

cppEDM Version 1.13 June 17, 2022

cppEDM is a C++ implementation of empirical dynamic modeling (EDM) algorithms. It is designed as an application programming interface (API) to functions in the libEDM.a library.

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Introduction

[cppEDM](#) is a C++ implementation of empirical dynamic modeling ([EDM](#)) algorithms. Primary algorithms are listed in table 1. The core code is an object oriented implementation supporting application programming interface (API) functions accepting parameters and returning data objects. EDM functions are accessed from a user-compiled library created from C++ source files and a unix-like compiler supporting the C++11 standard. [cppEDM](#) has Python and R interfaces implemented in the [pyEDM](#) and [rEDM](#) packages.

Algorithm	API Interface	Reference
Simplex projection	<code>Simplex()</code>	Sugihara and May (1990)
Sequential Locally Weighted Global Linear Maps (S-map)	<code>SMap()</code>	Sugihara (1994)
Predictions from multivariate embeddings	<code>Simplex()</code> , <code>SMap()</code>	Dixon et. al. (1999)
Convergent cross mapping	<code>CCM()</code>	Sugihara et. al. (2012)
Multiview embedding	<code>Multiview()</code>	Ye and Sugihara (2016)

Convenience functions to prepare and evaluate data are listed in table 2.

Function	Purpose	Parameter Range
<code>Embed()</code>	Timeseries delay dimensional embedding	User defined
<code>EmbedDimension()</code>	Evaluate prediction skill vs. embedding dimension	$E = [1, 10]$
<code>PredictInterval()</code>	Evaluate prediction skill vs. forecast interval	$T_p = [1, 10]$
<code>PredictNonlinear()</code>	Evaluate prediction skill vs. SMap nonlinear localisation	$\theta = 0.01, 0.1, 0.3, 0.5, 0.75, 1, 1.5, 2, 3, 4, 5, 6, 7, 8, 9$
<code>ComputeError()</code>	Pearson correlation, MAE, RMSE	

Installation

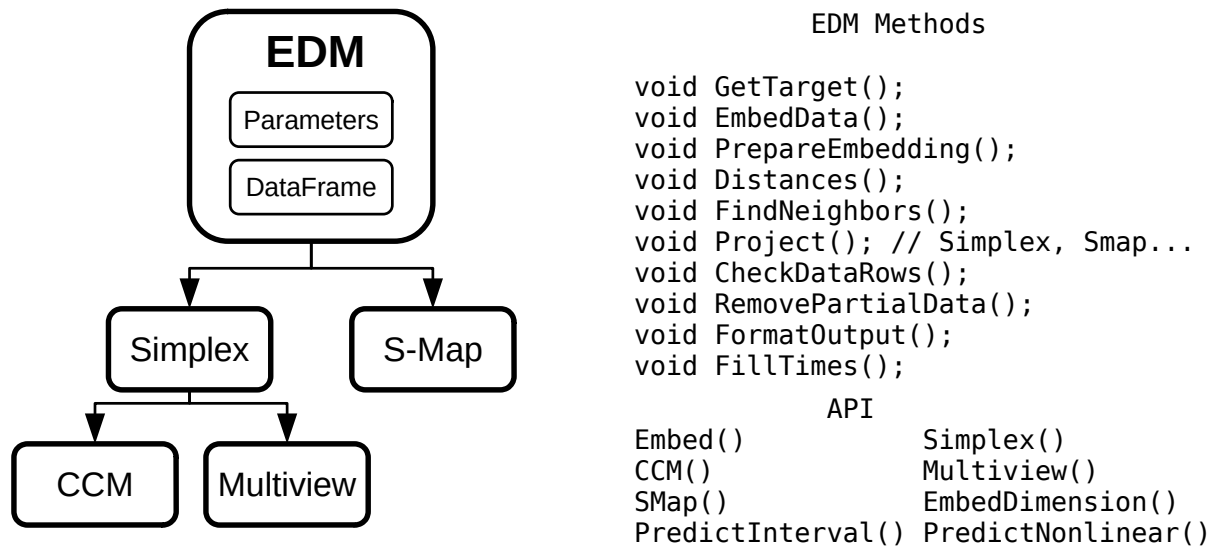
[cppEDM](#) is available at github.com/SugiharaLab/cppEDM.

[cppEDM](#) requires a C++11 standard compiler, and the LAPACK library. The `libEDM.a` library can be built by running "make" in the `cppEDM/src/` directory. This copies `libEDM.a` into the `cppEDM/lib/` directory, where it can be linked to user applications.

Once `libEDM.a` is built, there are a series of test applications in the `cppEDM/tests/` directory. The applications can be built with the "make" command, and executed at the command line. API examples can also be found in `cppEDM/etc/Test.cc`.

