Arithmetic Means and Geometric Means

LESSON 4

Objectives

Determines the Arithmetic Means

Determines the Geometric Means

Solves problems involving sequences

What is an Arithmetic Mean?

The Arithmetic Mean, or simply the mean or average when the context is clear, is the sum of a collection of numbers divided by the number of numbers in the collection.

Example 1:
Give an arithmetic mean between 1 and 9

Solution:

Simply, add 1 and 9 which gives us 10. Then divide 10 by the total number of terms that we added which is 2 terms, so 10 divided by 2 is 5.

Example 1:
Give an arithmetic mean between 1 and 9

Solution:

Therefore, 5 is an arithmetic mean between 1 and 9 and is written like this

1, 5, 9

Example 1:
Give an arithmetic mean between 1 and 9

Solution:

Therefore, the arithmetic mean between 1 and 9 is 5.

Note: There should be common difference in the generated sequence for it to be arithmetic.

Example 2:
Give 3 arithmetic means between 1 and 9.

Solution:

We will do the same process earlier. Add 1 and 9, then divide by 2 and we'll get 5. Now we have something like this

1, __, 5, __, 9

Example 2:
Give 3 arithmetic means between 1 and 9

Solution:

We need to get 2 more means and you can start to any side you want. Let's start in the left side first. Add 1 and 5, then divide by 2 and we'll get 3. Then do the same to the other side.

Example 2:
Give 3 arithmetic means between 1 and 9.

Solution:

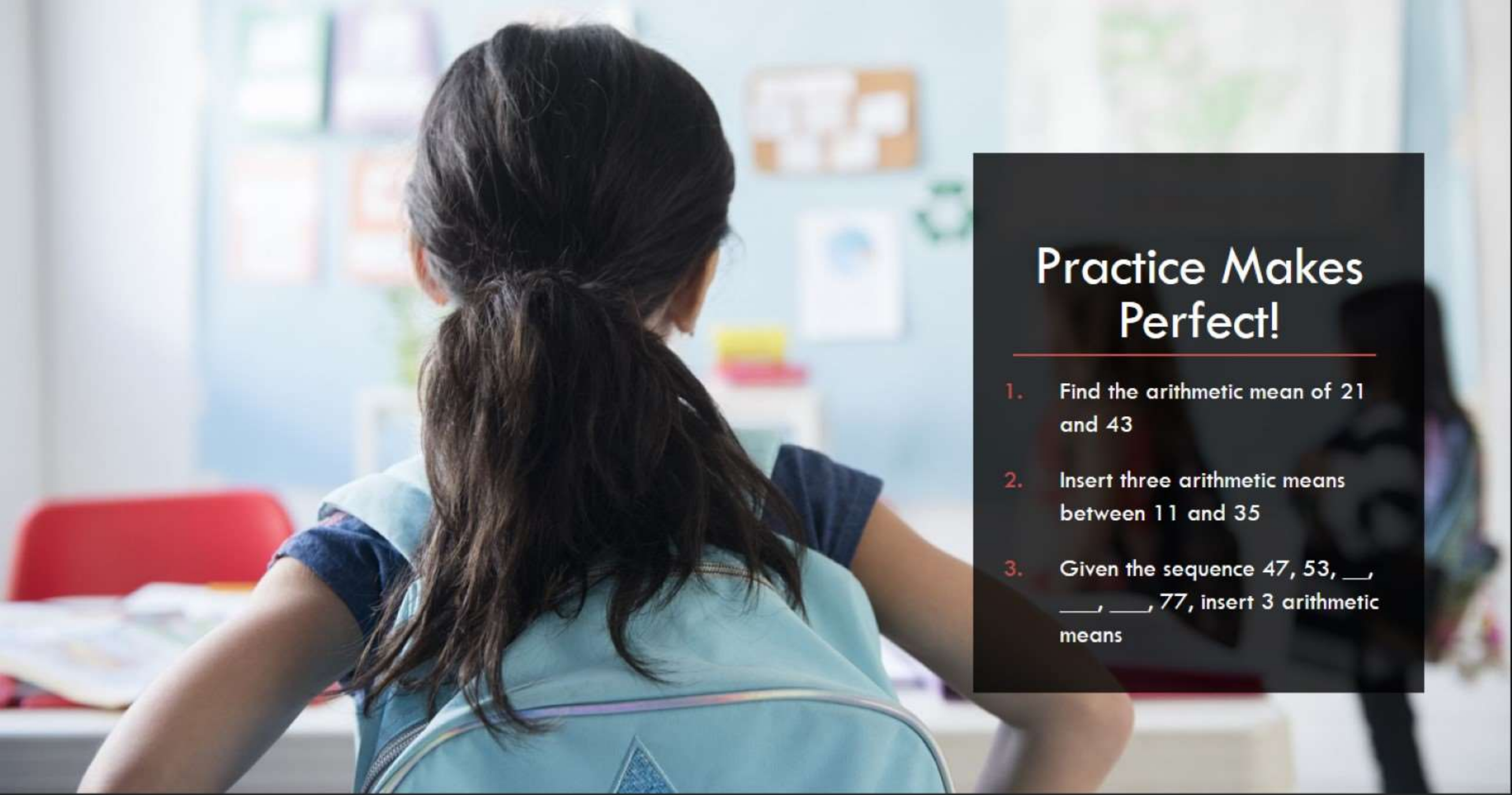
Add 5 and 9, then divide by 2 and we'll get 7. Now we have something like this

1, 3, 5, 7, 9

Example 2:
Give 3 arithmetic means between 1 and 9

Solution:

Therefore, the 3 arithmetic means between 1 and 9 are 3, 5 and 7.



