Two Wheeled Balancing Robot

Suhan N Shetty
SR No 11256
ME Mecahnical Engg
IISc,Bangalore

Project Guide :
Prof.A Ghosal
Department of Mechanical Engg
IISc,Bangalore

1 MOTIVATION

A two wheeled balancing robot is a self balancing robot controlled by an embedded processor which executes a control algorithm. It uses feedback from sensors to maintain vertical balance.

2 TOOLS USED

2.1 Hardwares

2.1.1 NI myRIO

Processor and FPGA type : Xilinx Z-7010

Processor Speed: 667 MHz

Processor Cores: 2

FPGA clock rate : 40 MHz

2.1.2 Accelerometer

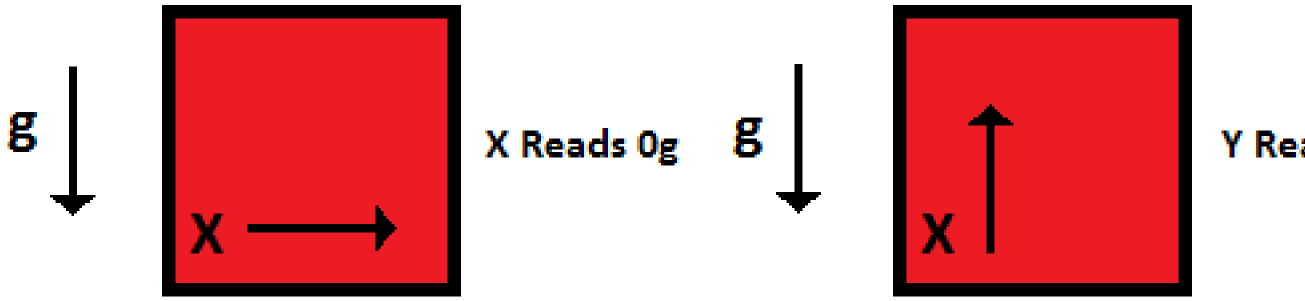
An accelerometer is an electronic device that will measure acceleration forces in g (gravity). These forces may be static or they could be dynamic caused by moving or vibrating the accelerometer. The following figure shows that when holding an accelerometer stationary and flat, it will read $0g$, when holding it at 90 degrees it will read $-1g$ and $1g$ at 90 degrees. Using trigonometry, the angle between g and x can be calculated in degrees . The NI myRIO-1900 contains an onboard three-axis accelerometer. The accelerometer samples each axis continuous and updates a readable register with the result.

Number of axes : 3

Range : $\mp$ 8g

Resolution : 8 bits

Sample Rate : 800 S/s

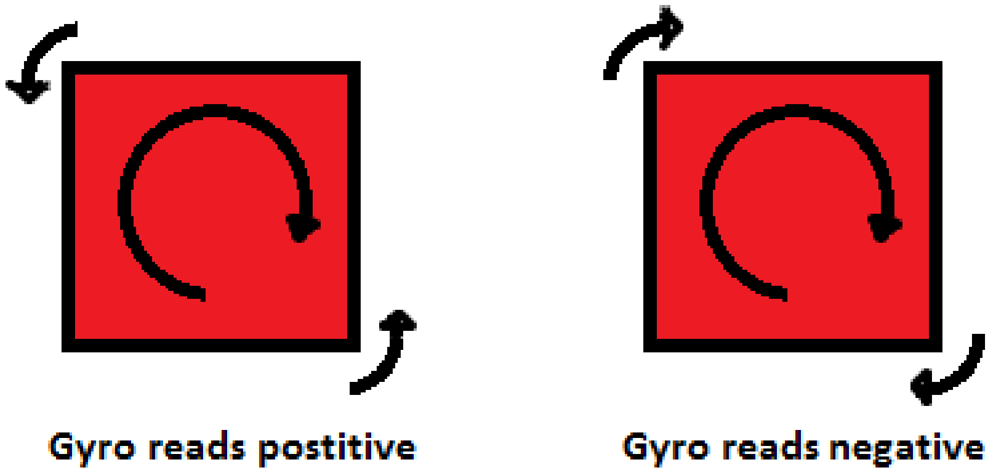


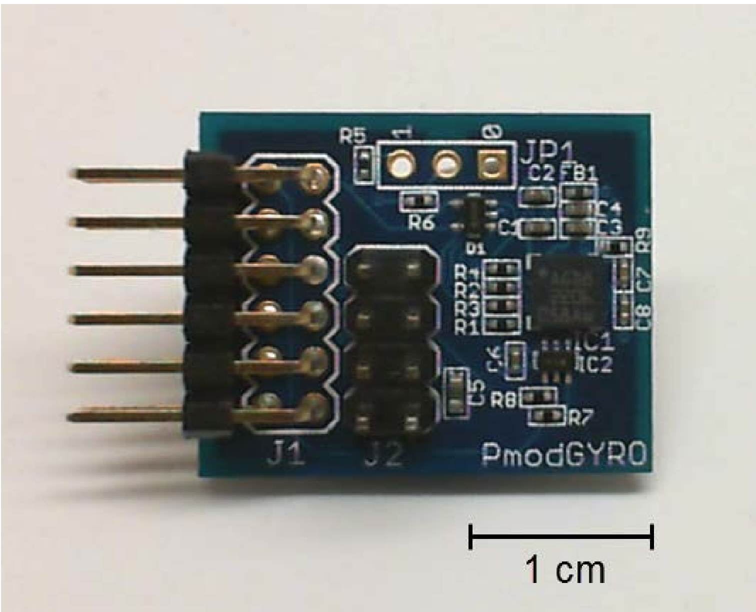
2.1.3 Gyroscope

The gyroscope that is used here is an electronic device that will measure angular velocity. It will read 0 when stationary and reads positive or negative when rotating as shown in the following figure. A common output is degrees per second. The two wheeled balancing robot uses a PmodGYRO sensor. PmodGYRO is a peripheral module featuring the STMicroelectronics L3G4200D MEMS motion sensor. The L3G4200D provides a three-axis digital output gyroscope with built in temperature sensor.