Class Objects

cppEDM is a C++ Object Oriented (OO) implementation. The primary data class is EDM, from which the algorithm classes are derived. The class hierarchy is:



Two C++ class objects are used for data access and parameter coordination, the `DataFrame` and `Parameters` classes, described below.

DataFrame

The `DataFrame` class is the fundamental data object of cppEDM. It stores data in a contiguous block of memory using the C++ `valarray` type in a row-major format.

A `DataFrame` can be initialised with data from a csv file by calling the `DataFrame` constructor with `path` and `fileName` parameters. Data input files are assumed to be in csv format. The files are required to have a single line header with column names.

It is assumed that the first column of the csv file is a vector of times or time indices for each observation (row). All subsequent columns are expected to be numeric. However, a `DataFrame` can be created with the `noTime = true` parameter to read in numeric data with no time in the first column. Such data frames should not be passed to prediction functions.

The `WriteData(path, file)` class method can be called explicitly to write data to a csv format file. If the `DataFrame` does not have column names, then column names are created as V1, V2...

Primary `DataFrame` access functions are listed in table 3.

DataFrame Method	Parameters	Type	Purpose
(row, column)	size_t row size_t column	double or int	Access data element
DataFrame(path, file)	string path string fileName	DataFrame<double>	Create DataFrame from csv file
WriteData(path, file)	string outputFilePath string outputFileName		Write DataFrame to file
Elements()		valarray	Access data valarray
NColumns()		size_t	Get number of columns
NRows()		size_t	Get number of rows
size()		size_t	Get number of elements
ColumnNames()		vector< string >	Access column names
ColumnNameToIndex()		map<string, size_t>	Access column name to index map
MaxRowPrint()		size_t	Access maximum number of rows to ostream
Column(col)	size_t col	valarray	Get data vector at column
Row(row)	size_t row	valarray	Get data vector at row
VectorColumnName(column)	string column	valarray	Get data vector at column with name
ColumnMajorData()		valarray	Elements() in column major format.
DataFrameFromColumnIndex (columns)	vector<size_t> columns	DataFrame<double>	Get DataFrame subset from column indices
DataFrameFromColumnNames (columns)	vector<string> columns	DataFrame<double>	Get DataFrame subset from column names
DataFrameFromRowIndex (rows)	vector<size_t> rows	DataFrame<double>	Get DataFrame subset from row indices
WriteRow(row, array)	size_t row std::valarray<T> array		Write valarray to row
WriteColumn(col, array)	size_t col valarray<T> array		Write valarray to column

Parameters

The `Parameters` class is used to store and access API function parameters in a unified object. Generally this is an internal object that does not need to be instantiated, accessed or dynamically modified. API parameter names and purpose are listed in table 4.

Parameter	Type	Default	Purpose
<code>pathIn</code>	string	<code>"./"</code>	Input data file path
<code>dataFile</code>	string	<code>""</code>	Data file name
<code>pathOut</code>	string	<code>"./"</code>	Output file path
<code>predictFile</code>	string	<code>""</code>	Prediction output file
<code>smapFile</code>	string	<code>""</code>	SMap coefficient output file
<code>lib</code>	string	<code>""</code>	library start : stop row indices
<code>pred</code>	string	<code>""</code>	prediction start : stop row indices
<code>E</code>	int	<code>0</code>	Embedding dimension
<code>Tp</code>	int	<code>0 or 1</code>	Prediction interval (rows)
<code>knn</code>	int	<code>0</code>	Number nearest neighbors
<code>tau</code>	int	<code>-1</code>	Embedding offset (time series rows)
<code>theta</code>	double	<code>0</code>	SMap localisation
<code>exclusionRadius</code>	int	<code>0</code>	Prediction vector exclusion row radius
<code>columns</code>	string	<code>""</code>	Column names or indices for prediction
<code>target</code>	string	<code>""</code>	Target library column name or index
<code>embedded</code>	bool	<code>false</code>	Is data an embedding?
<code>const_pred</code>	bool	<code>false</code>	Include non-projected forecast data
<code>verbose</code>	bool	<code>false</code>	Echo messages
<code>validLib</code>	vector<bool>	<code>[]</code>	Conditional Embedding (CE)
<code>generateSteps</code>	int	<code>0</code>	Simplex, SMap feedback prediction
<code>parameterList</code>	bool	<code>false</code>	Return Parameters map
<code>multiview</code>	int	<code>0</code>	Number of ensembles, $0 = \sqrt{N}$
<code>D</code>	int	<code>0</code>	Multiview dimension
<code>trainLib</code>	bool	<code>true</code>	Multiview use lib as training library
<code>excludeTarget</code>	bool	<code>false</code>	Multiview exclude target from combos
<code>libSizes</code>	string	<code>""</code>	CCM library sizes
<code>sample</code>	int	<code>0</code>	CCM number of random samples
<code>random</code>	bool	<code>true</code>	CCM use random samples?
<code>replacement</code>	bool	<code>false</code>	CCM sample with replacement?
<code>includeData</code>	bool	<code>false</code>	CCM include all projections in return
<code>seed</code>	unsigned	<code>0</code>	CCM RNG seed, $0 = \text{random seed}$

