ECON501 2017

lab02

A **differential/difference equation (DE)** is an equation relating an unknown function and some of its derivatives. Many natural laws in engineering, chemistry, biology, physics, economics, etc. can be modeled using DEs.

DIFFERENCE EQUATIONS

$\Delta y_t = ay_t + b$ or $Y(t+1) + cy(t) = d$,

Solving general first-order linear difference equations

- Method 1: by Conjecture- Verify
- Method 2: More Powerful Solution Method: Iteration- Verify
- Method 3: the particular solution and the complementary function/solution of the general first-order difference equation.

- Show $L(T+1) = (1+n)\,L(t)$

What do we mean by SOLVING a difference equation?

Solving a difference equation means (this will be the same for differential equations later on) transforming the difference equation so that the new equation tells us the value of the variable y at any point in time t.

That is, we are looking for a mathematical expression $y_t = f(t)$.

When solving a DE, our primary concern is separation of variables.

"I attempt only to separate the indeterminate x and it's differential dx, from the indeterminates y and dy, which deserves the prize in this investigation, for otherwise the construction of the solution to the differential equation won't be achieved."

- Johann Bernoulli

NONLINEAR DIFFERENCE EQUATIONS

Most nonlinear difference equations DO NOT have a known algebraic solution $y_t = G(t)$

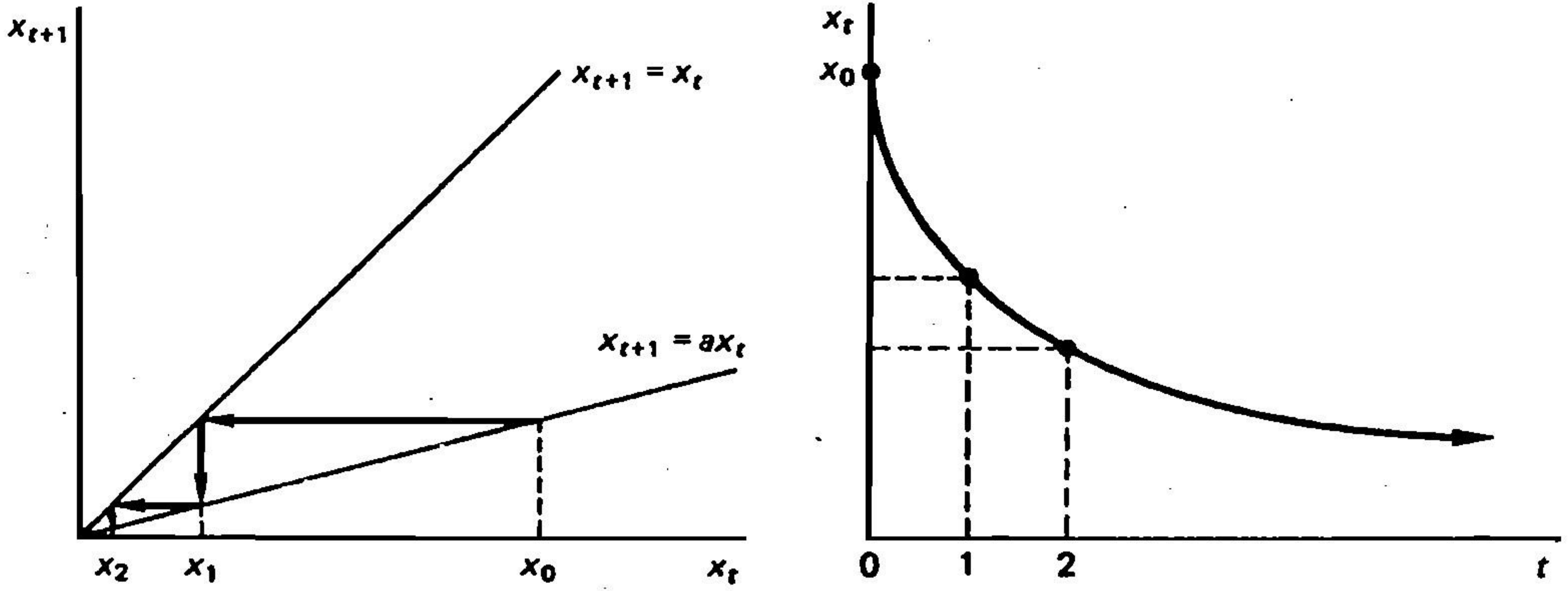
- A phase diagram of a single difference equation is a graph plotting y_{t+1} against y_t
- Phase diagrams identify the dynamic properties of a variable in different phases or “regions” of its domain

