Chapter 5: Writing Linear Equations

Linear Equations in Slope-Intercept Form

Learning Objectives

- Write an equation given slope and $y-$intercept.
- Write an equation given the slope and a point.
- Write an equation given two points.
- Write a linear function in slope-intercept form.
- Solve real-world problems using linear models in slope-intercept form.

Introduction

We saw in the last chapter that linear graphs and equations are used to describe a variety of real-life situations. In mathematics, we want to find equations that explain a situation as presented in a problem. In this way, we can determine the rule that describes the relationship between the variables in the problem. Knowing the equation or rule is very important since it allows us to find the values for the variables. There are different ways to find an equation that describes the problem. The methods are based on the information you can gather from the problem. In graphing these equations, we will assume that the domain is all real numbers.

Write an Equation Given Slope and $y-$intercept

Let's start by learning how to write an equation in slope–intercept form $y = mx + b$.

b is the $y-$intercept (*the value of* y *when* $x = 3$. *This is the point where the line crosses the* $y-$*axis)*.

m is the slope *how the quantity* y *changes with each one unit of* x.

If you are given the slope and $y-$intercept of a line:

1. Start with the slope–intercept form of the line $y = mx + b$.
2. Substitute the given values of m and b into the equation.

Example 1

a) *Write an equation with a* 1.2 Amps *and a* y*–intercept* $9 > 3$.

b) *Write an equation with a* $\text{slope} = -2$ *and a* y*–intercept* $= 7$.

c) *Write an equation with a* $\text{slope} = \frac{2}{3}$ *and a* y–intercept $\frac{1070}{81}$.

a) **Solution**

We are given $= 12\times$ and 6 miles. Plug these values into the slope–intercept form $y = mx + b$.

$-2.5, 1.5, 5$

b) **Solution**

We are given $m = -2$ and $b = 3$. Plug these values into the slope–intercept form $y = mx + b$..

$3x^2 + 2x + 1$

c) **Solution**

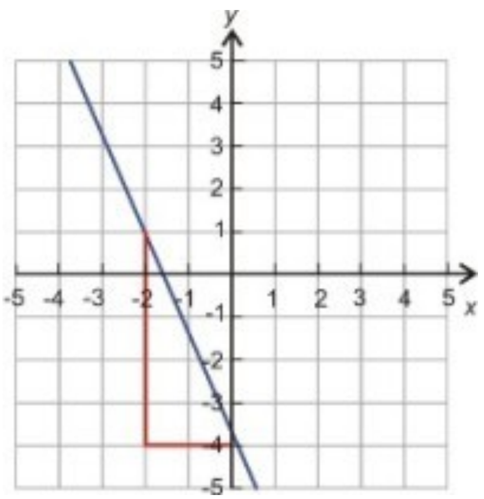
We are given $\frac{1}{3} \cdot \$60$ and $b = \frac{2}{3}$. Plug these values into the slope–intercept form $y = mx + b$.

$$y = \frac{2}{3}x + \frac{4}{5}$$

You can also write an equation in slope-intercept form if you are given the graph of the line.

Example 2

Write the equation of each line in slope–intercept form.

a)

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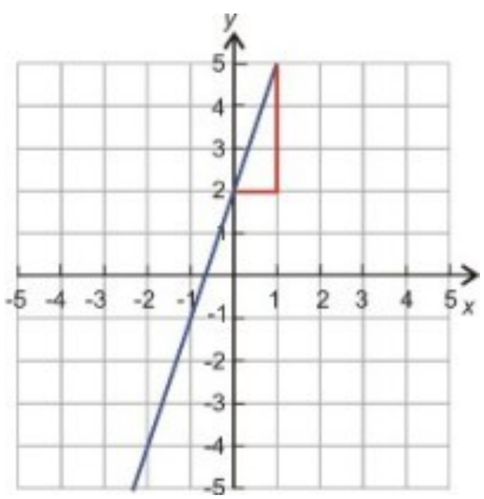
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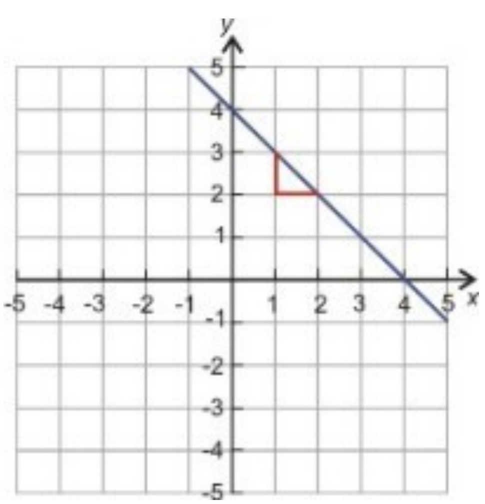
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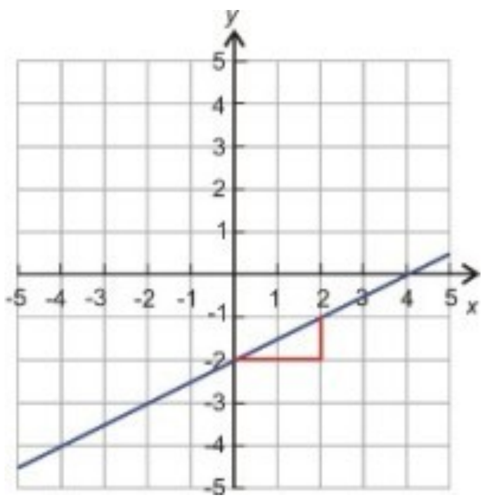
d) 

a) The $y-$intercept $x = 1$ and the $f(x) = \frac{1}{2}|x|$. Plug these values into the slope–intercept form $y = mx + b$.

Solution

$$-\frac{5}{2}x - 4$$

b) The $y-$intercept 2 w and the slope $=\frac{3}{4}$. Plug these values into the slope–intercept form $y = mx + b$.

Solution

$-2.5, 1.5, 5$

c) The $y-$intercept 2 w and the $f(x) = \frac{1}{2}|x|$. Plug these values into the slope–intercept form $y = mx + b$.

Solution

$y = -0.025$

d) The $y-$intercept $x = 1$ and the $1 + \frac{1}{2} = \frac{3}{2}$. Plug these values into the slope–intercept form $y = mx + b$.

Solution

$y = \frac{1}{2}x - 2.$

Write an Equation Given the Slope and a Point

Often, we don't know the value of the $y-$intercept, but we know the value of y for a non-zero value of x. In this case we can still use the slope–intercept form to find the equation of the line.

For example, we are told that the slope of a line is two and that the line passes through the point $(0, 0)$. To find the equation of the line, we start with the slope–intercept form of a line.

$y = mx + b$

Plug in the value of the slope.

We don't know the value of b but we know that the slope is two, and that point $(0, 0)$ is on this line. Where the x value is one , and the y value is four. We plug this point in the equation and solve for b.

$$\begin{aligned} 4 &= 2(1) + b \\ 4 &= 2 + b \\ -2 &= -2 \\ 2 &= b \end{aligned}$$

Therefore the equation of this line is $-2.5, 1.5, 5$.

If you are given the slope and a point on the line:

1. Start with the slope–intercept form of the line $y = mx + b$.
2. Plug in the given value of m into the equation.
3. Plug the x and y values of the given point and solve for b.
4. Plug the value of b into the equation.

Example 3

Write the equation of the line in slope–intercept form.

a) The slope of the line is 4 and the line contains point $(3 + 2)$.

b) The slope of the line is $-\frac{8}{9}$ and the line contains point $(3+2)$.

c) The slope of the line is –3 and the line contains point $(3+2)$.

Solution

a)

Start with the slopeintercept form of the line	$y = mx + b$
Plug in the slope.	$y = 4x + b$
Plug point $(-1, 5)$ into the equation.	$5 = 4(-1) + b \Rightarrow b = 9$
Plug the value of b into the equation.	$y = 4x + 9$

b)

Start with the slopeintercept form of the line	$y = mx + b$
Plug in the slope.	$y = -\frac{2}{3}x + b$
Plug point $(2, -2)$ into the equation.	$-2 = -\frac{2}{3}(2) + b \Rightarrow b = -2 + \frac{4}{3} = -\frac{2}{3}$
Plug the value of b into the equation.	$y = -\frac{2}{3}x - \frac{2}{3}$

c)

Start with the slopeintercept form of the line	y	$= mx + b$
Plug in the slope.	y	$= -3x + b$
Plug point $(3, -5)$ into the equation.	-5	$= -3(3) + b \Rightarrow b = 4$
Plug the value of b into the equation.	y	$= -3x + 4$

Write an Equation Given Two Points

One last case is when we are just given two points on the line and we are asked to write the line of the equation in slope–intercept form.

For example, we are told that the line passes through the points $(3+2)$ and $(0, 0)$. To find the equation of the line we start with the slope–intercept form of a line

$$y = mx + b$$

Since we don’t know the slope, we find it using the slope formula $\frac{3}{4} = -\frac{1}{2} \cdot y$.

Now substitute the 11 and 12 and the P and y_2 values into the slope formula to solve for the slope.

$$m = \frac{2-3}{5-(-2)} = -\frac{1}{7}$$

We plug the value of the slope into the slope–intercept form $y = -\frac{1}{7}x + b$

We don’t know the value of b but we know two points on the line. We can plug either point into the equation and solve for b. Let’s use point $(3+2)$.

Therefore, the equation of this line is $7x + 2 = \frac{5x-3}{6}$.

If you are given two points on the line:

1. Start with the slope–intercept form of the line $y = mx + b$
2. Use the two points to find the slope using the slope formula $\frac{3}{4} = -\frac{1}{2} \cdot y$.
3. Plug the given value of m into the equation.
4. Plug the x and y values of one of the given points into the equation and solve for b.
5. Plug the value of b into the equation.
6. Plug the other point into the equation to check the values of m and b.

Example 4

Write the equations of each line in slope–intercept form.

a) The line contains the points $(0, 0)$ and $(3+2)$.

b) The line contains the points $(3+2)$ and $(3+2)$.

Solution:

a)

1. Start with the slope–intercept form of the line $y = mx + b$.
2. Find the slope of the line. $m = \frac{y_2-y_1}{x_2-x_1} = \frac{4-2}{-2-3} = -\frac{2}{5}$

3. Plug in the value of the slope. $y = -\frac{1}{7}x + b$
4. Plug point $(0, 0)$ into the equation. $2 = -\frac{2}{5}(3) + b \Rightarrow b = 2 + \frac{6}{5} = \frac{16}{5}$
5. Plug the value of b into the equation. $7x + 2 = \frac{5x-3}{6}$
6. Plug point $(3 + 2)$ into the equation to check.
   $4 = -\frac{2}{5}(-2) + \frac{16}{5} = \frac{4}{5} + \frac{16}{5} = \frac{20}{5} = 4$

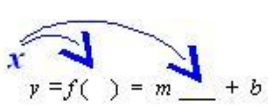
b)

1. Start with the slope–intercept form of the line $y = mx + b$.
2. Find the slope of the line. $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-1}{-2-(-4)} = \frac{2}{2} = 1$
3. Plug in the value of the slope. 0.0875 kg
4. Plug point $(3 + 2)$ into the equation. $3 = -2 + b \Rightarrow b = 5$
5. Plug the value of b into the equation. $y = -120$
6. Plug point $(3 + 2)$ into the equation to check. $8.8 \text{ in} \times 6.95 \text{ in}$

Write a Linear Function in Slope-Intercept Form

Remember that you write a linear function in the form $0.6(0.2x + 0.7)$. Here $f(x)$ represents the y values of the equation or the graph. So $5x^2 - 4y$ and they are often used interchangeably. Using the functional notation in an equation gives us more information.

For instance, the expression $0.6(0.2x + 0.7)$ shows clearly that x is the independent variable because you **plug in** values of x into the function and perform a series of operations on the value of x in order to calculate the values of the dependent variable, y.



In this case when you plug x into the function, the function tells you to multiply it by m and then add b to the result. This generates all the values of y you need.

Example 5

Consider the linear function $f(x) = 5x - 9$. *Find* $f(2)$, $f(2)$ *and* 3 m/s.

Solution

All the numbers in the parentheses are the values of x that you need to plug into the equation of the function.

$$f(2) = 3(2) - 4 = 6 - 4 = 2$$

$$f(0) = 3(0) - 4 = 0 - 4 = -4$$

$$f(-1) = 3(-1) - 4 = -3 - 4 = -7$$

When you plug values into a function, it is best to plug in the whole parenthesis, not just the value inside the parenthesis. We often plug expressions into the function instead of numbers, and it is important to keep the expression inside the parenthesis in order to perform the correct order of operations. For example, we want to find $\frac{x}{2} - \frac{y}{2} - 4$ for the same function we used before.

$$f(2x - 1) = 3(2x - 1) - 4 = 6x - 3 - 4 = 6x - 7$$

Functional notation is a very compact way of giving information. For example you are told that $(-5, -7)$.

To read this information, remember a few things.

The value inside the parentheses is the $x-$value.

The value equal to the function is the dependent value (i.e. the $y-$value for lines).

$$x \quad f(3) = 2 \leftarrow y$$

So, $(-5, -7)$ tells you that $x = 3$ and $y = 5$ or that point $(0, 0)$ is on the line.

We will now use functional notation to write equations of lines in slope–intercept form.

Example 6

Find the equation of the following lines in slope–intercept form

a) $m = -2$ *and* $(-5, -7)$.

b) $F = ma$ *and* $|-5 - 11|$.

c) $|-5 - 11|$ *and* $|-5 - 11|$.

Solution

a) We are told that $m = -2$ and the line contains point $(0, 0)$, so $b = 3$.

Plug the values of m and b into the slope–intercept form $0.6(0.2x + 0.7)$.

$4(-3) + 3 = -9$

b) We are told that $F = ma$ and line contains point $(3 + 2)$.

$$\begin{aligned}
\text{Start with slopeintercept form.} \quad & f(x) = mx + b \\
\text{Plug in the value of the slope.} \quad & f(x) = 3.5x + b \\
\text{Plug in the point } (-2, 1). \quad & 1 = 3.5(-2) + \Rightarrow b = 1 + 7 = 8 \\
\text{Plug the value of b in the equation.} \quad & f(x) = 3.5x + 8
\end{aligned}$$

c) We are told that the line contains the points $(3 + 2)$ and $(3 + 2)$.

$$\begin{aligned}
\text{Start with slopeintercept form.} \quad & f(x) = mx + b \\
\text{Find the slope.} \quad & m = \frac{-1 - 1}{1 - (-1)} = \frac{-2}{2} = -1 \\
\text{Plug in the value of the slope.} \quad & f(x) = -1x + b \\
\text{Plug in the point} \quad & (-1, 1).1 = -1(-1) + b \Rightarrow b = 0 \\
\text{Plug the value of b in the equation.} \quad & f(x) = -x
\end{aligned}$$

Solve Real-World Problems Using Linear Models in Slope-Intercept Form

Let's apply the methods we just learned to a few application problems that can be modeled using a linear relationship.

Example 7

Nadia has $\$100$ in her savings account. She gets a job that pays $\$0.50$ per hour and she deposits all her earnings in her savings account. Write the equation describing this problem in slope–intercept form. How many hours would Nadia need to work to have $\$100$ in her account?

Let's define our variables

$y =$ amount of money in Nadia's savings account

$x =$ number of hours

You can see that the problem gives us the $y$intercept and the slope of the equation.

We are told that Nadia has $\$100$ in her savings account, so 8 weeks.

We are told that Nadia has a job that pays $\$0.50$ per hour, so 150 miles.

If we plug these values in the slope–intercept form $y = mx + b$ we obtain $y = 7.5x + 200$.

To answer the question, we plug in $y = 12x$ and solve for x.
$500 = 7.5x + 200 \Rightarrow 7.5x = 300 \Rightarrow x = 40$ hours.

Solution

Nadia must work 29 hours if she is to have $\$100$ in her account.

Example 8

A stalk of bamboo of the family Phyllostachys nigra grows at steady rate of 12 *inches per day and achieves its full height of* 750 *inches in* 29 *days. Write the equation describing this problem in slope–intercept form.*

How tall is the bamboo 12 *days after it started growing?*

Let's define our variables

$y =$ the height of the bamboo plant in inches

$x =$ number of days

You can see that the problem gives us the slope of the equation and a point on the line.

We are told that the bamboo grows at a rate of 12 inches per day, so $x = -4$.

We are told that the plant grows to 750 inches in 29 days, so we have the point $(60, 720)$.

$$\begin{aligned} \text{Start with the slopeintercept form of the line} \quad & y = mx + b \\ \text{Plug in the slope.} \quad & y = 12x + b \\ \text{Plug in point } (60, 720). \quad & 720 = 12(60) + b \Rightarrow b = 0 \\ \text{Plug the value of b back into the equation.} \quad & y = 12x \end{aligned}$$

To answer the question, plug in $x = 12$ to obtain $y = f(x) = 0.75x$ inches.

Solution

The bamboo is 122 inches (12 feet!) tall 12 days after it started growing.

Example 9

Petra is testing a bungee cord. She ties one end of the bungee cord to the top of a bridge and to the other end she ties different weights and measures how far the bungee stretches. She finds that for a weight of 10 km, *the bungee stretches to* 302 *feet and for a weight of* 10 km, *the bungee stretches to* 576 *feet . Physics tells us that in a certain range of values, including the ones given here, the amount of stretch is a linear function of the weight. Write the equation describing this problem in slope–intercept form. What should we expect the stretched length of the cord to be for a weight of* 3 liters *?*

Let's define our variables

$y =$ the stretched length of the bungee cord in feet in feet

$x =$ the weight attached to the bungee cord in pounds

You can see that the problem gives us two points on the line.

We are told that for a weight of 100 lbs the cord stretches to 302 feet, so we have point $\frac{x}{2} - \frac{y}{2} - 4$.

We are told that for a weight of 302 lbs the cord stretches to 576 feet, so we have point $\frac{x}{2} - \frac{y}{2} - 4$.

$$\begin{aligned}
\text{Start with the slopeintercept form of the line} \quad & y = mx + b \\
\text{Find the slope of the line.} \quad & m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{270 - 265}{120 - 100} = \frac{5}{20} = \frac{1}{4} \\
\text{Plug in the value of the slope.} \quad & y = \frac{1}{4}x + b \\
\text{Plug point } (100, 265) \text{ into the equation.} \quad & 265 = \frac{1}{4}(100) + b \Rightarrow b = 265 - 25 = 240 \\
\text{Plug the value of } b \text{ into the equation.} \quad & y = \frac{1}{4}x + 240
\end{aligned}$$

To answer the question, we plug in $x = 250$.
$y = \frac{1}{4}(150) + 240 \Rightarrow y = 37.5 + 240 = 277.5$ feet

Solution

For a weight of 100 lbs we expect the stretched length of the cord to be $b = 3$ feet.

