Macroeconomics I
Problem Set 5

Margarita Kaginian
Sugarkhuu Radnaa
Ömer Erhan Erbiş
Taha Direk

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1) Exogenous state variables are : $[A_t, e_t]$ We can choose u_t instead of e_t easily. Once we know value of A_{t-1}, e_{t-1} (from previous period) and the shock to A in period t, ϵ_t^A, all the other variables can be derived using the system.

2) Equilibrium conditions (the output is in "main_labor_model.m" file):

   1. budget constraint: $c_t = e_t w_t + \Psi_t$
   2. marginal utility of consumption: $\lambda_t = 1/c_t$
   3. output: $y_t = A_t l_t$
   4. productivity: $A_t - A = \rho(A_{t-1} - A)$
   5. labor demand: $l_t = \frac{y_t}{x_t}$
   6. labor market clearing: $l_t = e_t$
   7. unemployment: $e_t = 1 - u_t$
   8. employment dynamic: $e_t = (1 - \vartheta)e_{t-1} + f_{t-1}u_{t-1}$
   9. labor firm profit: $\Upsilon_t = x_t + w_t$
   10. labor firm profit: $J_t = \Upsilon_t - (1 - \vartheta)E_t\{\beta_{t,t+1}, J_{t+1}\}$
   11. wage equation: $w_t = \eta x_t - (1 - \eta)\pi$
   12. match: $m_t = \chi u_t^{\xi} v_t^{1-\xi}$
   13. vacancy fill prob: $q_t = \frac{m_t}{v_t}$
   14. get employed probability: $f_t = \frac{m_t}{v_t}$
   15. search profit zero condition: $\kappa = q_t E_t\{\beta_{t,t+1}, J_{t+1}\}$
   16. stochbetat: $\beta_{t,t+j} = \beta\frac{\lambda_{t+j}}{\lambda_t}$
   17. output market clearing - double counting: $y_t = c_t + \kappa v_t$
   18. firm profit: $\Psi_t = \Upsilon_t e_t + \kappa v_t$

In FOC approximation we use as a consumption Euler equation the marginal utility of consumption: $\lambda_t = 1/c_t$. This property can be seen from the solving a household problem:

$$\int_0^1 E_t\left\{\sum_{j=0}^{\infty}\beta^j[\log(c_{i,t+j}) - \zeta I(i \in e_{t+j})]\right\}di$$

s.t. $c_t = e_t w_t + \Psi_t$

The Lagrange function then :

$$L = \int_0^1 E_t\left\{\sum_{j=0}^{\infty}\beta^j[\log(c_{i,t+j}) - \zeta I(i \in e_{t+j})]\right\}di - \sum_{j=0}^{\infty}\lambda_{t+j}(c_{t+j} - e_t w_t - \Psi_t)$$

We will maximize it by $c_t, c_{t+1}, ..$

At first rewrite the lagrange function:

$$L = \int_0^1 E_t\left\{\sum_{j=0}^{\infty}\beta^j[\log(c_{i,t+j})\right\}di - \int_0^1 E_t\left\{\zeta I(i \in e_{t+j})]\right\}di - \sum_{j=0}^{\infty}\lambda_{t+j}(c_{t+j} - e_t w_t - \Psi_t)$$

We can make an assumption that in equilibrium all the family members will have equal consumptions amount in period t: $c_{it} = c_t$ for all i. Then we can omit integral from the first part of lagrange as c_t doesn't depend on i and total mass of agents equals to 1.

$$L = E_t\left\{\sum_{j=0}^{\infty}\beta^j[\log(c_{i,t+j})\right\}di - \int_0^1 E_t\left\{\zeta I(i \in e_{t+j})]\right\} - \sum_{j=0}^{\infty}\lambda_{t+j}(c_{t+j} - e_t w_t - \Psi_t)$$

Now we can take derivative by c_t for period t ($j = 0$, as we have the expectation of future outcomes by period t it means that this period has already occured and we can omit the expectation for it): $\frac{1}{c_t} - \lambda_t = 0$ And here we get the marginal utility of consumption: $\lambda_t = 1/c_t$

3) At first to calculate $\bar{A}$ we minimize the function $y-1$ (as it is stated in the task that we should target a steady-state level of output of unity) on the interval of $A \in [1, 10]$. That gives $A = 1.0576$ [1].