Number of axes : 3

Resolution: 250/500/2000 dps





2.1.4 Motor

Gear ratio : 1:19

Rated voltage: 12 V

Rated torque: 295 g-cm

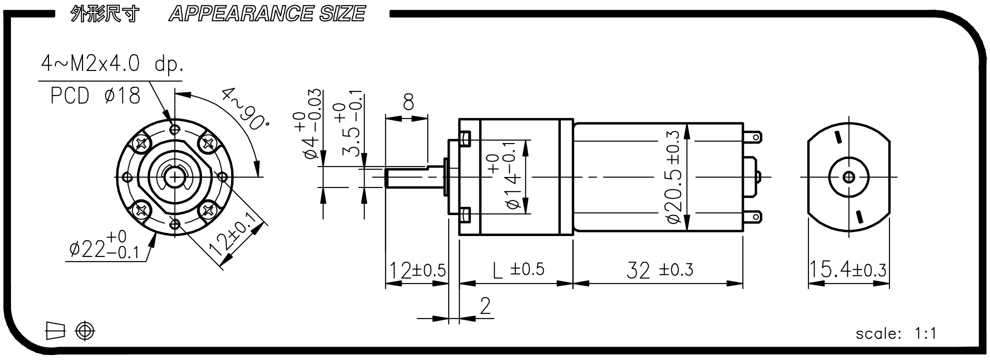
Rated speed: 405 rpm

IG-22GM

01&02 TYPE

REDUCTION RATIO	L	REDUCTION RATIO	L
1/4	14.40	1/198~1/455	25.35
1/14~1/19	18.05	1/742~1/1996	29.00
1/53~1/104	21.70		

外形尺寸 APPEARANCE SIZE



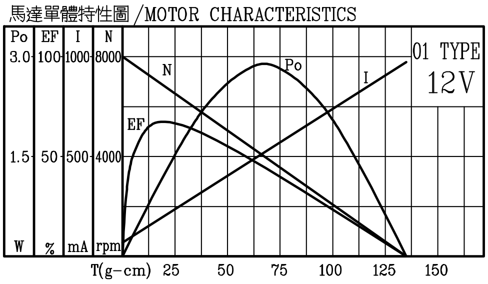
GEARED MOTOR TORQUE/SPEED

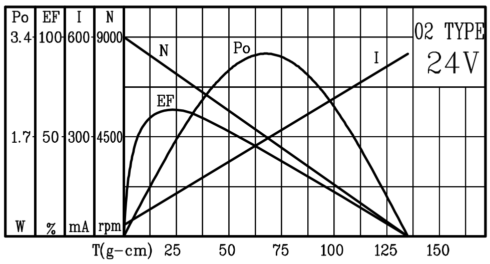
	減速比 Reduction ratio	1/4	1/14	1/16	1/19	1/53	1/62	1/72	1/84	1/104	1/198	1/231	1/270	1/316	1/370	1/455	1/742	1/1014	1/1249	1/1621	1/1996
12V	定格扭力(g-cm) Rated torque	77	215	250	295	695	810	950	1100	1370	2100	2500	2500	2500	2500	2500	3000	3000	3000	3000	3000
	定格回轉數(rpm) Rated speed	1497	470	407	348	127	109	93	79	64	34	29	25	22	19	15.5	9.5	7.4	6.0	4.6	3.8
24V	定格扭力(g-cm) Rated torque	77	215	250	295	695	810	950	1100	1370	2100	2500	2500	2500	2500	2500	3000	3000	3000	3000	3000
	定格回轉數(rpm) Rated speed	1650	518	450	384	140	120	103	88	71	37	32	28	23.5	21	17.5	10.5	8.0	6.6	5.0	4.2

馬達單體型式 /MOTOR DATA

定格電壓 Rated volt (V)	定格扭力 Rated torque (g-cm)	定格回轉數 Rated speed (rpm)	定格電流 Rated current (mA)	無負荷回轉數 No load speed (rpm)	無負荷電流 No load current (mA)	定格出力 Rated output (W)	重量 Weight (g)
12	22	6700	≤ 200	8000	≤ 70	1.5	32.0
24	22	7400	≤ 110	9000	≤ 40	1.7	32.0

馬達單體特性圖 /MOTOR CHARACTERISTICS





April 2010

2.1.5 H-Bridge

An H-bridge motor drive uses four power MOSFETs to direct current through a DC motor in one direction or the other, thereby allowing the motor controller to reverse the motor direction as needed. It provides a 2A H-bridge circuit for voltages up to 12V.The HB5 works with power supply voltages from 2.5V to 5V.

2.2 SOFTWARE USED

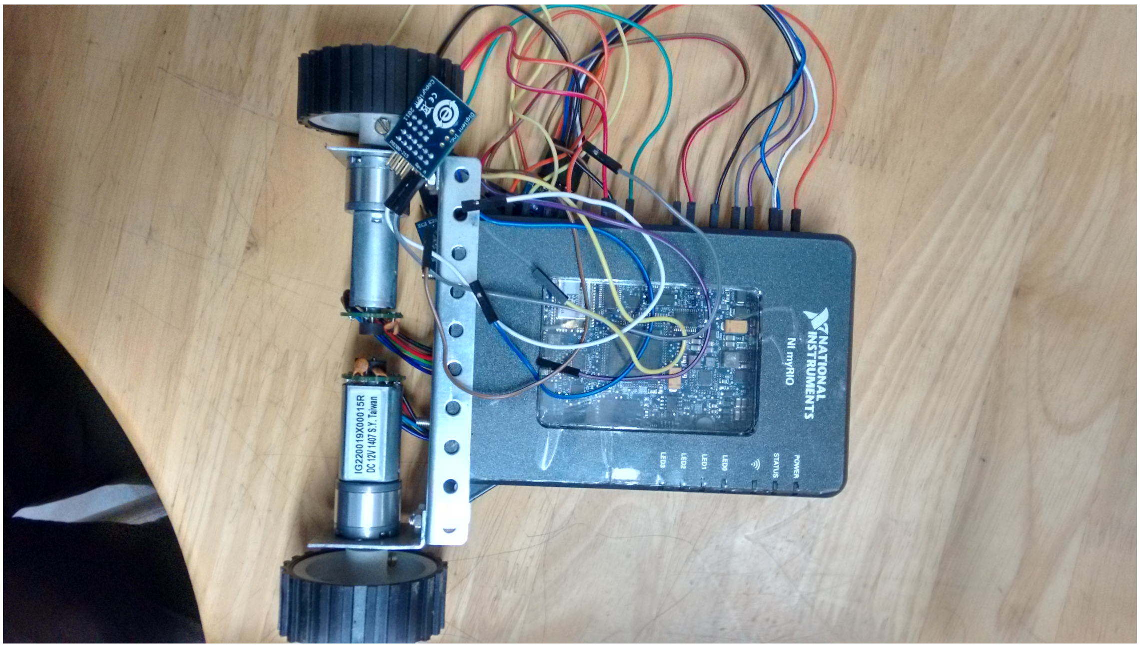
subsubsectionLabVIEW Labview is a dataflow visual programming language commonly used for data acquisition, instrumentation control, and industrial automation. LabVIEW includes extensive support for interfacing to devices, instruments, cameras, and other devices. LabVIEW includes built-in support for NI hardware platforms such as myRIO and CompactRIO, with a large number of device-specific blocks for such hardware, the Measurement and Automation Exploration (MAX) and Virtual Instrument Software Architecture (VISA) toolsets.

3 PROCEDURE

1. Build the robot
2. Sensor caliberation and PWM
3. Model the system
4. Controller design
5. Simulation
6. Controller implementation

4 THE ROBOT ARCHITECTURE

The robot is built upon the mounting frame of the motors and wheels. A picture of the robot is shown below.



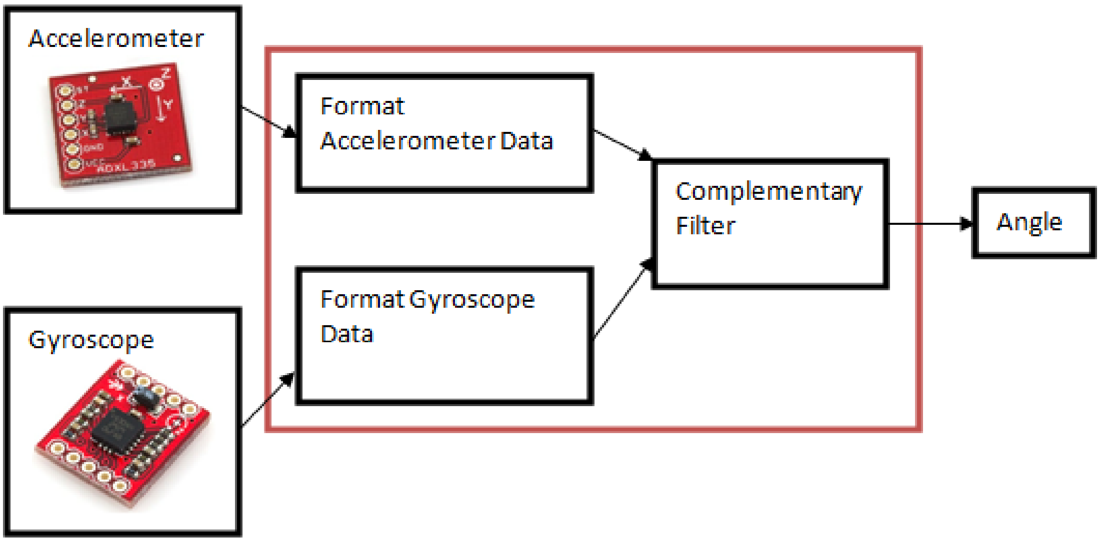
5 SENSOR CALIBERATION AND PWM

5.1 SENSOR DATA ACQUISITION

The gyro is carefully placed at appropriate position such that the Z-axis points along the direction of motion of the robot.The setup is such that the Z axes of onboard accelerometer also points in the same direction as that of gyro.The x,y, and z direction acceleration of the accelerometer is read. The Y angular velocity of the gyro gives the rate of tilt.