Application Programming Interface (API)

Embed

Create a data block of Takens (1981) time-delay embedding from each of the columns in the csv file or DataFrame. The columns parameter can be a list of column names, or a list of column indices. If columns is a list of indices, then column names are created as V1, V2...

Note: The returned DataFrame will have $|\tau| * (E - 1)$ fewer rows than the input data from the removal of partial vectors as a result of the embedding.

Note: The returned DataFrame does not have the time column.

```
//-----  
// Overload 1: Explicit data file path/name  
//-----  
DataFrame< double > Embed ( std::string path      = "",  
                           std::string dataFile = "",  
                           int             E      = 0,  
                           int             tau     = -1,  
                           std::string columns = "",  
                           bool            verbose = false );  
  
//-----  
// Overload 2: DataFrame provided  
//-----  
DataFrame< double > Embed ( DataFrame< double > dataframe,  
                           int             E      = 0,  
                           int             tau     = -1,  
                           std::string columns = "",  
                           bool            verbose = false );  
  
//-----  
// Called from Embed to create the time-delay embedding  
//-----  
DataFrame< double > MakeBlock ( DataFrame< double > dataframe,  
                               int             E,  
                               int             tau,  
                               std::vector<std::string> columnNames,  
                               bool            deletePartial = false );
```

Simplex

Simplex projection of the input data file or DataFrame. See the Parameters table for parameter definitions.

`Simplex()` returns a `SimplexValues` structure:

```
struct SimplexValues {  
    DataFrame< double >          predictions;  
    std::map< std::string, std::string > parameterMap;  
};
```

The predictions DataFrame has 3 columns "Time", "Observations", "Predictions". nan values are inserted where there is no observation or prediction. The parameterMap returns a map of parameters used in the predictions if `parameterList = true`.

Parameters

`lib` and `pred` specify [start stop] row indices of the input data for the library and predictions.

If `embedded` is `false` the data columns are embedded to dimension `E` with time offset `tau`. If `embedded` is `true` the data columns are assumed to be a multivariable data block.

If `knn` is not specified, and `embedded` is `false`, it is set equal to `E+1`. If `embedded` is `true`, `knn` is set equal to the number of columns + 1.

`exclusionRadius` defines the number of library rows excluded from the state-space library with respect to a temporal "radius" from the prediction state. If `exclusionRadius = 1`, library state-space points from observation time series that are within ± 1 sequential observation row of the prediction state are not included in the library. Note that units of the radius are time series rows, not time values.

`validLib` implements conditional embedding (CE). It is a boolean vector the same length as the number of time series rows. A `false` entry means that the state-space vector derived from the corresponding time series row will not be included in the state-space library.

If `generateSteps > 0`, then `Simplex` and `SMap` operate in feedback generative mode. The values of `pred` are over-riden to start at the end of the data. At each step one prediction is made, added to the columns data, a new time-delay embedded is created, and the cycle repeated for `generateSteps`. Feedback generation only operates on a univariate time series that is time-delay embedded. The columns and target variables must be the same.

If `parameterList = true`, then `parameterMap` is populated.

```

//-----
// Overload 1: Explicit data file path/name
//-----
SimplexValues Simplex( std::string pathIn          = "./data/",
                       std::string dataFile       = "",
                       std::string pathOut        = "./",
                       std::string predictFile    = "",
                       std::string lib           = "",
                       std::string pred          = "",
                       int E                     = 0,
                       int Tp                   = 1,
                       int knn                  = 0,
                       int tau                  = -1,
                       int exclusionRadius       = 0,
                       std::string columns      = "",
                       std::string target       = "",
                       bool embedded            = false,
                       bool const_pred         = false,
                       bool verbose            = true,
                       std::vector<bool> validLib = std::vector<bool>(),
                       int generateSteps      = 0,
                       bool parameterList     = false );

//-----
// Overload 2: DataFrame reference provided
//-----
SimplexValues Simplex( DataFrame< double > & dataFrameIn,
                       std::string pathOut      = "./",
                       std::string predictFile  = "",
                       std::string lib         = "",
                       std::string pred        = "",
                       int E                   = 0,
                       int Tp                  = 1,
                       int knn                 = 0,
                       int tau                 = -1,
                       int exclusionRadius     = 0,
                       std::string columns     = "",
                       std::string target      = "",
                       bool embedded           = false,
                       bool const_pred        = false,
                       bool verbose           = true,
                       std::vector<bool> validLib = std::vector<bool>(),
                       int generateSteps      = 0,
                       bool parameterList     = false );

```


SMap

SMap projection of the input data file or DataFrame. See the Parameters table for parameter definitions. If nan are detected in the columns or target, those rows are removed prior to any time-delay embedding. This may violate the presumes of Takens theorem.

