最优化方法作业10. 2021210929 方言

1. 令 $\varphi(x) = a(x-x^{(1)})^2 + b(x-x^{(1)}) + c$

则: $\begin{cases} \varphi(x^{(1)}) = c = f_1 \\ \varphi(x^{(2)}) = a(x^{(2)}-x^{(1)})^2 + b(x^{(2)}-x^{(1)}) + c = f_2 \\ \varphi'(x^{(1)}) = b = f_1' \end{cases}$

解得: $\begin{cases} a = \left(f_2 - f_1 - f_1'(x^{(2)}-x^{(1)})\right) / (x^{(2)}-x^{(1)})^2 \\ b = f_1' \\ c = f_1 \end{cases}$

$\varphi'(x) = 2a(x-x^{(1)}) + b$, $\varphi''(x) = 2a$

1° 若 $a=0$. 则 $\varphi'(x) = f_1'$, $\varphi''(x) = 0$.

当 $f_1' = 0$ 且 $f_1 = f_2$ 时. $\bar{x}$ 可取 $x^{(1)}$ 至 $x^{(2)}$ 任意一点. 均为极小点,

2° 当 $a \neq 0$. 则 $\varphi'(\bar{x}) = 0 \Rightarrow \bar{x} = x^{(1)} - \frac{b}{2a}$, $\varphi''(\bar{x}) = 2a$

当 $a = (f_2-f_1)/(x^{(2)}-x^{(1)})^2 - f_1'/(x^{(2)}-x^{(1)}) > 0$ 时. $\varphi''(\bar{x}) > 0$

此时 $\bar{x}$ 为极小点.

当 $a < 0$ 时. $\varphi(x)$ 无极小点,

2. $f(x) = (6+x_1+x_2)^2 + (2-3x_1-3x_2-x_1x_2)^2$

则 $\frac{\partial f}{\partial x_1} = 2(6+x_1+x_2) - 2(3+x_2)(2-3x_1-3x_2-x_1x_2)$

$= 2(10x_1 + 8x_2 + 6x_1x_2 + 3x_2^2 + x_1x_2^2)$

$\frac{\partial f}{\partial x_2} = 2(10x_2 + 8x_1 + 6x_1x_2 + 3x_1^2 + x_1^2x_2)$

$\Rightarrow \frac{\partial^2 f}{\partial x_1^2} = 2(10 + 6x_2 + x_2^2)$ $\frac{\partial^2 f}{\partial x_2^2} = 2(10 + 6x_1 + x_1^2)$

$\frac{\partial^2 f}{\partial x_1 \partial x_2} = 2(8 + 6x_1 + 6x_2 + 2x_1x_2)$

因此. 牛顿方向: $d = -\nabla^2 f(\hat{x})^{-1} \nabla f(\hat{x})$

$\nabla^2 f(\hat{x}) = \begin{pmatrix} 164, & -56 \\ -56, & 4 \end{pmatrix}$ $\nabla^2 f(\hat{x})^{-1} = \frac{-1}{2480} \begin{pmatrix} 4 & 56 \\ 56 & 164 \end{pmatrix}$

则 $d = (22/31, -126/31)^T$

最速下降方向: $d = -\nabla f(\hat{x}) = -(-344, 56)^T = (344, -56)^T$