不确定规划课程作业 8

方言
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1 Construct ξ and η

构造不确定空间 $(\Gamma, L, \mathcal{M})$ 为 $\{\gamma_1, \gamma_2, \gamma_3, \gamma_4\}$

$$\mathcal{M}\{\Lambda\} = \begin{cases} 0, & \Lambda = \emptyset \\ 1, & \Lambda = \Gamma \\ c, & \gamma_1 \in \Lambda \neq \Gamma \\ 1-c, & \gamma_1 \notin \Lambda \neq \emptyset \end{cases}$$

下证 $\mathcal{M}\{\Lambda\}$ 为不确定测度:

由于 $\mathcal{M}\{\Gamma\} = 1$，$\mathcal{M}\{\emptyset\} = 0$

取 $\gamma_1 \in \Lambda$，则 $\gamma_1 \notin \Lambda^C$，此时有 $\mathcal{M}\{\Lambda\} + \mathcal{M}\{\Lambda^C\} = c + 1 - c = 1$

由此满足 Normality Axiom 和 Duality Axiom

取一个有限的事件序列 $\Lambda_1, \Lambda_2, \ldots$，显然:

$$\mathcal{M}\{\bigcup_{i=1}^{\infty} \Lambda_i\} \leq \sum_{i=1}^{\infty} \mathcal{M}\{\Lambda_i\}$$

由此满足次可加性

因此，$\mathcal{M}\{\Lambda\}$ 为不确定测度

此时可以定义两个不确定集:

$$\xi(\gamma) = \begin{cases} [0,3], & \gamma = \gamma_1 \text{ or } \gamma = \gamma_2 \\ [1,2], & \gamma = \gamma_3 \text{ or } \gamma = \gamma_4 \end{cases}$$

$$\eta(\gamma) = \begin{cases} [0,3], & \gamma = \gamma_1 \text{ or } \gamma = \gamma_3 \\ [1,2], & \gamma = \gamma_2 \text{ or } \gamma = \gamma_4 \end{cases}$$

下证 ξ 和 η 是相互独立的:

分别取 $B_1 = [0,3], B_2 = [1,2]$ 和 $B_1 = [1,2], B_2 = [0,3]$，可得:

$$
\begin{aligned}
&\mathcal{M}\{(\xi \subset B_1) \cap (\eta \subset B_2)\} = \mathcal{M}\{\gamma_2\} = 1 - c = \mathcal{M}\{(\xi \subset B_1)\} \wedge \mathcal{M}\{(\eta \subset B_2)\} \\
&\mathcal{M}\{(\xi^C \subset B_1) \cap (\eta \subset B_2)\} = \mathcal{M}\{\gamma_4\} = 1 - c = \mathcal{M}\{(\xi^C \subset B_1)\} \wedge \mathcal{M}\{(\eta \subset B_2)\} \\
&\mathcal{M}\{(\xi \subset B_1) \cap (\eta^C \subset B_2)\} = \mathcal{M}\{\gamma_1\} = c = \mathcal{M}\{(\xi \subset B_1)\} \wedge \mathcal{M}\{(\eta^C \subset B_2)\} \\
&\mathcal{M}\{(\xi^C \subset B_1) \cap (\eta^C \subset B_2)\} = \mathcal{M}\{\gamma_3\} = 1 - c = \mathcal{M}\{(\xi^C \subset B_1)\} \wedge \mathcal{M}\{(\eta^C \subset B_2)\}
\end{aligned}
$$

$$
\begin{aligned}
&\mathcal{M}\{(\xi^C \subset B_1) \cup (\eta \subset B_2)\} = \mathcal{M}\{\gamma_1, \gamma_2, \gamma_4\} = c = \mathcal{M}\{(\xi^C \subset B_1)\} \vee \mathcal{M}\{(\eta \subset B_2)\} \\
&\mathcal{M}\{(\xi^C \subset B_1) \cup (\eta \subset B_2)\} = \mathcal{M}\{\gamma_2, \gamma_3, \gamma_4\} = 1 - c = \mathcal{M}\{(\xi^C \subset B_1)\} \vee \mathcal{M}\{(\eta \subset B_2)\} \\
&\mathcal{M}\{(\xi^C \subset B_1) \cup (\eta \subset B_2)\} = \mathcal{M}\{\gamma_1, \gamma_2, \gamma_3\} = c = \mathcal{M}\{(\xi^C \subset B_1)\} \vee \mathcal{M}\{(\eta \subset B_2)\} \\
&\mathcal{M}\{(\xi^C \subset B_1) \cup (\eta \subset B_2)\} = \mathcal{M}\{\gamma_1, \gamma_3, \gamma_4\} = c = \mathcal{M}\{(\xi^C \subset B_1)\} \vee \mathcal{M}\{(\eta \subset B_2)\}
\end{aligned}
$$

