№1

$$dy = f'(x)dx = \left(\sqrt{1+x^2} \cdot \operatorname{arctg} x\right)' dx = \left(\left(\sqrt{1+x^2}\right)' \cdot \operatorname{arctg} x + \sqrt{1+x^2} \cdot (\operatorname{arctg} x)'\right) dx =$$

$$\left(\frac{1}{2\sqrt{1+x^2}} \cdot 2x \cdot \operatorname{arctg} x + \sqrt{1+x^2} \cdot \frac{1}{1+x^2}\right) dx = \left(\frac{x \cdot \operatorname{arctg} x}{\sqrt{1+x^2}} + \frac{1}{\sqrt{1+x^2}}\right) dx = \frac{x \cdot \operatorname{arctg} x + 1}{\sqrt{1+x^2}} dx$$

№2

$$\left(\ln^2\left(5^{\cos 3x} + \operatorname{arcctg}\sqrt{1+x^3}\right)\right)' = 2\left(\ln\left(5^{\cos 3x} + \operatorname{arcctg}\sqrt{1+x^3}\right)\right) \cdot \left(\ln\left(5^{\cos 3x} + \operatorname{arcctg}\sqrt{1+x^3}\right)\right)'$$

$$\left(\ln\left(5^{\cos 3x} + \operatorname{arcctg}\sqrt{1+x^3}\right)\right)' = \frac{\left(5^{\cos 3x} + \operatorname{arcctg}\sqrt{1+x^3}\right)'}{5^{\cos 3x} + \operatorname{arcctg}\sqrt{1+x^3}}$$

$$\left(5^{\cos 3x} + \operatorname{arcctg}\sqrt{1+x^3}\right)' = \left(5^{\cos 3x}\right)' + \left(\operatorname{arcctg}\sqrt{1+x^3}\right)' =$$

$$\ln 5 \cdot 5^{\cos 3x} \cdot (\cos 3x)' - \frac{\left(\sqrt{1+x^3}\right)'}{1+1+x^3} = -\ln 5 \cdot 5^{\cos 3x} \cdot \sin 3x \cdot 3 - \frac{3x^2}{2\sqrt{1+x^3}\left(2+x^3\right)}$$

(Последовательно подставить снизу вверх и получить ответ).

№3

$$y'' = \left(\frac{1}{2x+1}\right)'' = \left(\left(\frac{1}{2x+1}\right)'\right)' = \left(\frac{-2}{(2x+1)^2}\right)' =$$

$$-2\left(\frac{1}{(2x+1)^2}\right)' = -2\left(\frac{-1 \cdot (4x^2+4x+1)'}{(2x+1)^4}\right) = 8\left(\frac{2x+1}{(2x+1)^4}\right) = \frac{8}{(2x+1)^3}$$