最优化方法作业13　　2021210929　　方言.

1. ① $x^{(1)} = \begin{pmatrix} 2 \\ 0 \end{pmatrix}$　起作用约束为 $I=\{3\}$.

则: $A_1 = (0 \quad 1)$　$b_1 = (0)$　$A_2 = \begin{pmatrix} 1 & 0 \\ 1 & -2 \end{pmatrix}$　$b_2 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

又由于 $f(x) = x_1^2 + x_2^2 + 2x_2 + 5$

$\nabla f(x) = (2x_1, \ 2x_2 + 2)^T$

此时 $M = (0 \quad 1)$

则 $P = I - M^T(MM^T)^{-1}M = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$

则 $d^{(1)} = -P\nabla f(x^{(1)}) = -\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 4 \\ 2 \end{pmatrix} = \begin{pmatrix} -4 \\ 0 \end{pmatrix} \neq 0$.

$\Rightarrow \quad \hat{d} = A_2 d^{(1)} = \begin{pmatrix} 1 & 0 \\ 1 & -2 \end{pmatrix}\begin{pmatrix} -4 \\ 0 \end{pmatrix} = \begin{pmatrix} -4 \\ -4 \end{pmatrix}$

$\hat{b} = b_2 - A_2 x^{(1)} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} - \begin{pmatrix} 1 & 0 \\ 1 & -2 \end{pmatrix}\begin{pmatrix} 2 \\ 0 \end{pmatrix} = \begin{pmatrix} -2 \\ -2 \end{pmatrix}$

此时可得: $\lambda_{max} = \min\{\frac{1}{2}, \frac{1}{2}\} = \frac{1}{2}$

求解: $\min f(x^{(1)} + \lambda d^{(1)})$　s.t. $0 \le \lambda \le \frac{1}{2}$

$x^{(1)} + \lambda d^{(1)} = \begin{pmatrix} 2 \\ 0 \end{pmatrix} + \lambda\begin{pmatrix} -4 \\ 0 \end{pmatrix} = \begin{pmatrix} 2-4\lambda \\ 0 \end{pmatrix}$.

则 $f(x^{(1)} + \lambda d^{(1)}) = (2-4\lambda)^2 + 5$　, $0 \le \lambda \le \frac{1}{2}$

故最优解 $\lambda_1 = \frac{1}{2}$.　$x^{(2)} = x^{(1)} + \lambda_1 d^{(1)} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

② 此时. 起约束作用为 $I=\{1,2,3\}$.　$A_1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 1 & -2 \end{pmatrix}$　$b_1 = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

令 $M = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$. 则 $P = I - M^T(MM^T)^{-1}M = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

此时 $d^{(2)} = -P\nabla f(x^{(2)}) = 0$.

令 $W = (MM^T)^{-1}M\nabla f(x^{(2)}) = \begin{pmatrix} 0 \\ 2 \end{pmatrix} \geq 0$. 故 $x^{(2)}$ 为 KKT点,

因此. 最优解为 $\bar{x} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$. 最优值 $f_{min} = 5$