ECON501 2017

lab05

2.17. Social security in the Diamond model. Consider a Diamond economy where g is zero, production is Cobb–Douglas, and utility is logarithmic.

(*a*) **Pay-as-you-go social security.** Suppose the government taxes each young individual an amount T and uses the proceeds to pay benefits to old individuals; thus each old person receives $(1 + n)T$.

(*i*) How, if at all, does this change affect equation (2.60) giving k_{t+1} as a function of k_t?

(*ii*) How, if at all, does this change affect the balanced-growth-path value of k?

(*iii*) If the economy is initially on a balanced growth path that is dynamically efficient, how does a marginal increase in T affect the welfare of current and future generations? What happens if the initial balanced growth path is dynamically inefficient?

(*b*) **Fully funded social security.** Suppose the government taxes each young person an amount T and uses the proceeds to purchase capital. Individuals born at t therefore receive $(1 + r_{t+1})T$ when they are old.

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http://listinet.com/bibliografia-comuna/Cdu339-52C2.pdf

P57 3 Romer, 2.16. Social Security

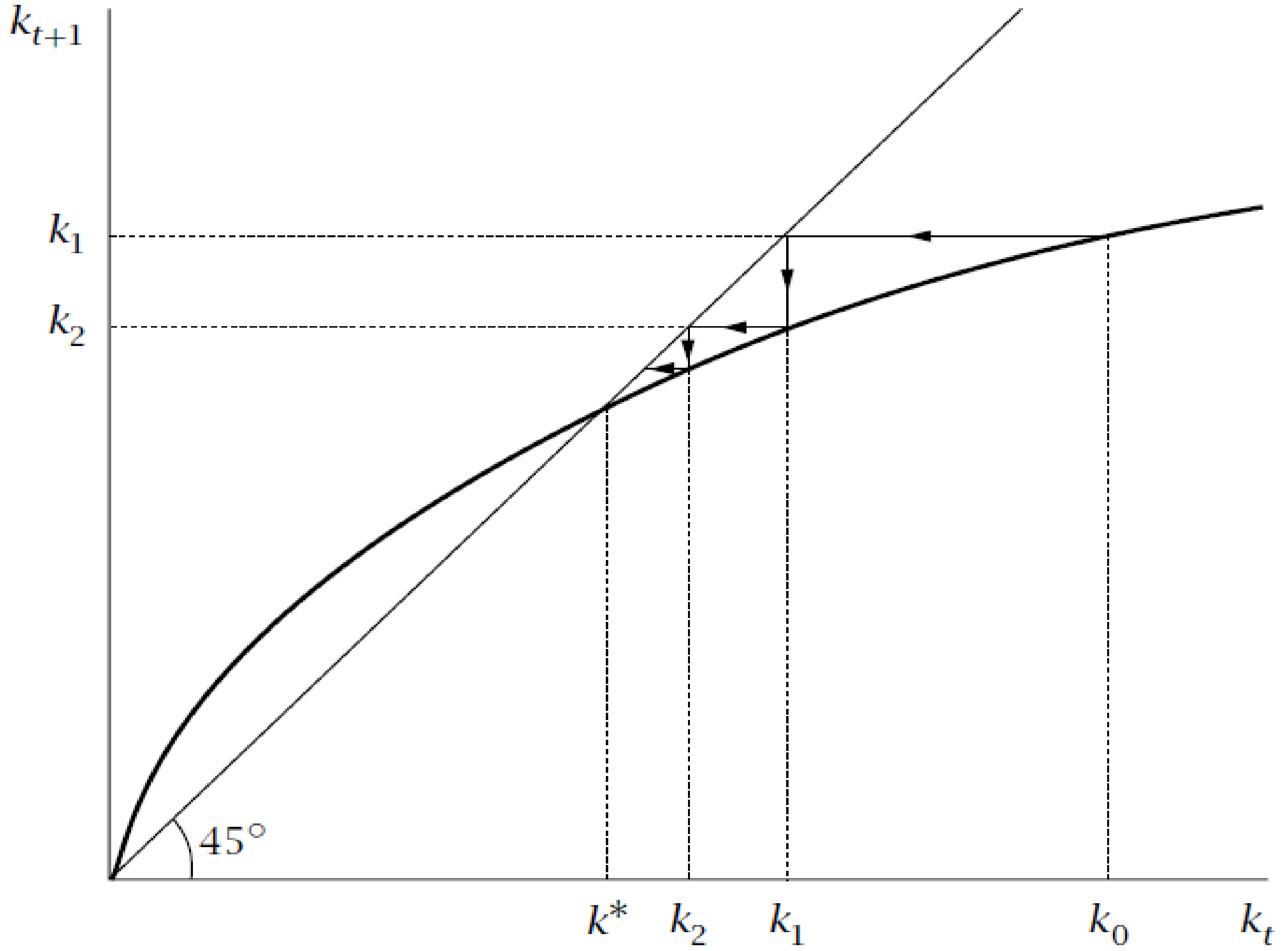

FIGURE 2.11 The dynamics of *k*

To answer these questions, we need to describe how k_{t+1} depends on k_t. Unfortunately, we can say relatively little about this for the general case. We therefore begin by considering the case of logarithmic utility and Cobb–Douglas production. With these assumptions, (2.59) takes a particularly sim-

Logarithmic Utility and Cobb–Douglas Production

When θ is 1, the fraction of labor income saved is $1/(2+\rho)$ (see equation [2.55]). And when production is Cobb–Douglas, $f(k)$ is k^{α} and $f'(k)$ is $\alpha k^{\alpha-1}$. Equation (2.59) therefore becomes

$$k_{t+1} = \frac{1}{(1+n)(1+g)} \frac{1}{2+\rho}(1-\alpha)k_t^{\alpha}. \tag{2.60}$$

Figure 2.11 shows k_{t+1} as a function of k_t. A point where the k_{t+1} function intersects the 45-degree line is a point where k_{t+1} equals k_t. In the case we are considering, k_{t+1} equals k_t at $k_t = 0$; it rises above k_t when k_t is small; and it then crosses the 45-degree line and remains below. There is thus a unique balanced-growth-path level of k (aside from $k = 0$), which is denoted k^*.

k^* is globally stable: wherever k starts (other than at 0, which is ruled out by the assumption that the initial capital stock is strictly positive), it

