Plotting Two-dimensional Differential Equations

The DEplot routine from the DEtools package is used to generate plots that are defined by differential equations. This worksheet details some of the options that are available, in sections on *Interface* and *Options*.

In order to access the routines in the **DEtools** package by their short names, the **with** command has been used.

```
> with(plots):
> with(DEtools):
```

Autonomous systems are automatically determined for plotting phase portraits. For instance,

>

Textbook Example 24.12 Both roots Negative Stable Node

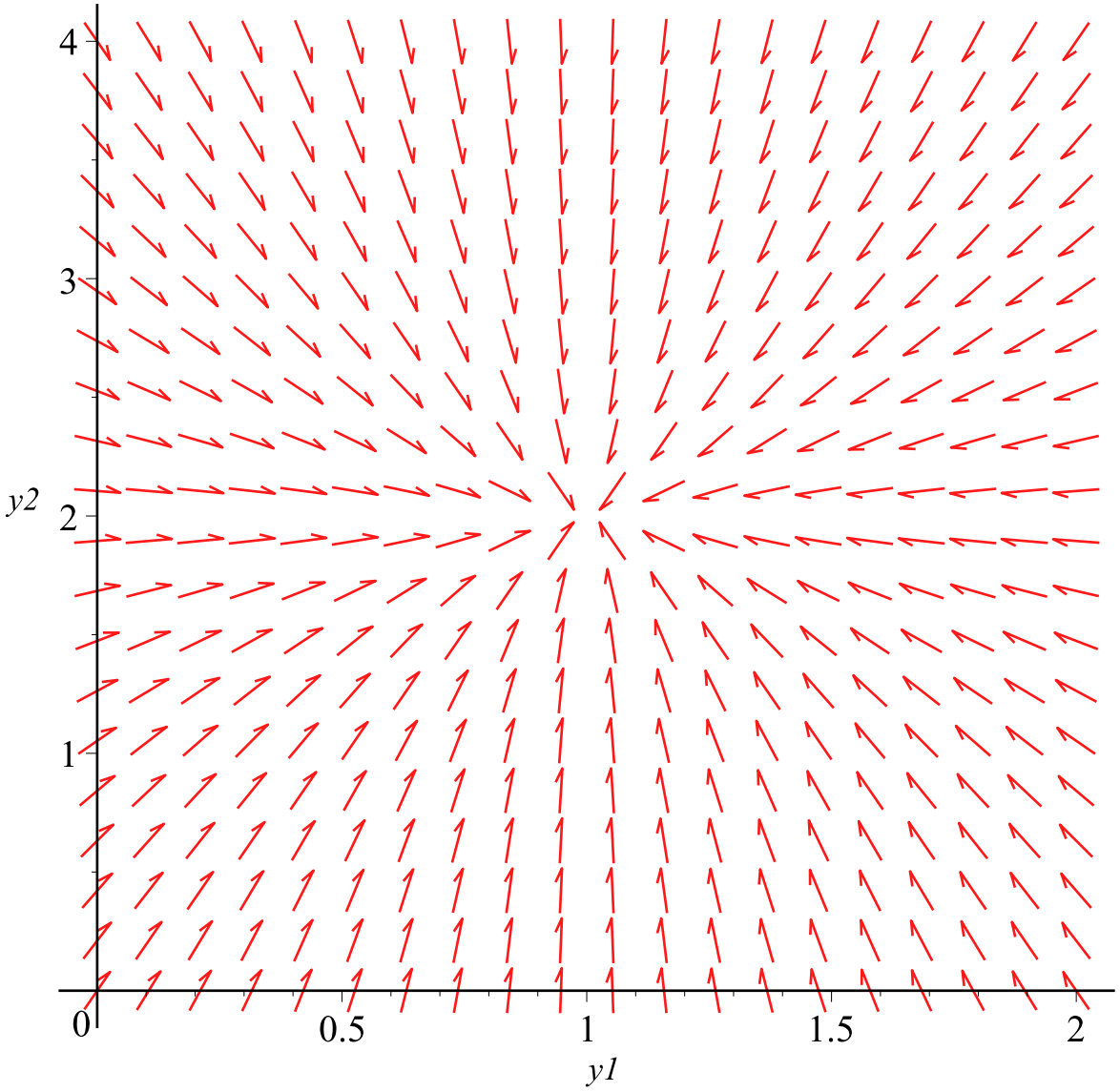
```
> DE2412 := [diff(y1(t),t) = -2*y1(t)+2, diff(y2(t),t) = -3*y2(t)+6];
```

$$DE2412 := \left[\frac{d}{dt} y1(t) = -2\, y1(t) + 2,\ \frac{d}{dt} y2(t) = -3\, y2(t) + 6\right] \quad \textbf{(1.1)}$$

