

Geometric Sequence and Series

LESSON 3

Objectives

- Illustrates Geometric Sequence
- Determines the sum of the terms of a geometric sequence
- Differentiates arithmetic sequence and geometric sequence





What is a Geometric Sequence?

A Geometric Sequence, also known as a geometric progression, is a sequence of numbers where each term after the first is found by multiplying the previous one by a fixed, non-zero number called the common ratio

The n th term, a_n , of a geometric sequence with first term a_1 and common ratio r is given by the formula

$$a_n = a_{n-1} \cdot r \quad \text{or} \quad a_n = a_1 \cdot r^{n-1}$$

Example 1:

Find the 8th term of a geometric sequence 1, 2, 4, 8, ...

Solution:

In the given, the first term which is a_1 is 1. To get the common ratio, we will divide the 2nd term with the 1st term and the 3rd term with the 2nd term (they must have a common answer). In the example, 2 divided by 1 is 2 and 4 divided by 2 is 2. Since 2 is equal to 2, then the common ratio is 2.



Example 1:

Find the 8th term of a geometric sequence 1, 2, 4, 8, ...



Solution:

Now we will use the formula for finding the n th term of the geometric sequence. It will be

$$a_8 = 1 \cdot 2^{(8-1)}$$

$$a_8 = 1 \cdot 128$$

$$a_8 = 128$$

Therefore, the 8th term of the geometric sequence 1, 2, 4, 8, ... is 128.

Example 2:

Find the fourth term of the geometric sequence with the first term 5, and the common ratio -2

Solution:

This time, both first term and the ratio are given. We can now proceed directly to the formula


$$a_4 = 5 \cdot (-2)^{(4-1)}$$

$$a_4 = 5 \cdot -8$$

$$a_4 = -40$$

Therefore, the 4th term of the geometric sequence with the first term 5 and common ratio -2 is -40 .





Practice Makes Perfect!

1. Find the 10th term of the sequence 1, 3, 9, 27, ...
2. Find the 11th term of the sequence with the first term 11 and a common ratio of -2 .



What is a Geometric Series?

A Geometric Series is the sum of n -terms of a geometric sequence. The geometric series is given by the formula

$$S_n = a_1(1 - r^n)/(1 - r)$$

Example 1:

Find the sum of the first 12 terms of a geometric sequence
3, -9, 27, -81, ...

Solution:

From the given, we have the first term which is 3, and the common ratio which is -3 . We can now use the formula for finding the geometric series. It looks like this

$$S_{12} = 3[1 - (-3)^{12}]/[1 - (-3)]$$

$$S_{12} = 3(1 - 531441)/(1 + 3)$$

$$S_{12} = 3(-531440)/4$$

$$S_{12} = -398580$$



Example 1:

Find the sum of the first 12 terms of a geometric sequence
 $3, -9, 27, -81, \dots$



Solution:

Therefore, the sum of the first 12 terms of a geometric sequence $3, -9, 27, -81, \dots$ is -398580 .

Example 2:

Find the sum of the first 8 terms of a sequence with first term 4 and common ratio of -5

Solution:

From the given, we have $a_1 = 4$ and $r = -5$. Using the formula for geometric series, we have

$$S_8 = 4[1 - (-5)^8]/[1 - (-5)]$$

$$S_8 = 4(1 - 390625)/(1 + 5)$$

$$S_8 = 4(-390624)/6$$

$$S_8 = -260416$$




Example 2:

Find the sum of the first 8 terms of a sequence with first term 4 and common ratio of -5



Solution:

Therefore, the sum of the first 8 terms of a sequence with first term 4 and common ratio of -5 is -260416 .



Practice Makes Perfect!

1. Find the sum of the first 8 terms of the sequence 6, 18, 54, 162, ...
2. Find the sum of the first 11 terms of the sequence with first term 4 and common ratio of -2

What is the difference between an Arithmetic Sequence and Geometric Sequence?

Once again, a sequence is a set of numbers, called terms, arranged in some particular order. An arithmetic sequence is a sequence with the difference between two consecutive terms constant.. A geometric sequence is a sequence with the ratio between two consecutive terms constant.

Simply speaking, Arithmetic sequence has a common difference(add or subtract) while Geometric sequence has a common ratio(multiply or divide)

