THE ICS DEMPSTER-SHAFER HOW TO

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Abstract. This document is a manual with a basic and superficial overview of the theory behind and the use of Dempster-Shafer mathematics and the Evidential Reasoning Algorithm for Multiple Attribute Decision Analyses. The manual is intended for master thesis students that are using Architecture Theory Diagrams (ATD) and needs a basic mathematical tool to manage uncertainties in assessments when used in multi attribute decision analyses. The manual is partly built on an example, where the property data quality is described in an ATD. Included in the manual are instructions for the use of Excel spreadsheets to implement the algorithms. The excel sheet described can be obtained by one of the authors.

Keywords. Enterprise Architecture, Architectural Analysis, Multiple Criteria Analysis, Dempster-Shafer Theory, Dempster-Shafer Applications

INTRODUCTION

This paper aims at giving an introduction to the application of the Dempster-Shafer Evidential Reasoning Algorithm on Architectural Theory Diagrams and the associated uncertainties made in the assessments. Together with this introduction, an excel spreadsheet is enclosed in order to detail the calculations. Readers interested in more in-depth treatments on the subject of Dempster-Shafer mathematics and MADA as they are used in this manual are referred to [1] and [2]. Readers interested in a more advanced approach, extending beyond the scope of this manual, dealing with multiple attribute framework with uncertain weights are referred to [3]. Readers interested in the background to and syntax of ATDs are referred to [4].

The Evidential Reasoning Algorithm is used on so called Multiple Attribute Decision Analyses (MADA), where a complex *general property* which is usually difficult to assess directly is broken down and operationalized by using well-defined, measurable concepts that together constitute the general property. These concepts should be easy to assess. The result of such a breakdown is a multiple attribute framework having the form of a tree structure, with assessable *basic attributes* at the lowest level. The assessment of the basic attributes can then be aggregated to an assessment of the general property.

Upon assessment of basic attributes, there is always a certain level of uncertainty. Elevated information search cost is a common source of uncertainty, as it is simply too expensive to collect all the information required for a perfectly certain assessment. The Dempster-Shafer mathematics are designed to aggregate the uncertainties in the basic attributes to a total uncertainty of the total assessment. The mathematics don't aid in providing more accurate assessment of basic attributes – but it can surely be used to determine the level of uncertainty of the results obtained!

Architectural Theory Diagrams (ATD) are used to make architectural theory explicit and easy to evaluate. This paper is based on the evaluation of the general property *data quality* using an ATD. To make the assessment, the concept of data quality is operationalized in a simple ATD by defining it in terms of the properties *contextual* and *intrinsic*. These intermediate properties are in turn broken down into several basic attributes, e.g. *relevance*, *completeness*. See Figure 1.

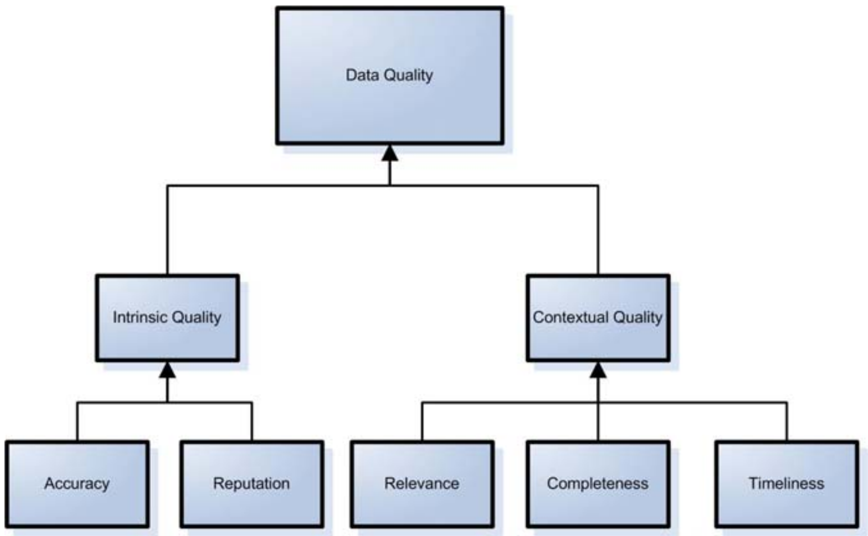


Figure 1. Evaluation of data quality represented as an ATD with three levels.

