ECON501 2017

lab11

OUTLINE： ESTIMATING THE MODEL

1. The dataset
2. Prior distribution
3. The estimation routine
4. produce and interpret figures

THE NEOCLASSICAL GROWTH MODEL

A representative household's problem is

$$\max_{\{c_t, l_t\}_{t=0}^{\infty}} E_0 \sum_{t=1}^{\infty} \beta^{t-1} \frac{\left(c_t^{\theta}(1-l_t)^{1-\theta}\right)^{1-\tau}}{1-\tau}$$

subject to the resource constraint

$$c_t + i_t = e^{z_t} k_t^{\alpha} l_t^{1-\alpha}$$

the law of motion for capital

$$k_{t+1} = i_t + (1-\delta) k_t$$

and the stochastic process for productivity

$$z_t = \rho z_{t-1} + s\epsilon_t$$

with $\epsilon_t \sim N(0, \sigma^2)$.

The system of equations characterizing an equilibrium is comprised of equations (1), (2) and (3), the Euler intertemporal condition

$$\frac{\left(c_t^\theta(1-l_t)^{1-\theta}\right)^{1-\tau}}{c_t} = \beta E_t\left[\frac{\left(c_{t+1}^\theta(1-l_{t+1})^{1-\theta}\right)^{1-\tau}}{c_{t+1}}(1+\alpha e^{z_t}k_t^{\alpha-1}l_t^\alpha-\delta)\right] \tag{4}$$

and an optimality condition for supply of labor

$$\frac{1-\theta}{\theta}\frac{c_t}{1-l_t} = (1-\alpha)e^{z_t}k_t^\alpha l_t^{-\alpha}. \tag{5}$$

Use (1) and (2) to get

$$k_{t+1} = e^{z_t}k_t^\alpha l_t^{1-\alpha} - c_t + (1-\delta)k_t. \tag{6}$$

An equilibrium is characterized by a system of four equations (3), (4), (5) and (6).

MODEL;

```
model;
    (c^the*(1-lab)^(1-the))^(1-tau)/c=bet*((c(+1)^the*(1-lab(+1))^(1-the))^(1-
tau)/c(+1))*(1+alp*exp(z(+1))*k^(alp-1)*lab(+1)^(1-alp)-del);
    c=the/(1-the)*(1-alp)*exp(z)*k(-1)^alp*lab^(-alp)*(1-lab);
    k=exp(z)*k(-1)^alp*lab^(1-alp)-c+(1-del)*k(-1);
    z=rho*z(-1)+s*e;
end;
```

PARAMETERS

```
parameters bet del alp rho the tau s;

bet   = 0.987;  % discount factor
the   = 0.357;  % share of consumption in utility function
del   = 0.012;  % appreciation rate
alp   = 0.4;    % share of capital in production function
tau   = 2;      % intertemporal preference parameter
rho   = 0.95;   % coefficient for AR(1) stochastic process of technology
s     = 0.007;  % standard deviation of productivity shock
```

ESTIMATION

Provided that you have observations on some endogenous variables, it is possible to use Dynare to estimate some or all parameters.

Both maximum likelihood and Bayesian techniques are available.

Using Bayesian methods, it is possible to estimate DSGE models, VAR models, or a combination of the two techniques called DSGE-VAR.

ESTIMATION

We perform a Bayesian estimation of the general model, using simulated series.

The simulated data series for consumption are used to estimate some unknown parameters of the model.

Suppose we would like to estimate the preference parameters θ and τ, and the stochastic process for productivity, summarized by two parameters ρ and σ.

OBSERVED VARIABLES

Note that in order to avoid stochastic singularity, you must have at least as many shocks or measurement errors in your model as you have observed variables.

The estimation using a first order approximation can benefit from the block decomposition of the model .

This command lists the name of observed endogenous variables for the estimation procedure. These variables must be available in the data file

```
%---------------------------------------------------------------
% Variables observed
%---------------------------------------------------------------
varobs c;
```

THE DATASET

Observed variables are declared after varobs. You can include the dataset in the following ways:

– As matlab savefile (*.mat). Names of variables have to correspond to the ones declared under varobs.

– As m-file. Again names of variables have to correspond to the ones declared under varobs.

