**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

Ans) Given, *μ* = 45 minutes and *σ* = 8 minutes

Work starts after 10 minutes of the drop off that is Time left for work to complete is 1 hour – 10 minutes = 50 minutes.

Probability that the service manager can meet his commitment is P ≤ 50

Z score for P ≤ 50 is

Z = (X - µ)/

Z = (50 – 45) / 8 = 0.734

Probability that the service manager cannot meet his commitment is

P (X > 50) = 1 – P (X ≤ 50) = 1 – 0.734 = 0.26

(or)

1 - stats.norm.cdf(50,45,8)

= 0.2659

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.

Given statement P(x>44) >P(38 ≤x ≤44)

P(x>44) = 1 – P(x≤44)

P(x>44)

1-stats.norm.cdf(44,38,6)

1-0.8413= 0.1587 = 15.87%

P(38 ≤x ≤44)

stats.norm.cdf(44,38,6)-stats.norm.cdf(38,38,6)

P(38 ≤x ≤44)= 0.3413 = 34.13%

From above, the probability of employees being between the age 38 and 44 are more than that of the employees of age above 44.

Hence, the given statement is FALSE.

1. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

employees under the age of 30 = P(x<30)

stats.norm.cdf(30,38,6)

0.0912

P(x<30)= 0.0912

Total employees = 400

Employees under the age of 30 = 0.0912\*400 = 36.48

Hence, A training program for employees under the age of 30 at the center would be expected to attract about 36 employees is TRUE.

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

Ans ) Given, *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables

2X1 ~ N(2 μ, 2σ2)

X1+X2 ~ N(2 μ, 2σ2)

Both are Normally distributed

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

Solution: Given X is normally distributed and X ~ N(100, 202) and two values a and b

probability of the random variable taking a value between them is 0.99.

µ = 100 and = 20

stats.norm.interval(0.99,loc=100,scale=20)

(48.48341392902199, 151.516586070978)

Hence, Value of a = 48.48341 , value of b = 151.5165

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

Given Division 1 Profit1 ~ N(5, 32) and division 2 Profit2 ~ N(7, 42)

The total collective mean of both the divisions is Mean = 5 + 7 = 12$ Million

Mean in rupees = 12\*45 = 540 million

The total collective SD of both the divisions is SD2= SD12+SD22 = 9+16

SD = 25

SD in rupees = 25\*45 = 225 million

Range is

np.round(stats.norm.interval(0.95,540,225),2)

([ 99.01, 980.99])

1. Specify the 5th percentile of profit (in Rupees) for the company

stats.norm.cdf(0.5,540,225)

0.008247434450918537

1. Which of the two divisions has a larger probability of making a loss in a given year?

Probability of division 1 making a loss for the given year is

stats.norm.cdf(0,5,3)

0.04779

Probability of division 1 making a loss for the given year is

stats.norm.cdf(0,7,4)

0.04005

Hence, The Probability of division 1 making a loss for the given year is more than that of the Probability of division 1 making a loss for the given year.