The 6 steps of Dempster-Shafer

This section presents how the Dempster-Shafer (D-S) theory may be used for evidential reasoning (ER) as stated by Yang [1]. The ER approach summarized below provides a rational process to generate an overall assessment by aggregating subjective judgments. We will describe the application of D-S-theory on one sub-tree; the reasoning is easily generalized to an entire tree consisting of several sub-trees.

Suppose there is a simple two level evaluation hierarchy with a general property at the upper level and *L* associated basic attributes at the lower level. The ER approach can then be summarized in the following 6 steps.

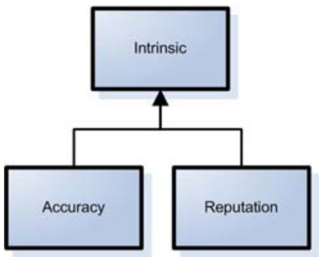
Step 1. Definition and representation of a multiple attribute decision problem.

i) Define a set of L basic attributes as follows:

$$E = \{\varepsilon_1 \varepsilon_2 \ldots \varepsilon_i \ldots \varepsilon_L\} \quad (1)$$

Suppose the breakdown is complete; the *L* basic attributes include all the factors influencing the assessment of the general attribute.

Example: Suppose we want to evaluate the property *intrinsic* data quality and that intrinsic data quality is causally determined by the basic attributes *accuracy* and *reputation*. (L=2)

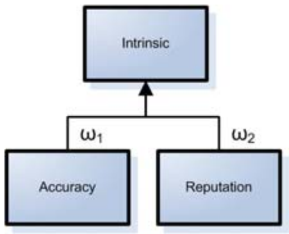


ii) Estimate the relative weights of the attributes where ω_i is the relative weight for basic attribute ε_i and is normalized so that $0 \leq \omega_i \leq 1$ and equation (2) are fulfilled.

$$\sum_{i=1}^{L} \omega_i = 1 \quad (2)$$

Please note that the sum of the weights must be one for each sub-tree.

Example: Accuracy is given the weight ω_1=0.35 whereas reputation has the weight ω_2=0.65. Please note that the sum of the weights must be one for each sub-tree. In this case, $\omega_1 + \omega_2$ = 0.35+0.65 = 1.



iii) Define *N* distinctive evaluation grades H_n as a complete set of standards for assessing each option on all attributes in the ATD.

$$H = \{H_1 H_2 ... H_n ... H_N\} \tag{3}$$

Example: The basic attributes can be assessed according to the grades *poor, average* or *good. (N=3)*

H_1=poor; H_2=average, H_3=good

iv) For each attribute ε_i and evaluation grade H_n a degree of belief β_n is assigned. The degree of belief denotes the source's level of confidence when assessing the level of fulfillment of a certain property.

Example: The basic attribute *accuracy*, is assessed to be *poor* by an observer, with a degree of belief $\beta_{1,1}$ of 40%, *average* $\beta_{2,1}$ by 50 %, and *good* to the degree of belief $\beta_{3,1}$ of 0 %.

Step 2: Basic probability assignments for each basic attribute.