SMap() returns a SMapValues structure:

```
struct SMapValues {  
    DataFrame< double > predictions;  
    DataFrame< double > coefficients;  
    std::map< std::string, std::string > parameterMap;  
};
```

The predictions DataFrame has 3 columns "Time", "Observations", "Predictions". nan values are inserted where there is no observation or prediction. If predictFile is provided the predictions will be written to it in csv format.

The coefficients DataFrame will have E+2 columns. The first column is the "Time" vector, the remaining E+1 columns are the SMap SVD fit coefficients. The first column "C0" is the bias term, following coefficients are $\partial \text{columns}[i] / \partial \text{target}$.

The parameterMap returns a map of parameters used in the predictions if parameterList = true.

Parameters

lib and pred specify [start, stop] row indices of the input data for the library and predictions.

If embedded is false the data columns are embedded to dimension E with offset tau. If embedded is true the data columns are assumed to be a multivariable data block. If smapFile is provided the coefficients will be written to it in csv format.

If a multivariate data set is used (number of columns > 1) it must use embedded = true with E equal to the number of columns. This prevents the function from internally time-delay embedding the multiple columns to dimension E. If the internal time-delay embedding is performed, then state-space columns will not correspond to the intended dimensions in the matrix inversion, coefficient assignment, and prediction. In the multivariate case, the user should first prepare the embedding (using Embed() for time-delay embedding if desired), then pass this embedding to SMap with appropriately specified columns, E, and embedded = true.

If knn is not specified, it is set equal to the library size. If knn is specified, it must be greater than E.

exclusionRadius defines the number of library rows excluded from the state-space library with respect to a temporal "radius" from the prediction state. If exclusionRadius = 1, library state-space points from observation time series that are within ± 1 sequential observation row of the prediction state are not included in the library. Note that units of the radius are time series rows, not time values.

`validLib` implements conditional embedding (CE). It is a boolean vector the same length as the number of time series rows. A `false` entry means that the state-space vector derived from the corresponding time series row will not be included in the state-space library.

The default solver for the SMap coefficient matrix is the LAPACK SVD function `dge1ss()`.

If `generateSteps > 0`, then `Simplex` and `SMap` operate in feedback generative mode. The values of `pred` are over-riden to start at the end of the data. At each step one prediction is made, added to the `columns` data, a new time-delay embedded is created, and the cycle repeated for `generateSteps`. Feedback generation only operates on a univariate time series that is time-delay embedded. The `columns` and `target` variables must be the same.

If `parameterList = true`, `parameterMap` is populated.

SMap

```
//-----  
// Overload 1: Explicit data file path/name  
//-----  
SMapValues SMap( std::string pathIn          = "./data/",  
                  std::string dataFile       = "",  
                  std::string pathOut        = "./",  
                  std::string predictFile    = "",  
                  std::string lib            = "",  
                  std::string pred           = "",  
                  int E                     = 0,  
                  int Tp                    = 1,  
                  int knn                   = 0,  
                  int tau                   = -1,  
                  double theta              = 0,  
                  int exclusionRadius        = 0,  
                  std::string columns        = "",  
                  std::string target         = "",  
                  std::string smapFile       = "",  
                  std::string derivatives    = "", // Not implemented  
                  bool embedded              = false,  
                  bool const_pred           = false,  
                  bool verbose              = true,  
                  std::vector<bool> validLib = std::vector<bool>(),  
                  int generateSteps         = 0,  
                  bool parameterList        = false );  
  
//-----  
// Overload 2: DataFrame reference provided  
//-----  
SMapValues SMap( DataFrame< double > &dataFrameIn,  
                  std::string pathOut        = "./",  
                  std::string predictFile    = "",  
                  std::string lib            = "",  
                  std::string pred           = "",  
                  int E                     = 0,  
                  int Tp                    = 1,  
                  int knn                   = 0,  
                  int tau                   = -1,  
                  double theta              = 0,  
                  int exclusionRadius        = 0,  
                  std::string columns        = "",  
                  std::string target         = "",  
                  std::string smapFile       = "",  
                  std::string derivatives    = "", // Not implemented  
                  bool embedded              = false,  
                  bool const_pred           = false,  
                  bool verbose              = true,  
                  std::vector<bool> validLib = std::vector<bool>(),  
                  int generateSteps         = 0,  
                  bool parameterList        = false );
```