5.2 TILT ANGLE MEASUREMENT

When looking at the characteristics of both gyroscopes and accelerometers they have their own strengths and weakness and in order to gain amore accurate representation of orientation, these individual sensors can be combined through a sensor fusion algorithm. Accelerometers and gyroscopes can be used together for a more accurate reading of orientation.A calculated tilt angle from an accelerometer has a slow response time, while the integrated angle from the gyroscope is subjected to drift over a period of time. The accelerometer data is useful for long term while the gyroscope data is useful in the short term.
A complementary filter can be used for sensor fusion and it is fairly simple to implement compared to alternatives like a Kalman filter.The complementary filter is designed in such a way that the strength of one sensor will be used to overcome the weaknesses of the other sensor which is complementary to each other. It will make use of the gyro in short period, and then the low pass filtered data from the accelerometer is used to correct the drift of the angle over long period of time. The offset of the gyro sensor will also be continuously updated and corrected.This will result in a drift free and fast responding estimated tilt angle. The block diagram below shows how the gyro and accelerometer are fused together.



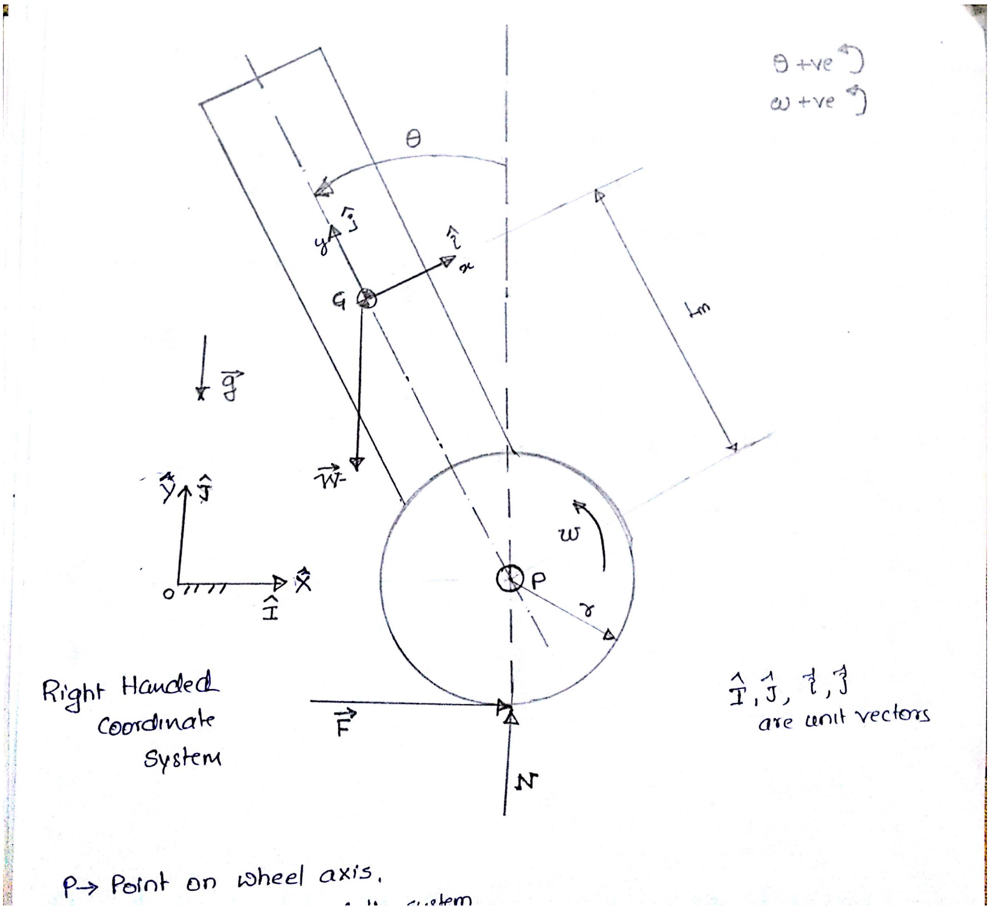
$$EstimatedAngle = a*(PreviousAngle+(Gyro*dt))+(1-a)*(Accelerometer)$$

A larger filter coefficient a trusts the integrated gyro signal for longer. A smaller filter coefficient merges the accelerometer signal faster.

5.3 PWM AND SPEED CONTROL OF MOTOR

In an analog circuit, motor speed is controlled by varying the input voltage to a circuit. In a digital circuit, however, only a logic high or logic low signal can be applied to the motor. Therefore, there are only two ways to control a motor digitally: use a variable resistance circuit to control the motor voltage, or, pulse the power to the motor. Since variable resistance circuitry is expensive, complicated, and wastes much energy in the form of heat, the better solution is pulse width modulation (PWM). Pulse width modulation is a digital method of transmitting an analog signal.The H-bridge is used as a voltage amplification and direction control circuit that is used to format the signal to the appropriate motor voltage and polarity to spin the motor. While voltage is being applied, the motor is driven by the changing magnetic forces. When voltage is stopped, momentum causes the motor to continue spinning a while. At a high enough frequency, this process of powering and coasting enables the motor to achieve a smooth rotation that can easily be controlled through digital logic.

6 SYSTEM MODELLING



6.1 Notation

Two right handed coordinate systems are introduced, a fixed frame with unit vectors $(\hat{I}, \hat{J}, \hat{K})$ and a body frame with unit vectors $(\hat{i}, \hat{j}, \hat{k})$.

L : Point on wheel axis

G : Center of mass of the system

L: Perpendicular Distance of CG from the axes

w: Angular velocity of wheel

θ : Angle of the robot w.r.t the vertical

r: Radius of the wheel

N: Normal force on the wheel

F: Traction on the wheels

J_p: Mass moment of inertia of the robot about wheel axis

M: Total mass of the system

6.2 System Parameters

For the robot the constant values are as follows;

$r = 0.027m$

$L = 0.05m$

$M = 0.53kg$

$J_p = 2.8e - 03kg$

Motor Specification:

$Armature Resistance, R_m = 13.72\Omega$

$Torgue Constant, K_t = 0.0123NmA^{-1}$

$Voltage Constant, K_v = 0.0123Vsrad^{-1}$

6.3 Calculations

From (1) we have the equation for net moment of force about an accelerating point P in vector form is given by:

$$\vec{M}_p = \frac{d\vec{H}_{p_{rel}}}{dt} + \vec{\rho} \times M\vec{a}_{P/O} \tag{1}$$

where

$\vec{M}_p$: Net moment of force about P

$\vec{H}_{Prel}$: Relative angular momentum of the system with respect to P

$\vec{P}$: Position vector from P to the CG

$\vec{a}_{P/O}$: Absolute acceleration of P

$\vec{a}_{G/O}$: Absolute acceleration of the CG

$\vec{a}_{G/P}$: Relative acceleration of the CG with respect to P

$$\vec{F} = M\vec{a}_{G/O} \tag{2}$$

We have

$$\begin{aligned}\vec{a}_{G/O} &= \vec{a}_{G/P} + \vec{a}_{P/O} \\ \vec{a}_{P/O} &= -r\frac{d\vec{w}}{dt} \\ \vec{a}_{G/P} &= -L\dot{\theta}\hat{i} - L\ddot{\theta}\hat{j}\end{aligned}$$

and

$$\begin{aligned}\hat{i} &= \sin\theta\hat{I} + \cos\theta\hat{J} \\ \hat{j} &= -\sin\theta\hat{I} + \cos\theta\hat{J}\end{aligned}$$