[1] The file "findAbar.m" does the job.

$$
\begin{aligned}
\bar{A} &= 1.0576\\
\bar{x} &= \bar{A}\\
\bar{w} &= 0.4\bar{x} + (1-0.4)1.02\\
\bar{\Upsilon} &= \bar{x} - \bar{w}\\
\bar{J} &= \frac{\bar{\Upsilon}}{1-(1-0.0265)0.997}\\
E_t\{\beta_{t,t+1}, J_t\} &= 0.997\\
\bar{q} &= \frac{0.24}{E_t\{\beta_{t,t+1}, J_{t+1}\}}\\
\bar{f} &= (0.38\bar{q}^{(0.5-1)})^{1/0.5}\\
\bar{e} &= \frac{\bar{f}}{\bar{f}+0.0265}\\
\bar{u} &= 1-\bar{e}\\
\bar{m} &= \bar{f}\bar{u}\\
\bar{v} &= \left(\frac{\bar{m}}{0.38\bar{u}^{0.5}}\right)^{1/(1-0.5)}\\
\bar{l} &= \bar{e}\\
\bar{y} &= \bar{A}\bar{l}\\
\bar{c} &= \bar{y} - 0.24^{\bar{v}}\\
\bar{\lambda} &= \frac{1}{\bar{c}}\\
\bar{\Psi} &= \bar{\Upsilon}\bar{e} - 0.24^{\bar{v}}
\end{aligned}
$$

The steady state is constructed then with parameters:

$$\bar{A} = 1.0576$$
$$\bar{x} = 1.0576$$
$$\bar{e} = 0.9455$$
$$\bar{c} = 0.9808$$
$$\bar{w} = 1.0350$$
$$\bar{\Psi} = 0.0022$$
$$\bar{\lambda} = 1.0195$$
$$stoch\_\beta = 0.997$$
$$\bar{y} = 1$$
$$\bar{l} = 0.9455$$
$$\bar{x} = 1.0576$$
$$\bar{u} = 0.0545$$
$$\bar{f} = 0.46$$
$$\bar{\upsilon} = 0.0226$$
$$\bar{J} = 0.7668$$
$$\bar{v} = 0.0798$$
$$\bar{m} = 0.0251$$
$$\bar{q} = 0.3139$$

4) [2] $E_t f(y_{t-1}, y, x_{t-1}, x) = 0$. The solution is of the form $x_{t-1} = h(x, \sigma) + \sigma \eta e_{t-1}$ and $y = g(x, \sigma)$ To calculate the FOC approximation we use the notation in Schmitt-Grohe and Uribe (JEDC, 2004) and a modified version of solab.m by Klein method. To calculate the FOC approximation we compute the matrices gx and hx.

   The first-order approximations to the functions g and h around the point $(x, \sigma) = (\bar{x}, 0)$, where $\bar{x} = h(\bar{x}, 0)$, are:

$$x_t = h(x, \sigma) = \bar{x} + hx(x_{t-1} - \bar{x})$$

   and

$$y_t = g(x, \sigma) = \bar{y} + gx(x_{t-1} - \bar{x})$$

   where $\bar{y} = g(\bar{x}, 0)$.

5) In a simulation with 10,000 draws, we get the following (different runs will give slightly different result):

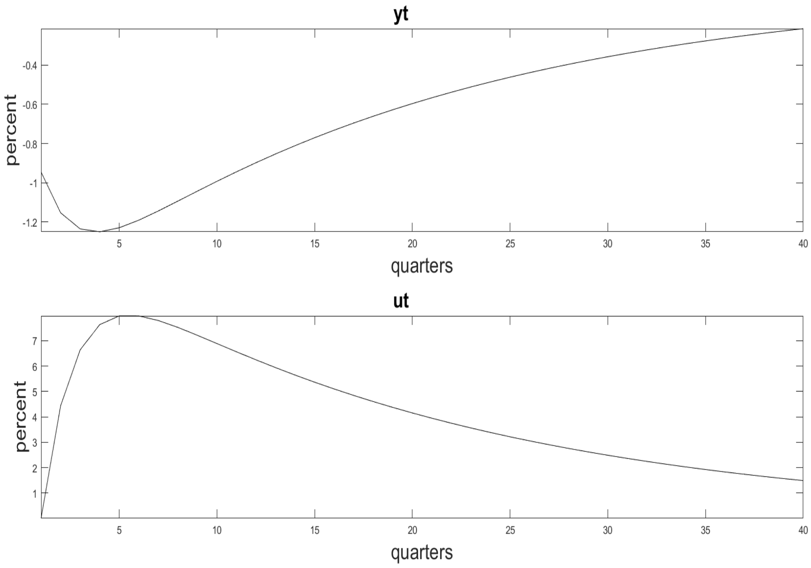
[2]Denominations: y_t - is a vector of control variables: [c_t, w_t, Ψ_t,λ_t, $\beta_{t,t+j}$, y_t, l_t, x_t, u_t, f_t, υ_t, J_t, v_t, m_t, q_t] , x - is a vector of state variables: [e_t, A_t]