SMap

```
//-----  
// Overload 3: Data path/file with external solver object, init to default SVD  
//-----  
SMapValues SMap( std::string pathIn          = "./data/",  
                  std::string dataFile       = "",  
                  std::string pathOut        = "./",  
                  std::string predictFile    = "",  
                  std::string lib            = "",  
                  std::string pred           = "",  
                  int E                      = 0,  
                  int Tp                    = 1,  
                  int knn                   = 0,  
                  int tau                   = -1,  
                  double theta              = 0,  
                  int exclusionRadius        = 0,  
                  std::string columns        = "",  
                  std::string target         = "",  
                  std::string smapFile       = "",  
                  std::string derivatives    = "", // Not implemented  
                  std::valarray<double> (*solver)(DataFrame < double >,  
                                                    std::valarray < double >) = &SVD,  
                  bool embedded              = false,  
                  bool const_predict        = false,  
                  bool verbose              = true,  
                  std::vector<bool> validLib = std::vector<bool>(),  
                  int generateSteps         = 0,  
                  bool parameterList        = false );  
  
//-----  
// Overload 4: DataFrame with external solver object, init to default SVD  
//-----  
SMapValues SMap( DataFrame< double > &dataFrameIn,  
                  std::string pathOut        = "./",  
                  std::string predictFile    = "",  
                  std::string lib            = "",  
                  std::string pred           = "",  
                  int E                      = 0,  
                  int Tp                    = 1,  
                  int knn                   = 0,  
                  int tau                   = -1,  
                  double theta              = 0,  
                  int exclusionRadius        = 0,  
                  std::string columns        = "",  
                  std::string target         = "",  
                  std::string smapFile       = "",  
                  std::string derivatives    = "", // Not implemented  
                  std::valarray<double> (*solver)(DataFrame < double >,  
                                                    std::valarray < double >) = &SVD,  
                  bool embedded              = false,  
                  bool const_predict        = false,  
                  bool verbose              = true,  
                  std::vector<bool> validLib = std::vector<bool>(),  
                  int generateSteps         = 0,  
                  bool parameterList        = false );
```

CCM

Convergent cross mapping of columns against target via Simplex. Normally, one column and one target are specified. The column time series is time-delay embedded to dimension E, cross mapped with the target time series. The target time series is then embedded to E and cross mapped against the column as the "target" time series, not an embedding.

If there are multiple columns and embedded is false, each column is time-delay embedded to dimension E creating an N-columns * E dimensional "mixed" embedding. If embedded is true, no time-delay embedding is done, creating a multivariate embedding of the specified columns. The same logic applies if multiple target are specified for the "reverse" mapping. If embedded is false, each target is time-delay embedded to dimension E creating an N-target * E dimensional "mixed" embedding cross mapped to the first column as the cross map target. If embedded is true, no time-delay embedding is done, creating a multivariate embedding of the specified target(s).

Cross mappings are performed between column : target, and, target : column in separate threads. Threading can be disabled in the makefile by removing -DCCM_THREADED.

Note: The entire prediction vector is used in the Simplex prediction at each library subset size. See the Parameters table for parameter definitions.

CCM() returns a CCMValues structure:

```
struct CCMValues {
    DataFrame< double > AllLibStats; // unified mean libsize, rho, RMSE, MAE
    CrossMapValues      CrossMap1;
    CrossMapValues      CrossMap2;
    std::map< std::string, std::string > parameterMap;
};
struct CrossMapValues {
    DataFrame< double > LibStats; // mean libsize, rho, RMSE, MAE
    DataFrame< double > PredictStats; // each predict libsize, rho, RMSE, MAE
    std::forward_list< DataFrame< double > > Predictions;
};
```

CCMValues::LibStats DataFrame has 3 columns. The first column is "LibSize", the second and third columns are Pearson correlation coefficients for "column : target" and "target : column" cross mapping.

If the includeData parameter is true, then the CrossMapValues structures CrossMap1 and CrossMap2 in CCMValues are populated with the DataFrame PredictStats and list of DataFrames Predictions. LibStats is used for internal storage, and is empty upon return.

Parameters

The libSizes parameter is a string of whitespace or comma separated library sizes. If the string has 3 values, and, if the third value is less than the second value, then the three values are interpreted as a sequence generator specifying "start stop increment" row values, i.e. "10 80 10" will evaluate library sizes from 10 to 80 in increments of 10.

If random is true, sample observations are randomly selected from the subset of each library size. If replacement is true, observations are selected with replacement. If seed=0 a random seed is generated for the random number generator. Otherwise, seed is used to initialise the random number generator.

