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Homework 2

4.5-1

a. $T(n) = 2T\left(\frac{n}{4}\right) + 1$

$a = 2,\ b = 4,\ f(n) = 1$

$1 = \ O\left(n^{\log_4 2-\epsilon}\right) = O(n^{1/2-\epsilon})$? Yes, for some $\epsilon > 0$

Case 1 apply. Therefore

$$T(n) = \Theta\left(n^{\frac{1}{2}}\right) = \Theta\left(\sqrt{n}\right)$$

b. $T(n) = 2T\left(\frac{n}{4}\right) + \sqrt{n}$

$a = 2,\ b = 4,\ f(n) = \sqrt{n}$

$\sqrt{n} = \ O\left(n^{\log_4 2-\epsilon}\right) = O(n^{1/2-\epsilon})$? No

$\sqrt{n} = \ \Theta\left(n^{\log_4 2}\right) = \Theta\left(n^{\frac{1}{2}}\right)$? Yes

Case 2 apply. Therefore

$$T(n) = \Theta\left(n^{\frac{1}{2}}\lg(n)\right) = \Theta\left(\sqrt{n}\lg(n)\right)$$

c. $T(n) = 2T\left(\frac{n}{4}\right) + n$

$a = 2,\ b = 4,\ f(n) = n$

$n = \ O\left(n^{\log_4 2 - \epsilon}\right) = O(n^{1/2-\epsilon})$? No

$n = \ \Theta\left(n^{\log_4 2}\right) = \Theta\left(n^{\frac{1}{2}}\right)$? No

$n = \Omega\left(n^{\log_4 2 + \epsilon}\right) = \Omega\left(n^{1/2+\epsilon}\right)$? Yes, for some $\epsilon > 0$

For some $c < 1$ & $LARGE\ n,$ $2\left(\frac{n}{4}\right) \leq cn \rightarrow \frac{n}{2} \leq cn$? Yes, for $c \leq \frac{1}{2} < 1$

Case 3 apply. Therefore

$$T(n) = \Theta(f(n)) = \Theta(n)$$

d. $T(n) = 2T\left(\frac{n}{4}\right) + n^2$

$a = 2,\ b = 4,\ f(n) = n^2$

$n^2 = \ O\left(n^{\log_4 2 - \epsilon}\right) = O(n^{1/2-\epsilon})$? No

$n^2 = \ \Theta\left(n^{\log_4 2}\right) = \Theta\left(n^{\frac{1}{2}}\right)$? No

$n^2 = \Omega\left(n^{\log_4 2 + \epsilon}\right) = \Omega\left(n^{1/2+\epsilon}\right)$? Yes, for some $\epsilon > 0$

For some $c < 1$ & $LARGE\ n,$ $2\left(\frac{n^2}{16}\right) \leq cn^2 \rightarrow \frac{n^2}{8} \leq cn^2$?Yes, for $c \leq \frac{1}{8} < 1$

Case 3 apply. Therefore

$$T(n) = \Theta(f(n)) = \Theta(n^2)$$

4-1

a. $T(n) = 2T\left(\frac{n}{2}\right) + n^4$

$a = 2,\ b = 2,\ f(n) = n^4$

$n^4 = O\left(n^{\log_2 2-\epsilon}\right) = O(n^{1-\epsilon})$? No

$n^4 = \Theta\left(n^{\log_2 2}\right) = \Theta(n)$? No

$n^4 = \Omega\left(n^{\log_2 2+\epsilon}\right) = \Omega(n^{1+\epsilon})$? Yes, for some $\epsilon > 0$

For some $c < 1$ & $LARGE\ n,$ $2\left(\frac{n^4}{16}\right) \leq cn^4 \rightarrow \frac{n^4}{8} \leq cn^4$?Yes, for $c \leq \frac{1}{8} < 1$