Let $m_{n,i}$ be a basic probability mass representing the degree to which the *i*:th basic attribute ε_i supports a hypothesis that the general attribute is assessed to the *n*:th evaluation grade H_n.

$$m_{n,i} = \omega_i \beta_{n,i} \tag{4}$$

Example: The probability mass of *accuracy* (ε_1) being *poor* (H_1) $m_{1,1}$ is calculated with the values of the weight ω_1=0.35 and the degree of belief $\beta_{1,1}$ assigned in step 1:

$$m_{1,1} = \omega_1 \beta_{1,1} = 0{,}35 \times 0{,}4 = 0{,}14$$

The probability mass of *accuracy* (ε_1) being *average* (H_2) $m_{2,1}$ is:

$$m_{2,1} = \omega_1 \beta_{2,1} = 0{,}35 \times 0{,}5 = 0{,}175$$

And the probability mass of *accuracy* (ε_1) being *good* (H_3) $m_{3,1}$ is:

$$m_{3,1} = \omega_1 \beta_{3,1} = 0{,}35 \times 0 = 0$$

In a similar way, we calculate the probability mass of *reputation* (ε_2) using the values of the weight ω_2=0.65 and the degree of belief $\beta_{n,2}$.
Reputation's probability mass for being *poor* is:

$$m_{1,2} = \omega_2 \beta_{1,2} = 0{,}65 \times 0{,}1 = 0{,}065$$

Reputation's probability mass for being *average* is:

$$m_{2,2} = \omega_2 \beta_{2,2} = 0{,}65 \times 0{,}75 = 0{,}4875$$

Reputation's probability mass for being *good* is:

$$m_{3,2} = \omega_2 \beta_{3,2} = 0{,}65 \times 0{,}15 = 0{,}0975$$

In the enclosed excel spreadsheet, these values and calculations are shown as in Table 1 below.

Table 1: Spreadsheet with assigned weights, belief degrees and calculated probability masses.

Evaluation Grade H_1=poor, H_2=average, H_3=good	Weight	Belief				Probability Mass					
	ω_i	$\beta_{1,i}$	$\beta_{2,i}$	$\beta_{3,i}$	β_H	$m_{1,i}$	$m_{2,i}$	$m_{3,i}$	$m_{H,i}$	$\bar{m}_{H,i}$	$\tilde{m}_{H,i}$
Intrinsic *(Intermediate property)*											
Accuracy *(Operational property)*	0,35	0,4	0,5	0	0,1	0,14	0,175	0			
Reputation *(Operational property)*	0,65	0,1	0,75	0,15	0	0,065	0,4875	0,0975			

Let $m_{H,i}$ be the remaining probability mass unassigned to each basic attribute ε_i, $m_{H,i}$ is calculated as follows:

$$m_{H,i} = 1 - \sum_{n=1}^{N} m_{n,i} = 1 - \omega_i \sum_{n=1}^{N} \beta_{n,i} \tag{5}$$

Decompose $m_{H,i}$ into $\overline{m}_{H,i}$ and $\tilde{m}_{H,i}$ as follows:

$$\overline{m}_{H,i} = 1 - \omega_i$$
$$\tilde{m}_{H,i} = \omega_i \left(1 - \sum_{n=1}^{N} \beta_{n,i} \right) \tag{6}$$

With

$$m_{H,i} = \overline{m}_{H,i} + \tilde{m}_{H,i} \tag{7}$$

Example: The remaining probability mass of *accuracy* (ε1) being poor (H_1) $m_{1,1}$ is calculated with ω_1 =0,35.

$$\overline{m}_{H,i} = 1 - \omega_i$$
$$\overline{m}_{1,1} = 1 - \omega_1 = 1 - 0{,}35 = 0{,}65$$
$$\overline{m}_{1,2} = 1 - \omega_2 = 1 - 0{,}65 = 0{,}35$$
$$\tilde{m}_{1,1} = \omega_1 \left(1 - \sum_{n=1}^{N} \beta_{n,1} \right) = \omega_1 \left(1 - \left(\beta_{1,1} + \beta_{2,1} + \beta_{3,1} \right) \right) =$$
$$= 0{,}35 \left(1 - 0{,}4 - 0{,}5 - 0 \right) = 0{,}035$$
$$\tilde{m}_{1,2} = \omega_2 \left(1 - \sum_{n=1}^{N} \beta_{n,2} \right) = 0{,}65 \left(1 - 0{,}1 - 0{,}75 - 0{,}15 \right) = 0$$

Table 2 below shows how the unassigned probability masses are calculated in the spreadsheet.