Combining the above four equations, we get

$\vec{a}_{G/O} = (-r\dot{w} + L\dot{\theta}^2\sin\theta - L\ddot{\theta}\cos\theta)\hat{I} + (-L\ddot{\theta}\sin\theta - L\dot{\theta}^2\cos\theta)\hat{J}$

Making small angle approximations we get

$$\sin\theta \approx \theta$$

$$\cos\theta \approx 1$$

Substituting this in the above equation for $\vec{a}_{G/O}$, we get

$\vec{a}_{G/O} = (-r\dot{w} + L\dot{\theta}^2\theta - L\ddot{\theta})\hat{I} + (-Lth\ddot{e}ta\theta - L\dot{\theta}^2)\hat{J}$

Also, $\theta\dot{\theta} \approx 0$

Hence,

$$\vec{a}_{G/O} = (-r\dot{w} - L\ddot{\theta})\hat{I} + (-L\ddot{\theta}\theta - L\dot{\theta}^2)\hat{J} \tag{3}$$

Therefore,

$$\vec{F} = M[(-r\dot{w} - L\ddot{\theta})\hat{I} + (-L\ddot{\theta}\theta - L\dot{\theta}^2)\hat{J}] \tag{4}$$

The Moment of force about P is given by the equation

$$\vec{M}_P = \vec{r} \times \vec{F} + \vec{L} \times \vec{W} \tag{5}$$

Where,

$\vec{r}$: Vector from P to point of contact of the wheel ($\vec{r} = -r\hat{J}$)

$\vec{L}$: Vector from P to CG

$\vec{W}$: Weight as vector ($\vec{W} = -Mg\hat{J}$)

and $\vec{F} = M\vec{a}_{G/O}$

$\vec{M}_P$ can also be given by,

$$\vec{M}_P = \dot{\vec{H}}_{Prel} + \vec{L} \times M\vec{a}_{P/O} \tag{6}$$

Combining (5) and (6) we get

$$\vec{r} \times \vec{F} + \vec{L} \times \vec{W} = \dot{\vec{H}}_{Prel} + \vec{L} \times M\vec{a}_{P/O} \tag{7}$$

Where,

$$\begin{aligned} (\vec{r} \times \vec{F}) &= (-r\hat{J} \times M\vec{a}_{G/O)}) \\ &= Mr(-r\dot{w} - L\ddot{\theta})\hat{K} \end{aligned}$$

$$\begin{aligned} (\vec{L} \times \vec{W}) &= (L\hat{j} \times (-Mg)\hat{J}) \\ &= MgL\theta\hat{K} \end{aligned}$$

$$\begin{aligned} (\vec{L} \times M\vec{a}_{P/O}) &= (L\hat{j} \times M(-r\dot{w})\hat{I} \\ &= MrL\dot{w}\hat{K} \end{aligned}$$

$$\dot{\vec{H}}_{p_{rel}} = J_P \ddot{\theta} \hat{K}$$

Substituting the above terms in (7) we get

$$-(Mr^2 + MrL)\dot{w} = [(J_p + MrL)\ddot{\theta} - MgL\theta]$$

Taking Laplace transform of the above equatuation we get

$$\frac{\theta(s)}{w(s)} = -\frac{(Mr^2 + MrL)s}{(J_P + MrL)s^2 - MgL} \tag{8}$$

Let

$$A = \frac{Mr^2 + MrL}{J_P + MrL} \qquad B = \frac{MgL}{J_P + MrL} \tag{9}$$

Then

$$\frac{\theta(s)}{w(s)} = -\frac{As}{s^2 - B} \tag{10}$$

Electrical Circuit Modelling of Motor

We have two motors which runs the two wheels.We model the motor circuit as an RLC circuit. Each wheel is modelled as follows :
Motor Circuit Parameters:

L_i : Motor inductance

R_m : Armature resistance

V : Applied voltage

Applying KVL,

$$V = R_m i + L_m \frac{di}{dt} + K_v w_m \tag{11}$$

where the term $(K_v w_m)$ accounts for the back emf generated, and

w_m = rotor speed in rad/s

Let n be the gear ratio of the motor, then

$w_m = nw$

For our case we will assume, $\quad L_m \frac{di}{dt} \approx 0$
Then

$$V = R_m i + K_v w_m \tag{12}$$

We have the relation between rotor torque (T_{motor}) and armature current

$$T_{motor} = K_v i \tag{13}$$

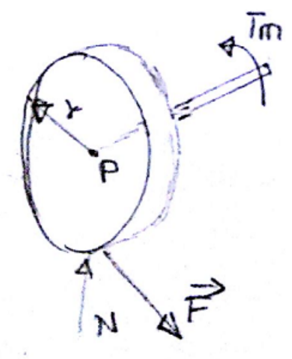
We can determine motor torque as follows
Net moment of force about the wheel axis

$$n\vec{T_{motor}} + \vec{r} \times \frac{\vec{F}}{2} = J_e q\dot{w}\hat{K}$$

$$i.e.\ n\vec{T_{motor}} = J_{eq}\dot{w}\hat{K} - \vec{r} \times \frac{\vec{F}}{2}$$

$$i.e.\ n\vec{T_{motor}} = [J_{eq}\dot{w} + \frac{Mr^2\dot{w} + MrL\ddot{\theta}}{2}]\hat{K}$$

where J_{eq} is mass moment inertia of the wheel and rotor about the axis and



$T_m = nT_{motor}$

$$nT_{motor} = J_{eq}\dot{w} + \frac{M}{2}(r^2\dot{w} + rL\ddot{\theta}) \tag{14}$$

Substituting (14) in (13) we get

$$i = \frac{T_{motor}}{K_v} = \frac{J_{eq}\dot{w} + \frac{M}{2}(r^2\dot{w} + rL\ddot{\theta})}{nK_v}$$

Substituting this and ($w_m = nw$) in (12)

$$V = R_m(\frac{T_{motor}}{K_v}) + K_v(nw)$$

$$nV = \frac{R_m}{K_v}(J_{eq} + \frac{Mr^2}{2})\dot{w} + n^2K_v w + \frac{MR_mLr\ddot{\theta}}{2K_v}$$

Let

$$C = \frac{R_m}{K_v}(J_{eq} + \frac{Mr^2}{2}) \qquad D = n^2K_v \qquad E = \frac{MR_mLr}{2K_v} \tag{15}$$

i.e.

$$nV = C\dot{w} + Dw + E\ddot{\theta}$$

Applying Laplace transform to this equation we get

$$V(s) = sCw(s) + Dw(s) + s^2E\theta(s)$$

Rearranging

$$n\frac{V(s)}{\theta(s)} = (D + sC)\frac{w(s)}{\theta(s)} + s^2E$$

substituting for $\frac{w(s)}{\theta(s)}$ from (10) in the above equation we get

$$n\frac{V(s)}{\theta(s)} = \frac{(D + sC)(s^2 - B) - s^3AE}{-sA}$$

i.e.

$$\frac{\theta(s)}{V(s)} = -\frac{-nAs}{s^3(C - AE) + s^2D - sBC - BD} \tag{16}$$

This is the transfer function of the system
Substituting the parameters:

$M = 0.53kg$ $\qquad R_m = 13.7$ ohm $\qquad n = 19$
$r = 0.027m$ $\qquad K_v = 0.0123$ Vs rad^{-1}
$J_P = 2.8 \times 10^{-03}$ kg m^2 $\qquad J_{eq} = 4 \times 10^{-05}$ kg m^2

we get
$A = 0.31$
$B = 74$
$C = 0.26$
$D = 4.4$
$E = 0.4$

Substituting this in (16) we get the plant transfer function :

$$L(s) = \frac{\theta(s)}{V(s)} = \frac{-5.96s}{0.13s^3 + 4.4s^2 - 19.2s - 328.4} \tag{17}$$

7 CONTROLLER DESIGN

We have the plant transfer function :

$$L(s) = \frac{\theta(s)}{V(s)} = \frac{-5.96s}{0.13s^3 + 4.4s^2 - 19.2s - 328.4}$$