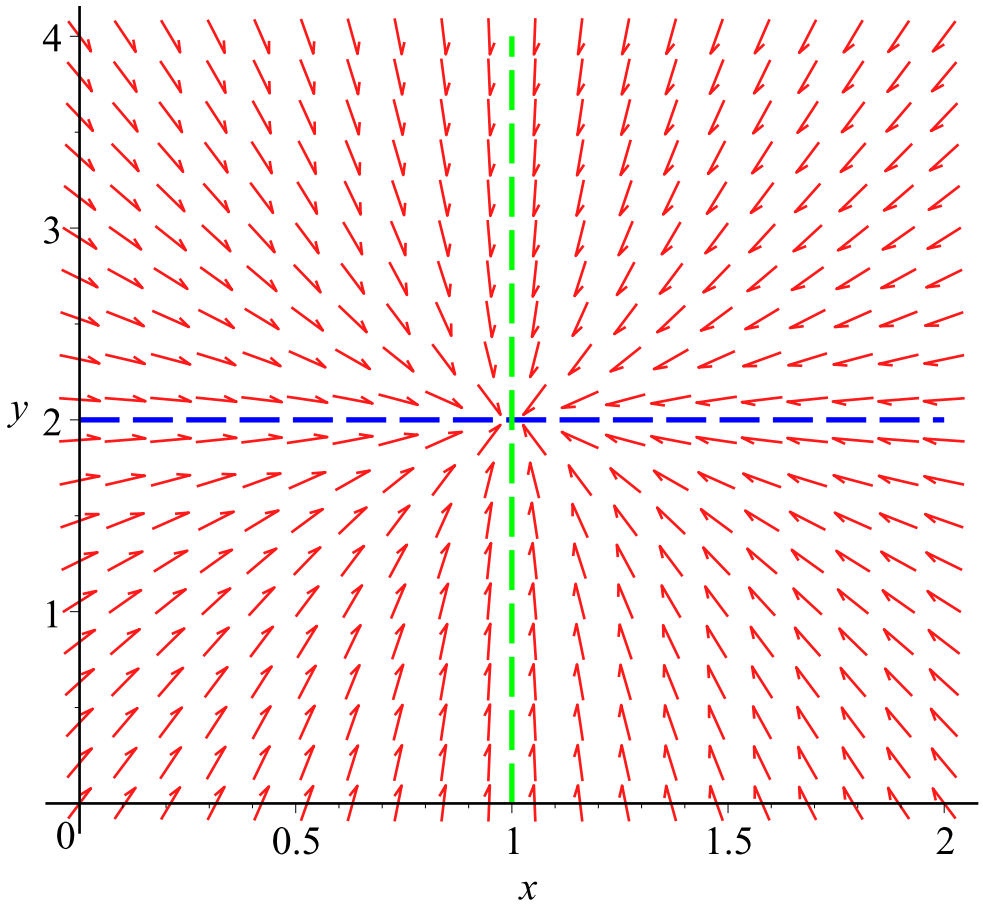
```
> dsolve(DE2412);
```

$$\{y1(t) = 1 + e^{-2t}\_C2,\ y2(t) = 2 + e^{-3t}\_C1\} \quad \textbf{(1.2)}$$

```
> p3 := DEplot(DE2412, [y1(t), y2(t)], t = 0..10, y1 = 0..2, y2 = 0..4);
p3 :=
```