Table 2: Spreadsheet with remaining, unassigned probability masses calculated.

Evaluation Grade H_1=poor, H_2=average, H_3=good	Weight	Belief				Probability Mass					
	ω_i	$\beta_{1,i}$	$\beta_{2,i}$	$\beta_{3,i}$	β_H	$m_{1,i}$	$m_{2,i}$	$m_{3,i}$	$m_{H,i}$	$\overline{m}_{H,i}$	$\tilde{m}_{H,i}$
Intrinsic *(Intermediate property)*											
Accuracy *(Operational property)*	0,35	0,4	0,5	0	0,1	0,14	0,175	0	0,685	0,65	0,035
Reputation *(Operational property)*	0,65	0,1	0,75	0,15	0	0,065	0,4875	0,0975	0,35	0,35	0

Step 3: Combined probability assignments for a general attribute

In this step, the assessments of the basic attributes that constitute the general property are aggregated to form a single assessment of the general property. The probability masses assigned to the various assessment grades as well as the probability mass left unassigned are denoted by:

$m_{n,I(L)}(n = 1,...., N)$, $\bar{m}_{H,I(L)}, \tilde{m}_{H,I(L)}$,

and

$m_{H,I(L)}$

Let I(1)=1, then we get

$m_{n,I(1)} = m_{n,1}(n = 1,..., N),\ \bar{m}_{H,I(1)} = \bar{m}_{H,1}, \tilde{m}_{H,I(1)} = \tilde{m}_{H,1}$

and

$m_{H,I(1)} = m_{H,1}$.

The combined probability masses can be generated by aggregating all the basic probability assignments using the following recursive ER algorithm:

$$\{H_n\}: \tag{8}$$
$$m_{n,I(i+1)} = K_{I(i+1)}\left[m_{n,I(i)} \cdot m_{n,i+1} + m_{H,I(i)} \cdot m_{n,i+1} + m_{n,I(i)} \cdot m_{H,i+1}\right]$$
n=1,…,N

We continue to let i=1, which leads to that in (8) the term

$m_{n,1}, m_{n,2}$ measures the degree of both attributes ε_1 and ε_2 supporting the general attribute *y* to be assessed to H_n.

The term $m_{n,1}, m_{H,2}$ measures the degree of only ε_1 supporting y to be assessed to H_n.

The term $m_{H,1}m_{n,2}$ measures the degree of only ε_2 supporting y to be assessed to H_n.

$$\{H\}:$$
$$m_{H,I(i)} = \tilde{m}_{H,I(i)} + \bar{m}_{H,I(i)} \tag{9}$$
$$\tilde{m}_{H,I(i+1)} = K_{I(i+1)}\left[\tilde{m}_{H,I(i)} \cdot \tilde{m}_{H,i+1} + \bar{m}_{H,I(i)} \cdot \tilde{m}_{H,i+1} + \tilde{m}_{H,I(i)} \cdot \bar{m}_{H,i+1}\right] \tag{10}$$
$$\bar{m}_{H,I(i+1)} = K_{I(i+1)}\left[\bar{m}_{H,I(i)} \cdot \bar{m}_{H,i+1}\right] \tag{11}$$
$$K_{I(i+1)} = \left[1 - \sum_{t=1}^{N}\sum_{\substack{j=1\\ j\neq t}}^{N} m_{t,I(i)} \cdot m_{j,i+1}\right]^{-1} \tag{12}$$
$$i = \{1,2,..., L-1\}$$

In (10)**Fel! Hittar inte referenskälla.**, the term $\tilde{m}_{H,1}\tilde{m}_{H,2}$ measures the degree to which y cannot be assessed to any individual grades due to the incomplete assessments for both ε_1 and ε_2.

