ES5205 – Project 2 Report

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Introduction and Motivation

The study of exoplanets has become a central focus in astrophysics, providing key insights into planetary formation, composition, and the potential for habitability. One of the fundamental aspects of exoplanet characterization involves measuring their mass, radius, and density, as these properties offer valuable clues about their composition and structure.

The aim of this project is to derive the mass, radius, and density of the exoplanet HD 189733 b using data from two different observational techniques: radial velocity and transit. By analyzing radial velocity data, we can determine the planet's mass based on its gravitational influence on its host star. The transit data, on the other hand, provides the planet's radius through the dimming of starlight as the planet passes in front of its star. Combining these two sets of data allows for the calculation of the planet's density, which is extremely useful in understanding its internal composition.

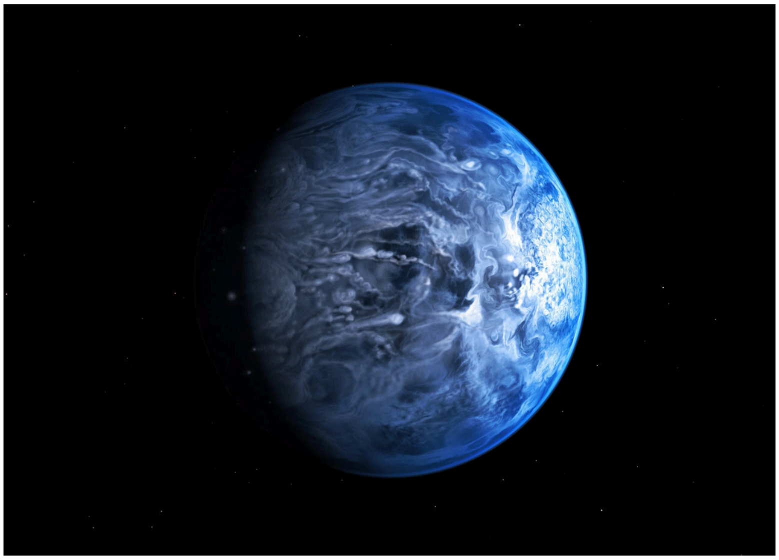


Figure 1. HD 189733 b Artist Rendition (NASA)

Bayesian methods have been used for the analysis to account for uncertainties in the measurements and provide more accurate estimates for the planet's properties. HD 189733 b has both radial velocity and transit data which makes it an ideal candidate for this project. The properties of this planet have been compared to similar exoplanets to assess how this exoplanet fits within the spectrum of planetary characteristics. We also compare the measured mass and radius to the M-R relation from Chen & Kipping (2016), to further compare our findings.

This project deepened our understanding of the measurement process for exoplanet properties and the role of observational uncertainties in shaping our interpretation of distant worlds.

Methods

Assumptions

It was assumed that the error bars on the transit data hold 100% of the potential uncertainty values (rather than 68% as in a normal distribution). This will be expanded upon when the "Monte Carlo" inspired uncertainty analysis is discussed below.

The densities have been measured assuming a homogeneous planet composition even though planets have complex internal structures.

These assumptions help simplify the analysis and make calculations computationally feasible while still providing meaningful insights.

Calculations

1. Transit Method:

Data was downloaded from NEA and the transit light curve was curvefit using NEA EXOFAST.