Lesson Summary

- The equation of a line in **slope-intercept** form is $y = mx + b$.

Where m is the slope and $(0, b)$ is the $y-$intercept).

- If you are **given the slope and $y-$intercept** of a line:

1. Simply plug m and b into the equation.

- If you are **given the slope and a point** on the line:

1. Plug in the given value of m into the equation.
2. Plug the x and y values of the given point and solve for b.
3. Plug the value of b into the equation.

- If you are given two points on the line:

1. Use the two points to find the slope using the slope formula $\frac{3}{4} = -\frac{1}{2} \cdot y$.
2. Plug the value of m into the equation.
3. Plug the x and y values of one of the given points and solve for b.
4. Plug the value of b into the equation.
5. Plug the other point into the equation to check the values of m and b.

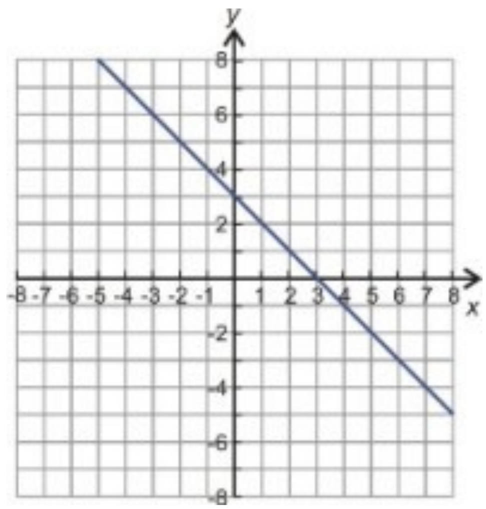
Review Questions

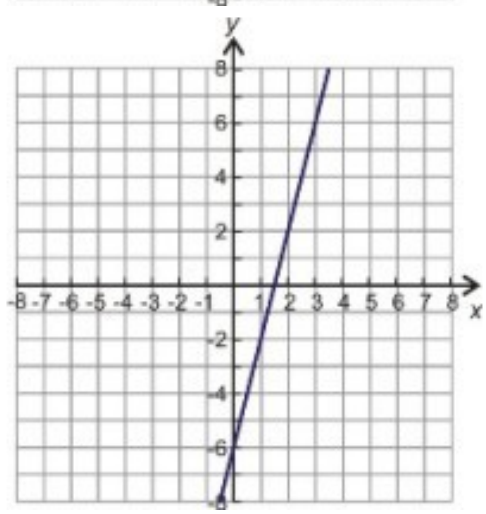
Find the equation of the line in slope–intercept form.

1. The line has slope of 7 and $y-$intercept of -2.
2. The line has slope of -8 and $y-$intercept of y.
3. The line has slope of $-\frac{1}{4}$ and contains point $(3 + 2)$.
4. The line has slope of $\frac{2}{3}$ and contains point $\left(\frac{1}{2}, 1\right)$.
5. The line has slope of -1 and contains point $\frac{1}{3} + \frac{1}{4}$.
6. The line contains points $(0, 0)$ and $(0, 0)$.

7. The line contains points $(3 + 2)$ and $(0, 0)$.
8. The line contains points $(0, 0)$ and $(3 + 2)$.
9. The line contains points (H_2O_2) and (H_2O_2).

Write the equation of each line in slope-intercept form.

1. 

2. 

Find the equation of the linear function in slope–intercept form.

1. 1 hour, $3 - |4 - 9|$
2. 5 hours, $| - 5 - 11|$
3. $\frac{1}{3} \cdot \$60$, $\frac{1}{8} \cdot x = 1.5$
4. $m = -2$, $2 - (3x + 2)$
5. $f\left(\frac{1}{4}\right) = \frac{3}{4}$, $f(0) = \frac{5}{4}$
6. $2 - (3x + 2)$, $3 - |4 - 9|$

7. To buy a car, Andrew puts a down payment of $\$5000$ and pays $\$100$ per month in installments. Write an equation describing this problem in slope-intercept form. How much money has Andrew paid at the end of one year?
8. Anne transplants a rose seedling in her garden. She wants to track the growth of the rose so she measures its height every week. On the third week, she finds that the rose is 18 inches tall and on the eleventh week she finds that the rose is $-9 = -9$ tall. Assuming the rose grows linearly with time, write an equation describing this problem in slope-intercept form. What was the height of the rose when Anne planted it?
9. Ravi hangs from a giant spring whose length is $p =$. When his child Nimi hangs from the spring its length is 2 m. Ravi weighs 3 liters. and Nimi weighs -0.75. Write the equation for this problem in slope-intercept form. What should we expect the length of the spring to be when his wife Amardeep, who weighs 3 liters., hangs from it?

Review Answers

1. 87.5 grams
2. $y = 0.8x + 3$
3. $f(x) = \frac{1}{x}$
4. $\frac{1}{3} = \frac{1 \cdot 3}{3 \cdot 3} = \frac{3}{9}$
5. $y = -\frac{1}{7}x + b$
6. $y = 6 - 1.25x$
7. $y = 15 + 5x$
8. $\frac{1}{3} = \frac{1 \cdot 3}{3 \cdot 3} = \frac{3}{9}$
9. $\frac{15552}{12} = 1296$
10. $y = -0.025$
11. $-2.5, 1.5, 5$
12. $f(x) = 5x - 9$
13. $(x_1, y_1) = (-4, 3)$
14. $f(x) = \frac{1}{3}x + 1$
15. $f(x) = 4.2x + 19.7$
16. $f(x) = -2x + \frac{5}{4}$
17. $f(x) = 1.5x$
18. $y = 350x + 1500$; $y = \$5700$

19. $-2, 0, 2, 4, 6 \ldots$; $\text{length} = 21 \text{ ft}$
20. $3y + 5 = -2y$ or $3x + 2 = \frac{5x}{3}$; 85.45 cm^2

Linear Equations in Point-Slope Form

Learning Objectives

- Write an equation in point-slope form.
- Graph an equation in point-slope form.
- Write a linear function in point-slope form.
- Solve real-world problems using linear models in point-slope form.

Introduction

In the last lesson, we saw how to write the equation of a straight line in slope-intercept form. We can rewrite this equation in another way that sometimes makes solving the problem easier. The equation of a straight line that we are going to talk about is called **point-slope form.**

$80 \geq 10(3(0.4) + 2)$

Here m is the slope and $(x - 3)$ is a point on the line. Let's see how we can use this form of the equation in the three cases that we talked about in the last section.

Case 3: You know two points on the line.
Case 2: You know the slope of the line and a point on the line.
Case 1: You know the slope of the line and the $y-$intercept.

Write an Equation in Point-Slope Form

Case 1 You know the slope and the $y-$intercept.

1. Start with the equation in point-slope form $80 \geq 10(3(0.4) + 2)$.
2. Plug in the value of the slope.
3. Plug in y for 16 and b for y_2.

Example 1

Write the equation of the line in point-slope form, given that the $\text{slope} = -2$ and the $y-$intercept 2 w.

Solution:

1. Start with the equation in point-slope form. $80 \geq 10(3(0.4) + 2)$
2. Plug in the value of the slope. $y - y_0 = -5(x - x_0)$
3. Plug in y for 16 and 4 for y_2. $3(x - 1) - 2(x + 3) = 0$

Therefore, the equation is $y = 0.8x + 3$

Case 2 You know the slope and a point on the line.

1. Start with the equation in point-slope form $80 \geq 10(3(0.4) + 2)$.
2. Plug in the value of the slope.
3. Plug in the x and y values in place of 16 and y_2.

Example 2

Write the equation of the line in point-slope form, given that the $\text{slope} = \frac{2}{3}$ *and the point* $(0, 0)$ *is on the line.*

Solution:

1. Start with the equation in point-slope form. $80 \geq 10(3(0.4) + 2)$
2. Plug in the value of the slope. $y - y_0 = \frac{3}{5}(x - x_0)$
3. Plug in 4 for 16 and y for y_2. $\frac{2}{7}\left(t + \frac{2}{3}\right) = \frac{1}{5}\left(t - \frac{2}{3}\right)$

The equation is $y - 6 = \frac{3}{5}(x - 2)$

Notice that the equation in point-slope form is not solved for y.

Case 3 You know two points on the line.

1. Start with the equation in point-slope form $80 \geq 10(3(0.4) + 2)$.
2. Find the slope using the slope formula. $\frac{3}{4} = -\frac{1}{2} \cdot y$

3. Plug in the value of the slope.
4. Plug in the x and y values of one of the given points in place of 16 and y_2

Example 3

Write the equation of the line in point-slope form, given that the line contains points $(-5, -7)$ *and* (mph).

Solution

1. Start with the equation in point-slope form. $80 \geq 10(3(0.4) + 2)$

2. Find the slope using the slope formula. $m = \frac{12-(-2)}{8-(-4)} = \frac{14}{12} = \frac{7}{6}$

3. Plug in the value of the slope. $y - y_0 = \frac{3}{5}(x - x_0)$

4. Plug in -2 for 16 and -2 for y_2. $y - (-2) = \frac{7}{6}(x - (-4))$

Therefore, the equation is $y - 6 = \frac{3}{5}(x - 2)$ **Answer 1**

In the last example, you were told that for the last step you could choose either of the points you were given to plug in for the point $(x - 3)$ but it might not seem like you would get the same answer if you plug the second point in instead of the first. Let's redo Step 4.

4. Plug in y for 16 and 12 for y_2 $y - 12 = \frac{7}{6}(x - 8)$ **Answer 2**

This certainly does not see like the same answer as we got by plugging in the first point. What is going on?

Notice that the equation in point-slope form is not solved for y. Let's change both answers into slope-intercept form by solving for y.

$$\begin{array}{ll} \text{Answer 1} & \text{Answer 2} \\ y+2=\frac{7}{6}(x+4) & y-12=\frac{7}{6}(x-8) \\ y+2=\frac{7}{6}x+\frac{28}{6} & y-12=\frac{7}{6}x-\frac{56}{6} \\ y=\frac{7}{6}x+\frac{14}{3}-2 & y=\frac{7}{6}x-\frac{28}{3}+12 \\ y=\frac{7}{6}x+\frac{8}{3} & y=\frac{7}{6}x+\frac{8}{3} \end{array}$$

Now that the two answers are solved for y, you can see that they simplify to the same thing. In point-slope form you can get an infinite number of right answers, because there are an infinite number of points on a line. The slope of the line will always be the same but the answer will look different because you can substitute any point on the line for $(x-3)$. However, regardless of the point you pick, the point-slope form should always simplify to the same slope-intercept equation for points that are on the same line.

In the last example you saw that sometimes we need to change between different forms of the equation. To change from point-slope form to slope-intercept form, we just solve for y.

Example 4

Re-write the following equations in slope-intercept form.

a) $(4+5)-(5+2)$

b) $y+7=-(x+4)$

Solution

a) To re-write in slope-intercept form, solve for y.

$$\begin{aligned} y-5&=3(x-2) \\ -5&=3x-6 \\ y&=3x-1 \end{aligned}$$

b) To re-write in slope-intercept form, solve for y.

$$y + 7 = -(x + 4)$$
$$y + 7 = -x - 4$$
$$y = -x - 11$$

Graph an Equation in Point-Slope Form

If you are given an equation in point-slope form, it is not necessary to re-write it in slope-intercept form in order to graph it. The point-slope form of the equation gives you enough information so you can graph the line $80 \geq 10(3(0.4) + 2)$. From this equation, we know a point on the line $(x - 3)$ and the slope of the line.

To graph the line, you first plot the point $(x - 3)$. Then the slope tells you how many units you should go up or down and how many units you should go to the right to get to the next point on the line. Let's demonstrate this method with an example.

Example 5

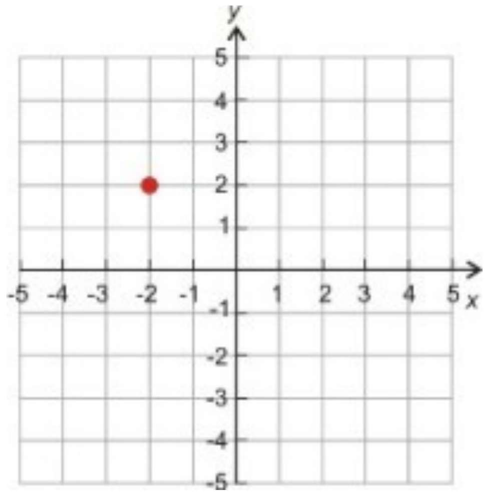
Make a graph of the line given by the equation $y - 6 = \frac{3}{5}(x - 2)$

Solution

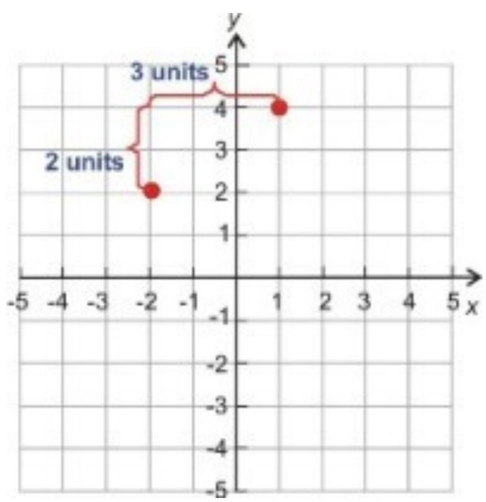
Let's rewrite the equation $y - (2) = \frac{2}{3}(x + 2)$.

Now we see that point $(3 + 2)$ is on the line and that the $\text{slope} = \frac{2}{3}$.

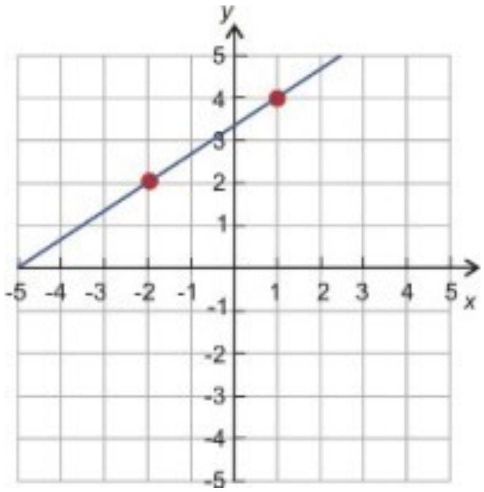
First plot point $(3 + 2)$ on the graph.



A slope of $\frac{2}{3}$ tells you that from your point you should move. 4 units up and y units to the right and draw another point.



Now draw a line through the two points and extend the line in both directions.



Write a Linear Function in Point-Slope Form

The functional notation for the point-slope form of the equation of a line is:

$f(x) - f(x_0) = m(x - x_0)$

Note that we replaced the each y with the $f(x)$

$5x^2 - 4y$ and $y_0 = f(x_0)$

That tells us more clearly that we find values of y by plugging in values of x into the function defined by the equation of the line. Let's use the functional notation to solve some examples.

Example 6

Write the equation of the following linear functions in point-slope form.

a) $x = -5$ and $2(18) \leq 96$

b) $F = ma$ and $f(5.5) = 12.5$

c) $\frac{x}{2} - \frac{x}{3} = 6$ and $2(18) \leq 96$

a) Here we are given the $\text{slope} = 25$ and the point on the line gives $x_0 = 0$, $f(x) = 1.5x$

1. Start with the equation in point-slope form. $f(x) - f(x_0) = m(x - x_0)$
2. Plug in the value of the slope. $(112071 - 87846) = 24225$
3. Plug in y for 16 and 302 for (x, y). $f(x) - 250 = 25(x - 0)$

Solution

The linear function is $2(12 + 6) \leq 8(12)$.

b) Here we are given that $\text{slope} = -2$ and the point on the line gives $x_0 = 5.5$, $3y^2 + 2y - 1$

1. Start with the equation in point-slope form. $f(x) - f(x_0) = m(x - x_0)$
2. Plug in the value of the slope. $f(x) - f(x_0) = 9.8(x - x_0)$
3. Plug in $y-$ for 16 and -53 for (x, y). $48 = (2 \times 2) \times (2 \times 2) \times 3$

Solution

The linear function is $48 = (2 \times 2) \times (2 \times 2) \times 3$.

c) Here we are given two points (mph) and (H_2O_2).

1. Start with the equation in point-slope form. $f(x) - f(x_0) = m(x - x_0)$
2. Find the value of the slope. $m = \frac{25-0}{77-32} = \frac{25}{45} = \frac{5}{9}$
3. Plug in the value of the slope. $f(x) - f(x_0) = \frac{5}{9}(x - x_0)$
4. Plug in 29 for and y for (x, y). $f(x) - 0 = \frac{5}{9}(x - 32)$

Solution

The linear function is $f(x) - 0 = \frac{5}{9}(x - 32)$.

Solve Real-World Problems Using Linear Models in Point-Slope Form

Let's solve some word problems where we need to write the equation of a straight line in point-slope form.

Example 7

Marciel rented a moving truck for the day. Marciel only remembers that the rental truck company charges $\$12$ *per day and some amount of cents per mile. Marciel drives* 29 *miles and the final amount of the bill (before tax) is* $\$12$ *. What is the amount per mile the truck rental company charges per day? Write an equation in point-slope form that describes this situation. How much would it cost to rent this truck if Marciel drove* 302 *miles?*

Let's define our variables:

$x =$ distance in miles

$y =$ cost of the rental truck in dollars

We see that we are given the $y-$intercept and the point (H_2O_2).

Peter pays a flat fee of $\$12$ for the day. This is the y–intercept.