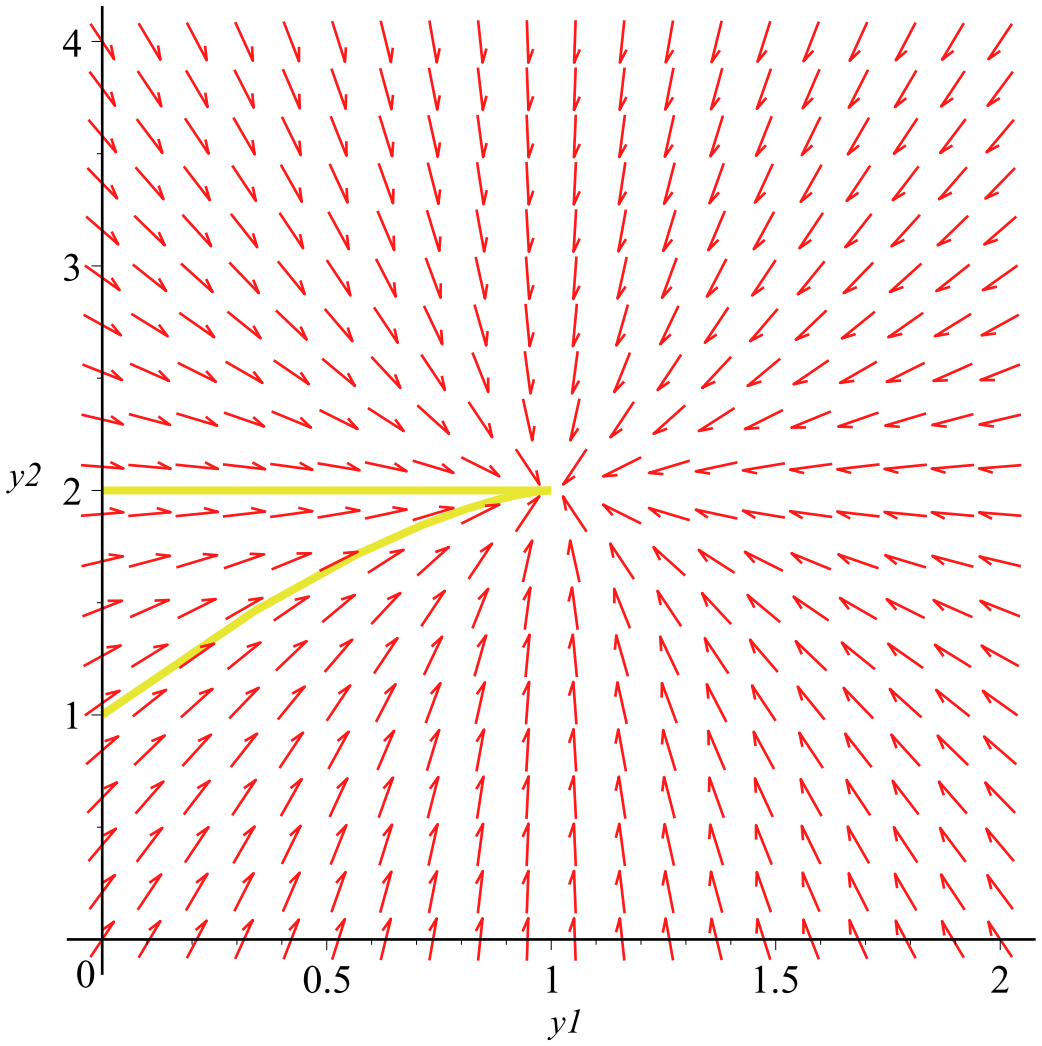



```
> # Horizontal line y=1

  p1 := implicitplot(y = 2, x = 0 ..2, y = 0 ..4,  colour = blue, linestyle = 3, thickness = 2) :

  # Vertical line x=Pi/2

  p2 := implicitplot(x = 1, x = 0 ..2, y = 0 ..4, colour = green, linestyle = 3, thickness = 2) :
> display(p1, p2, p3);
```



Direction field with solution curves

```
> DEplot(DE2412, [y1(t), y2(t)], t = 0..10, y1 = 0..2, y2 = 0..4, [[y1(0) = 1,   y2(0) = 2],
      [y1(0) = 0, y2(0) = 2], [y1(0) = 0, y2(0) = 1]], animatecurves = true);
```



Textbook Example 24.13 Both Roots Positive Unstable Node

> $DE2413 := \left[\frac{d}{dt} y1(t) = 2 \cdot y1(t) - 2, \frac{d}{dt} y2(t) = 3 \cdot y2(t) - 6\right];$