ESTIMATION: PRIOR DISTRIBUTION

This block lists all parameters to be estimated and specifies bounds and priors as necessary.

Four common prior distributions used in the literature:

– Beta distribution for parameters between 0 and 1.

– Gamma distribution for parameters restricted to be positive.

– InverseGamma distribution for the standard deviation of the shocks.

– Normal distribution.

Each line corresponds to an estimated parameter. In a Bayesian estimation, each line follows this syntax:

```
%----------------------------------------------------------------
%  Estimate parameters with setting prior
%----------------------------------------------------------------
estimated_params;
stderr e, inv_gamma_pdf, 0.95,30;  % the stochastic process for productivity
rho, beta_pdf,0.93,0.02;
the, normal_pdf,0.3,0.05;  % the preference  parameters
tau, normal_pdf,2.1,0.3;
end;
```

THE ESTIMATION ROUTINE

The command estimation triggers the estimation of the model:

1. The likelihood function of the model is evaluated by the Kalman Filter.

2. Posterior mode is computed.

3. The distribution around the mode is approximated by a Markov Chain Monte Carlo algorithm.

4. Diagnostics, impulse response functions, moments are printed.

SOME OPTIONS INCLUDE:

– datafile= FILENAME specifies the filename.

– nobs number of observation used.

– first_obs specifies the first observation to be used.

– mode_compute specifies the optimizer. For example:

- ∗ 0: switch mode computation off
- ∗ 1: fmincon
- ∗ 4: csminwel

– nodiagnostic

– The Dynare userguide offers a very good description of all options available.