We have the PID controller designed in frequency domain which satisfactorily balances the robot whose transfer function is given by :

$$C(s) = \frac{V(s)}{\theta_{err}(s)} = \frac{-145s^2 - 5600s - 60000}{s^2 + 100s} \tag{18}$$

where

$$\theta_{err} = \theta_s - \theta$$

and θ_s is the reference angle which is 0

This is a PID Controller with

Proprtional Gain, $P = -50$

Integral Gain, $I = -600$

Differential Gain, $DN = -0.95 \times 100$

The form of the PID is :

$$C(s) = P + \frac{I}{s} + \frac{sD}{\frac{s}{N} + 1}$$

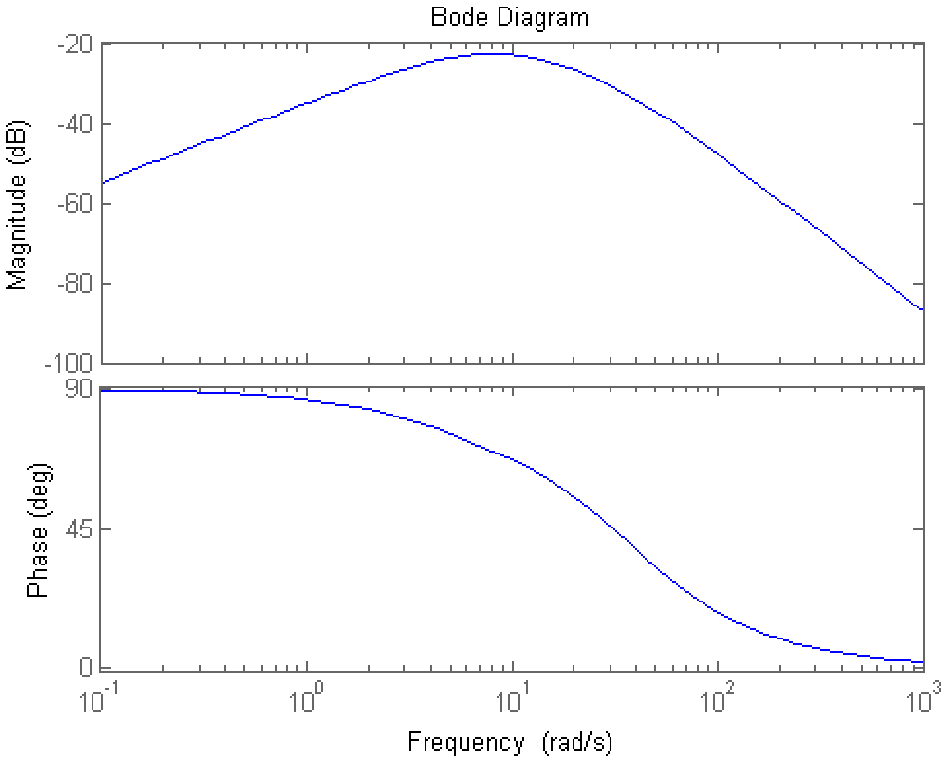
The open loop transfer function is given by

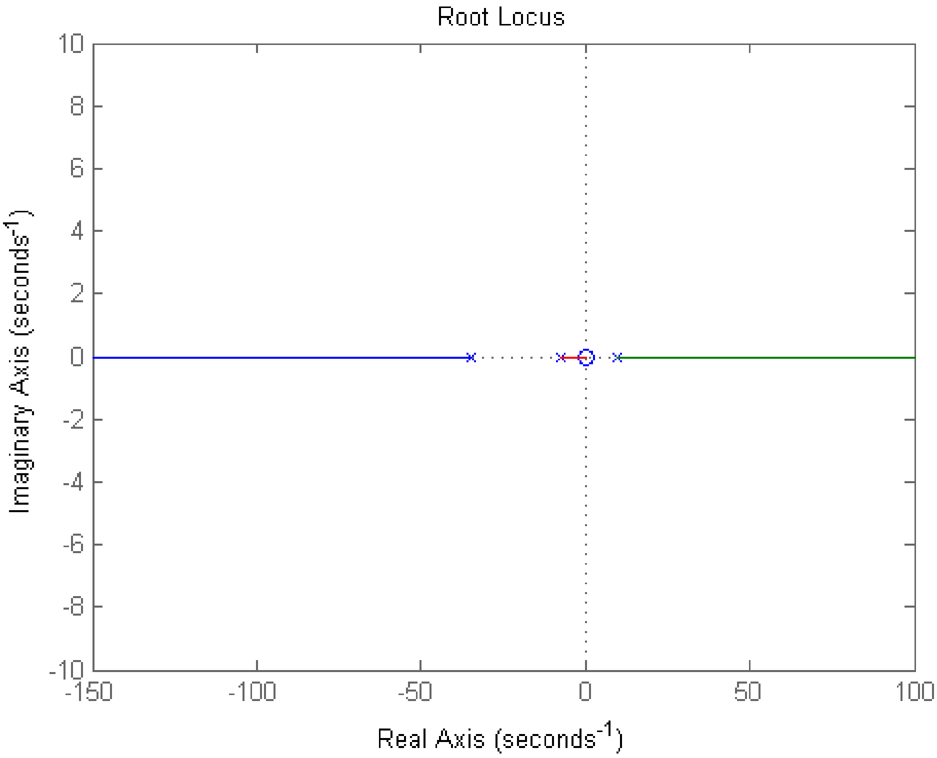
$$T(s) = \frac{\theta(s)}{\theta_{err}(s)} = C(s)L(s)$$

The MATLAB code file is shown below :

```
% clc;clear all;
M=0.53;r=0.027;L=0.05;Jp=2.8e-3;Rm=13.7;Kv=0.0123;Jeq=4e-5;n=19;g=9.81;
A=(M*r^2+M*r*L)/(Jp+M*r*L)
B=M*g*L/(Jp+M*r*L)
C=(Rm/Kv)*(Jeq+0.5*M*r^2)
D=Kv*n^2
E=(Rm*M*r*L)/(2*Kv)
s=tf('s');
L=(-A*n*s)/((C-A*E)*s^3+D*s^2-B*C*s-B*D)
[nl dl]=tfdata(L);
numL=nl{1};
denL=dl{1};
bode(L)
figure(2)
rlocus(L)
figure(3)
```

```
A =

    0.3134


B =

   73.9482


C =

    0.2597


D =

    4.4403


E =

    0.3985


L =

                   -5.955 s
  ---------------------------------------
  0.1348 s^3 + 4.44 s^2 - 19.21 s - 328.4

Continuous-time transfer function.
```


Bode Diagram
Magnitude (dB)
Phase (deg)
Frequency (rad/s)