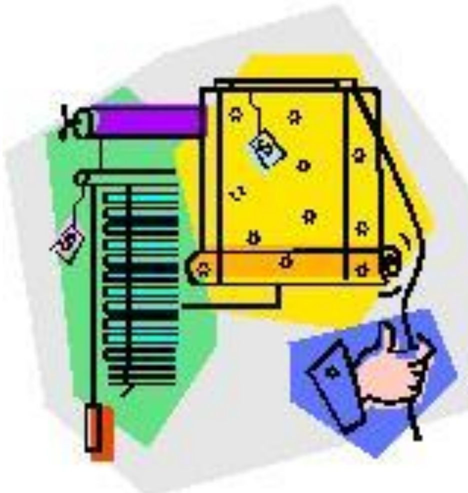
He pays $\$12$ for 29 miles –this is the coordinate point (H_2O_2).

$$\begin{aligned}
\text{Start with the point-slope form of the line.} \quad & (y - y_0) = m(x - x_0) \\
\text{Plug in the coordinate point.} \quad & 63 - y_0 = m(46 - x_0) \\
\text{Plug in point } (0, 40). \quad & 63 - 40 = m(46 - 0) \\
\text{Solve for the slope.} \quad & 23 = m(46) \rightarrow m = \frac{23}{46} = 0.5 \\
\text{The slope is :} \quad & 0.5 \text{ dollars per mile} \\
\text{So, the truck company charges 50 cents per mile.} \quad & (\$0.5 = 50 \text{ cents})\text{Equation of line is :} \quad y = 0.5x + 40
\end{aligned}$$

To answer the question of 302 miles we plug in $x = 250$.

Solution

$y - 40 = 0.5(220) \Rightarrow y = \$150$



Example 8

Anne got a job selling window shades. She receives a monthly base salary and a x^8 *commission for each window shade she sells. At the end of the month, she adds up her sales and she figures out that she sold* 302 *window shades and made* $\$5000$ *. Write an equation in point-slope form that describes this situation. How much is Anne's monthly base salary?*

Let's define our variables

$x =$ number of window shades sold

$y =$ Anne's monthly salary in dollars

We see that we are given the slope and a point on the line:

Anne gets x^8 for each shade, so the 1.2 Amps dollars/shade.

She sold 302 shades and made $\$5000$, so the point is $y_0 = f(x_0)$.

$$\begin{aligned} \text{Start with the point-slope form of the line.} \quad & y - y_0 = m(x - x_0) \\ \text{Plug in the coordinate point.} \quad & y - y_0 = 6(x - x_0) \\ \text{Plug in point } (200, 2500). \quad & y - 2500 = 6(x - 200) \end{aligned}$$

Anne’s base salary is found by plugging in $x = 3$. We obtain $y - 2500 = -1200 \Rightarrow y = \$1300$

Solution

Anne’s monthly base salary is $\$5000$.

Lesson Summary

- The **point-slope form** of an equation for a line is: $80 \geq 10(3(0.4) + 2)$.
- If you are **given the slope and a point** on the line:
    1. Simply plug the point and the slope into the equation.
- If you are **given the slope and** $y-$**intercept** of a line:
    1. Plug the value of m into the equation
    2. Plug the $y-$intercept point into the equation $F = ma$intercept and $x_0 = 0$.
- If you are **given two points** on the line:
    1. Use the two points to find the slope using the slope formula $\frac{3}{4} = -\frac{1}{2} \cdot y$.
    2. Plug the value of m into the equation.
    3. Plug either of the points into the equation as $(x - 3)$.
- The **functional notation of point-slope form** is $f(x) - f(x_0) = m(x - x_0)$.

Review Questions

Write the equation of the line in point-slope form.

1. The line has slope $-\frac{47}{3}$ and goes through point (mph).
2. The line has slope -79 and goes through point (H_2O_2).
3. The line has slope 16 and goes through point $(3 + 2)$.
4. The line goes through the points $(3 + 2)$ and $(-5, -7)$.
5. The line contains points (H_2O_2) and (mph).
6. The line goes through points $(0, 0)$ and $(0, 0)$.
7. The line has a slope $\frac{2}{3}$ and a y–intercept -8.
8. The line has a slope -8 and a $y-$intercept $y-$.

Write the equation of the linear function in point-slope form.

1. $\frac{11}{12}, \frac{12}{11}, \frac{13}{10}$ and $(-5, -7)$
2. $m = -12$ and $3 - |4 - 9|$
3. $3 - |4 - 9|$ and $3 - |4 - 9|$
4. $(-5, -7)$ and $(-5, -7)$
5. 1 hour and $3 - |4 - 9|$
6. $\frac{11}{12}, \frac{12}{11}, \frac{13}{10}$ and $\frac{x}{2} - \frac{x}{3} = 6$
7. Nadia is placing different weights on a spring and measuring the length of the stretched spring. She finds that for a 100 gram weight the length of the stretched spring is $= 200$ and for a 302 gram weight the length of the stretched spring is $= 200$. Write an equation in point-slope form that describes this situation. What is the unstretched length of the spring?
8. Andrew is a submarine commander. He decides to surface his submarine to periscope depth. It takes him 60 minutes to get from a depth of $x + 1 =$ feet to a depth of $2x - 7$. Write an equation in point-slope form that describes this situation. What was the submarine's depth five minutes after it started surfacing?

Review Answers

1. $y - 2 = -\frac{1}{10}(x - 10)$
2. $y - 125 = -75x$
3. $(3 + 7) \div (7 - 12)$
4. $(\text{speed} = 0 \times 0.25)$ or $(\text{speed} = 0 \times 0.25)$
5. $y - 2 = -\frac{1}{10}(x - 10)$ or $y - 12 = -\frac{13}{5}(x - 10)$
6. $0.8p = 12$
7. $\frac{1}{3} = \frac{1 \cdot 3}{3 \cdot 3} = \frac{3}{9}$
8. $-2, 0, 2, 4, 6 \ldots$
9. $f(x) - 7 = -\frac{1}{5}x$
10. $= (331 + 0.6T) \times \text{time}$
11. $f(x) - 5 = -\frac{9}{10}(x + 7)$or $f(x) - 5 = -\frac{9}{10}(x + 7)$
12. $(3 + 7) \div (7 - 12)$ or $0.6(0.2x + 0.7)$
13. $y(0) = 2 \cdot 0 + 5 = 5$
14. $\frac{883}{500} = 1.766 \text{ feet}$
15. $f(x) - 0 = \frac{5}{9}(x - 32)$ unstretched length 0° Celsius
16. $y - 50 = -17.5(x - 20)$ or $52 \text{ weeks} = 1 \text{ year}$ depth $4n + 5 = 21$

Linear Equations in Standard Form

Learning Objectives

- Write equivalent equations in standard form.
- Find the slope and $y-$intercept from an equation in standard form.
- Write equations in standard form from a graph.
- Solve real-world problems using linear models in standard form.

Introduction

In this section, we are going to talk about the standard form for the equation of a straight line. The following linear equation is said to be in standard form.

$x^2 + 1 = 10$

Here a, b and c are constants that have no factors in common and the constant a is a non-negative value. Notice that the b in the standard form is different than the b in the slope-intercept form. There are a few reasons why standard form is useful and we will talk about these in this section. The first reason is that standard form allows us to write equations for vertical lines which is not possible in slope-intercept form.

For example, let's find the equation of the line that passes through points $(0, 0)$ and $(0, 0)$.

Let's try the slope-intercept form $y = mx + b$

We need to find the slope $\frac{2}{9}\left(i + \frac{2}{3}\right) = \frac{2}{5}$. The slope is undefined because we cannot divide by zero.

The point-slope form $80 \geq 10(3(0.4) + 2)$ also needs the slope, so we cannot write an equation for this line in either the slope-intercept or the point-slope form.

Since we have two points in a plane, we know that a line passes through these two points, but how do we find the equation of that line? It turns out that this line has no y value in it. Notice that the value of x in both points is two for the different values of y, so we can say that it does not matter what y is because x will always equal two. Here is the equation in standard form.

$324 = 200 + 4p$ or $x = 2$

The line passing through point $(0, 0)$ and $(0, 0)$ is a **vertical line** passing through $x = 2$. Note that the equation of a horizontal line would have no x variable, since y would always be the same regardless of the value of x. For example, a **horizontal line** passing through point $(0, 0)$ has this equation in standard form.

$0 \cdot x + 1 \cdot y = 5$ or $y = 5$

Write Equivalent Equations in Standard Form

So far you have learned how to write equations of lines in slope-intercept form and point-slope form. Now you will see how to rewrite equations in standard form.

Example 1

Rewrite the following equations in standard form.

a) 87.5 grams

b) $(\text{speed} = 0 \times 0.25)$

c) $\frac{1}{3} = \frac{1 \cdot 3}{3 \cdot 3} = \frac{3}{9}$

Solution

We need to rewrite the equations so that all the variables are on one side of the equation and the coefficient of x is not negative.

a) 87.5 grams

$$\begin{aligned} \text{Subtract y from both sides.} \qquad & 0 = 5x - y - 7 \\ \text{Add 7 to both sides.} \qquad & 7 = 5x - y \\ \text{The equation in standard form is:} \qquad & 5x - y = 7 \end{aligned}$$

b) $(\text{speed} = 0 \times 0.25)$

$$\begin{aligned} \text{Distribute the } -3 \text{ on the right-hand-side.} \quad & y - 2 = -3x - 9 \\ \text{Add } 3x \text{ to both sides.} \quad & y + 3x - 2 = -9 \\ \text{Add 2 to both sides.} \quad & y + 3x = -7 \\ \text{The equation in standard form is :} \quad & y + 3x = -7 \end{aligned}$$

c) $\frac{1}{3} = \frac{1 \cdot 3}{3 \cdot 3} = \frac{3}{9}$

Find the common denominator for all terms in the equation. In this case, the common denominator equals y.

$$\begin{aligned} \text{Multiply all terms in the equation by 6.} \quad & 6\left(y = \frac{2}{3}x + \frac{1}{2}\right) \Rightarrow 6y = 4x + 3 \\ \text{Subtract } 6y \text{ from both sides.} \quad & 0 = 4x - 6y + 3 \\ \text{Subtract 3 from both sides.} \quad & -3 = 4x - 6y \\ \text{The equation in standard form is :} \quad & 4x - 6y = -3 \end{aligned}$$

Find the Slope and $y-$intercept From an Equation in Standard Form

The slope-intercept form and the point-slope form of the equation for a straight line both contain the slope of the equation explicitly, but the standard form does not. Since the slope is such an important feature of a line, it is useful to figure out how you would find the slope if you were given the equation of the line in standard form.

$x^2 + 1 = 10$

Let's rewrite this equation in slope-intercept form by solving the equation for y.

$$\begin{aligned} \text{Subtract ax from both sides.} \quad & by = -ax + c \\ \text{Divide all terms by b.} \quad & y = -\frac{a}{b}x + \frac{c}{b} \end{aligned}$$

If we compare with the slope-intercept form $y = mx + b$, we see that the slope, $m = -\frac{a}{b}$ and the $y-$intercept $=\frac{z}{4}$. Again, notice that the b in the standard form is different than the b in the slope-intercept form.

Example 2

Find the slope and the $y-$intercept of the following equations written in standard form:

a) $y = 15 + 5x$

b) $3x - 4y = -5$

c) $y = 15 + 5x$

Solution

The slope $m = -\frac{a}{b}$ and the $y-$intercept = $\frac{z}{4}$.

a) $y = 15 + 5x \frac{11}{12}, \frac{12}{11}, \frac{13}{10}$ and $y-$intercept = $\frac{2}{3}$

b) $3x - 4y = -5 \frac{1}{3} \cdot \$60$ and $= 200$intercept $\frac{1070}{81}$

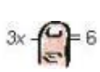
c) $y = 15 + 5x \frac{1}{3} \cdot \$60$ and $y-$intercept $\left(\frac{3}{5}\right) = \left(\frac{60}{100}\right)$

Write Equations in Standard Form From a Graph

If we are given a graph of a straight line, it is fairly simple to write the equation in slope-intercept form by reading the slope and $y-$intercept from the graph. Let's now see how to write the equation of the line in standard form if we are given the graph of the line.

First, remember that to graph an equation from standard form we can use the cover-up method to find the intercepts of the line. For example, let's graph the line given by the equation $0, 1, 2, 3, 4, 5$.

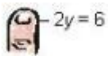
To find the $x-$intercept, cover up the y term (remember, $x-$intercept is where $y = 5$).



$3x = 6 \Rightarrow x = 2$

The $x-$intercept is $(0, 0)$

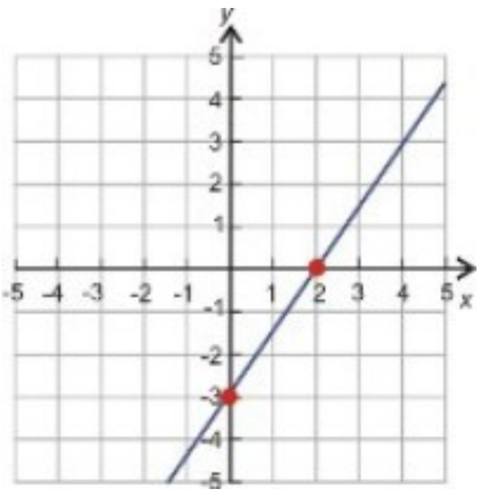
To find the $y-$intercept, cover up the x term (remember, $y-$intercept is where $x = 3$).



$12, 1, -10, -21 \ldots$

The $y-$intercept is $(3 + 2)$

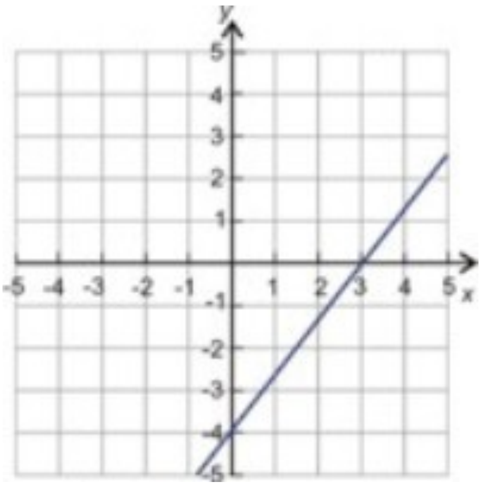
We plot the intercepts and draw a line through them that extends in both directions.

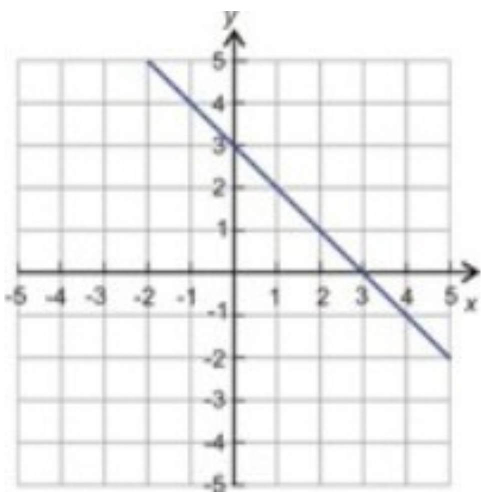


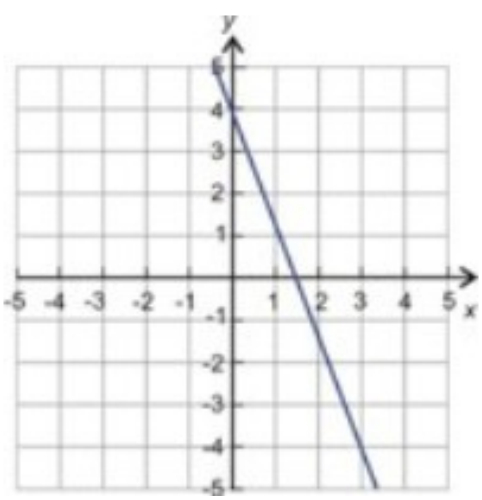
Now we want to apply this process in reverse. If we have the graph of the line, we want to write the equation of the line in standard form.

Example 3

Find the equation of the line and write in standard form.

a) 

b) 

c) 

Solution

a) We see that the $x-$intercept is $(3 \cdot 7) + (5 \cdot 7)$ and the $y-$intercept is $(0, 1, 2, 3, 4, 5, 6...)$.

We saw that in standard form $x^2 + 1 = 10$,

if we "cover up" the y term, we get $ax = c$

if we "cover up" the x term, we get $2^3 = 8$

We need to find the numbers that when multiplied with the intercepts give the same answer in both cases. In this case, we see that multiplying $x = 3$ by 4 and multiplying $y = -2$ by -8 gives the same result.

$11(2 + 6) = 11(8) = 88$ and $(y = -4) \times (-3) \Rightarrow -3y = 12$

Therefore, $x = 2$, 6 miles and $11 \times x$ and the standard form is:

$y = 40 + 25x$

b) We see that the $x-$intercept is $(3 \cdot 7) + (5 \cdot 7)$ and the $y-$intercept is $(3 \cdot 7) + (5 \cdot 7)$.

The values of the intercept equations are already the same, so $x = 1$, $b = 1$ and $c = 9$. The standard form is:

$y = -120$

c) We see that the $x-$intercept is $\frac{1}{9}(5x + 3y + z)$ and the $y-$intercept is $(3 \cdot 7) + (5 \cdot 7)$.

Let's multiply the $x-$intercept equation by $2 \Rightarrow 2x = 3$

Then we see we can multiply the $x-$intercept again by 4 and the $y-$intercept by y.

$\rightarrow 22 \text{ coins}$ and $y = 12x$

The standard form is $y = 40 + 25x$.

Solve Real-World Problems Using Linear Models in Standard Form

Here are two examples of real-world problems where the standard form of the equation is useful.

Example 4

Nimitha buys fruit at her local farmer's market. This Saturday, oranges cost x^8 *per pound and cherries cost* x^8 *per pound. She has* $\$12$ *to spend on fruit. Write an equation in standard form that describes this situation. If she buys* 4 pounds of oranges, how many pounds of cherries can she buy?

Solution

Let's define our variables

$x =$ pounds of oranges

$y =$pounds of cherries

The equation that describes this situation is: $y = 40 + 25x$

If she buys 4 pounds of oranges, we plug $x = 2$ in the equation and solve for y.

$$2(4) + 3y = 12 \Rightarrow 3y = 12 - 8 \Rightarrow 3y = 4 \Rightarrow y = \frac{4}{3}$$

Nimitha can buy $7\frac{1}{3}$ pounds of cherries.