The Base Graph:

1. Draw a cartesian plane with y_t on the horizontal axis and y_{t+1} on the vertical axis
2. Draw the 45° line
3. Plot the difference equation $y_{t+1} = g(y_t)$
4. Intersections of $g(y_t)$ with the 45° line identify steady states

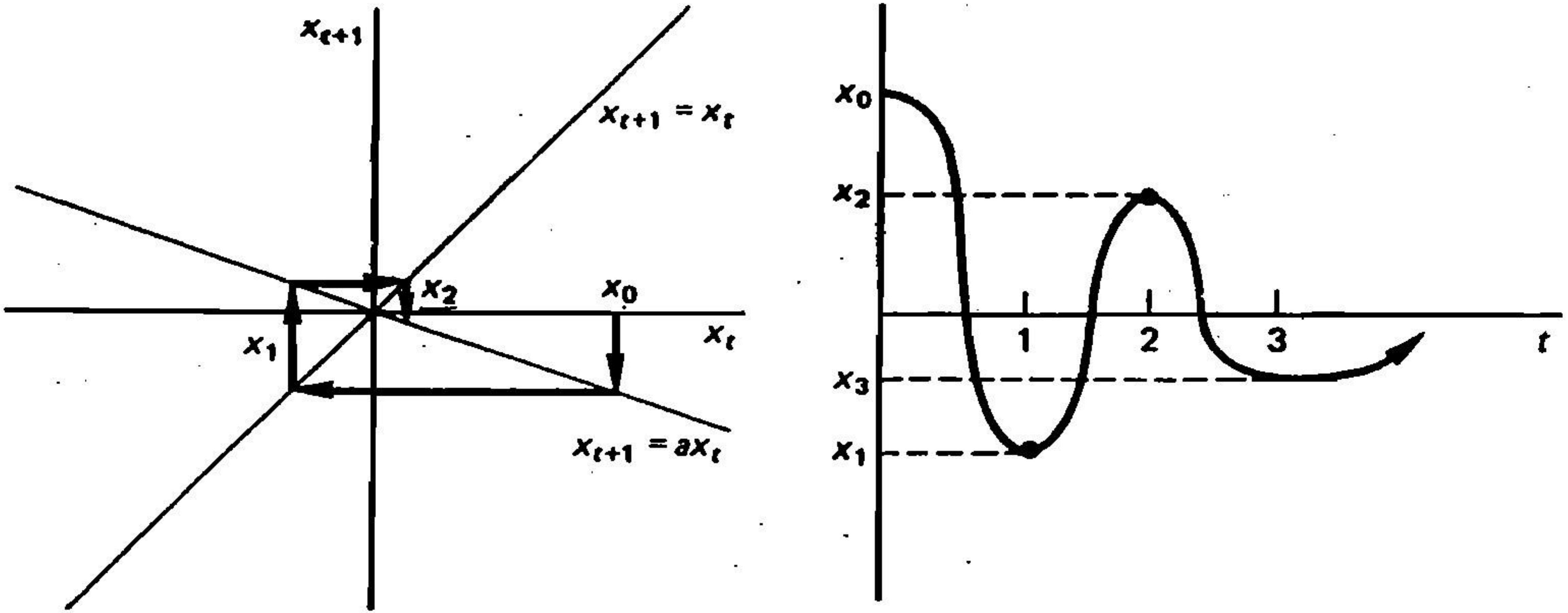
   $(y_{t+1} = y_t)$

Case 1: $a \in (0,1)$. The system converges smoothly to the origin, which is the only "steady state" of the equation; once x becomes zero, it remains zero forever. Suppose, for instance, that $a = 0.5$ and $x_0 = 16$. Then $x_1 = (0.5)(16) = 8$, $x_2 = 4$, $x_3 = 2$, $x_4 = 1$, etc. As $t \to \infty$, x_t clearly converges to zero.