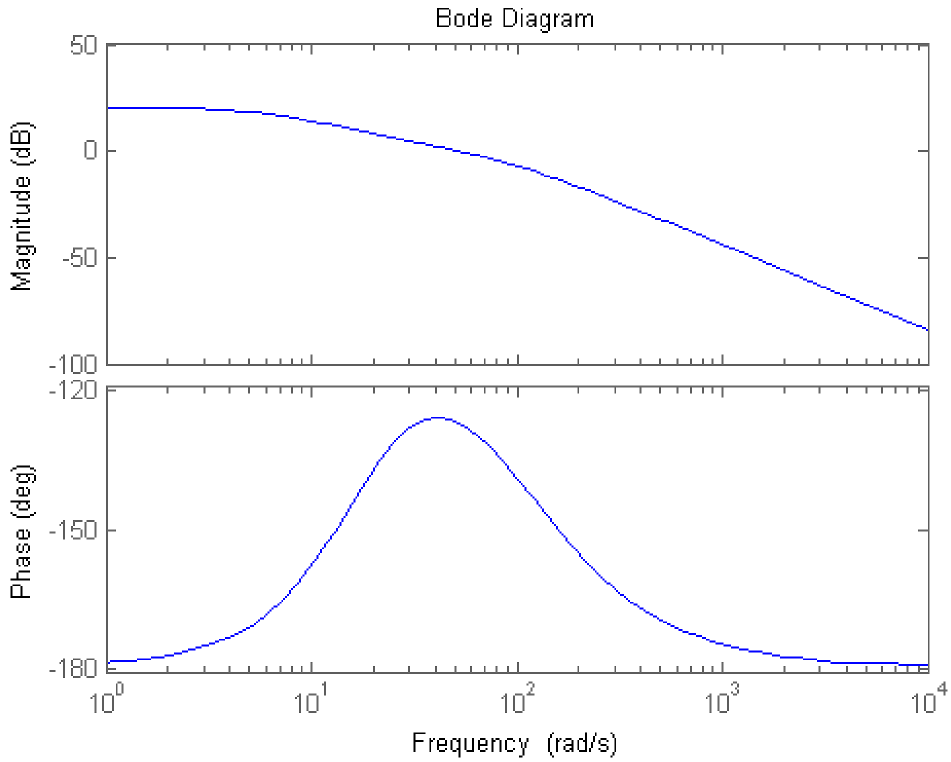

Root Locus
Imaginary Axis (seconds-1)
Real Axis (seconds-1)

Controller Design

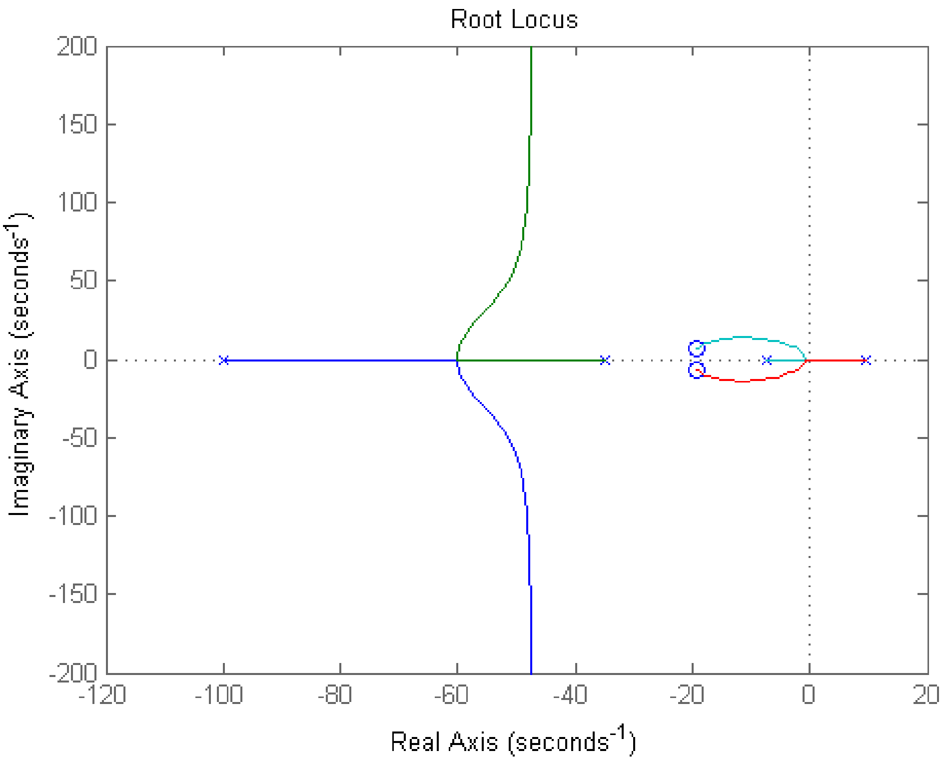
```
P=-50
I=-600
D=-1.9*0.5
N=100
C=I/s+P+D*s/(s/N+1)

L1=L*C;
bode(L1)
```

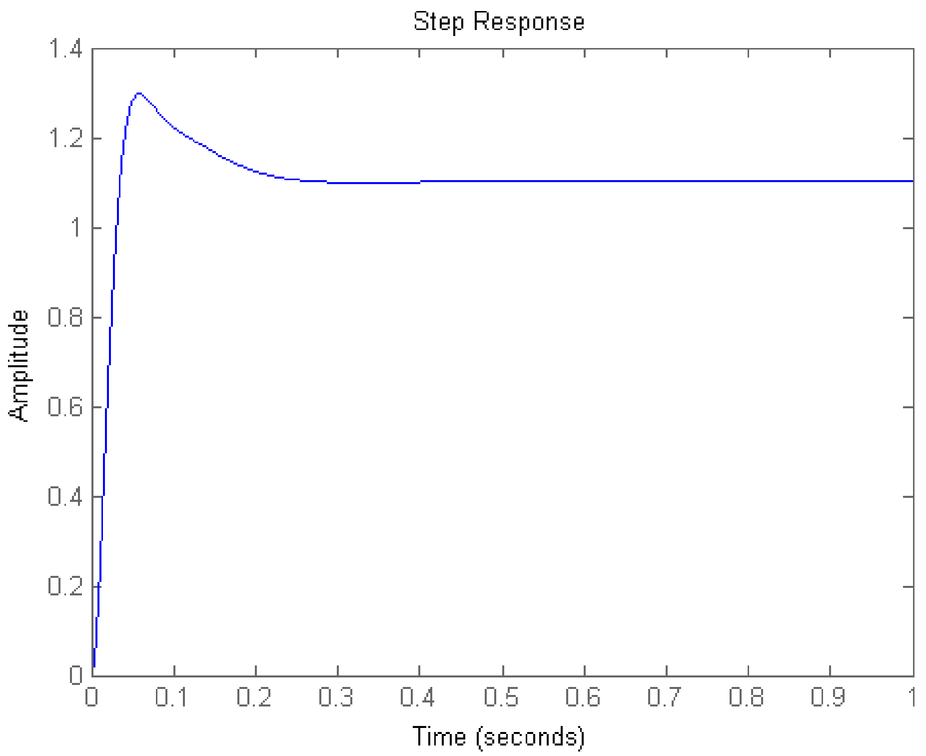
```
P =

   -50


I =

  -600


D =

   -0.9500
```

```
N =

   100


C =

  -145 s^2 - 5600 s - 60000
  -------------------------
         s^2 + 100 s

Continuous-time transfer function.
```




```
[n d]=tfdata(C);
numC=n{1};
denC=d{1};
bode(L1)
figure(3)
rlocus(L1)
figure(4)
M=L1/(1+L1);
step(M,[0:0.0001:1])
```


Root Locus
Imaginary Axis (seconds^-1)
Real Axis (seconds^-1)
200
150
100
50
0
-50
-100
-150
-200
-120
-100
-80
-60
-40
-20
0
20


Step Response
Amplitude
Time (seconds)
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9

Discrete Domain

```
Ts=0.01 %sampling time
Cd=c2d(C,Ts)
[ncd dcd]=tfdata(Cd);
numCd=ncd{1};
denCd=dcd{1};
Ld=c2d(L,Ts)
```

```
Ts =

    0.0100


Cd =

  -145 z^2 + 252.4 z - 111.2
  --------------------------
    z^2 - 1.368 z + 0.3679

Sample time: 0.01 seconds
Discrete-time transfer function.


Ld =

  -0.001987 z^2 + 0.0002065 z + 0.001781
  --------------------------------------
    z^3 - 2.733 z^2 + 2.45 z - 0.7194

Sample time: 0.01 seconds
Discrete-time transfer function.
```