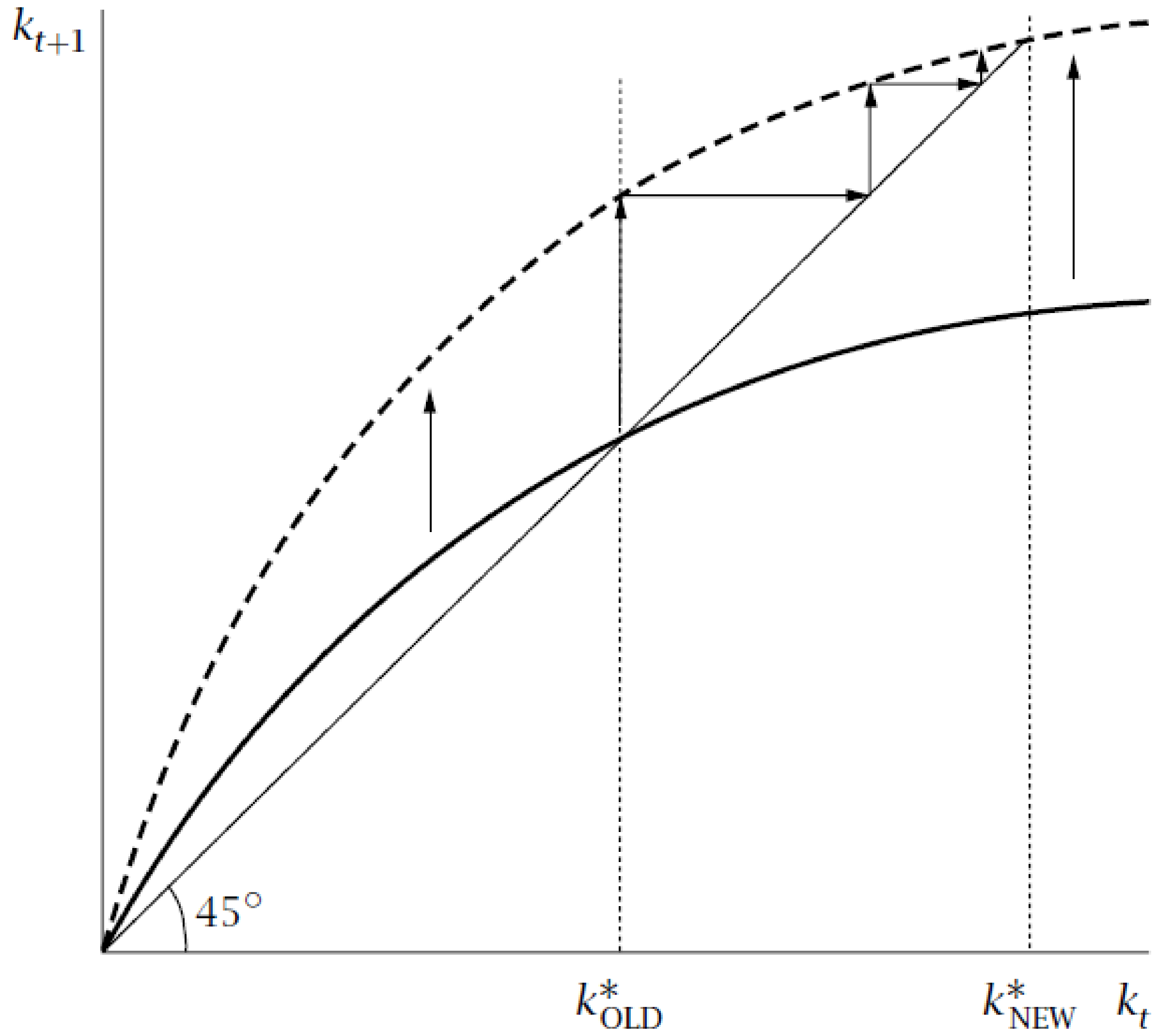

FIGURE 2.12 The effects of a fall in the discount rate

consumption per worker

n·k ? n·f(k)

effective labor. On a balanced growth path with $g = 0$, total consumption per unit of effective labor is output per unit of effective labor, $f(k)$, minus break-even investment per unit of effective labor, $nf(k)$. The golden-rule capital stock therefore satisfies $f'(k_{GR}) = n$. $f'(k^*)$ can be either more or less than $f'(k_{GR})$. In particular, for α sufficiently small, $f'(k^*)$ is less than $f'(k_{GR})$—the capital stock on the balanced growth path exceeds the golden-rule level.