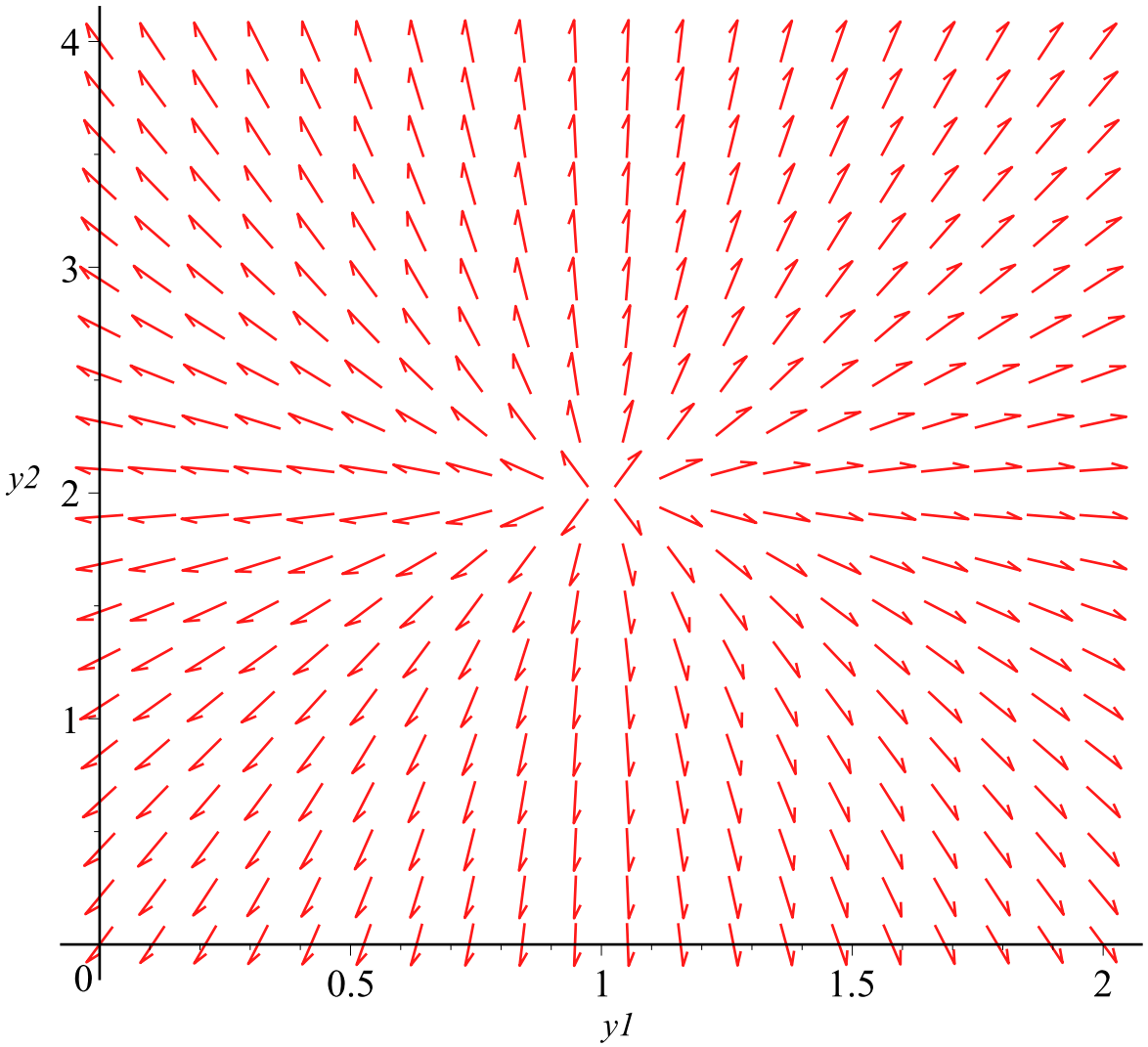
$$DE2413 := \left[\frac{d}{dt} y1(t) = 2\, y1(t) - 2, \frac{d}{dt} y2(t) = 3\, y2(t) - 6\right] \quad \textbf{(2.1)}$$

> $dsolve(DE2413);$

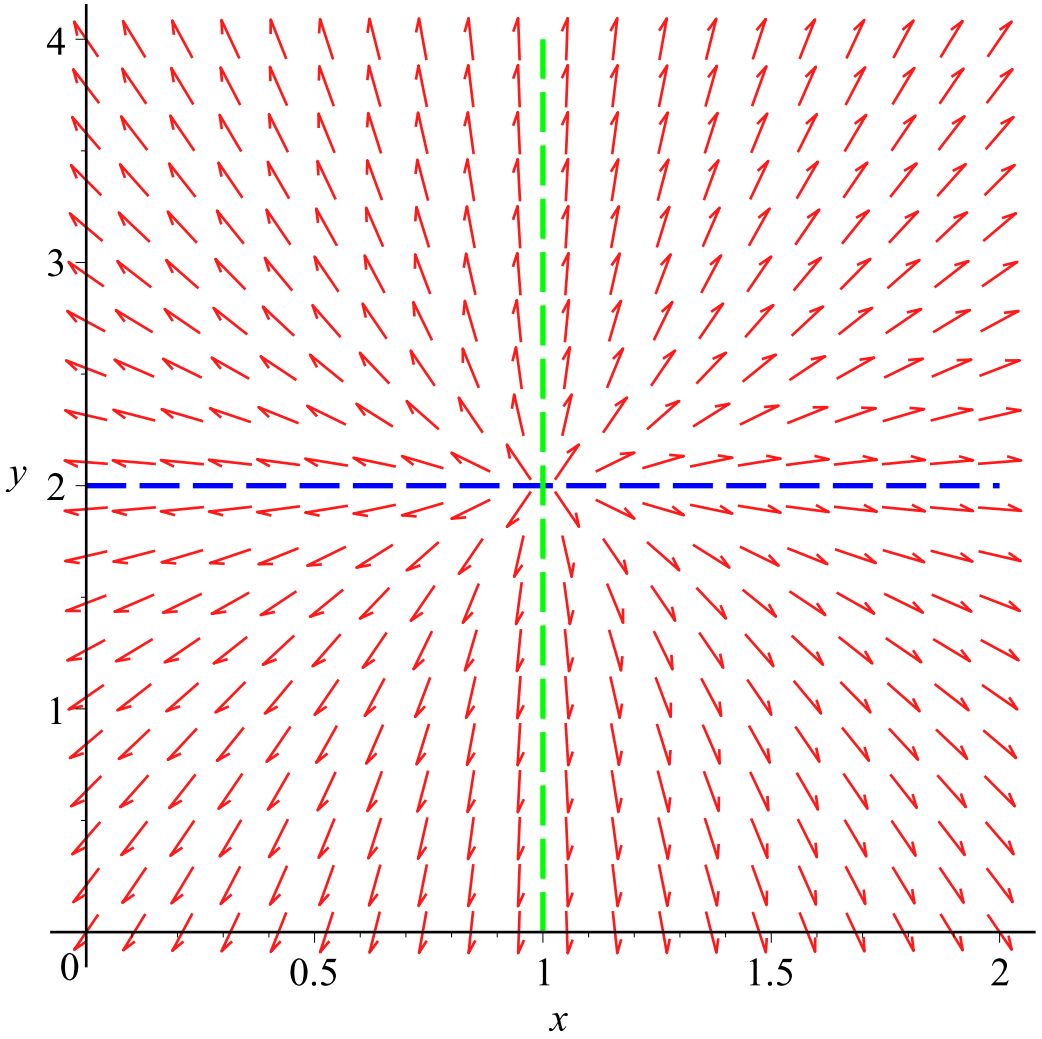
$$\{y1(t) = 1 + e^{2t}\_C2, y2(t) = 2 + e^{3t}\_C1\} \quad \textbf{(2.2)}$$

```
> p3 := DEplot(DE2413, [y1(t), y2(t)], t = 0..10, y1 = 0..2, y2 = 0..4);
```

