B.C. $k_{t+1} + c_t = e^{z_t} k_t^{\alpha} l_t^{1-\alpha} + (1-\delta) k_t$

$$\mathcal{L} = \sum_{t=0}^{\infty} \left\{ (\beta^{t-1} \frac{(c_t^{\theta}(1-l_t)^{1-\theta})^{1-\tau}}{1-\tau} + \lambda_t \left[e^{z_t} k_t^{\alpha} \cdot l_t^{1-\alpha} - c_t + (1-\delta) k_t - k_{t+1} \right] \right\}$$

F.O.C.

$$\frac{\partial \mathcal{L}}{\partial c_t} = 0 \Rightarrow \beta^{t-1} \frac{(c_t^{\theta}(1-l_t)^{1-\theta})^{1-\tau}}{1-\tau} \cdot \frac{\theta(1-\tau)}{c_t} - \lambda_t = 0 \quad \cdots\cdots \text{⓪}$$

$$\frac{\partial \mathcal{L}}{\partial l_t} = 0 \Rightarrow \beta^{t-1} \frac{(c_t^{\theta}(1-l_t)^{1-\theta})^{1-\tau}}{1-\tau} \frac{(1-\theta)(1-\tau)(-1)}{(1-l_t)} + \lambda_t e^{z_t} k_t^{\alpha} (1-\alpha) l_t^{-\alpha} = 0 \quad \cdots \text{①}$$

$$\frac{\partial \mathcal{L}}{\partial k_{t+1}} = 0 \Rightarrow \lambda_{t+1} \left[e^{z_t} \alpha k_{t+1}^{\alpha-1} l_{t+1}^{1-\alpha} + (1-\delta) \right] - \lambda_t = 0. \quad \cdots \text{②}$$

combine ⓪, ②. cancell out λ_t and λ_{t+1}

intertemporal Euler equ.

$$\lambda_{t+1} = \beta^{t} \frac{(c_{t+1}^{\theta}(1-l_{t+1})^{1-\theta})^{1-\tau}}{1-\tau} \cdot \frac{\theta(1-\tau)}{c_{t+1}}$$

$$\left[e^{z_t} \alpha k_{t+1}^{\alpha} l_{t+1}^{1-\alpha} + 1 - \delta \right] = \beta^{t-1} \frac{(c_t^{\theta}(1-l_t)^{1-\theta})^{1-\tau}}{1-\tau} \cdot \frac{\theta(1-\tau)}{c_t}$$

combine (0), (1). cancell out λ_t.

$$\cancel{\beta^{t-1} \frac{(c_t^{\theta}(1-l_t)^{1-\theta})^{1-\tau}}{1-\tau}} \frac{(1-\theta)\cancel{(1-\tau)}(-1)}{(1-l_t)} = \cancel{\beta^{t-1} \frac{(c_t^{\theta}(1-l_t)^{1-\theta})^{1-\tau}}{1-\tau}} \cdot \frac{\theta\cancel{(1-\tau)}}{c_t} \cdot e^{z_t} k_t^{\alpha} (1-\alpha) l_t^{-\alpha}$$

$$\frac{(1-\theta)}{\theta} \cdot \frac{c_t}{(1-l_t)} = (1-\alpha) e^{z_t} k^{\alpha} \cdot l_t^{-\alpha}$$