	Std	Rel. std (to Y)
Consumption	0.6928	0.8294
Unemployment	5.1291	6.1400

The unemployment is about 6 times more volatile than the output and the consumption is about 80% of the output volatility. Although volatility of unemployment looks high, it is a percent of a small number. For employment, the standard deviation is about 0.3 which is still, like the standard RBC considered in the class, way too low compared to the U.S. economy statistic of 1.79.

6) Since the employment is fully determined by the previous period variables, the initial period impact on employment of productivity shock is zero. Hence output, on impact, changes by the magnitude of productivity (negative 1 percent) times labor (less than 1). Hence, initial drop in output is less than 1 percent as shown in Figure 1. Moreover, the unemployment doesn't change at all in the first period.
However, starting from the next period, the firm decision will kick in and will push the unemployment up and, hence, worsen the output. It takes four periods for output to bottom out. While productivity always increases after its initial drop, the output and the unemployment will drop for some time before the recovery in productivity eventually lifts them up and put them back to the direction towards their steady state.

Figure 1. The impulse response of output and unemployment to a contractionary 1 percent productivity shock



7) It is given:

$$w_t = \eta x_t + (1 - \eta)\pi$$

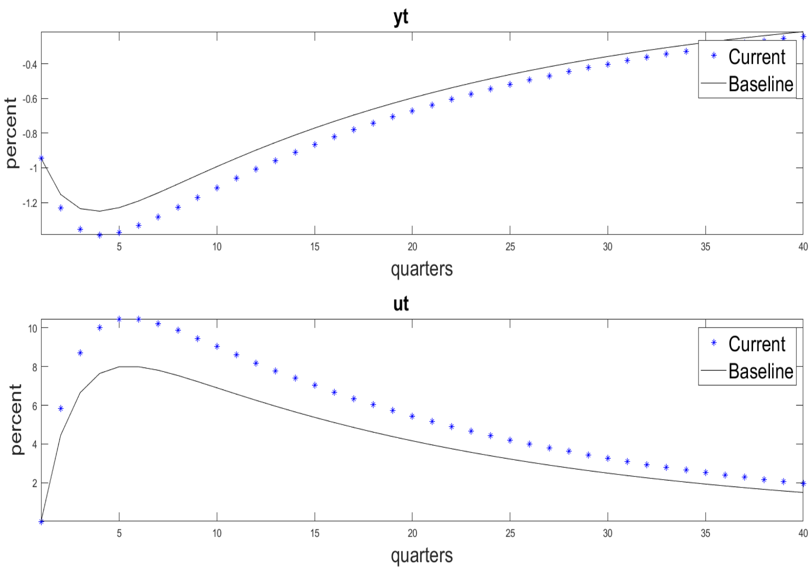
From previous results, the steady state wage is 1.0350. Since $\bar{x} = A$,

$$\pi = \frac{\bar{w} - \eta * A}{(1 - \eta)} = \frac{1.0350 - 0.2 * 1.0576}{(1 - 0.2)} = 1.02935$$

Since equilibrium steady state wage is unchanged, it should not affect the steady state unemployment (we understand 'average' as the steady state) and any other steady state levels.

Regarding the standard deviation, it increases with the new parameterization (using the HP filter with $\gamma = 1600$). In the old setting, the standard error was about 5 percent. However, in the new setting, it increased on average about 6.6 percent.

Figure 2. Wage inflexibility increases volatility in output and unemployment. The impulse response of output and unemployment in two models (1 percent negative productivity shock, marked blue is the new model)



Looking at the IRFs in Figure 2 to the negative productivity shock (symmetric for the positive shock since the model was linearized) reveals that the response of output and unemployment are indeed larger over all horizons.