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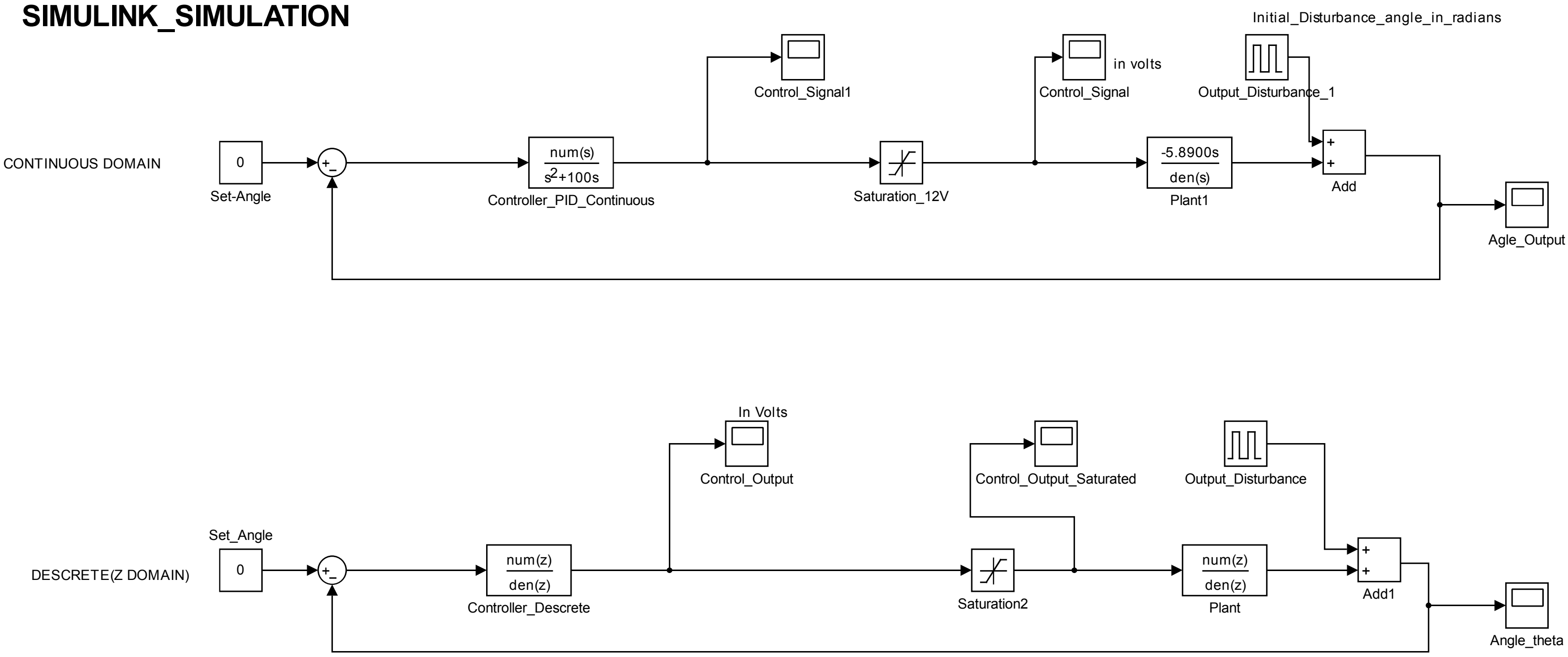
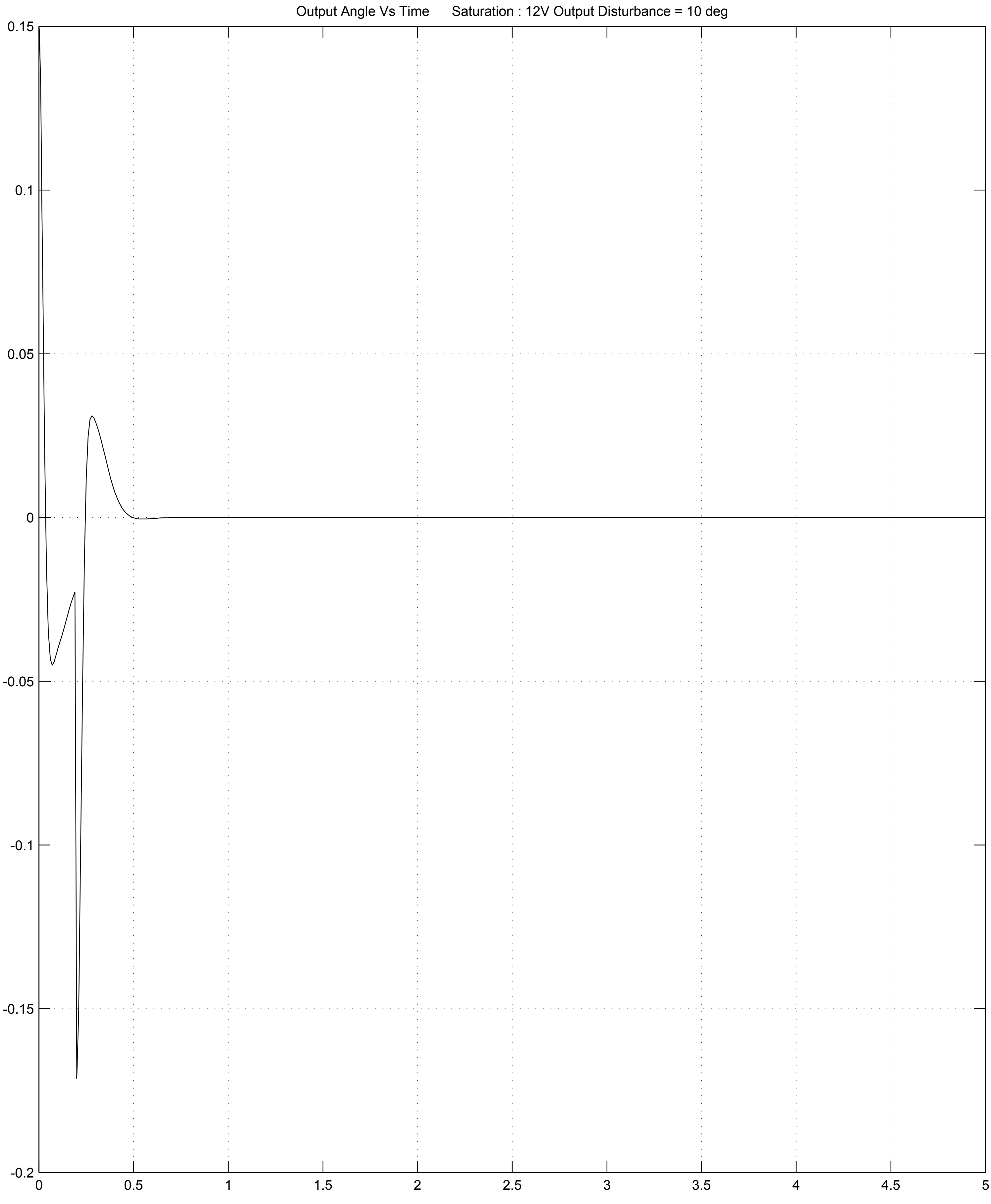
8 SIMULATION