INTERPRET FIGURES

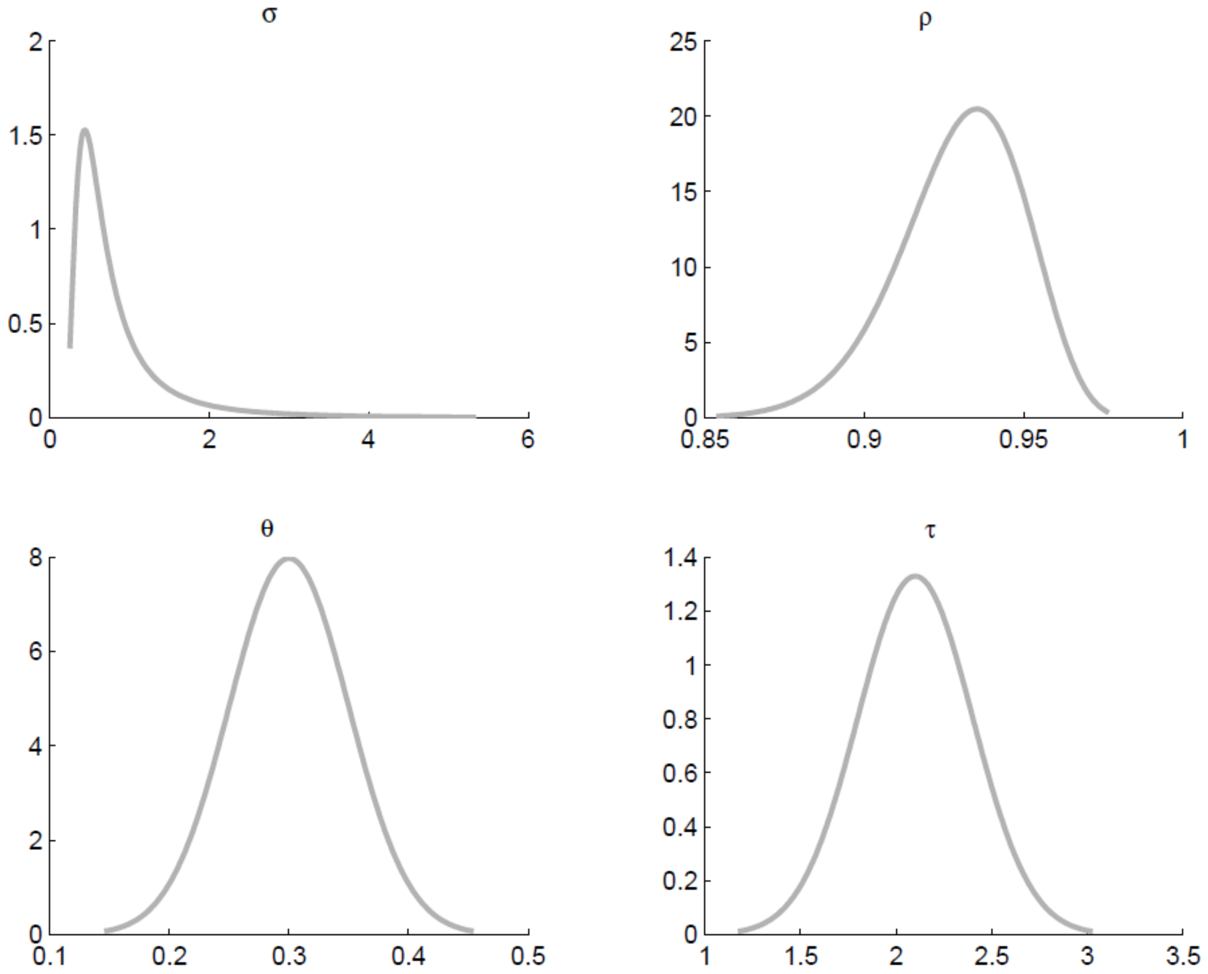
Prior vs. Posterior

• For each parameter Dynare plots the prior and the posterior distribu-
tion in one figure.

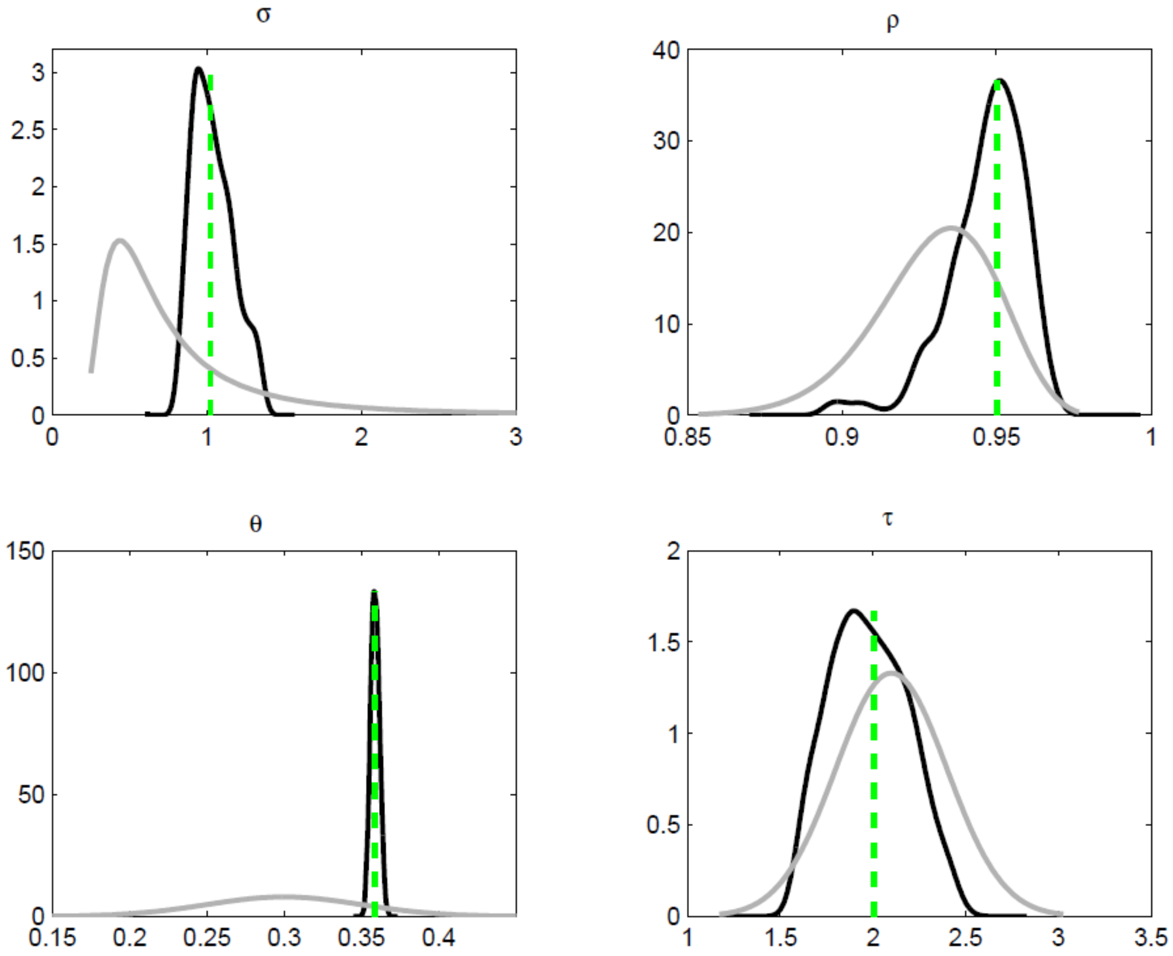
• The grey line represents the prior, the black line the posterior. Both should be different from each other. In case there are not the parameter is not identified.