The term $\tilde{m}_{H,1}\bar{m}_{H,2}$ measures the degree to which *y* cannot be assessed due to the incomplete assessments for ε_1 only;.

The term $\bar{m}_{H,1}\tilde{m}_{H,2}$ measures the degree to which *y* cannot be assessed due to the incomplete assessments for ε_2 only.

In (11), the term $\bar{m}_{H,1}\bar{m}_{H,2}$ measures the degree to which *y* has not yet been assessed to individual grades due to the relative importance of ε_1 and ε_2 after ε_1 and ε_2 have been aggregated.

$K_{I(2)}$ as calculated by (12) is used to normalize $m_{n,I(2)}$ and $m_{H,I(2)}$ so that

$$\sum_{n=1}^{N} m_{n,I(2)} + m_{H,I(2)} = 1$$

Example: *Accuracy* is assessed as shown above. *Reputation* is rated as *poor* with a belief of 10 %, *average* with a belief of 75 % and *good* with a belief of 15 %. We are now using the recursive ER algorithm to aggregate the probability masses of the basic attributes to the intermediate property *intrinsic data quality*. Recall from the examples above that the equation (4) gave:

$$m_{1,1} = 0{,}14; \quad m_{2,1} = 0{,}175; \quad m_{3,1} = 0$$

$$m_{1,2} = 0{,}065; \quad m_{2,2} = 0{,}4875; \quad m_{3,2} = 0{,}0975$$

These probability masses combined with the unassigned probability masses calculated using equations (6) and (7) gave:

$$m_{H,1} = 0{,}685; \quad \bar{m}_{H,1} = 0{,}65; \quad \tilde{m}_{H,1} = 0{,}035;$$

$$m_{H,1} = 0{,}35; \quad \bar{m}_{H,1} = 0{,}35; \quad \tilde{m}_{H,1} = 0;$$

Putting these values into equations (8) and (12) above, this gives us an assessment of the general property intrinsic data quality for the grade *poor:*

$$\{H_1\}:$$

$$m_{1,I(2)} = K_{I(2)}\left[m_{1,1} \cdot m_{1,2} + m_{H,1} \cdot m_{1,2} + m_{1,2} \cdot m_{H,2}\right] = 0{,}1154$$

$$i = \{1,2,\ldots,\mathrm{L}-1\}; \quad \mathrm{L} = \text{number of attributes} = 2$$

The computations for the grades *average* and *good* are similar to the intrinsic data quality property.

The degree to which intrinsic data quality cannot be assessed to any particular grade, due to the incomplete assessments of ε_1 and ε_2 is computed as follows

(bearing in mind that $m_{H,I(i)} = \tilde{m}_{H,I(i)} + \bar{m}_H$):

$$\{H\}:$$

$$K_{I(2)} = \left[1 - \sum_{t=1}^{3}\sum_{\substack{j=1 \\ j\neq 1}}^{3} m_{t,I(i)} \cdot m_{j,i+1}\right]^{-1} = 1{,}12402$$

$$\tilde{m}_{H,I(2)} = 1{,}12402 \cdot [\tilde{m}_{H,I(1)} \cdot \tilde{m}_{H,2} + \bar{m}_{H,I(1)} \cdot \tilde{m}_{H,2} + \tilde{m}_{H,I(1)} \cdot \bar{m}_{H,2}] =$$
$$= 0{,}0138$$

$$\bar{m}_{H,I(2)} = 1{,}12402 \cdot [\bar{m}_{H,I(1)} \cdot \bar{m}_{H,2}] = 0{,}2557 \Rightarrow$$
$$\Rightarrow m_{H,I(2)} = 0{,}2557 + 0{,}0138 = 0{,}2695$$

which give us all information necessary for the next step.