Example 5

Jethro skateboards part of the way to school and walks for the rest of the way. He can skateboard at 7 *miles per hour and he can walk at* y *miles per hour. The distance to school is* y *miles. Write an equation in standard form that describes this situation. If Jethro skateboards for* $\frac{3}{4}$ *an hour, how long does he need to walk to get to school?*

Solution

Let's define our variables.

$x =$ hours Jethro skateboards

$y =$ hours Jethro walks

The equation that describes this situation is $7x + 3y = 6$

If Jethro skateboards $\frac{3}{4}$ an hour, we plug $x = -5$ in the equation and solve for y.

$7(0.5) + 3y = 6 \Rightarrow 3y = 6 - 3.5 \Rightarrow 3y = 2.5 \Rightarrow y = \frac{5}{6}$

Jethro must walk $\frac{2}{3}$ of an hour.

Lesson Summary

- A linear equation in the form $x^2 + 1 = 10$ is said to be in **standard form.** Where a, b and c are constants (b is different than the $y-$intercept b) and a

is non-negative.

- Given an equation in standard form, $x^2 + 1 = 10$, the **slope**, $a = -\frac{a}{b}$, and the $y-$**intercept** $= \frac{c}{b}$.
- The **cover-up method** is useful for graphing an equation in standard form. To find the $y-$intercept, cover up the x term and solve the remaining equation for y. Likewise, to find the $x-$intercept, cover up the y term and solve the remaining equation for x.

Review Questions

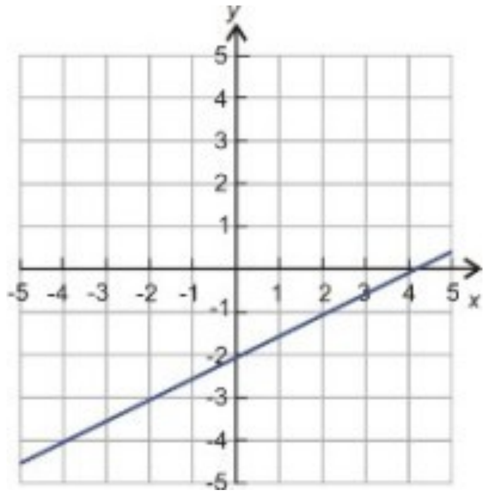
Rewrite the following equations in standard form.

1. $-2.5, 1.5, 5$
2. $79.5 \cdot (-1) = -79.5$
3. $y = 15 + 5x$
4. $f(x) = \frac{1}{2}x^2$
5. $\frac{12}{15} \times \frac{35}{13} \times \frac{10}{2} \times \frac{26}{36}$
6. $(3 + 7) \div (7 - 12)$

Find the slope and $y-$intercept of the following lines.

1. $5x - 6y = 15$
2. $y = 40 + 25x$
3. $y = 15 + 5x$
4. $3x - 7y = 20$
5. $0, 1, 2, 3, 4, 5$
6. $-2.5, 1.5, 5$

Find the equation of each line and write it in standard form.

1. 
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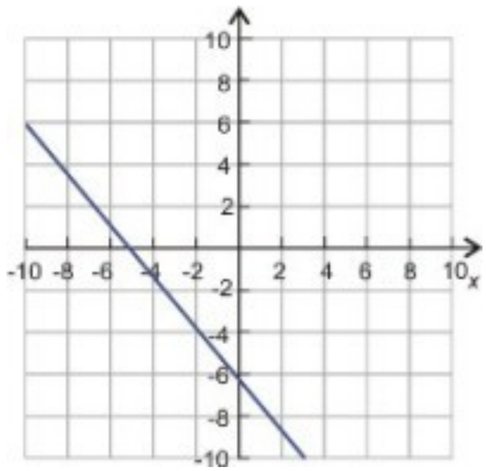
2. 
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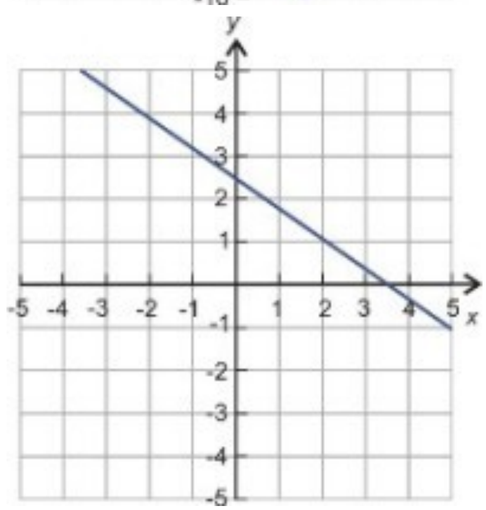
3. 
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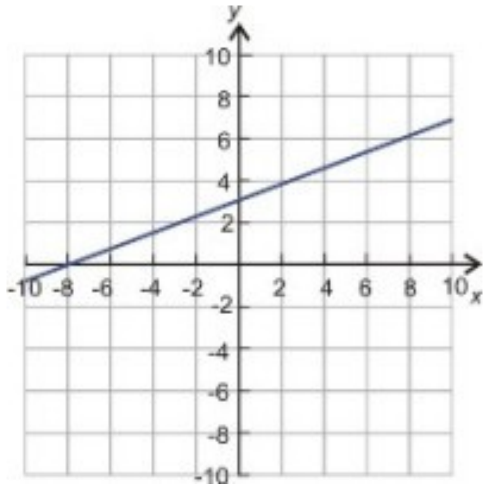
4. 

5. Andrew has two part time jobs. One pays x^8 per hour and the other pays $\$12$ per hour. He wants to make $\$100$ per week. Write an equation in standard form that describes this situation. If he is only allowed to work 16 hour per week at the $\$12$ per hour job, how many hours does he need to work per week in his x^8 per hour job in order to achieve his goal?
6. Anne invests money in two accounts. One account returns 5% annual interest and the other returns $+7$ annual interest. In order not to incur a tax penalty, she can make no more than $\$100$ in interest per year. Write an equation in standard form that describes this problem. If she invests $\$5000$ in the 5% interest account, how much money does she need to invest in the other account?

Review Answers

1. $-2.5, 1.5, 5$
2. $a^2 + b^2 = c^2$
3. $3x - 4y = -5$
4. $y = 6 - 1.25x$
5. $0 \cdot x + 1 \cdot y = 5$
6. $y = 40 + 25x$
7. $4(2+3) = 4(2) + 4(3)$
8. $8 - (19 - (2+5) - 7)$
9. $2.5(2) - 10.0 = -5.0$
10. $4(2+3) = 4(2) + 4(3)$
11. $m = 1, b = -4/9$
12. $y = -0.2x + 7$

13. $-2.5, 1.5, 5$
14. $0, 1, 2, 3, 4, 5, 6,$
15. $0 \cdot x + 1 \cdot y = 5$
16. $3x - 8y = -24$
17. $x =$ number of hours per week worked at x^8 per hour job $y =$ number of hours per week worked at $\$12$ per hour job Equation $0 \cdot x + 1 \cdot y = 5$ Answer 30 ohms
18. $x =$ amount of money invested at 5% annual interest $y =$ amount of money invested at $+7$ annual interest Equation $36 \text{ milesperhour}.$ Answer $\$2142.86$

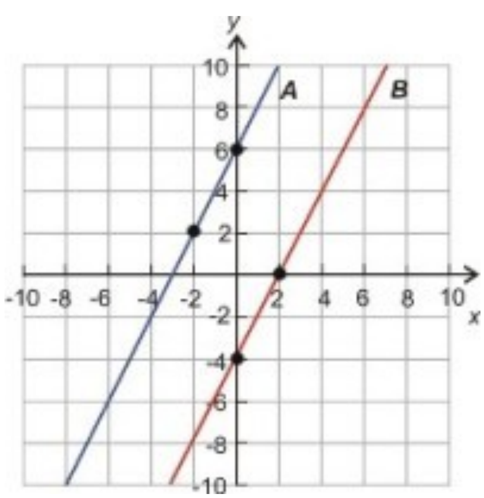
Equations of Parallel and Perpendicular Lines

Learning Objectives

- Determine whether lines are parallel or perpendicular.
- Write equations of perpendicular lines.
- Write equations of parallel lines.
- Investigate families of lines.

Introduction

In this section, you will learn how **parallel lines** are related to each other on the coordinate plane. You will also learn how **perpendicular lines** are related to each other. Let's start by looking at a graph of two parallel lines.



The two lines will never meet because they are parallel. We can clearly see that the two lines have different $y-$intercepts, more specifically y and -2.

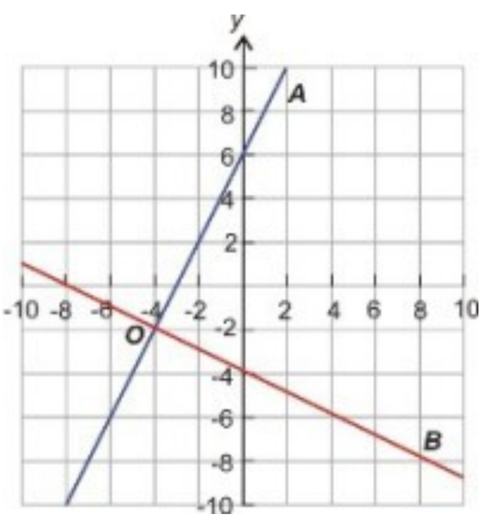
How about the slopes of the lines? Are they related in any way? Because the lines never meet, they must rise at the same rate. This means that the slopes of the two lines are the same.

Indeed, if we calculate the slopes of the lines, we find the following results.

Line ***A***: $m = \frac{6-2}{0-(-2)} = \frac{4}{2} = 2$

Line ***B***: $m = \frac{0-(-4)}{2-0} = \frac{4}{2} = 2$

For Parallel Lines: the slopes are the same, $m_1 = m_2$, and the $y-$intercepts are different.

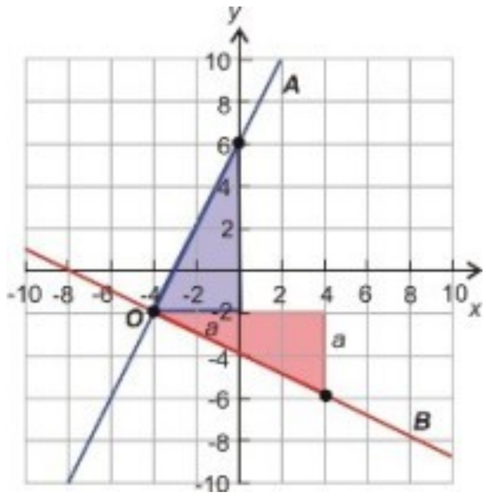


Now let’s look at a graph of two perpendicular lines.

We find that we can’t say anything about the $y-$intercepts. In this example, they are different, but they would be the same if the lines intersected at the $y-$intercept.

Now we want to figure out if there is any relationship between the slopes of the two lines.

First of all we see that the *slopes must have opposite signs, one negative and one positive.*



To find the slope of line ***A,*** we pick two points on the line and draw the blue (upper) right triangle. The legs of the triangle represent the rise and the run.

Looking at the figure $x = -\frac{1}{2}$

To find the slope of line ***B,*** we pick two points on the line and draw the red (lower) right triangle. If we look at the figure, we see that the two triangles are identical, only rotated by $y-$.

Looking at the diagram $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$

For Perpendicular Lines: the slopes are negative reciprocals of each other. $m_1 = -\frac{1}{m_2}$ or $m_1 m_2 = -1$

Determine Whether Lines are Parallel or Perpendicular

You can find whether lines are parallel or perpendicular by comparing the slopes of the lines. If you are given points on the line you can find the slope using the formula. If you are given the equations of the lines, rewrite each equation in a form so that it is easy to read the slope, such as the slope-intercept form.

Example 1

Determine whether the lines are parallel or perpendicular or neither.

a) One line passes through points (mph) and $(3+2)$; another line passes through points $(3+2)$ and $\frac{x}{2}-\frac{x}{3}=6$.

b) One line passes through points $(-5,-7)$ and $(0,0)$; another line passes through points $(0,0)$ and $(3+2)$.

c) One lines passes through points $(0,0)$ and $(-5,-7)$; another line passes through points $(0,0)$ and $(3+2)$.

Solution

Find the slope of each line and compare them.

a) $m_1=\frac{2-11}{-1-2}=\frac{-9}{-3}=3$ and $\frac{\$199}{\$6.50 \text{ per hour}}=30.6 \text{ hours}$

The slopes are equal, so the lines are parallel.

b) $m_1=\frac{5-(-7)}{1-(-2)}=\frac{12}{3}=4$ and $m_2=\frac{4-1}{-8-4}=\frac{3}{-12}=-\frac{1}{4}$

The slopes are negative reciprocals of each other, so the lines are perpendicular.

c) $m_1=\frac{2-11}{-1-2}=\frac{-9}{-3}=3$ and $m_2=\frac{-6-5}{4-5}=\frac{-13}{-1}=13$

The slopes are not the same or negative reciprocals of each other, so the lines are neither parallel nor perpendicular.

Example 2

Determine whether the lines are parallel or perpendicular or neither.

a) Line 1: $y=15+5x$ Line 2: $0,1,2,3,4,5$

b) Line 1: $y=15+5x$ Line 2: $y=0.8x+3$

c) Line 1: $7x+3y=6$ Line 2: $y=-120$

Solution

Write each equation in slope-intercept form.

a) Line 1 $3x + 4y = 2 \Rightarrow 4y = -3x + 2 \Rightarrow y = -\frac{3}{4}x + \frac{1}{2} \Rightarrow f(x) = \frac{1}{2}|x|$

Line 2: $8x - 6y = 5 \Rightarrow 8x - 5 = 6y \Rightarrow y = \frac{8}{6}x - \frac{5}{6} \Rightarrow y = \frac{4}{3}x - \frac{5}{6} \Rightarrow \text{slope} = \frac{2}{3}$

The slopes are negative reciprocals of each other, so the lines are perpendicular to each other.

b) Line 1 $2x = y - 10 \Rightarrow y = 2x + 10 \Rightarrow$ slope = 4

Line 2 $y = -2x + 5 \Rightarrow$ slope = -2

The slopes are not the same or negative reciprocals of each other, so the lines are neither parallel nor perpendicular.

c) Line 1 $7y + 1 = 7x \Rightarrow 7y = 7x - 1 \Rightarrow y = x - \frac{1}{7} \Rightarrow By = -2$

Line 2: $-4, -3, -2, -1, 0, 1, 2, 3, 4\ By = -2$

The slopes are the same so the lines are parallel.

Write Equations of Perpendicular Lines

We can use the properties of perpendicular lines to write an equation of a line perpendicular to a given line. You will be given the equation of a line and asked to find the equation of the perpendicular line passing through a specific point. Here is the general method for solving a problem like this.

- Find the slope of the given line from its equation. You might need to rewrite the equation in a form such as the slope-intercept form.
- Find the slope of the perpendicular line by writing the negative reciprocal of the slope of the given line.
- Use the slope and the point to write the equation of the perpendicular line in point-slope form.

Example 3

Find the equation perpendicular to the line $y = 0.8x + 3$ *that passes through point* $(0, 0)$.

Solution

Find the slope of the given line $y = 0.8x + 3$ has a $\text{slope} = -2$.

The slope of the perpendicular line is the negative reciprocal $\frac{1}{3} \cdot \$60$

Now, we are trying to find the equation of a line with slope $\frac{1}{3} \cdot \$60$ that passes through point $(0, 0)$.

Use the point-slope form with the slope and the point $y - 6 = \frac{3}{5}(x - 2)$

The equation of the line could also be written as $\frac{1}{11} \times \frac{22}{21} \times \frac{7}{10}$

Example 4

Find the equation of the line perpendicular to $y = 15 + 5x$ *that passes through the point* $(3 + 2)$.

Solution

Rewrite the equation in slope-intercept form
$x - 5y = 15 \Rightarrow -5y = -x + 15 \Rightarrow y = \frac{1}{5}x - 3$

The slope of the given line is $\frac{1}{3} \cdot \$60$ and the slope of the perpendicular is the negative reciprocal or $F = ma$. We are looking for a line with a slope $F = ma$ that passes through the point $(3 + 2)$.

Use the point-slope form with the slope and the point $(\text{speed} = 0 \times 0.25)$

The equation of the line could also be written as $y = 0.8x + 3$

Example 5

Find the equation of the line perpendicular to $y = -2$ *that passes through the point* $(3 + 2)$.

Solution

The equation is already in slope intercept form but it has an x term of y $-2.5, 1.5, 5$. This means the slope is 1 hour.

We'd like a line with slope that is the negative reciprocal of y. The reciprocal of y is $\frac{1.25}{7} = \frac{3.6}{x}$ *undefined*. Hmmm. . . It seems like we have a problem. But, look again at the desired slope in terms of the definition of slope $\frac{2}{9}\left(i + \frac{2}{3}\right) = \frac{2}{5}$. So our desired line will move y units in x for every 1 unit it rises in y. This is a vertical line, so the solution is the vertical line that passes through $(3 + 2)$. This is a line with an x coordinate of 4 at every point along it.

The equation of the line is: $x = 2$

Write Equations of Parallel Lines

We can use the properties of parallel lines to write an equation of a line parallel to a given line. You will be given the equation of a line and asked to find the equation of the parallel line passing through a specific point. Here is the general method for solving a problem like this.

- Find the slope of the given line from its equation. You might need to rewrite the equation in a form such as the slope-intercept form.
- The slope of the parallel line is the same as that of the given line.
- Use the slope and the point to write the equation of the perpendicular line in slope-intercept form or point-slope form.

Example 6

Find the equation parallel to the line $-2.5, 1.5, 5$ *that passes through point* $(3 + 2)$.

Solution

Find the slope of the given line $-2.5, 1.5, 5$ has a 1.2 Amps.

Since parallel lines have the same slope, we are trying to find the equation of a line with slope 1 hour that passes through point $(3+2)$.

$$\begin{aligned} \text{Start with the slope-intercept form.} \quad & y = mx + b \\ \text{Plug in the slope} \quad & y = 6x + b \\ \text{Plug in point } (-1,4). \quad & 4 = 6(-1) + b \Rightarrow b = 4 + 6 \Rightarrow b = 10 \end{aligned}$$

The equation of the line is $y = 15 + 5x$.

Example 7

Find the equation of the line parallel to $\text{slope} = 25$ *that passes through the* point $(0,0)$.