• The dotted green line represents the value at the posterior mode. Ide-
ally the mode is in the center of the posterior distribution.

THE PRIORS USED IN ESTIMATION



POSTERIOR DISTRIBUTION



Parameter	Distribution	Mean	Std.Dev.
ρ	Beta	0.93	0.02
θ	Normal	0.3	0.05
τ	Normal	2.1	0.3
σ	Inv. Gamma	0.95	inf.

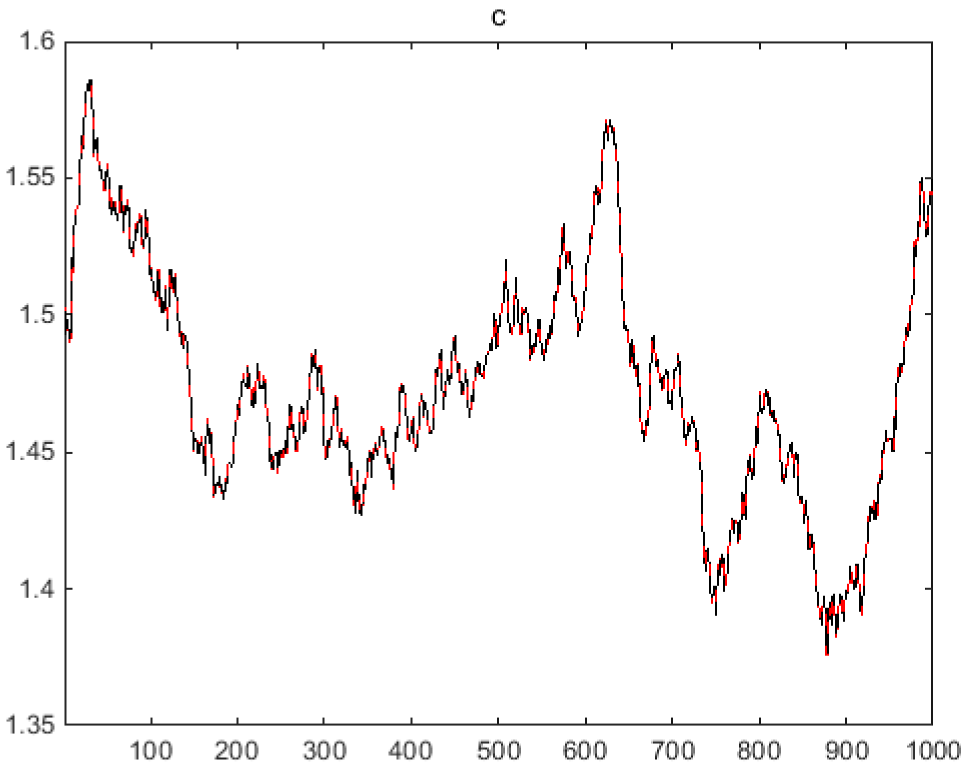
Table 3: Priors

	prior mean	post. mean	conf.	interval	prior dist	prior std
ρ	0.930	0.9480	0.9319	0.9645	beta	0.0200
θ	0.300	0.3589	0.3540	0.3645	norm	0.0500
τ	2.100	2.0046	1.6532	2.3356	norm	0.3000
σ	0.950	1.0227	0.8321	1.2175	invg	Inf

Table 4: Posterior.