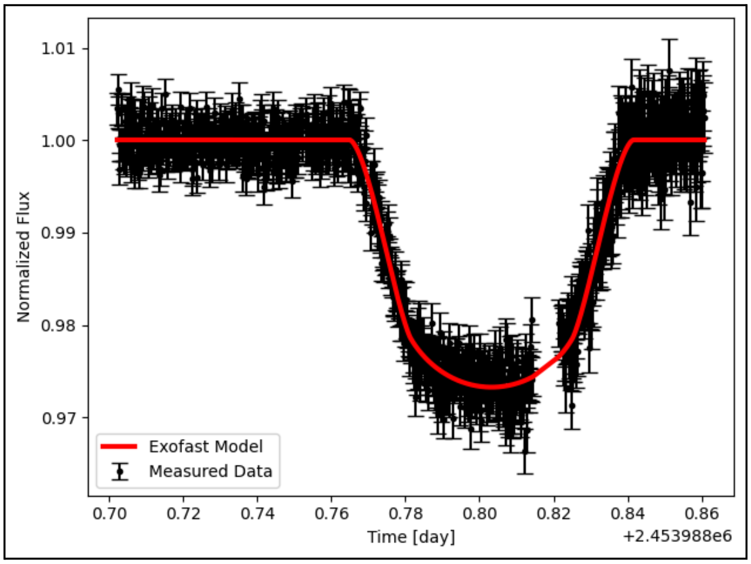


Figure 2. Transit Light Curve EXOFAST Curvefit

This gave a measured transit depth value of 0.024686348 (normalized flux) and also included the error bars for each data point. From here, the team wanted to do further analysis on the uncertainty associated with this measurement. Nine additional normalized flux vectors were generated by randomly selecting a number between the error bars for each data point.

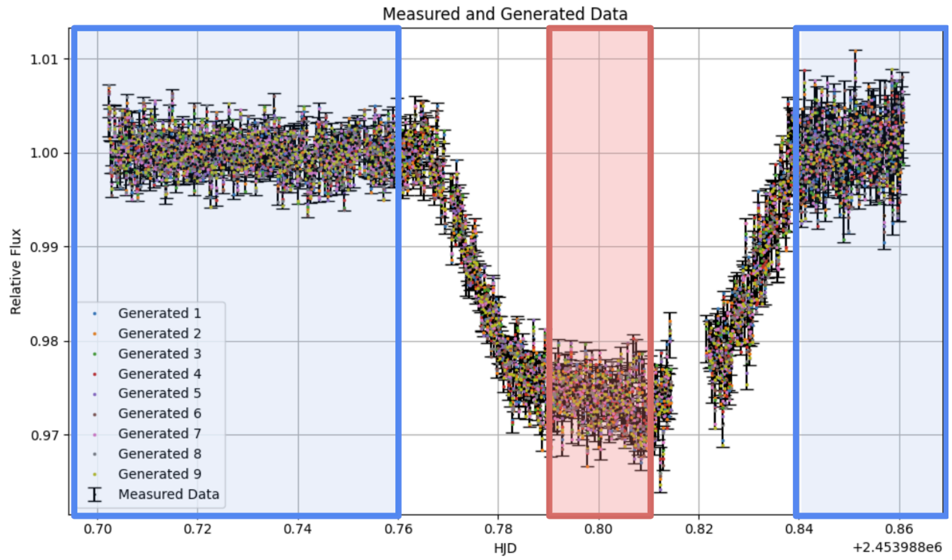


Figure 3. Transit Generated Data

The transit depths associated with each generated data set were then measured. The team chose a simple approach in which the average value of the data points in the red box was averaged and subtracted from the average value of the points found in the blue bounds. This is a potential point of improvement in this analysis as it is a simplified way to approximate the transit depth and neglects to take into account the influence of all of the points not located within the selected bounds. This resulted in an average transit depth of 0.02607105 with a standard deviation of 0.00047800. Acknowledging the less accurate estimation of the generated data, the team chose the EXOFAST measured value as the final answer for the transit depth with the uncertainty from the Monte Carlo analysis:

$$\delta = 0.24686 + / - 0.00048$$

We then used the following equation to calculate the the radius of the planet from the transit depth

$$\delta = (R_p/R_S)^2 ,$$

Where δ is the transit depth, R_P is the radius of the exoplanet, and R_S is the radius of the parent star. This makes sense since the flux of a celestial object is proportional to its cross-sectional area. If we rearrange the equation above, we can solve for R_P.

$$R_P = R_S\sqrt{\delta}$$

2. Radial Velocity:

The HIRES 10m radial velocity curve dataset was downloaded from NEA for the team to investigate the radial velocity data of HD 189733 and approximate the mass of HD 189733 b. The data was preprocessed and curvefit in EXOFAST. With the resulting EXOFAST parameters and orbital period information, the dataset was conditioned in Python with the following techniques:

a) Renormalization: The measurement uncertainty vector $\boldsymbol{\varepsilon}$ was scaled by an error renormalization factor β to enforce a near-ideal reduced χ^2 statistic of $\chi^2_\upsilon \cong 1.04$ for stronger goodness-of-fit:

$$\boldsymbol{\varepsilon}' = \beta\boldsymbol{\varepsilon}$$

b) Centering: The RV data vector $\boldsymbol{v}$ was centered by removing DC offset γ to mitigate systemic instrumentation bias:

$$\boldsymbol{v}' = \boldsymbol{v} - \gamma$$

c) Phase folding and standardization: The independent x-axis time vector $\boldsymbol{t}$ was transformed to phase by wrapping about a single period using the modulo operation. The time data were standardized using orbital parameters P, T_c, and T_p to result in the standardized phase vector $\boldsymbol{\tau}$:

$$\boldsymbol{\tau} = \left(\frac{\boldsymbol{t}-T_c}{P} + \frac{T_p-T_c}{P} + 0.25\right) \% \ 1$$

The parameter values for those of interest generated by EXOFAST and the useful orbital parameters from NEA utilized in data conditioning are given below:

$$\beta = 16.416735$$

$$\gamma = -11.804028$$

$$P = 2.218666\ days$$

$$T_c = 2453935.570977\ days$$

$$T_p = 2453935.531264\ days$$

Putting this together by plotting the conditioned radial velocity dataset $(\boldsymbol{\tau}, \boldsymbol{v}')$ with renormalized error $\boldsymbol{\varepsilon}'$ and curve-fit model, we obtain Figure 4:

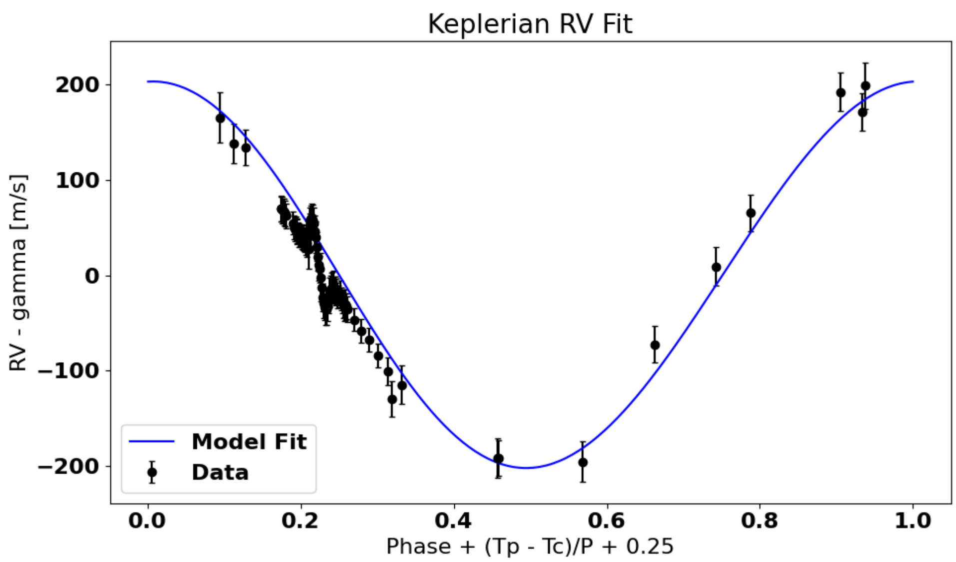


Figure 4. Conditioned RV Data Plot with Curve Fit

The team proceeded to generate nine Monte Carlo trial datasets as performed with the transit dataset. The estimated radial velocity semi-amplitude K was obtained by taking the mean and standard deviation of the ten entries composed of the initial dataset model fit semi-amplitude and the nine stochastic trial model fit semi-amplitude values.