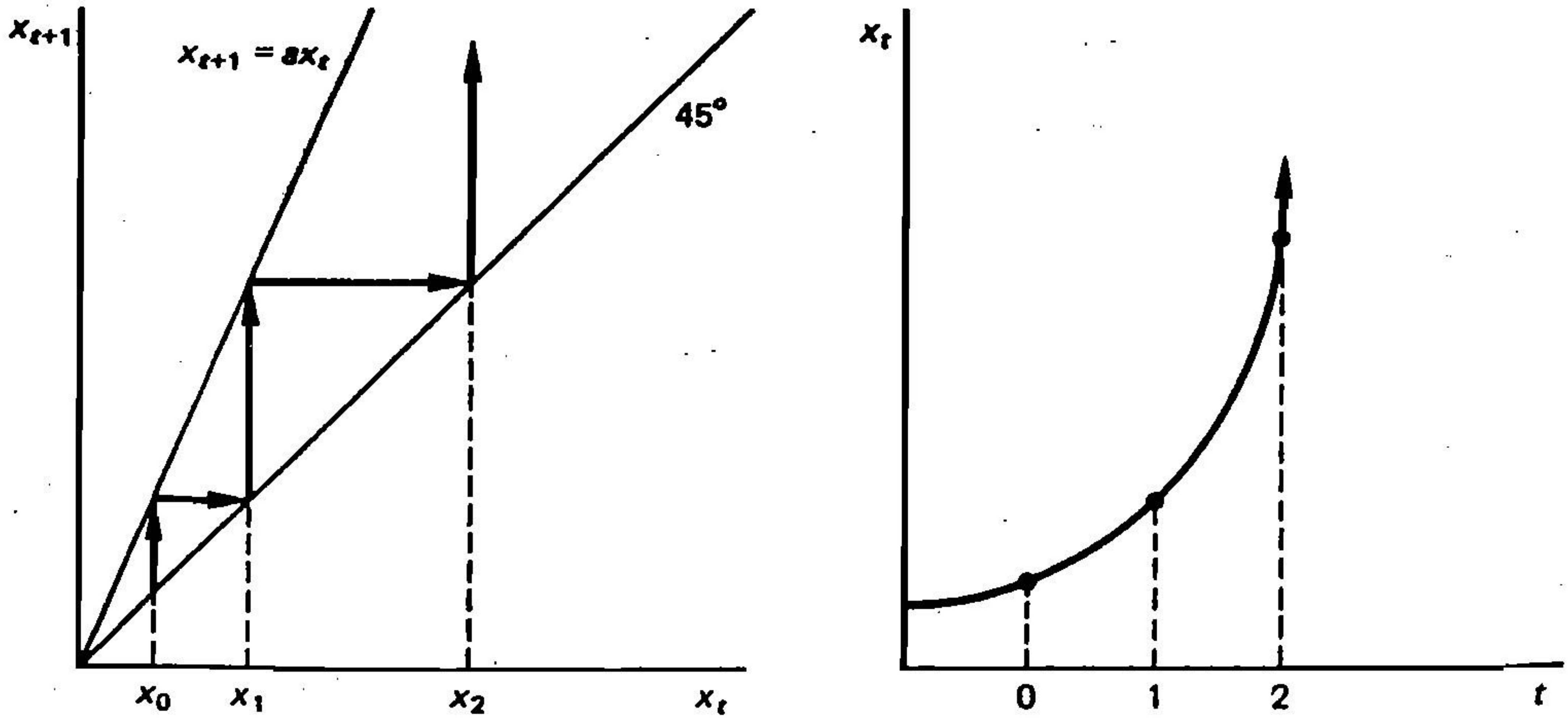
(a)

Case 2: $a \in (-1,0)$. The system converges again to zero, but now positive and negative values of x_t alternate in a pattern of damped oscillations. See figure 2.1(b) for the phase diagram and time path.