Case 3 apply. Therefore

$$T(n) = \Theta(f(n)) = \Theta(n^4)$$

b. $T(n) = T\left(\frac{7n}{10}\right) + n$

$a = 1,\ b = \frac{10}{7},\ f(n) = n$

$n = O\left(n^{\log_{\frac{10}{7}} 1-\epsilon}\right) = O(n^{0-\epsilon})$? No

$n = \Theta\left(n^{\log_{\frac{10}{7}} 1}\right) = \Theta(1)$? No

$n = \Omega\left(n^{\log_{\frac{10}{7}} 1+\epsilon}\right) = \Omega(n^{0+\epsilon})$? Yes, for some $\epsilon > 0$

For some $c < 1$ & $LARGE\ n,$ $\frac{7n}{10} \leq cn$?Yes, for $c \leq \frac{7}{10} < 1$

Case 3 apply. Therefore

$$T(n) = \Theta(f(n)) = \Theta(n)$$

c. $T(n) = 16T\left(\frac{n}{4}\right) + n^2$

$a = 16,\ b = 4,\ f(n) = n^2$

$n^2 = \ O\left(n^{\log_4 16-\epsilon}\right) = O(n^{2-\epsilon})$? No

$n^2 = \ \Theta\left(n^{\log_4 16}\right) = \Theta(n^2)$? Yes

Case 2 apply. Therefore

$$T(n) = \Theta\left(n^{\log_4 16}\lg(n)\right) = \Theta(n^2\lg(n))$$

d. $T(n) = 7T\left(\frac{n}{3}\right) + n^2$

$a = 7,\ b = 3,\ f(n) = n^2$

$n^2 = \ O\left(n^{\log_3 7-\epsilon}\right) = O(n^{1.77-\epsilon})$? No

$n^2 = \ \Theta\left(n^{\log_3 7}\right) = \Theta(n^{1.77})$? No

$n^2 = \Omega\left(n^{\log_3 7+\epsilon}\right) = \Omega(n^{1.77+\epsilon})$? Yes, for some $\epsilon > 0$

For some $c < 1$ & $LARGE\ n$, $\frac{7n^2}{9} \leq cn^2$?Yes, for $c \leq \frac{7}{9} < 1$

Case 3 apply. Therefore

$$T(n) = \Theta(f(n)) = \Theta(n^2)$$

e. $T(n) = 7T\left(\frac{n}{2}\right) + n^2$

$a = 7,\ b = 2,\ f(n) = n^2$

$n^2 = \ O\left(n^{\log_2 7-\epsilon}\right) = O(n^{2.81-\epsilon})$? Yes, for some $\epsilon > 0$

Case 1 apply. Therefore

$$T(n) = \Theta\left(n^{\log_2 7}\right) = \Theta(n^{2.81})$$

f. $T(n) = 2T\left(\frac{n}{4}\right) + \sqrt{n}$

$a = 2,\ b = 4,\ f(n) = \sqrt{n}$

$\sqrt{n} = \ O\left(n^{\log_4 2-\epsilon}\right) = O(n^{1/2-\epsilon})$? No

$\sqrt{n} = \ \Theta\left(n^{\log_4 2}\right) = \Theta\left(n^{\frac{1}{2}}\right)$? Yes

Case 2 apply. Therefore

$$T(n) = \Theta\left(n^{\frac{1}{2}}\lg(n)\right) = \Theta\left(\sqrt{n}\lg(n)\right)$$

g. $T(n) = T(n-2) + n^2$

$= T(n-4) + (n-2)^2 + n^2$

$= T(n-2k) + \sum_{i=0}^{k-1}(n-2i)^2$ for $n - 2k \geq 0$

$= T(0) + \sum_{i=0}^{\frac{n}{2}-1}(n-2i)^2$

$= \sum_{i=0}^{\frac{n}{2}-1}(n^2 - 4ni + 4i^2) + C$

$= \left(\frac{n}{2}-1\right).n^2 - \frac{1}{2}.\frac{n}{2}.\left(\frac{n}{2}-1\right).4n + \frac{1}{6}.\left(\frac{n}{2}-1\right).\frac{n}{2}.(n-1).4 + C$

$T(n) = n^3 + \cdots + C$

Therefore

$$T(n) = \Theta(n^3)$$