p3 :=

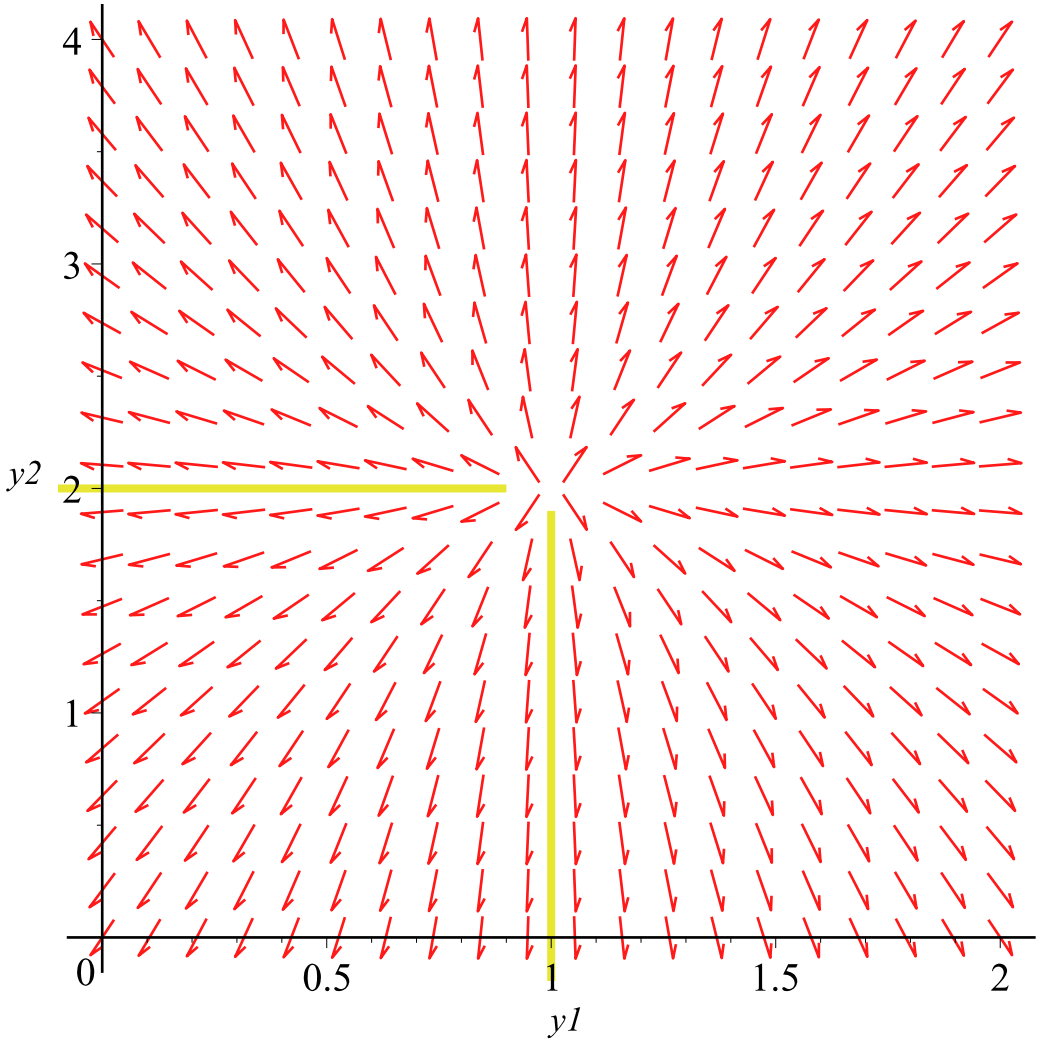



```
> # Horizontal line y=1

  p1 := implicitplot(y = 2, x = 0..2, y = 0..4, colour = blue, linestyle = 3, thickness = 2):

  # Vertical line x=Pi/2

  p2 := implicitplot(x = 1, x = 0..2, y = 0..4, colour = green, linestyle = 3, thickness = 2):
> display(p1, p2, p3);
```