Table 3 gives an example of how the aggregated probability masses appear in the excel spreadsheet.

Table 3: Spreadsheet with aggregated probability masses.

Evaluation Grade H_1=poor, H_2=average, H_3=good	Weight	Belief				Probability Mass						Constant	Probability Mass (aggregation)					
	ω_i	$\beta_{1,i}$	$\beta_{2,i}$	$\beta_{3,i}$	β_H	$m_{1,i}$	$m_{2,i}$	$m_{3,i}$	$m_{H,i}$	$\bar{m}_{H,i}$	$\tilde{m}_{H,i}$	$K_{I(i+1)}$	$m_{1,I(i+1)}$	$m_{2,I(i+1)}$	$m_{3,I(i+1)}$	$m_{H,I(i+1)}$	$\bar{m}_{H,I(i+1)}$	$\tilde{m}_{H,I(i+1)}$
Intrinsic *(Intermediate property)*																		
Accuracy *(Operational property)*	0,35	0,4	0,5	0	0,1	0,14	0,175	0	0,685	0,65	0,035							
Reputation *(Operational property)*	0,65	0,1	0,75	0,15	0	0,065	0,4875	0,0975	0,35	0,35	0	1,12402	**0,1154**	**0,5401**	**0,0751**	**0,2695**	**0,2557**	**0,0138**

Step 4: Calculation of the combined degrees of belief for a general property.

Let β_n denote the combined degree of belief that the general property assessed to the grade H_n, which is generated by combining the assessments for all the associated basic attributes ε_i.
β_n is then calculated by:

$$
\begin{aligned}
&\{H_n\}:\ \beta_n = \frac{m_{n,I(L)}}{1-\bar{m}_{H,I(L)}} \qquad n = 1, 2, \ldots, N \\
&\{H\}:\ \beta_H = \frac{\tilde{m}_{H,I(L)}}{1-\bar{m}_{H,I(L)}}
\end{aligned}
\tag{13}
$$

Example: Using (13) and with the values calculated in step 3, we get the combined degrees of belief for intrinsic data quality.

$$\beta_{1,1} = \frac{m_{1,I(L)}}{1-\bar{m}_{H,I(L)}} = \frac{0{,}1154}{1-0{,}2557} = 0{,}155$$

$$\beta_{2,1} = \frac{m_{2,I(L)}}{1-\bar{m}_{H,I(L)}} = \frac{0{,}5401}{1-0{,}2557} = 0{,}7257$$

$$\beta_{3,1} = \frac{m_{3,I(L)}}{1-\bar{m}_{H,I(L)}} = \frac{0{,}0751}{1-0{,}2557} = 0{,}1009$$

$$\beta_{H} = \frac{\tilde{m}_{H,I(L)}}{1-\bar{m}_{H,I(L)}} = \frac{0{,}0138}{1-0{,}2557} = 0{,}0185$$

Table 4 below shows the appearance of the spreadsheet where the aggregated belief degrees have been calculated.

Table 4: Spreadsheet with aggregated belief degrees.

Evaluation Grade H_1=poor, H_2=average, H_3=good	Weight	Belief				Probability Mass						Constant	Probability Mass (aggregation)					
	ω_i	$\beta_{1,i}$	$\beta_{2,i}$	$\beta_{3,i}$	β_H	$m_{1,i}$	$m_{2,i}$	$m_{3,i}$	$m_{H,i}$	$\bar{m}_{H,i}$	$\tilde{m}_{H,i}$	$K_{I(i+1)}$	$m_{1,I(i+1)}$	$m_{2,I(i+1)}$	$m_{3,I(i+1)}$	$m_{H,I(i+1)}$	$\bar{m}_{H,I(i+1)}$	$\tilde{m}_{H,I(i+1)}$
Intrinsic *(Intermediate property)*		**0,155**	**0,7257**	**0,1009**	**0,0185**													
Accuracy *(Operational property)*	0,35	0,4	0,5	0	0,1	0,14	0,175	0	0,685	0,65	0,035							
Reputation *(Operational property)*	0,65	0,1	0,75	0,15	0	0,065	0,4875	0,0975	0,35	0,35	0	1,12402	0,1154	0,5401	0,0751	0,2695	0,2557	0,0138