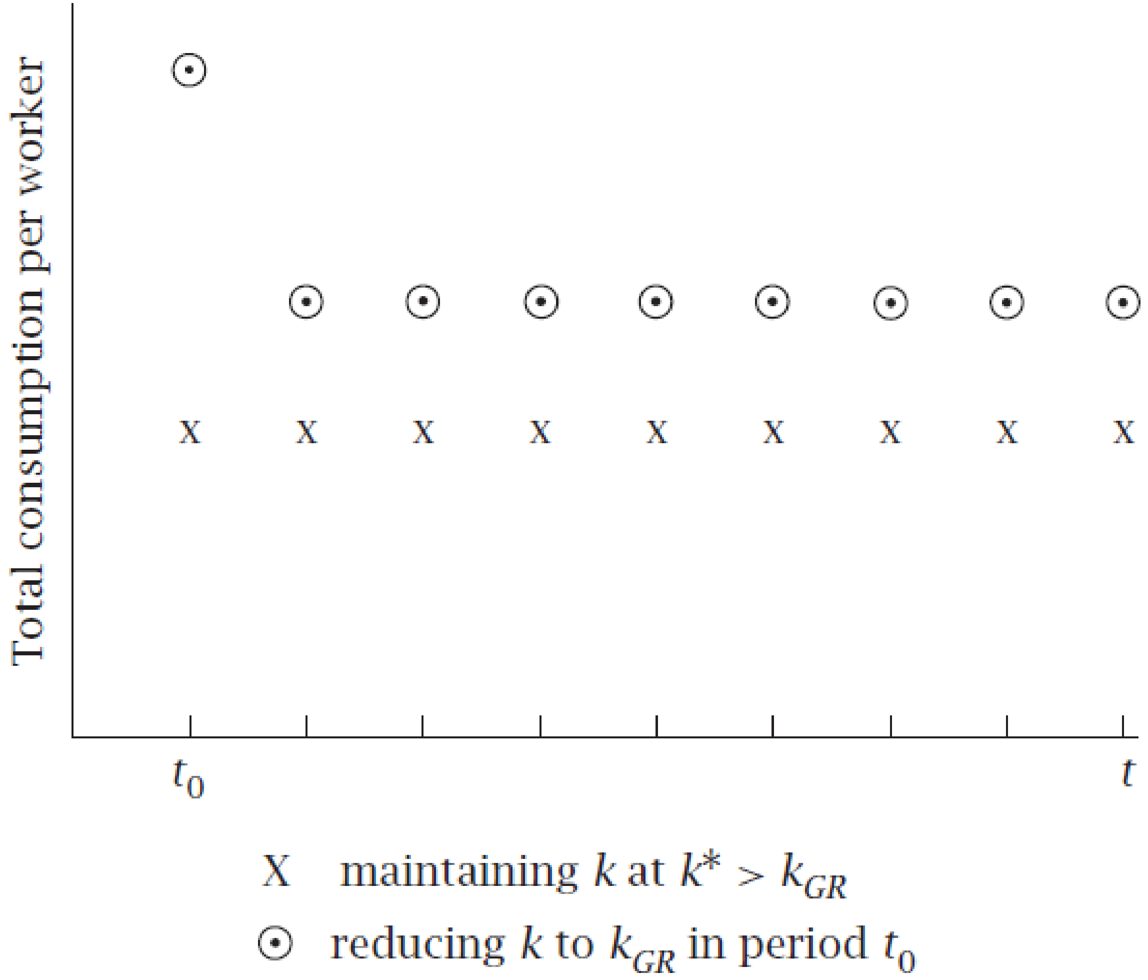

FIGURE 2.14 How reducing k to the golden-rule level affects the path of consumption per worker

$f(k^*) - nk^*$. And since k^* is greater than k_{GR}, $f(k^*) + (k^* - k_{GR}) - nk_{GR}$ is even larger than $f(k_{GR}) - nk_{GR}$. The path of total consumption under this policy is shown by the circles in Figure 2.14. As the figure shows, this policy makes more resources available for consumption in every period than the policy of maintaining k at k^*. The planner can therefore allocate consumption between the young and the old each period to make every generation better off.

ically, the possibility of inefficiency in the Diamond model stems from the fact that the infinity of generations gives the planner a means of providing for the consumption of the old that is not available to the market. If individuals in the market economy want to consume in old age, their only choice is to hold capital, even if its rate of return is low. The planner, however, need not have the consumption of the old determined by the capital stock and its rate of return. Instead, he or she can divide the resources available for consumption between the young and old in any manner. The planner can take, for example, 1 unit of labor income from each young person and transfer it to the old. Since there are $1+n$ young people for each old person, this increases the consumption of each old person by $1+n$ units. The planner can prevent this change from making anyone worse off by requiring the next generation of young to do the same thing in the following period, and then continuing this process every period. If the marginal product of capital is less than n—that is, if the capital stock exceeds the golden-rule level—this way of transferring resources between youth and old age is more efficient than saving, and so the planner can improve on the decentralized allocation.

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(i) Now recall that old people "eat" their capital and therefore K_{t+1} consists only of the savings of young people: $K_{t+1} = S_t L - T$, or per unit of effective labor, recalling that for the Cobb-Douglas production function $w = (1-\alpha)k^\alpha$ and that in this problem A is constant,