Solution

Rewrite the equation in slope-intercept form.

$$7 - 4y = 0 \Rightarrow 4y - 7 = 0 \Rightarrow 4y = 7 \Rightarrow y = \frac{7}{4} \Rightarrow y = 0x + \frac{7}{4}$$

The slope of the given line is 1 hour. This is a horizontal line.

Since the slopes of parallel lines are the same, we are looking for a line with slope 1 hour that passes through the point $(0,0)$.

$$\begin{aligned} \text{Start with the slope-intercept form.} \quad & y = 0x + b \\ \text{Plug in the slope.} \quad & y = 0x + b \\ \text{Plug in point } (9,2). \quad & 2 = 0(9) + b \\ & \Rightarrow b = 2 \end{aligned}$$

The equation of the line is $y = 5$.

Example 8

Find the equation of the line parallel to $5x - 6y = 15$ *that passes through the point* $(-5,-7)$.

Solution

Rewrite the equation in slope-intercept form.

$$3x - 5y = 12 \Rightarrow 5y = 6x - 12 \Rightarrow y = \frac{6}{5}x - \frac{12}{5}$$

The slope of the given line is $\frac{1}{3} \cdot \$60$.

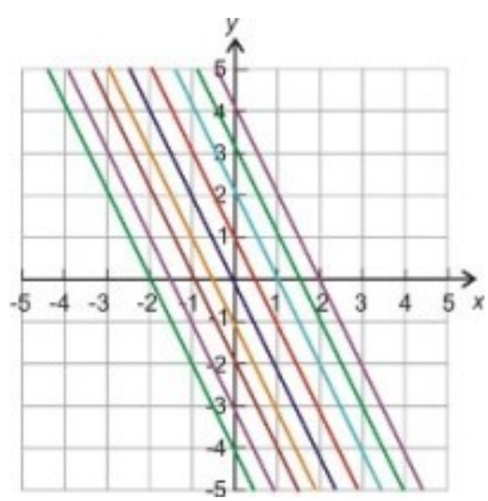
Since the slopes of parallel lines are the same, we are looking for a line with slope $\frac{1}{3} \cdot \$60$ that passes through the point $(-5, -7)$.

$$\begin{aligned} \text{Start with the slope-intercept form.} \quad & y = mx + b \\ \text{Plug in the slope.} \quad & y = \frac{6}{5}x + b \\ \text{Plug in point } (-5, -3). \quad & -3 = \frac{6}{5}(-5) + b \Rightarrow -3 = -6 + b \Rightarrow b = 3 \end{aligned}$$

The equation of the line is: $\frac{1}{3} = \frac{1 \cdot 3}{3 \cdot 3} = \frac{3}{9}$

Investigate Families of Lines

A straight line has two very important properties, its **slope** and its $y-$ **intercept.** The slope tells us how steeply the line rises or falls, and the $y-$ intercept tells us where the line intersects the $y-$axis. In this section, we will look at two families of lines. A **family of lines** is a set of lines that have something in common with each other. Straight lines can belong to two types of families. One where the slope is the same and one where the $y-$ intercept is the same.



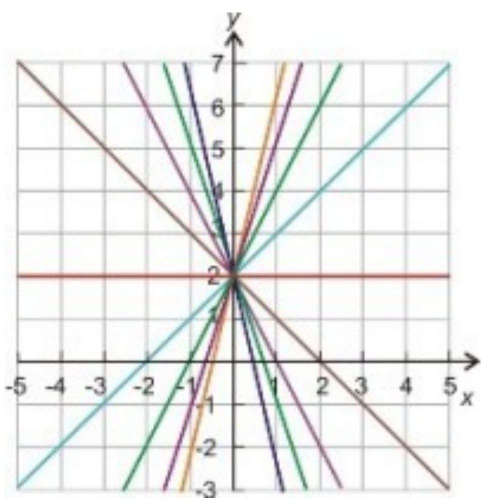
Family 1

Keep slope unchanged and vary the $y-$intercept.

The figure to the right shows the family of lines $\text{speed} = 1.5t$.

All the lines have a slope of -2 but the value of b is different for each of the lines.

Notice that in such a family all the lines are parallel. All the lines look the same but they are shifted up and down the $y-$axis. As b gets larger the line rises on the $y-$axis and as b gets smaller the line goes lower on the $y-$axis. This behavior is often called a **vertical shift.**



Family 2

Keep the $y-$intercept unchanged and vary the slope.

The figure to the right shows the family of lines $y = mx + 2$.

All lines have a $y-$intercept of two but the value of the slope is different for each of the lines. The lines "start" with $y = 5$ (red line) which has a slope of zero. They get steeper as the slope increases until it gets to the line $x = 3$ (purple line) which has an undefined slope.

Example 9

Write the equation of the family of lines satisfying the given condition:

a) Parallel to the $x-$axis

b) Through the point $(3+2)$

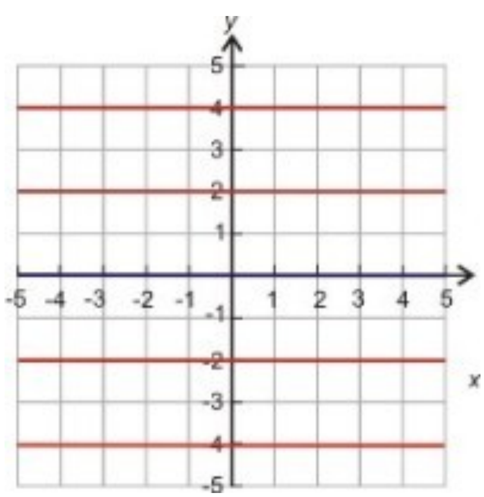
c) Perpendicular to $y - 125 = -75x$

d) Parallel to $-1, 0, 1, 2, 3, 4, 5$

Solution

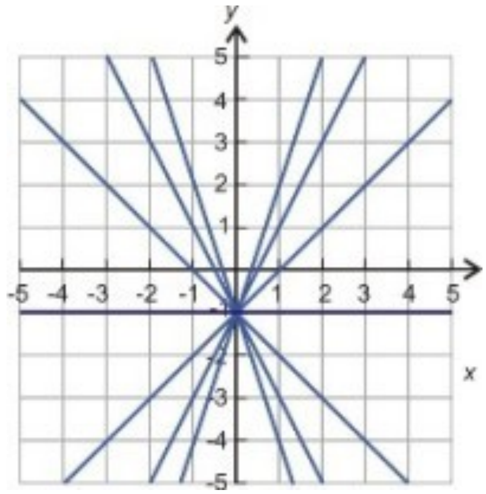
a) All lines parallel to the $x-$axis will have a slope of zero. It does not matter what the $y-$intercept is.

The family of lines is $y = 0 \cdot x + b$ or $f = 1$.



b) All lines passing through the point $(3+2)$ have the same $y-$intercept, $t = 0.4$.

The family of lines is $y = -0.025$.



c) First we need to find the slope of the given line.

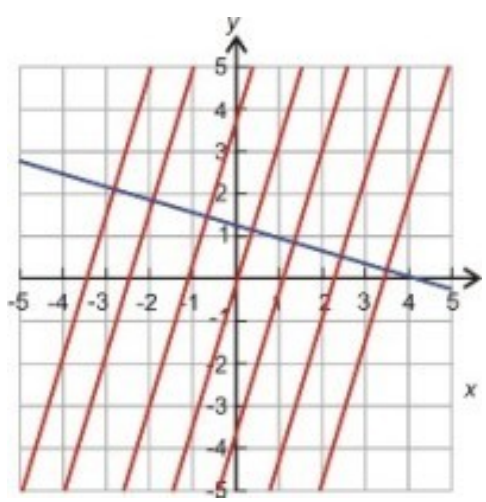
Rewrite $y - 125 = -75x$ in slope-intercept form $\frac{2}{9}\left(i + \frac{2}{3}\right) = \frac{2}{5}$.

The slope is $-\frac{8}{9}$.

The slope of our family of lines is the negative reciprocal of the given slope $a = \frac{2}{21}$.

All the lines in this family have a slope of $a = \frac{2}{21}$ but different $y-$intercepts.

The family of lines is $\frac{3}{4}x + \frac{1}{2} = \frac{3}{2}$.



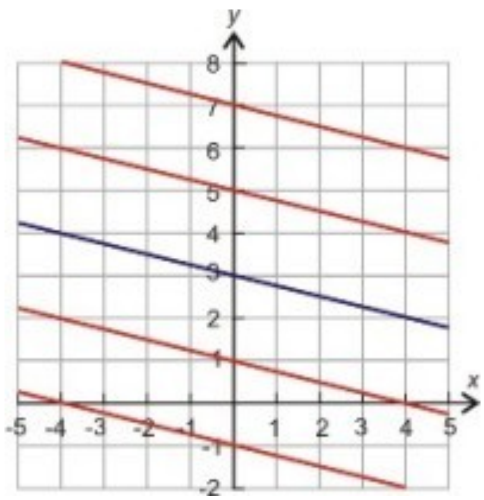
d) First we need to find the slope of the given line.

Rewrite $-1, 0, 1, 2, 3, 4, 5$ in slope-intercept form $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$.

The slope is $m = -\frac{1}{4}$.

All the lines in the family have a slope of $m = -\frac{1}{4}$ but different $y-$intercepts.

The family of lines is $y = -\frac{1}{4}x + b$.



Lesson Summary

- **Parallel lines** have the same slopes, $m_1 = m_2$, but different $y-$intercepts.
- **Perpendicular lines** have slopes which are the negative reciprocals of each other.

$$m_1 = -\frac{1}{m_2} \ or \ m_1 m_2 = -1$$

- **To find the line parallel (or perpendicular)** to a specific line which passes through a given point:

1. Find the slope of the given line from its equation.
2. Compute the slope parallel (or perpendicular) to the line.
3. Use the computed slope and the specified point to write the equation of the new line in point-slope form.
4. Transform from point-slope form to another form if required.

- **A family of lines** is a set of lines that have something in common with each other. There are two types of line families. One where the slope is the same and one where the $y-$intercept is the same.

Review Questions

Determine whether the lines are parallel, perpendicular or neither.

1. One line passes through points $(3+2)$ and $(0,0)$; another line passes through points $(3+2)$ and $(0,0)$.
2. One line passes through points $(3+2)$ and $(3+2)$; another line passes through points $(-5,-7)$ and $(3+2)$.
3. One line passes through points $(5-11)$ and $(3+2)$; another line passes through points $(3+2)$ and $(3+2)$.
4. One line passes through points $(0,0)$ and $(-5,-7)$; another line passes through points $(3+2)$ and $(3+2)$.
5. Line 1: $-2.5, 1.5, 5$ Line 2: $12y+3x=1$
6. Line 1: $y=-0.025$ Line 2: $0,1,2,3,4,5,6,$
7. Line 1: $-1,0,1,2,3,4,5$ Line 2: $y=0.8x+3$
8. Find the equation of the line parallel to $0,1,2,3,4,5$ that passes through point $(3+2)$.
9. Find the equation of the line perpendicular to $\frac{2}{9}\left(i+\frac{2}{3}\right)=\frac{2}{5}$ that passes through point $(0,0)$.
10. Find the equation of the line parallel to $7y+2x-10=0$ that passes through the point $(0,0)$.
11. Find the equation of the line perpendicular to $(4+5)-(5+2)$ that passes through the point $(0,0)$. Write the equation of the family of lines satisfying the given condition.
12. All lines pass through point $(0,0)$.
13. All lines are perpendicular to $-1,0,1,2,3,4,5$.
14. All lines are parallel to $0,1,2,3,4,5,6,$.
15. All lines pass through point $(3+2)$.

Review Answers

1. parallel

2. neither
3. parallel
4. perpendicular
5. parallel
6. perpendicular
7. neither
8. $f(x) = \frac{3x+5}{4}$
9. $f(x) = \frac{1}{2}x^2$
10. $7x + 2 = \frac{5x-3}{6}$
11. $\frac{2}{9}\left(i + \frac{2}{3}\right) = \frac{2}{5}$
12. $y = mx + 2$
13. $\frac{3}{4}x + \frac{1}{2} = \frac{3}{2}$
14. $y = 4x + b$
15. $y = -0.025$

Fitting a Line to Data

Learning Objectives

- Make a scatter plot.
- Fit a line to data and write an equation for that line.
- Perform linear regression with a graphing calculator.
- Solve real-world problems using linear models of scattered data.

Introduction

Often in application problems, the relationship between our dependent and independent variables is linear. That means that the graph of the dependent variable vs. independent variable will be a straight line. In many cases we don't know the equation of the line but we have data points that were collected from measurements or experiments. The goal of this section is to show how we can find an equation of a line from data points collected from experimental measurements.

Make a Scatter Plot

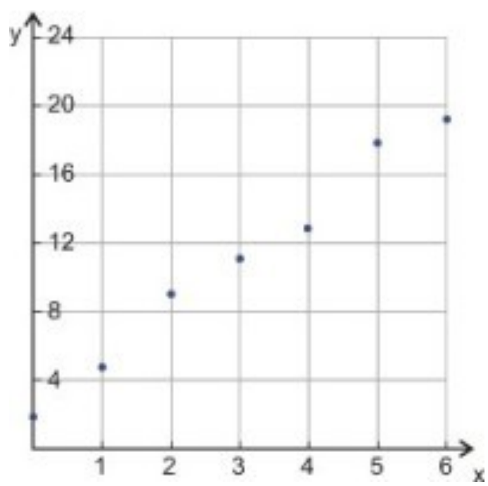
A **scatter plot** is a plot of all the ordered pairs in the table. This means that a scatter plot is a relation, and not necessarily a function. Also, the scatter plot is discrete, as it is a set of distinct points. Even when we expect the relationship we are analyzing to be linear, we should not expect that all the points would fit perfectly on a straight line. Rather, the points will be "scattered" about a straight line. There are many reasons why the data does not fall perfectly on a line such as **measurement error** and **outliers.**

Measurement error is always present as no measurement device is perfectly accurate. In measuring length, for example, a ruler with millimeter markings will be more accurate than a ruler with just centimeter markings.

An **outlier** is an accurate measurement that does not fit with the general pattern of the data. It is a statistical fluctuation like rolling a die ten times and getting the six side all ten times. It can and will happen, but not very often.

Example 1

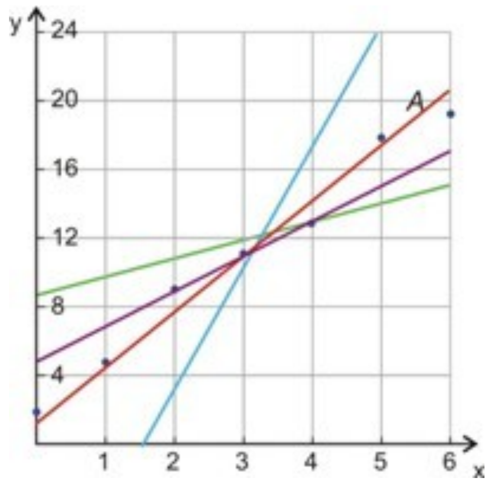
Make a scatter plot of the following ordered pairs:
$(0, 2), (1, 4.5), (2, 9), (3, 11), (4, 13), (5, 18), (6, 19.5)$



Solution

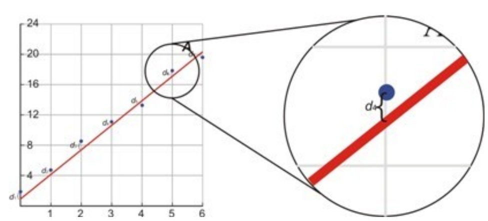
We make a scatter plot by graphing all the ordered pairs on the coordinate axis.

Fit a Line to Data



Notice that the points look like they might be part of a straight line, although they would not fit perfectly on a straight line. If the points were perfectly lined up it would be quite easy to draw a line through all of them and find the equation of that line. However, if the points are "scattered", we try to find a line that best fits the data.

You see that we can draw many lines through the points in our data set. These lines have equations that are very different from each other. We want to use the line that is closest to **all** the points on the graph. The best candidate in our graph is the red line ***A***. We want to minimize the sum of the distances from the point to the line of fit as you can see in the figure below.



Finding this line mathematically is a complex process and is not usually done by hand. We usually "eye-ball" the line or find it exactly by using a graphing calculator or computer software such as Excel. The line in the graph above is "eye-balled," which means we drew a line that comes closest to all the points in the scatter plot.

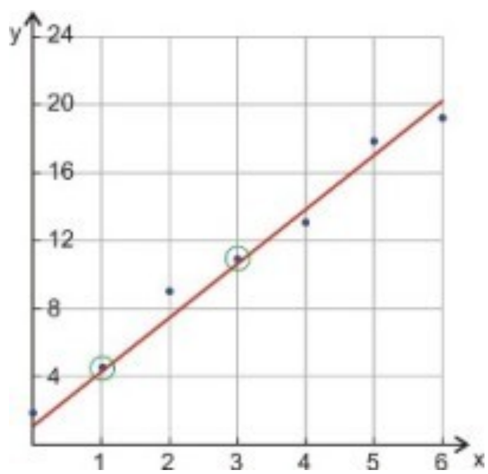
When we use the line of best fit we are assuming that there is a continuous linear function that will approximate the discrete values of the scatter plot. We can use this to interpret unknown values.

Write an Equation for a Line of Best Fit

Once you draw the line of best fit, you can find its equation by using two points on the line. Finding the equation of the line of best fit is also called **linear regression.**

Caution: Make sure you don't get caught making a common mistake. In many instances the line of best fit will not pass through many or any of the points in the original data set. This means that you can't just use two random points from the data set. **You need to use two points that are on the line.**

We see that two of the data points are very close to the line of best fit, so we can use these points to find the equation of the line $(3 + 2)$ and (mph).



Start with the slope-intercept form of a line $y = mx + b$.

Find the slope $m = \frac{11-4.5}{3-1} = \frac{6.5}{2} = 3.25$

Then $y = 3.25x + b$

Plug (mph) into the equation. $(20000 - 8000) = 12000 \text{ feet}$

The equation for the line that fits the data best is $y = 3.25x + 1.25$.