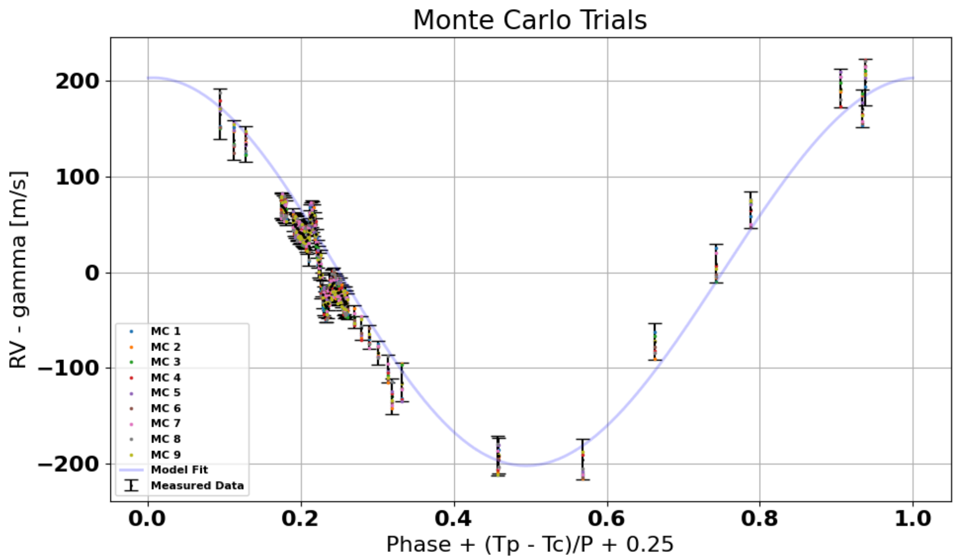


Figure 5. RV Monte Carlo Trials Data Plot

The generated data for each Monte Carlo trial is shown above in Figure 5. The estimated result obtained from this dataset has a value:

$$\overline{K} = 202.84 \pm 1.79\, m/s$$

Recalling the radial velocity semi-amplitude formula:

$$K = \left(\frac{2\pi G}{P}\right)^{1/3} \frac{M_p \sin i}{\left(M_\star + M_p\right)^{2/3}} \frac{1}{\left(1-e^2\right)^{1/2}}$$

We rearrange to solve for planetary mass:

$$M_p \sin i = K\left(\frac{P}{2\pi G}\right)^{-1/3}\left(M_\star + M_p\right)^{2/3} \frac{1}{\left(1-e^2\right)^{1/2}}$$

Given that HD 189733 b is a transiting exoplanet, we make the simplifying assumption:

$$i \approx \frac{\pi}{2} \Rightarrow \sin i \approx 1$$

And obtain our equation to estimate the mass of HD 189733 b:

$$M_p = K\left(\frac{P}{2\pi G}\right)^{1/3}\left(M_\star + M_p\right)^{2/3}\left(1 - e^2\right)^{1/2}$$

The team conducted the mass estimation with these parameter values sourced from NEA:

$$P = 0.031\, AU$$

$$M_{\star} = 0.82785146527300\, M_{\odot}$$

$$e = 0.019434$$

3. Density:

We assumed that the exoplanet has a homogeneous composition and as a result, we can use the equation below to find the density of the exoplanet.

$$\rho = \frac{M}{V},$$

where ρ is the density of the exoplanet, M is the mass of the exoplanet, and V is the volume of the exoplanet.

Results

Using the transit depth and equations discussed above, we estimated the radius of the planet. After pulling the stellar radius from NEA, the radius of HD 189733 b was calculated to be:

$$R_{P,\, est} \sim 85,270 \pm 11,860\ km$$

With the NEA values being:

$$R_{P,\, NEA} \sim 79,892 \pm 2,740\ km$$

So it can be seen that our analysis did a good job of approximating the NEA value with an increase in the error bounds due to our unidealized uncertainty analysis.

Following the radial velocity Monte Carlo estimation and equations discussed prior, we estimated the planetary mass. The mean mass estimate of HD 189733 b was computed as:

$$\overline{M_p} = 365.59 \pm 3.08\, M_{\oplus}$$

And the NEA published mass is:

$$M_{p,NEA} = 359 \pm 25\, M_{\oplus}$$

This result deviates from the NEA published mass by 1.6 ± 7.0 percent error. In terms of Jovian mass, the mass of HD 189733 b is:

$$\overline{M_p} = 1.15 \pm 0.01\, M_{♃}$$

Compared to the NEA value:

$$M_{p,NEA} = 1.13 \pm 0.08\, M_{♃}$$

Returning to the estimated radial velocity semi-amplitude discussed earlier, the team's estimate,