Step 1-4 can now be employed for the other sub-tree with contextual data quality as the general property, see Figure 1. Then, finally, using steps 1-4 once more, the actual data quality can be assessed. In this sub-tree, data quality is the general property and, the contextual and intrinsic data qualities are the basic attributes. Table 5 below shows the assigned values and the aggregated belief degrees for the entire data quality ATD.

Table 5: Spreadsheet with aggregated belief degrees for the entire data quality ATD.

Evaluation Grade ε_1=poor, ε_2=average, ε_3=good	Weight	Belief				Probability Mass						Constant	Probability Mass (aggregation)					
	ω_i	$\beta_{1,i}$	$\beta_{2,i}$	$\beta_{3,i}$	β_H	$m_{1,i}$	$m_{2,i}$	$m_{3,i}$	$m_{H,i}$	$\bar{m}_{H,i}$	$\tilde{m}_{H,i}$	$K_{I(i+1)}$	$m_{1,I(i+1)}$	$m_{2,I(i+1)}$	$m_{3,I(i+1)}$	$m_{H,I(i+1)}$	$\bar{m}_{H,I(i+1)}$	$\tilde{m}_{H,I(i+1)}$
Data Quality *(Abstract property)*		**0,2695**	**0,6097**	**0,0872**	**0,0336**													
Intrinsic *(Intermediate property)*	0,6	0,155	0,7257	0,1009	0,0185	0,093	0,4354	0,0605	0,4111	0,4	0,0111							
Contextual *(Intermediate property)*	0,4	0,534	0,2979	0,0823	0,0857	0,2136	0,1192	0,0329	0,6343	0,6	0,0343	1,16499	0,1942	0,4393	0,0628	0,3038	0,2796	0,0242
Intrinsic *(Intermediate property)*																		
Accuracy *(Operational property)*	0,35	0,4	0,5	0	0,1	0,14	0,175	0	0,685	0,65	0,035							
Reputation *(Operational property)*	0,65	0,1	0,75	0,15	0	0,065	0,4875	0,0975	0,35	0,35	0	1,12402	0,1154	0,5401	0,0751	0,2695	0,2557	0,0138
Contextual *(Intermediate property)*																		
Relevance *(Operation property)*	0,45	0,6	0,2	0,05	0,15	0,27	0,09	0,0225	0,6175	0,55	0,0675							
Completeness *(Operational property)*	0,25	0,25	0,45	0,3	0	0,0625	0,1125	0,075	0,75	0,75	0	1,07174	0,2765	0,1576	0,0695	0,4963	0,4421	0,0543
Timeliness *(Operational property)*	0,3	0,55	0,35	0	0,1	0,165	0,105	0	0,73	0,7	0,03	1,0797	0,3556	0,1984	0,0548	0,3912	0,3341	0,0571

Step 5: Representation of the distributed overall assessment and calculation of the expected utility.

In this step, the utilities of the respective assessment grades $H_{1...n}$ are estimated via utility functions $u(H_n)$. This estimation can be accomplished for instance by means of a range of methods and techniques that can be utilized for this purpose. In this document however we will not dwell on the subject of utility estimations, rather we assume that the utilities of the respective assessments grade can be appreciated in a linear fashion. The interested reader is kindly refered to works in game and decision theory, or economics.

Example: The evaluation grades' utilities are estimated to be.

H_1={poor}=0

H_2={average}=0,5

H_3={good}=1

This estimation is not based on any empirical data as to what respondents mean when they say poor, average or good respectively.

