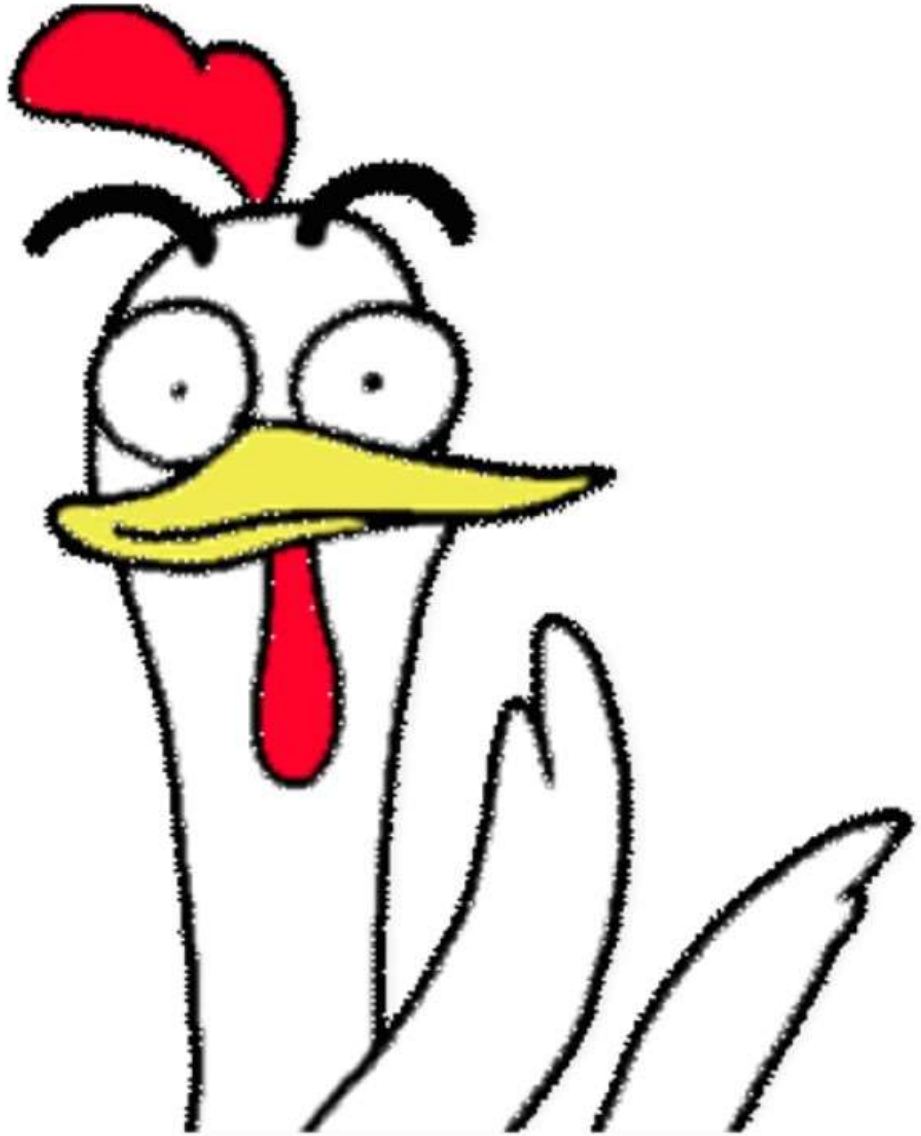
Arithmetic Sequence and Series

Lesson 2



Objectives

- Illustrates Arithmetic Sequence
- Determines the nth term and the sum of the terms of a given arithmetic sequence

What is an Arithmetic Sequence?

An Arithmetic Sequence, also known as arithmetic progression, is a sequence of numbers such that the difference between the consecutive terms is constant. Difference here means the second minus the first.

The nth term of the sequence $(a_{n)}$ with first term a_1 and common difference d is given by the formula

$$a_n = a_1 + (n-1)d$$

Example 1:
Find the 8th term of an arithmetic sequence 5, 7, 9, 11, …

Solution:

In the given, the first term which is a_1 is 5. To get the common difference, we will subtract the 2nd term with the 1st term and the 3rd term with the 2nd term(they must have a common answer). In the example, 7 minus 5 is 2 and 9 minus 7 is 2. Since 2 is equal to 2, then the common difference is 2. Now we will use the formula for finding the nth term of the arithmetic sequence.

Example 1:
Find the 8th term of an arithmetic sequence 5, 7, 9, 11, …

Solution:

It will be

$$a_8 = 5 + (8 - 1)2$$

$$a_8 = 5 + (7)2$$

$$a_8 = 19$$

Therefore the 8th term of the arithmetic sequence 5, 7, 9, 11, … is 19.

Example 2:
Find the fourth term of the arithmetic sequence with the first term 10, and the common difference –2.

Solution:

This time, both first term and the difference are given. We can now proceed directly to the formula

$$a_4 = 10 + (4-1)(-2)$$

$$a_4 = 10 + (3)(-2)$$

$$a_4 = 4$$

Therefore, the 4th term of the arithmetic sequence with the first term 10 and common difference –2 is 4.

Practice Makes Perfect!

1. Find the 9th term of the sequence 1, 7, 13, 19, …

2. Find the 11th term of the sequence with the first term 11 and a common difference of –1.

What is an Arithmetic Series?

An Arithmetic Series is the sum of all the n-terms of an arithmetic sequence.

The arithmetic series is given by the formula

$$S_n = (n/2)(a_1 + a_n)$$

Example 1:

Find the sum of the first 20 terms of the arithmetic sequence 17, 22, 27, 32, ...

Solution:

From the given, we have the first term which is 17 and common difference is 5. We have to find the 20th term before we can use the formula for finding the sum of n-terms of the sequence. To get the 20th term, we will use the formula for finding the nth term of the arithmetic sequence.

Example 1:
Find the sum of the first 20 terms of the arithmetic sequence 17, 22, 27, 32, ...

Solution:

It will be

$$a_{20} = 17 + (20-1)5$$

$$a_{20} = 10 + (19)5$$

$$a_{20} = 105$$

105 is our 20th term.

Example 1:

Find the sum of the first 20 terms of the arithmetic sequence 17, 22, 27, 32, ...

Solution:

Now we have our 20th term so we can then proceed to solve for the sum of the first 20 terms of the sequence. It will be

$$S_{20} = (20/2)(17 + 105)$$

$$S_{20} = (10)(122)$$

$$S_{20} = 1220$$

Therefore, the sum of the first 20 terms of the arithmetic sequence 17, 22, 17, 32, ... is 1220.

Example 2:
Find the sum of the first 8 terms of a sequence with first term 18 and common difference of –5.

Solution:

From the given, we have $a_1 = 18$ and $d = -5$ but there the 8^{th} term is not given. We have to find it first using the same method we did in previous example. It will be

$$a_8 = 18 + (8-1)(-5)$$

$$a_8 = 18 + (7)(-5)$$

$$a_8 = -17$$

Then, it will be

$$. S_8 = (8/2)(18 - 17)$$

$$S8 = (4)(1)$$

$$S_8 = 4$$

Therefore, the sum of the first 8 terms of a sequence with first term 18 and common difference of -5 is 4.

Practice Makes Perfect!

1. Find the sum of the first 8 terms of the sequence 6, 18, 30, 42, …

2. Find the sum of the first 11 terms of the sequence with the first term 7 and common difference of –2