

# Binary Exponentiation

## What is Binary Exponentiation?

**Binary Exponentiation** or **Exponentiation by squaring** is the process of calculating a number raised to the power another number (AB) in **Logarithmic** time of the exponent or power, which speeds up the execution time of the program.

## Why to Use Binary Exponentiation?

Whenever we need to calculate (AB), we can simple calculate the result by taking the **result** as 1 and multiplying **A** for exactly **B** times. The time complexity for this approach is O(B) and will fail when values of B in order of  $10^8$  or greater. **This is when we can use Binary exponentiation because it can calculate the result in O(log(B)) time complexity, so we can easily calculate the results for larger values of B in order of  $10^{18}$  or less.**

## Idea Behind Binary Exponentiation:

When we are calculating (AB), we can have 3 possible positive values of B:

- **Case 1:** If B = 0, whatever be the value of A, our result will be 1.
- **Case 2:** If B is an even number, then instead of calculating (AB), we can calculate  $(A^2)^{(B/2)}$  and the result will be same.
- **Case 3:** If B is an odd number, then instead of calculating (AB), we can calculate

$$(A * (A^{(B-1)/2})^2),$$

## Recursive Implementation of Binary Exponentiation:

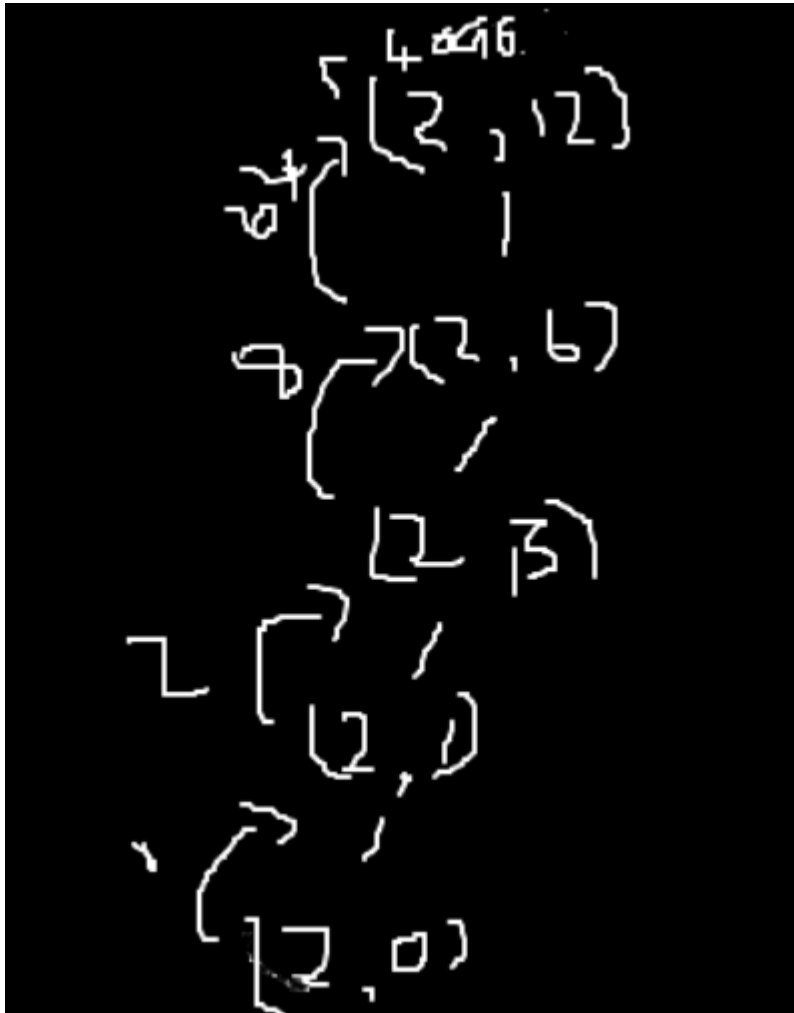
```
import java.io.*; class GFG {  
static long power(long A, long B) { `if (B == 0) return 1;
```