由此可知 ξ 和 η 是相互独立的
由于 ξ 和 η 属于全序不确定集，其存在隶属函数 μ 和 ν，满足 $\mu(x) = \mathcal{M}\{x \in \xi\}$ 和 $\nu(x) = \mathcal{M}\{x \in \eta\}$，由此可得:

$$
\mu(x) = \begin{cases} 1, & x \in [1,2] \\ c, & x \in [0,1) \cup (2,3] \\ 0, & \text{otherwise} \end{cases}
$$

$$
\nu(x) = \begin{cases} 1, & x \in [1,2] \\ c, & x \in [0,1) \cup (2,3] \\ 0, & \text{otherwise} \end{cases}
$$

因此，$\mu \equiv \nu$，且 $\mathcal{M}\{\xi \subset \eta\} = \mathcal{M}\{\gamma_1, \gamma_3, \gamma_4\} = c$，满足条件

2 Is it possible to re-do (1) when c is below 0.5 ?

当 $c < 0.5$ 时，由于 $\mathcal{M}\{\xi \subset \eta\} = \inf_{x\in\mathbb{R}}(1 - \mu(x)) \vee \nu(x)$
若满足 $\mu \equiv \nu$，则 $\mathcal{M}\{\xi \subset \eta\} = \inf_{x\in\mathbb{R}}(1 - c) \vee c = 1 - c$，不满足 $\mathcal{M}\{\xi \subset \eta\} = c$
若满足 $\mathcal{M}\{\xi \subset \eta\} = c$，则需要:

$$
\mu(x) = \begin{cases} 1, & x \in [1,2] \\ 1 - c, & x \in [0,1) \cup (2,3] \\ 0, & \text{otherwise} \end{cases}
$$

此时不满足 $\mu \equiv \nu$
综上，当 $c < 0.5$ 时无法同时满足两个条件

3 Is it stupid to think that $\xi \subset \eta$ if and only if $\mu(x) \le \nu(x)$ for all x ?

定义:

$$
\xi(\gamma) = \begin{cases} [0,3], & \gamma = \gamma_3 \text{ or } \gamma = \gamma_4 \\ [1,2], & \gamma = \gamma_1 \text{ or } \gamma = \gamma_2 \end{cases}
$$

$$\eta(\gamma) = \begin{cases} [0,3], & \gamma = \gamma_1 \text{ or } \gamma = \gamma_3 \\ [1,2], & \gamma = \gamma_2 \text{ or } \gamma = \gamma_4 \end{cases}$$

则可以求出隶属函数:

$$\mu(x) = \begin{cases} 1, & x \in [1,2] \\ 1-c, & x \in [0,1) \cup (2,3] \\ 0, & \text{otherwise} \end{cases}$$

$$\nu(x) = \begin{cases} 1, & x \in [1,2] \\ c, & x \in [0,1) \cup (2,3] \\ 0, & \text{otherwise} \end{cases}$$

不妨取 $c = 0.2$，则 $\mathcal{M}\{\xi \subset \eta\} = \mathcal{M}\{\gamma_1, \gamma_2, \gamma_3\} = 0.2 = \inf_{x\in\mathbb{R}}(1-\mu(x)) \vee \nu(x)$
但此时，当 $x \in [0,1) \cup (2,3]$，$\mu(x) = 0.8 > \nu(x) = 0.2$，不满足题目条件

4 Is it stupid to think that $\xi = \eta$ if and only if $\mu(x) = \nu(x)$ for all x ?

定义:

$$\xi(\gamma) = \begin{cases} [0,3], & \gamma = \gamma_3 \text{ or } \gamma = \gamma_4 \\ [1,2], & \gamma = \gamma_1 \text{ or } \gamma = \gamma_2 \end{cases}$$

$$\eta(\gamma) = \begin{cases} [0,3], & \gamma = \gamma_1 \text{ or } \gamma = \gamma_3 \\ [1,2], & \gamma = \gamma_2 \text{ or } \gamma = \gamma_4 \end{cases}$$

则可以求出隶属函数:

$$\mu(x) = \begin{cases} 1, & x \in [1,2] \\ 1-c, & x \in [0,1) \cup (2,3] \\ 0, & \text{otherwise} \end{cases}$$

$$\nu(x) = \begin{cases} 1, & x \in [1,2] \\ c, & x \in [0,1) \cup (2,3] \\ 0, & \text{otherwise} \end{cases}$$

不妨取 $c = 0.2$，则 $\mathcal{M}\{\xi = \eta\} = \mathcal{M}\{\gamma_2, \gamma_3\} = 0.8 = \inf_{x\in\mathbb{R}}(1-\mu(x)) \vee \mu(x)$
但此时，当 $x \in [0,1) \cup (2,3]$，$\mu(x) = 0.8 \neq \nu(x) = 0.2$，不满足题目条件