

1 Some Theory

Theory Exercise 1 *Proof the following*
If T is a contraction then $T^n x$ is a Cauchy sequence.

Theory Exercise 2 *Consider the growth model with only capital where productivity z is stochastic and $\ln z$ follows an AR-1 process such that $E(\ln z' | \ln z) = \mu(1 - \rho) + \rho \ln z$. Assume $\delta = 1$. Show that the value function can be written as $V(k, z) = A + B \ln k + C \ln z$.*

2 Basics

Exercise 1 (a.Value Function Iteration) *Assume household income can be either 10/9 or 1/9. The probability to move from high income to low income is 4/90 and the probability to move from low income to high income is 2/5. The household can borrow and lend at a 4/90 interest rate and has log utility in consumption. The borrowing constraint is -9/4. Its discount factor is 0.95. Solve the consumption savings model by value-function iteration. Consider first only on-grid choices and then off-grid choices. Plot value and policy functions Use a 30-point log-grid for asset holdings between the borrowing limit and 6.*

Exercise 1 (b.Value Function Iteration) *Redo Exercise 1 a. but use Broyden's rootfinding procedure instead of VFI - compare running times and numbers of iterations.*

Exercise 1 (c. Policy Function Iteration) *Same as Exercise 1 a., but solve using policy function iteration.*

3 Endogenous grid method

Exercise 2 (Endogenous Grid Method) *Assume household income can be either 10/9 or 1/9. The probability to move from high income to low income is 4/90 and the probability to move from low income to high income is 2/5. The household can borrow and lend at a 4/90 interest rate and has log utility in consumption. The borrowing constraint is -9/4. Its discount factor is 0.95. Solve the consumption savings model using the endogenous grid method and plot value and policy functions and compare time to compute to off-grid VFI! Use a 100-point log-grid for asset holdings between the borrowing limit and 6.*

4 Young's method & Solving an Aiyagari economy

Exercise 3 (Bewley Model) *Re-use the codes/model from exercise 2, but now assume that $r = 0$, the asset they save in bears no interest (money) and that*

households have a CRRA utility function with risk aversion 2. Further assume the household cannot borrow. Set the maximum asset holdings to 3. Solve the model with EGM, then

1. Simulate an agent over $T=100,000$ periods of time and calculate the average asset holding of the agent in periods $t \geq 10,000$.
2. Create a transition matrix from the policy functions. Use this to calculate the ergodic distribution of the model. Calculate the expected asset holdings from that ergodic distribution.
3. Compare the histograms of the two distributions and aggregate money demand.

Exercise 4 (Hugget model) Extend Exercise 2 to calculate the equilibrium interest rate. Use Young's method and MATLAB's `fzero` function to find a root in the excess demand for bonds function. Assume that households have a CRRA utility function with risk aversion 2, set the maximum asset holdings to 10. Plot the demand and supply for funds.

Exercise 5 (Aiyagari model) Extend Exercise 4 to calculate the equilibrium interest rate (same as before), but now assume that the asset is physical capital and households have a CRRA utility function with risk aversion 4, set the maximum asset holdings to 20. Plot the demand and supply for funds. The production function is

$$F(K, N) = K^\alpha N^{1-\alpha}. \quad (1)$$

Let $\delta = .1$, $\beta = 0.95$, $\alpha = 0.36$.

5 A HANC model

Exercise 6 Take an Aiyagari setup: Aggregate output is given by

$$F(Z, K, N) = ZK^\alpha N^{1-\alpha}. \quad (2)$$

Aggregate productivity, Z follows a log AR-1 process with autocorrelation $\rho_Z = 0.75$ and standard deviation $\sigma = 0.01$. Other parameters are: depreciation, $\delta = 1/10$; discount factor, $\beta = 0.95$; capital share, $\alpha = 0.36$; risk aversion, $\gamma = 1$.

Agents can be employed (productivity 10/9) or unemployed. If unemployed they move with 2/5 probability to employment and if employed they lose their job with prob. 4/90. When unemployed, they receive 50% of the employed wage financed by a corresponding labor tax.

1. Solve for the steady state without aggregate risk.
2. Solve using Reimers's method.