$$k_{t+1} = \frac{K_{t+1}}{A(1+n)L_t} = \frac{1}{1+n}\frac{S_t}{A} = \frac{1}{1+n}\left(\frac{1-\alpha}{2+\rho}k_t^\alpha - \frac{Z_t T}{A}\right).$$

Note that if $T = 0$ we get our original equation from the standard Diamond model. Since $Z_t > 0$, we know that k_{t+1} decreases for every k_t.

(ii) We can look again at Figure 2 to convince ourselves that this leads to the decrease in the BGP level of capital. Intuitively, less money is available for savings every period, therefore the lower level of capital per unit of effective labor can be sustained on the BGP.

(iii) If the economy is dynamically efficient ($k^* < k^{GR}$), the (marginal) decrease in k^* will decrease consumption for all future generations, only current generation of old people will benefit from the introduction of the PAYG system, all future generations will be worse off. However, if the economy was dynamically inefficient ($k^* > k^{GR}$), the (marginal) decrease in k^* will increase consumption for all future generations and current generation of old will still gain. Therefore PAYG system can improve welfare in this case.

which says that the FF social security system causes the one-to-one reduction in personal savings. This makes sense because the social security brings the same return as private savings and therefore is a perfect substitute for private savings. We can now guess the answer to this question, because all this implies that total savings (private plus government) are not affected by the introduction of FF social security system and therefore the behavior and the BGP level of capital will not change. But let us derive this more formally.

(i) (ii) Now the capital stock will be held not only by households but also by government and thus

$$K_{t+1} = S_t L_t + T L_T,$$

or, per unit of effective labor,

$$k_{t+1} = \frac{1}{1+n}\left[\frac{1}{2+\rho}w_t - \frac{T}{A}\right] + \frac{1}{1+n}\frac{T}{A} = \frac{1}{1+n}\frac{1}{2+\rho}(1-\alpha)k_t^{\alpha},$$

which is exactly the same as without social security (there is no T in the equation) and our guess was indeed correct: nothing happens to k_{t+1} equation and therefore nothing happens to the BGP level of capital.