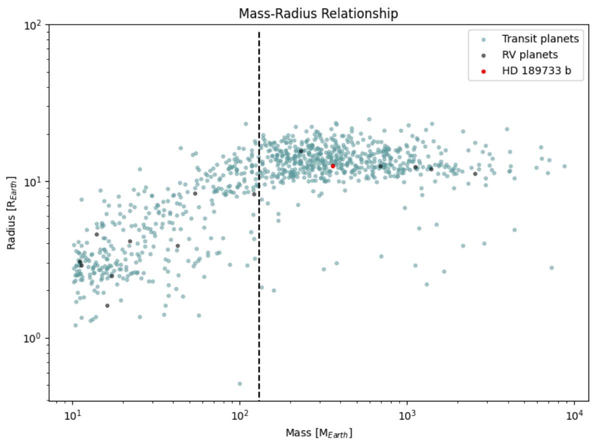
$$\overline{K} = 202.84 \pm 1.79\, m/s,$$

yields a 1.1 ± 3.1 percent error and closely matches the NEA value:

$$K = 205.0 \pm 6.0\, m/s$$

These results represent strong estimations that capture the state-of-the-art value for the planetary mass of HD 189733 b within error. Notably, the RV results show comparatively smaller uncertainty than the NEA values. This is likely a limitation of the uncertainty analysis present in the estimation. The team believes that the method resulting in the NEA uncertainty value is more robust and better propagates and accounts for systematic errors in instrumentation and observation. The uncertainty in the team's estimate is largely a function of the inherent stochastic behavior of the Monte Carlo trials and does not strongly include systematic bias analysis. Regardless, the computed result is a relatively precise estimate of the planetary mass.

Conclusions



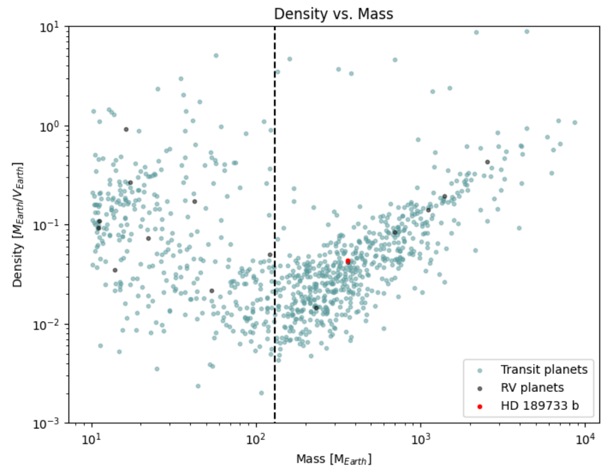


Figure 6. Mass-radius relationship of exoplanets detected using radial velocity and transit methods

Figure 7. Density vs. mass relationship of exoplanets

Figure 6 displays the mass-radius relationship of ice-giant planets and gas-giant exoplanets. The cadet blue points represent the exoplanets that have been detected using the transit method, the black points depict the exoplanets that have been detected using the radial velocity method, and the red point represents the exoplanet HD 189733 b. The black dashed line shows us the divider between ice-giants (to the left) and gas-giants (to the right). This plot shows us that HD 189733 b, with an estimated mass of 1.15 ± 0.01 M_J and estimated radius of 1.12 ± 0.04 $M_{J,}$ is a gas-giant.

Figure 7 displays the Density vs. Mass plot for ice-giant and gas-giant exoplanets, it has the same color scheme as figure 7. In this plot, you can clearly see the transition from ice-giants to gas-giants by the change in slope of the points. By assuming a constant density, we were able to find the density of HD 189733 b to be ~ 1000 kg m^{-3}.

From Chen and Kipping (2016), we find that a more accurate estimation of the mass and radius of HD 189733 b gives us a mass estimate of 1.15 ± 0.039 M_J and a radius estimate of 1.151 ± 0.038 R_J. This is pretty close to our results which supports our analysis and calculations.