Direction field with solution curves

```
> DEplot(DE2413, [y1(t), y2(t)], t = 0..10, y1 = 0..2, y2 = 0..4, [[y1(0) = 1,   y2(0) = 2],
      [y1(0) = 1, y2(0) = 2 - 0.1], [y1(0) = 1 - 0.1, y2(0) = 2]], animatecurves = true);
```



Textbook Example 24.14 Roots of opposite sign saddle point

> $DE2414 := \left[\frac{d}{dt} y1(t) = y2(t) - 2, \frac{d}{dt} y2(t) = \frac{1}{4} \cdot y1(t) - \frac{1}{2} \right];$

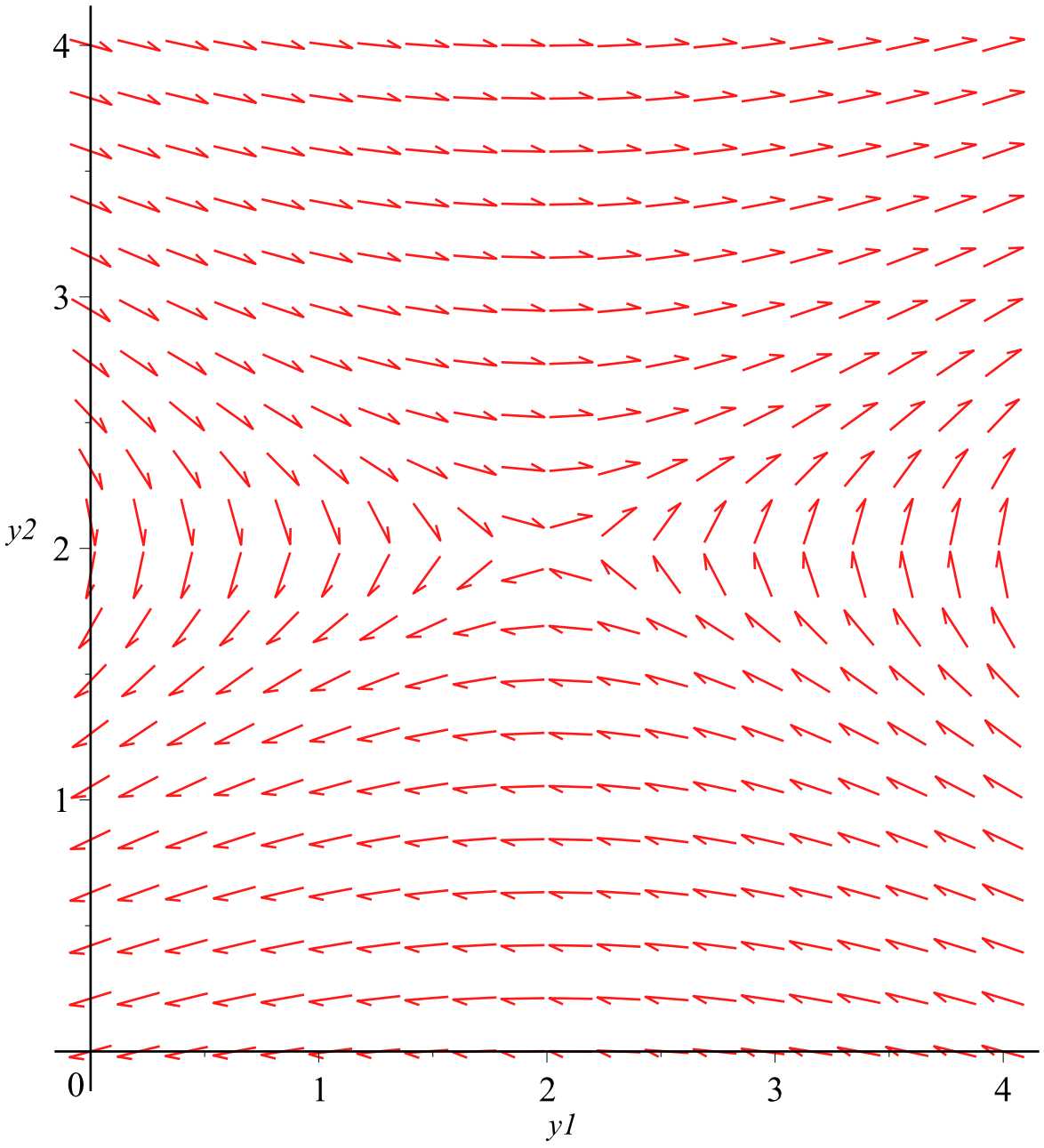
$$DE2414 := \left[\frac{d}{dt} y1(t) = y2(t) - 2, \frac{d}{dt} y2(t) = \frac{1}{4} y1(t) - \frac{1}{2} \right] \quad \textbf{(3.1)}$$