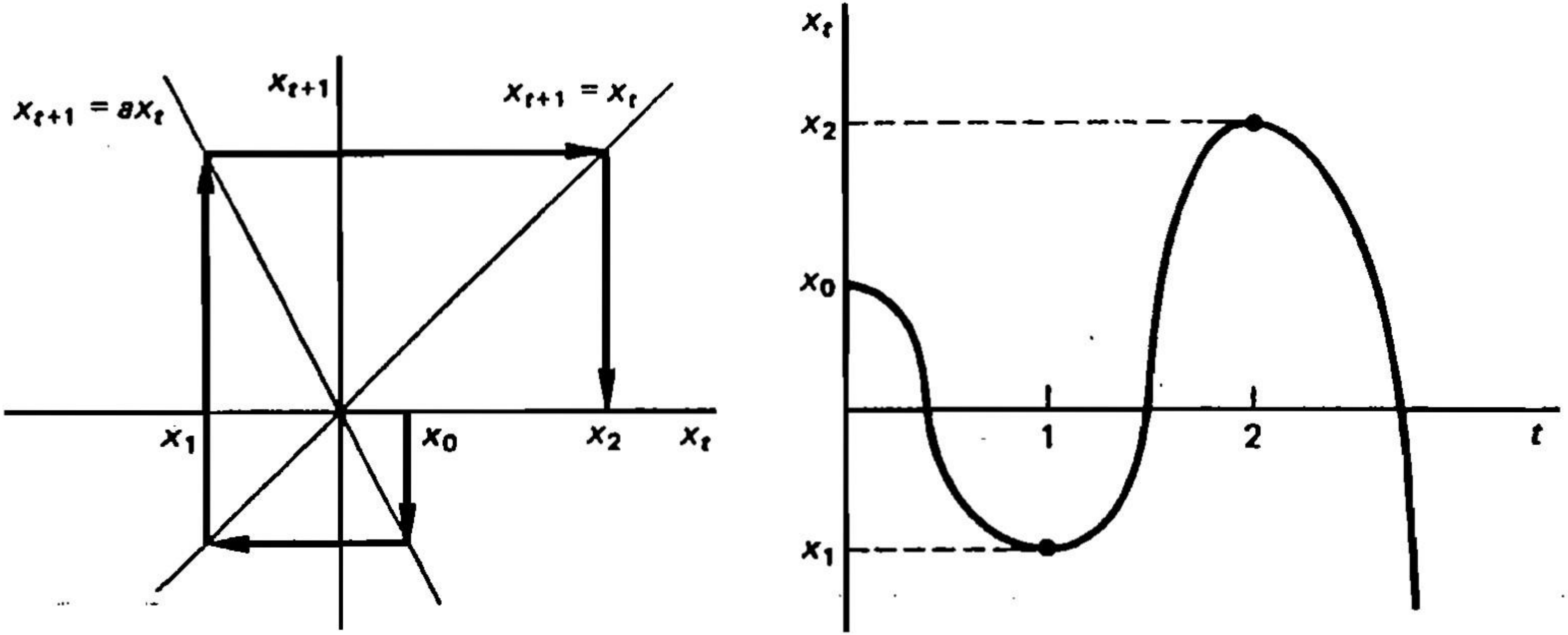
(b)

Case 3: $a \in (1,\infty)$. Here the system "explodes." If $x_0 > 0$, then $x_t \to \infty$ as $t \to \infty$; if $x_0 < 0$, then $x_t \to -\infty$ as $t \to \infty$. See figure 2.1(c).