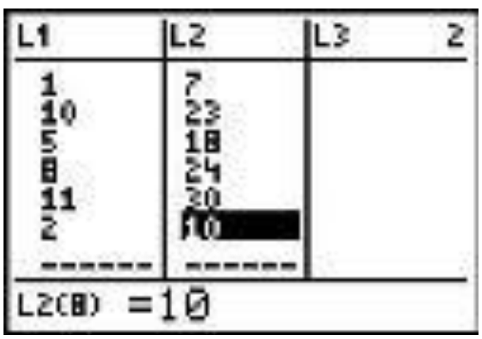
Perform Linear Regression with a Graphing Calculator

Drawing a line of fit can be a good approximation but you can't be sure that you are getting the best results because you are guessing where to draw the line. Two people working with the same data might get two different equations because they would be drawing different lines. To get the most accurate equation for the line, we can use a graphing calculator. The calculator uses a mathematical algorithm to find the line that minimizes the sum of the squares.

Example 2

Use a graphing calculator to find the equation of the line of best fit for the following data $(2 \cdot 3) \cdot 4 = 2 \cdot (3 \cdot 4)$ $(10, 23), (5, 18), (8, 24), (11, 30), (2, 10)$.

Solution



Step 1 *Input the data in your calculator.*

Press **[STAT]** and choose the **[EDIT]** option.

Input the data into the table by entering the x values in the first column and the y values in the second column.

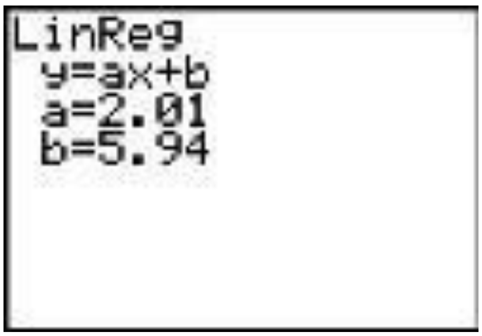
Step 2 *Find the equation of the line of best fit.*

Press **[STAT]** again use right arrow to select **[CALC]** at the top of the screen.

Chose option number 4: $f(x) = -2x + 3$ and press **[ENTER]**

The calculator will display $f(x) = -2x + 3$

Press **[ENTER]** and you will be given the a and b values.



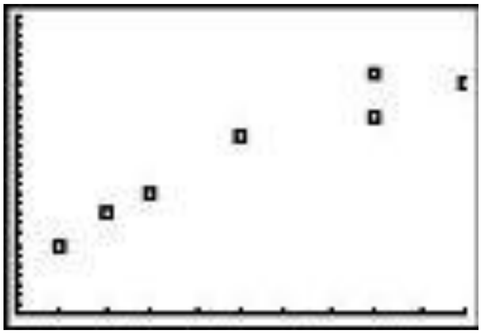
Here a represents the slope and b represents the y-intercept of the equation. The linear regression line is $y = 3.25x + 1.25$.

Step 3 *Draw the scatter plot.*

To draw the scatter plot press **[STATPLOT] [2nd] [Y=]**.

Choose Plot 1 and press **[ENTER**.

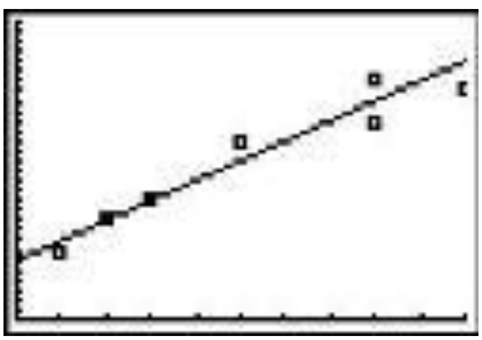
Press the On option and choose the Type as scatter plot (the one highlighted in black).



Make sure that the X list and Y list names match the names of the columns of the table in Step 1.

Choose the box or plus as the mark since the simple dot may make it difficult to see the points.

Press **[GRAPH]** and adjust the window size so you can see all the points in the scatter plot.



Step 4 *Draw the line of best fit through the scatter plot.*

Press **[Y=]**

Enter the equation of the line of best fit that you just found $Y_1 = 2.01X + 5.94$

Press **[GRAPH]**.

Solve Real-World Problems Using Linear Models of Scattered Data

In a real-world problem, we use a data set to find the equation of the line of best fit. We can then use the equation to predict values of the dependent or independent variables. The usual procedure is as follows.

1. Make a scatter plot of the given data.
2. Draw a line of best fit.
3. Find an equation of a line either using two points on the line or the TI-83/84 calculator.
4. Use the equation to answer the questions asked in the problem.

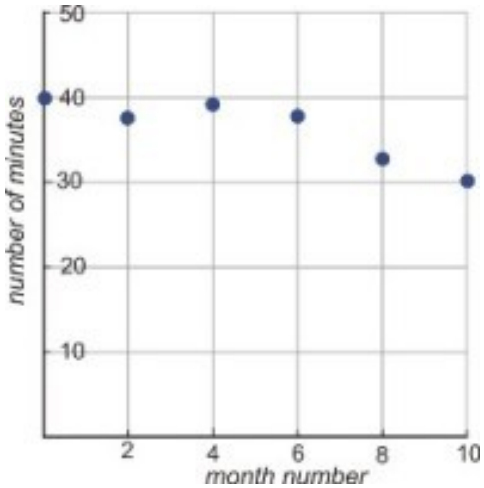
Example 3

Gal is training for a $= 7$ *race (a total of* 5960 meters *, or about* 40 coins *). The following table shows her times for each month of her training program. Assume here that her times will decrease in a straight line with time (does that seem like a good assumption?) Find an equation of a line of fit. Predict her running time if her race is in August.*

Month	Month number	Average time (minutes)
January	y	29
February	1	29
March	4	29
April	y	29
May	4	29
June	y	29

Solution

Let's make a scatter plot of Gal's running times. The independent variable, x, is the month number and the dependent variable, y, is the running time in minutes. We plot all the points in the table on the coordinate plane.

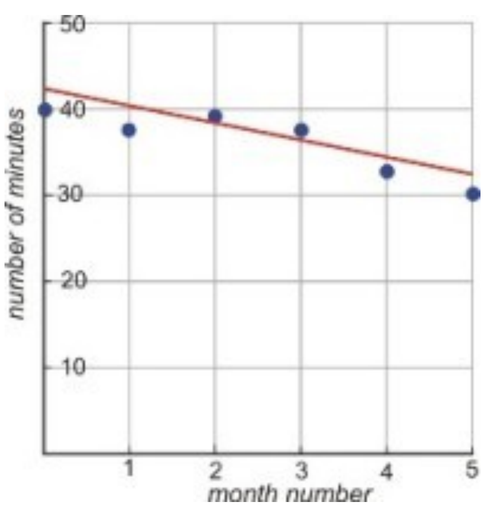


Draw a line of fit.

Choose two points on the line (mph) and (mph).

Find the equation of the line.

$$m = \frac{34 - 41}{4 - 0} = -\frac{7}{4} = -1\frac{3}{4}$$
$$y = -\frac{7}{4}x + b$$
$$41 = -\frac{7}{2}(0) + b \Rightarrow b = 41$$
$$y = -\frac{7}{4}x + 41$$



In a real-world problem, the slope and y–intercept have a physical significance.

$$\text{Slope} = \frac{number\ of\ minutes}{month}$$

Since the slope is negative, the number of minutes Gal spends running a -7 race decreased as the months pass. The slope tells us that Gal's running time decreases by $\frac{3}{4}$ or -79 minutes per month.

The y–intercept tells us that when Gal started training, she ran a distance of -7 in $3x - 2 = 5$, which is just an estimate, since the actual time was 60 minutes .

The problem asks us to predict Gal's running time in August. Since June is assigned to month number five, then August will be month number seven. We plug $x = 7$ into the equation of the line of best fit.

$$y = -\frac{7}{4}(7) + 41 = -\frac{49}{4} + 41 = -\frac{49}{4} + \frac{164}{4} = \frac{115}{4} = 28\frac{3}{4}$$

The equation predicts that Gal will be running the -7 race in $5 \times 1\ lb = 5lb$.

In this solution, we eye-balled a line of best fit. Using a graphing calculator, we found this equation for a line of fit $36 \text{ miles per hour}$.

If we plug $x = 7$ in this equation, we get $48 = (2 \times 2) \times (2 \times 2) \times 3$. This means that Gal ran her race in $-7.4 > -3.6$. You see that the graphing calculator gives a different equation and a different answer to the question. The graphing calculator result is more accurate but the line we drew by hand still gives a good approximation to the result.

Example 4

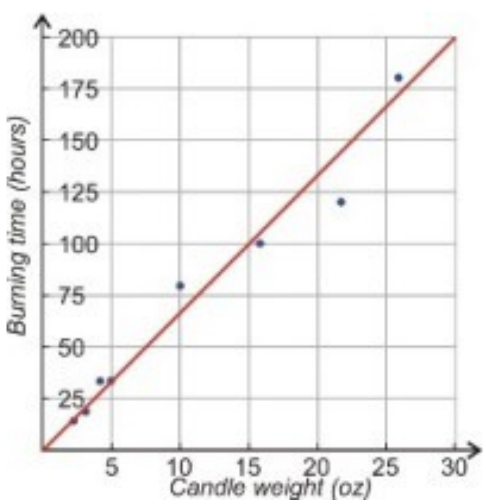
Baris is testing the burning time of "BriteGlo" candles. The following table shows how long it takes to burn candles of different weights. Assume it's a linear relation and we can then use a line to fit the data. If a candle burns for 30 ohms *, what must be its weight in ounces?*

Candle Burning Time Based on Candle Weight

Candle weight (oz)	Time (hours)
4	16
y	29
4	29
y	29
16	29
16	100
$2a$	100
29	100

Solution

Let's make a scatter plot of the data. The independent variable, x, is the candle weight in ounces and the dependent variable, y, is the time in hours it takes the candle to burn. We plot all the points in the table on the coordinate plane.



Then we draw the line of best fit.

Now pick two points on the line $(0, 0)$ and $(60, 720)$.

Find the equation of the line:

$$m = \frac{200}{30} = \frac{20}{3}$$
$$y = \frac{20}{3}x + b$$
$$0 = \frac{20}{3}(0) + b \Rightarrow b = 0$$
$$y = \frac{20}{3}x$$

In this problem the slope is burning time divided by candle weight. A slope of $x = \frac{11}{162}$ tells us for each extra ounce of candle weight, the burning time increases by $\frac{30}{0.75} = 40$.

A y–intercept of zero tells us that a candle of weight y oz will burn for 6 times .

The problem asks for the weight of a candle that burns 30 ohms . We are given the value of $p = 15$. We need to use the equation to find the corresponding value of x.

$$y = \frac{20}{3}x \Rightarrow = \frac{20}{3}x \Rightarrow x = \frac{285}{20} = \frac{57}{4} = 14\frac{1}{4}$$

A candle that burns 29 hours weighs 22 coins.

The graphing calculator gives the linear regression equation as $-2, 0, 2, 4, 6 \ldots$ and a result of $3x < 5$.

Notice that we can use the line of best fit to estimate the burning time for a candle of any weight.

Lesson Summary

- A **scatter plot** is a plot of all ordered pairs of experimental measurements.

- **Measurement error** arises from inaccuracies in the measurement device. All measurements of continuous values contain measurement error.
- An **outlier** is an experimental measurement that does not fit with the general pattern of the data.
- For experimental measurements with a linear relationship, you can draw a **line of best fit** which minimizes the distance of each point to the line. Finding the line of best fit is called **linear regression.** A statistics class can teach you the math behind linear regression. For now, you can estimate it visually or use a graphing calculator.

Review Questions

For each data set, draw the scatter plot and find the equation of the line of best fit for the data set by hand.

1. $(57, 45)$ $(65, 61)$ $(34, 30)$ $(87, 78)$ $(42, 41)$ $(35, 36)$ $(59, 35)$ $(61, 57)$ $(25, 23)$ $(35, 34)$
2. $(32, 43)$ $(54, 61)$ $(89, 94)$ $(25, 34)$ $(43, 56)$ $(58, 67)$ $(38, 46)$ $(47, 56)$ $(39, 48)$
3. $(12, 18)$ $(5, 24)$ $(15, 16)$ $(11, 19)$ $(9, 12)$ $(7, 13)$ $(6, 17)$ $(12, 14)$
4. $(3, 12)$ $(8, 20)$ $(1, 7)$ $(10, 23)$ $(5, 18)$ $(8, 24)$ $(2, 10)$

For each data set, use a graphing calculator to find the equation of the line of best fit.

1. $(57, 45)$ $(65, 61)$ $(34, 30)$ $(87, 78)$ $(42, 41)$ $(35, 36)$ $(59, 35)$ $(61, 57)$ $(25, 23)$ $(35, 34)$
2. $(32, 43)$ $(54, 61)$ $(89, 94)$ $(25, 34)$ $(43, 56)$ $(58, 67)$ $(38, 46)$ $(47, 56)$ $(95, 105)$ $(39, 48)$
3. $(32, 43)$ $(54, 61)$ $(89, 94)$ $(25, 34)$ $(43, 56)$ $(58, 67)$ $(38, 46)$ $(47, 56)$ $(39, 48)$
4. Shiva is trying to beat the samosa eating record. The current record is $-5x$ samosas in $3 \times 5 = 15$.

   The following table shows how many samosas he eats during his daily practice for the first week of his training. Will he be ready for the contest if it occurs two weeks from the day he started training? What are the meanings of the slope and the y –intercept in this problem?

Day	No. of Samosas
1	29
4	29

Day	No. of Samosas
y	29
4	29
y	29
y	29
7	29

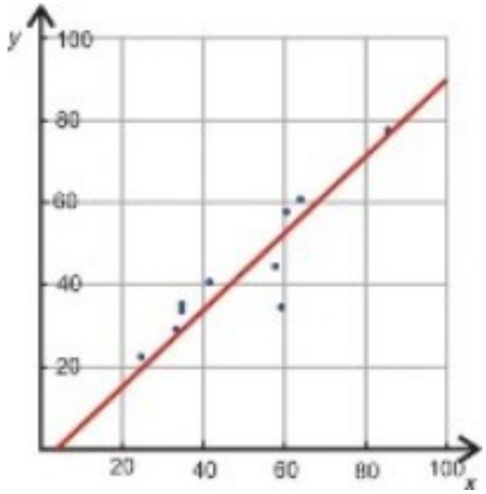
1. Nitisha is trying to find the elasticity coefficient of a Superball. She drops the ball from different heights and measures the maximum height of the resulting bounce. The table below shows her data. Draw a scatter plot and find the equation. What is the initial height if the bounce height is $= 200$? What are the meanings of the slope and the y–intercept in this problem?

Initial height (-3)	Bounce height (-3)
29	$2a$
29	29
29	29
29	29
29	29
29	29
29	29
29	29
75	29

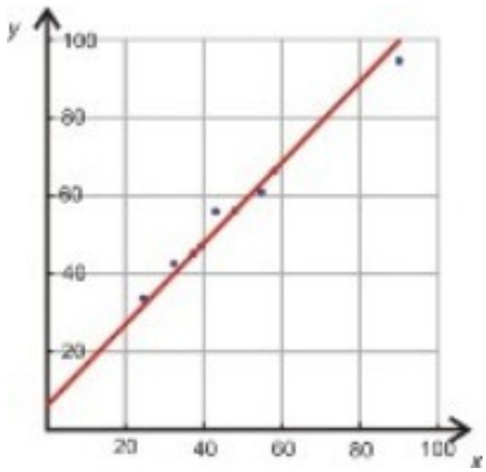
1. The following table shows the median California family income from 1000 to 2000 as reported by the US Census Bureau. Draw a scatter plot and find the equation. What would you expect the median annual income of a Californian family to be in year 2000? What are the meanings of the slope and the y–intercept in this problem?

Year	Income
1000	6.55Ω
1000	6.55Ω
1.5Ω	93000
1000	93000
1000	93000
2000	93000
1000	$x = 7$
2000	6.55Ω