Under the assumption that the assessment of the general property is *complete*, the utility of the assessed property is calculated according to

$$u = \sum_{n=1}^{N} \beta_n u(H_n) . \qquad (14)$$

Example: Assuming the utility for each assessment grade and the beliefs for each assessment grade for the general property data quality are **completely** assessed as follows (using (14)):

$u(H_1)=0$; $\beta_1=0{,}2695$

$u(H_2)=0{,}5$ $\beta_2= 0{,}6097$

$u(H_3)=1$ $\beta_3= 0{,}0872$

The total expected utility of this (complete) assessment is thus:

u=0,2695*0+0,6097*0,5+0,0872*1=0,3921

The complete assessment of a property is a rare occurrence in this resource-scarce world, the cost of collecting data is very high, and we therefore look towards the case where we have incomplete assessments, with unassigned belief, see Step 6 in the next chapter.

Step 6: Calculation of the utility interval of incomplete assessments.

Provided we have some belief β_H left unassigned in the assessments we can somewhat arbitrarily calculate an *utility interval* for the general property being assessed. This interval is calculated as follows:

$$u_{\max} = \sum_{n=1}^{N-1} \beta_n u(H_n) + (\beta_N + \beta_H) u(H_N) \quad (15)$$

$$u_{\min} = (\beta_1 + \beta_H) u(H_1) + \sum_{n=2}^{N} \beta_n u(H_n) \quad (16)$$

$$u_{avg} = \frac{u_{\max} + u_{\min}}{2} \quad (17)$$

Example: Assuming that the utility for each assessment grade and that the belief degrees for each assessment grade for the general property data quality are **incompletely** assessed as follows:

$u(H_1)=0$ $\beta_1=0{,}2695$

$u(H_2)=0{,}5$ $\beta_2= 0{,}6097$

$u(H_3)=1$ $\beta_3= 0{,}0872$

with a degree of belief left unassigned:

$\beta_H=0{,}0336$

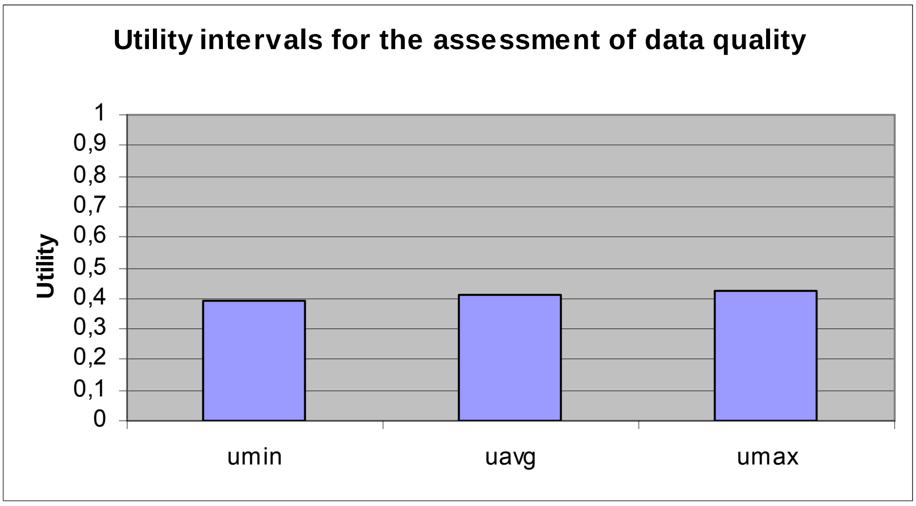
the utility interval can then be caluclated using eq. (15)-(17) as:

$u_{max}=0{,}2695*0+0{,}6097*0{,}5+(0{,}0872+0{,}0336)*1=0{,}4257$

$u_{min}=(0{,}2695+0{,}0336)*0+0{,}6097*0{,}5+0{,}0872*1=0{,}3921$

$u_{avg}=(0{,}4257+0{,}3921)s/2=0{,}4089$

This can be presented in a diagram as shown below.



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