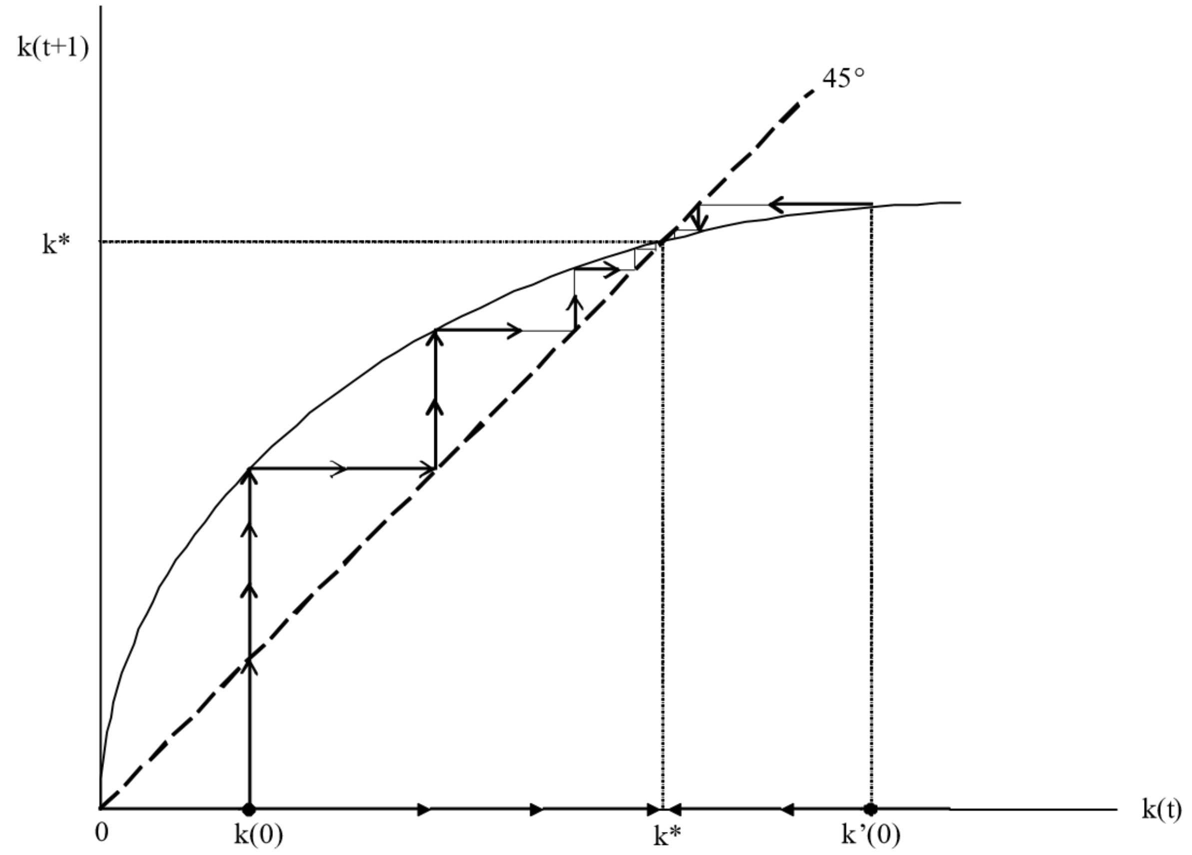
(c)

Case 4: $a \in (-\infty, -1)$. In this case, all solutions to equation (2.3) are explosive oscillations, as shown in figure 2.1(d).