If random is false, sample is ignored and contiguous library rows up to the current library size are used. Note this is not Convergent Cross Mapping (CCM).

```
//-----
// Overload 1: Explicit data file path/name
//-----
CCMValues CCM( std::string pathIn      = "./data/",
               std::string dataFile   = "",
               std::string pathOut    = "./",
               std::string predictFile = "",
               int          E          = 0,
               int          Tp         = 0,
               int          knn        = 0,
               int          tau        = -1,
               int          exclusionRadius = 0,
               std::string columns     = "",
               std::string target     = "",
               std::string libSizes    = "",
               int          sample     = 0,
               bool         random     = true,
               bool         replacement = false,
               unsigned     seed       = 0, // seed=0: use RNG
               bool         embedded   = false,
               bool         includeData = false,
               bool         parameterList = false,
               bool         verbose    = true );

//-----
// Overload 2: DataFrame reference provided
//-----
CCMValues CCM( DataFrame< double > &dataFrameIn,
               std::string pathOut    = "./",
               std::string predictFile = "",
               int          E          = 0,
               int          Tp         = 0,
               int          knn        = 0,
               int          tau        = -1,
               int          exclusionRadius = 0,
               std::string columns     = "",
               std::string target     = "",
               std::string libSizes    = "",
               int          sample     = 0,
               bool         random     = true,
               bool         replacement = false,
               unsigned     seed       = 0, // seed=0: use RNG
               bool         embedded   = false,
               bool         includeData = false,
               bool         parameterList = false,
               bool         verbose    = true );
```

Multiview

Multiview embedding and forecasting of the input data file or DataFrame. See the Parameters table for parameter definitions.

`Multiview()` returns a `MultiviewValues` structure:

```
struct MultiviewValues {
    DataFrame< double > ComboRho;
    DataFrame< double > Predictions;
    std::map< std::string, std::vector< std::string > > ColumnNames;
    std::map< std::string, std::string > parameterMap;
};
```

The ComboRho DataFrame has D+3 columns. The first D columns are the the column indices in the input DataFrame embedding (not the input DataFrame) and are applied to multivariate Simplex predictions. The last three columns are "rho", "MAE", "RMSE" corresponding to the prediction Pearson correlation, maximum absolute error and root mean square error.

ColumnNames is a map of string vectors listing the multiview embedding column names of the input DataFrame embedding.

The Predictions DataFrame has 3 columns "Time", "Observations", "Predictions". nan values are inserted where there is no observation or prediction. If predictFile is provided Predictions will be written to it in csv format.

Parameters

D represents the number of variables to combine for each assessment, if not specified, it is the number of columns.

E is the embedding dimension of each variable. If E = 1, no time delay embedding is done.

multiview is the number of top-ranked D-dimensional predictions to "average" for the final prediction. Corresponds to parameter k in Ye & Sugihara with default $k = \sqrt{C}$ where C is the number of combinations $C(n,D)$ available from the embedding columns taken D at-a-time.

trainLib specifies whether projections used to rank the column combinations are done in-sample (pred = lib, the default), or, using the lib and pred specified as input options (trainLib false).

lib and pred specify [start, stop] row indices of the input data for the library and predictions.

If knn is not specified, it is set equal to D+1.

```
//-----
// Overload 1: Explicit data file path/name
//-----
MultiviewValues Multiview( std::string pathIn      = "./",
                           std::string dataFile    = "",
                           std::string pathOut     = "./",
                           std::string predictFile = "",
                           std::string lib         = "",
                           std::string pred        = "",
                           int D                   = 0,
                           int E                   = 1,
                           int Tp                  = 1,
                           int knn                 = 0,
                           int tau                 = -1,
                           std::string columns     = "",
                           std::string target      = "",
                           int multiview           = 0,
                           int exclusionRadius     = 0,
                           bool trainLib           = true,
                           bool excludeTarget      = false,
                           bool parameterList      = false,
                           bool verbose            = false,
                           unsigned nThreads       = 4 );
```

```
//-----
// Overload 2: DataFrame provided
//-----
MultiviewValues Multiview( DataFrame< double >,
                           std::string pathOut     = "./",
                           std::string predictFile = "",
                           std::string lib         = "",
                           std::string pred        = "",
                           int D                   = 0,
                           int E                   = 1,
                           int Tp                  = 1,
                           int knn                 = 0,
                           int tau                 = -1,
                           std::string columns     = "",
                           std::string target      = "",
                           int multiview           = 0,
                           int exclusionRadius     = 0,
                           bool trainLib           = true,
                           bool excludeTarget      = false,
                           bool parameterList      = false,
                           bool verbose            = false,
                           unsigned nThreads       = 4 );
```


EmbedDimension

Evaluate Simplex prediction skill for embedding dimensions from 1 to maxE (default 10). The returned DataFrame has columns "E" and "rho". See the Parameters table for parameter definitions.

