < j \leq n$。

4 Let ξ be an uncertain variable. Are ξ and $1 - \xi$ independent? Please justify your answer.

证明:

1) 当 ξ 为常量时，$1 - \xi$ 也为常量。由于常量与任何不确定变量都是独立的，因此此时 ξ 和 $1 - \xi$ 是独立的。

2) 当 ξ 不是常量时，将不确定变量 $1 - \xi$ 记为 η。令 $\xi(\gamma) = \gamma$，则 $\eta(\gamma) = 1 - \xi(\gamma) = 1 - \gamma$，因此:

$$\mathcal{M}\{(\xi(\gamma) < 0.4) \cap (\eta(\gamma) < 0.4)\} = \mathcal{M}\{(0 < \gamma < 0.4) \cap (0.6 < \gamma < 1)\} = \mathcal{M}\{\emptyset\} = 0$$

由于:

$$\mathcal{M}\{\xi(\gamma) < 0.4\} = \mathcal{M}\{0 < \gamma < 0.4\} = 0.4$$

$$\mathcal{M}\{\eta(\gamma) < 0.4\} = \mathcal{M}\{0.6 < \gamma < 1\} = 0.4$$

则:

$$\mathcal{M}\{(\xi(\gamma) < 0.4) \cap (\eta(\gamma) < 0.4)\} = 0 \neq 0.4 = \mathcal{M}\{\xi(\gamma) < 0.4\} \wedge \mathcal{M}\{\eta(\gamma) < 0.4\}$$

因此，ξ 与 η 不独立，即 ξ 与 $1 - \xi$ 不独立。

5 Construct 100 independent uncertain variables.

考虑 100 个不确定空间，分别记为 $(\Gamma_i, \mathcal{L}_i, \mathcal{M}_i),\ i=1,2,...,100$，在其中分别取不确定变量 $\xi_1(\gamma_1), \xi_2(\gamma_2), ..., \xi_{100}(\gamma_{100})$，则在乘积不确定空间中，对任意 $\mathbb{R}$ 上的布雷尔集 $B_1, B_2, ..., B_{100}$:

$$
\begin{aligned}
&\mathcal{M}\{\bigcap_{i=1}^{100}(\xi_i \in B_i)\}\\
&= \mathcal{M}\{(\gamma_1, \gamma_2, ..., \gamma_{100})|\xi_1(\gamma_1) \in B_1, \xi_2(\gamma_2) \in B_2, ..., \xi_{100}(\gamma_{100}) \in B_{100}\}\\
&= \mathcal{M}\{(\gamma_1|\xi_1(\gamma_1) \in B_1) \times (\gamma_2|\xi_2(\gamma_2) \in B_2) \times ... \times (\gamma_{100}|\xi_{100}(\gamma_{100}) \in B_{100})\}\\
&= \mathcal{M}_1\{(\gamma_1|\xi_1(\gamma_1) \in B_1)\} \wedge \mathcal{M}_2\{(\gamma_2|\xi_2(\gamma_2) \in B_2)\} \wedge ... \wedge \mathcal{M}_{100}\{(\gamma_{100}|\xi_{100}(\gamma_{100}) \in B_{100})\}\\
&= \mathcal{M}_1\{\xi_1 \in B_1\} \wedge \mathcal{M}_2\{\xi_2 \in B_2\} \wedge ... \wedge \mathcal{M}_{100}\{\xi_{100} \in B_{100}\}\\
&= \mathcal{M}\{\xi_1 \in B_1\} \wedge \mathcal{M}\{\xi_2 \in B_2\} \wedge ... \wedge \mathcal{M}\{\xi_{100} \in B_{100}\}\\
&= \bigwedge_{i=1}^{100} \mathcal{M}\{\xi_i \in B_i\}
\end{aligned}
$$

因此，不确定变量 $\xi_1(\gamma_1), \xi_2(\gamma_2), ..., \xi_{100}(\gamma_{100})$ 是独立的。