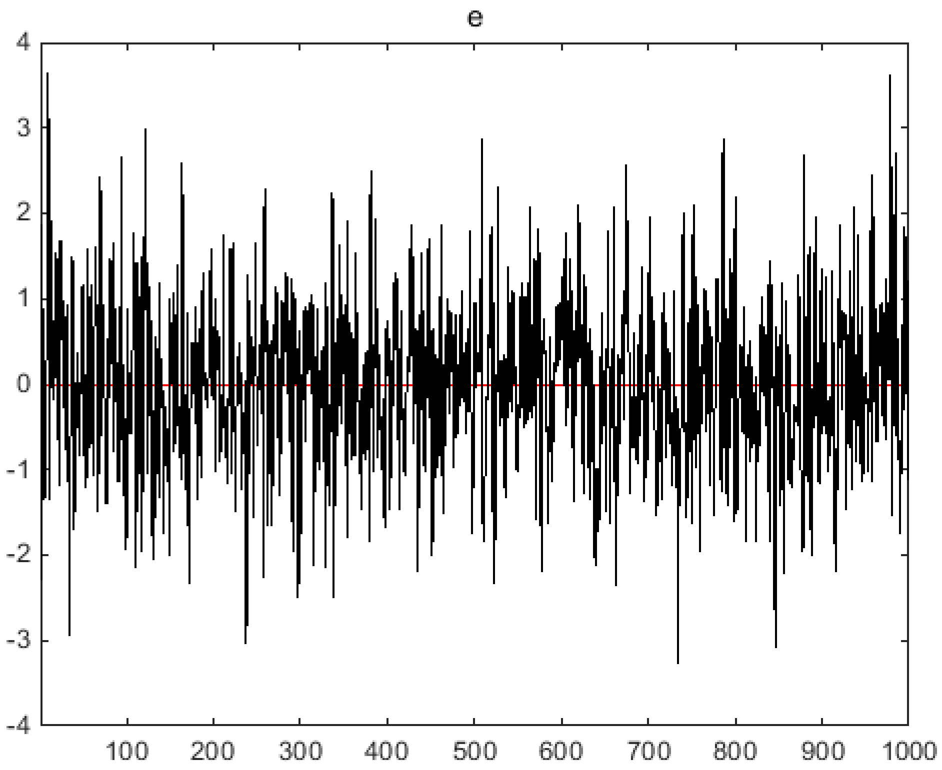
FILTERED AND SMOOTHED VARIABLES

smoother computes posterior distribution of smoothed endogenous variables and shocks, i.e. infers about the unobserved state variables using all available information up to T (see figure 4): $C_{t|T} = E[C_t \mid I_T]$



The plot of smoothed shocks is always produced.

It also serves as a check for the estimation → the shock realizations should be around zero. (see figure 3)



- $\rightarrow$ The average acceptance rate and therefore the speed of convergence depend on the scaling parameter c.
- Recommended is an accepted rate of about 0.23 (see Roberts et al. (1994) for a formal derivation). The optimal scale factor has to be found by trial and error.
- *mh_jscale* sets the scaling parameter.
- *mh_init_scale* allows for a wider distribution for the first draw.

MARKOV CHAIN MECHANISM

- Given θ^{i-1}, draw the parameter vector θ from a joint normal distribution (proposal distribution):

$$\theta^i \sim \mathcal{N}(\theta^i, c^2\Sigma)$$

  where Σ denotes the inverse Hessian evaluated at the posterior mode and c a scaling factor.

- Denote the logobjective function as $l(\theta)$. The draw is then accepted with probability:

$$min(1, exp(l(\theta^i) - l(\theta^{i-1})))$$

- Repeat this until the distribution has converged to the target distribution.

State Space Model

http://quant-econ.net/py/linear_models.html

Kalman Filter

http://quant-econ.net/py/kalman.html

Markov chain Monte Carlo (Metropolis-Hastings)

http://quant-econ.net/py/finite_markov.html

https://en.wikipedia.org/wiki/Markov_chain_Monte_Carlo

https://en.wikipedia.org/wiki/Metropolis%E2%80%93Hastings_algorithm

https://theoreticalecology.wordpress.com/2010/09/17/metropolis-hastings-mcmc-in-r/