Note: nThreads defines the number of worker threads for the 10 embeddings. The maximum number of threads is maxE.

```
//-----  
// Overload 1: Explicit data file path/name  
//-----  
DataFrame<double> EmbedDimension( std::string pathIn      = "./data/",  
                                  std::string dataFile    = "",  
                                  std::string pathOut     = "./",  
                                  std::string predictFile  = "",  
                                  std::string lib         = "",  
                                  std::string pred        = "",  
                                  int maxE               = 10,  
                                  int Tp                 = 1,  
                                  int tau                = -1,  
                                  int exclusionRadius     = 0,  
                                  std::string columns    = "",  
                                  std::string target     = "",  
                                  bool embedded          = false,  
                                  bool verbose           = true,  
                                  std::vector<bool> validLib =  
                                      std::vector<bool>(),  
                                  unsigned nThreads      = 4 );
```

```
//-----  
// Overload 2: DataFrame reference provided  
//-----  
DataFrame<double> EmbedDimension( DataFrame< double > &dataFrameIn,  
                                  std::string pathOut      = "./",  
                                  std::string predictFile  = "",  
                                  std::string lib          = "",  
                                  std::string pred         = "",  
                                  int maxE                = 10,  
                                  int Tp                  = 1,  
                                  int tau                 = -1,  
                                  int exclusionRadius      = 0,  
                                  std::string columns      = "",  
                                  std::string target       = "",  
                                  bool embedded            = false,  
                                  bool verbose             = true,  
                                  ustd::vector<bool> validLib =  
                                      std::vector<bool>(),  
                                  nsigned nThreads        = 4 );
```

PredictInterval

Evaluate Simplex prediction skill for forecast intervals from 1 to maxTp (default 10). The returned DataFrame has columns "Tp" and "rho". See the Parameters table for parameter definitions.

Note: nThreads defines the number of worker threads for the 10 prediction interval forecasts. The maximum number of threads is maxTp.

```
//-----  
// Overload 1: Explicit data file path/name  
//-----  
DataFrame<double> PredictInterval( std::string pathIn      = "./data/",  
                                   std::string dataFile     = "",  
                                   std::string pathOut      = "./",  
                                   std::string predictFile   = "",  
                                   std::string lib          = "",  
                                   std::string pred         = "",  
                                   int maxTp               = 10,  
                                   int E                  = 0,  
                                   int tau                = -1,  
                                   int exclusionRadius     = 0,  
                                   std::string columns     = "",  
                                   std::string target      = "",  
                                   bool embedded          = false,  
                                   bool verbose           = true,  
                                   std::vector<bool> validLib =  
                                       std::vector<bool>(),  
                                   unsigned nThreads      = 4 );  
  
//-----  
// Overload 2: DataFrame reference provided  
//-----  
DataFrame<double> PredictInterval( DataFrame< double > &dataFrameIn,  
                                   std::string pathOut      = "./",  
                                   std::string predictFile   = "",  
                                   std::string lib          = "",  
                                   std::string pred         = "",  
                                   int maxTp               = 10,  
                                   int E                  = 0,  
                                   int tau                = -1,  
                                   int exclusionRadius     = 0,  
                                   std::string columns     = "",  
                                   std::string target      = "",  
                                   bool embedded          = false,  
                                   bool verbose           = true,  
                                   std::vector<bool> validLib =  
                                       std::vector<bool>(),  
                                   unsigned nThreads      = 4 );
```

PredictNonlinear

Evaluate SMap prediction skill for localisation parameters θ specified in the string theta. Default values of theta are "0.01 0.1 0.3 0.5 0.75 1 1.5 2 3 4 5 6 7 8 9". Default knn (0) sets to size of the library. Smaller knn values can be specified.

The returned DataFrame has columns "theta" and "rho". See the Parameters table for parameter definitions.