> $dsolve(DE2414);$

$$\left\{ y1(t) = e^{\frac{1}{2}t}\_C2 + e^{-\frac{1}{2}t}\_C1 + 2,\ y2(t) = \frac{1}{2}e^{\frac{1}{2}t}\_C2 - \frac{1}{2}e^{-\frac{1}{2}t}\_C1 + 2 \right\} \quad \textbf{(3.2)}$$

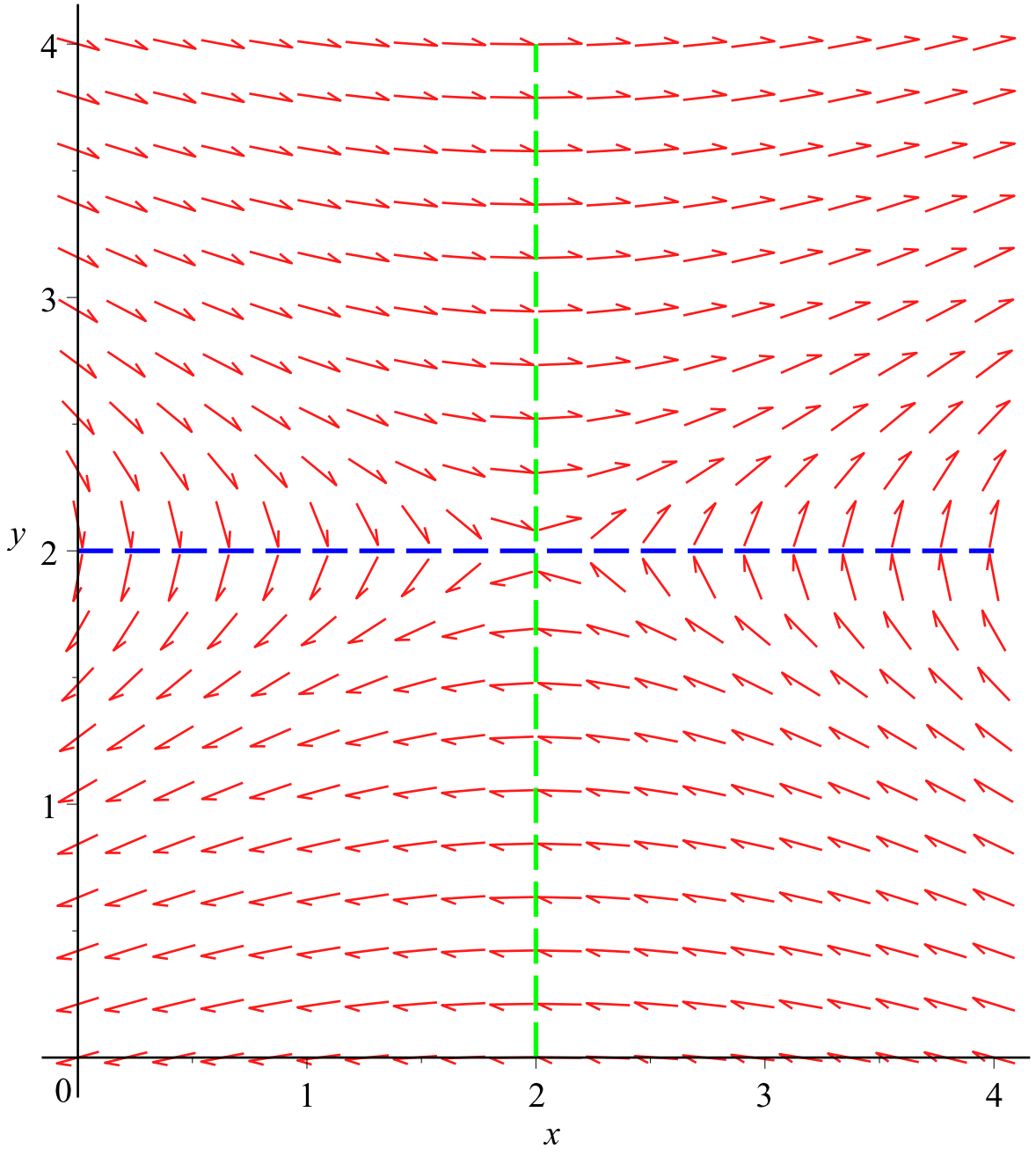
```
> p3 := DEplot(DE2414, [y1(t), y2(t)], t = 0..10, y1 = 0..4, y2 = 0..4);
```