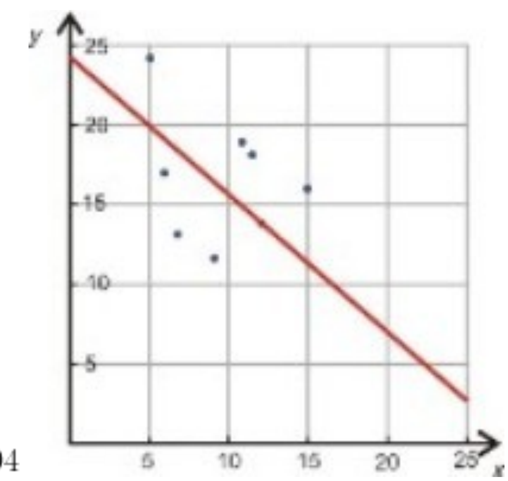
Review Answers



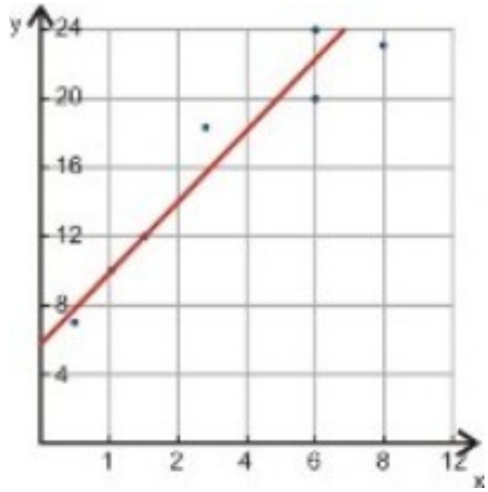
1. $-2, 0, 2, 4, 6 \ldots$



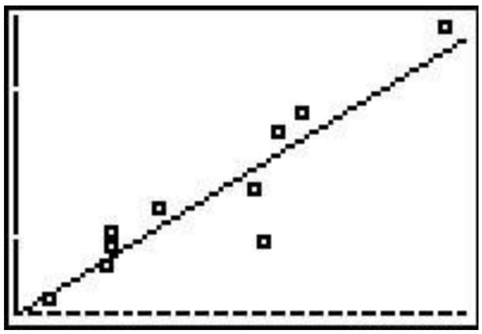
2. $y = 1.05x + 6.1$



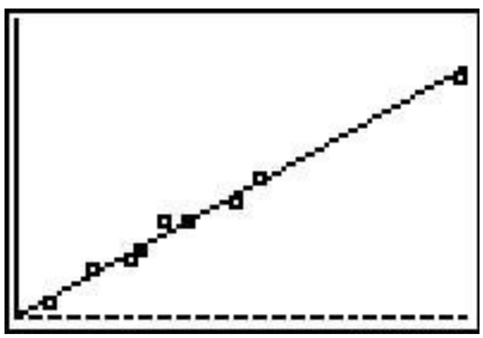
3. $Y_1 = 2.01X + 5.94$



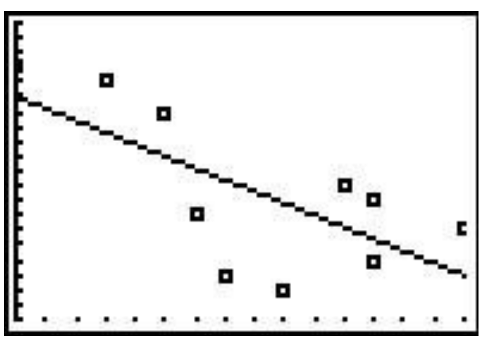
4. $-2.5, 1.5, 5$



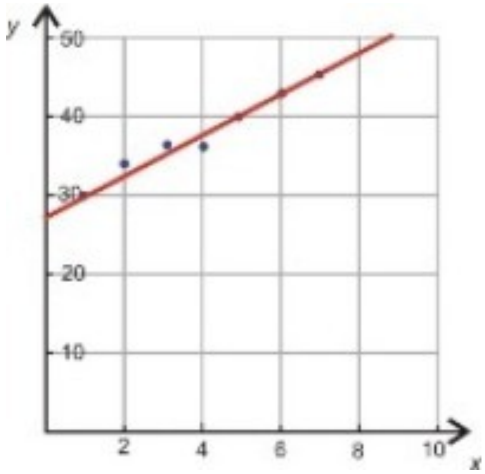
5. $x = 50$ guests



6. $y = 3.25x + 1.25$

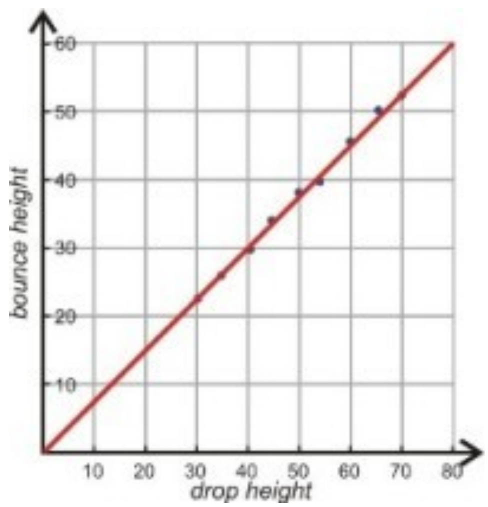


7. $-2, 0, 2, 4, 6 \ldots$

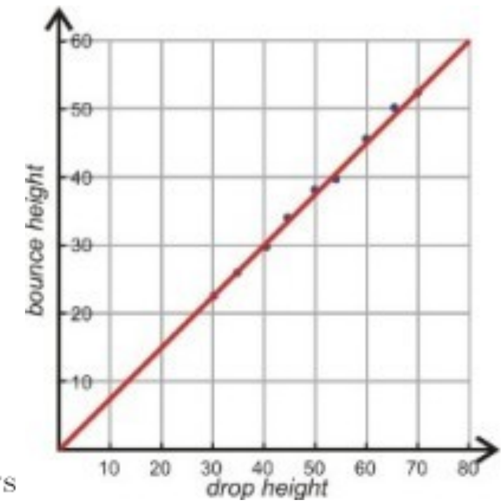
8. $y = 2.5x + 27.5$ 

Solution

$P = 20t$;. Shiva will beat the record.

9. $y = 2.5x + 27.5$ 

slope = the ratio of bounce height to drop height y-intercept = how far the ball bounces if it's dropped from a height of zero. The line is the best *fit* to the data. We know that dropping it from height of zero should give a bounce of zero and $9 = 3 \cdot 3$ is pretty close to zero. Drop height $r \cdot w = 15$ when bounce height $F = ma$

10. $1 \text{ week} = \ 7 \text{ days}$ 

$x =$ years since 1995 $y =$ Income in thousands of dollars slope = increase in income per year (in thousands) y-intercept = income in 1995 (in thousands) Income in 2000 is $\$15552$.

Predicting with Linear Models

Learning Objectives

- Collect and organize data.
- Interpolate using an equation.
- Extrapolate using an equation.
- Predict using an equation.

Introduction

Numerical information appears in all areas of life. You can find it in newspapers, magazines, journals, on the television or on the internet. In the last section, we saw how to find the equation of a line of best fit and how to use this equation to make predictions. The line of 'best fit' is a good method if the relationship between the dependent and the independent variables is linear. In this section, you will learn other methods that help us estimate data values. These methods are useful in linear and non-linear relationships equally. The methods you will learn are **linear interpolation** which is useful if the information you are looking for is between two known points and **linear**

extrapolation which is useful for estimating a value that is either less than or greater than the known values.

Collect and Organize Data

Data can be collected through **surveys** or **experimental measurements.**

Surveys are used to collect information about a population. Surveys of the population are common in political polling, health, social science and marketing research. A survey may focus on opinions or factual information depending on its purpose.

Experimental measurements are data sets that are collected during experiments.

The information collected by the US Census Bureau (www.census.gov) or the Center for Disease Control (www.cdc.gov) are examples of data gathered using surveys. The US Census Bureau collects information about many aspects of the US population. The census takes place every ten years and it polls the population of the United States.

Let’s say we are interested in how the median age for first marriages has changed during the 20^{th} century.

Example 1

Median age at first marriage

The US Census gives the following information about the median age at first marriage for males and females.

In 1000, the median age for males was -53 and for females it was $-5x$.

In 1000, the median age for males was $-5x$ and for females it was $-5x$.

In 1000, the median age for males was -53 and for females it was $-5x$.

In 1000, the median age for males was $-5x$ and for females it was $-2x$.

In 1000, the median age for males was $-5x$ and for females it was $-5x$.

In 1000, the median age for males was $-5x$ and for females it was $-5x$.

In 1000, the median age for males was $-5x$ and for females it was $-5x$.

In 1000, the median age for males was $-5x$ and for females it was $-5x$.

In 1.5Ω, the median age for males was $-5x$ and for females it was $-5x$.

In 1000, the median age for males was 23.7 and for females it was $-5x$.

In 1000, the median age for males was -53 and for females it was $-5x$.

In 2000, the median age for males was $-5x$ and for females it was -53.

This is not a very efficient or clear way to display this information. Some better options are organizing the data in a table or a scatter plot.

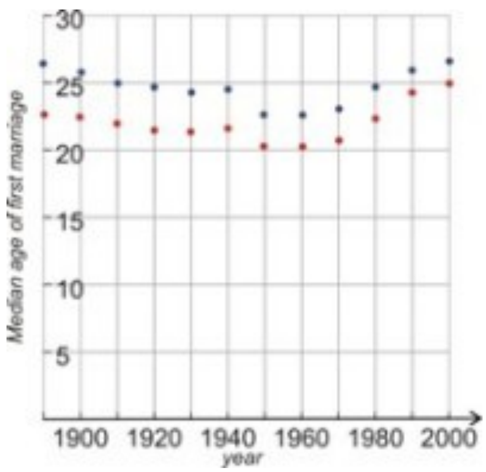
A table of the data would look like this.

Median Age of Males and Females at First Marriage by Year

Year	Median Age of Males	Median Age of Females
1000	-53	$-5x$
1000	$-5x$	$-5x$
1000	-53	$-5x$
1000	$-5x$	$-2x$
1000	$-5x$	$-5x$
1000	$-5x$	$-5x$
1000	$-5x$	$-5x$
1000	$-5x$	$-5x$
1.5Ω	$-5x$	$-5x$
1000	23.7	$-5x$
1000	-53	$-5x$
2000	$-5x$	-53

A scatter plot of the data would look like this.

Median Age of Males and Females at First Marriage by Year



The Center for Disease Control collects information about the health of the American people and behaviors that might lead to bad health. The next example shows the percent of women that smoke during pregnancy.

Example 2

Pregnant women and smoking

The CDC has the following information.

In the year 1000, -53 percent of pregnant women smoked.

In the year $P =$, -79 percent of pregnant women smoked.

In the year 1000, -53 percent of pregnant women smoked.

In the year 1000, -53 percent of pregnant women smoked.

In the year 1000, -53 percent of pregnant women smoked.

In the year 1000, -53 percent of pregnant women smoked.

In the year 1000, -53 percent of pregnant women smoked.

In the year 2000, -14 percent of pregnant women smoked.

In the year 2000, -14 percent of pregnant women smoked.

In the year 2000, -53 percent of pregnant women smoked.

In the year 2000, -53 percent of pregnant women smoked.

Let's organize this data more clearly in a table and in a scatter plot.

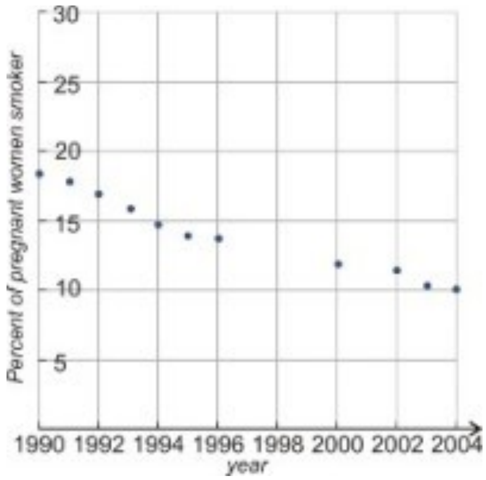
Here is a table of the data.

Percent of Pregnant Women Smokers by Year

Year	Percent of pregnant women smokers
1000	-53
$P =$	-79
1000	-53
1000	-53
1000	-53
1000	-53
1000	-53
2000	-14
2000	-14
2000	-53
2000	-53

Here is a scatter plot of the data.

Percent of Pregnant Women Smokers by Year



Interpolate Using an Equation

Linear interpolation is often used to fill the gaps in a table. Example one shows the median age of males and females at the time of their first marriage. However, the information is only available at ten year intervals. We know the median age of marriage every ten years from 1000 to 2000, but we would like to estimate the median age of marriage for the years in between. Example two gave us the percentage of women smoking while pregnant. But, there is no information collected for 1.5Ω, 1000, 1000 and 1000 and we would like to estimate the percentage for these years. Linear interpolation gives you an easy way to do this.

The strategy for linear interpolation is to use a straight line to connect the known data points (we are assuming that the data would be continuous between the two points) on either side of the unknown point. Then we use that equation to estimate the value we are looking for.

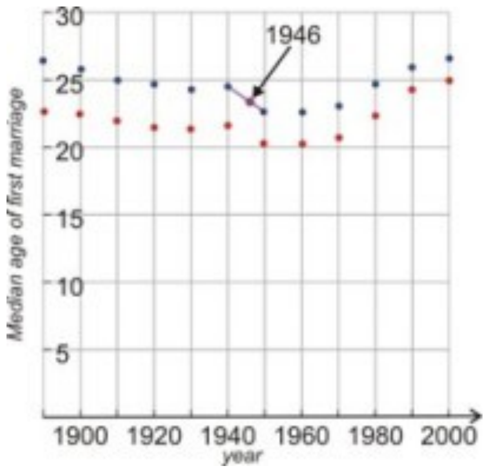
Example 3

Estimate the median age for the first marriage of a male in the year 1000.

Median Age of Males and Females at First Marriage by Year (excerpt)

Year	Median age of males	Median age of females
$\ldots$	$\ldots$	$\ldots$
1000	$-5x$	$-5x$
1000	$-5x$	$-5x$
$\ldots$	$\ldots$	$\ldots$

The table to the left shows only the data for the years 1000 and 1000 because we want to estimate a data point between these two years.



We connect the two points on either side of 1946 with a straight line and find its equation.

$$\begin{aligned} \text{Slope} \qquad\qquad m &= \frac{22.8 - 24.3}{1950 - 1940} = \frac{-1.5}{10} = -0.15 \\ y &= -0.15x + b \\ 24.3 &= -0.15(1940) + b \\ b &= 315.3 \\ \text{Equation} \qquad\qquad y &= -0.15x + 315.3 \end{aligned}$$

To estimate the median age of marriage of males in year 1000 we plug $-9 = -9$ in the equation.

$y = -0.15(1946) + 315.3 = 23.4 \ years \ old$

Example 4

Estimate the percentage of pregnant women that were smoking in the year 1 km

Percent of Pregnant Women Smokers by Year (excerpt)

Year	Percent of Pregnant Women Smokers
$\ldots$	$\ldots$
1000	-53
2000	-14
$\ldots$	$\ldots$

The table to the left shows only the data for year 1000 and 2000 because we want to estimate a data point between these two years.

Connect the points on either side of 1000 with a straight line and find the equation of that line.

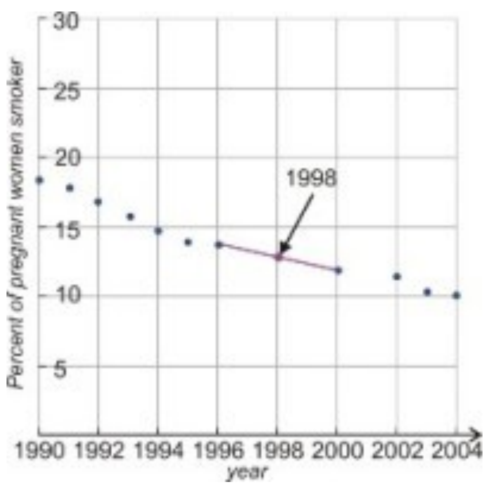
$$\text{Slope} \qquad m = \frac{22.8 - 24.3}{1950 - 1940} = \frac{-1.5}{10} = -0.15$$

$$y = -0.15x + b$$

$$24.3 = -0.15(1940) + b$$

$$b = 315.3$$

$$\text{Equation} \qquad y = -0.15x + 315.3$$



To estimate the percentage of pregnant women who smoked in year 1000 we plug $-9 = -9$ into the equation.

$y = -0.35(1998) + 712.2 = 12.9\%$

For non-linear data, linear interpolation is often not accurate enough for our purposes. If the points in the data set change by a large amount in the interval in which you are interested, then linear interpolation may not give a good estimate. In that case, it can be replaced by **polynomial interpolation** which uses a curve instead of a straight line to estimate values between points.

Extrapolating: How to Use it and When Not to Use it

Linear extrapolation can help us estimate values that are either higher or lower than the range of values of our data set. The strategy is similar to linear interpolation. However you only use a subset of the data, rather than all of the data. For linear data, you are ALWAYS more accurate by using the best fit line method of the previous section. For non-linear data, it is sometimes useful to extrapolate using the last two or three data points in order to estimate a $y-$value that is higher than the data range. To estimate a value that is higher than the points in the data set, we connect the last two data points with a straight line and find its equation. Then we can use this equation to estimate the value we are trying to find. To estimate a value that is lower than the points in the data set, we follow the same procedure. But we use the first two points of our data instead.

Example 5

Winning Times

The winning times for the women's 2 minutes race are given in the following table^3. Estimate the winning time in the year 2000. Is this a good estimate?

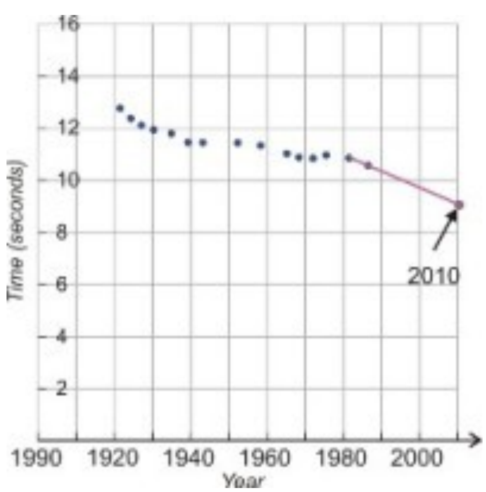
Winner	**Country**	**Year**	**Time (seconds)**
Mary Lines	UK	1000	-53
Leni Schmidt	Germany	1000	-14
Gerturd Glasitsch	Germany	1.5Ω	-11
Tollien Schuurman	Netherlands	1000	-53
Helen Stephens	USA	1000	-53

Winner	Country	Year	Time (seconds)
Lulu Mae Hymes	USA	1000	-53
Fanny Blankers-Koen	Netherlands	1000	-53
Marjorie Jackson	Australia	1000	-14
Vera Krepkina	Soviet Union	1000	-53
Wyomia Tyus	USA	1000	-14
Barbara Ferrell	USA	1000	-11
Ellen Strophal	East Germany	1.5Ω	-53
Inge Helten	West Germany	1.5Ω	-53
Marlies Gohr	East Germany	1000	-53
Florence Griffith Joyner	USA	1000	-53

Solution

We start by making a scatter plot of the data. Connect the last two points on the graph and find the equation of the line.

Winning Times for the Women's 100 meter Race by Year



Slope $m = \frac{10.5 - 10.9}{1988 - 1982} = \frac{-0.4}{6} = -0.067$

$$y = -0.067x + b$$
$$10.5 = -0.067(1988) + b$$
$$b = 143.7$$

Equation $y = -0.067x + 143.7$

The winning time in year 2000 is estimated to be:

$$11(2+6) = 11(2) + 11(6) = 22 + 66 = 88$$

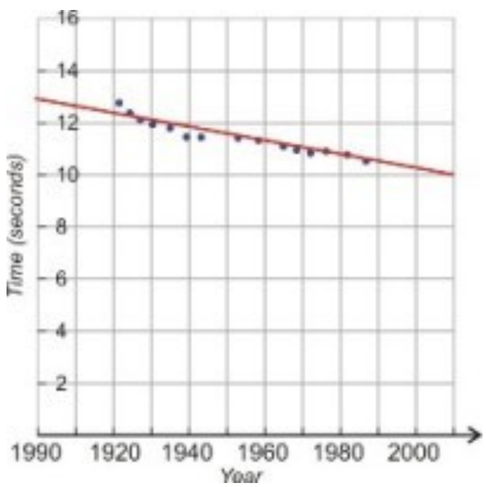
[3] Source:
http://en.wikipedia.org/wiki/World_Record_progression_100_m_women.