Note: nThreads defines the number of worker threads for the θ value forecasts.

```
//-----  
// Overload 1: Explicit data file path/name  
//-----  
DataFrame<double> PredictNonlinear( std::string pathIn      = "./data/",  
                                     std::string dataFile    = "",  
                                     std::string pathOut      = "./",  
                                     std::string predictFile  = "",  
                                     std::string lib          = "",  
                                     std::string pred         = "",  
                                     std::string theta        = "",  
                                     int E                   = 0,  
                                     int Tp                  = 1,  
                                     int knn                 = 0,  
                                     int tau                  = -1,  
                                     int exclusionRadius      = 0,  
                                     std::string columns      = "",  
                                     std::string target       = "",  
                                     bool embedded           = false,  
                                     bool verbose            = true,  
                                     std::vector<bool> validLib =  
                                         std::vector<bool>(),  
                                     unsigned nThreads       = 4 );  
  
//-----  
// Overload 2: DataFrame reference provided  
//-----  
DataFrame<double> PredictNonlinear( DataFrame< double > &dataFrameIn,  
                                     std::string pathOut      = "./",  
                                     std::string predictFile  = "",  
                                     std::string lib          = "",  
                                     std::string pred         = "",  
                                     std::string theta        = "",  
                                     int E                   = 0,  
                                     int Tp                  = 1,  
                                     int knn                 = 0,  
                                     int tau                  = -1,  
                                     int exclusionRadius      = 0,  
                                     std::string columns      = "",  
                                     std::string target       = "",  
                                     bool embedded           = false,  
                                     bool verbose            = true,  
                                     std::vector<bool> validLib =  
                                         std::vector<bool>(),  
                                     unsigned nThreads       = 4 );
```

ComputeError

Compute Pearson correlation coefficient, maximum absolute error (MAE) and root mean square error (RMSE) between two vectors.

`ComputeError()` returns a `VectorError` struct:

```
struct VectorError {
    double rho;
    double RMSE;
    double MAE;
};

//-----
//-----
VectorError ComputeError( std::valarray< double > obsIn,
                          std::valarray< double > predIn );
```

Application Notes

All data input files are assumed to be in csv format. The files are required to have a single line header with column names.

It is assumed that the first column of the csv file is a vector of times or time indices for each observation (row). All subsequent columns are expected to be numeric. However, a `DataFrame` can be created with the `noTime = true` parameter to read in numeric data with no time in the first column. Such data frames should not be passed to prediction functions.

`SMap()` should be called with `DataFrame` that have columns explicitly corresponding to dimensions E . Subsequently, if a multivariate data set is used, it should Not be called with an embedding from `Embed()` since `Embed()` will add lagged coordinates for each variable. These extra columns will not correspond to the intended dimensions in the matrix inversion and prediction reconstruction. In this case, use the `embedded` parameter set to `true` so that the columns selected correspond to the proper dimension.

See `etc/Notes` for details regarding the building an application (`etc/Test.cc`) on Windows.

See `etc/Notes` for an explanation of differences between `Simplex()` and `CCM()` default embeddings, and the effect this has on predictions.

Example Application

This application is assumed to be located in the etc/ directory. Otherwise, adjust the -I and -L compiler flags and the pathIn argument accordingly. The file etc/Test.cc shows sample invocations for several API functions.

```
// g++ TestApp.cc -o TestApp -std=c++11 -g -I../src -L../lib -lstdc++ -lEDM -llapack
```

```
#include "API.h"
```

```
int main( int argc, char *argv[] ) {
    try {
        //-----
        // embedded = false : Simplex embeds all columns to E = 3
        //-----
        DataFrame<double> dataFrame =
            Simplex( "../data/",          // pathIn
                    "block_3sp.csv",      // dataFile
                    "./",                 // pathOut
                    "Block3sp_E3.csv",     // predictFile
                    "1 100",               // lib
                    "101 195",             // pred
                    3,                     // E
                    1,                     // Tp
                    0,                     // knn
                    -1,                    // tau
                    0,                     // exclusionRadius
                    "x_t y_t z_t",         // columns
                    "x_t",                 // target
                    false,                 // embedded
                    false,                 // const_predict
                    true );                // verbose

        dataFrame.MaxRowPrint() = 12; // Set number of rows to print
        std::cout << dataFrame;

        VectorError ve = ComputeError(
            dataFrame.VectorColumnName( "Observations" ),
            dataFrame.VectorColumnName( "Predictions" ) );

        std::cout << "rho " << ve.rho << " RMSE " << ve.RMSE
                    << " MAE " << ve.MAE << std::endl << std::endl;
    }

    catch ( const std::exception & e ) {
        std::cout << "Exception caught in main:\n";
        std::cout << e.what() << std::endl;
        return -1;
    }
    catch (...) {
        std::cout << "Unknown exception caught in main.\n";
        return -1;
    }

    std::cout << "Normal termination.\n";

    return 0;
}
```

Code Notes

- 1) The OSX XCode compiler/linker seems to be incompatible with the C++11 standard implementation allowing template classes to be distributed into declarations (.h) and implementation (.cc). To support OSX, DataFrame.h contains both declarations and implementations. See: etc/libstdc++_Notes.txt.
- 2) The code relies heavily on class and data containers without explicit heap allocation. This facilitates garbage collection. If the code encounters massive data objects/large problems, this may warrant investigation.
- 3) SMap uses the LAPACK (<http://www.netlib.org/lapack/explore-html/index.html>) routine `dgeiss()` to solve the local linear maps by singular value decomposition.

References

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