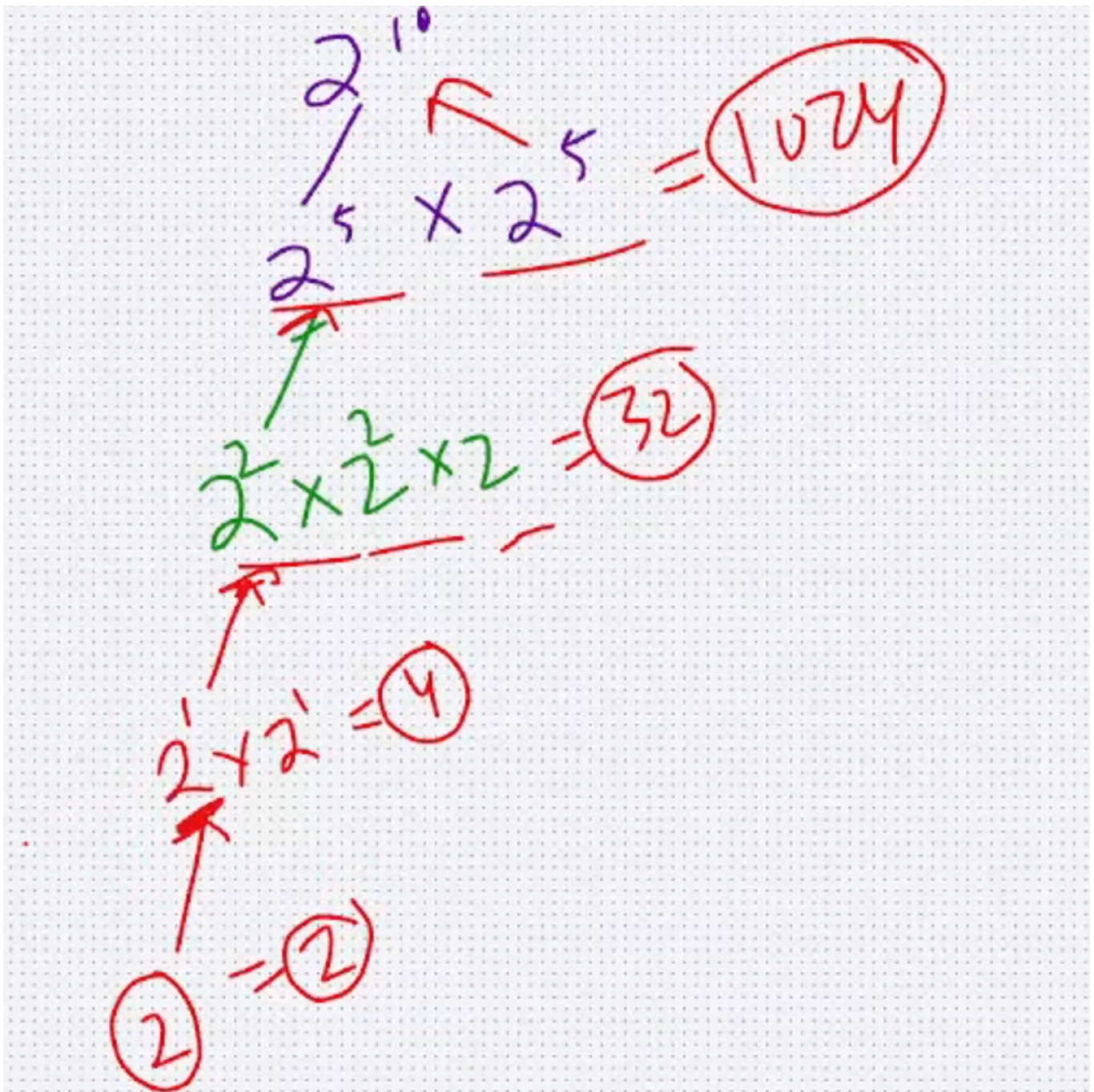
```
    long res = power(A, B / 2);  
  
    if (B % 2 == 1)  
        return res * res * A;  
    else  
        return res * res;
```

```
}
```

```
public static void main(String[] args) {  
    System.out.println(power(2, 12));  
}
```

```
}
```





## Iterative Implementation of Binary Exponentiation:

```
public class Main { public static long power(long a, long b) {
    long result = 1; while (b > 0) {
    if ((b & 1) == 1) { result = a;`
    } a = a;
    b >>= 1; }
    return result; }
```

```
public static void main(String[] args) {
    System.out.println(power(2, 12));
```

```
}
```

```
}
```

## Compute a large number modulo M:

**$(A \cdot B) \bmod M = ((A \bmod M) \cdot (B \bmod M)) \bmod M$**

```
`import java.util.*;
```

```
public class Main { // Constant representing the modulo value
```

```
`static final int mod = 1000000007;
```

```
    // Function to calculate the power of a number (a) raised to the power of b
    modulo mod
```

```
    public static long power(long a, long b) {
```

```
        long result = 1;
```

```
        while (b > 0) {
```

```
            // If the current bit of b is set, multiply the result by a
```

```
            if ((b & 1) == 1)
```

```
                result = (result * a) % mod;
```

```
            // Square the value of a and reduce it modulo mod
```

```
            a = (a * a) % mod;
```

```
            // Right shift b to move to the next bit
```

```
            b >>= 1;
```

```
        }
```

```
        return result;
```

```
    }
```

```
    public static void main(String[] args) {
```

```
        // Output the result of 2^42 modulo mod
```

```
        System.out.println(power(2, 42));
```

```
    }
```

```
}
```