p3 :=



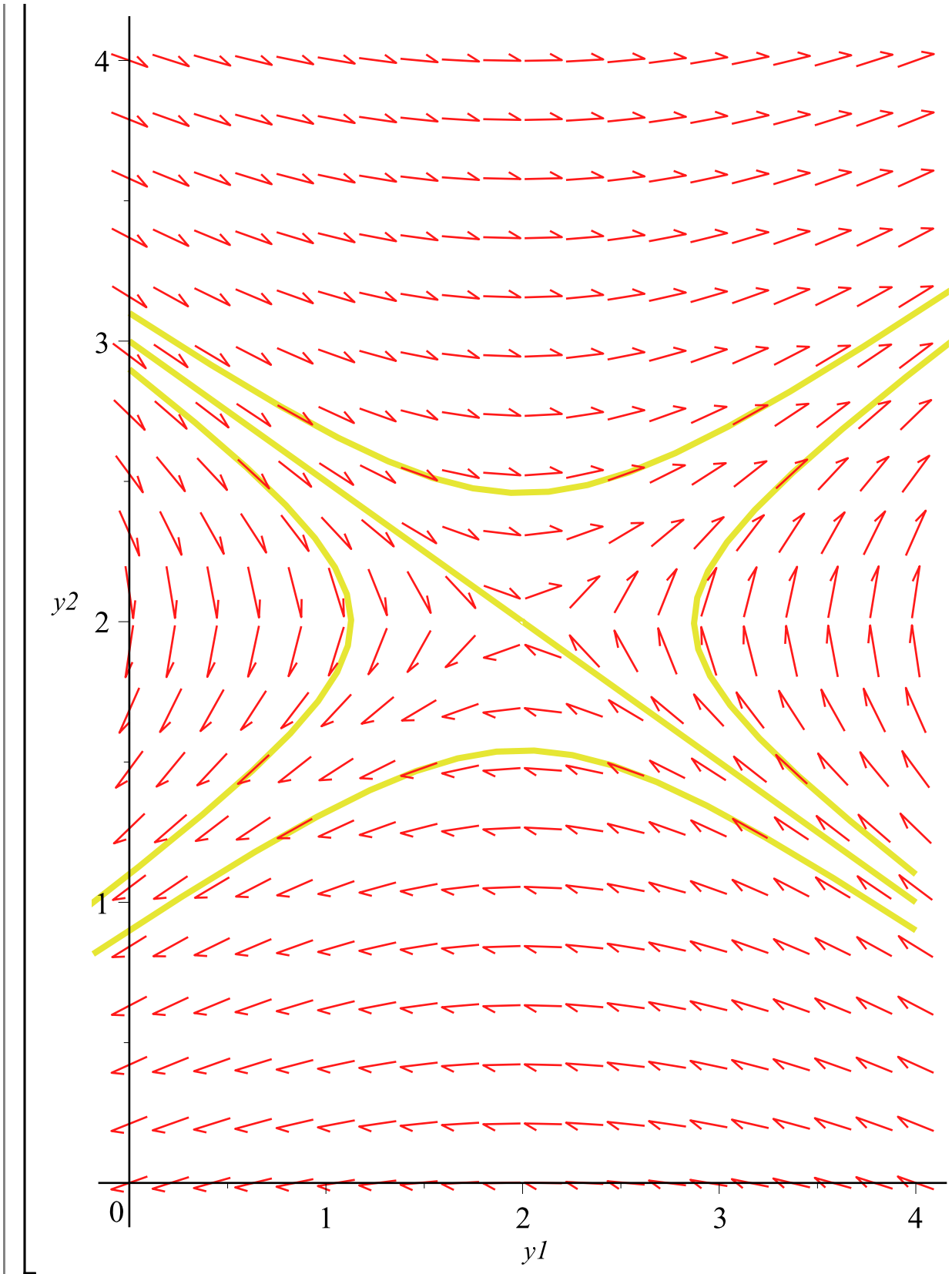

```
> # Horizontal line y=1

  p1 := implicitplot(y = 2, x = 0..4, y = 0..4, colour = blue, linestyle = 3, thickness = 2):

  # Vertical line x=Pi/2
```

```
p2 := implicitplot(x = 2, x = 0..4, y = 0..4, colour = green, linestyle = 3, thickness = 2):
> display(p1, p2, p3);
```



Direction field with solution curves

```
> DEplot(DE2414, [y1(t), y2(t)], t = 0..20, y1 = 0..4, y2 = 0..4, [[y1(0) = 0,   y2(0) = 3],
    [y1(0) = 0, y2(0) = 3 + 0.1], [y1(0) = 0, y2(0) = 3 − 0.1],   [y1(0) = 4,   y2(0) = 1],
    [y1(0) = 4, y2(0) = 1 + 0.1], [y1(0) = 4, y2(0) = 1 − 0.1]   ], animatecurves = true);
```


4
3
2
1
0
y2
1
2
3
4
y1