The rationale is that when the wage is less responsive to the negative productivity shock, it hurts firm's profit much larger than the original setting. Productivity of the worker has declined, but the wage is not decreasing as much as it did in the original setting. The firm will react by decreasing its search effort in line with the reduced profit, thus resulting in higher unemployment. Again, in positive shock, firm will overshoot the original setting in symmetric way. [3].

8) We introduce a variable τ_t as instructed [4]. For this additional variable, we need an additional equation to pin down its dynamic. It should be the case that the unemployment is fixed over time. We can use this property as an additional equation as:

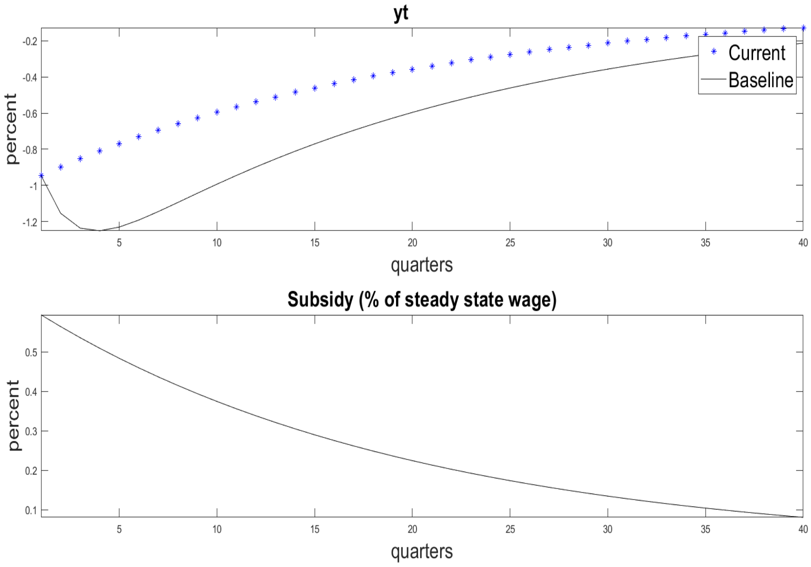
$$u_{t+1} = u_t$$

[3]The code is in "eta_labor_sim.m" file. We did the dozens of replicaitons using the same file. Ideally we could run 100 or 1000 similar simulations to achieve the average standard deviations.

[4]The codes are in the files "wsub_labor_model.m" and "wsub_labor_sim.m".

The steady state of τ_t is 0 as given [5]. The behaviour of the model could differ depending on who bears the tax burden, but, in the steady state it is zero [6]. Hence, the steady state of the model will be unchanged.

Figure 3. The subsidy decreases output volatility. The impulse response of output and subsidy to a contractionary 1 percent productivity shock



As shown in Figure 3. The response of output is less (does not drop as much) than the original (Baseline) both in the initial period and along the way. Thanks to the subsidy by the government, the firm are able to keep employing its workers so that it can produce higher output in the face of negative productivity shock.
The subsidy, as percentage of steady state wage, initially jumps to about 0.6 percent. It gradually decreases as productivity improves and the firm is able to offer higher wages to its workers so that, if not fully, the part of the assistance from the government to keep its employees hired fully is eliminated (i.e., the firm can pay that part on its own) [7].

In relation to the question 7, the subsidy allows more flexibility in the wage thus avoiding larger output volatility.

[5] One technical issue is regarding the exogeneity of e_t that we have applied so far. It is now determined out of the system at its steady state level given the fixed unemployment. Thus, we should move it out of the exogenous state variables. Therefore, the system now has only A_t as the exogenous state variable.

[6] We left the tax out of the model and aware that it must be wrong to do so, but do not know whom to levy upon the tax. Approximately, the consumption should follow the output closely (now it differs notably as can be seen in the IRFs), but depending on the how the tax is levied (lump sum etc) it can affect firm's behaviour thus change the eventual outcome. But we believe that this effect is marginal to the current discussion although it can in general important

[7] We add another variable w_firm_t and use its definition for the equation. The response of the subsidy is calculated as the difference in responses between employee's wage and firm's wage (i.e., in line with $subsidy_t = w_t - w\_firm_t$)