(d)

THE STEADY STATE: STABLE AND UNSTABLE



CALCULATION

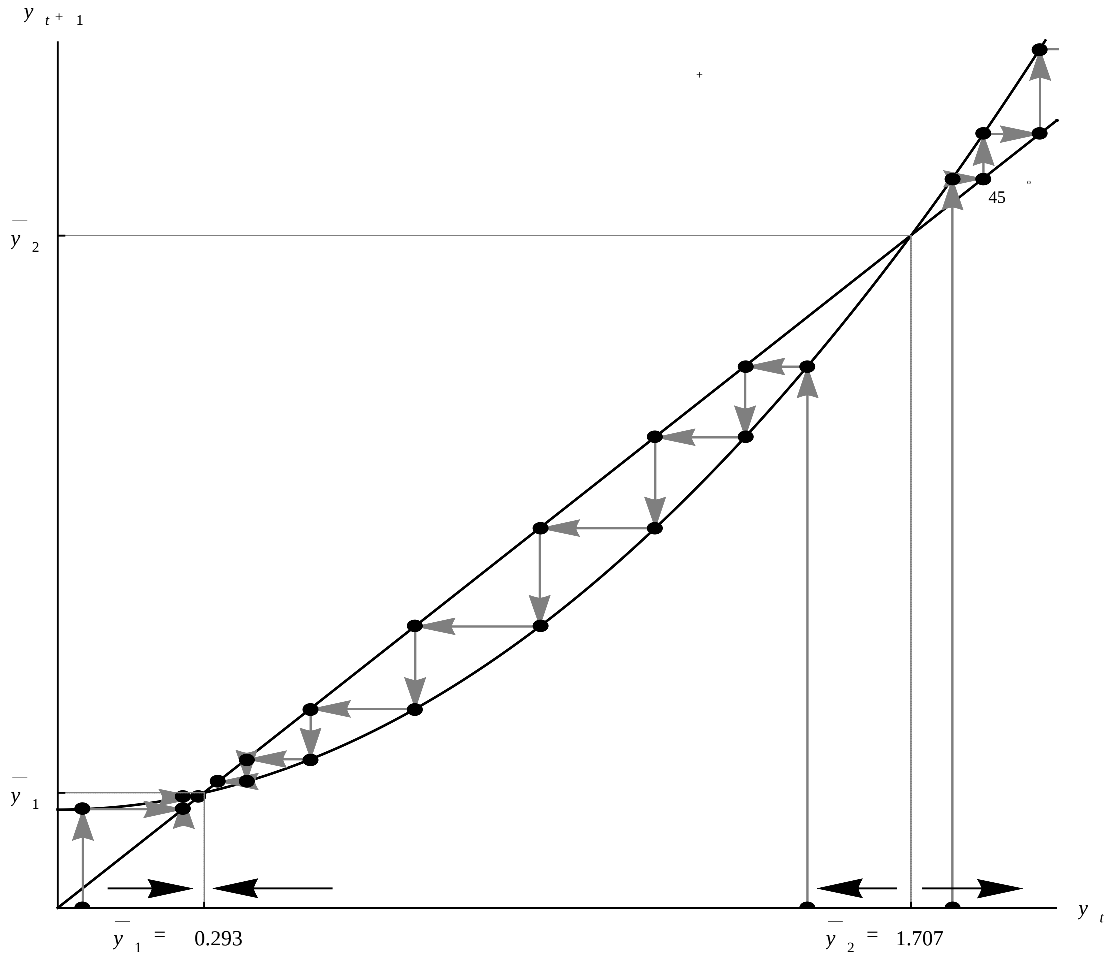
See Juliabox.

STABILITY PROPERTIES OF STEADY STATES

The local stability of a Steady State is determined by examining the derivative $g'(y)$ evaluated at $\bar{y}$. The normal cases are:

- If $|g'(\bar{y})| < 1$, the steady state is locally stable
- If $|g'(\bar{y})| > 1$, the steady state is locally unstable
- If $|g'(\bar{y})| = 1$, the steady state is semi-stable (convergence from one side and divergence from the other)
- If $g'(\bar{y}) < 0$, y follows an oscillatory time path
- If $g'(\bar{y}) > 0$, y evolves monotonically

CONVERGE AND DIVERGE



“it may be used to qualitatively visualize solutions, or to numerically approximate them.”