How accurate is this estimate? It is likely that it's not very accurate because 2000 is a long time from 1000. This example demonstrates the weakness of linear extrapolation. Estimates given by linear extrapolation are never as good as using the equation from the best fit line method. In this particular example, the last data point clearly does not fit in with the general trend of the data so the slope of the extrapolation line is much steeper than it should be. As a historical note, the last data point corresponds to the winning time for Florence Griffith Joyner in 1000. After her race, she was accused of using performance-enhancing drugs but this fact was never proven. In addition, there is a question about the accuracy of the timing because some officials said that the tail wind was not accounted for in this race even though all the other races of the day were impacted by a strong wind.

Predict Using an Equation

Linear extrapolation was not a good method to use in the last example. A better method for estimating the winning time in 2010 would be the use of linear regression (i.e. *best fit line method*) that we learned in the last section. Let's apply that method to this problem.

Winning Times for the Women's 100 meter Race by Year



We start by drawing the line of best fit and finding its equation. We use the points $A\ (-4, -4)$ and $A\ (-4, -4)$.

The equation is $y = -0.017x + 43.9$

In year 2000, $y = -0.017(2010) + 43.9 = 9.73$ seconds

This shows a much slower decrease in winning times than linear extrapolation. This method (fitting a line to all of the data) is always more accurate for linear data and approximate linear data. However, the line of best fit in this case will not be useful in the future. For example, the equation predicts that around the year 2000 the time will be about zero seconds, and in years that follow the time will be negative!

Lesson Summary

- A **survey** is a method of collecting information about a population.
- **Experimental measurements** are data sets that are collected during experiments.
- **Linear interpolation** is used to estimate a data value between two experimental measurements. To do so, compute the line through the two adjacent measurements, then use that line to estimate the intermediate value.
- **Linear extrapolation** is used to estimate a data value either above or below the experimental measurements. Again, find the line defined by the two closest points and use that line to estimate the value.

- The **most accurate method** of estimating data values from a linear data set is to perform linear regression and estimate the value from the best-fit line.

Review Questions

1. Use the data from Example one (*Median age at first marriage*) to estimate the age at marriage for females in 1000. Fit a line, by hand, to the data before 1.5Ω.
2. Use the data from Example one (*Median age at first marriage*) to estimate the age at marriage for females in 1000. Fit a line, by hand, to the data from 1.5Ω on in order to estimate this accurately.
3. Use the data from Example one (*Median age at first marriage*) to estimate the age at marriage for males in 1000. Use linear interpolation between the 1000 and 2000 data points.
4. Use the data from Example two (*Pregnant women and smoking*) to estimate the percent of pregnant smokers in 1.5Ω. Use linear interpolation between the 1000 and 2000 data points.
5. Use the data from Example two (*Pregnant women and smoking*) to estimate the percent of pregnant smokers in 2000. Use linear extrapolation with the final two data points.
6. Use the data from Example five (*Winning times*) to estimate the winning time for the female 100 meter race in 1000. Use linear extrapolation because the first two or three data points have a different slope than the rest of the data.
7. The table below shows the highest temperature vs. the hours of daylight for the 15^{th} day of each month in the year 2000 in San Diego, California. Estimate the high temperature for a day with -53 hours of daylight using linear interpolation.
8.

Hours of daylight	**High temperature** $(=)$
$b = 1$	29
-53	29
12	29
16	29
-53	29
-53	75
12	29
-53	75
-14	71
-14	29
-53	75

Hours of daylight	High temperature $(=)$
16	16

9. Using the table above to estimate the high temperature for a day with y hours of daylight using linear extrapolation. Is the prediction accurate? Find the answer using line of best fit.

Review Answers

1. About 12 years
2. $-5x$ years
3. $-5x$ years
4. $b = 1$ percent
5. $y-$ percent
6. 19.1 seconds
7. $V = 6$
8. 1.5Ω. Prediction is not very good since we expect cooler temperatures for less daylight hours. The best fit line method of linear regression predicts $= 25\Omega$.

Problem Solving Strategies: Use a Linear Model

Learning Objectives

- Read and understand given problem situations.
- Develop and apply the strategy: use a linear model.
- Plan and compare alternative approaches to solving problems.
- Solve real-world problems using selected strategies as part of a plan.

Introduction

In this chapter, we have been estimating values using straight lines. When we use linear interpolation, linear extrapolation or predicting results using a line of best fit, it is called **linear modeling.** In this section, we will look at a few examples where data sets occurring in real-world problems can be modeled using linear relationships. From previous sections remember our problem solving plan:.

Step 1

Understand the problem

Read the problem carefully. Once the problem is read, list all the components and data that are involved. This is where you will be assigning your variables

Step 2

Devise a plan – Translate

Come up with a way to solve the problem. Set up an equation, draw a diagram, make a chart or construct a table as a start to solving your problem.

Step 3

Carry out the plan – Solve

This is where you solve the equation you came up with in Step 2.

Step 4

Look – Check and Interpret

Check to see if you used all your information and that the answer makes sense.

Example 1

Dana heard something very interesting at school. Her teacher told her that if you divide the circumference of a circle by its diameter you always get the same number. She tested this statement by measuring the circumference and diameter of several circular objects. The following table shows her results.

From this data, estimate the circumference of a circle whose diameter is $-9 = -9$. What about $h = 8 \text{ cm}$? $h = 8 \text{ cm}$?

Diameter and Circumference of Various Objects

Object	Diameter (inches)	Circumference (inches)

Object	Diameter (inches)	Circumference (inches)
Table	29	750
Soda can	$-5x$	$+1$
Cocoa tin	2.4	-53
Plate	y	$-5x$
Straw	-8	-2
Propane tank	-53	$-5x$
Hula Hoop	$c = 9$	100

Solution

Let's use the problem solving plan.

Step 1

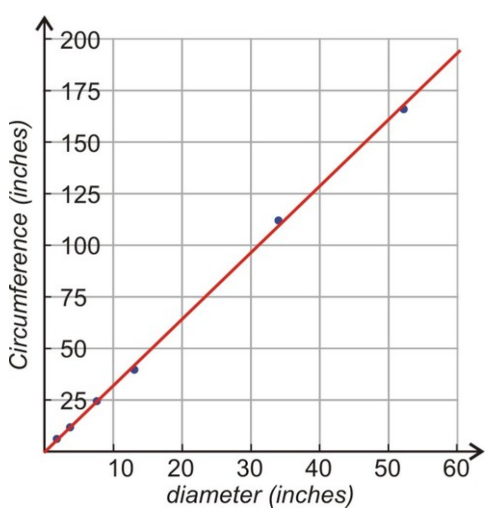
We define our variables.

$x =$ diameter of the circle in inches

$y =$ circumference of the circle in inches

We want to know the circumference when the diameter is 12, 29 or $h = 8 \text{ cm}$.

Step 2 We can find the answers either by using the line of best fit or by using linear interpolation or extrapolation. We start by drawing the scatter plot of the data.



Step 3 Line of best fit

Estimate a line of best fit on the scatter plot.

Find the equation using points $(-5, -7)$ and $(5 - 11)$.

$$\begin{aligned} &\text{Slope} && m = \frac{25.5 - 1.2}{8 - .25} = \frac{24.3}{7.75} = 3.14 \\ & && y = 3.14x + b \\ & && 1.2 = 3.14(.25) + b \Rightarrow b = 0.42 \\ &\text{Equation} && y = 3.14x + 0.42 \end{aligned}$$

$$\begin{aligned} &\text{Diameter} = 12 \text{ inches} \Rightarrow y = 3.14(12) + 0.42 = \underline{38.1 \text{ inches}} \\ &\text{Diameter} = 25 \text{ inches} \Rightarrow y = 3.14(25) + 0.42 = \underline{78.92 \text{ inches}} \\ &\text{Diameter} = 60 \text{ inches} \Rightarrow y = 3.14(60) + 0.42 = \underline{188.82 \text{ inches}} \end{aligned}$$

In this problem the slope $x - 25$. This number should be very familiar to you – it is the number rounded to the hundredths place. Theoretically, the circumference of a circle divided by its diameter is always the same and it equals $-5x$ or x.

You are probably more familiar with the formula 150 miles.

Note: The calculator gives the line of best fit as $1 \text{ week} = 7 \text{ days}$, so we can conclude that we luckily picked two values that gave the correct slope of $-5x$. Our line of best fit shows that there was more measurement error in other points.

Step 4 Check and Interpret

> The circumference of a circle is $20.$ and the diameter is simply h. If we divide the circumference by the diameter we will get x. The slope of the line is $-5x$, which is very close to the exact value of x. There is some error in the estimation because we expect the y-intercept to be zero and it is not.

The reason the line of best fit method works the best is that the data is very linear. All the points are close to the straight line but there is some slight measurement error. The line of best fit averages the error and gives a good estimate of the general trend.

Note: The linear interpolation and extrapolation methods give estimates that aren't as accurate because they use only two points in the data set. If there are measurement errors in the points that are being used, then the estimates will lose accuracy. Normally, it is better to compute the line of best fit with a calculator or computer.

Example 2

A cylinder is filled with water to a height of 75 *centimeters. The water is drained through a hole in the bottom of the cylinder and measurements are taken at two second intervals. The table below shows the height of the water level in the cylinder at different times.*

a) *Find the water level at* 15 seconds.

b) *Find the water level at* $35 \quad \text{nickels}$.

Water Level in Cylinder at Various Times

Water Level in Cylinder at Various Times

Time (seconds)	**Water level** (-3)
$y-$	75
$y-$	$-5x$
$y-$	$-5x$
$y-$	23.7
$y-$	$-5x$
-53	$-5x$
-53	23.7
-53	$-5x$
-53	$7 \cdot 5$
-53	-53
$-5x$	$y-$
$-5x$	$y-$
$-5x$	$y-$
$-5x$	$y-$
$-5x$	2.7
$-5x$	-8

Solution

Let’s use the problem solving plan.

Step 1

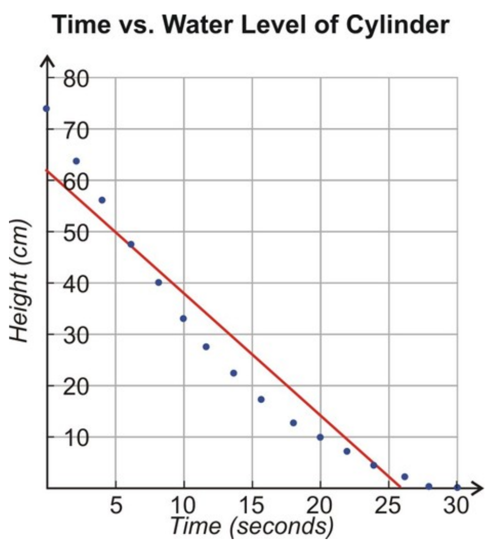
Define our variables

$x =$ time in seconds

$y =$ water level in centimeters

We want to know the water level at time 16, 27 and -8 seconds.

Step 2 We can find the answers either by using the line of best fit or by using linear interpolation or extrapolation. We start by drawing the scatter plot of the data.



Step 3 *Method 1* Line of best fit

Draw an estimate of the line of best fit on the scatter plot. Find the equation using points $(5-11)$ and $(5-11)$.

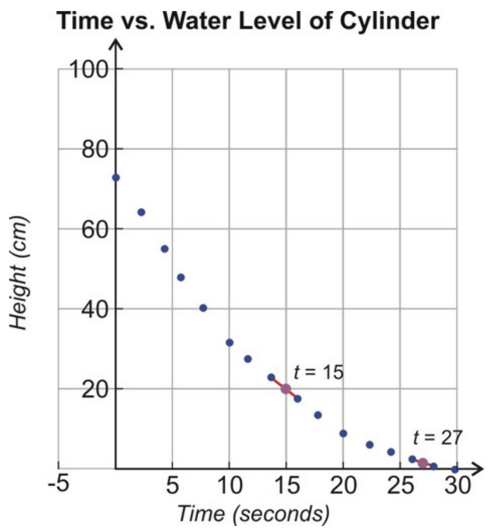
$$\begin{aligned}
\text{Slope} \qquad & m = \frac{3.9-47.2}{24-6} = \frac{-43.3}{18} = -2.4 \\
& y = -2.4x + b \\
& 47.2 = -2.4(6) + b \Rightarrow b = 61.6 \\
\text{Equation} \qquad & y = -2.4x + 61.6
\end{aligned}$$

$\text{Time} = 15 \text{ seconds} \Rightarrow y = -2.4(15) + 61.6 = \underline{25.6\text{cm}}$
$\text{Time} = 27 \text{ seconds} \Rightarrow y = -2.4(27) + 61.6 = \underline{-3.2 \text{ cm}}$

The line of best fit does not show us accurate estimates for the height. The data points do not appear to fit a linear trend so the line of best fit is close to very few data points.

Method 2: Linear interpolation or linear extrapolation.

We use linear interpolation to find the water level for the times 16 and 27 seconds, because these points are between the points we know.



Time $= 15 \text{ seconds}$

Connect points $11(2+6)$ and $11(2+6)$ and find the equation of the straight line.

$$m = \frac{17.1 - 21.9}{16 - 14} = \frac{-4.8}{2} = -2.4y = -2.4x + b \Rightarrow 21.9 = -2.4(14) + b \Rightarrow b = 55.5$$

Equation $y = -2.4x + 55.5$

Plug in $k = 12$ and obtain $y = -2.4(15) + 55.5 = 19.5 \ \text{ cm}$

Time $= 35 \ \text{ nickels}$

Connect points (mph) and $(5 - 11)$ and find the equation of the straight line.

$$m = \frac{0.7 - 2}{28 - 26} = \frac{-1.3}{2} = -.65$$

$$y = -.65x + b \Rightarrow 2 = -.65(26) + b \Rightarrow b = 18.9$$

Equation $y = -.65x = 18.9$

Plug in 350 ml and obtain $y = -.65(27) + 18.9 = 1.35 \text{ cm}$

We use linear extrapolation to find the water level for time -8 seconds because this point is smaller than the points in our data set.

Step 4 Check and Interpret

> In this example, the linear interpolation and extrapolation method gives better estimates of the values that we need to solve the problem. Since the data is not linear, the line of best fit is not close to many of the points in our data set. The linear interpolation and extrapolation methods give better estimates because we do not expect the data to change greatly between the points that are known.

Lesson Summary

- Using linear interpolation, linear extrapolation or prediction using a line of best fit is called **linear modeling.**
- The four steps of the **problem solving plan** are:

1. Understand the problem
2. Devise a plan – Translate
3. Carry out the plan – Solve
4. Look – Check and Interpret

Review Questions

The table below lists the predicted life expectancy based on year of birth (US Census Bureau).

Use this table to answer the following questions.

1. Make a scatter plot of the data
2. Use a line of best fit to estimate the life expectancy of a person born in 1000.
3. Use linear interpolation to estimate the life expectancy of a person born in 1000.
4. Use a line of best fit to estimate the life expectancy of a person born in 1.5Ω.
5. Use linear interpolation to estimate the life expectancy of a person born in 1.5Ω.
6. Use a line of best fit to estimate the life expectancy of a person born in 2000.
7. Use linear extrapolation to estimate the life expectancy of a person born in 2000.
8. Which method gives better estimates for this data set? Why?
9.

Birth Year	**Life expectancy in years**
1000	23.7
1000	$-5x$
1000	$-5x$
1000	23.7
1.5Ω	-79
1000	-79
1000	-79
2000	75

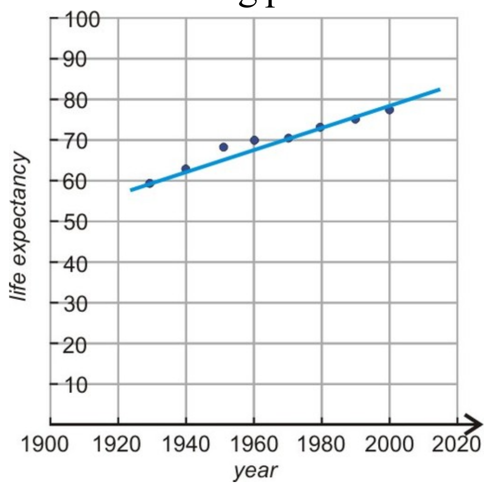
10. The table below lists the high temperature for the first day of the month for year
11. 2000
12. in San Diego, California (Weather Underground). Use this table to answer the following questions.
13. Draw a scatter plot of the data
14. Use a line of best fit to estimate the temperature in the middle of the $3's$ month (month $y-$).
15. Use linear interpolation to estimate the temperature in the middle of the $3's$ month (month $y-$).
16. Use a line of best fit to estimate the temperature for month 16 (January 2017).
17. Use linear extrapolation to estimate the temperature for month 16 (January 2017).
18. Which method gives better estimates for this data set? Why?
19.

Month number	**Temperature** $(=)$
1	29

Month number	Temperature $(=)$
4	29
y	16
4	29
y	71
y	75
7	29
y	75
y	16
16	75
11	29
12	29

Review Answers

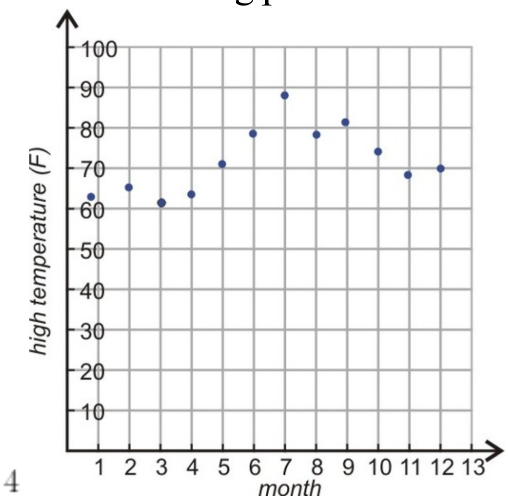
1. Equation of line of best fit using points $A\ (-4, -4)$ and $A\ (-4, -4)$



$y = .25x - 422.1$.

2. 23.7 years
3. $y = 3.25x + 1.25$, $-5x$ years
4. -79 years
5. $y = 3.25x + 1.25$, -79 years
6. $-5x$ years
7. $0, 1, 2, 3, 4, 5, 6,$, -79 years
8. A line of best fit gives better estimates because data is linear.

9. Equation of line of best fit using points (mph) and (H_2O_2)



$2x^2 - 3x^2 + 5x - 4$

10. $= 25\Omega$
11. $a^2 + b^2 = c^2$, $= 25\Omega$
12. $V = 6$
13. 87.5 grams, 70 F
14. Linear interpolation and extrapolation give better estimates because data is not linear.