Practice Makes Perfect!

1. Find the arithmetic mean of 21 and 43
2. Insert three arithmetic means between 11 and 35
3. Given the sequence 47, 53, __, ___, ___, 77, insert 3 arithmetic means

What is a Geometric Mean?

In mathematics, the geometric mean is a mean or average, which indicates the typical value of a set of numbers by using the product of their values (as opposed to the arithmetic mean which uses their sum). The geometric mean is defined as the nth root of the product of n numbers.

Example 1:
Find the geometric mean between 2 and 8

Solution:

First, we will multiply the two terms 2 and 8 which gives us 16. Then, we will get the nth root of that product. Since we used 2 terms(which is 2 and 8) only, then we will get the square root(n = 2) of the product. The square root of 16 is 4 or -4.

Example 1:
Find the geometric mean between 2 and 8

Solution:

It is written like this

2, 4, 8 or 2, -4, 8

Therefore, 4 or –4 is the geometric mean between 2 and 8.

Note: There should be common ratio in the generated sequence for it to be geometric

Example 2:
Find 3 geometric means between 1 and 81

Solution:

We will do the same with the previous process. Multiplying 1 and 81 will give us 81. Then the square root of 81 is 9 and -9. Then, it will look like this

1, ___, 9, ___, 81

and

1, ___, -9, ___, 81

Example 2:
Find 3 geometric means between 1 and 81

Solution:

Next, we will do the same with the left side and then right side of the first sequence. Multiplying 1 and 9 is 9 and then the square root of 9 is 3 and –3. Multiplying 9 and 81 is 729 and then the square root of 729 is 27 and –27.

Example 2:
Find 3 geometric means between 1 and 81

Solution:

The first sequence will look like this

1, 3, 9, 27, 81

And

1, -3, 9, -27, 81

For the second sequence. Multiplying 1 and –9 is –9 then the square root of –9 is imaginary.

Example 2:
Find 3 geometric means between 1 and 81

Solution:

Multiplying –9 and 81 is –729 and the square root of –729 is imaginary. This is not a proper sequence.

Therefore, the 3 geometric means are 3, 9, 27 or –3, 9, -27.

Practice Makes Perfect!

1. Find the geometric mean of 9 and 16
2. Insert three geometric means between 5 and 1280
3. Given the sequence 2, 10, __, ___, ___, 6250, insert 3 geometric means

How can we apply sequences in problems?

SOME WORD PROBLEMS CAN BE SOLVED USING SEQUENCES.

HERE ARE SOME EXAMPLES

Example 1:

Martin received 12 rare stamps as a gift from his grandfather, so he decided to start a stamp collection. From the following week onward, Martin added 4 new stamps to his collection each week. How many stamps will Martin have after 5 weeks?

Solution:

Since it is mentioned that the number of stamps is added by 4 as week goes by, then it means that we're dealing with arithmetic sequence. From here on, we can use the formula for arithmetic sequence

$$a_5 = 12 + (5 - 1)4$$

Example 1:

Martin received 12 rare stamps as a gift from his grandfather, so he decided to start a stamp collection. From the following week onward, Martin added 4 new stamps to his collection each week. How many stamps will Martin have after 5 weeks?

Solution:

We used A5 because we have to find the number of stamps after 5 weeks, then our common difference is 4 and the first term is 12. Simplifying this will give us

$$a_5 = 12 + 4(4)$$
$$a_5 = 12 + 16$$
$$a_5 = 28$$

Therefore, Martin will have 28 stamps after 5 weeks.

Solution:

The situation can be modeled by a geometric sequence with an initial term of 284. The student population will be 104% of the prior year, so the common ratio is 1.04.

Let a be the student population and n be the number of years after 2013. Using the explicit formula for a geometric sequence we get

$$a_n = 284(1.04)^n$$

Example 2:

In 2013, the number of students in a small school is 284. It is estimated that the student population will increase by 4% each year. What will be the population of students after 2020?

Solution:

We can find the number of years since 2013 by subtracting.

$$2020 - 2013 = 7$$

We are looking for the population after 7 years. We can substitute 7 for n to estimate the population in 2020.

$$a_7 = 284\,(1.04)^7$$
$$a_7 = 374$$

Therefore, the student population will be about 374 in 2020.

Example 2:

In 2013, the number of students in a small school is 284. It is estimated that the student population will increase by 4% each year. What will be the population of students after 2020?

Practice Makes Perfect!

1. A Theater has 30 seats in the first row of the center section. Each row behind the first row gains 2 additional seats. How many seats are there in the 5th row in the center section?

2. A research laboratory is to begin experimentation with bacteria that doubles every 4 hours. The laboratory starts with 200 bacteria. How many bacteria will be present at the end of the 12th hour?