Individual Contributions

Technical Work

Transit Modeling + Calculations + Plotting	Andy
RV Modeling + Calculations + Plotting	Jackson
Density Modeling + Calculations + Plotting	Suhas
NEA Comparison Plot	Suhas

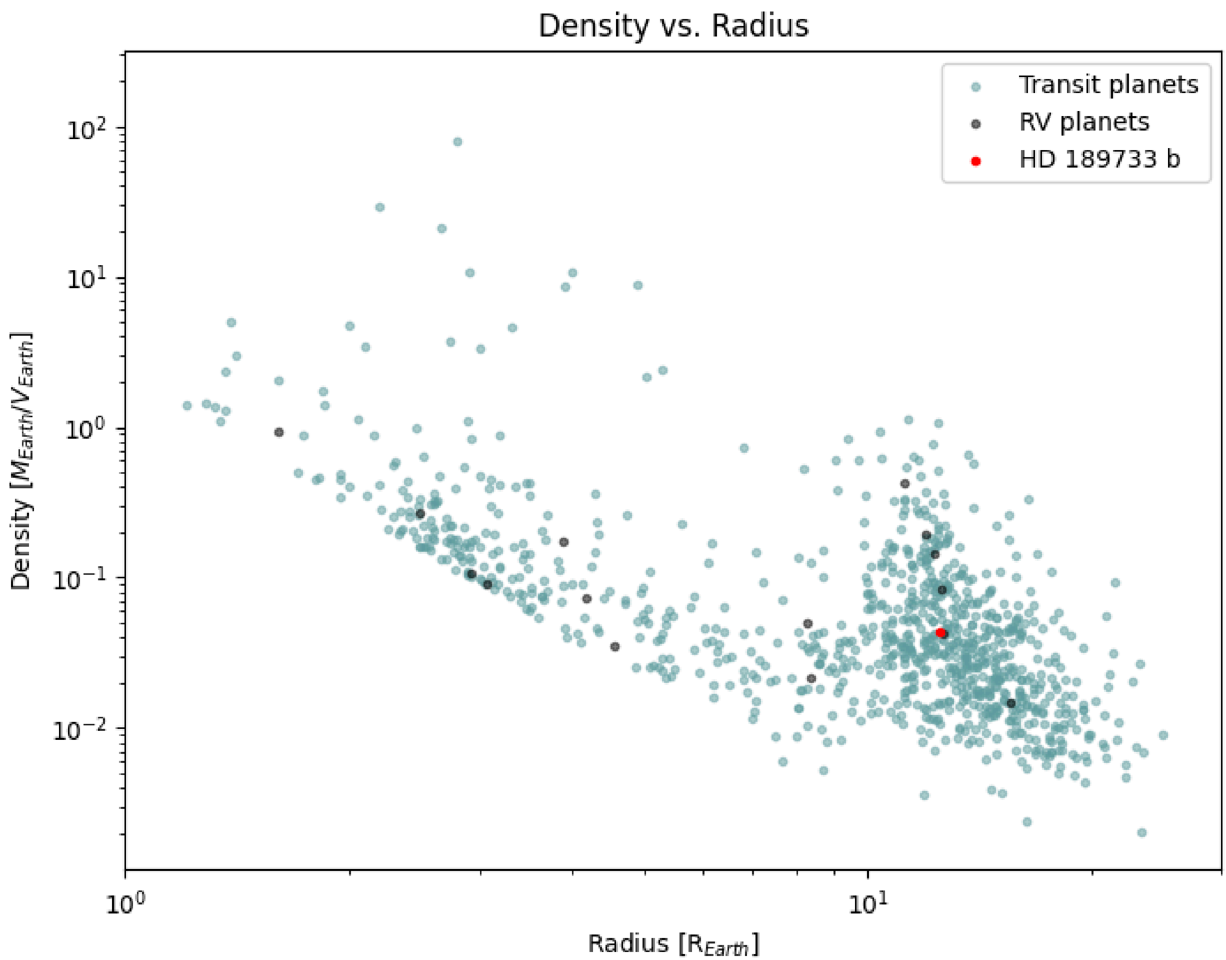
Project Presentation

Introduction	Suhas
Transit Methods/Results	Andy
RV Methods/Results	Jackson
Conclusions	Suhas

Project Report

Introduction	Suhas
Transit Methods/Results	Andy
RV Methods/Results	Jackson
Conclusions	Suhas

Attachments



References

Chen, J. and Kipping D (2016). Probabilistic Forecasting of the Masses and Radii of Other Worlds. The Astrophysical Journal. https://arxiv.org/abs/1603.08614

AI Statement

For the transit modelling, Google Gemini was used within Google Colab to help understand and write in Python syntax.

ChatGPT o3-mini was used to optimize and debug the Python radial velocity curve fit methods.