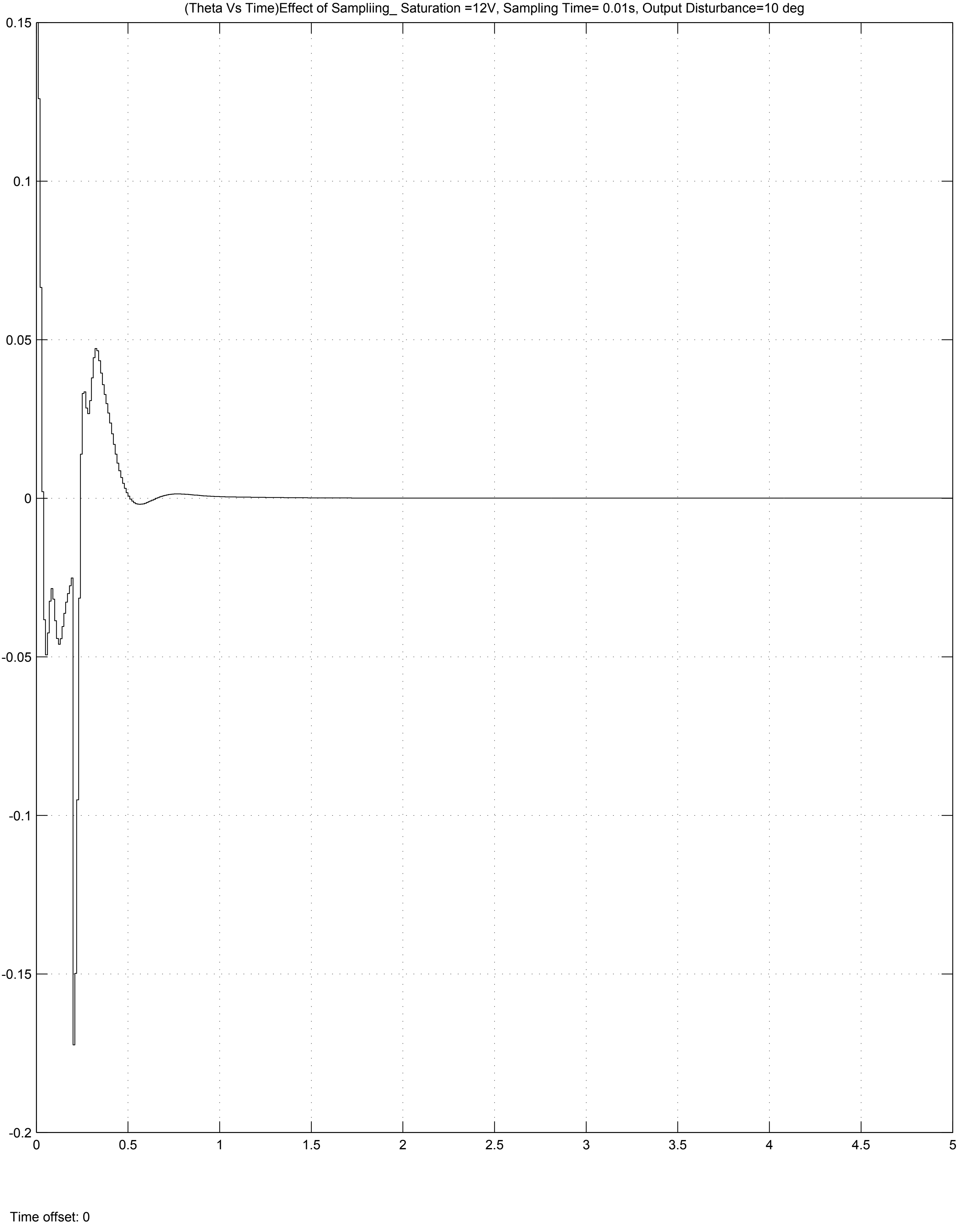
The effect of sampling and output disturbance to the system is studied using simulink. The similink results for a controller output saturation of 12V and an output disturbance of 10 degrees are shown below

SIMULINK_SIMULATION

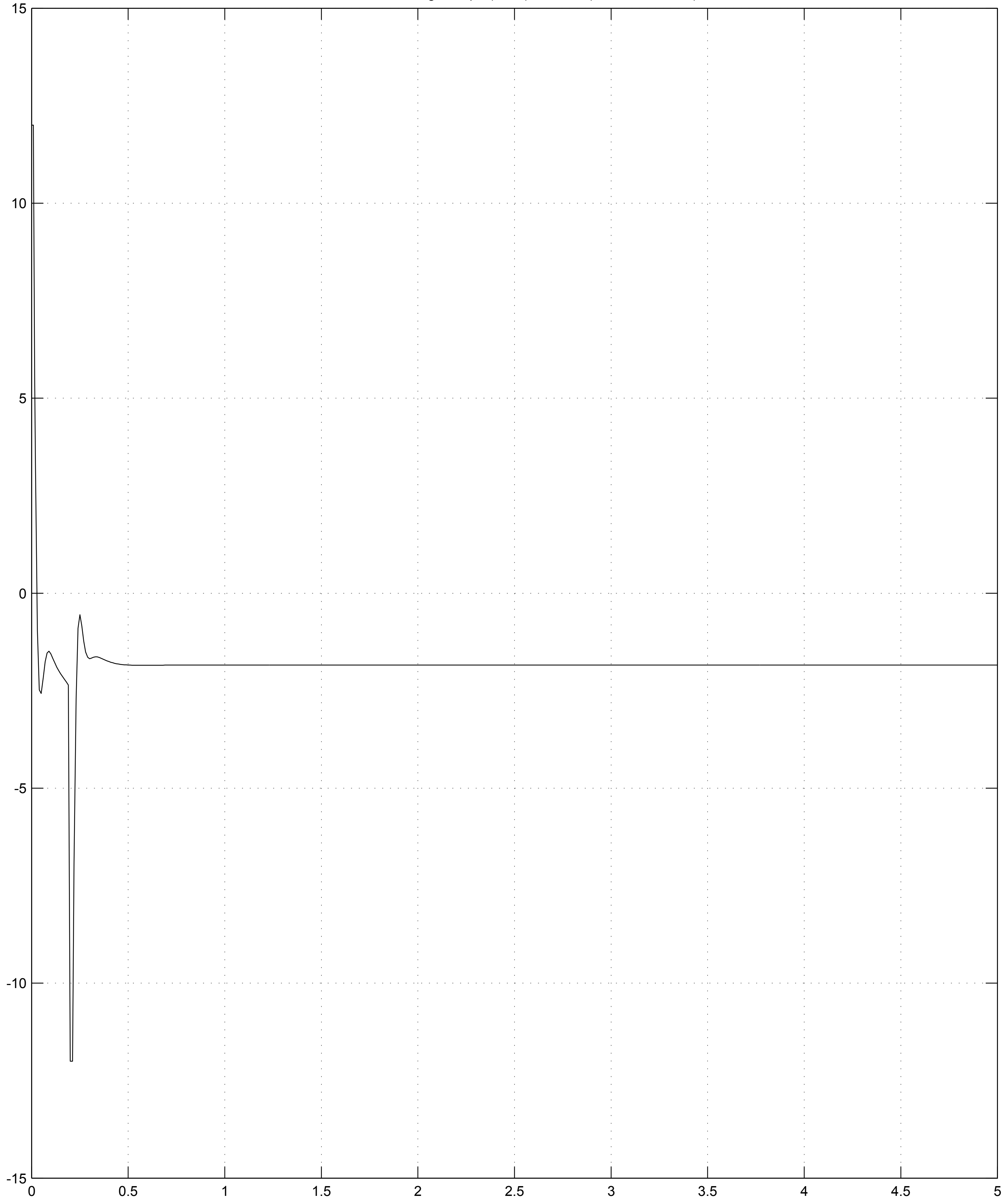




Time offset: 0


(Theta Vs Time)Effect of Sampliing_ Saturation =12V, Sampling Time= 0.01s, Output Disturbance=10 deg
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
Time offset: 0

Controller Voltage Output(Volts) Vs Time (After Saturation)



Time offset: 0

9 IMPLEMENTATION

The designed PID controller has been implemented successfully. The error angle is measured from the output angle reading from the sensors. The error angle is processed in the PID loop. The output of the controller is supplied as voltage input to the motors using PWM.

10 CONCLUSION

The PID contoller stabilises the robot satisfactorily. Since the system modelling of the robot was good not much tuning was required for the controller while implementing. Eventhough the robot experienced a lot of external disturbance due to wires connected to it,the disturbance rejection was still satifactory.Ideally the robot should move back and forth and stop once it stabilises itself at equilibrium configuration(zero vertical angle).Although our robot moves back and forth and tries itself to reach equilibrium configuration,it doesnt really halt at the equilibrium position.This might be due to lot of wires that are attached externally to the robot which always have a pulling effect on the robot.It might also be due to the oscillations in the angle measured near vertical configuration which is because of poor complementary filter.